

# Buying high and selling low: Stock repurchases and persistent asymmetric information<sup>1</sup>

Philip Bond, University of Washington

Hongda Zhong, University of Minnesota

June 2014

<sup>1</sup>Author email addresses: [apbond@uw.edu](mailto:apbond@uw.edu) and [zhong070@umn.edu](mailto:zhong070@umn.edu). We thank Brendan Daley, David Dicks, Diego Garcia, Paolo Fulghieri, Ohad Kadan, Robert McDonald, Raj Singh, Gunter Strobl, and seminar audiences at the University of Minnesota, Washington University in St Louis, the Pacific Northwest Finance Conference, the Jackson Hole Finance Group Conference, the Finance Theory Group Conference, the Financial Intermediation Research Society, and the Western Finance Association, for some very helpful comments. Any errors are our own.

## **Abstract**

We investigate the consequences of allowing for repeated capital market transactions in a model with asymmetric information between a firm and its investors. All firms in the model possess a profitable project that they need to raise cash to undertake. However, equilibria exist in which firms return cash to investors via share repurchases. Consistent with managerial accounts, some firms directly profit from repurchasing their stock. The ultimate source of these profits is that other firms buy “high” in order to improve the terms of subsequent stock issues, which is again consistent with empirical evidence. Only equilibria with repurchases satisfy a mild refinement. Repurchases lower social welfare by reducing the fraction of firms that invest, even though repurchasing itself carries no deadweight cost. Our model generates a number of empirical predictions.

An important idea in corporate finance is that firms have more information about their future cash flows than investors. A large body of research has studied the consequences of this asymmetric information for a firm’s capital market transactions. However, the vast majority of such papers have restricted firms to a single round of capital market transactions.<sup>1</sup> In this paper, we study the implications of relaxing this assumption for what is arguably the best-known corporate finance model based on asymmetric information, namely Myers and Majluf’s (1984) model of equity financing to fund an investment.<sup>2</sup>

Our main finding is that allowing for multiple capital market transactions in Myers and Majluf generates the following equilibrium dynamics. Some firms repurchase their stock for strictly less than its fair value, consistent with managerial claims that repurchases are driven by undervaluation.<sup>3</sup> Other firms repurchase stock in order to lower the cost of subsequent equity issuance, consistent with empirical evidence (see Billet and Xue (2007)).<sup>4</sup>

Moreover, these dynamics are present in *all* equilibria satisfying a standard and arguably mild refinement: specifically “Never Dissuaded Once Convinced,” henceforth NDOC, Osborne and Rubinstein (1990), and discussed in detail below.

At first sight, the ability of firms to strictly profit from trading on their superior information would appear to violate the no-trade theorem (see, e.g., Milgrom and Stokey (1982)). Many existing models of share repurchases avoid this problem by introducing an assumption that firms (exogenously) care directly about an interim share price.<sup>5</sup> Our model avoids this assumption. Instead, in our model some firms strictly profit from repurchases because other inferior firms also repurchase, and make losses. This second group of firms “buy high” when

---

<sup>1</sup>In exceptions such as Lucas and McDonald (1990, 1998), Chowdry and Nanda (1994), and Hennessy, Livdan and Miranda (2010), a firm’s informational advantage only lasts one period. In contrast, in our paper the information asymmetry is persistent. In Constantinides and Grundy (1989), which we discuss in detail below, firms engage in two rounds of transactions, but the second transaction is a deterministic function of the first.

<sup>2</sup>As we detail below, we focus on the version of this model where firms know more about the value of their existing assets, but have no informational advantage with respect to growth options.

<sup>3</sup>Brav et al (2005) survey managers. A very large fraction of managers agree (Table 6) that the “Market price of our stock (if our stock is a good investment, relative to its true value)” is an important factor.

<sup>4</sup>Related, in Brav et al (2005), a very large fraction of managers agree (Table 3) that “Repurchase decisions convey information about our company to investors.”

<sup>5</sup>See discussion of related literature below.

they repurchase, i.e., buy their stock for more than it is worth.

Why does this second group of inferior firms repurchase at a loss? They do so in order to improve the terms at which they can subsequently issue stock to finance a profitable investment. This is consistent with the empirical findings of Billet and Xue (2007). Nonetheless, and as is standard in models of this type, even the improved issuance terms are still associated with a negative price response at issue (this is the “selling low” of the title). These firms can be viewed as “manipulating” their stock price: after they repurchase, their stock price increases, and although the price then declines with the issue announcement, the issue price is still higher than it would otherwise be.

Repurchases do not carry any deadweight loss in our model; in this, our model is very different from much of the prior literature, which assumes that payouts generate a deadweight loss either via increased taxes or via an increased need for (exogenously) costly external financing.<sup>6</sup> Nonetheless, the repurchases strictly lower social welfare (meaning the total amount of profitable investment), in the sense that social welfare is lower in an equilibrium with repurchases than in an equilibrium of a benchmark one-period model without repurchases. The reason is that firms that issue to finance the profitable investment are forced to first repurchase to signal their quality, and this repurchase generates a loss (which, as discussed above, makes it possible for other firms to strictly profit from repurchases). Consequently, equilibrium repurchases raise the net cost of financing for firms that eventually invest; this in turn reduces the amount of equilibrium investment. Note that because repurchases have no deadweight loss, this welfare result is fundamentally different from the commonly-made observation (see, e.g., Arrow (1973)) that social welfare would be higher if a costly signal were prohibited.

*Related literature:*

Grullon and Ikenberry (2000) offer a good survey of the literature on repurchasing.

---

<sup>6</sup>See discussion of related literature below. Note that Brav et al’s (2005) survey of managers finds little support for the idea that repurchases are made to signal that a firm can bear such costs. For example, only a small fraction of managers (Table 3) say that “We use repurchases to show we can bear costs such as borrowing costly external funds or passing up investment...”

The idea that firms repurchase their stock to signal they are good is related to the old idea that *retaining* equity is a useful signal (Leland and Pyle (1977)). Also related, Example 1 of Brennan and Krause (1987) has a good firm simultaneously repurchasing debt and issuing equity. The debt repurchase allows the firm to signal that it is good.

Our paper is related to the literature on signaling in static payout models. In one branch of this literature (e.g., Bhattacharya (1980), Vermaelen (1984), Miller and Rock (1985)), good firms repurchase to show that they have (or expect to have) high cash flow. Bad firms do not mimic because they have low cash flow, and so paying out cash necessitates either costly external financing or distorts investment. An important assumption in this branch of the literature is that a firm's objective (exogenously) includes the interim share price. Regarding this assumption, Allen and Michaely (2003) write "why would a management care so much about the stock price next period? Why is its horizon so short that it is willing to 'burn money' (in the form of a payout) just to increase the value of the firm now, especially when the true value will be revealed next period?" In contrast to this literature, we do not exogenously assume that the firm cares about the interim share price.

In a second branch of the literature (e.g., John and Williams (1985), Ambarish, John and Williams (1987), Williams (1988)), firms pay out cash in a costly way, typically by issuing dividends, which are tax-inefficient. Firms then issue equity to finance an investment. Good firms pay out, while bad firms do not. Because of this separation, good firms are able raise the funds they need for investment in a less dilutive way. Bad firms do not mimic good firms because they would pay the same cost (inefficient cash pay outs), but benefit less because dilution is less costly to them than it is to good firms. The economic function of pay outs in these models is that they destroy value. This raises the question of whether other value-destroying actions would make better signals, and led the literature to consider multi-dimensional signaling models (see, e.g., Ofer and Thakor (1987), Vishwanathan (1995); we briefly consider the robustness of our analysis to multi-dimensional signaling in Section 8). Because repurchases are generally regarded as a tax efficient way of making pay outs, and

hence do not destroy value, the main focus of this branch of the literature is on dividends rather than repurchases.

Constantinides and Grundy (1989) study a model in which firms issue securities to fund an investment, and can commit to return any excess cash in the form of a repurchase. They give conditions under which full separation of firms is possible, and show that the commitment to repurchase plays an important role in supporting this separating equilibrium. Because the equilibria they study are fully separating, no firm profits from the repurchase transaction. Moreover, it is important that the original security issued differs from equity. In contrast, we study a case in which firms cannot commit to future transactions, and transact in the same security (equity) at all dates. We show that all equilibria satisfying NDOC entail some firms making strictly positive profits from stock repurchases.

An important assumption in any model of repurchasing based on signaling, including ours, is that a firm's repurchase decision is actually observable. Although regulatory mandates force this to be true in many markets, there has been some debate in the literature about the observability of repurchases in the United States. For example, in an early study of repurchases, Barclay and Smith (1988) find evidence that the announcement of a repurchase program is followed by an increased bid-ask spread, which they interpret as an increase in adverse selection, which they in turn interpret as investors being unsure about whether or not they are trading against the firm. However, in general subsequent research has not supported this original finding (see the discussion in Grullon and Ikenberry (2000)).

A relatively small literature studies dynamic models of trade under asymmetric information. Noldeke and van Damme (1990) and Swinkels (1999) study a labor market model where education acts as a signal. Fuchs and Skrzypacz (2013) study trade of a single indivisible asset that is more highly valued by buyers than the seller. They focus on whether more trading opportunities increase or reduce welfare. Kremer and Skrzypacz (2007) and Daley and Green (2011) study a similar model in which information arrives over time. In contrast to these papers, in our model both sales and repurchases are possible; trade is in

divisible shares; and the gains from trade arise from the possibility of financing a profitable investment. Perhaps closest to the current paper are Morellec and Schurhoff (2011) and Strebulaev, Zhu and Zryumov (2014). Both papers study dynamic models in which a firm with long-lived private information chooses a date to raise outside financing and invest. In both papers, issue and investment are tied together (by assumption), and the combination of repurchases with subsequent equity issue—which is our main focus—is not examined. Instead, the main results of both papers concern the timing of investment. Finally, a contemporaneous paper by Ordonez, Perez-Reyna and Yogo (2013) studies a dynamic model of debt issuance.

In a model with moral hazard in place of adverse selection, DeMarzo and Urošević (2006) study the dynamics of a large shareholder selling off his stake in a firm.

Bond and Eraslan (2010) study trade between differentially-informed parties in common-values setting. The no-trade theorem does not apply because the eventual owner of the asset takes a decision that affects the asset’s final cash flow. Trade affects the information available to the party making the decision. In the current paper, trade of the asset (i.e., shares) at date 1 instead affects a firm’s ability to raise finance at date 2. Related, Huang and Thakor (2013) give a model in which the gains from repurchase stem from reducing disagreement among a firm’s shareholders.

