Exclusive contracts and market dominance*

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Abstract

We propose a new theory of exclusive dealing. The theory is based on the assumption that a dominant firm has a competitive advantage over its rivals, and that the buyers' willingness to pay for the product is private information. In this setting, we show that the dominant firm can impose contractual restrictions on buyers without having to compensate them. This implies that exclusive dealing contracts can be both profitable and anticompetitive. We discuss the general implications of the theory for competition policy and illustrate by example how it applies to real world antitrust cases.

Keywords: Exclusive dealing; Non-linear pricing; Antitrust; Dominant firm

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1 Introduction

Exclusive dealing contracts have long raised antitrust concerns. However, existing anticompetitive theories rely on assumptions that do not always fit real antitrust cases. In this paper, we propose a new theory of competitive harm, which is arguably more broadly applicable than existing ones. We illustrate the applicability of the theory by example and discuss its more general implications for competition policy.

At the centre of our analysis lies a simple anticompetitive story. Two or more firms are active in an industry and compete by supplying substitute goods. One firm, so the story goes, can raise its market share and prices by contractually restricting buyers' purchases from its competitors. This increases the firm's profits at the expense of buyers and rivals alike.

This story is intuitive, easy to understand, and probably relevant to many antitrust cases. However, it is difficult to reproduce in rigorous models. A major obstacle is the so-called Chicago critique. The critique holds, firstly, that buyers must be compensated for accepting exclusive deals, and secondly, that rivals may respond by offering similar contracts. This, according to the critics, undermines the profitability of exclusive contracts.

We overcome this critique by making two simple, and often realistic, assumptions. The first is that firms cannot extract the buyers' surplus fully, for the simple reason that they do not know exactly how large it is. In other words, the buyers' willingness to pay for the product is private information. The second assumption is that one firm stands out from the others, enjoying a dominant position vis-à-vis its rivals.[1] By itself, dominance is benign in our model. It arises because one firm benefits from a competitive advantage over its rivals, in terms of higher quality and/or lower cost. However, we show that if the competitive advantage is large enough, it allows the dominant firm to use exclusive contracts for anticompetitive purposes.

The intuition is as follows. When exclusive contracts are banned, firms compete for each marginal unit of a buyer's demand. In the competition for marginal units, the dominant firm cannot take advantage of the information rents left on inframarginal units. In contrast, with exclusive contracts firms compete for the entire volume demanded by a buyer – i.e., they compete in "utility space."[2] This provides the dominant firm with an opportunity to leverage on the rents that it must inevitably leave to the buyers.

[1] Besides often being true, this case is certainly of relevance for antitrust policy. The vast majority of antitrust cases involve dominant firms that control a substantial share of the market in the face of smaller rivals. In fact, some degree of asymmetry is a prerequisite for competition policy intervention. For example, violation of Section 2 of the Sherman Act requires "the possession of monopoly power in the relevant market" (*United States v. Grinnel Corp.*). Likewise, infringement of Article 102 of the Treaty of Rome, i.e. the abuse of a dominant position, obviously requires that a firm actually enjoys a dominant position.

[2] The notion of competition in utility space was introduced by Bliss (1988) and used by Armstrong and Vickers (2001) and Rochet and Stole (2002), among others. These authors focus on models of one-stop shopping where exclusivity is not imposed contractually but arises for technological reasons.

In fact, when the competitive advantage is large enough, the information rents that buyers obtain when dealing exclusively with the dominant firm may be greater than their outside option, i.e. the surplus that they could obtain by trading exclusively with the dominant firm's competitors. This may be true even if the dominant firm charges monopoly prices while its competitors price at cost. In this case, the dominant firm can impose exclusivity without compensating the buyers, and rivals cannot respond. In other words, the dominant firm is totally immune from competition in utility space. This argument completely overcomes the Chicago critique.

However, one further difficulty remains. O'Brien and Shaffer (1997) and Bernheim and Whinston (1998) have shown that under complete information, profit maximisation requires the maximisation of total surplus from trade.[3] If exclusion is inefficient, because rivals supply imperfect substitutes for which there is demand, exclusive dealing must reduce the total surplus. But then exclusive contracts cannot be profitable: at best, they are redundant.

This neutrality result does not apply to our setting because the dominant firm does not exactly know the buyers' willingness to pay and hence cannot fully extract the surplus. As is well known, it is then profitable to distort the contracts that apply in certain states of demand in order to extract more surplus in others. When contracts cannot be conditioned on rivals' volumes, the optimal distortion is the standard one, i.e., to reduce own sales below the efficient level in low-demand states. This however entails sacrificing profits. In contrast, with exclusive contracts (or other contracts that reference rivals) a firm can reduce its rivals' volumes in low-demand states. Like the standard distortion, this allows to extract more rents in high-demand states. Unlike the standard distortion, however the dominant firm can now also increase its profits even in low-demand states, as the goods are substitutes.

Since the distortion is more highly profitable, more distortion is created. This implies that when the dominant firm's competitive advantage is large exclusive dealing contracts are not only profitable but also anticompetitive. They harm the buyers, who will suffer from stronger distortion, and the rivals, who must absorb a large part of this distortion.

When the competitive advantage is small, however, a countervailing effect may come into play. Whereas product differentiation softens competition for marginal units, it does not soften competition in utility space. In utility space, product diversity is in fact irrelevant: all that counts is the amount of rent left to buyers. This tends to make competition in utility space tougher than competition for marginal units when firms are relatively symmetric.

This procompetitive effect is the focus of Calzolari and Denicolò (2013).

[3] The intuition is fairly simple. Under complete information a firm would use non-linear pricing to extract all the surplus in excess of what the buyer can obtain by trading with the firm's competitors (which supply substitute goods). Consequently, the firm would offer a contract that maximises such surplus – a property known as bilateral efficiency. In the absence of contractual externalities, under mild regularity conditions such bilateral efficiency conditions would translate into global efficiency. O'Brien and Shaffer (1997) also highlight the fact that the surplus from trade is maximised even if buyers possess a certain bargaining power, as long as bargaining is efficient.

That paper studies a symmetric duopoly where no firm has any competitive advantage over the other. The anticompetitive mechanism analysed in this paper is then mute, as it crucially hinges on such a competitive advantage. Nevertheless, under incomplete information each firm still has a unilateral incentive to offer exclusive contracts so as to try to force the rival to absorb the distortion. Because of that unilateral incentive, firms are now caught in a prisoners' dilemma. They end up competing in utility space, where they are perfectly homogeneous and thus price competition becomes extremely intense. In this case, therefore, exclusive contracts reduce prices and profits.[4]

The rest of the paper proceeds as follows. After setting out the model in section 2, section 3 focuses on the baseline case in which the dominant firm faces a competitive fringe. The fringe always prices at cost and thus the intensity of competition cannot be further enhanced. This allows us to abstract from the countervailing effect mentioned above, and to focus on the new anticompetitive mechanism central to this paper.

In section 4, we briefly discuss the case of a duopoly where the dominant firm faces a smaller, less efficient competitor which, however, enjoys a degree of market power. This brings the countervailing effect into the picture. The anticompetitive effect prevails when the competitive advantage is large, the procompetitive effect when it is small. We also analyse the case in which the dominant firm can use market-share discounts. We find that exclusive dealing is optimal in some states of demand, and market-share contracts in others.

Section 5 highlights the model's empirical predictions. Our theory naturally applies to situations where a dominant firm controls a substantial share of the market and competes with a smaller rival (or group of rivals). A recent antitrust case that fits this description is the *Intel* case. From 2001 to 2007, Intel had a market share of more than 80%, while its strongest competitor, AMD, controlled a mere 15% of the market. Over the period in question, Intel allegedly used various types of loyalty discounts for anticompetitive purposes. We use this example to illustrate what kind of empirical evidence may verify, or falsify, our theory. We argue that the theory fits this case quite well.

Finally, section 6 summarises the main arguments, discusses broader implications for competition policy, and hints at some possible directions for future research. We end this introduction by briefly reviewing some related literature. After presenting our main results, a fuller discussion of the literature is offered in section 5.

Much of the literature regarding the possible anticompetitive effects of exclusive contracts focuses on the case of complete information and avoids the neutrality result mentioned above by assuming some kind of contractual externalities. Contractual externalities typically arise when the scope for negotiation is limited. For example, a potential entrant may not be able to contract with the buyers until after they have been signed up by the incumbent, as in Aghion

[4] Calzolari and Denicolò (2013) then go on to argue that firms can coordinate their pricing strategies to some extent so as to relax competition. However, in a non-cooperative equilibrium the scope for coordination is limited, so the final effect of exclusive contracts is to enhance competition.

and Bolton (1987), Rasmusen, Ramseyer and Wiley (1991) and Segal and Whinston (2000). Or firms may be unable to contract with buyers due to enter the market only at a later date, as in Bernheim and Whinston (1998, sect. IV). Contractual externalities may also arise when buyers compete in downstream markets, as in Asker and Bar-Isaac (2014). In some models, both mechanisms are simultaneously at work (see e.g. Fumagalli and Motta, 2006; Simpson and Wickelgren, 2007; and Wright, 2009).

In this paper we pursue a different line of inquiry, where the non-neutrality of exclusive contracts is due to the difficulty of extracting the buyers' rent. In an earlier paper, Mathewson and Winter (1987) assumed that surplus extraction is imperfect because firms are restricted to linear pricing. However, in their complete information setting two-part tariffs would restore neutrality. With adverse selection, on the contrary, surplus extraction is imperfect even if firms are not restricted to simple contracts. The adverse selection approach has been pioneered by Martimort (1996), who posits symmetric firms and models the exclusive dealing case by assuming that each firm has access to a different retailer. Unfortunately, these assumptions effectively rule out competition for exclusives and the possibility of foreclosure outcomes. Majumdar and Shaffer (2009) are closer in spirit to our own approach. However, they focus on market-share discounts and posit that demand may take on only two possible values.

2 Model

We consider a one-period model of price competition. There are two substitute goods, A and B. Good A is supplied by firm A. In our baseline model we assume that good B is supplied by a competitive fringe. The case in which good B is supplied by firm B (the duopoly model) is presented in section 4.

A buyer who buys q_A units of good A and q_B units of good B obtains a benefit, measured in monetary terms, of $u\left(q_A, q_B, \theta\right)$. We may think of buyers as downstream firms, with u as their gross profits,[5] or as final consumers, with u as their utility function. The function u is symmetric, smooth and concave in quantities. It initially increases in q_A and q_B, but in view of our normalisation of costs (see below) we assume that there exists a finite satiation point. The goods are imperfect substitutes, in the sense that $u_{q_i q_i}\left(q_A, q_B, \theta\right) < u_{q_i q_j}\left(q_A, q_B, \theta\right) < 0$, where subscripts denote partial derivatives. This implies that buyers have a preference for variety. The reservation payoff, $u\left(0, 0, \theta\right)$, is normalised to zero.

The one-dimensional parameter θ is the buyer's private information; it is distributed over an interval $[\theta_{\min}, \theta_{\max}]$ according to a distribution function $F(\theta)$ with density $f(\theta)$. We assume that higher values of θ correspond to higher demand and we make the single-crossing assumption $u_{\theta q_i}\left(q_A, q_B, \theta\right) \geq 0$.

[5] In this interpretation, our analysis literally requires that downstream firms operate in separate markets and do not interact strategically with each other. Otherwise, contractual externalities might arise, as discussed above. However, our insights should also be applicable to situations in which downstream firms compete, as long as they have some market power.

We assume that firm A (the dominant firm) has a competitive advantage in terms of lower cost, better quality, or a combination of the two. Firm A's marginal production cost is normalised to zero. With cost asymmetry, the unit production cost of product B is $c \geq 0$.[6] With asymmetric demand, the buyer's payoff becomes $u(q_A, q_B, \theta) - cq_B$ (with B's cost now set to zero). In this case, parameter c can be interpreted as an index of vertical product differentiation, with product A being of better quality, and hence more in demand, than product B. The two formulations are analytically equivalent, and to fix ideas in what follows we shall stick to the cost interpretation.

To make the analysis interesting, we assume that total exclusion is inefficient. In this setting, this requires that $q_B^{fb}(\theta_{\max}) > 0$, where

$$\left\{q_A^{fb}(\theta), q_B^{fb}(\theta)\right\} = \arg\max_{q_A, q_B} \left[u(q_A, q_B, \theta) - cq_B\right] \tag{1}$$

are the full information, first best quantities. This condition places an upper bound on c. To simplify the exposition, we assume that the market is uncovered. A sufficient condition for this is that $q_A^{fb}(\theta_{\min}) = 0$ and $q_A^{fb}(\theta) > 0$ for all $\theta > \theta_{\min}$.[7]

Firms compete by simultaneously and independently offering menu of contracts. We distinguish two different modes of competition according to the type of contract that the firms may offer. With simple *non-linear pricing*, the payment to each firm depends only on its own quantity. A strategy for firm i then is a function $P_i(q_i)$ in which q_i is the quantity firm i is willing to supply and $P_i(q_i)$ is the corresponding total payment it asks. With *exclusive contracts*, a strategy for firm i comprises two price schedules, $P_i^E(q_i)$ and $P_i^{NE}(q_i)$. The former applies to exclusive contracts ($q_j = 0$), the latter to non exclusive ones ($q_j > 0$).[8] In section 4, we shall also allow for *market-share contracts*, whereby a firm can more freely condition its payment request on its competitors' sales volume: $P_i = P_i(q_i, q_j)$.

Buyers observe the firms' offers and the realisation of demand θ and then choose the quantities $\{q_A(\theta), q_B(\theta)\}$ that maximise their net payoff. Buyers have no bargaining power, but are large enough that firms can monitor whether they purchase from their competitors. To avoid any issues of equilibrium existence when firms compete for exclusives, we assume that if buyers are indifferent in terms of monetary payoff, they prefer to trade with firm A. We focus on subgame perfect equilibria, ruling out equilibria that rely on weekly dominated strategies.

[6] We abstract from fixed costs, and hence from economies of scale. As long as all firms remain active, this is with no loss of generality. Furthermore, in the competitive fringe model one can interpret c as the minimum average cost of a number of identical firms, thus allowing for economies of scale at firm level.

[7] In fact it suffices that there exists a $\theta \in [\theta_{\min}, \theta_{\max}]$ such that $q_A^{fb}(\theta) = 0$. If this holds, one can always choose $\theta_{\min}$ as the largest θ for which $q_A^{fb}(\theta) = 0$ and re-scale the distribution function accordingly.

[8] To guarantee a solution to the buyer's maximisation problem, we assume that each price schedule P_i must be non decreasing in q_i (a free disposal assumption which also implies that price schedules must be differentiable almost everywhere), that it satisfies $P_i(0) = 0$, and that it is upper semi-continuous.

3 Results

In this section, we discuss the equilibrium of the baseline model in an informal manner. Formal propositions and the derivation of the results can be found in Appendix 1.

To facilitate intuition, we use a series of figures that depict the equilibrium quantity schedules $q_A(\theta)$ and $q_B(\theta)$, and the associated price schedules. Figures are drawn for the case of a uniform-quadratic model, in which the parameter θ is assumed to be uniformly distributed over the interval $[0,1]$ and the buyer's payoff is taken to be:

$$u\left(q_A, q_B, \theta\right) = \theta(q_A + q_B) - \frac{1-\gamma}{2}(q_A^2 + q_B^2) - \gamma q_A q_B.\text{[9]} \tag{2}$$

With this specification, the equilibrium quantity schedules are piecewise linear, and the equilibrium price schedules are piecewise quadratic.[10] However, the qualitative properties of the equilibrium are more general and require only mild regularity conditions, as set out in the Appendix.

3.1 Preliminaries

Clearly, the competitive fringe will always price at cost (i.e., $P_B(q_B) = cq_B$), without imposing any exclusivity clause. Given this passive behaviour, finding the model's equilibrium is tantamount to finding the dominant firm's optimal strategy $q_A(\theta)$. As it turns out, with both non-linear pricing and exclusive contracts the dominant firm simply uses combinations of the following strategies:

(i) *Monopoly*: $q^m(\theta)$ and $P^m(q)$.[11] This is the outcome that would prevail if product B was not supplied at all. The schedules $q^m(\theta)$ and $P^m(q)$ are

[9] When buyers are final consumers, (2) implies that under linear pricing the demand functions would be:

$$p_i = \theta - (1-\gamma)q_i - \gamma q_j.$$

If, on the contrary, buyers are downstream firms, assuming that such firms are local monopolies and are restricted to linear pricing, and normalising their costs to zero, (2) is the downstream profit function associated with the following demand functions for the final products:

$$p_i = \theta - \frac{1-\gamma}{2}q_i - \frac{\gamma}{2}q_j.$$

In both cases, the parameter γ captures the degree of substitutability among the products: it ranges from $\frac{1}{2}$ (perfect substitutes) to 0 (independent goods). The assumed relationship between own and cross price effects on demand serves only to guarantee that changes in γ do not affect the size of the market. As Shubik and Levitan (1980) have argued, this rules out spurious effects in the comparative statics analysis.

[10] The explicit solutions are reported in Annex 2.

[11] Note that for any quantity schedule $q(\theta)$, the price schedule that implements it is only defined *modulo* a constant term. However, when the market is uncovered the marginal buyer's demand is negligible. Hence, the price schedules which apply to the marginal buyer cannot involve any fixed fee or subsidy – a property first noted by Wilson (1994). (The marginal buyer is the lowest type purchasing a positive quantity of the good.) In contrast, the price schedules that apply to non-marginal buyers may involve constant terms, which are specified in the Appendix.

the solution to the standard monopolistic non-linear pricing problem, which is obtained by simply setting $q_B = 0$ in the buyer's payoff function $u(q_A, q_B, \theta)$.

(*ii*) *Limit pricing*: $q^{\lim}(\theta)$ and $P^{\lim}(q)$. The limit pricing quantity $q^{\lim}(\theta)$ is implicitly defined by the condition

$$u_{q_B}(q_A, 0, \theta) = c. \tag{3}$$

When the dominant firm supplies this quantity, the competitive fringe is just foreclosed even without exclusive dealing arrangements.

(*iii*) *Common representation*: $q_A^{cr}(\theta)$ and $P_A^{cr}(q)$. Whereas the first two strategies require, or imply, that $q_B = 0$, in this case the dominant firm accommodates the fringe. Therefore, it must now account for the fact that buyers purchase a substitute product. The equilibrium is still obtained as the solution to a problem of "monopolistic" non-linear pricing, but buyers now behave as if they had an "indirect" payoff function

$$v(q_A, \theta) = \max_{q_B} \left[u\left(q_A, q_B, \theta\right) - P_B(q_B) \right], \tag{4}$$

which is the maximum payoff that can obtained in state θ by purchasing q_A and then trading optimally with the fringe. The indirect payoff function is similar to residual demand in models of linear pricing. Given $q_A^{cr}(\theta)$, the equilibrium quantity of product B is

$$q_B^{cr}(\theta) = \arg\max_{q_B} \left[u\left(q_A^{cr}(\theta), q_B, \theta\right) - P_B(q_B) \right]. \tag{5}$$

3.2 Large competitive advantage

Armed with these definitions, we can now proceed. We focus in particular on the case in which the competitive advantage is relatively large.[12] Panel a of Figure 1 depicts the equilibrium quantities under non-linear pricing when $c \geq c^m$.[13] In this case, in low-demand states buyers are effectively captive, so the dominant firm can engage in monopoly pricing. As demand increases, however, the buyer's temptation to also purchase product B increases. However, if buyers purchased a positive amount of product B, their demand for product A would decrease, as the products are substitutes. To prevent this, the dominant firm therefore engages in limit pricing, raising the sales of product A just to the point where the marginal willingness to pay for product B equals the competitive fringe's cost c. Finally, when demand gets even higher, foreclosing the competitive fringe becomes too costly. The dominant firm therefore accommodates the competitive fringe, and in equilibrium buyers purchase both products.

[12] Remember, however, that the condition whereby total exclusion is inefficient places an upper bound on c. For example, with the uniform-quadratic specification (2), the upper bound is $\frac{1-2\gamma}{1-\gamma}$.

[13] The threshold c^m is defined as the lowest c such that there exists at least one type θ for which $q^m(\theta) > q^{\lim}(\theta)$. In the uniform-quadratic specification, $c^m = \frac{1}{2}$. Notice that this is lower than the upper bound $\frac{1-2\gamma}{1-\gamma}$, provided that the goods are not too strong substitutes.

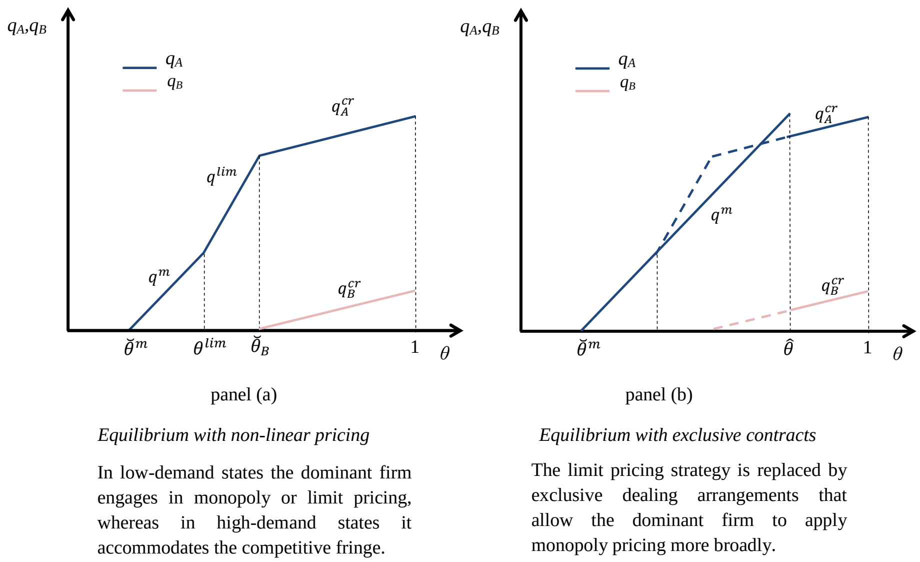


Figure 1: Quantities with large competitive advantage.

