Algebra refresher

A. Blanchet

Exercise 1

Let

$$E := \{(x,y,z) \in \mathbb{R}^3 \ : \ y+z=0 \quad \text{et} \quad x+y-z=0 \quad \text{et} \quad x-2z=0\}$$

and

$$F := \text{Vect}\left\{ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \\ -2 \end{pmatrix} \right\} .$$

1. Prove that E and F are vector subspace of $\mathbb{R}^3$.
2. Give a spanning family of E.
3. Give a basis and the geometric nature of E.
4. Give a spanning family of F.
5. Give a basis and the geometric nature of F.
6. Give a basis and the geometric nature of $E \cap F$.

Exercise 2

Let

$$E := \{(x,y,z) \in \mathbb{R}^3 \ : \ y+z=0 \quad \text{et} \quad x+y-z=0 \quad \text{et} \quad x-2z=0\}$$

and

$$F := \text{Vect}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\} .$$

1. Prove that F and E are sumpplementary in $\mathbb{R}^3$.
2. determine the projection on E of direction F.

Exercise 3

Consider the mapping

$$\begin{array}{rrcl} f : & \mathbb{R}^3 & \to & \mathbb{R}^3 \\ & (x,y,z) & \mapsto & (y+z, x+y, x+2y+z) . \end{array}$$

1. Determine in the caconical basis the matricial representation of f.

2. Determine a system of cartesian equations of $\mathrm{Ker}\,(f)$.
3. Determine a spanning family of $\mathrm{Im}\,(f)$.
4. Determine a linearly independent family of $\mathrm{Im}\,(f)$.
5. Determine a system of cartesian equations of $\mathrm{Im}\,(f)$.
6. Discuss the solutions of $f(x, y) = (1, 1, 1)$.
7. Discuss the solutions of $f(x, y) = (1, 1, 2)$.
8. Discuss the number of solutions of $f(x, y) = (1, a, b)$ depending on the prameters $(a, b) \in \mathbb{R}^2$.
9. Determine the solutions to $f(x, y) = (0, 0, 0)$.

Exercise 4

Let

$$\Theta := \{P \in \mathbb{R}_n[X] \, : \, P(0) = 0\} \, .$$

1. Prove that Θ is a vector space.
2. determine a basis of Θ.