## 1 Example

Firms have cash 1, and the opportunity to invest 9 at date 2 in a project that subsequently yields 11. Hence firms need to raise additional funds of 8 in order to invest. Firms can either repurchase (buy) or issue (sell) shares at each of dates 1 and 2. All uncertainty is resolved at date 3, and firms act to maximize their date 3 share price. The initial number of shares is normalized to 1.

Firm assets-in-place  $a$  are distributed over  $[0, 40]$ , with a density that satisfies the fol-

lowing properties. First, there is a probability  $\frac{1}{4}$  that the assets-in-place lie in each of the intervals  $[0, 2]$ ,  $[2, 4]$ ,  $[4, 21]$ , and  $[21, 40]$ . Second, the conditional expectation of  $a$  satisfies  $E[a | a \in [0, 2]] = 1$ ,  $E[a | a \in [2, 4]] = 2.2$ ,  $E[a | a \in [21, 40]] = 37.8$ , while  $a$  is uniform over  $[4, 21]$ .<sup>7</sup>

If date 2 transactions are exogenously ruled out, this setting is simply a version of Myers and Majluf with a continuum of firm types. We first describe an equilibrium of this benchmark.<sup>8</sup> Firms  $a \leq 4.834$  raise funds 8 by issuing  $\frac{8}{4.667}$  shares at a price  $P^{MM} = 4.667$ , and then invest. Firms  $a > 4.834$  do nothing. To see that the price  $P^{MM}$  is fair, note the expected value of  $a$  conditional on  $a \leq 4.834$  is 1.667;<sup>9</sup> and that  $P^{MM}$  solves

$$P^{MM} = \frac{11 + 1.667}{1 + \frac{8}{P^{MM}}}.$$

Given the issue price  $P^{MM}$ , the date 3 share price of firm  $a = 4.834$  is 5.834 if it does nothing, and is  $\frac{11+4.834}{1+\frac{8}{4.667}} = 5.834$  if it issues and invests. Hence firms with  $a < 4.834$  strictly prefer to issue and invest, while firms with  $a > 4.834$  find issue too dilutive, and strictly prefer to do nothing.

The focus of our paper is the case in which transactions are possible at both dates 1 and 2. In this case, the following is a Perfect Bayesian equilibrium (PBE), illustrated in Figure 1:

- At date 1, firms with assets-in-place in either  $[2, 4]$  or  $[21, 40]$  spend all their cash 1 to repurchase  $\frac{1}{22}$  shares for a price  $P_1 = 22$ . The remaining firms do nothing.
- At date 2, firms with assets-in-place below 2 raise funds 8 by issuing 2 shares at a price  $P_2^D = 4$ , and invest. Firms with assets-in place in  $[2, 4]$  raise funds 9 by issuing  $\frac{9}{4.4}$

---

<sup>7</sup>Note that there are an infinite number of distributions satisfying these properties. We also stress that these properties are chosen only to produce a reasonably simple numerical example. Finally, uniformity over  $[4, 21]$  is used only to compute the equilibrium of the one-period benchmark.

<sup>8</sup>The equilibrium described entails firms either raising just enough outside financing to fund the investment, or else doing nothing. Other equilibria exist in which issuing firms raise strictly more funds than required. However, all equilibria of the benchmark are characterized by a cutoff firm type such that firms below this cutoff issue and invest, while firms above this cutoff do nothing; see Proposition 2 below.

<sup>9</sup>That is, the conditional expectation is  $\frac{\frac{1}{4} + \frac{1}{4} \cdot 2.2 + \frac{1}{4} \cdot \frac{.834}{17} \cdot 4.417}{\frac{1}{4} + \frac{1}{4} + \frac{1}{4} \cdot \frac{.834}{17}} = 1.667$ .



shares at a price  $P_2^{RI} = 4.4$ , and invest. The remaining firms do nothing.

We verify this is an equilibrium. First, conditional on firms behaving this way, the repurchase and issues prices are fair, as follows. The date 2 issue-after-repurchase price  $P_2^{RI} = 4.4$  is fair, since it solves

$$P_2^{RI} = \frac{E[a|a \in [2, 4]] + 11}{1 - \frac{1}{22} + \frac{9}{P_2^{RI}}}.$$

The date 2 direct issue price  $P_2^D = 4$  is fair, since it solves

$$P_2^D = \frac{E[a|a \in [0, 2]] + 11}{1 + \frac{8}{P_2^D}}.$$

The date 1 repurchase price is fair, since with probability 1/2 the date 2 price will be  $P_2^{RI} = 4.4$  and with probability 1/2 it will be  $\frac{E[a|a \in [21, 40]]}{1 - \frac{1}{22}} = 39.6$ , and so, conditional on date 1 repurchase, the expected date 2 price is 22.

Second, firms respond optimally to the stated repurchase and issue prices. If a firm repurchases then issues, it has  $1 - \frac{1}{22} + \frac{9}{4.4} = 3$  shares outstanding at date 3. If a firm issues directly, it has  $1 + \frac{8}{4} = 3$  shares outstanding at date 3. Hence the date 3 share price of a firm with assets-in-place  $a$  under both these alternatives is

$$\frac{11 + a}{3},$$

while the date 3 share price from repurchasing at date 1 and then doing nothing is

$$\frac{a}{1 - \frac{1}{22}} = \frac{22}{21}a$$

and the date 3 share price from doing nothing at both dates is simply

$$1 + a.$$

Out of these three alternatives, firms with assets-in-place below 4 obtain the highest payoff

from either repurchasing and then investing, or directly issuing and investing; they are indifferent between the two options. Firms with assets-in-place between 4 and 21 obtain the highest payoff from doing nothing. Finally, firms with assets-in-place above 21 obtain the highest payoff from repurchasing at date 1 and then doing nothing.<sup>10</sup>

*Discussion:*

Firms with assets-in-place  $a > 21$  repurchase shares for strictly less than their true value,  $a + 1$ , and so make strictly positive profits. The reason investors accept the lower price is that these firms pool with worse firms (namely, firms with  $a$  between 2 and 4). But this raises the question of why these worse firms are prepared to repurchase. They do so in order to improve the terms at which they can subsequently issue. If instead they attempt to issue equity directly, they obtain a worse price: specifically, they issue shares at a price 4 rather than 4.4.

The intermediate interval of firms with between 4 and 21 find issue too dilutive, as in Myers and Majluf, and also find repurchase too expensive.

Firms with  $a > 21$  strictly profit from their repurchase transactions, even though these transactions fail to create any value. The ultimate source of these profits is that the investing firms with  $a \leq 4$  end up paying a premium to raise capital. By this, we mean that if firms  $a \leq 4$  could all credibly pool and issue directly, the issue price  $P$  would satisfy  $P = \frac{11 + \frac{1}{2}(1+2.2)}{1 + \frac{8}{P}}$ , i.e.,  $P = 3 + 1.6 = 4.6$ , and so the payoff of each firm  $a < 4$  would be  $\frac{11+a}{1+4.6}$ , which is higher than they get in the above equilibrium.

A related observation is that the equilibrium of the Myers and Majluf setting, where repurchase is impossible, entails investment by firms with assets-in-place between 0 and a cutoff level strictly above 4. In other words, repurchases lower total surplus in the economy (see Section 6). Nonetheless, and as we show below, when repurchase is possible, any equilibrium that satisfies NDOC features some repurchase.

---

<sup>10</sup>We have established that firms act optimally when their choice set is limited to the four equilibrium strategies. This still leaves open the possibility that a firm could profitably deviate to some strategy other than these four strategies. Off-equilibrium beliefs that deter such deviations are specified in the proofs of Propositions 3 and 5.

## 2 Model and preliminary results

Our model is essentially the same as Myers and Majluf (1984). The only substantive difference is that whereas Myers and Majluf consider a firm's interactions with the equity market at just one date, we consider two possible dates. As we will show, this additional feature generates equilibrium share repurchases.

There are four dates,  $t = 0, 1, 2, 3$ ; an all-equity firm, overseen by a manager; and at each of dates 1 and 2, a large number of risk-neutral investors who trade the firm's stock. We normalize the date-0 number of shares to 1.

At date 0, the manager of the firm privately learns the value of the firm's existing assets ("assets-in-place"). Write  $a$  for the expected value of these existing assets, where  $a \in [\underline{a}, \bar{a}]$ . Let  $\mu$  be a measure on  $[\underline{a}, \bar{a}]$ , which determines the distribution of assets-in-place  $a$ . We assume  $a$  has full support on  $[\underline{a}, \bar{a}]$ , and has no atoms. In addition to assets  $a$ , the firm has cash (or other marketable securities) with a value  $S$ .

At the end of date 2, the firm has an opportunity to undertake a new project. (In Section 7, we extend the model to allow for a choice of investment timing, with the firm able to invest at either date 1 or date 2.) The project requires an initial investment  $I$  and generates an expected cash flow  $I + b$ . For simplicity, we assume that  $b$  is common knowledge; in other words, we focus on a version of the Myers and Majluf environment in which asymmetric information is about assets-in-place, not investment opportunities. Throughout, we assume  $I > S$ , so that the firm needs to raise external financing to finance the investment  $I$ .

At each of dates  $t = 1, 2$ , the firm can issue new equity and/or repurchase existing equity. Equity issues and repurchases take place as follows. The manager makes a public offer to buy or sell a fixed dollar amount  $s_t$  of shares, where  $s_t > 0$  corresponds to share repurchases and  $s_t < 0$  corresponds to share issues. Investors respond by offering a quantity of shares in exchange. In other words, if  $s_t > 0$  each investor offers a number of shares he will surrender in exchange for  $s_t$ ; and if  $s_t < 0$ , each investor offers a number of shares he will accept in return for paying the firm  $-s_t$ .

(Note that both  $a$  and  $I + b$  are expected values, so our model allows for very volatile cash flows. In particular, we assume that there is enough cash flow volatility that it is impossible for firms to issue risk free debt. In general, the choice between risky debt and equity under asymmetric information is non-obvious; see Fulghieri, Garcia and Hackbarth (2014) for a recent characterization. In Section 8 we discuss the robustness of our analysis to allowing for other securities.)

At date 3, the true value of the firm is realized, including the investment return, and the firm is liquidated.