Now, consider how the equilibrium changes with exclusive contracts (panel b). Notice that limit pricing is a second best strategy from the dominant firm's viewpoint: it is less profitable than monopoly pricing, and must be adopted only because of the competitive pressure from the fringe. The role of exclusive contracts is to eliminate such pressure.

Of course, buyers have the option of refusing exclusive deals and trading with the competitive fringe only. In this case, however, this option does not really constrain the dominant firm. This is so because buyers obtain an information rent even under monopoly. When $c \geq c^m$, this is actually larger than the rent that they could obtain by trading with the competitive fringe only. As a result, exclusive dealing effectively shelters the dominant firm from the competitive pressure from the fringe in utility space, at no cost.

To put it differently, by using exclusive contracts the dominant firm can turn the competition for each marginal unit of demand into competition for the entire volume demanded by a buyer. This allows the dominant firm to better exploit its competitive advantage. By leveraging on the rents that it inevitably must leave on inframarginal units, the dominant firm can keep selling the monopoly quantity in states of greater demand, without having to resort to limit pricing. Thus, *exclusive contracts allow dominant firms to more profitably foreclose competitors from a segment of the market.* This is perhaps the main conclusion of this paper.

Partial foreclosure. To further elaborate, note that when $c \geq c^m$ the competitive advantage is so large that in principle the dominant firm could impose exclusive dealing, and sell monopoly quantities at monopoly prices, for all states of demand. However, it turns out that the most profitable strategy is to eventually accommodate the competitive fringe, allowing high-demand types to purchase both products.

The intuitive reason for this is that exclusive dealing reduces total surplus, thus imposing an unnecessary cost on buyers. (Formally, $u(q_A(\theta), 0, \theta) \leq v(q_A(\theta), \theta)$, with strict inequality whenever exclusion is inefficient.) However, exclusive dealing allows better screening. In particular, high-demand types value the opportunity to purchase both products more than low-demand ones do, and therefore have more to lose from accepting exclusive contracts. This suggests that it may be profitable to impose exclusivity dealing only in low-demand states.[14]

That this must indeed be so follows from a no-distortion-at-the-top property. Remember that with full information the efficient outcome is always achieved (O'Brien and Shaffer, 1997; Bernheim and Whinston, 1998). When demand is high, the dominant firm must then find it profitable to maximise the surplus and extract it as best as it can, just as it would have done under complete information. Thus, exclusive dealing is not imposed in high-demand states. As a result, the competitive fringe is not driven out of the market completely.

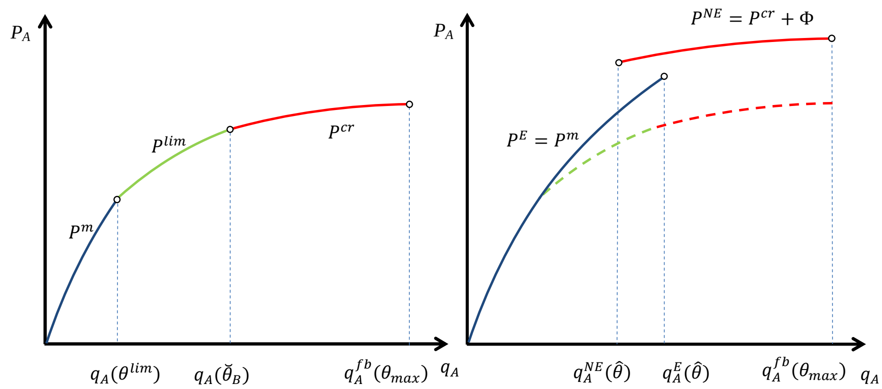
Separation. Remarkably, in high-demand states buyers obtain exactly the same quantities as they would do in the non-linear pricing equilibrium. This follows from a property of "hybrid" optimal screening problems such as ours, which we call the *separation property*.

The dominant firm's screening problem is hybrid in that it involves both continuous and discrete choices. Indeed, the firm effectively controls, through its price schedules $P_A^E(q_A)$ and $P_A^{NE}(q_A)$, two variables: its own quantity (a continuous variable), and whether the rivals' quantity can be positive or must necessarily be nil (a binary variable). By using the Revelation Principle, the problem becomes a hybrid optimal control problem, where the programmer can choose among different control systems, corresponding to the different possible values of the discrete variable. Therefore, to find the solution one needs to choose a sequence of control systems, the switching points, and the control function $q_A(\theta)$ for each system that maximise the firm's profit.[15]

Generally speaking, the optimal choice of the continuous variable depends on the value taken by the binary variable. Should the binary variable be constant

[14] From this viewpoint, exclusive dealing resembles the strategy of damaging one's goods analysed by Deneckere and McAfee (1996). One difference, however, is that with exclusive contracts most of the cost of the damage is borne by the firm's rivals, which makes the strategy more attractive.

[15] This technical complication does not arise in the symmetric framework of Calzolari and Denicolò (2013). In that case, the structure of equilibrium exclusive contracts is fully pinned down by competitive forces, since competition in utility space is fierce. When, on the other hand, the dominant firm enjoys a large competitive advantage, and thus is totally or partially sheltered from rivals in utility space, it has room for designing its exclusive contracts so as to better screen the buyers.



panel (a)

Equilibrium with non-linear pricing

Under non-linear pricing, the equilibrium price schedule is continuous, and smooth, across the three possible equilibrium regimes.

panel (b)

Equilibrium with exclusive contracts

With exclusive contracts, prices are higher than in the non-linear pricing equilibrium. When the buyer switches to common representation, prices jump up.

Figure 2: Prices with large competitive advantage.

for all possible states of demand, there would be two separate regimes with distinct solutions, which can be obtained using standard techniques. However, the actual solution to the firm's screening problem may involve switches, with the binary variable taking one value for certain states of demand and another value for other states. In principle, this might affect the solution for the continuous variable within each regime. However, the separation property ensures that this does not happen. The solution depends only on the state of demand and on the value of the binary variable that applies to that state, and does not depend on the number or order of the switches. In short, under exclusive contracts $q_A(\theta)$ is either $q^m(\theta)$ or $q_A^{cr}(\theta)$.

Prices. Even though in high-demand states equilibrium quantities are the same as in the non-linear pricing equilibrium, the payments in the two cases are different. In particular, with exclusive contracts the dominant firm adds a fixed fee to its non-exclusive tariff (marginal prices cannot change as they must support the same quantities). This fixed fee can be interpreted as a "tax" levied on product variety. This is illustrated in Figure 2.

The figure also shows that with exclusive contracts, prices also increase for certain intermediate demand states, as the dominant firm can keep charging monopoly prices without resorting to limit pricing. In short, the dominant firm's prices clearly increase.

Now compare tariffs $P_A^E(q_A)$ and $P_A^{NE}(q_A)$ in panel b of Figure 2. It appears that at the switching point between the exclusive and non-exclusive regimes, the average price goes up. On the other hand, the marginal price goes down. In other words, the curve $P_A^{NE}(q_A)$ is flatter than the curve $P_A^E(q_A)$. Intuitively, lower non-exclusive marginal prices serve to compete more effectively for marginal units. The upward jump in the average prices, on the other hand, serves as an inducement to buyers to accept exclusive deals. Thus, the dominant firm effectively offers loyalty discounts. Compared to the non-linear price equilibrium tariff, however, such "discounts" result from a surcharge on non-exclusive deals, rather than from any discount on exclusive deals.

Notice also that our normalisation of costs implies that $P_A^E(q_A)$ and $P_A^{NE}(q_A)$ are the marginal profits earned on a buyer who buys q_A units of the good in the two regimes. Therefore, at the switching point the marginal profit is higher under common representation. This means that the dominant firm should actually welcome a switch to common representation, although (or, rather, precisely because) it sets its prices so as to penalise such a switch.

Product variety. The dominant firm forecloses the competitive fringe more extensively with exclusive contracts than by using limit pricing: with reference to Figure 1, $\hat{\theta} > \breve{\theta}_B$. This implies that the dominant firm's market share tends to increase.

In the light of the foregoing analysis, this result is clear. Since exclusive dealing is more profitable than limit pricing as a foreclosure strategy, the dominant firm uses it more widely. As a result, fewer buyers purchase both goods when exclusive contracts are permitted than when they are prohibited: consequently, there is less product variety.

Welfare effects. We can now summarise our results and note the implications for welfare. That the dominant firm benefits from exclusive contracts follows from a simple revealed preference argument. More specifically, we have seen that the dominant firm's market share rises, while at the same time it can also raise its prices.

In a competitive fringe model, the dominant firm's competitors cannot really be harmed, as they would just break even anyway. However, buyers are harmed in terms of both higher prices and less variety. To be precise, in low-demand states buyers are unaffected; in intermediate-demand states they obtain the monopoly quantity rather than either the limit pricing quantities or the common representation quantities; and in high-demand states, where quantities do not change, they are negatively affected by the fixed fee added to the non-linear pricing equilibrium tariff. Clearly, since the quantities obtained in lower-demand states are more heavily distorted, the firm can extract higher rents in higher-demand states.[16,17]

The impact of exclusive contracts on social welfare is also negative. This follows immediately from the fact that the equilibrium quantities are never greater than under non-linear pricing, and are strictly lower in certain intermediate-demand states. In such states, buyers obtain the monopoly quantity rather than either the limit pricing or the common representation quantities. This entails a greater distortion, and is thus detrimental to social welfare.

* * *

Qualitatively similar results hold when $c^{\lim} \leq c < c^m$.[18] There are only two relatively minor changes. Firstly, under non-linear pricing the monopoly region (i.e. for low-demand states) disappears, and the marginal buyer is served under limit pricing. Secondly, the dominant firm cannot implement the monopoly solution for all buyers who sign exclusive contracts, as the monopoly tariff now lies above the fringe's price (i.e. the cq line) when q is small. In other words, the competitive pressure from the competitive fringe forces the dominant firm to set the exclusive prices at $\min[P^m(q), cq]$, and hence the exclusive quantity at $\max[q^m(\theta), q^e(\theta)]$, where $q^e(\theta)$ is the solution to

$$u_{q_A}(q_A, 0, \theta) = c. \tag{6}$$

However, in low-demand states buyers would have purchased the limit pricing quantity $q^{\lim}(\theta)$ under non-linear pricing. Since this is higher than $q^e(\theta)$, as the goods are imperfect substitutes, buyers are still harmed, and social welfare is negatively affected. We can therefore conclude that exclusive contracts are unambiguously anticompetitive in this case as well.

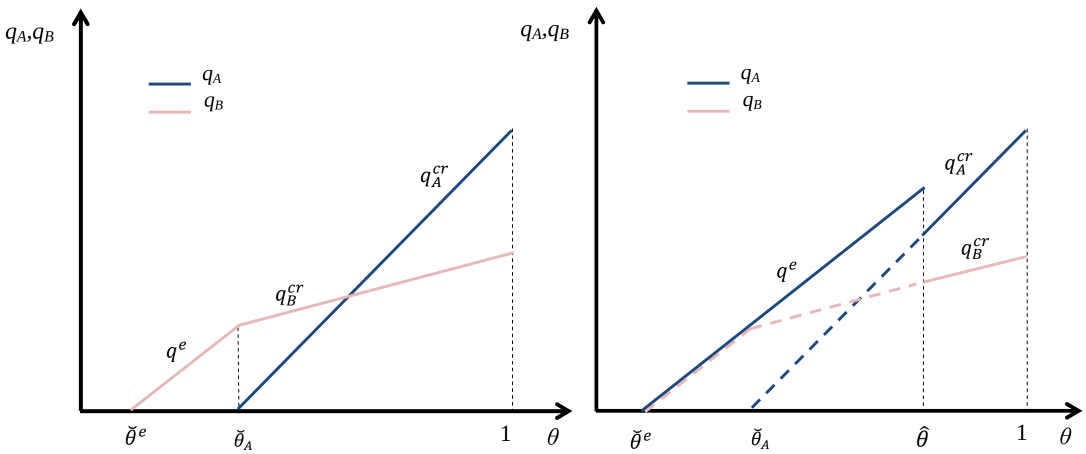
3.3 Small competitive advantage

The case where the competitive advantage is small (i.e. $c < c^{\lim}$) is somewhat different, since firm A's dominance is due not so much to its competitive ad-

[16] It may be tempting to view buyers who sign exclusive contracts as imposing a negative externality on those who do not, similarly to what happens in models of "naked exclusion." However, this analogy would be misleading. In our model, there are in fact no contractual externalities: a buyer's payoff depends only on his trades with the firms, and not on other buyers' trades. This is quite different from models of naked exclusion. In those models, buyers who do not sign exclusive contracts gain if other buyers do not sign either, provoking entry. It should also be pointed out that in our model exclusive contracts would be profitable and anticompetitive even if the contracts intended for high-demand buyers remained the same as in the non-linear pricing equilibrium.

[17] Note that when buyers are downstream firms, the extent to which their gains or losses are shifted onto final consumers may depend on how prices change exactly. Generally speaking, higher upstream prices tend to translate into higher downstream prices, so final consumers should also suffer from exclusive contracts when downstream firms do. However, if the only change is an increase in a fixed fee, final consumers may be unaffected.

[18] The threshold $c^{\lim}$ is the lowest c such that at least one type θ exists for which $q_A^{cr}(\theta) > q^{\lim}(\theta)$. In the uniform-quadratic specification, $c^{\lim} = \frac{1-2\gamma}{2-3\gamma}$. This implies that the region $c^{\lim} \leq c < c^m$ is non-empty for all possible values of the product differentiation parameter γ, even accounting for the restriction $c < \frac{1-2\gamma}{1-\gamma}$ implied by the condition $q_B^{fb}(\theta_{\max}) > 0$.



panel (a)

Equilibrium with non-linear pricing

In low-demand states only product *B* is bought as the competitive fringe prices more aggressively than the dominant firm.

panel (b)

Equilibrium with exclusive contracts

Using exclusive dealing contracts, the dominant firm can replace the competitive fringe in the low-demand segment of the market.

Figure 3: Quantities with small competitive advantage.

vantage (which is indeed low) as to the lack of market power of product B's suppliers. As a result, under non-linear pricing low-demand types purchase product B only (see panel a of Figure 3).[19] This cannot occur when product B is offered by a duopolist that also possesses market power. As a result, in the duopoly model the case of limited competitive advantage results in economic effects that are quite different from those found in the competitive fringe model, as we shall see below.

Besides being socially inefficient, the fact that low-demand types purchase product B only is disappointing from the point of view of the dominant firm. If only the dominant firm could replace the competitive fringe in the low-demand segment of the market, it would save the production cost and increase its profits by the same amount. However, to achieve that result the dominant firm would have to undercut the competitive fringe. With non-linear pricing, this would improve the buyers' contractual options in high-demand states: buyers could now purchase, at a unit price of c, a certain amount of product A in addition to any amount of product B. This possibility would reduce the rent that the firm can extract in high-demand states, offsetting any gains in low-demand ones.

With exclusive contracts, however, the dominant firm can undercut the competitive fringe in low-demand states under an exclusivity clause. This leaves the

[19] The reason for this is simple. Product A is less costly to produce, or of higher quality. However, product B is supplied competitively, whereas the dominant firm exercises its market power in the market for product A. When c is sufficiently low, this latter effect must prevail.

buyers' contractual options in high-demand states unchanged. (Note that buyers can already purchase any amount of one product, namely B, at a unit price of c.) It follows that the dominant firm can now replace the competitive fringe in the low-demand segment of the market, without losing any profit in the high-demand segment. Figure 3 (panel b) illustrates this.

Again, buyers are clearly harmed by exclusive contracts. The fact that the equilibrium quantities, which are already inefficiently low under non-linear pricing, are further reduced has a negative impact on social welfare. Now, however, exclusive contracts have a positive productive efficiency effect. That is, replacing the competitive fringe in low-demand states reduces total production costs. This may make the total welfare effect of exclusive contracts uncertain.

In the limiting case $c = 0$, exclusive contracts become irrelevant. The anti-competitive mechanism analysed in this paper crucially hinges on the dominant firm's competitive advantage.

4 Extensions

In this section, we discuss two model extensions: (i) the case in which the dominant firm faces competition from one small rival firm, which, however, exercises a certain market power; and (ii) the case in which the dominant firm can offer both exclusive and market-share contracts.[20]

4.1 Duopoly

In principle, the case in which there is only one supplier of product B (which we call firm B) might open up new possibilities. Unlike the competitive fringe, firm B can actively respond to the dominant firm's attempt at foreclosing it. In particular, it may offer exclusive contracts in turn, lower its non-exclusive prices, or both. Therefore, the party offering exclusive contracts, and whose contracts are accepted in equilibrium, is now endogenous.

The analysis of the duopoly model is developed in Annex 3. As it turns out, when the dominant firm's competitive advantage is large the results do not change substantially from the competitive fringe model. Qualitatively, the equilibrium quantities are the same as in Figure 1. However, there are a few notable differences that we shall now discuss.

First of all, firm B too may now offer exclusive contracts. If it does,[21] it prices exclusive deals at cost. However, the dominant firm either undercuts firm B in Bertrand-like fashion, or else it engages in monopoly pricing – whichever leads to lower prices. As a result, only the exclusive contracts offered by the dominant firm are accepted in equilibrium. In practice, this implies that only A's exclusive contracts can be observed.

[20] Both extensions may be useful when applying our analysis to the *Intel* case, where AMD arguably possessed some market power, and Intel allegedly used different types of contracts that reference rivals' volumes.

[21] If $c \geq c^m$, it is in fact irrelevant whether firm B offers exclusive contracts or not.

Secondly, firm B now exercises whatever market power it has thanks to product differentiation, by charging supra-competitive non-exclusive prices. As a result, the common representation quantities are different than in the baseline model. To be precise, under duopoly $q_A^{cr}(\theta)$ is greater, and $q_B^{cr}(\theta)$ smaller.[22]

Thirdly, firm B now actively resists being foreclosed. Besides pricing its exclusive contracts as aggressively as it can, it also cuts its non-exclusive prices. Compared to the non-linear pricing tariff, firm B offers a lump-sum subsidy to buyers who switch to common representation. Firm A, in contrast, charges them a fixed fee, precisely as it does in the competitive fringe model. Clearly, firms are competing for market shares. Firm A tries to induce buyers into accepting exclusive dealing arrangements, and firm B to opt for common representation. However, the fact that firms use lump-sum transfers for competing at the extensive margin implies that the common representation quantities can be exactly the same as in the non-linear pricing equilibrium. This is similar to the baseline model.

Finally, firm B now obtains a positive profit (gross of fixed costs). Hence, it can be definitely harmed by exclusive contracts. Indeed, firm B is harmed both because its sales shrink, and because it must reduce its prices in order to resist being foreclosed from an even larger segment of the market.

Things are quite different when the dominant firm's competitive advantage is small. In this case, there is a countervailing procompetitive effect that may outweigh the anticompetitive effect that we have analysed so far. This procompetitive effect has been analysed in Calzolari and Denicolò (2013), where we focused on the symmetric case $c = 0$.[23]

To better understand these different results, remember that firms may compete for marginal units (if exclusive contracts are banned) or for the entire volume demanded by a buyer (if exclusive contracts are permitted). Which form of competition is tougher depends on the relative strength of two opposing effects. On the one hand, in utility space the dominant firm can leverage on the rents that it must inevitably leave to buyers. This allows the dominant firm to make more effective use of its competitive advantage, and to be better protected from competition. On the other hand, in utility space competition is not softened by product differentiation. This implies that the dominant firm's rivals tend to price more aggressively.

The latter effect is mute in the competitive fringe model, where the fringe always prices at cost. However, it comes into play in the duopoly model. On the other hand, when $c = 0$ the anticompetitive effect vanishes, as we have seen above. Therefore, in a duopoly the procompetitive effect must prevail when the competitive advantage is relatively small. This explains why the results found in Calzolari and Denicolò (2013) are robust to a small degree of asymmetry, but are completely reversed when the dominant firm's competitive advantage is large enough. Annex 3 provides a detailed analysis, which also covers intermediate cases in which exclusive contracts have mixed effects.[24]

[22] For the same reason, $q^{\lim}(\theta)$ is lower than in the competitive fringe model.

[23] We also considered the special case of independent products when c is positive but small.

[24] Another difference between the cases of large and small competitive advantage in the

4.2 Market-share contracts

Besides offering exclusive contracts, the dominant firm might employ other contracts that reference rivals' volume. For example, it may offer market-share discounts, i.e. discounts that depend on its share of a buyer's total purchases. To account for this possibility, we allow for the fact that the dominant firm may freely condition its payment request on its market share, or, equally, on the competitive fringe's volume: $P_A = P_A(q_A, q_B)$. The analysis of this case is set out in Annex 4.

Naturally, this pricing strategy requires that the dominant firm observe with some precision not only whether q_B is positive or nil, but also the exact value of q_B. Thus, market-share contracts are observationally demanding.[25] However, we find that, if market-share contracts are feasible, they will be offered, and signed, in certain states of demand.

To be precise, in low-demand states *de facto* or contractual exclusivity still prevails. However, the transition from exclusive dealing to unconstrained common representation is now smoother. In other words, as demand rises exclusive dealing is no longer imposed on buyers, who therefore start purchasing both products. However, the dominant firm now uses market-share discounts to reduce the incentive to purchase product B. As demand increases, the dominant firm allows for a greater and greater share of product B. However, the quantity of product B is always lower than in the non-linear pricing equilibrium, except when demand is highest (this is, once again, a no-distortion-at-the-top property).

Compared to the non-linear pricing equilibrium, market-share contracts are clearly anticompetitive.[26] The comparison with exclusive contracts is less clear: quantities are more heavily distorted in high-demand states, but less so in intermediate ones.

duopoly model is the scope for multiple equilibria. Generally speaking, multiple equilibria are endemic in models in which firms interact strategically by making contractual offers that are destined not to be accepted in equilibrium. For example, under duopoly there are always equilibria in which both firms charge exorbitant non-exclusive prices, forcing buyers into exclusive dealing agreements. However, such equilibria may be ruled out on the grounds that they rely on weekly dominated strategies. When the competitive advantage is large, this is in fact the only source of multiplicity. When the competitive advantage is small, on the other hand, firms face non obvious coordination problems that enlarge the scope for multiplicity. For a more detailed discussion of this, see Annex 3.

[25] In practice, market-share contracts are often cast in terms of critical market-share thresholds. This may facilitate verification and hence enforcement.

[26] This result differs from Majumdar and Shaffer (2009), who show that in a two-state model the first-best solution may be achieved with market-share contracts, but not with simple non-linear prices. This is because the reservation payoff is type dependent. However, with a continuum of types it is generally impossible to reproduce the full information solution even with type-dependent participation constraints (see, for instance, Jullien, 2000).

5 Applications

In this section we illustrate how the analysis developed so far can be applied to a real-world antitrust case. In doing so, we compare our theory with the alternative explanations of exclusive contracts proposed in the literature.

For the purposes of our illustration we use the *Intel* case. Intel produces microprocessors, i.e. integrated circuits that are used by Original Equipment Manufacturers (OEMs) to produce computers. From 2001 to 2007, Intel allegedly entered into exclusive dealing arrangements and market-share contracts with various OEMs, including Dell, HP, NEC and Lenovo. The case has been the subject of extensive litigation both in Europe and in the US.[27]

We choose this example for three main reasons. Firstly, the case is important in its own right: the microprocessor industry is essential to the modern economy, Intel is a large company, and the litigation has sparked a heated debate.[28] Secondly, the aforementioned lawsuits have together resulted in a detailed account of Intel's relations with OEMs. Although we do not have access to confidential case material, the public evidence is more detailed than for many other cases.[29] Thirdly, the market structure seems broadly consistent with our assumptions. In particular, Intel was a dominant firm, but not a monopolistic one;[30] dominance was largely attributable to Intel's competitive advantage over its competitors, in terms of both lower costs and higher perceived quality;[31] the products were imperfect substitutes and buyers had a preference for variety;[32] and finally there is indirect evidence of the importance of asymmetric information.[33]

[27] In Europe the case has gone all the way to the European Court, whose decision is still pending, and has resulted in the largest fine ever seen in the history of European competition policy so far, over a billion Euro. In the US, a private antitrust action by AMD and two court challenges by the FTC and the New York State General Attorney were all eventually settled. Intel agreed to stop the practices and pay AMD damages of \$1.25 billion. Related lawsuits were also brought in individual European countries, Japan and Korea.

[28] It has also inspired a considerable amount of academic research: see for instance DeGraba (2013), DeGraba and Simpson (2013), Johnson (2011), Wright (2011), and Chen and Sappington (2011).

[29] In what follows, we shall mainly refer to the following documents: the European Commission's Decision C(2009) 3726 of 13 May 2009 (the *EU decision*); the General Court's Judgement T-286/09 of 12 June 2014 (the *EU appeal*); and the New York Attorney General's Complaint of 3 November 2009 (the *NYAG complaint*).

[30] Intel consistently boasted a market share of more than 80% over the period in question but faced ongoing competition from AMD (with a market share around 15%) and a few other smaller competitors. See, for instance, DeGraba and Simpson (2013). The fact that the incumbent faces actual rather than potential competition is common to many antitrust cases. Models where an incumbent competes with a potential entrant are not directly applicable to such cases.

[31] Goettler and Gordon (2011) estimate that final consumers were willing to pay almost \$200 more for the Intel brand than for the AMD brand, on top of any price difference reflecting the objectively measurable performances of the microprocessors in question. They also report that Intel benefited from lower unit costs during the period in question.

[32] There is extensive evidence that OEMs valued the possibility of offering broad product lines that included AMD based products very highly: see, for instance, *EU decision* paras 183-6; *NYGA complaint*, paras 92,152-6.

[33] Intel did not make any take-it-or-leave-it offer, but proposed contracts in which prices were conditional on various factors such as volumes, participation in joint marketing campaigns, and

The anticompetitive mechanism analysed in this paper implies that exclusive contracts increase the dominant firm's market share and allow it to raise its prices. Unfortunately, in the microprocessor industry technological change is so fast that identifying the effects of Intel's contractual practices on prices and market shares is a daunting task.[34] On the face of it, the evolution of market shares seems consistent with our theory.[35] However, there are so many confounding factors that this fact, by itself, is inconclusive. Credible causal statements require an empirical structural model that incorporates Intel's pricing strategies, which is beyond the scope of this paper.

The validity of the model's qualitative predictions can be assessed more easily. Firstly, the model predicts that the smaller firm does not offer exclusive contracts – or, if it does, that only the exclusive contracts offered by the dominant firm are accepted in equilibrium. Indeed, there is no evidence in the public records that any OEM had signed exclusive contracts with AMD. This observation may raise doubts about the validity, in this particular case, of the various pro-efficiency explanations of exclusive contracts that have been proposed in the literature. Such explanations may be plausible when exclusive contracts are employed by all active firms, but may be viewed with a certain scepticism when exclusive contracts are used by the dominant firm only.[36]

Secondly, Intel's contracts with OEMs were informal and implicit. They could be terminated at will, or at very short notice, without any contractual penalty being incurred.[37] This does not affect the applicability of our theory, in which contracts play no commitment role. However, it may be a problem for theories that rely on players' commitment to signed contracts.[38] It is even more problematic for the applicability of theories in which penalties for breach of exclusive dealing are crucial for the profitability of such arrangements.[39]

Thirdly, the amount of the market foreclosed by Intel was not particularly

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the meeting of exclusivity (or market-share) requirements. Such conditional pricing strategies may naturally be viewed as a means of inducing buyers into self-selecting in situations of asymmetric information.

[34] The identification of possible price effects is especially difficult. Listed prices are observable, but Intel negotiated substantial discounts with individual OEMs. Annual average prices may be recovered from OEMs' balance sheets, but the exact pricing schemes were confidential. Even the investigations conducted by European and American antitrust authorities revealed such schemes only partially.

[35] After reaching a low in 2001, Intel's market share recovered in the subsequent years, falling again in 2006 when Dell broke the exclusive dealing agreement. Meanwhile, the relative quality of AMD's microprocessors was improving (Goettler and Gordon, 2011, p. 1147), suggesting that the temporary increase in Intel's market share may be attributable to its more aggressive contractual practices.

[36] See Whinston (2008) for an excellent discussion of the possible pro-efficiency rationales for exclusive arrangements. In fact Intel did not invoke any such rationale to justify its practices.

[37] *Eu decision* para 190; *NYGA complaint* para 85.

[38] Commitment is important in theories based on the limited scope for negotiations, such as those of Rasmusen, Ramseyer and Wiley (1990) or Chen and Shaffer (2014). Should negotiations that initially could not take place become subsequently feasible, the parties would have an incentive to renegotiate – a point that has been forcefully made by Spier and Whinston (1995).

[39] For instance, Aghion and Bolton (1987).

large. In its appeal lodged with the European General Court, Intel pointed out that its contracts could only foreclose less than 14% of the market.[40] Intel argued that this would leave plenty of room for a competitor to achieve economies of scale and prosper. However, this does not pose any problems for our theory, which does not view exclusive contracts as a means to raise rivals' costs.

Fourthly, it seems clear that AMD was never under any real threat of shutdown.[41] This makes life more difficult for theories that presume some form of recoupment,[42] because it rules out intertemporal recoupment. Since recoupment is not necessarily intertemporal (for example, it might take place in a related market), the observation is not dispositive. However, it should be pointed out that the issue does not arise in our analysis, where exclusive contracts are directly profitable and do not entail any sacrifice of profits.

Other antitrust cases share with the *Intel* case the fact that the dominant firm's competitors did not offer exclusive contracts, that the contracts were short-term, that the amount of the market foreclosed was relatively small, and that rivals were not actually driven out of the market.[43] However, the *Intel* case provides a rare opportunity to assess what is possibly the most distinctive prediction of our theory. This is that buyers may switch from one form of contract to the other without disrupting the dominant firm's strategy. Such switches may occur naturally following changes in the realisation of demand and/or in the size of the competitive advantage.

It should be pointed out, first of all, that Intel did use a variety of contractual formats. Over the same period of time, Dell agreed to an exclusive dealing arrangement, HP and NEC agreed to market-share contracts with different market-share targets (95% for HP and 80% for NEC), and other OEMs

[40] *EU appeal*, paras 183-94.

[41] AMD's market share never fell below 12% over the period in question (DeGraba and Simpson, 2013). Stock price data also confirm that AMD always remained a viable concern (Wright, 2011).

[42] For example, the European Commission argued that if Intel's discounts had been attributed to "contestable" sales only, they would have entailed below-cost pricing: see *EU decision* paras 1002-1508.

[43] See, for instance, *Omega v. Gilbarco*, *3M v. Appleton*, and *United States v. Dentsply.* This last case is one of the few recent instances in the US of enforcement agency challenges to exclusive dealing arrangements. Dentsply was the leading manufacturer of artificial teeth. By the end of the '90s, it had a market share of 65-70% in terms of units, and of 75-80% in terms of value. It faced competition from a fringe comprising a dozen competitors, the largest of which controlled less than 5% of the market. In 1993, Dentsply entered into exclusive or near-exclusive agreements with a certain number of dental dealers. These are intermediaries that distribute the products to end users, i.e., dental laboratories. The DOJ recognised that one reason why rival manufacturers failed to prosper was their failure to adapt their products to the preferences of American consumers. Nevertheless, it challenged the exclusive agreements under Section 2 of the Sherman Act. Both the district court and the appeal court agreed that: (i) Dentsply was the only manufacturer using exclusive agreements, and its proffered procompetitive justifications were pretextual; (ii) Dentsply's dealers were at liberty to terminate their contractual relations with Dentsply at any time and without penalty; (iii) rivals were not at risk of being driven out of the market. However, the courts disagreed on the entity of the foreclosed portion of the market. The district court ruled for the defendant on the grounds that the amount foreclosed was small. The appeal court disagreed on this point, and overruled the district court's decision.

to quantity-discounts contracts unconditional upon AMD's volumes.[44] More to the point, we possess detailed information on the long, complicated process that eventually led Dell to switch from exclusive dealing to common representation in 2006. Our model makes several specific predictions regarding such switches:

(i) non-exclusive contracts are signed in high-demand states, and exclusive dealing contracts in intermediate or low-demand states;[45]

(ii) the dominant firm charges a fixed fee to buyers who switch to common representation;

(iii) as a result, the average price charged by the dominant firm goes up;

(iv) however, the marginal price goes down, as the dominant firm must now compete for marginal units;

(v) overall, the profit is greater at the switching point under common representation than it is under exclusive dealing;

(vi) the dominant firm's competitor offers lump-sum discounts to buyers who switch to common representation.

When it comes to these more specific predictions, the success of our model is partial. While there is some evidence consistent with results (iii), (iv) and (vi), other evidence seems to be at odds with results (i), (ii) and (v).

On the positive side, the European Commission provides evidence that the average price went up after Dell broke the exclusive dealing agreement. However, Intel disputed this conclusion, arguing that under common representation it would have had a stronger incentive to lower its marginal prices. We do not have access to Intel's submission, nor to the evidence supporting its claims. However, our theory predicts that under common representation Intel would indeed charge a higher average price, but also a lower marginal price. This might reconcile the seemingly opposing claims of the European Commission and Intel.

There is also evidence consistent with result (vi). In particular, when HP first considered launching AMD based products in a certain segment of the market, AMD offered to deliver a significant amount of CPUs to HP for free.[46] This offer is equivalent to a lump-sum subsidy.

On the negative side, a first problem is that the allegedly anticompetitive contracts were signed by some of the largest buyers, such as Dell, HP and Lenovo. In our model, on the other hand, efficiency should prevail in high-demand states. Secondly, the evidence suggests that Intel did not charge fixed fees under common representation. Rather, it may have offered lump-sum subsidies to buyers that opted for exclusive dealing.[47] Finally, Intel did not welcome

[44] *EU appeal* para 35.

[45] In fact, in low-demand states exclusive outcomes can also be achieved without explicit exclusive agreements.

[46] *NYGA complaint* para 160.

[47] The lump-sum nature of Intel's discounts is specifically emphasised by DeGraba and Simpson (2013). Their conclusion that Intel's discounts were at least partially of a lump-sum nature seems solid, even though Intel's pricing schemes were deliberately opaque (and all-units discounts are not exactly equivalent to lump-sum discounts).

Dell's switch to common representation, as it should have done if profits had jumped up at the switching point (as our theory predicts). On the contrary, Intel was seriously concerned with the possibility of such a switch.[48]

Are these problems fatal, or just inherent to one particular version of the theory? To answer this question, note that so far we have implicitly assumed that exclusion is efficient when demand is low, and inefficient when it is high. This follows from the fact that the dominant firm's competitive advantage has been taken as constant. In relative terms, therefore, the advantage is greater when the willingness to pay is low. The result that exclusive contracts should not be signed in high-demand states thus inevitably follows from the no-distortion-at-the-top property.

However, our theory can be extended to the opposing case, where exclusion is efficient when demand is high, but inefficient when it is low. This does not change the basic anticompetitive mechanism, but reverses certain specific results – precisely those that do not seem to fit the facts of the *Intel* case.

Consider, for instance, the following payoff function:

$$u\left(q_A, q_B, \theta\right) = (1+\theta)q_A + (1+b\theta)q_B - \frac{1-\gamma}{2}(q_A^2 + q_B^2) - \gamma q_A q_B. \qquad (7)$$

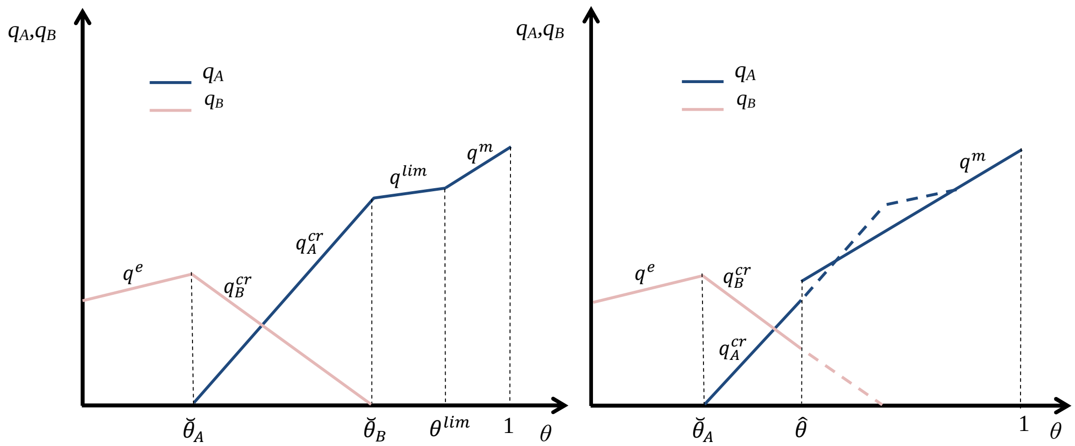
With this specification, the dominant firm's competitive advantage is captured by setting $b < 1$. Hence, the competitive advantage is larger in high-demand states. As a result, if b is small enough exclusion may now be efficient in high-demand states, but inefficient in low-demand ones.

This suggests that the model may now predict exclusive outcomes for large buyers, and common representation outcomes for small ones. This conjecture is confirmed in Annex 5. Figure 4 (panel a) shows the non-linear pricing equilibrium when b is small. When demand is low, buyers purchase product B only, as the competitive fringe prices more aggressively than the dominant firm. As θ increases, the demand for product A increases more strongly than that of product B. As a result, we now have common representation first, followed by limit pricing and, eventually, by monopoly.

The effects of exclusive contracts are illustrated in panel b. The competitive fringe is foreclosed in high-demand states. To be precise, when demand is very high *de facto* exclusivity is already obtained under simple non-linear pricing. However, for intermediate values of demand, exclusive dealing must be imposed contractually. Finally, in low-demand states buyers will either be served under common representation, or else purchase product B only.

As in the baseline model, exclusive contracts here serve as a better substitute for limit pricing. However, this alternative specification of demand reverses the model's result regarding who signs exclusive contracts and who does not. It also changes other results that are at odds with the facts of the *Intel* case. In the

[48] The evidence on this point is overwhelming: see for instance *EU decision* para 240, *NYGA complaint* para 116.



panel (a)

Equilibrium with non-linear pricing

Only product A is bought in high-demand states, where the dominant firm's competitive advantage is higher. In low-demand states, in contrast, only product B is bought as the competitive fringe prices more aggressively.

panel (b)

Equilibrium with exclusive contracts

In high-demand states, the dominant firm uses exclusive contracts as a better substitute for limit pricing. Exclusive dealing now prevails in high-demand states, common representation in low-demand ones.

Figure 4: Quantities with large competitive advantage in high-demand states.

equilibrium depicted in Figure 4, the dominant firm earns more, at the switching point, under exclusive dealing than under common representation. And it offers lump-sum subsidies for exclusivity, rather than imposing a lump-sum tax on product variety.

This version of the model also predicts that the amount of the lump-sum discounts will increase as the size of the competitive advantage decreases. This theoretical framework seems consistent with the history of Intel's contractual relationships with Dell.[49] Before 2001, Intel's competitive advantage was large, and Dell procured all its microprocessors from Intel without any contractual restriction applying. Around the turn of the millennium, however, Intel's competitive advantage began to be whittled away.[50] This process may help explain why exclusivity had to be imposed contractually in 2001 (when for the first time Intel negotiated with Dell discounts conditional up on exclusivity, the so-called 'mother-of-all-programs'), why Intel's lump-sum discounts to Dell were repeatedly increased from 2001 to 2006, and why Dell eventually switched to common representation in 2006.

The foregoing discussion shows that our theory is broadly consistent with many important facets of the *Intel* case. Of course, no abstract model can account for all features of a concrete antitrust case. For example, the microprocessor industry is highly innovative; buyers actively compete against one

[49] For a relatively brief account of such history see *NYGA complaint* paras 74-148.
[50] See Goettler and Gordon (2011, p. 1147).

another; and demand may be affected by dynamic effects. Our framework abstracts from these important aspects, which lie are at the centre of other models instead.

In particular, various papers have proposed a "downstream competition" theory, in which competition among downstream firms erodes any benefits they could obtain from reduced marginal prices for their inputs. As a result, downstream firms may be willing to coordinate on an equilibrium in which they obtain lump-sum subsidies from the upstream incumbent in exchange for keeping a potential entrant out: see, for instance, Asker and Bar-Isaac (2014), DeGraba (2013) and DeGraba and Simpson (2013).

Other models focus on different effects. For example, in order to capture dynamic demand effects Johnson (2011) has developed a model in which the precise quality of product B is unknown to final consumers. Since downstream firms can signal the product's quality by patronising the product, the dominant firm has an incentive to sign them up in order to prevent the signal being sent. Chen and Sappington (2011), on the other hand, focus on innovation. They show that fully or partially exclusive contracts may deter a potential entrant's R&D and hence impede its entry.

All these models provide useful insights. However, there is one final difference between our theory and most of the existing ones: the latter all predict that when a large buyer such as Dell breaks an exclusive dealing agreement, the structure of the market will change radically. The exact reason may vary: a competitor may achieve economies of scale and enter the market, a semi-collusive agreement may break down, or the real quality of a product may be revealed to final consumers. However, in any case the market ought to become more competitive. In our framework, on the other hand, such a switch is a natural occurrence that does not disrupt the dominant firm's pricing strategy and does not radically change the structure of the market. On the face of it, this picture seems to be more consistent with the evolution of the microprocessor industry at the time that Dell switched to common representation.

6 Conclusion

Summary. In this paper, we have developed a new theory of exclusive dealing. The theory supports the simple anticompetitive story that a dominant firm may profitably use exclusive contracts to increase its market share and prices, harming buyers (in terms of higher prices and reduced variety) and rivals alike. The theory rests on two assumptions. The first assumption is that firms cannot fully extract the buyers' surplus because they are imperfectly informed about demand. The second is that the dominant firm has a sizeable competitive advantage over its rivals, in terms of lower costs, higher quality, or a combination of the two.

Not only are these assumptions often realistic, but the model's predictions are also consistent with the characteristic facts of many antitrust cases. In addition to a dominant firm that controls a substantial share of the market and has entered into some kind of exclusive arrangement with its customers, these often involve one or more smaller competitors, that have been active in the industry for some time and in principle could themselves use exclusive contracts, but apparently have chosen not to. Our theory can explain these characteristic facts and, more specifically, it can also help explain several important aspects of the *Intel* case, some of which may prove difficult to account for using other theories.