Write  $P_3$  for the date-3 liquidation share price, and write  $P_1$  and  $P_2$  for the transaction price of the shares at dates  $t = 1, 2$ . Because the number of investors trading at each of dates 1 and 2 is large, competition among investors implies that the date  $t$  share price is

$$P_t = E[P_3 | \text{date } t \text{ information, including firm offer } s_t]. \quad (1)$$

The manager's objective is to maximize the date 3 share price, namely

$$P_3 = \frac{S - s_1 - s_2 + a + b\mathbf{1}_{\text{investment}}}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}}, \quad (2)$$

where  $\mathbf{1}_{\text{investment}}$  is the indicator function associated with whether the firm undertakes the new project, and the denominator reflects the number of shares outstanding at date 3. Note that in the case that only share issues are possible, the manager's objective function coincides with the one specified in Myers and Majluf (1984), which is to maximize the utility of existing ("passive") shareholders. In our setting, where repurchases are possible, the manager's objective function can be interpreted as maximizing the value of passive shareholders, who neither sell nor purchase the firm's stock at dates 1 and 2. Alternatively, the manager's objective can be motivated by assuming that the manager himself has an equity stake in the firm, and is restricted from trading the firm's shares on his own account.<sup>11</sup>

---

<sup>11</sup>Note that if the manager also put weight on a high date 1 share price this would further increase the

For use throughout, observe that (1) and (2), together with the fact that the firm invests whenever it has sufficient funds, imply that the date 2 share price conditional on  $s_1$  and  $s_2$  is

$$P_2(s_1, s_2) = \frac{S - s_1 + E[a|s_1, s_2] + b\mathbf{1}_{S-s_1-s_2 \geq I}}{1 - \frac{s_1}{P_1}}. \quad (3)$$

Iterating, (1) and (3), together with the law of iterated expectations, imply that the date 1 share price conditional on  $s_1$  is

$$P_1(s_1) = S + E[a + b\mathbf{1}_{S-s_1-s_2 \geq I} | s_1]. \quad (4)$$

From (3) and (4), the payoff of firm  $a$  from  $(s_1, s_2)$  is

$$\frac{S - s_1 - s_2 + a + b\mathbf{1}_{S-s_1-s_2 \geq I}}{\left(1 - \frac{s_1}{P_1}\right) \left(1 - \frac{s_2}{S-s_1+E[a|s_1, s_2]+b\mathbf{1}_{S-s_1-s_2 \geq I}}\right)} = \frac{S - s_1 - s_2 + a + b\mathbf{1}_{S-s_1-s_2 \geq I}}{\left(1 - \frac{s_1}{S+E[a+b\mathbf{1}_{S-s_1-s_2 \geq I}|s_1]}\right) \left(1 - \frac{s_2}{S-s_1+E[a|s_1, s_2]+b\mathbf{1}_{S-s_1-s_2 \geq I}}\right)}. \quad (5)$$

We characterize the perfect Bayesian equilibria (PBE) of this game. We restrict attention to pure strategy equilibria in which all investors hold the same beliefs off-equilibrium. We focus on equilibria in which all firms play a best response (as opposed to equilibria in which almost all firms play a best response).<sup>12</sup>

Finally, we state here a simple monotonicity result, which we use repeatedly:

**Lemma 1** *If in equilibrium firms  $a'$  and  $a''$  conduct capital transactions  $(s'_1, s'_2)$  and  $(s''_1, s''_2)$ , with  $S - s'_1 - s'_2 > S - s''_1 - s''_2$ , then  $a' < a''$ .*

---

manager's incentives to repurchase equity. On the other hand, it is important for our analysis that the manager does not fully internalize the welfare of date 0 shareholders who sell at date 1: in particular, our analysis requires that if a manager is able to repurchase shares at less than their true value, then he does so. As discussed in the text, one justification is that the manager seeks to maximize the value of his own equity stake. A second justification is that when a firm repurchases its own stock, it may not be its existing shareholders who sell shares to the firm; instead, the firm's repurchase offer may be filled by short-sellers of the firm's stock. Attaching zero welfare weight to short-sellers is analogous to the Myers and Majluf assumption of attaching zero welfare weight to new purchasers of the firm's shares.

<sup>12</sup>Given a perfect Bayesian equilibrium in which almost all firms play a best response, one can easily construct an equilibrium in which all firms play a best response by switching the actions of the measure zero set of firms who originally did not play a best response. Because only a measure zero set of firms are switched, the original set of beliefs remain valid.

An immediate corollary of Lemma 1 is:

**Corollary 1** *In any equilibrium, there exists  $a^* \in [\underline{a}, \bar{a}]$  such that all firms  $a < a^*$  invest and all firms  $a > a^*$  do not invest.*

### 3 One-period benchmark

Before proceeding to our main analysis, we characterize the equilibrium of the benchmark model in which firms can only issue or repurchase shares at date 1, with the date 2 issue/repurchase decision  $s_2$  exogenously set to 0. The main conclusion of this section is that the Myers and Majluf conclusion holds: only the lowest asset firms issue and invest, and repurchases play no meaningful role. In other words, the addition of the possibility of repurchases to the Myers and Majluf environment is, by itself, inconsequential. Instead, our results further below are driven by the possibility of firms engaging in capital transactions at multiple dates.

The key reason that the firms do not take advantage of repurchases in a one-period model is the no-trade theorem (Milgrom and Stokey (1982)). Even though firms enjoy an informational advantage relative to investors, they are unable to profit from this advantage.

**Proposition 1** *In the single stage benchmark game, the set of firms who repurchase and strictly profit relative to doing nothing is of measure 0.*

Proposition 1 establishes that, in the one-period benchmark, a firm's ability to repurchase its own stock plays no meaningful role. Accordingly, the equilibria of the one-period benchmark coincide with those of the standard Myers and Majluf (1984) setting, as formally established by the next result:

**Proposition 2** *In any equilibrium, there exists  $a^* \in (\underline{a}, \bar{a}]$  such that almost all firms below  $a^*$  issue the same amount  $s^*$  and invest, while almost all firms above  $a^*$  receive the same payoff as doing nothing (i.e.,  $P_3 = a + S$ ).*

Proposition 2 characterizes properties an equilibrium must possess. However, it does not actually establish the existence of an equilibrium. However, this is easily done. In particular, fix any  $s^*$  such that  $S - s^* \geq I$ , and define  $a^*$  by

$$a^* = \max \left\{ a \in [\underline{a}, \bar{a}] : \frac{S - s^* + a^* + b}{1 - \frac{s^*}{S + E[a|a \in [\underline{a}, a^*] + b]}} \geq S + a^* \right\}.$$

Then there is an equilibrium in which all firms with assets below  $a^*$  issue and raise an amount  $-s^*$ , while firms with assets above  $a^*$  do nothing. Off-equilibrium-path beliefs are such that any other offer to issue (i.e.,  $s < 0$  and  $s \neq s^*$ ) is interpreted as coming from the worst type  $\underline{a}$ , and any offer to repurchase (i.e.,  $s > 0$ ) is interpreted as coming from the best type  $\bar{a}$ .

Observe that if  $\frac{I + \bar{a} + b}{1 + \frac{I - S}{S + E[\bar{a}] + b}} \geq S + \bar{a}$ , this benchmark model has an equilibrium in which the socially efficient outcome of all firms investing is obtained. In order to focus attention on the case in which asymmetric information causes a social loss, for the remainder of the paper we assume instead that

$$\frac{I + \bar{a} + b}{1 + \frac{I - S}{S + E[\bar{a}] + b}} < S + \bar{a}, \quad (6)$$

so that there is no equilibrium of the benchmark model in which all firms invest. For use below, note that (6) implies

$$\bar{a} > E[\bar{a}] + b > \underline{a} + b. \quad (7)$$

## 4 Analysis of the dynamic model

We now turn to the analysis of the full model, in which firms can engage in capital transactions at multiple dates.

### 4.1 Existence of a repurchase equilibrium

We first show that there is nothing “special” about the example we presented above. For all parameter values satisfying (6), there exists an equilibrium in which the best firms strictly

profit from repurchasing, while worse firms repurchase their stock for more than it is worth—i.e., “buy high”—in order to improve the terms at which they can subsequently issue shares to finance the investment.

**Proposition 3** *An equilibrium exists in which a strictly positive mass of firms pool and repurchase at date 1. A strict subset of these these firms make strictly positive profits from the repurchase, and do nothing at date 2. The remaining repurchasing firms repurchase their stock for more than it is worth, and then issue enough shares to finance investment at date 2.*

The proof of Proposition 3 is constructive. The equilibrium constructed is either similar to the above example; or else features all firms repurchasing at date 1, with a strict subset then issuing equity to fund investment at date 2.

## 4.2 Necessity of repurchases

As is common with games of asymmetric information, our model has multiple equilibria. However, we next show that the properties stated in Proposition 3 are possessed by *any* equilibrium satisfying a refinement known as “Never Dissuaded Once Convinced” (NDOC) (Osborne and Rubinstein (1990)). Hence the NDOC refinement selects precisely equilibria that feature repurchases.

NDOC is a consistency condition on how beliefs evolve over time. Once investors are 100% sure that the firm’s type belongs to some set  $A$ , NDOC states that subsequent beliefs put positive probability only on firm types within  $A$ .<sup>13</sup> This restriction is highly intuitive and is typically regarded as mild; see, for example, Rubinstein (1985) and Grossman and Perry (1986), or more recently, its use as Assumption 1 in Ely and Valimaki (2003) and as Condition R in Feinberg and Skrzypacz (2005).

More formally, in our context, NDOC states that date 2 investor beliefs after observing firm actions  $(s_1, s_2)$  must satisfy the following: (I) if  $s_1$  is an equilibrium action, then date

---

<sup>13</sup>NDOC is stronger than the standard PBE definition, because it applies to off-equilibrium beliefs.



2 beliefs assign probability 1 to the firm's type lying in the set of firms who play  $s_1$  in equilibrium, and (II) if  $s_1$  is not an equilibrium action, and date 1 beliefs assign probability 1 to some subset  $A$  of firm types, date 2 beliefs likewise assign probability 1 to the same subset  $A$ .

**Proposition 4** *Any equilibrium satisfying NDOC has the properties stated in Proposition 3, and in particular, features strictly profitable repurchases.*

The economics behind Proposition 4 is as follows. Under assumption (6), the best firms do not invest in equilibrium.<sup>14</sup> Consequently, if they do not repurchase, these firms do not make any profits, and the final payoff of a high-value firm  $a$  is simply  $S + a$ . Consequently, for repurchases to be unattractive in equilibrium for the top firm  $\bar{a}$ , investors must charge at least  $S + \bar{a}$  to surrender their shares; in turn, this requires investors to believe that (off-equilibrium) repurchase offers come from very good firms. But given these beliefs, a low-value firm could profitably deviate from its equilibrium strategy by repurchasing at date 1, thereby triggering beliefs that it is very good, and then (by NDOC) issue at a high price at date 2.