Our analysis, however, is not necessarily an alternative to existing anticompetitive explanations of exclusive contracts, but may well complement them. It is well understood that incumbents may have a variety of reasons for foreclosing rivals: for instance, they may want to deprive rivals of economies of scale, reduce their incentives to innovate, or exploit positive dynamic externalities in demand. These motives may even justify foreclosure strategies entailing a short-term loss of profits. Our contention here is that exclusive contracts can be directly profitable. If this is so, then the aforementioned reasons may simply induce the dominant firm to behave more aggressively.

Implications for policy. The analysis developed in this paper does not call for a radical change in the current antitrust treatment of exclusive dealing arrangements, which as such is based on the rule of reason. However, it may suggest that different factors should be considered for the purposes of antitrust evaluation. For example, in our model the length of the contracts is irrelevant, since contracts are not used for commitment purposes. Also, significant anticompetitive effects may be produced even if the entity of the foreclosed portion of the market is relatively limited, as the anticompetitive mechanism does not rest on raising rivals' costs.

In contrast, our analysis suggests an approach where the crucial factor is the size of the dominant firm's competitive advantage. This determines whether the dominant firm's rivals can compete for exclusives effectively (in which case procompetitive effects are possible) or not (in which case anticompetitive effects are more likely). Interestingly, this approach seems to have been foreshadowed in some recent decisions by US courts.[51]

Further extensions. The oft-mentioned analogy between exclusive dealing and tying suggests that a variant of our theory may apply to tying. Indeed, this paper offers two main insights. First of all, a dominant firm that benefits from a competitive advantage over its rivals but must leave information rents to the buyers, can impose contractual restrictions without having to compensate the buyers. Secondly, such restrictions can help better screen the buyers. In this paper, we have focused on contractual restrictions on what buyers *can* purchase from *rivals*. However, restrictions may be placed on what buyers *must* purchase from *the dominant firm*.

[51] For example, the appeal court in the *United States v. Dentsply* case, in ruling against the defendant, noted the limited ability of Dentsply's competitors to offer exclusive contracts themselves. Conversely, in *RJR v. Philip Morris* the district court upheld PM's program of loyalty discounts arguing, among other things, that RJR had successfully engaged in similar programs.

Many other possible extensions of the model may be worth considering, but for brevity we mention just three. Firstly, it seems important to extend the analysis to the case of competing buyers. In many cases, including that of *Intel*, the assumption that buyers are local monopolies, or final consumers, is unrealistic. In such cases, it is important to understand how competition among buyers might affect the behaviour of upstream firms. Secondly, in our model the equilibrium price schedules may be fairly complex. Firms sometimes use simpler pricing schemes, and it would be interesting to study how this affects the structure of optimal exclusive dealing arrangements. Thirdly, our analysis has been confined, for reasons of tractability, to the case of one-dimensional heterogeneity. In reality, buyers are likely to differ in many respects, and this might generate a richer set of predictions.

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Appendix 1
Baseline model: the derivation of equilibrium

This Appendix derives the equilibrium of the baseline model. As noted in the main text, the competitive fringe always prices at cost (i.e., $P_B(q_B) = cq_B$), without imposing any exclusivity clause. Therefore, we can focus on the dominant firm's optimal pricing strategy. We first characterise the equilibrium with non-linear pricing, and then with exclusive contracts.

Non-linear pricing

Consider first the optimal pricing strategy when the dominant firm is restricted to simple non-linear pricing. The firm maximises its profit $\int_{\theta_{\min}}^{\theta_{\max}} P_A(q_A(\theta))f(\theta)d\theta$, where $q_A(\theta)$ is chosen by the buyer so as to maximise his net payoff. (Our assumptions guarantee that this maximisation problem is well defined and has a unique solution.) By invoking the Revelation Principle, we can reformulate the problem as if the firm could control $q_A(\theta)$ directly (i.e. a direct mechanism). Define the indirect payoff function

$$v(q_A, \theta) = \max_{q_B \geq 0} \left[u\left(q_A, q_B, \theta\right) - cq_B \right],$$

and use the change of variables $U(\theta) = v(q_A(\theta), \theta) - P_A(q_A(\theta))$. The firm's objective function then becomes

$$\int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta), \theta) - U(\theta) \right] f(\theta)d\theta,$$

where $\tilde{\theta} \geq \theta_{\min}$ is the lowest type served by the firm (chosen optimally). Provided that the indirect payoff function satisfies the single-crossing condition $v_{\theta q_A}(q_A, \theta) \geq 0$ (this is verified in the proof of Proposition 1), the incentive compatibility constraint $q_A(\theta) = \arg\max_{q_A \geq 0}[v(q_A, \theta) - P_A(q_A)]$ is equivalent to the requirements that $U'(\theta) = v_\theta(q_A, \theta)$ and that $q_A(\theta)$ is non-decreasing. The participation constraint is $U(\theta) \geq v(0, \theta)$, and hence is type dependent. The program then becomes

$$\begin{aligned} & \max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta), \theta) - U(\theta) \right] f(\theta)d\theta \\ \text{s.t.} \quad \frac{dU}{d\theta} &= v_\theta(q_A(\theta), \theta) \\ U(\theta) &\geq v(0, \theta) \end{aligned} \tag{8}$$

and $q_A(\theta)$ non-decreasing. This is an optimal control program with $q_A(\theta)$ as the control variable and $U(\theta)$ as the state variable. Once the optimal quantity has been found, one can then recover the tariff that supports it.

By a standard integration by parts, the firm's problem can be rewritten as

$$\max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta), \theta) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q_A, \theta) \right] f(\theta) d\theta,$$

where the term inside square brackets is usually referred to as the "virtual surplus."

As we proceed, we shall impose several regularity conditions that serve to simplify the analysis. The first is:

H1. *For all values of* c, *the virtual surplus function*

$$v(q_A, \theta) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q_A, \theta)$$

is globally concave in q_A *and has increasing differences in* q_A *and* θ.

This assumption guarantees that the solution to the dominant firm's problem can be found by pointwise maximisation of the virtual surplus function. If it fails, an ironing procedure is needed, and the solution exhibits bunching. The condition can be reformulated in terms of the primitives of the model, but it involves third derivatives the economic interpretation of which is not obvious. However, it is easily met in the uniform-quadratic specification (this is true, in fact, for all the regularity conditions that we shall introduce).

Like the indirect payoff function, the virtual surplus function has two branches corresponding to the cases in which the quantity

$$\tilde{q}_B(q_A, \theta) = \arg \max_{q_B \geq 0} \left[u\left(q_A, q_B, \theta\right) - c q_B \right]$$

is 0 or is strictly positive, and a kink in between. The maximum may occur along either branches, or at the kink. When the maximum occurs on the branch with $\tilde{q}_B(q_A, \theta) = 0$, it is the monopoly solution $q^m(\theta)$. When the maximum occurs on the branch corresponding to $\tilde{q}_B(q_A, \theta) > 0$, we obtain the common representation outcome $q_A^{cr}(\theta)$ (this may be a slight misuse of terminology as $q_A^{cr}(\theta)$ could in fact be nil.) As for the kink, it is implicitly defined by the condition $u_{q_B}(q_A(\theta), 0, \theta) = c$, which coincides with condition (3) in the main text. Therefore, this is the limit pricing solution $q^{\lim}(\theta)$. (Proving that $q^{\lim}(\theta)$ is monotone, and hence implementable, is a straightforward matter.)

To reduce the number of cases that need to be taken into consideration, we rule out multiple intersections between the curve $q^{\lim}(\theta)$ and the curves $q^m(\theta)$ and $q_A^{cr}(\theta)$. While this is not really necessary for our results, it simplifies the exposition considerably.

H2. *The curves* $q^m(\theta)$ *and* $q^{\lim}(\theta)$*, and the curves* $q^{\lim}(\theta)$ *and* $q_A^{cr}(\theta)$*, intersect at most once.*

The dominant firm can actually engage in monopoly or limit pricing only if the competitive advantage parameter c exceeds critical thresholds denoted by c^m and $c^{\lim}$ respectively (with $c^m > c^{\lim}$). More precisely, c^m is the lowest c such that there exists at least one type θ for which $q^m(\theta) > q^{\lim}(\theta)$, and $c^{\lim}$ is the lowest c such that there exists at least one type θ for which $q_A^{cr}(\theta) > q^{\lim}(\theta)$. The existence of these thresholds is guaranteed as $q^{\lim}(\theta)$ decreases with c and vanishes if c is large enough, $q_A^{cr}(\theta)$ increases with c, and $q^m(\theta)$ is independent of c (the proof that $c^m > c^{\lim}$ is provided in the proof of Proposition 1.)

Proposition 1 *In the competitive fringe model, there is a unique non-linear pricing equilibrium where the competitive fringe prices at cost ($P_B = cq_B$) and:*

- *when $0 \leq c \leq c^{\lim}$, firm A offers the price schedule*

$$P_A(q) = P_A^{cr}(q);$$

- *when $c^{\lim} \leq c \leq c^m$, firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^{\lim}(q) & \text{for } 0 \leq q \leq q^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \text{ constant} & \text{for } q \geq q^{\lim}(\breve{\theta}_B), \end{cases}$$

  where $\breve{\theta}_B$ is the highest type such that $q_B^{cr}(\theta) = 0$ and the constant guarantees the continuity of the price schedule;

- *when $c \geq c^m$, firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^m(q) & \text{for } 0 \leq q \leq q^m(\theta^{\lim}) \\ P^{\lim}(q) + \text{ constant} & \text{for } q^m(\theta^{\lim}) \leq q \leq q^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \text{ constant} & \text{for } q \geq q^{\lim}(\breve{\theta}_B), \end{cases}$$

  where $\theta^{\lim}$ is the solution to $q^m(\theta) = q^{\lim}(\theta)$ and the constants guarantee the continuity of the price schedule.

Proof. See Annex 1.

The corresponding equilibrium quantities are:

- when $c \leq c^{\lim}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_A \\ q_A^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_A \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}^e \\ q^e(\theta) & \text{for } c \leq \theta \leq \breve{\theta}_A \\ q_B^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_A, \end{cases}$$

- when $c^{\lim} \leq c \leq c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}^e \\ q^{\lim}(\theta) & \text{for } \breve{\theta}^e \leq \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_B, \end{cases}$$

- when $c \geq c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}^m \\ q^m(\theta) & \text{for } \breve{\theta}^m \leq \theta \leq \theta^{\lim} \\ q^{\lim}(\theta) & \text{for } \theta^{\lim} \leq \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \theta \geq \breve{\theta}_B, \end{cases}$$

where the threshold $\breve{\theta}_A$ is implicitly defined as the largest θ such that $q_A^{cr}(\theta) = 0$, and thus is the marginal buyer for product A (just as $\breve{\theta}_B$ is the marginal buyer for product B) under common representation; $\breve{\theta}^m$ is the marginal buyer under monopoly; and $\breve{\theta}^e$ is defined by the condition $q^e(\theta) = 0$, and hence is the marginal buyer under competitive (and limit) pricing.

Exclusive contracts

When exclusive contracts are permitted, the dominant firm can control not only $q_A(\theta)$, but also whether $q_B(\theta)$ may be positive or must be nil. In general, the firm can set $q_B(\theta)$ to nil (i.e. impose an exclusivity clause) for some types, and allow $q_B(\theta)$ to be positive for others. However, it is convenient to first consider the firm's optimal pricing when constrained to offer only non-exclusive contracts, next consider the firm's optimal pricing when constrained to offer only exclusive contracts, and then to address how the firm should combine these two.

When the dominant firm is constrained to offer only non-exclusive contracts, the problem it faces is identical to (8), since in the competitive fringe model $P_B^E(q_B) = P_B^{NE}(q_B) = cq_B$. The solution, which has been characterised in Proposition 1, is denoted by $q_A^{NE}(\theta)$.

Next consider the program in which the firm imposes an exclusivity clause on all buyers. Since $q_B(\theta)$ is thereby set to zero, the firm's problem is

$$\begin{aligned} &\max_{q_A(\theta)} \int_{\breve{\theta}}^{\theta_{\max}} \left[u(q_A(\theta), 0, \theta) - U(\theta)\right] f(\theta) d\theta \\ \text{s.t.} \quad \frac{dU}{d\theta} &= u_\theta(q_A(\theta), 0, \theta) \\ U(\theta) &\geq v(0, \theta) \end{aligned} \tag{9}$$

and $q_A(\theta)$ non-decreasing. We use $q_A^E(\theta)$ to denote the solution to problem (9).

Compared with problem (8), the indirect payoff function $v(q_A(\theta), \theta)$ is replaced by $u(q_A(\theta), 0, \theta)$. (Note that the regularity condition H1 applies to $u(q_A(\theta), 0, \theta)$ as well, since $v(q_A(\theta), \theta) = u(q_A(\theta), 0, \theta)$ when c is large enough.) If the monopoly tariff lies below the cq line, the dominant firm can implement the monopoly solution $q^m(\theta)$. In this case, the participation constraint in problem (9) is non-binding. Otherwise, the dominant firm must undercut the competitive fringe, pricing at c and selling $q^e(\theta)$ units of its product.

Again, to reduce the number of cases to be considered, we rule out multiple intersections between the relevant curves.

H3. *The curves $q^m(\theta)$ and $q^e(\theta)$ intersect at most once.*

Notice that $q^m(\theta_{\max}) > q^e(\theta_{\max})$ by the no-distortion-at-the-top property. Therefore, if the curves do intersect, the curve $q^e(\theta)$ must cut $q^m(\theta)$ from above. It follows that the solution to problem (9) may either coincide with $q^m(\theta)$, or it may be formed by two branches, i.e. $q^e(\theta)$ for low types and $q^m(\theta)$ for high types. Clearly, the first pattern will emerge if the dominant firm's competitive advantage is large enough. To be precise, the condition is $c \geq c^m$. The reason why the critical threshold is again c^m is that since the goods are imperfect substitutes, it is clear from the definitions that $q^e(\theta) \leq q^{\lim}(\theta)$, with equality only when both quantities vanish. The lowest c such that $q^m(\theta)$ always exceeds $q^e(\theta)$ must therefore coincide with the lowest c such that $q^m(\theta)$ can exceed $q^{\lim}(\theta)$.

In general, however, the dominant firm may want to impose exclusivity dealing on subsets of buyers only. Therefore, it must solve a *multi-stage optimal control problem* involving two different control systems, (8) and (9), and the possibility of switching from one system to the other. To solve this problem, one needs to choose a sequence of control systems, the switching points, and the control function $q_A(\theta)$ for each system that maximise the firm's profit. An additional incentive compatibility constraint that must be met is that the control system chosen for a given type must guarantee that type a (weakly) greater utility than the other. This reflects the fact that the firm simply offers both exclusive and non-exclusive contracts, and buyers can freely choose which type of contract to sign.

Generally speaking, multi-stage control problems are difficult to solve because the solution for each control system may depend on the number and order of the switches. However, in Calzolari and Denicolò (2014) we prove the following *separation property*: for any possible sequence of control systems and switching points, the optimal control function for the multi-stage problem coincides with $q_A^{NE}(\theta)$ whenever problem (8) applies, and with $q_A^E(\theta)$ whenever problem (9) applies. The separation property requires only mild regularity conditions (to be precise, that $q_A^{NE}(\theta)$ and $q_A^E(\theta)$ are strictly increasing and that the participation constraint is only binding for the lowest types) which are met in our problem. The separation property guarantees that the solution to the multistage problem is either $q_A^{NE}(\theta)$ or $q_A^E(\theta)$.

It remains to decide which buyers are served under exclusive dealing, and which under common representation. From the no-distortion-at-the-top property, we know that the solution for high-demand types must be nearly efficient, which rules out exclusive dealing. However, exclusive dealing can be optimal for low-demand buyers, whose quantities are distorted more heavily. Our last simplifying assumption guarantees that the solution to the multi-stage control problem involves a unique switch, which must then necessarily be from exclusive to non-exclusive dealing.

H4. $v_\theta(q_A^{NE}(\theta), \theta) > u_\theta(q_A^E(\theta), 0, \theta)$.

In specific examples, such as the uniform-quadratic model, it is easy to verify that H4 holds. More generally, H4 will hold if, for example, $u_{\theta q_i}$ is constant provided that aggregate sales are greater under non exclusivity than under exclusivity.

Consider now the optimal switching point. The non-exclusive and exclusive tariffs are denoted by $P_A^{NE}(q_A)$ and $P_A^E(q_A)$, respectively. The following conditions must hold:

$$u(q_A^E(\hat{\theta}), 0, \hat{\theta}) - P_A^E(q_A^E(\hat{\theta})) = v(q_A^{NE}(\hat{\theta}), \hat{\theta}) - P_A^{NE}(q_A^{NE}(\hat{\theta})), \tag{10}$$

and

$$\frac{P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_A^E(q_A^E(\hat{\theta}))}{v_\theta(q_A^{NE}(\hat{\theta}), \hat{\theta}) - u_\theta(q_A^E(\hat{\theta}), 0, \hat{\theta})} = \frac{1 - F(\hat{\theta})}{f(\hat{\theta})}. \tag{11}$$

Condition (10) simply reflects the new incentive compatibility constraint mentioned above. Condition (11), on the other hand, follows from the firm's optimisation. To prove it, let Φ be a parallel shift in the price schedules that apply for $\theta \geq \hat{\theta}$. Notice that a change in Φ will not affect the quantities nor the switching points to the right of $\hat{\theta}$. Therefore, a small increase $d\Phi$ in Φ will increase profits by $\left[1 - F(\hat{\theta})\right] d\Phi$. On the other hand, a change in Φ will change $\hat{\theta}$. By implicit differentiation, the associated change in profits is

$$\frac{P_A^{NE}(q_A^{NE}(\hat{\theta})) - P_A^E(q_A^E(\hat{\theta}))}{v_\theta(q_A^{NE}(\hat{\theta}), \hat{\theta}) - u_\theta(q_A^E(\hat{\theta}), 0, \hat{\theta})} f(\hat{\theta}) d\Phi.$$

At an optimum, profit must be locally constant, so condition (11) follows.

Note that (11) implies that profits jump up at a switching point. The economic intuition is simple. Consider an increase in the constant term of the tariff that applies for $\theta > \hat{\theta}$. (Note that if the entire price schedule to the right of $q(\hat{\theta})$ is shifted up by a constant, local incentive compatibility is preserved, and hence the equilibrium quantities in that interval do not change.) Clearly, this move has a direct, positive effect on profits extracted from higher types, and an indirect effect due to the resulting increase in $\hat{\theta}$. The indirect effect would vanish if $P_A^{NE}(q_A^{NE}(\hat{\theta})) = P_A^E(q_A^E(\hat{\theta}))$. At the optimum, however, the indirect effect must be negative, as it must exactly offset the positive, direct effect. This implies that profitability must be greater to the right than to the left of the switching point. For example, when the denominator of the left-hand side of (11) is positive, the system optimally switches from exclusivity to non-exclusivity, and the dominant firm obtains higher profits, at the margin, by serving type $\hat{\theta}$ under common representation than under exclusivity.

Note that while the equilibrium outcome is still unique, it can now be supported by different price schedules. The reason for this is that when the dominant firm offers both exclusive and non-exclusive contracts, some contracts are destined not to be accepted and may therefore be specified arbitrarily to some extent at least. Accordingly, the following proposition specifies only the relevant parts of the equilibrium price schedules.

Proposition 2 *With exclusive contracts, the competitive fringe model presents a unique equilibrium outcome where $P_B^E(q_B) = P_B^{NE}(q_B) = cq_B$ for all $q_B \geq 0$. Furthermore, there exists a threshold $\bar{c} < c^m$ such that :*

- *when $c \leq \bar{c}$, firm A offers the price schedules:*

$$\begin{array}{ll} P_A^E(q) = cq & \text{for } 0 \leq q \leq q^e(\hat{\theta}) \\ P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A & \text{for } q \geq q^e(\hat{\theta}) \end{array}$$

where Φ_A is a constant term;

- *when $\bar{c} \leq c \leq c^m$, firm A offers the price schedules:*

$$P_A^E(q) = \begin{cases} cq & \text{for } 0 \leq q \leq q_A^m(\bar{\theta}) \\ P^m(q) + \ \textit{constant} & \text{for } q_A^m(\bar{\theta}) \leq q \leq q_A^m(\hat{\theta}) \end{cases}$$

where $\bar{\theta}$ is the solution to $q^e(\bar{\theta}) = q_A^m(\bar{\theta})$, and the constant guarantees the continuity of the price schedule, and

$$P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A \qquad \text{for } q \geq q_A^m(\hat{\theta});$$

- *when $c \geq c^m$, firm A offers the price schedules:*

$$\begin{array}{ll} P_A^E(q) = P^m(q) & \text{for } 0 \leq q \leq q_A^m(\hat{\theta}) \\ P_A^{NE}(q) = P_A^{cr}(q) + \Phi_A & \text{for } q \geq q_A^m(\hat{\theta}). \end{array}$$

In each case, $\hat{\theta}$ and Φ_A are determined by the equilibrium conditions (10) and (11).

Proof. See Annex 1.