A second important implication of Proposition 4 is that the equilibrium outcome of the one-period benchmark economy is not an equilibrium outcome of the full model under NDOC. At first sight, this might seem surprising: one might imagine that one could take the equilibrium of the one-period economy and then assign off-equilibrium beliefs to make other actions, and in particular repurchases, unattractive. However, the dynamic nature of the model makes this impossible. The reason is that, as just illustrated, to deter repurchases, off-equilibrium beliefs must assign a large weight to a repurchasing-firm being a high type; but given these beliefs, a deviating firm can issue at attractive terms at date 2. In brief, under NDOC it is impossible to assign off-equilibrium beliefs that deter *both* date 1 repurchase and date 2 issue.

---

<sup>14</sup>Formally, this is established in Corollary A-2 in the appendix.

### 4.3 Existence of a repurchase equilibrium satisfying NDOC

A drawback of the NDOC restriction is that, for some games, it eliminates *all* equilibria: see Madrigal et al (1987). To see the issue, consider again the example of Section 1. In the equilibrium described, if a firm does nothing at date 1, the NDOC restriction implies that investors must believe the firm has a type  $a \leq 21$ , regardless of the firm's action at date 2. This in turn means that any firm that does nothing at date 1 is able to repurchase shares at date 2 for a price of  $1 + 21 = 22$  (or less). In particular, firms with  $a > 21$  would make strictly positive profits by doing nothing at date 1, and then repurchasing at date 2.

It is important to note that—despite this concern—the actions described in the example of Section 1 are consistent with an equilibrium satisfying NDOC. The reason is that the deviation just discussed—namely doing nothing and then repurchasing—gives a firm a payoff of  $\frac{a}{1-\frac{1}{22}}$  if investors associate the strategy of do-nothing-then-repurchase with the belief that a firm is type  $a = 21$ . (Note that this belief satisfies NDOC.) But this payoff is no better than the equilibrium payoff of firms  $a > 21$ , and so is not a strictly profitable deviation.

Hence the example shows that for at least some parameter values our model possesses equilibria that satisfy NDOC, and existence is not a concern. Despite this, we are unable to establish a general existence result. However, there are two straightforward perturbations of our model under which we are able to guarantee equilibrium existence:

**Proposition 5** *There exists an equilibrium satisfying NDOC if either:*

(I) *There is a probability  $\alpha > 0$  that a firm is exogenously unable to conduct any capital market transaction at date 1.*

(II) *The maximum repurchase size is  $\bar{S}$ , and  $\bar{S}$  is sufficiently small.*

*Moreover, under each of these model perturbations, Proposition 4 continues to hold, i.e., any equilibrium satisfying NDOC has the properties stated in Proposition 3.*

Perturbation (I) of Proposition 5 is motivated by the observation that the act of doing nothing at date 1 has “too much” signaling power in the above example. After all, it is

easy to imagine that a firm does nothing at date 1 for some exogenous reason; for example, perhaps its manager failed to get approval for either an issue or repurchase. In this case, NDOC does not impose any restriction on investor beliefs about firms that do nothing at date 1, and the equilibrium constructed in Proposition 3 is an equilibrium of this perturbed game. In contrast, NDOC continues to have bite for firms that repurchase at date 1 and then issue at date 2: this is why Proposition 4 continues to hold. Finally, note that the exogenous probability  $\alpha$  can be made arbitrarily small.

Perturbation (II) is motivated by the fact that there may exist limits on how much a firm can repurchase. For example, not all of the firm’s “cash”  $S$  may be immediately available for repurchase transactions. Instead, only an amount  $\bar{S}$  may be truly liquid, while the remaining portion  $S - \bar{S}$  can be liquidated before the investment  $I$  must be made.<sup>15</sup> Existence is guaranteed in this case for the same reason that the example satisfies NDOC: when the maximal repurchase size is small, the deviation of doing-nothing at date 1 and then repurchasing at date 2 does not generate strictly higher profits than the strategy of the equilibrium established in Proposition 3, namely repurchasing immediately at date 1.

## 5 Stock price reactions

A large empirical literature has examined stock price reactions to repurchase and issuance announcements; see, e.g., Allen and Michaely (2003) for a survey. As documented by this literature, repurchase announcements are associated with price increases, and issue announcements are associated with price declines.

Our model provides a natural explanation of both these announcement effects. Issue announcements generate negative price responses because lower-value firms issue. This is the “selling low” of the paper’s title, and is very much in line with the existing literature (again, see Allen and Michaely (2003)).

Repurchase announcements generate positive price reactions. The reason is that some

---

<sup>15</sup>See Duchin et al (2013) for a detailed empirical analysis of the nature of firms’ cash holdings.

of the firms repurchasing are high-value firms. This is an effect present in several existing models in the literature. With respect to this previous literature, the innovation of our paper is to obtain this effect without exogenously assuming that firms care about the interim stock price. Specifically, the reason high-value firms repurchase in our model is that they pool with low-value firms, and so are able to repurchase at an attractive price.

The reason low-value firms repurchase—and do so at a price that is high for them—is that by doing so they reduce the price of subsequent equity issues. This is one of the primary empirical implications of our model. Billet and Xue (2007) find evidence for this effect. They compare the issuance price reactions of firms that previously repurchased stock with the issuance price reactions of firms that did not previously repurchase. The price decline of the former group is smaller, consistent with our model.

The following result (which holds independently of NDOC) formalizes these predictions of our model:

**Proposition 6** *Let  $s_1 \geq 0$  be a date 1 repurchase decision used by a positive measure of firms. Then:*

*(A, price drops at issue) A positive-measure subset of these firms issue an amount  $s_2$  such that  $S - s_1 - s_2 \geq I$  at date 2, at a price  $P_2 \leq P_1$ . Moreover, the date-2 price of non-issuing firms exceeds  $P_1$ . Both relations are strict whenever  $\Pr(s_2|s_1) < 1$ .*

*(B, repurchase increases subsequent issue price) Suppose that a positive measure of firms issue  $s'_1 < 0$  at date 1. Then there exists  $s'_2$  such that  $s'_2 \leq 0$ ,  $S - s'_1 - s'_2 \geq I$ ,  $\Pr(s'_2|s'_1) = 1$ , and  $P_2(s'_1, s'_2) = P_1(s'_1) \leq P_2(s_1, s_2)$ . Likewise, if  $(0, s'_2)$  with  $s'_2 < 0$  is played by a positive measure of firms, then  $P_2(0, s'_2) \leq P_2(s_1, s_2)$ . Both price relations are strict if  $s_1 > 0$  and  $\Pr(s_2|s_1) < 1$ .*

*(C, price increases at repurchase) If a positive measure of firms take no action at date 1, then  $P_1(s_1) \geq P_1(0)$ , with the inequality strict if  $s_1 > 0$  and  $\Pr(s_2|s_1) < 1$ .*

Our model also generates cross-sectional predictions between, on the one hand, the *size* of repurchases and issues, and on the other hand, the price response associated with these

transactions. These predictions emerge in equilibria of the model in which multiple repurchase and issue levels coexist (in contrast to the example, which features just one repurchase level).<sup>16</sup>

As one would expect, larger repurchases are associated with higher repurchase prices, since they are conducted by firms that are, on average, better. Similarly, larger issues are associated with lower issue prices. Both predictions are consistent with empirical evidence: see, for example, Ikenberry, Lakonishok and Vermaelen (1995) for evidence on repurchases, and Asquith and Mullins (1986) for evidence on issues.

**Proposition 7** (*A, repurchases*) Consider an equilibrium in which  $s'$  and  $s'' > s'$  are repurchase levels, with associated prices  $P'$  and  $P''$ , and such that there exist firms  $a'$  and  $a''$  where firm  $a'$  (respectively,  $a''$ ) repurchases  $s'$  (respectively,  $s''$ ) and does not conduct any other capital transaction at any other date. Then (i)  $P'' \geq P'$ , (ii)  $s''/P'' > s'/P'$ , and (iii)  $a'' > a'$ . In particular, repurchase size is positively correlated with repurchase price.

(*B, issues*) Let  $(s'_1, s'_2)$  and  $(s''_1, s''_2)$  be equilibrium strategies such that  $S - s''_1 - s''_2 > S - s'_1 - s'_2$ . Then  $P_2(s'_1, s'_2) > P_2(s''_1, s''_2)$ . In particular, if  $s'_2 < 0$  and  $s''_2 < 0$ , then greater cumulative issue is associated with lower date 2 issue prices.<sup>17</sup>

## 6 Welfare

As we have established, our economy features equilibria in which some firms repurchase. Here, we ask how social welfare in such equilibria compares with social welfare in the equilibrium of the one-period benchmark. Because capital market transactions do not have any deadweight cost, social welfare is simply proportional to the fraction of firms that invest.<sup>18</sup>

We obtain the following strong result (which holds independently of NDOC):

<sup>16</sup>One can show, via numerical simulation, that such equilibria exist.

<sup>17</sup>It is also possible to establish that  $s'_1 > s''_1$ , i.e., greater cumulative issue is associated with smaller initial repurchases. A proof is available upon request.

<sup>18</sup>If each investor holds a diversified portfolio of shares, this welfare measure coincides with the Pareto welfare ranking.

**Proposition 8** *Consider any equilibrium featuring repurchases, and a finite number of actions.<sup>19</sup> Then there exists an equilibrium of the benchmark one-period model that has strictly high welfare, and no repurchases.<sup>20</sup>*

The example illustrates the basic economics of this result. In the equilibrium of the example, some high-value firms strictly profit from repurchasing their stock for less than its true value. Because investors break even in expectation, the ultimate source of these profits is low-value firms who initially pool with high-value firms and repurchase, in order to reduce the cost of subsequent issues. Low-value firms lose money on the repurchase leg of this transaction. In the one-period benchmark, repurchases do not arise (Proposition 1), and low-value firms do not have to endure this loss-making leg. This allows them to issue at better terms, which in turn means that a greater fraction of firms find issuance (and investment) preferable to non-issuance.

Despite this relatively simple intuition, the proof of Proposition 8 is long and involved. The main complication stems from the need to deal with equilibria that feature many different repurchase and issue levels.

At least since Arrow (1973), it has been understood that the possibility of economic agents signaling their type by undertaking a socially costly action may result in lower welfare relative to a situation in which signaling is prohibited or otherwise impossible.<sup>21</sup> In our setting, however, repurchases carry no deadweight cost, yet welfare is still reduced.

## 7 Extension: Investment timing

In our main model, the investment project can only be undertaken at date 2. Here, we consider an extension in which the investment can be undertaken at either date 1 or date 2

---

<sup>19</sup>This restriction is made for simplicity, to avoid mathematical complication. The result covers equilibria with an arbitrarily large (but finite) number of equilibrium actions.

<sup>20</sup>In particular, if the one-period benchmark has a unique equilibrium in the class of equilibria with  $S - s_1 = I$ , then welfare in this equilibrium exceeds welfare in any equilibrium of the full model.

<sup>21</sup>For a recent result along these lines, see Hoppe, Moldovanu and Sela (2009).

(though not both). We focus on the benchmark case in which the project available is exactly the same at each of the two dates.

Investment at date 1 moves both the cash outflow associated with investment ( $I$ ) and the subsequent benefits ( $I + b$ ) forward by one period. If the discount rate is positive, this means that date 1 investment is more expensive, but generates greater benefits, relative to investment at date 2. In our main model we normalize the discount rate to 0; or more precisely, the objects  $S, s_1, s_2, I, b, a$  are all expressed as date 3 future values. To incorporate the effect of the investment timing choice on investment costs and benefits, we write the investment cost at dates 1 and 2 as  $I_1$  and  $I_2$  respectively, and likewise write the present value generated as  $b_1$  and  $b_2$  respectively. Hence  $\frac{I_1}{I_2} = \frac{b_1}{b_2} \geq 1$ , where both ratios equal the one-period interest rate.

The flexibility of investment timing introduces an additional dimension in which firms can signal their type. In particular, if  $b_1 > b_2$ , then delaying investment is costly, and so there may exist equilibria in which bad firms issue and invest at date 1, while good firms signal their type by waiting until date 2 to issue and invest. (See Morellec and Schurhoff (2011) for an analysis dedicated to this issue.) However, when  $b_1$  and  $b_2$  are sufficiently close, i.e., when the effect of discounting is small, one can show that no equilibrium of this type exists, and the best firms never invest in equilibrium. Intuitively, waiting to invest is not a strong enough signal to support separation. In this case, the economic forces behind our result that any equilibrium satisfying NDOC features repurchases (Proposition 4) remain unchanged. Formal proofs of the analogues of Propositions 3-5 are available upon request.

Consequently, the extension of our model to endogenous investment timing leaves our main results unchanged, at least when discount rates are not too high. At the same time, endogenous investment timing introduces a new effect into our model: namely that repurchases are associated with an inefficient delay of investment. Specifically, if repurchases are exogenously ruled-out, the one-period benchmark equilibrium remains an equilibrium of the

two-period model, with all investment conducted at date 1.<sup>22</sup> But when repurchases are feasible, any equilibrium satisfying NDOC features at least some investment at date 2. Hence, there are three distinct costs associated with investment: (i) inefficient delayed investment (the new effect of this section); (ii) the cross-subsidy from investing firms to repurchase-only firms (the effect stressed in the main model); and (iii) the cross-subsidy from better investing firms to worse investing firms (the standard Myers and Majluf effect).

## 8 Robustness

We have restricted attention to the case in which firms can only signal via equity repurchases. However, we do not believe this restriction is critical, as follows.

Our main equilibrium characterization result is that that any equilibrium satisfying NDOC must feature repurchases (Proposition 4). A key step ingredient in this result is that in any candidate equilibrium without repurchases, the best firms would obtain their reservation payoff of  $S + a$ . As discussed, this property implies that repurchases can only be deterred in equilibrium if off-equilibrium beliefs associate a repurchase offer with a high firm type. The dynamic setting, combined with NDOC, then implies that a firm that deviates and repurchases could issue at very good terms the following period, thereby undercutting the proposed equilibrium without repurchases.

This argument still works even if additional signaling possibilities are introduced, provided that any candidate equilibrium without repurchases has the best firms receiving their reservation payoffs. Indeed, the extension of Section 7 in which investment timing can potentially serve as a signal illustrates exactly this. Moreover, it may be possible to extend this argument to cover cases in which the best firms receive more than their reservation payoff, since in such a case, it is still necessary to assign very favorable beliefs to any firm that attempts to repurchase. Finally, note that in this generalization firms may repurchase a *different* security from equity; however, under the conditions described, some firms will

---

<sup>22</sup>Again, this is for the case in which  $b_1$  and  $b_2$  are sufficiently close.



repurchase some form of risky security.

## 9 Conclusion

We investigate the consequences of allowing for repeated capital market transactions in a model with asymmetric information between a firm and its investors. All firms in the model possess a profitable project that they need to raise cash to undertake. However, we show that there always exist equilibria in which firms return cash to investors via share repurchases. Consistent with managerial accounts, some repurchasing firms profit from repurchasing their stock. The ultimate source of these profits is that other firms buy “high” in order to improve the terms of subsequent stock issues, which is again consistent with empirical evidence. Moreover, only equilibria that feature repurchases satisfy the relatively mild NDOC restriction on off-equilibrium beliefs. Repurchases lower social welfare by reducing the fraction of firms that invest, even though repurchasing itself carries no deadweight cost. Our model generates a number of empirical predictions.

## References

- Allen, Franklin and Roni Michaely (2003). “Payout Policy” Handbook of the Economics of Finance, Vol. 1, Part A, pp. 337-429.
- Ambarish, Ramasastry, Kose John and Joseph Williams (1987). “Efficient Signalling with Dividends and Investments” Journal of Finance, Vol. 42, No. 2, pp. 321-343
- Arrow, Kenneth (1973) “Higher Education as a Filter.” Journal of Public Economics, Vol. 2, pp. 193-216.
- Barclay and Smith (1988) “Corporate payout policy: Cash dividends versus open-market repurchases.” Journal of Financial Economics 22, pp. 61- 82.
- Bhattacharya, Sudipto (1980) “Nondissipative Signaling Structures and Dividend Policy.” Quarterly Journal of Economics, Vol. 95, No. 1, pp. 1-24.

Billet and Xue (2007) “Share Repurchases and the Need for External Finance” *Journal of Applied Corporate Finance*, Vol. 19, Issue 3, pp 42–55.

Bond, Philip and Hulya Eraslan (2010). “Information-based trade” *Journal of Economic Theory*, Vol. 145, No. 5, pp. 1675-1703.

Brav, Alon, John Graham, Campbell Harvey and Roni Michaely (2005) “Payout Policy in the 21st Century.” *Journal of Financial Economics*.

Brennan, Michael and Alan Kraus (1987) “Efficient Financing Under Asymmetric Information.” *Journal of Finance*, Vol. 42, No. 5, pp. 1225-1243.

Constantinides and Grundy (1989) “Optimal Investment with Stock Repurchases and Financing as Signals” *Review of Financial Studies*, Vol. 2, No. 4, pp. 445-465

Daley, Brendan and Brett Green (2011) “Waiting for News in the Market for Lemons.” forthcoming in *Econometrica*.

DeMarzo, Peter and Branko Urošević (2006) “Ownership Dynamics and Asset Pricing with a Large Shareholder.” *Journal of Political Economy* , Vol. 114, No. 4, pp. 774-815.

Duchin, Ran, Thomas Gilbert, Jarrad Harford, and Chris Hrdlicka (2013) “Precautionary Savings with Risky Assets: When Cash Is Not Cash.” Working Paper.

Ely, Jeffrey C and Juuso Valimäki (2003) “Bad Reputation.” *The Quarterly Journal of Economics* (2003) 118 (3): 785-814.

Feinberg, Yossi and Andrzej Skrzypacz (2005) “Uncertainty about Uncertainty and Delay in Bargaining” *Econometrica*, Vol. 73, No. 1 (January, 2005), 69–91.

Fuchs, William and Andrzej Skrzypacz (2013) “Costs and Benefits of Dynamic Trading in a Lemons Market.” Working Paper.

Fulghieri, Paolo, Diego Garcia and Dirk Hackbarth (2014). “Asymmetric information and the pecking (dis)order.” Working Paper.

Grossman, Sanford J. and Motty Perry (1986). “Perfect Sequential Equilibrium.” *Journal of Economic Theory*, Vol. 39, pp. 97-119.

Grullon, Gustavo and David Ikenberry (2000) “What do we know about stock repurchases?”

Journal of Applied Corporate Finance, Vol. 13, No. 1, pp. 31-51.

Hennessy, Christopher, Dmitry Livdan, and Bruno Miranda (2010) “Repeated Signaling and Firm Dynamics” *Review of Financial Studies*, 23, No. 5, pp. 1981-2023.

Hoppe, Heidrun, Benny Moldovanu and Aner Sela (2009) “The Theory of Assortative Matching Based on Costly Signals.” *Review of Economic Studies*, 76, 253–281.

Ikenberry, Lakonishok and Vermaelen (1995) “Market Underreaction to Open Market Share Repurchases.” *Journal of Financial Economics* 39, pp.181-208.

John and Williams (1985) “Dividends, Dilution, and Taxes: A Signalling Equilibrium.” *The Journal of Finance*, Vol. 40, No. 4, pp. 1053-1070.

Huang, Sheng and Anjan V. Thakor (2013) “Investor Heterogeneity, Investor-Management Disagreement and Share Repurchases.” *Review of Financial Studies*, Vol. 26.,No. 10, pp. 2453-2491.

Kremer, Ilan and Andrzej Skrzypacz (2007) “Dynamic Signaling and Market Breakdown.” *Journal of Economic Theory*, 133 (1), pp. 58-82.

Leland, Hayne and David Pyle (1977) “Informational Asymmetries, Financial Structure, and Financial Intermediation.” *Journal of Finance*, Vol. 32, No. 2, pp. 371-387.

Lucas, Deborah and Robert McDonald (1990) “Equity Issues and Stock Price Dynamics.” *Journal of Finance*, Vol. 45, No. 4, pp. 1019-1043.

Madrigal, Vicente, Tommy C. Tan and Sergio Ribeiro da Costa Werlang (1987) “Support Restrictions and Sequential Equilibrium” *Journal of Economic Theory* 43, pp. 329-334.

Milgrom, Paul and Nancy Stokey (1982) “Information, Trade and Common Knowledge.” *Journal of Economic Theory*, Vol. 26, No. 1, pp. 17-27.

Miller and Rock (1985) “Dividend Policy under Asymmetric Information”, *The Journal of Finance*, Vol. 40, No. 4., pp. 1031-1051

Morellec, Erwan and Norman Schurhoff (2011) “Corporate investment and financing under asymmetric information.” *Journal of Financial Economics* 99, pp 262–288.

Myers and Majluf (1984) “Corporate Financing and Investment Decisions When Firms Have

Information That Investors Do Not Have.” *Journal of Financial Economics* 13, pp 187-221.

Noldeke, Georg and Eric van Damme (1990) “Signalling in a Dynamic Labour Market.” *Review of Economic Studies*, 57 (1), pp. 1-23.