The equilibrium quantities are:

- when $c \leq \bar{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \check{\theta}^e \\ q^e(\theta) & \text{for } \check{\theta} \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

- when $\bar{c} \leq c \leq c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \check{\theta}^e \\ q^e(\theta) & \text{for } \check{\theta}^e \leq \theta \leq \theta^+ \\ q^m(\theta) & \text{for } \theta^+ \leq \theta \leq \hat{\theta} \\ q^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

when $c \geq c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \theta^m \\ q^m(\theta) & \text{for } \theta^m \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \theta > \hat{\theta} \end{cases}$$

where θ^+ is the solution to $q^e(\theta) = q^m(\theta)$ (this is unique by H3).

Annex 1
Proofs for the baseline model

(NOT FOR PUBLICATION)

Proof of Proposition 1. Since the competitive fringe will always price at cost, to prove the proposition it suffices to show that the dominant firm's equilibrium pricing strategy is indeed optimal. To do so, we shall focus on direct mechanisms and hence find the optimal quantity $q_A(\theta)$, showing that it coincides with the equilibrium quantity reported in Appendix 1. It is then straightforward to conclude that the price schedules that support these quantities, which are the equilibrium price schedules, are indeed optimal.

To begin with, consider the indirect payoff function $v(q_A, \theta)$, as defined by (4) when $P_B(q_B) = cq_B$. This is piecewise smooth, with two branches corresponding to the cases in which the quantity $\tilde{q}_B(q_A, \theta) = \arg\max_{q_B \geq 0} \left[u\left(q_A, q_B, \theta\right) - cq_B\right]$ is 0 or is strictly positive, and a kink between the two branches. It can be easily checked that $v(q_A, \theta)$ is globally concave in q_A. It also satisfies the single-crossing condition $v_{\theta q_A}(q_A, \theta) \geq 0$, since we have

$$v_\theta(q_A, \theta) = u_\theta\left(q_A, \tilde{q}_B(q_A, \theta), \theta\right)$$

and hence:

$$\begin{aligned} v_{\theta q_A} &= u_{\theta q_A} + \frac{d\tilde{q}_B(q_A, \theta)}{dq_A} u_{\theta q_B} \\ &= u_{\theta q_A} - \frac{u_{q_B q_A}}{u_{q_B q_B}} u_{\theta q_B} \geq 0, \end{aligned}$$

where the inequality follows by the fact that the goods are imperfect substitutes.

The single-crossing condition guarantees that the participation constraint binds only for the marginal buyer, whom we indicate here as $\tilde{\theta}$, and that firm A's optimisation program (8) can be written as

$$\max_{q_A(\theta)} \int_{\tilde{\theta}}^{\theta_{\max}} \left[v(q_A(\theta), \theta) - U(\theta)\right] f(\theta) d\theta$$

$$\begin{aligned} \text{s.t. } \frac{dU}{d\theta} &= v_\theta(q_A, \theta) \\ U(\tilde{\theta}) &= v(0, \tilde{\theta}) \end{aligned}$$

By a standard integration by parts, the problem reduces to finding the function $q_A(\theta)$ that pointwise maximises the indirect virtual surplus:

$$s(q_A, \theta) = v(q_A, \theta) - \frac{1 - F(\theta)}{f(\theta)} v_\theta(q_A, \theta).$$

Like the indirect payoff function, the indirect virtual surplus has two branches and a kink at $q_A = q^{\lim}(\theta)$. Generally speaking, for any θ the maximum can occur in either one of the two quadratic branches, or at the kink. By definition, $q^m(\theta) = \arg\max_{q_A} s(q_A, \theta)$ when the maximum lies on the first branch, and $q_A^{cr}(\theta) = \arg\max_{q_A} s(q_A, \theta)$ when it lies on the second. Furthermore, the kink is implicitly defined by the condition $u_{q_B}\left(q^{\lim}(\theta), 0, \theta\right) = c$ and hence occurs at $q^{\lim}(\theta)$.

Since $q_A(\theta)$ must pointwise maximise the virtual surplus, we can conclude that $q_A(\theta) = q_A^m(\theta)$ if the maximum is achieved on the upper branch, $q_A(\theta) = q_A^{cr}(\theta)$ if the maximum is achieved on the lower branch, and $q_A(\theta) = q^{\lim}(\theta)$ if the maximum is achieved at the kink. By assumption H1, $s(q_A, \theta)$ is globally concave in q_A. This implies that if $q^m(\theta) > q^{\lim}(\theta)$, then $s(q_A, \theta)$ is increasing at the kink and the maximum is achieved at $q^m(\theta)$. If instead $q^m(\theta) < q^{\lim}(\theta)$, then $s(q_A, \theta)$ is decreasing to the right of the kink, and one must further distinguish between two cases. If $q_A^{cr}(\theta) > q^{\lim}(\theta)$, then $s(q_A, \theta)$ is increasing to the left of the kink and so the maximum is achieved at the kink, $q^{\lim}(\theta)$. If instead $q_A^{cr}(\theta) < q^{\lim}(\theta)$, the maximum is achieved to the left of the kink and is $q_A^{cr}(\theta)$.

It remains to find out when each type of solution applies. By H2, the condition $q^m(\theta) > q^{\lim}(\theta)$ is equivalent to $\theta < \theta^{\lim}$. Since $q^m(\theta)$ is positive only for $\theta > \theta^m$, the monopoly solution is obtained if and only if the interval $\theta^m \leq \theta \leq \theta^{\lim}$ is not empty. This is true if only if $c > c^m$ (recall that c^m is defined as the lowest c such that $q^m(\theta) > q^{\lim}(\theta)$ for some θ). In this case, then, we have $q_A(\theta) = q^m(\theta)$ for $\theta^m \leq \theta \leq \theta^{\lim}$. Of course, the corresponding equilibrium quantity of good B must be nil.

Now suppose that $\theta > \theta^{\lim}$, so that $q^m(\theta) < q^{\lim}(\theta)$. In this case, the solution depends on whether $q_A^{cr}(\theta)$ is larger or smaller than $q_A^{\lim}(\theta)$. The limit pricing solution can emerge only if $q_A^{cr}(\theta) > q_A^{\lim}(\theta)$. By H2, the condition $q_A^{cr}(\theta) > q^{\lim}(\theta)$ reduces to $\theta < \breve{\theta}_B$. Since $q^{\lim}(\theta)$ is positive only for $\theta > \breve{\theta}$, the limit pricing solution is obtained if and only $\breve{\theta} < \breve{\theta}_B$. This condition is equivalent to $c \geq c^{\lim}$ (recall that $c^{\lim}$ is the lowest c such that $q_A^{cr}(\theta) > q^{\lim}(\theta)$ for some θ). When this condition holds, there exists an interval of types to whom the limit pricing solution applies. Again, the corresponding equilibrium quantity of good B must be nil.

Finally, consider the case in which $\theta \geq \breve{\theta}_B$, so that $q_A^{cr}(\theta) \leq q^{\lim}(\theta)$ and the maximum is achieved on the lower branch of the virtual surplus function. Here, we must distinguish between two sub-cases, depending on whether the solution is interior, or is a corner solution at $q_A(\theta) = 0$. Clearly, the solution is interior, and is $q_A^{cr}(\theta)$, when $\theta \geq \breve{\theta}_A$. In this case, the corresponding equilibrium quantity of good B is $q_B^{cr}(\theta) = \tilde{q}_B(q_A^{cr}(\theta), \theta)$. Now, notice that when $c < c^{\lim}$ we have $\breve{\theta}_B < \breve{\theta}_A$, whereas the inequality is reversed when $c \geq c^{\lim}$. This means that if $c \geq c^{\lim}$ and the maximum is achieved in the lower branch, it must necessarily be an interior solution. However, when $c < c^{\lim}$ we have $\breve{\theta}_B < \breve{\theta}_A$. In this case, for $\breve{\theta} \leq \theta \leq \breve{\theta}_A$, we have a corner solution for q_A, and the corresponding equilibrium quantity of good B is $q^e(\theta)$; for $\theta \geq \breve{\theta}_A$, the solution is again interior.

This completes the derivation of the optimal quantities in all possible cases. It is then easy to check that they coincide with the equilibrium quantities reported above, and that they are implemented by the equilibrium price schedules.

Notice that since equilibrium quantities are everywhere continuos, the equilibrium price schedules must be continuous. (In fact, it can be verified that the equilibrium price schedules are also everywhere smooth.) Finally, notice that the constant terms that guarantee continuity are all negative, i.e. fixed subsidies.

To complete the proof, we finally show that $c^m > c^{\lim}$. For the purposes of this proof, let us define $\theta^m(c)$ and $\theta^{\lim}(c)$ as follows: $\theta^m(c)$ is such that $q^{\lim}(\theta^m) = q^m(\theta^m)$, and $\theta^{\lim}(c)$ such that $q^{\lim}(\theta^{\lim}) = q_A^{cr}(\theta^{\lim})$. Then, it is clear that c^m satisfies $q^{\lim}(\theta^m(c^m)) = q^m(\theta^m(c^m)) = 0$, and $c^{\lim}$ satisfies $q^{\lim}(\theta^{\lim}(c^{\lim})) = q_A^{cr}(\theta^{\lim}(c^{\lim})) = 0$.

We distinguish between two cases: $\theta^m(c) \leq \theta^{\lim}(c)$ and $\theta^m(c) > \theta^{\lim}(c)$. If $\theta^m(c^m) \leq \theta^{\lim}(c^m)$ then

$$q^{\lim}(\theta^{\lim}(c^m)) = q_A^{cr}(\theta^{\lim}(c^m)) \geq q^{\lim}(\theta^m(c^m)) = q^m(\theta^m(c^m)) = 0,$$

where the inequality simply follows from the fact that all quantities are increasing in θ. Furthermore, since $q^{\lim}$ is decreasing in c and q_A^{cr} is increasing in c, it follows that $\theta^{\lim}$ is increasing in c. This immediately implies that $c^{\lim} < c^m$.

If instead $\theta^m(c^m) > \theta^{\lim}(c^m)$, then the equilibrium never entails monopoly pricing and hence the comparison between $c^{\lim}$ and c^m is irrelevant (strictly speaking, c^m is not even well defined). To see why this is so, notice that if $\theta^m > \theta^{\lim}$ monopoly pricing would only arise for intermediate types, with both lower and higher types buying both products under common representation. But this is impossible as it would entail multiple intersections between $q^m(\theta)$ and $q_A^{cr}(\theta)$, thus contradicting assumption H2. ■

Proof of Proposition 2. The strategy of proof is the same as for Proposition 1. Obviously, the competitive fringe will always price at cost, i.e. $P_B^E(q_B) = P_B^{NE}(q_B) = cq_B$. As for firm A, we shall focus on direct mechanisms and hence look for the optimal quantity $q_A(\theta)$, showing that it coincides with the equilibrium quantity reported above.

The separation property implies that the solution to the dominant firm's problem is formed by appropriately joining the solutions to the maximisation program (8) and (9). By assumption H4, the former applies to high-demand states ($\theta < \hat{\theta}$), the latter to low-demand ones ($\theta > \hat{\theta}$).

The solution to problem (8) has already been characterised in Proposition 1. Therefore, we start by focusing on problem (9). This is a standard monopolistic non-linear pricing problem with a utility function $u(q_A, 0, \theta)$, except that buyers now have a type-dependent reservation utility

$$U_A^R(\theta) = \max\left[u(0, q, \theta) - cq\right].$$

Thus, the problem becomes

$$\begin{aligned} &\max_{q_A(\theta)} \int_0^1 \left[u(q_A(\theta), 0, \theta) - U(\theta)\right] f(\theta) d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= u_\theta(q_A(\theta), 0, \theta) \\ U(\theta) &\geq U_A^R(\theta). \end{aligned} \tag{A1.1}$$

Its solution is given in the following.

Lemma 1 *When $c \geq c^m$, the solution to problem (A1.1) is*

$$q_A(\theta) = \begin{cases} 0 & \text{for } 0 \leq \theta \leq \breve{\theta}^m \\ q^m(\theta) & \text{for } \theta \geq \breve{\theta}^m. \end{cases}$$

When instead $c \leq c^m$, the solution is

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}^e \\ q^e(\theta) & \text{for } \breve{\theta}^e \leq \theta \leq \theta^+ \\ q^m(\theta) & \text{for } \theta \geq \theta^+. \end{cases}$$

Proof. Consider first the unconstrained problem. Clearly, its solution is $q_A^m(\theta)$.

When $c \geq c^m$, we have $U^m(\theta) \geq U_A^R(\theta)$ for all θ, so the unconstrained solution applies. To show this, notice first of all that it follows from our definitions that

$$q^e(\theta) \leq q^{\lim}(\theta),$$

with equality only when both quantities vanish. Thus, $\breve{\theta}^e$ is the largest θ such that $q^e(\theta) = q^{\lim}(\theta) = 0$. The condition $c \geq c^m$ guarantees that $\breve{\theta}^m \geq \breve{\theta}^e$. By H3, this implies that $q_A^m(\theta) > q^e(\theta)$ for $\theta > \theta^m$. Since

$$U^m(\theta) = \int_{\breve{\theta}^m}^{\theta} u_\theta(q_A^m(s), 0, s) ds$$

whereas

$$U_A^R(\theta) = \int_{\breve{\theta}^m}^{\theta} u_\theta(q^e(s), 0, s) ds$$

it follows by the sorting condition $u_{\theta q_A} \geq 0$ that the participation constraint is always satisfied.

Now suppose that $c < c^m$, so that the type-dependent participation constraint must bind for a non-empty set of types. To deal with this constraint, we use the results of Jullien (2000), and in particular Proposition 3. To apply that proposition, we must show that our problem satisfies the conditions of Weak Convexity, Potential Separation, Homogeneity, and Full Participation. Weak Convexity requires that $U^m(\theta)$ is more strongly convex than $U_A^R(\theta)$. This is implied by assumption H3. Following Jullien (2000), define the modified virtual surplus function

$$s^E(g, q_A, \theta) = u(q_A, 0, \theta) - \frac{g - F(\theta)}{f(\theta)} u_\theta(q_A, 0, \theta)$$

where the "weight" $g \in [0,1]$ accounts for the possibility that the participation constraint may bind over any subset of the support of the distribution of types. Pointwise maximisation of the modified virtual surplus function yields

$$\ell^E(g,\theta) = \arg\max_{q_A} s^E(g, q_A, \theta).$$

Potential Separation requires that $\ell^E(g,\theta)$ is non-decreasing in θ, which is obviously true. Homogeneity is obvious, as it requires that $U_A^R(\theta)$ can be implemented by a continuous and non decreasing quantity; in our case, this is by construction $q^e(\theta)$. Finally, the condition of Full Participation requires that in equilibrium all types $\theta > \check{\theta}^e$ obtain positive quantities, which is obvious given that their reservation utility is strictly positive.

Proposition 3 in Jullien (2000) then implies that the solution to problem (5) is

$$q_A(\theta) = \begin{cases} q^e(\theta) & \text{for } \check{\theta}^e \le \theta \le \theta^+ \\ q^m(\theta) & \text{for } \theta \ge \theta^+, \end{cases}$$

and obviously $q_A(\theta) = 0$ for $\theta \le \check{\theta}^e$. ■

Next, we proceed to the characterisation of the optimal switching point, $\hat{\theta}$. To begin with, observe that condition H4 guarantees that the equilibrium rent function $U(\theta)$ is steeper under non-exclusivity than under exclusivity. This implies that the solution to the hybrid optimal control problem involves a unique switch from problem (9) (which applies to low-demand types) to problem (8) (which applies to high-demand types).

The next lemma says that the switch must be from exclusive dealing to a common representation equilibrium. In other words, at the switching point the solution to problem (9) is given by the common representation quantities $q_A^{cr}(\theta), q_B^{cr}(\theta) > 0$. This rules out the possibility that the switch occurs for types who obtain the monopoly or limit pricing quantity of product A.

Lemma 2 *When $\theta > \hat{\theta}$, both $q_A(\theta)$ and $q_B(\theta)$ are strictly positive.*

Proof. From condition (11), it is clear that when $v_\theta(q_A^{NE}(\theta),\theta) > u_\theta(q_A^E(\theta),0,\theta)$ (which is guaranteed by H4) it must be $P_A^{NE}(q_A^{cr}(\hat{\theta})) > P_A^E(q_A^E(\hat{\theta}))$, so the dominant firm extracts more rents, at the margin, from buyers who accept non-exclusive contracts than from those who accept exclusive ones. From this, it follows immediately that that $q_A^{NE}(\hat{\theta}) > 0$ (otherwise, $P_A^{NE}(q_A^{NE}(\hat{\theta}))$ must be nil). The proof that also $q_B^{NE}(\hat{\theta}) > 0$ is equally simple. If the solution to problem (4) entails $q_B(\theta) = 0$, it must be either $\max[q_A^m(\theta), q^e(\theta)]$ or $q_A^{\lim}(\theta)$. In the former case, the dominant firm would obtain the same rent from buyers who accept non-exclusive contracts as from those who accept the exclusive one; in the latter, it would actually obtain less. Since we have just shown that it must obtain more, these two cases are not possible. ■

While for $\theta > \hat{\theta}$ we always have the common representation quantities, for $\theta < \hat{\theta}$ we can have either the monopoly quantity $q_A^m(\theta)$ or the quantity $q^e(\theta)$.

The former case arises when $c > \bar{c}$, the latter when $c \leq \bar{c}$, where the threshold $\bar{c}$ is implicitly defined as the solution to $\hat{\theta}(c) = \theta^{+}(c)$ and hence satisfies $\bar{c} < c^{\lim}$.

This completes the derivation of the equilibrium quantities in all possible cases. It is then easy to check that these equilibrium quantities are implemented by the price schedules reported in the statement of the Proposition. ■

Annex 2
Uniform-quadratic model

(NOT FOR PUBLICATION)

Here we provide the explicit solutions for the uniform-quadratic specification (2) and check that all our regularity assumptions are satisfied in this case.

Equilibrium quantities are:

$$q^m(\theta) = \frac{2\theta - 1}{1-\gamma},$$

$$q^{\lim}(\theta) = \frac{\theta - c}{\gamma},$$

$$q^e(\theta) = \frac{\theta - c}{1-\gamma}$$

$$q_A^{cr}(\theta) = 2\theta - 1 + c\frac{\gamma}{1-2\gamma},$$

and

$$q_B^{cr}(\theta) = \theta\frac{1-2\gamma}{1-\gamma} + \frac{\gamma}{1-\gamma} - c\frac{1-\gamma}{1-2\gamma}.$$

The critical thresholds are $c^m = \frac{1}{2}$, $c^{\lim} = \frac{1-2\gamma}{2-3\gamma}$, $\breve{\theta}^m = \frac{1}{2}$, $\theta^{\lim} = \frac{c(1-\gamma)-\gamma}{1-3\gamma}$, $\breve{\theta}^e = c$, $\breve{\theta}_A = \frac{1}{2} + c\frac{\gamma}{2(1-2\gamma)}$, $\breve{\theta}_B = c\frac{(1-\gamma)^2}{(1-2\gamma)^2} - \frac{\gamma}{1-2\gamma}$, and $\theta^+ = 1 - c$.

The price schedules are

$$P^m(q) = \frac{1}{2}q - \frac{1-\gamma}{4}q^2,$$

$$P^{\lim}(q) = cq - \left(\frac{1}{2} - \gamma\right)q^2,$$

and

$$P_A^{cr}(q_A) = \frac{1-2\gamma+c\gamma}{2(1-\gamma)}q_A - \frac{1-2\gamma}{4(1-\gamma)}q_A^2.$$

The explicit expressions for $\hat{\theta}$, $\bar{c}$ and $\Phi_A > 0$ are complicated and are reported in a Mathematica file which is available from the authors upon request.

We now verify that conditions H1-H4 are met. Consider condition H1 first. The indirect payoff function is :

$$v(q_A, \theta) = \begin{cases} \theta q_A - \dfrac{1-\gamma}{2}q_A^2 & \text{if } q_A \geq q^{\lim}(\theta) \\ A_0 + A_1 q_A + A_2 q_A^2 & \text{if } q_A \leq q^{\lim}(\theta), \end{cases}$$

where

$$A_0 = \frac{(\theta - c)^2}{2(1-\gamma)}, \quad A_1 = \frac{c\gamma + \theta(1-2\gamma)}{1-\gamma}, \quad \text{and } A_2 = -\frac{1-2\gamma}{2(1-\gamma)}.$$

On both branches, the coefficients of the quadratic terms are negative. Furthermore,

$$\begin{aligned}
\left.\frac{\partial v(q_A,\theta)}{\partial q_A}\right|_{q_A<q_A^{\lim}(\theta)} &= \frac{c\gamma+\theta(1-2\gamma)}{1-\gamma}-\frac{1-2\gamma}{(1-\gamma)}q_A^{\lim}(\theta) \\
&\geq \left.\frac{\partial v(q_A,\theta)}{\partial q_A}\right|_{q_A>q_A^{\lim}(\theta)} = \theta-(1-\gamma)\,q_A^{\lim}(\theta),
\end{aligned}$$

so the function v is globally concave in q_A. Since the additional term in the virtual surplus function,$(1-\theta)v_\theta(q_A,\theta)$, is linear in q_A, $s(q_A,\theta)$ is also globally concave in q_A.