Ofer, Aharon R. and Anjan V. Thakor (1987) “A Theory of Stock Price Responses to Alternative Corporate Cash Disbursement Methods: Stock Repurchases and Dividends.” *Journal of Finance*, Vol. 42, No. 2, pp. 365-394.

Ordóñez, Guillermo, David Perez-Reyna, and Motohiro Yogo (2013) “Debt: Deleveraging or Default.” Working Paper.

Osborne, Martin J and Ariel Rubinstein (1990) “Bargaining and Markets.” Academic Press Inc. 1990

Rubinstein, Ariel (1985) “A Bargaining Model with Incomplete Information About Time Preferences.” *Econometrica*, Vol. 53, No. 5, pp. 1151-1172.

Strebulaev, Ilya, Haoxiang Zhu and Pavel Zryumov (2014) “Dynamic Information Asymmetry, Financing, and Investment Decisions” Working Paper.

Swinkels, Jeroen M. (1999) “Educational Signalling With Preemptive Offers.” *Review of Economic Studies*, 66 (4), pp. 949-970.

Vermaelen, Theo (1984) “Repurchase Tender Offers, Signaling, and Managerial Incentives.” *Journal of Financial and Quantitative Analysis*, Vol. 19, No. 2, pp. 163-181.

Viswanathan, S. (1995) “A Multiple-Signaling Model of Corporate Financial Policy.” *Research in Finance*, 12, pp. 1-35.

Williams (1988) “Efficient Signalling with Dividends, Investment, and Stock Repurchases” *The Journal of Finance*, Vol. 43, No. 3, pp 737-747

## Appendix

**Proof of Lemma 1:** Suppose to the contrary that  $a' \geq a''$ . Since firms  $a'$  and  $a''$  follow different strategies,  $a' > a''$ . Let  $P'_1$  and  $P'_2$  (respectively,  $P''_1$  and  $P''_2$ ) be the prices associated

with  $s'_1$  and  $s'_2$  (respectively,  $s''_1$  and  $s''_2$ ). Also, let  $\mathbf{1}'$  and  $\mathbf{1}''$  be the investment decisions of firms  $a'$  and  $a''$ .

From the equilibrium conditions,

$$\frac{a'' + S - s''_1 - s''_2 + b\mathbf{1}''}{1 - \frac{s''_1}{P''_1} - \frac{s''_2}{P''_2}} \geq \frac{a'' + S - s'_1 - s'_2 + b\mathbf{1}'}{1 - \frac{s'_1}{P'_1} - \frac{s'_2}{P'_2}}. \quad (\text{A-1})$$

By supposition, and given optimal investment decisions, the numerator of the LHS is strictly smaller than the numerator of RHS. Hence the denominator of the LHS must also be strictly smaller, i.e.,

$$1 - \frac{s''_1}{P''_1} - \frac{s''_2}{P''_2} < 1 - \frac{s'_1}{P'_1} - \frac{s'_2}{P'_2}. \quad (\text{A-2})$$

Also from the equilibrium conditions,

$$\frac{a' + S - s'_1 - s'_2 + b\mathbf{1}'}{1 - \frac{s'_1}{P'_1} - \frac{s'_2}{P'_2}} \geq \frac{a' + S - s''_1 - s''_2 + b\mathbf{1}''}{1 - \frac{s''_1}{P''_1} - \frac{s''_2}{P''_2}}.$$

From (A-2),

$$\frac{a' - a''}{1 - \frac{s'_1}{P'_1} - \frac{s'_2}{P'_2}} < \frac{a' - a''}{1 - \frac{s''_1}{P''_1} - \frac{s''_2}{P''_2}},$$

which implies

$$\frac{a'' + S - s'_1 - s'_2 + b\mathbf{1}'}{1 - \frac{s'_1}{P'_1} - \frac{s'_2}{P'_2}} > \frac{a'' + S - s''_1 - s''_2 + b\mathbf{1}''}{1 - \frac{s''_1}{P''_1} - \frac{s''_2}{P''_2}},$$

contradicting the equilibrium condition (A-1) and completing the proof.

**Proof of Corollary 1:** Suppose to the contrary that the claim does not hold, i.e., there exists an equilibrium in which there are firms  $a'$  and  $a'' > a'$  where  $a''$  invests and  $a'$  does not invest. Since investment decisions are optimal, the capital transactions of firms  $a'$  and  $a''$ , say  $(s'_1, s'_2)$  and  $(s''_1, s''_2)$ , must satisfy  $S - s'_1 - s'_2 < I \leq S - s''_1 - s''_2$ . This contradicts Lemma 1, completing the proof.

**Proof of Proposition 1:** Suppose otherwise. Let  $s_1(a)$  be the strategy of firm  $a$ , and  $A^{rep} = \{a : s_1(a) > 0\}$  be the set of firms who repurchase in equilibrium. By supposition,

$\mu(A^{rep}) > 0$ . On the one hand, a firm prefers repurchasing to doing nothing if and only if  $\frac{a+S-s_1}{1-P_1(s_1)} \geq a+S$ , or equivalently,  $P_1(s_1) \leq a+S$ . Since by supposition a strictly positive mass of repurchasing firms have a strict preference for repurchasing,

$$E[P_1(s_1(a)) - (a+S) | a \in A^{rep}] < 0.$$

On the other hand, investors only sell if  $P_1(s_1) \geq E\left[\frac{a+S-s_1}{1-P_1(s_1)} | s_1\right]$ , or equivalently,  $P_1(s_1) \geq E[a|s_1] + S$ . By the law of iterated expectations, this implies

$$E[P_1(s_1(a)) - (a+S) | a \in A^{rep}] \geq 0.$$

The contradiction completes the proof.

**Proof of Proposition 2:** Fix an equilibrium. From Proposition 1, there cannot be a positive mass of firms who repurchase and obtain  $P_3 > a+S$ . By a parallel proof, there cannot be a positive mass of firms who issue, do not invest, and obtain  $P_3 > a+S$ . By (4), any issue  $s$  that is enough for investment is associated with the price  $P_1(s) = S + E[a|s] + b$ . Given these observations, standard arguments then imply that there exists some  $\varepsilon > 0$  such that almost all firms in  $[\underline{a}, \underline{a} + \varepsilon]$  issue and invest: if an equilibrium does not have this property, then these firms certainly have the incentive to deviate and issue and invest, since this is profitable under any investor beliefs. So by Corollary 1, there exists  $a^* > \underline{a}$  such that all firms in  $[\underline{a}, a^*]$  issue and invest.

Finally, suppose that contrary to the claimed result that different firms in  $[\underline{a}, a^*]$  issue different amounts. Given Lemma 1, it follows that there exists  $\check{a} \in (\underline{a}, a^*)$  such that any firm in  $[\underline{a}, \check{a}]$  issues strictly more than any firm in  $(\check{a}, a^*)$ . Hence there must exist firms  $a' \in [\underline{a}, \check{a})$  and  $a'' \in (\check{a}, a^*)$  such that

$$P_1(s(a')) \leq S + a' + b < S + a'' + b \leq P_1(s(a'')).$$

Since  $-s(a') > -s(a'')$ , this combines with the equilibrium condition for firm  $a'$  to deliver the following contradiction, which completes the proof:

$$\frac{S - s(a'') + a' + b}{1 - \frac{s(a'')}{P_1(s(a''))}} \leq \frac{S - s(a') + a' + b}{1 - \frac{s(a')}{P_1(s(a'))}} \leq \frac{S - s(a'') + a' + b}{1 - \frac{s(a'')}{P_1(s(a'))}} < \frac{S - s(a'') + a' + b}{1 - \frac{s(a'')}{P_1(s(a''))}}.$$

### Proof of Proposition 3:

*Preliminaries:*

Given any date 1 repurchase level  $s_1 > 0$ , define  $a^*(s_1)$  to be the smallest solution of

$$\frac{1}{1 + \frac{I - S + s_1}{S - s_1 + E[a|a \leq a^*] + b}} (I + a^* + b) - (S - s_1 + a^*) = 0. \quad (\text{A-3})$$

We first show that  $a^*(s_1)$  is well-defined, decreasing in  $s_1$ , and lies in  $(\underline{a}, \bar{a})$ . The proof is as follows. The LHS of (A-3) is strictly positive at  $a^* = \underline{a}$ . The LHS of (A-3) is strictly decreasing in  $s_1$  for any  $a^* > \underline{a}$ . Consequently, (6) implies that the LHS of (A-3) is strictly negative at  $a^* = \bar{a}$ . Existence of  $a^*(s_1)$  follows by continuity. The other two properties are immediate.

Observe that at  $s_1 = 0$  and  $a_1 = \underline{a}$ ,

$$\frac{1}{1 - \frac{s_1}{S + \bar{a}}} \frac{1}{1 + \frac{I - S + s_1}{S - s_1 + E[a|a \leq a_1] + b}} (I + a_1 + b) > S + a_1. \quad (\text{A-4})$$

By continuity, choose  $\bar{a}_1 > \underline{a}$  and  $\bar{s}_1 > 0$  such that inequality (A-4) holds for all  $(a_1, s_1) \in [\underline{a}, \bar{a}_1] \times [0, \bar{s}_1]$ .

Note that  $a^*(s_1) > \bar{a}_1$ .

Fix  $s_1 \in (0, \min\{\bar{s}_1, \frac{S}{2}\})$  sufficiently small such that

$$\max \left\{ \frac{I - S + s_1}{S - s_1 + E[a|a < a^*(\frac{S}{2})] + b}, \frac{I - S + s_1}{S - s_1 + E[a|a < \bar{a}_1] + b} \right\} \leq \frac{I - S}{S + \underline{a} + b}. \quad (\text{A-5})$$

Given  $s_1$ , we explicitly construct an equilibrium. There are two cases, corresponding to whether  $S + a^*(s_1)$  is larger or smaller than  $S + E[a] + b \Pr(a \leq a^*(s_1))$ . In the first case,

all firms repurchase  $s_1$  at date 1, and then a strict subset of firms issue  $I + s_1 - S$  at date 2. In the second case, some firms repurchase  $s_1$  at date 1, with a strict subset then issuing  $I + s_1 - S$  at date 2; while other firms do nothing at date 1, with a strict subset then issuing  $I - S$  at date 2. In both cases, any off-equilibrium repurchase offer triggers investor beliefs that the firm is type  $\bar{a}$ , while any off-equilibrium issue offer triggers beliefs that the firm is type  $\underline{a}$ .

*Case 1:*  $S + a^*(s_1) \geq S + E[a] + b \Pr(a \leq a^*(s_1))$ .

In this (easier) case, we show there is an equilibrium in which at date 1 all firms repurchase  $s_1$ ; and at date 2 firms  $a \leq a^*(s_1)$  issue  $I - S + s_1$  and invest, while other firms do nothing at date 2. The date 1 repurchase price  $P_1$  and date 2 issue price  $P_2$  in such an equilibrium are

$$\begin{aligned} P_1 &= S + E[a] + b \Pr(a \leq a^*(s_1)) \\ P_2 &= \frac{S - s_1 + E[a|a \leq a^*(s_1)] + b}{1 - \frac{s_1}{P_1}}. \end{aligned}$$

Hence the payoff for a firm  $a$  from repurchase-issue is

$$\frac{1}{1 - \frac{s_1}{P_1} + \frac{I-S+s_1}{P_2}}(I + a + b) = \frac{1}{1 - \frac{s_1}{P_1}} \frac{I + a + b}{1 + \frac{I-S+s_1}{S-s_1+E[a|a \leq a^*(s_1)]+b}}. \quad (\text{A-6})$$

By (A-5) and  $a^*(s_1) > \bar{a}_1$ , the payoff (A-6) is at least

$$\frac{1}{1 - \frac{s_1}{P_1}} \frac{I + a + b}{1 + \frac{I-S}{S+\underline{a}+b}} > \frac{I + a + b}{1 + \frac{I-S}{S+\underline{a}+b}}.$$

The RHS of this inequality is the payoff to issuing directly given out-of-equilibrium beliefs in which direct issue is associated with the worst firm  $\underline{a}$ . Hence all firms prefer the equilibrium repurchase-issue strategy to the off-equilibrium direct issue strategy.

Firms  $a \geq a^*(s_1)$  prefer repurchase-do-nothing to do-nothing. To see this, simply note



that the payoff for a firm  $a$  from repurchase-do-nothing is  $\frac{S-s_1+a}{1-\frac{s_1}{P_1}}$ , which exceeds the payoff from do-nothing, i.e.,  $S+a$ , if and only if  $P_1 \leq S+a$ . Since we are in Case 1, this condition is satisfied for all firms  $a \geq a^*(s_1)$ .

Firms  $a \geq a^*(s_1)$  prefer repurchase-do-nothing to repurchase-issue by the definition of  $a^*(s_1)$ .

Likewise, firms  $a \leq a^*(s_1)$  prefer repurchase-issue to repurchase-do-nothing by the definition of  $a^*(s_1)$ .

Finally, firms  $a \leq a^*(s_1)$  prefer repurchase-issue to do-nothing because this is true for firm  $a^*(s_1)$ ; and is also true for firm  $\underline{a}$ , since this firm prefers direct issue to do-nothing. Since all payoffs are linear in firm type, it then follow that all firms between  $\underline{a}$  and  $a^*(s_1)$  likewise prefer repurchase-issue to do-nothing.

*Case 2:*  $S + a^*(s_1) < S + E(a) + b \Pr(a \leq a^*(s_1))$ .

In this case, we show there exists  $a_1$  and  $a_2$ , along with a partition  $A_0, A_1$  of  $[\underline{a}, a_1]$ , such that the following is an equilibrium: At date 1 firms  $A_1 \cup [a_2, \bar{a}]$  repurchase  $s_1$ , while other firms do nothing; and at date 2 firms  $A_1$  issue  $I - S + s_1$  and invest, firms  $A_0$  directly issue  $I - S$  (without previously repurchasing), and the remaining firms do nothing.

In such an equilibrium, the date 1 repurchase price  $P_1$  and date 2 issue price  $P_2$  following repurchase are

$$P_1 = S + \frac{E[a|A_1] \mu(A_1) + E[a|a \geq a_2] \mu([a_2, \bar{a}]) + b \mu(A_1)}{\mu(A_1) + \mu([a_2, \bar{a}])}$$

$$P_2 = \frac{S - s_1 + E[a|A_1] + b}{1 - \frac{s_1}{P_1}}.$$

We show that there exist  $a_1, a_2 \in [\underline{a}, \bar{a}]$  and  $a_1 < a_2$ , together with a partition  $A_0, A_1$  of  $[\underline{a}, a_1]$ , that solve the following system of equations (where  $P_1$  is as defined immediately above):

$$\frac{1}{1 - \frac{s_1}{P_1}} \frac{1}{1 + \frac{I-S+s_1}{S-s_1+E[a|A_1]+b}} (I + a_1 + b) = S + a_1 \quad (\text{A-7})$$

$$\frac{1}{1 - \frac{s_1}{P_1}} \frac{1}{1 + \frac{I-S+s_1}{S-s_1+E[a|A_1]+b}} = \frac{1}{1 + \frac{I-S}{S+E[a|A_0]+b}} \quad (\text{A-8})$$

$$P_1 = S + a_2 \quad (\text{A-9})$$

Condition (A-7) states that firm  $a_1$  is indifferent between repurchase-issue and do-nothing. Condition (A-8) states that firms are indifferent between repurchasing and then issuing, and issuing directly. Condition (A-9) states that firm  $a_2$  is indifferent between repurchase-do-nothing and do-nothing.

Notationally, define  $\gamma_0 \equiv \frac{\mu(A_0)}{\mu([\underline{a}, a_1])}$  and  $E_0 \equiv E[a|A_0]$ , and note that  $E[a|A_1] = \frac{E[a|a \leq a_1] - \gamma_0 E_0}{1 - \gamma_0}$ . The system of equations (A-7)-(A-9) has a solution if and only if the following system has a solution in  $\gamma_0, E_0, a_1$  and  $a_2$ :

$$\frac{1}{1 - \frac{s_1}{S+a_2}} \frac{1}{1 + \frac{I-S+s_1}{S-s_1 + \frac{E[a|a \leq a_1] - \gamma_0 E_0}{1 - \gamma_0} + b}} (I + a_1 + b) - (S + a_1) = 0 \quad (\text{A-10})$$

$$\frac{1}{1 + \frac{I-S}{S+E_0+b}} (I + a_1 + b) - (S + a_1) = 0 \quad (\text{A-11})$$

$$\frac{(E[a|a \leq a_1] - \gamma_0 E_0) \mu([\underline{a}, a_1]) + E[a|a \geq a_2] \mu([a_2, \bar{a}])}{(1 - \gamma_0) \mu([\underline{a}, a_1]) + \mu([a_2, \bar{a}])} + \frac{b(1 - \gamma_0) \mu([\underline{a}, a_1])}{(1 - \gamma_0) \mu([\underline{a}, a_1]) + \mu([a_2, \bar{a}])} - a_2 = 0 \quad (\text{A-12})$$

along with the additional restriction that  $E_0$  is consistent with  $\gamma_0$  and  $a_1$ . (At  $\gamma_0 = 0$  this consistency condition is simply that  $E_0$  lies in the interval  $[\underline{a}, a_1]$ . As  $\gamma_0$  increases, the lower bound of this interval increases and the upper bound decreases, with both continuous in  $\gamma_0$ .)

*Claim (i):* There exists  $\hat{a} \in [\bar{a}_1, \bar{a}]$  such that for  $\gamma_0 = 0$  and  $a_1 \in [\hat{a}, a^*(s_1)]$ , equation (A-10) has a unique solution in  $a_2$ , which we denote  $a_2(a_1)$ . Moreover,  $a_2(a_1)$  is continuous in  $a_1$ , with  $a_2(\hat{a}) = \bar{a}$  and  $a_2(a^*(s_1)) = a^*(s_1)$ , and  $a_2(a_1) \in (a_1, \bar{a})$  for  $a_1 \in (\hat{a}, a^*(s_1))$ .

*Proof of Claim (i):* The LHS of (A-10) is strictly decreasing in  $a_2$ , so if a solution exists it

is continuous. By the definition of  $a^*(s_1)$ , the LHS of (A-3) is positive for all  $a_1 \in [a, a^*(s_1)]$ , and strictly so except for at  $a_1 = a^*(s_1)$ . Consequently, the LHS of (A-10) evaluated at  $a_2 = a_1$  is greater than  $\frac{S-s_1+a_1}{1-\frac{s_1}{S+a_1}} - (S+a_1) = 0$ , and strictly so except for at  $a_1 = a^*(s_1)$ . So at  $a_1 = a^*(s_1)$  we have  $a_2(a_1) = a_1$ , while for  $a_1 < a^*(s_1)$  any solution to (A-10) must strictly exceed  $a_1$ .

Evaluated at  $a_1 = \bar{a}_1$  and  $a_2 = \bar{a}$ , the LHS of (A-10) is strictly positive, by (A-4). Evaluated at  $a_1 = a^*(s_1)$  and  $a_2 = \bar{a}$ , the LHS of (A-10) is

$$\frac{S - s_1 + a^*(s_1)}{1 - \frac{s_1}{S + \bar{a}}} - (S + a^*(s_1)) = (S + \bar{a}) \frac{S - s_1 + a^*(s_1)}{S - s_1 + \bar{a}} - (S + a^*(s_1)) < 0.$$

So by continuity, there exists  $\hat{a} \in (\bar{a}_1, a^*(s_1))$  such that, for all  $a_1 \in (\hat{a}, a^*(s_1))$ , the LHS of (A-10) evaluated at  $a_2 = \bar{a}$  is strictly negative, while at  $a_1 = \hat{a}$  it is exactly zero.

Consequently, for  $a_1 \in [\hat{a}, a^*(s_1)]$  equation (A-10) has a unique solution in  $a_2$ . The solution lies in the interval  $[a_1, \bar{a}]$ ; equals  $a_1$  when  $a_1 = a^*(s_1)$ ; equals  $\bar{a}$  when  $a_1 = \hat{a}$ ; and lies in  $(a_1, \bar{a})$  otherwise. This completes the proof of the Claim (i).

*Claim (ii):* There exists  $\bar{\gamma}_0 > 0$  such that (A-11) has a unique solution,  $E_0(a_1)$  say, when  $\gamma_0 \in [0, \bar{\gamma}_0]$  and  $a_1 \in [\hat{a}, a^*(s_1)]$ . Moreover, the solution  $E_0(a_1)$  is independent of  $\gamma_0$ , and is consistent with  $a_1$  and  $\gamma_0$ .