Conditions H2 and H3 are obviously met, as the quantity schedules are linear and thus can intersect at most once. Finally, to verify H4 notice that $u_\theta(q_A^E(\theta),0,\theta)=q^m(\theta)$ whereas $v_\theta(q_A^{NE}(\theta),\theta)$ is either $\max[q^m(\theta),q^{\lim}(\theta)]$ or

$$\frac{(\theta-c)}{(1-\gamma)}+\frac{(1-2\gamma)}{1-\gamma}q_A^{cr}(\theta)>q^m(\theta).$$

Annex 3
Duopoly

(NOT FOR PUBLICATION)

The duopoly model is more complex than the competitive fringe model. The main reason for this is that the solution to a firm's pricing problem does not yield directly the equilibrium, but only its best response to its rival's strategy. Finding the equilibrium requires finding a fixed point of the best response correspondence. Given the extra complexity, we focus on the uniform-quadratic specification of the model.[52]

Non-linear pricing

To find the non-linear pricing equilibrium, we adapt to the asymmetric case the solution procedure proposed by Martimort and Stole (2009) for the symmetric case (i.e. $c = 0$). This is a "guess and check" procedure that starts from the conjecture that the equilibrium price schedules are (piecewise) quadratic and then verifies it by identifying the coefficients of the price schedules.[53]

The non-linear pricing equilibrium turns out to be similar to the competitive fringe model: depending on the size of its competitive advantage c and the intensity of demand θ, the dominant firm can engage in monopoly pricing, limit pricing, or it can accommodate its rival. The exact structure of the equilibrium is as follows:

Proposition 3 *In the duopoly model, the following is a non-linear pricing equilibrium. Firm B offers the price schedule*

$$P_B(q) = P_B^{cr}(q)$$

and:

- *when* $c \leq \tilde{c}$,[54] *firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^{\lim}(q) & \text{for } 0 \leq q \leq q^{\lim}(\breve{\theta}_B) \\ P_A^{cr}(q) + \text{ constant} & \text{for } q \geq q^{\lim}(\breve{\theta}_B) \end{cases}$$

where $\breve{\theta}_B$ *is implicitly defined by the condition* $q_B^{cr}(\breve{\theta}_B) = 0$ *and the constant guarantees the continuity of the price schedule*;

[52] However, the analysis could be generalised using the techniques of Calzolari and Denicolò (2013).

[53] It is important to stress that this procedure makes a guess on the structure of the equilibrium, but does not restrict firms to quadratic price schedules. The drawback of the guess and check procedure is that it cannot find equilibria in which the price schedules do not conform to the guess, if there are any. However, this is not a serious problem for our purposes. If there were multiple non-linear pricing equilibria, for each there would exist a corresponding equilibrium with exclusive contracts, with the same comparative statics properties.

[54] To be precise, the threshold $\tilde{c}$ is the lowest c such that there exists at least one type θ for whom $q^m(\theta) > q^{\lim}(\theta)$.

- *when $c \geq \tilde{c}$, firm A offers the price schedule*

$$P_A(q) = \begin{cases} P^m(q) & \textit{for } 0 \leq q \leq q^m(\theta^{\lim}) \\ P^{\lim}(q) + \textit{ constant} & \textit{for } q^m(\theta^{\lim}) \leq q \leq q^{\lim}(\breve{\theta}_B) \\ P^{cr}(q) + \textit{ constant} & \textit{for } q \geq q^{\lim}(\breve{\theta}_B), \end{cases}$$

where $\theta^{\lim}$ is implicitly defined by the condition $q^m(\theta^{\lim}) = q^{\lim}(\theta^{\lim})$ and the constants guarantee the continuity of the price schedule.

The monopoly price schedule is exactly the same as in the competitive fringe model. The limit pricing schedule is similar, except that now the unit cost c is replaced by the marginal price that firm B charges for the first unit it offers, $P_B'^{cr}(0)$. As for the price schedules under common representation quantities, they are:

$$P_A^{cr}(q) = \alpha q + c\frac{\alpha\gamma}{1-2\gamma}q - \frac{\alpha}{2}q^2; \quad P_B^{cr}(q) = cq + \alpha\left[1 - \frac{c(1-\gamma)}{1-2\gamma}\right]q - \frac{\alpha}{2}q^2,$$

respectively, so $P_B'^{cr}(0) = c + \alpha\left[1 - \frac{c(1-\gamma)}{1-2\gamma}\right]$.

Proof. As usual, we start by reporting the equilibrium quantities, which are

- when $c \leq \tilde{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq P_B'^{cr}(0) \\ q^{\lim}(\theta) & \text{for } \breve{\theta}_A \leq \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1. \end{cases}$$

- when $c > \tilde{c}$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q^m(\theta) & \text{for } \frac{1}{2} < \theta \leq \theta^{\lim} \\ q^{\lim}(\theta) & \text{for } \theta^{\lim} < \theta \leq \breve{\theta}_B \\ q_A^{cr}(\theta) & \text{for } \theta > \breve{\theta}_B \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_B \\ q_B^{cr}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq 1. \end{cases}$$

where

$$P_B'^{cr}(0) = \alpha + c\left[1 - \frac{\alpha(1-\gamma)}{1-2\gamma}\right].$$

Like in Appendix 1, $\breve{\theta}_B$ is implicitly defined by the condition $q_B^{cr}(\breve{\theta}_B) = 0$ and θ^m by the condition $q^m(\theta^m) = q^{\lim}(\theta^m)$; now, however, the explicit expressions are different as $q_B^{cr}(\breve{\theta}_B)$ and $q^{\lim}(\theta^m)$ in the duopoly model differ from the competitive fringe model. They are:

$$\breve{\theta}_B = \alpha + c\frac{(1-\gamma)(1-\alpha)}{1-2\gamma}$$

and

$$\theta^m = \frac{(1-\gamma)}{1-3\gamma} P_B'^{cr}(0) - \frac{\gamma}{1-3\gamma}.$$

To prove the proposition, we must show that the equilibrium price schedules satisfy the best response property. Given its rival's price schedule, a firm is faced with an optimal non-linear pricing problem that can be solved by invoking the Revelation Principle and thus focusing on direct mechanisms. The strategy of the proof is to show that for each firm $i = A, B$ the optimal quantities $q_i(\theta)$, given $P_{-i}(q_{-i})$, coincide with the equilibrium quantities reported above. It is then straightforward to conclude that the price schedules that support these quantities must be equilibrium price schedules.

Given $P_{-i}(q_{-i})$, firm i faces a monopolistic screening problem where type θ has an indirect payoff function

$$v^i(q_i, \theta) = \max_{q_{-i} \geq 0} \left[u(q_i, q_{-i}, \theta) - P_{-i}(q_{-i}) \right],$$

which accounts for any benefit he can obtain by optimally trading with its rival. Since u is quadratic and $P_{-i}(q_{-i})$ is piecewise quadratic, v_i is also piecewise quadratic. It may have kinks, but we shall show that any such kink preserve concavity, so the indirect payoff function is globally concave.

Provided that the single-crossing condition holds, firm i's problem reduces to finding a function that pointwise maximises the "indirect virtual surplus"

$$s^i(q_i, \theta) = v^i(q_i, \theta) - c_i q_i - (1-\theta) v_\theta^i,$$

where c_i is zero for $i = A$ and c for $i = B$. It is then easy to verify ex post that the maximiser $q_i(\theta)$ satisfies the monotonicity condition.

Consider, then, firm A's best response to the equilibrium price schedule of firm B, $P_B(q_B)$. The indirect payoff function is piecewise quadratic, with two branches corresponding to the case in which $\arg\max_{q_B \geq 0} \left[u(q_A, q_B, \theta) - P_B(q_B) \right]$ is 0 or is strictly positive, and a kink between the two branches:

$$v^A(q_A, \theta) = \begin{cases} \theta q_A - \dfrac{1-\gamma}{2} q_A^2 & \text{if } q_B = 0 \text{ or, equivalently, } q_A \geq q^{\lim}(\theta) \\ A_0 + A_1 q_A + A_2 q_A^2 & \text{if } q_B > 0 \text{ or, equivalently, } q_A < q^{\lim}(\theta). \end{cases}$$

The coefficients A_0, A_1 and A_2 can be calculated as

$$\begin{aligned} A_0 &= \frac{\left[(\theta - c)(1-2\gamma) - \alpha(1 - c(1-\gamma) - 2\gamma)\right]^2}{2(1-\gamma-\alpha)(1-2\gamma)^2}, \\ A_1 &= \gamma \frac{c(1-2\gamma) + \alpha(1 - c(1-\gamma) - 2\gamma)}{(1-\gamma-\alpha)(1-2\gamma)} + \theta \frac{1-2\gamma-\alpha}{1-\gamma-\alpha} \\ A_2 &= -\frac{1-2\gamma+\alpha(1-\gamma)}{2(1-\gamma-\alpha)} < 0. \end{aligned}$$

On both branches of the indirect payoff function, the coefficients of the quadratic terms are negative. In addition, it can be easily checked that

$$\left. \frac{\partial v^A(q_A, \theta)}{\partial q_A} \right|_{q_A \leq q_A^{\lim}(\theta)} \geq \left. \frac{\partial v^A(q_A, \theta)}{\partial q_A} \right|_{q_A > q_A^{\lim}(\theta)},$$

so the function $v^A(q_A,\theta)$ is globally concave in q_A. It can also be checked that the sorting condition $\frac{\partial^2 v^A}{\partial\theta\partial q_A} > 0$ is satisfied as

$$\frac{\partial^2 v^A}{\partial\theta\partial q_A} = \begin{cases} 1 & \text{if } q_A \geq q_A^{\lim}(\theta) \\ \dfrac{1-2\gamma-\alpha}{1-\gamma-\alpha} > 0 & \text{if } q_A < q_A^{\lim}(\theta). \end{cases}$$

We can therefore obtain A's best response by pointwise maximising the virtual surplus function $s^A(q_A,\theta)$. Like the indirect payoff function, the virtual surplus function is piecewise quadratic with a kink. The maximum can occur in either one of the two quadratic branches, or at the kink. To be precise:

$$\arg\max_{q_A(\theta)}[\sigma^A(q_A,\theta)] = \begin{cases} \dfrac{2\theta-1}{1-\gamma} & \text{if } \gamma < \frac{1}{3} \text{ and } \frac{1}{2} \leq \theta \leq \theta^{\lim} \\ \dfrac{\theta - P_B'^{cr}(0)\alpha}{\gamma} & \begin{array}{l} \text{if } \gamma < \frac{1}{3} \text{ and } \theta^{\lim} \leq \theta \leq \breve{\theta}_B \\ \text{or if } \gamma \geq \frac{1}{3} \text{ and } P_B'^{cr}(0) \leq \theta \leq \breve{\theta}_B \end{array} \\ \dfrac{\theta-\alpha}{1-\alpha} + \dfrac{c\gamma}{1-2\gamma} & \text{if } \theta \geq \breve{\theta}_B. \end{cases}$$

But these are precisely the monopoly, limit-pricing and common representation quantities. Note also that the case in which $\gamma < \frac{1}{3}$ and $\frac{1}{2} \leq \theta \leq \theta^m$ cannot arise if $c < \tilde{c}$. In this case, the optimum is never achieved on the upper branch of the indirect payoff function; in other words, firm A's best response never involves setting the quantity at the monopoly level. It is therefore apparent that firm A's best response is to offer precisely the equilibrium quantities. This can be achieved by offering the equilibrium price schedules. This verifies that firm A's equilibrium price schedule satisfies the best response property.

Consider now firm B. The procedure is the same as for firm A, but now we must distinguish between two cases, depending on whether A's price schedule comprises the lowest, monopoly branch or not.

Consider first the case in which there is no monopoly branch in A's price schedule. The indirect payoff function of a buyer when trading with firm B then is

$$v^B(q_B,\theta) = \begin{cases} \theta q_B - \frac{1-\gamma}{2} q_B^2 & \text{if } q_B \geq q_B^{\lim}(\theta) \\ \hat{B}_0 + \hat{B}_1 q_B + \hat{B}_2 q_B^2 & \text{if } \check{q}_B(\theta) \leq q_B < q_B^{\lim}(\theta) \\ B_0 + B_1 q_B + B_2 q_B^2 & \text{if } 0 < q_B \leq \check{q}_B(\theta) \end{cases}$$

where

$$\begin{aligned} q_B^{\lim}(\theta) &= \frac{\theta-\alpha}{\gamma} - \frac{\alpha c}{1-2\gamma} \\ \check{q}_B(\theta) &= \frac{\theta - \alpha - c(1-\alpha)}{\gamma} + \frac{\alpha c}{1-2\gamma}. \end{aligned}$$

The first branch corresponds to firm B acting as a monopolist. Along the second branch, firm B competes against firm A's limit-pricing price schedule. Clearly, neither case can occur in equilibrium. Finally, the third branch corresponds to the case in which firm A accommodates.

The coefficients of the lower branches of the indirect payoff functions are

$$\hat{B}_0 = \frac{(\theta - c)^2}{2\gamma}; \quad \hat{B}_1 = c; \quad \hat{B}_2 = -\frac{1-2\gamma}{2}$$

and

$$B_0 = \frac{2\theta - 1}{2(1-\gamma)}; \quad B_1 = \theta - \frac{\gamma}{1-\gamma}; \quad B_2 = -\frac{1-\gamma}{2}.$$

All branches are concave. Global concavity can be checked by comparing the left and right derivatives of $v^B(q_B, \theta)$ at the kinks, as we did for firm A. The sorting condition can also be checked as we did for firm A. We can therefore find B's best response by pointwise maximisation of the virtual surplus function.

It is easy to verify that there is never an interior maximum on the upper or intermediate branch of the virtual surplus function. This is equivalent to the intuitive result that firm B is active only when firm A supplies the common representation quantity $q_A^{cr}(\theta)$. Pointwise maximisation of the relevant branch of virtual surplus function then leads to

$$\arg\max[\sigma^B(q_B, \theta)] = \frac{\theta - \alpha}{1-\alpha} - c\frac{1-\gamma}{1-2\gamma}.$$

This coincides with $q_B^{cr}(\theta)$, thereby confirming that the equilibrium price schedule $P_B(q_B)$ is indeed its best response to firm A's strategy.

The case where firm A's price schedule comprises also the monopoly branch is similar. The indirect payoff function $v^B(q_B, \theta)$, and hence the virtual surplus $s^B(q_B, \theta)$, now comprise four branches (all quadratic). The equation of the fourth branch, which corresponds to $0 < q_A < q^m(\theta)$, is

$$v^B(q_B, \theta) = \tilde{B}_0 + \tilde{B}_1 q_B + \tilde{B}_2 q_B^2$$

where

$$\tilde{B}_0 = \frac{(2\theta - 1)^2}{4(1-\gamma)}; \quad \tilde{B}_1 = \frac{\theta + \gamma(1-3\gamma)}{1-\gamma}; \quad \tilde{B}_2 = -\frac{1-\gamma(2+\gamma)}{2(1-\gamma)}.$$

However, it turns out that the optimum still lies on the same branch as before and that it therefore entails a quantity equal to $q_B^{cr}(\theta)$. This observation completes the proof of the proposition. ■

Exclusive contracts

With exclusive contracts, there is scope for multiple equilibria, for reasons discussed in the main text. However, when the competitive advantage is large (to be precise, the threshold $\check{c}$ is $\frac{2(1-2\gamma)}{5(1-\gamma)+\sqrt{1-2\gamma+9\gamma^2}}$) there is only one equilibrium in undominated strategies.[55] We therefore start from this simpler case, i.e. $c \geq \check{c}$.

[55] The other equilibria involve exorbitant non-exclusive prices, whereas the exclusive prices are $P_B^E(q_B) = cq_B$ and $P_A^E(q_A) = \min[P^m(q_A), cq_A\ ]$.

The equilibrium is similar to the equilibrium of the competitive model in the large competitive advantage case. Like in the compeptitive fringe model, the non-exclusive tariffs coincide with the common representation tariffs that arise in the non-linear pricing equilibrium, except for constant terms. These constant terms, which we denote by Φ_A and Φ_B, are determined as follows.

Let $\hat{\theta}$ denote the critical buyer $\hat{\theta}$ who is just indifferent between exclusive and non-exclusive contracts. For this buyer, the following condition must hold

$$u(q_A^E(\hat{\theta}),0,\hat{\theta})-P_A^E(q_A^E(\hat{\theta})) = u(q_A^{cr}(\hat{\theta}),q_B^{cr}(\hat{\theta}),\hat{\theta})-P_A^{cr}(q_A^{cr}(\hat{\theta}))-P_B^{cr}(q_B^{cr}(\hat{\theta})-\Phi_A-\Phi_B. \tag{A2.1}$$

Clearly, an increase in Φ_i will increase $\hat{\theta}$.

Intuitively, when choosing Φ_A and Φ_B, both firms are trading off market share and profitability. Consider, for instance, firm B. Since its exclusive contracts are not accepted (and in any case would not be profitable), it must try to induce more high-demand buyers, who value product variety more highly, to reject the exclusive contracts offered by firm A and buy both products. To get such buyers to purchase both products, firm B must lower its non-exclusive prices by adding a negative term (a lump-sum subsidy) to the tariff $P_B^{cr}(q)$. Firm A, by contrast, will add a fixed fee to the tariff $P_A^{cr}(q)$. The fixed fee is sufficiently large that the dominant firm earns more, at the margin, from buyers who choose common representation than from those who choose exclusive dealing like in the competitive fringe model.

More formally, firm A's profit is

$$\int_{\tilde{\theta}}^{\hat{\theta}} P_A^E(q_A^E(\theta))d\theta + \int_{\hat{\theta}}^{1} \left[P_A^{cr}(q_A^{cr}(\theta)) + \Phi_A\right] d\theta,$$

and firm B's is

$$\int_{\hat{\theta}}^{1} \left[P_B^{cr}(q_B^{cr}(\theta)) - cq_B^{cr}(\theta) + \Phi_B\right] d\theta.$$

Since $\hat{\theta}$ is determined by (A2.1), the equilibrium conditions for Φ_A and Φ_B are:

$$\frac{P_A^{cr}(q_A^{cr}(\hat{\theta})) + \Phi_A - P_A^E(q_A^E(\hat{\theta}))}{q_A^{cr}(\hat{\theta}) + q_B^{cr}(\hat{\theta}) - q_A^E(\hat{\theta})} = 1-\hat{\theta}, \tag{A2.2}$$

$$\frac{P_B^{cr}(q_B^{cr}(\hat{\theta})) + \Phi_B - cq_B^{cr}(\hat{\theta})}{q_A^{cr}(\hat{\theta}) + q_B^{cr}(\hat{\theta}) - q_A^E(\hat{\theta})} = 1-\hat{\theta}. \tag{A2.3}$$

Conditions (A2.2) and (A2.3) are the duopoly counterpart of condition (11) in the competitive fringe model. The economic intuition is similar. It can be confirmed that in equilibrium $\Phi_A > 0$, $\Phi_B < 0$ and $\Phi_A + \Phi_B > 0$.

We are now ready to provide the characterisation of the equilibrium when exclusive contracts are permitted and the dominant firm's competitive advantage is large.

Proposition 4 *The following is an equilibrium in the duopoly model when firms can use exclusive contracts and the dominant firm's competitive advantage is large, i.e.* $c > \check{c}$.[56]

- *When* $\check{c} < c < c^m$ *the two firms offer the following exclusive price schedules*

$$\begin{aligned} P_B^E(q) =& \quad cq \\ P_A^E(q) =& \quad \begin{cases} cq & \text{for } q \leq q^e(\theta^m) \\ P_A^m(q) + \ \text{constant} & \text{for } q > q^e(\theta^m) \end{cases} \end{aligned}$$

where θ^m *is such that* $q^e(\theta^m) = q_A^m(\theta^m)$ *and the constant guarantees the continuity of the price schedule, and the following non-exclusive price schedules*

$$\begin{aligned} P_A^{NE}(q) =& \quad P_A^{cr}(q) + \Phi_A \qquad \text{for } q \geq q_A^{cr}(\hat{\theta}) \\ P_B^{NE}(q) =& \quad P_B^{cr}(q) + \Phi_B \qquad \text{for } q \geq q_B^{cr}(\hat{\theta}) \end{aligned}$$

where $\hat{\theta}$, Φ_A *and* Φ_B *are the solution to system (A2.1)-(A2.3).*

- *When* $c \geq c^m$ *the two firms offer the following price schedules*

$$P_A^E(q) = P^m(q)$$

(firm B may not offer any exclusive contract at all), and

$$\begin{aligned} P_A^{NE}(q) =& \quad P_A^{cr}(q) + \Phi_A \quad \text{for } q \geq q_A^{cr}(\hat{\theta}) \\ P_B^{NE}(q) =& \quad P_B^{cr}(q) + \Phi_B \quad \text{for } q \geq q_B^{cr}(\hat{\theta}) \end{aligned}$$

where $\hat{\theta}$, Φ_A *and* Φ_B *are defined as in the previous case.*

To avoid repetitions, it is convenient to take up this proposition after Proposition 4.

Now consider the case in which the dominant firm's competitive advantage is small ($c \leq \check{c}$). In this case, there is a multiplicity of equilibria that arises because the firms may or may not succeed in coordinating their strategies so as to extract the preference for variety and reduce the intensity of competition.

To understand the coordination problems that the firms face, consider the outcome of the competition for exclusives: firm B prices at cost, and firm A just undercuts it. Clearly, this is always a possible equilibrium. When the competitive advantage is small, however, both firms can obtain larger profits. This requires that the firms lower their non-exclusive prices in coordinated fashion, inducing some buyers to purchase both products. This move allows firms to extract the buyers' preference for variety.

[56] As we have already noted, if there were different equilibrium price schedules under common representation, $P_i^{cr}(q)$, for each of them there would be corresponding equilibria with exclusive contracts with the same structure as that described in Proposition 4. The same remark applies also to Proposition 5 below.

Notice that this coordination cannot take place when the competitive advantage is large. The reason for this is that the payoff function u by itself always entails a preference for variety, but the fact that $c > 0$ means that exclusion may be efficient. In particular, when demand is low efficiency requires that only good A must be produced. In other words, there is room for extracting the preference for variety only if the competitive advantage is not too large. The condition is precisely $c < \breve{c}$.

Assuming, then, that $c < \breve{c}$, notice that if firms manage to coordinate their non-exclusive prices as described above, a new opportunity of coordination arises. Since certain exclusive contracts will no longer be accepted in equilibrium, firms have no longer an incentive to undercut one another's exclusive prices; therefore, they can also increase exclusive prices so as to reduce the intensity of competition.

However, in all equilibria the effect of exclusive contracts is to reduce prices and profits. For brevity, we shall not provide a complete characterisation of the set of equilibria. Our objective here is to confirm that exclusive contracts are procompetitive when the competitive advantage is sufficiently small. To this end, we shall focus exclusively on the "most cooperative" equilibrium, where prices and profits are largest (given that the firms are actually playing a non-cooperative game). In such an equilibrium, the exclusive and non-exclusive price schedules must be determined simultaneously.

The conditions that must be satisfied in the most cooperative equilibrium are the following. Let $U^E(\theta)$ be the (type-dependent) reservation utility that buyer θ could obtain by choosing his most preferred exclusive contract. To extract the buyer's preference for variety, the firms must introduce non-exclusive price schedules implicitly defined by the condition:

$$\max_{q_A, q_B} \left[u(q_A, q_B, \theta) - P_A^{NE}(q_A) - P_B^{NE}(q_B) \right] = U^E(\theta).^{57} \tag{A2.4}$$

These price schedules apply to low-demand buyers; high-demand buyers will actually obtain more than $U^E(\theta)$ simply thanks to the competition in non-exclusive contracts. Notice that equation (A2.4) does not pin down $P_A^{NE}(q_A)$ and $P_B^{NE}(q_B)$ uniquely. This reflects the fact that the preference for variety can be split between the two firms in different ways. All that matters is that the total payment requested by the firms does not exceed what the buyer is willing to pay in order to purchase both goods. Since we look for the equilibrium in which firms' profits are largest, we shall focus on the case in which the firms maximise the rents that they extract. This requires maximisation of the total surplus $u(q_A, q_B, \theta) - cq_B$, subject to the constraint that buyers must obtain $U^E(\theta)$. Using the envelope theorem, the constraint can be rewritten as

$$q_A(\theta) + q_B(\theta) = q^E(\theta), \tag{A2.5}$$

where $q^E(\theta)$ is the optimal quantity under exclusivity. Notice that $q^E(\theta)$ de-

[57] Again, to avoid issue of equilibrium existence we assume that when buyers are indifferent in monetary terms, they prefer to purchase both goods.

pends on what exclusive prices are sustainable in the most cooperative equilibrium and hence must be determined jointly with all other variables.

Generally speaking, the more efficient firm must produce more than the less efficient one. In particular, the problem of total-surplus maximisation may have a corner solution in which some low-demand types must buy good A only. In this case, exclusive contracts must be accepted in equilibrium by those types, and so competition in utility space implies that exclusive prices must fall to c. Therefore, for low-demand types $q_A(\theta)$ must coincide with $q^e(\theta)$, and $q_B(\theta)$ must vanish.

When instead the total-surplus maximisation problem has an interior solution, which is

$$q_A(\theta) = \frac{1}{2}q^E(\theta) + \frac{c}{2(1-2\gamma)}; \quad q_B(\theta) = \frac{1}{2}q^E(\theta) - \frac{c}{2(1-2\gamma)}, \tag{A2.6}$$

buyers purchase both products. The corresponding exclusive contracts are not actually accepted in equilibrium, and so there may be room for coordinating also the exclusive prices. The reason for this is that exclusive contracts affect the equilibrium outcome even if they are not accepted. The less aggressively firms bid for exclusivity, the lower the buyer's payoff under exclusive dealing, and hence the greater the payments firms can obtain for non-exclusive contracts. Thus, raising the exclusive prices is good for the firms' profits.

Let us denote by an upper bar the highest exclusive prices that can be sustained in a non-cooperative equilibrium. To find them, we can assume, with no loss of generality, that both firms offer the same exclusive price schedule $\bar{P}^E(q)$.[58] By construction, low-type buyers must be just indifferent between exclusive and non-exclusive contracts (equation (A2.4)). Thus, any arbitrarily small discount would trigger a switch to an exclusive contract. In equilibrium, no such deviation can be profitable. This implies the following no undercutting conditions:

$$\begin{aligned} P^E(q^E(\theta)) &\leq P_A^{NE}(q_A^{cr}(\theta)); \\ P^E(q^E(\theta)) - cq^E(\theta) &\leq P_B^{NE}(q_B^{cr}(\theta)) - cq_B^{cr}(\theta), \end{aligned} \tag{A2.7}$$

which in the most cooperative equilibrium must hold as equalities.

The most cooperative equilibrium is found by solving the system of equations (A2.4)-(A2.7). Specifically, denote by $\bar{q}^E(\theta)$ the optimal quantity associated with the exclusive prices $\bar{P}^E(q)$, and by $\bar{q}_i^{cr}(\theta)$ the values of $q_i(\theta)$ given by (A2.6) when $q^E(\theta) = \bar{q}^E(\theta)$. Rewrite (A2.4) as

$$u(\bar{q}_A^{cr}(\theta), \bar{q}_B^{cr}(\theta), \theta) - \bar{P}_A^{NE}\left(\bar{q}_A^{cr}(\theta)\right) - \bar{P}_B^{NE}\left(\bar{q}_B^{cr}(\theta)\right) = u(0, \bar{q}^E(\theta), \theta) - \bar{P}^E\left(\bar{q}^E(\theta)\right)$$

and use the no-undercutting conditions (A2.7) to get

$$\bar{P}^E\left(q^E(\theta)\right) = \left[u(\bar{q}_A^{cr}(\theta), \bar{q}_B^{cr}(\theta), \theta) - u(0, \bar{q}^E(\theta), \theta)\right] + c\left[\bar{q}^E(\theta) - \bar{q}_B^{cr}(\theta)\right].$$

[58] We can prove that this does not entail any loss of generality by contradiction. Suppose to the contrary that one firm offered more attractive exclusive contracts than its rival. Since these contracts are not accepted in equilibrium, the firm could increase its exclusive prices without losing any profits on its exclusive contracts. In fact, the buyers' reservation utility would decrease, allowing both firms to increase their profits from non-exclusive contracts.

The term inside the first square brackets on right-hand side can be interpreted as the preference for variety, while the term inside the second square bracket is the cost saving. Using (A2.6), we finally get

$$\bar{P}^E(q) = \frac{c^2}{2(1-2\gamma)} + \frac{c}{2}q + \frac{1-2\gamma}{4}q^2, \tag{A2.8}$$

and

$$\bar{P}^{cr}_A(q_A) = -cq + (1-2\gamma)q^2 + cq^e(\hat{\theta}); \quad \bar{P}^{cr}_B(q_B) = 2cq + (1-2\gamma)q^2, \tag{A2.9}$$

where $\hat{\theta}$ is now the solution to $q^e(\hat{\theta}) = \bar{q}^{cr}_A(\hat{\theta})$ and the constant term in $\bar{P}^{cr}_A(q_A)$ guarantees smooth pasting from exclusive to non-exclusive contracts. The corresponding quantities are

$$\bar{q}^E(\theta) = \frac{2\theta - c}{3-4\gamma}, \tag{A2.10}$$

and

$$\bar{q}^{cr}_A(\theta) = \frac{2\theta - c}{2(3-4\gamma)} + \frac{c}{2(1-2\gamma)}; \quad \bar{q}^{cr}_B(\theta) = \frac{2\theta - c}{2(3-4\gamma)} - \frac{c}{2(1-2\gamma)}. \tag{A2.11}$$

We are now ready to provide the characterisation of the most cooperative equilibrium.

Proposition 5 *Suppose that the dominant firm's competitive advantage is small: $c \leq \check{c}$. Then, in the duopoly model the most cooperative equilibrium with exclusive contracts is as follows. Both firms offer the exclusive price schedules*

$$P^E_A(q) = P^E_B(q) = \begin{cases} cq & \text{for } q \leq q^e(\hat{\theta}) \\ \bar{P}^E(q) & \text{for } q > q^e(\hat{\theta}) \end{cases}$$

with firm A slightly undercutting firm B, though. Furthermore:

$$P^{NE}_A(q) = \begin{cases} \bar{P}^{cr}_A(q) & \text{for } q \leq \bar{q}^{cr}_A(\bar{\theta}) \\ P^{cr}_A(q) + \text{constant} & \text{for } q \geq \bar{q}^{cr}_A(\bar{\theta}) \end{cases}$$
$$P^{NE}_B(q) = \begin{cases} \bar{P}^{cr}_B(q) & \text{for } q \leq \bar{q}^{cr}_B(\bar{\theta}) \\ P^{cr}_B(q) + \text{constant} & \text{for } q \geq \bar{q}^{cr}_B(\bar{\theta}) \end{cases}$$

where $\hat{\theta}$ is the solution to $q^e(\hat{\theta}) = \bar{q}^{cr}_A(\hat{\theta})$ and $\bar{\theta}$ the solution to $\bar{q}^{cr}_A(\bar{\theta}) = q^{cr}_A(\bar{\theta})$ (and to $\bar{q}^{cr}_B(\bar{\theta}) = q^{cr}_B(\bar{\theta})$), and the constants guarantee the continuity of the price schedules.

Proof. The equilibrium quantities are:

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q^e(\theta) & \text{for } c \leq \theta \leq \hat{\theta} \\ \bar{q}^{cr}_A(\theta) & \text{for } \hat{\theta} \leq \theta \leq \bar{\theta} \\ q^{cr}_A(\theta) & \text{for } \bar{\theta} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ \bar{q}^{cr}_B(\theta) & \text{for } \hat{\theta} \leq \theta \leq \bar{\theta} \\ q^{cr}_B(\theta) & \text{for } \bar{\theta} \leq \theta \leq 1, \end{cases}$$

where $\hat{\theta}$ and $\bar{\theta}$, which are defined in the text of the Proposition, are given by

$$\begin{aligned}\hat{\theta} &= \frac{c(2-3\gamma)}{1-2\gamma},\\ \bar{\theta} &= \frac{c(1-2\gamma)+\alpha[3-c-2(2-c)\gamma]}{\alpha+2(1-2\gamma)}.\end{aligned}$$

The claim that this is the most cooperative equilibrium has justified above. Here, we just verify that this is indeed an equilibrium of the game. The logic of the proof is the same as for Proposition 3. We must show that for each firm the equilibrium price schedules satisfy the best response property. When calculating the best response, we take $\{P_{-i}^{E}(q), P_{-i}^{NE}(q)\}$ as given. Hence, we can invoke the Revelation Principle and focus on direct mechanisms. Proceeding this way, we must show that for each firm $i = A, B$ the optimal quantities $q_i(\theta)$ coincide with the equilibrium quantities reported above. It is then straightforward to conclude that the price schedules $P_i^E(q), P_i^{NE}(q)$ that support these quantities must be equilibrium price schedules.

Given its rival's exclusive and non exclusive price schedules, a firm must solve a monopolistic screening problem in which the buyer has an indirect payoff function

$$v^i(q_i,\theta) = \max_{q_{-i}\geq 0}\left[u\left(q_i, q_{-i}, \theta\right) - P_{-i}^{NE}(q_{-i})\right],$$

and a reservation utility

$$U_i^R(\theta) = \max_{q_{-i}}\left[u(0, q_{-i}, \theta) - P_{-i}^{E}(q_{-i})\right].$$

Since firm i can impose exclusivity clauses, it must solve a hybrid optimal control problem in which the two control systems are

$$\begin{aligned} &\max_{q_i}\int\left[v^i(q_i,\theta) - U(\theta) - c_i q_i\right]d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= v_\theta^i(q_i,\theta) \\ U(\theta) &\geq U_i^R(\theta) \end{aligned} \tag{A1.12}$$

if $q_{-i}(\theta) > 0$, and

$$\begin{aligned} &\max_{q_i}\int\left[u(q_i,0,\theta) - U(\theta) - c_i q_i\right]d\theta \\ \text{s.t. } \frac{dU}{d\theta} &= u_\theta(q_i,0,\theta) \\ U(\theta) &\geq U_i^R(\theta) \end{aligned} \tag{A2.13}$$

if $q_{-i}(\theta) = 0$. In both cases, $q_i(\theta)$ must be non-decreasing.

Problem (A2.13) is relevant only for the dominant firm. When it sets $q_B(\theta) = 0$, noting that problem (A2.13) coincides with problem (A.5) in the proof of

Proposition 2, we can apply Lemma 1 and conclude that

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \le c \\ q^e(\theta) & \text{for } c \le \theta \le 1-c \\ q_A^m(\theta) & \text{for } \theta \ge 1-c. \end{cases}$$

It is then easy to verify that $\hat{\theta}$ is now lower than $1-c$, so the only relevant part of the solution is $q^e(\theta)$.

Consider now problem (A2.12). Several properties of the solution to this problem must hold for both firms. By construction, the indirect payoff functions $v^i(q_i, \theta)$ are almost everywhere differentiable. At any point where the derivatives exist, by the envelope theorem we have

$$v_\theta^i(q_i, \theta) = q_i + \tilde{q}_{-i}(q_i, \theta),$$

where

$$\tilde{q}_{-i}(q_i, \theta) = \arg\max_{q_{-i}>0} \left[u\left(q_i, q_{-i}, \theta\right) - P_{-i}^{NE}(q_{-i})\right]$$

Generally speaking, the indirect payoff functions $v^i(q_i, \theta)$ have two branches, according to whether $\tilde{q}_{-i}(q_i, \theta) \le \bar{q}_{-i}(\bar{\theta})$ or $\tilde{q}_{-i}(q_i, \theta) \ge \bar{q}_{-i}(\bar{\theta})$ respectively. When $\tilde{q}_{-i}(q_i, \theta) \le \bar{q}_{-i}(\bar{\theta})$, we have $P_{-i}^{NE}(q_{-i}) = \bar{P}_{-i}^{cr}(q_{-i})$. When $\tilde{q}_{-i}(q_i, \theta) \ge \bar{q}_{-i}(\bar{\theta})$, we have $P_{-i}^{NE}(q_{-i}) = P_{-i}^{cr}(q_{-i})$ (plus a constant).

The indirect payoff functions $v^i(q_i, \theta)$ are continuous, almost everywhere differentiable, and satisfy $v_{\theta q_i}^i(q_i, \theta) > 0$. Continuity and a.e. differentiability follows directly from the definition of $v^i(q_i, \theta)$. To prove the sorting condition, observe that

$$v_{\theta q_i}^i(q_i, \theta) = 1 - \gamma \frac{\partial \tilde{q}_{-i}(q_i, \theta)}{\partial q_i} \ge 0.$$

Consider the two branches of the indirect payoff function in turn. When $\tilde{q}_{-i}(q_i, \theta) \le \bar{q}_{-i}(\bar{\theta})$,

$$v_{\theta q_i}^i(q_i, \theta) = 1 + \frac{\partial \tilde{q}_{-i}(q_i, \theta)}{\partial \theta}(-\gamma)^2 = \frac{3-6\gamma}{3-5\gamma} > 0.$$

When instead $\tilde{q}_{-i}(q_i, \theta) \ge \bar{q}_{-i}(\bar{\theta})$ the sorting condition is immediately verified since

$$v_{\theta q_i}(q_i, \theta) = \frac{1-\alpha-2\gamma}{1-\alpha-\gamma} \ge 0.$$

Now consider problem (A2.12). Because of the type-dependent participation constraint, following Jullien (2000) we define the modified virtual surplus function:

$$\sigma^i(g, q_i, \theta) = v^i(q_i, \theta) - (g - \theta)\, v_\theta^i(q_i, \theta)$$

where the "weight" $g \in [0, 1]$ accounts for the possibility that the participation constraint may bind for a whole set of types. Let

$$\ell_i(g, \theta) = \arg\max_{q_i \ge 0} \sigma^i(g, q_i, \theta)$$

be the maximiser of the modified virtual surplus function. This solution is still in implicit form, as it depends on the value of g, which is still to be determined. This can be done by exploiting Proposition 5.5 of Jullien (2000).

To apply that Proposition, we first prove the following lemma.

Lemma 3 *Problem (A2.12) satisfies the conditions of Potential Separation, Homogeneity and Weak Convexity.*

Proof. Potential Separation requires that $\ell_i(g,\theta)$ is non-decreasing in θ. This follows from the fact that the modified virtual surplus function has increasing differences. To show this, consider each branch of the indirect payoff function separately. First, when $\tilde{q}_{-i}(q_i,\theta) \leq \bar{q}_{-i}(\bar{\theta})$ we have

$$\sigma^i_{q_i\theta}(q_i,\theta) = v^i_{q_i\theta}(q_i,\theta) - \left[1 + \frac{\partial \tilde{q}_{-i}(q_i,\theta)}{\partial q_i}\right] \frac{d}{d\theta}\left(g - \theta\right).$$

The first term is positive, as we have just shown. The second term is positive because $\frac{d}{d\theta}\left(g-\theta\right) < 0$ and

$$1 + \frac{\partial \tilde{q}_{-i}}{\partial q_i} = \frac{1-2\gamma}{3-5\gamma} > 0.$$

Second, when $\tilde{q}_{-i}(q_i,\theta) \geq \bar{q}_{-i}(\bar{\theta})$ the indirect payoff function coincides, modulo a constant, with the one arising in the equilibrium with non-linear pricing. In this case, it is immediate to show that $\sigma^i_{q_i\theta}(q_i,\theta) > 0$. This completes the proof that problem (A2.12) satisfies the condition of Potential Separation.

Homogeneity requires that $U^R_i(\theta)$ can be implemented by a continuous and non decreasing quantity. This is obvious, since $U^R_i(\theta)$ is implemented by $q^E(\theta)$, where $q^E(\theta)$ is the optimal quantity given the exclusive price schedule $P^E_{-i}(q)$:

$$q^E(\theta) = \begin{cases} q^e(\theta) & \text{if } \theta \leq \hat{\theta} \\ \bar{q}^E(\theta) & \text{if } \theta > \hat{\theta}. \end{cases}$$

To prove Weak Convexity, we first show that $\ell_i(0,\theta) + \tilde{q}_{-i}(\ell_i(0,\theta),\theta) \geq q^E(\theta)$ for all $\theta \in [0,1]$. By definition,

$$\ell_i(0,\theta) = \arg\max_{q_i} \left[v^i(q_i,\theta) + \theta v^i_\theta(q_i,\theta)\right].$$

Thus, $\ell_i(0,\theta)$ is implicitly defined by the first order condition

$$v^i_{q_i}(q_i,\theta) + \theta v^i_{\theta q_i}(q_i,\theta) = 0.$$

Since $v^i_{\theta q_i}(q_i,\theta) > 0$, this implies that $v^i_{q_i}(q_i,\theta) < 0$, or $u_{q_i}(q_i,\tilde{q}_{-i}(q_i,\theta),\theta) < 0$. In other words, $\ell_i(0,\theta)$ exceeds the satiation consumption $u_{q_i}(q_i,\tilde{q}_{-i}(q_i,\theta),\theta) = 0$. The quantity $q^E(\theta)$, on the contrary, is lower than the satiation consumption. It follows that $\ell_i(0,\theta) + \tilde{q}_{-i}(\ell_i(0,\theta),\theta) \geq q^E(\theta)$.

In addition, Weak Convexity requires that the curve $q^E(\theta)$ cuts the curve $\ell_i(1,\theta) + \tilde{q}_{-i}(\ell_i(1,\theta),\theta) = q_A^{cr}(\theta) + q_B^{cr}(\theta)$ from above. Noting that $\ell_i(1,\theta) = q_i^{cr}(\theta)$, the fact that $q^E(\theta)$ can only cut the curve $q_A^{cr}(\theta) + q_B^{cr}(\theta)$ from above as

$$\frac{d[q_A^{cr}(\theta) + q_B^{cr}(\theta)]}{d\theta} \geq \frac{dq^E(\theta)}{d\theta},$$

irrespective of whether $q^E(\theta)$ is $q^e(\theta)$ or $\bar{q}^E(\theta)$. This finally proves Weak Convexity and hence the lemma. ■

With these preliminary results at hand, let us now consider the dominant firm's problem. The solution when $q_B(\theta) = 0$ has been already characterised. If $q_B(\theta) > 0$, Proposition 5.5 in Jullien (2000) guarantees that generally speaking the solution partitions the set of types into three sets: buyers who are excluded, buyers who obtain their reservation utility $U_A^R(\theta)$, and buyers whose payoff is strictly greater than $U_A^R(\theta)$. Clearly, the first set is always empty: if $q_B(\theta) > 0$, we always have $q_A(\theta) > 0$.

Next consider the second group of buyers. When the participation constraint binds, firm A can guarantee to each low-type buyer his reservation utility $U_A^R(\theta)$ in two ways. First, it can offer an exclusive price schedule that just matches that of firm B. Alternatively, it can implement, *via* non-exclusive prices, the quantities that satisfy the condition

$$\bar{q}_A^{cr}(\theta) + \bar{q}_B^{cr}(\theta) = \bar{q}^E(\theta),$$

which by the envelope theorem guarantees that the participation constraint is met as an equality. The maximum payment that firm A can requested for $\bar{q}_A^{cr}(\theta)$ is

$$\bar{P}_A^{cr}\left(\bar{q}_A^{cr}(\theta)\right) = -c\bar{q}_A^{cr}(\theta) + (1-2\gamma)\left[\bar{q}_A^{cr}(\theta)\right]^2 + cq^e(\hat{\theta}).$$

The second strategy is at least as profitable as the first one if

$$\bar{P}_A^{cr}\left(\bar{q}_A^{cr}(\theta)\right) \geq \bar{P}^E(\bar{q}^E(\theta)),$$

which is precisely the no-undercutting condition (A2.7), which holds by construction. This shows that offering $\bar{P}_A^{cr}(q_A)$ is indeed a best response for firm A when the participation constraint is binding.

Finally, when the participation constraint does not bind, the solution to firm A's program is obtained simply by setting $g = 1$. Assume that $\ell_A(1,\theta) \geq \bar{q}_A^{cr}(\bar{\theta})$ when $\theta > \hat{\theta}$ (this will be proven shortly). Since the modified virtual surplus function $\sigma_A(1, q_A, \theta)$ is exactly the same as in the non-linear pricing equilibrium, modulo a constant, the maximisers of the virtual surplus functions must coincide and the optimal quantity is

$$\ell_A(1,\theta) = q_A^{cr}(\theta).$$

Finally, the cutoff $\bar{\theta}$ is implicitly given by the condition

$$\bar{q}^E(\bar{\theta}) = \ell_A(1,\bar{\theta}) + \tilde{q}_B(\ell_A(1,\bar{\theta}),\bar{\theta}).$$

This also establishes that $\ell_A(1,\theta) \geq \bar{q}_A^{cr}(\bar{\theta})$ when $\theta > \hat{\theta}$.

To complete the verification of the best response property for firm A, it remains to consider the switch from exclusive to non-exclusive contracts. By the no-deviation condition (A2.7), which in the most cooperative equilibrium holds as an equality, firm A is just indifferent between imposing an exclusivity clause or not for all $\theta \leq \bar{\theta}$. Exclusive dealing arises just when $\bar{q}_B^{cr}(\theta) \leq 0$, which is equivalent to $\theta \leq \hat{\theta}$. Because firm A is indifferent between the exclusive and non-exclusive regimes, at the switching point a smooth-pasting condition must now hold, which implies that aggregate quantities must be continuous, and hence that $P_A^{NE}(\bar{q}_A^{cr}(\hat{\theta})) = P_A^E(\bar{q}_A^E(\hat{\theta}))$.

The problem faced by firm B is similar, except that firm B can never make a profit by selling under an exclusivity clause. Thus, we can focus on problem (A2.12). Proceeding as for firm A, one can show that the optimal quantity is $\bar{q}_B^{cr}(\theta)$ when the participation constraint $U(\theta) \geq U_A^R(\theta)$ is binding, and $q_B^{cr}(\theta)$ when it is not.

These arguments complete the proof that the solution to the problem of firm i coincides with $q_i(\theta)$ as shown in the text of the proposition. By construction, this solution can be implemented by firm i using the equilibrium price schedules $\left(P_i^E(q_i), P_i^{NE}(q_i)\right)$.

This solution is well defined when the three intervals $[c,\hat{\theta})$, $[\hat{\theta},\bar{\theta}]$ and $(\bar{\theta},1]$ are non-empty. This requires $c \leq \hat{\theta}$, $\hat{\theta} \leq \bar{\theta}$ and $\bar{\theta} \leq 1$. It is immediate to show that the first and the last of these inequality always hold. Thus, the solution is well defined if and only is $\hat{\theta} \leq \bar{\theta}$, which is equivalent to

$$c \leq \check{c} \equiv \frac{2(1-2\gamma)}{5(1-\gamma)+\sqrt{1-2\gamma+9\gamma^2}}. \quad \blacksquare$$

We can now finally proceed to the proof of Proposition 4.

Proof of Proposition 4. As usual, we start by reporting the equilibrium quantities, which are:

- when $\check{c} \leq c \leq c^m$,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq c \\ q^e(\theta) & \text{for } c \leq \theta \leq \theta^m \\ q_A^m(\theta) & \text{for } \theta^m \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1, \end{cases}$$

- when $c > c^m$

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q_A^m(\theta) & \text{for } \frac{1}{2} \leq \theta \leq \hat{\theta} \\ q_A^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta} \\ q_B^{cr}(\theta) & \text{for } \hat{\theta} \leq \theta \leq 1. \end{cases}$$

The strategy of the proof is the same as for Proposition 5. Many of the arguments are indeed the same as in previous proofs and thus need not be repeated here. In particular, notice that:

- first, when $c > \check{c}$, there is no longer any scope for coordinating exclusive prices (this was shown in the proof of Proposition 5. Hence, firm B always sets exclusive prices at the competitive level $P_B^E(q_B) = cq_B$. This implies that when firm A imposes an exclusivity clause, the buyers' reservation utility is exactly the same as in the competitive fringe model. It follows that the solution to problem (5) is still given by Lemma 3;

- second, without exclusivity the problems that are faced by the firms are exactly the same as in the proof of Proposition 3 when the participation constraint does not bind.

These remarks imply that Proposition 4 can be proved simply by combining arguments already presented in the proofs of Proposition 3 and Proposition 5. The only difference is that now the switch from the exclusive to the non-exclusive regime is the result of the interaction between the pricing choices of firm A and firm B. This point, however, has already been discussed in the main text, which shows that the equilibrium switching point must satisfy conditions (A2.1)-(A2.3). The explicit expressions for Φ_A and Φ_B are complicated and are reported in a Mathematica file that is available from the authors upon request. ■

References

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Annex 4
Market-share contracts

(NOT FOR PUBLICATION)

Since the dominant firm can freely condition its requested payment $P_A(q_A, q_B)$ on the competitive fringe's volume as well, now it effectively controls both quantities q_A, q_B. Firm A thus must solve a problem of multi-product monopolistic screening with type-dependent participation constraints.

For simplicity, we focus on the uniform-quadratic specification (2). In this case, it can be easily checked that the single-crossing conditions hold. Standard arguments then show that the problem can be formulated as follows:

$$\max_{q_A(\theta)\geq 0, q_B(\theta)\geq 0} \int_{\theta_{\min}}^{\theta_{\max}} \left[u\left(q_A, q_B, \theta\right) - P_B(q_B) - U(\theta)\right] f(\theta) d\theta$$

$$\begin{aligned} \text{s.t. } \frac{dU}{d\theta} &= u_\theta(q_A, q_B, \theta) \\ U(\theta) &\geq U^E(\theta), \end{aligned}$$

where $U^E(\theta) = u(0, q^e(\theta), \theta) - cq^e(\theta)$ is the net payoff that the buyer could obtain by trading with the competitive fringe only.

Let us consider first the relaxed problem in which the participation constraint is $U(\theta) \geq 0$. The associated virtual surplus function is

$$s(q_A, q_B, \theta) = v(q_A, q_B, \theta) - \frac{1 - F(\theta)}{f(\theta)} v_\theta(q_A, q_B, \theta).$$

With the uniform-quadratic specification, the virtual surplus function is strictly concave in q_A and q_B, and it has increasing differences in θ and q_A and q_B, respectively. Thus, the solution can be obtained by poitwise maximisation of the virtual surplus. The necessary and sufficient conditions for a maximum are

$$\begin{aligned} u_{q_A} - \frac{1 - F(\theta)}{f(\theta)} u_{\theta q_A} &\leq 0 \qquad \left[u_{q_A} - \frac{1 - F(\theta)}{f(\theta)} u_{\theta q_A}\right] q_A = 0 \\ u_{q_B} - c - \frac{1 - F(\theta)}{f(\theta)} u_{\theta q_B} &\leq 0 \qquad \left[u_{q_B} - c - \frac{1 - F(\theta)}{f(\theta)} u_{\theta q_B}\right] q_B = 0. \end{aligned}$$

From these we easily obtain the optimal quantities in the relaxed problem,

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \frac{1}{2} \\ q^m(\theta) & \text{for } \frac{1}{2} \leq \theta \leq \hat{\theta}^{ms} \\ q_A^{ms}(\theta) & \text{for } \hat{\theta}^{ms} \leq \theta \leq 1 \end{cases} \qquad q_B(\theta) = \begin{cases} 0 & \text{for } \theta \leq \hat{\theta}^{ms} \\ q_B^{ms}(\theta) & \text{for } \hat{\theta}^{ms} \leq \theta \leq 1, \end{cases}$$

where

$$q_A^{ms}(\theta) = 2\theta - 1 + c\frac{\gamma}{1 - 2\gamma}; \quad q_B^{ms}(\theta) = 2\theta - 1 - c\frac{1 - \gamma}{1 - 2\gamma},$$

and $\breve{\theta}^{ms}$ is implicitly defined by the condition $q_B^{ms}(\hat{\theta}^{ms}) = 0$; the explicit expression is $\hat{\theta}^{ms} = \frac{1+c-(2+c)\gamma}{2(1-2\gamma)}$.

Notice that $q_A^{ms}(\theta) \geq q_B^{ms}(\theta)$. Comparing the above quantities with those of the baseline model (see Annex 2) we observe that $q_A^{ms}(\theta) = q_A^{cr}(\theta)$, whereas $q_B^{ms}(\theta) \leq q_B^{cr}(\theta)$ with a strict inequality except at $\theta = 1$ (no distortion at the top). Thus, the marginal type $\breve{\theta}^{ms}$ is smaller than the marginal type $\breve{\theta}_B$ in the non-linear pricing equilibrium. The fact that $q_A^{ms}(\theta) = q_A^{cr}(\theta)$ is due to the linearity of the marginal payoff functions.[59]

When $c \geq c^m (= \frac{1}{2})$, it can be easily checked that the solution to the relaxed problem satisfies the type-dependent participation constraint, and thus is the final solution. When $c < c^m$, however, the participation constraint binds in low-demand states. We must then apply Proposition 5.5 in Jullien (2000). Proceeding as in the proof of Proposition 2, it is easy to show that the conditions of Homogeneity, Potential Separation and Weak Convexity are met. Jullien's result then guarantees that the solution partitions the set of types into three sets: buyers who are excluded, buyers who obtain their reservation utility $U^E(\theta)$, and buyers whose net payoff is strictly greater than $U^E(\theta)$.

This means that when $c < c^m$ the exclusive dealing branch of the $q_A(\theta)$ schedule is formed by two sub-branches, i.e. $q^e(\theta)$ for low types and $q^m(\theta)$ for intermediate types. This is similar to the equilibrium pattern that arises in the non-linear pricing equilibrium. In any case, equilibrium quantities are never greater than in the non-linear pricing equilibrium, and are strictly lower for a range of values of θ. This implies that market-share contracts are definitely anticompetitive with respect to that benchmark.

[59] Majumdar and Shaffer (2009) obtain the same property in a linear demand function example with two types..

Annex 5
Alternative specification of demand

(NOT FOR PUBLICATION)

In this Annex we derive the equilibrium of the model where the payoff function is

$$u\left(q_A, q_B, \theta\right) = (1+\theta)q_A + (1+b\theta)q_B - \frac{1-\gamma}{2}(q_A^2 + q_B^2) - \gamma q_A q_B, \tag{12}$$

and θ is uniformly distributed over the interval $[0, 1]$. For this analysis, we set $c = 0$ and capture the dominant firm's competitive advantage by setting $b < 1$.

In particular, for brevity we focus only on the case in which the competitive advantage is sufficiently large. To be precise, we assume $b \in [0, \bar{b}]$, where $\bar{b} = \frac{3\gamma-1}{1-\gamma}$. Note that the interval is non empty if the goods are sufficiently strong substitutes, i.e. $\gamma > 1/3$.

Non-linear pricing. The equilibrium can be found proceeding as in the baseline model. The competitive fringe always prices at cost, i.e. $P_B(q_B) = 0$. The indirect payoff function,

$$v(q_A, \theta) = \max_{q_B \geq 0} u\left(q_A, q_B, \theta\right),$$

has two branches, depending on whether $\tilde{q}_B(\theta) = \arg\max u\left(q_A(\theta), q_B, \theta\right)$ is strictly positive or is nil, and a kink in between. It can be easily checked that the single crossing condition holds, and that the indirect virtual surplus

$$s(q_A(\theta), \theta) = v(q_A(\theta), \theta) - (1-\theta)v_\theta(q_A, \theta),$$

is globally concave and has increasing difference in θ and q_A. Notice that we have a type-dependent participation constraint $U(\theta) \geq U^E(\theta)$,where $U^E(\theta) = u(0, q^e(\theta), \theta)$ is the net payoff that the buyer could obtain by trading with the competitive fringe only, and

$$q_B^e(\theta) = \frac{1+\theta b}{1-\gamma}$$

is the quantity that he would buy in that case.

Let us start from the relaxed problem in which the participation constraint is $U(\theta) \geq 0$. In this case, the solution can be found by pointwise maximisation of the indirect virtual surplus. The maximum can occur on the branch where $\tilde{q}_B(\theta) = 0$, in which case it is $q^m(\theta)$, on the branch where $\tilde{q}_B(\theta) > 0$, in which case it is $q_A^{cr}(\theta)$, or at the kink $q_A(\theta) = q^{\lim}(\theta)$, where $q^{\lim}(\theta)$ is implicitly defined by the condition $u_{q_B}\left(q^{\lim}(\theta),0,\theta\right)=0$. These quantities are

$$\begin{aligned} q^m(\theta) &= \frac{2\theta}{1-\gamma} \\ q_A^{cr}(\theta) &= \frac{2[1-\gamma(1+b)]\theta-(1-b)\gamma}{1-2\gamma} \\ q^{\lim}(\theta) &= \frac{1+b\theta}{\gamma}. \end{aligned}$$

The corresponding quantity of product B is $q_B(\theta)=\arg\max u\left(q_A(\theta),q_B,\theta\right)$. By construction, this is positive only when $q_A(\theta)=0$ or if $q_A(\theta)=q_A^{cr}(\theta)$, in which case it is

$$q_B^{cr}(\theta)=\begin{cases} \frac{1+\theta b}{1-\gamma} & \text{for } q_A(\theta)=0 \\ q_A^{cr}(\theta)+\frac{1-\gamma(1+b)-[2(1-\gamma)+b]\theta}{1-3\gamma+2\gamma^2} & \text{for } q_A(\theta)=q_A^{cr}(\theta). \end{cases}$$

It is immediate to check that the $q_A(\theta)$ schedules are monotone increasing, and hence implementable. The price schedules that implement those quantities are, respectively

$$\begin{aligned} P^m(q_A) &= q_A-\frac{1-\gamma}{4}q_A^2 \\ P_A^{cr}(q_A) &= \frac{2-(3+b)\gamma}{2(1-\gamma)}q_A-\frac{1-2\gamma}{4(1-\gamma)}q_A^2 \\ P^{\lim}(q_A) &= -\frac{1-b}{b}q_A+\frac{\gamma-b(1-\gamma)}{2b}q_A^2 \end{aligned}$$

The schedule $q_B^{cr}(\theta)$, in contrast, is non monotone. However, given firm A's pricing, it is obviously implemented by the price schedule $P_B(q_B)=0$.

Let $\breve{\theta}_i$ be the solutions to $q_i^{cr}(\theta)=0$. We have $\breve{\theta}_A=\frac{(1-b)\gamma}{2[1-\gamma(1+b)]}>0$, whereas the condition $b\leq\bar{b}$ implies that $\breve{\theta}_B<1$. In this case, therefore, the markets for each of the two products are uncovered: product A is not purchased in low-demand states, product B in high-demand ones. However, the market is covered in the sense that at least one good is bought in all states of demand. Both goods are bought when both common representation quantities are positive, i.e. for $\theta\in\left[\breve{\theta}_A,\breve{\theta}_B\right]$. This interval is not empty.

Notice that the condition $b\leq\bar{b}$ implies that $q^m(\theta)$ intersects $q^{\lim}(\theta)$ from below. The condition $q^m(\theta)>q^{\lim}(\theta)$ is then equivalent to $\theta>\theta^{\lim}$, where $\theta^{\lim}$ is the solution to $q^m(\theta)=q^{\lim}(\theta)$ and hence is $\theta^{\lim}=\frac{1-\gamma}{(2+b)\gamma-b}$. The condition $b\leq\bar{b}$ guarantees precisely that $\theta^{\lim}\leq 1$. That is, the condition guarantees the existence of a monopoly region.

With these preliminaries at hand, we can now proceed to the maximisation of the virtual surplus. Since $s(q_A(\theta),\theta)$ is concave, it is clear that if $q^m(\theta)>q^{\lim}(\theta)$ then $s(q_A,\theta)$ is increasing at the kink and the maximum is achieved at $q^m(\theta)$. This solution then applies when $\theta\geq\theta^{\lim}$. If instead $q^m(\theta)<q^{\lim}(\theta)$, i.e. for

$\theta < \theta^{\lim}$, then $s(q_A, \theta)$ is decreasing to the right of the kink, and one must further distinguish between two cases. If $q_A^{cr}(\theta) > q^{\lim}(\theta)$, then $s(q_A, \theta)$ is increasing to the left of the kink and so the maximum is achieved at the kink, $q^{\lim}(\theta)$. If instead $q_A^{cr}(\theta) < q^{\lim}(\theta)$, the maximum is achieved to the left of the kink and is $q_A^{cr}(\theta)$. Noting that by construction we have $q_A^{cr}(\theta) = q^{\lim}(\theta)$ when $\theta = \breve{\theta}_B$, we can conclude that for $\breve{\theta}_B \leq \theta \leq \theta^{\lim}$ we have the limit pricing solution, whereas for $\theta \leq \breve{\theta}_B$ we have common representation. However, for $\theta < \breve{\theta}_A$ we must have $q_A(\theta) = 0$, in which case only product B is sold, and the quantity is $q_B^e(\theta) > 0$.

Summarizing, the equilibrium quantities are

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_A \\ q_A^{cr}(\theta) & \text{for } \breve{\theta}_A \leq \theta \leq \breve{\theta}_B \\ q^{\lim}(\theta) & \text{for } \breve{\theta}_B \leq \theta \leq \theta^{\lim} \\ q^m(\theta) & \text{for } \theta \geq \theta^{\lim} \end{cases} \qquad q_B(\theta) = \begin{cases} q_B^e(\theta) & \text{for } \theta \leq \breve{\theta}_A \\ q_B^{cr}(\theta) & \text{for } \breve{\theta}_A \leq \theta \leq \breve{\theta}_B \\ 0 & \text{for } \theta \geq \breve{\theta}_B. \end{cases}$$

It is easy to see that the type-dependent participation constraint $U(\theta) \geq U^E(\theta)$ is always met, so the solution to the relaxed problem solves also the original problem, and hence is the non-linear pricing equilibrium.

Exclusive contracts. The analysis proceeds exactly as for the baseline model. The separation property guarantees that the solution to the dominant firm's problem is either $q^m(\theta)$ or the non-linear pricing solution that we have just characterised. (It can be confirmed that in the exclusive dealing region the type dependent participation constraint is met even if the dominant firm engages in monopoly pricing.) The analog of Lemma 2 in the proof of Proposition 2 guarantees that the switch among the two regimes can only occur in a region where in the non-linear pricing equilibrium both $q_A(\theta)$ and $q_B(\theta)$ are strictly positive.

The optimal switching point is still characterised by conditions (10) and (11). An argument identical to that proposed in the analysis of the baseline model confirms that at the switching point the average price jumps up, and hence so does the profit earned on the critical buyer $\hat{\theta}$.

Summarising, in equilibrium firm A offers the price schedules:

$$\begin{array}{ll} P_A^{NE}(q) = P_A^{cr}(q) & \text{for } 0 \leq q \leq q_A^m(\hat{\theta}) \\ P_A^E(q) = P^m(q) + \Phi_A & \text{for } q \geq q_A^m(\hat{\theta}). \end{array}$$

where $\hat{\theta}$ and Φ_A are determined by the equilibrium conditions (10) and (11). Now, however, Φ_A is negative. In other words, the dominant firm offers lump-sum subsidies to buyers who opt for exclusive dealing. (The explicit expression for $\hat{\theta}$ and Φ_A are complicated and are reported in a Mathematica file which is available from the authors upon request.) The competitive fringe will always price at cost. The associated equilibrium quantities are

$$q_A(\theta) = \begin{cases} 0 & \text{for } \theta \leq \breve{\theta}_A \\ q_A^{cr}(\theta) & \text{for } \breve{\theta}_A \leq \theta \leq \hat{\theta} \\ q^m(\theta) & \text{for } \theta > \hat{\theta} \end{cases} \qquad q_B(\theta) = \begin{cases} q_B^e(\theta) & \text{for } \theta \leq \breve{\theta}_A \\ q_B^{cr}(\theta) & \text{for } \breve{\theta}_A \leq \theta \leq \hat{\theta} \\ 0 & \text{for } \theta > \hat{\theta} \end{cases}$$

There is only one last remark to be made. Above we have reported, as usual, only the parts of the price schedules which correspond to quantities actually bought at equilibrium. Price offers that are destined not to be accepted in equilibrium are to some extent arbitrary. In this case, however, the dominant firm must make it sure that the non-exclusive contracts that it offers may not attract high-demand types. Even if the dominant firm does have some flexibility, the best way to do that is to truncate the non-exclusive price schedule at $q_A^m(\hat{\theta})$. In other words, buyers who sign non-exclusive contracts will never be offered quantities in excess of $q_A^m(\hat{\theta})$. This is consistent with the fact that in the Intel case buyers, such as Dell, who opted for exclusive dealing were given a preferential treatment in case of shortages.