*Proof of Claim (ii):* From Claim (i), (A-10) has a unique solution in  $a_2$  when  $\gamma_0 = 0$  and  $a_1 \in [\hat{a}, a^*(s_1)]$ . A necessary condition for (A-10) to have a solution is that the LHS of (A-10) is weakly negative at  $a_2 = \bar{a}$ . From (A-5), and the fact that  $a_1 \geq \hat{a} \geq \bar{a}_1$ , we know  $\frac{1}{1 + \frac{I-S}{S+a+b}} < \frac{1}{1 - \frac{s_1}{S+\bar{a}}} \frac{1}{1 + \frac{I-S+s_1}{S-s_1+E[a \leq a_1]+b}}$ . Hence the LHS of (A-11) is strictly negative when  $E_0 = \underline{a}$ . Conversely, the LHS of (A-11) is strictly positive when  $E_0 = a_1$ . Finally, noting that the LHS of (A-11) is strictly increasing in  $E_0$  completes the proof of Claim (ii).

Since (A-10) is strictly decreasing in  $a_2$ , it follows from Claims (i) and (ii) that there exist continuous functions  $a_2(a_1; \gamma_0)$ ,  $\hat{a}(\gamma_0)$ ,  $a^*(s_1; \gamma_0)$  of  $\gamma_0 \in [0, \bar{\gamma}_0]$  such that for all  $a_1 \in [\hat{a}(\gamma_0), a^*(s_1; \gamma_0)]$ , the unique solution of (A-10) and (A-11) is  $(a_2(a_1; \gamma_0), E_0(a_1))$ ; and

moreover,  $(a_2(a_1; 0), \hat{a}(0), a^*(s_1; 0)) = (a_2(a_1), \hat{a}, a^*(s_1))$ . Moreover, it is straightforward to see that for any  $\gamma_0 \in [0, \bar{\gamma}_0]$ ,  $a_2(a_1; \gamma_0)$  is continuous in  $a_1$ .

At  $\gamma_0 = 0$ , the LHS of (A-12) evaluated at  $(a_1, a_2, E_0) = (\hat{a}(\gamma_0), a_2(\hat{a}(\gamma_0); \gamma_0), E_0(\hat{a}(\gamma_0)))$  equals  $E[a|a \leq a_1] + b - \bar{a}$ , which is strictly negative by (7); while evaluated at  $(a_1, a_2, E_0) = (a^*(s_1; \gamma_0), a_2(a^*(s_1; \gamma_0); \gamma_0), E_0(a^*(s_1; \gamma_0)))$  it equals  $E[a] + b \Pr(a \leq a^*(s_1)) - a^*(s_1)$ , which is strictly positive since we are in Case 2. By continuity, the same two statements also hold for  $\gamma_0$  small but strictly positive. Fix any such  $\gamma_0$ . By continuity, there then exists  $(a_1, a_2(a_1; \gamma_0), E_0(a_1))$  that satisfies equations (A-10)-(A-12). This completes the treatment of this case, and hence the proof.

**Lemma A-1** *There is no equilibrium in which almost all firms invest.*

**Proof of Lemma A-1:** Suppose to the contrary that there is an equilibrium in which almost all firms invest. By assumption (6), it follows that there is a firm  $a'$  that invests and such that  $a' > E[a]$  and

$$S + a' > \frac{I + a' + b}{1 + \frac{I-S}{S+E[a]+b}}.$$

Let  $(s_1, s_2)$  be the strategy of firm  $a'$ , and let  $(P_1, P_2)$  be the associated prices. So the equilibrium condition for firm  $a'$  implies

$$\frac{S - s_1 - s_2 + a' + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}} \geq S + a' > \frac{I + a' + b}{1 + \frac{I-S}{S+E[a]+b}} \geq \frac{S - s_1 - s_2 + a' + b}{1 - \frac{s_1+s_2}{S+E[a]+b}},$$

where the final inequality makes use of  $-s_1 - s_2 \geq I - S$  (since firm  $a'$  invests) and  $a' > E[a]$ . Since any firm has the option of following strategy  $(s_1, s_2)$ , it follows that the equilibrium payoff of an arbitrary firm  $a$  is at least

$$\frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}} > \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1+s_2}{S+E[a]+b}}.$$

Consequently, the unconditional expected firm payoff is *strictly* greater than

$$E \left[ \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1 + s_2}{S + E[a] + b}} \right] = S + E[a] + b.$$

But this violates investor rationality (formally, it violates (4)), giving a contradiction and completing the proof.

**Corollary A-2** *In any equilibrium, there is a non-empty interval  $[\bar{a} - \delta, \bar{a}]$  of firms that do not invest.*

**Proof of Corollary A-2:** Immediate from Corollary 1 and Lemma A-1.

**Proof of Proposition 4:**

*Claim:* There is a non-empty interval  $[\bar{a} - \delta, \bar{a}]$  of firms that make strictly positive profits, i.e., obtain a payoff strictly in excess of  $S + a$ .

*Proof of Claim:* Suppose to the contrary that this is not the case. i.e., that one can find a firm  $a$  arbitrarily close to  $\bar{a}$  that has a payoff of  $S + a$ .

Consider any repurchase offer  $s_1 > 0$ . If  $P_1(s_1) < S + \bar{a}$ , then by supposition one can find a firm that could strictly increase its payoff by repurchasing  $s_1$ , a contradiction. Hence  $P_1(s_1) \geq S + \bar{a}$ . So from (4), the beliefs associated with  $s_1$  must be such that

$$E[a + b \mathbf{1}_{S - s_1 - s_2 \geq I} | s_1] \geq \bar{a}. \quad (\text{A-13})$$

There are two separate cases, which we deal with in turn. In the first case,  $E[a | s_1] = \bar{a}$ . By the NDOC restriction on beliefs, it follows that if the firm offers  $s_2 = S - s_1 - I < 0$  so that investment is possible, the firm's equilibrium payoff (5) is

$$\frac{I + a + b}{\left(1 - \frac{s_1}{S + \bar{a} + b}\right) \left(1 - \frac{S - s_1 - I}{S - s_1 + \bar{a} + b}\right)} = \frac{I + a + b}{I + \bar{a} + b} (S + \bar{a} + b) > S + a,$$

where the inequality follows from  $I > S$  and  $b > 0$ . Consequently, any firm  $a$  is able to

achieve a payoff strictly in excess of  $S + a$  by first repurchasing  $s_1$  and then at date 2 issuing enough shares to fund investment  $I$ . The contradiction completes the proof of the claim.

The remainder of the proof deals with the second case, in which  $E[a|s_1] < \bar{a}$ . In this case, inequality (A-13) implies that  $\Pr(s_2 \text{ s.t. } S - s_1 - s_2 \geq I|s_1) > 0$ , and hence that there exists  $s_2$  with  $S - s_1 - s_2 \geq I$  such that  $E[a + b|s_1, s_2] \geq \bar{a}$ . So by (3), firm  $a$ 's payoff from playing  $(s_1, s_2)$  is weakly greater than

$$\frac{S - s_1 - s_2 + a + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right) \left(1 - \frac{s_2}{S - s_1 + \bar{a}}\right)}.$$

By the equilibrium condition, the unconditional expected equilibrium payoff of a firm is at least

$$\frac{S - s_1 - s_2 + E[a] + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right) \left(1 - \frac{s_2}{S - s_1 + \bar{a}}\right)} \geq \frac{I + E[a] + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right) \left(\frac{I + \bar{a}}{S - s_1 + \bar{a}}\right)}, \quad (\text{A-14})$$

where the inequality follows from (7) and  $S - s_1 - s_2 \geq I$ .

Since  $P_1(s_1)$  is bounded below by  $S + \underline{a}$ , the term  $\frac{s_1}{P_1(s_1)}$  approaches 0 as  $s_1$  approaches 0. Consequently, the limiting value of the RHS of (A-14) is

$$(I + E[a] + b) \frac{S + \bar{a}}{I + \bar{a}}. \quad (\text{A-15})$$

Because the above argument holds for any initial choice of  $s_1 > 0$ , the unconditional expected equilibrium payoff of a firm is at least (A-15). Moreover, by (7), expression (A-15) is itself strictly greater than  $S + E[a] + b$ . But this violates investor rationality (formally, it violates (4)), giving a contradiction.

*Completing the proof:* By Corollary A-2 and the Claim, there exists  $\delta' > 0$  such that all firms in  $[\bar{a} - \delta', \bar{a}]$  make strictly positive profits and do not invest. Let  $\varepsilon > 0$  be the minimum profits made by a firm in this interval. (Note that the minimum is well-defined because a firm's equilibrium payoff is continuous in  $a$ : if this is not the case, there is a profitable deviation for some  $a$ .) Then choose  $\delta \in (0, \delta')$  sufficiently small such that, for all  $a \in [\bar{a} - \delta, \bar{a}]$ ,

$a + \varepsilon > \bar{a}, a + b > \bar{a}$ , and  $(S + a) \frac{\bar{a}}{a} < S + a + \varepsilon$ . To complete the proof, we show all firms in  $[\bar{a} - \delta, \bar{a}]$  repurchase, and make strictly positive profits from the repurchase transaction.

Suppose to the contrary that there exists some firm  $a \in [\bar{a} - \delta, \bar{a}]$  that either does not repurchase, or else makes weakly negative profits from the repurchase: formally, either  $s_1(a) \leq 0$ , or  $s_1(a) > 0$  with  $P_1(s_1(a)) \geq S + a$ ; and either  $s_2(a) \leq 0$ , or  $s_2(a) > 0$  with  $P_2(s_1(a), s_2(a)) \geq \frac{S - s_1(a) + a}{1 - \frac{s_1(a)}{P_1(s_1(a))}}$ .

We first show that firm  $a$ 's payoff is bounded above by

$$\frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{P_1(s_1(a))}}. \quad (\text{A-16})$$

If  $s_2(a) > 0$  this is immediate. Otherwise, (5) and the fact that by (3) (and using  $a \leq \bar{a}$  and the firm does not invest)  $P_2(s_1(a), s_2(a)) \leq \frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{P_1(s_1(a))}}$  together imply that the firm's payoff is bounded above by

$$\frac{S - s_1(a) - s_2(a) + a}{\left(1 - \frac{s_1(a)}{P_1(s_1(a))}\right) \left(1 - \frac{s_2(a)}{S - s_1(a) + \bar{a}}\right)} = \frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{P_1(s_1(a))}} \frac{S - s_1(a) - s_2(a) + a}{S - s_1(a) - s_2(a) + \bar{a}},$$

which is below expression (A-16).

If  $s_1(a) > 0$ , expression (A-16) is in turn bounded above by

$$\frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{S + a}} = (S + a) \frac{S - s_1(a) + \bar{a}}{S - s_1(a) + a} \leq (S + a) \frac{\bar{a}}{a}.$$

But this is less than  $S + a + \varepsilon$ , a contradiction.

Consequently, it must be the case that  $s_1(a) \leq 0$ . Observe that if  $P_1(s_1(a)) \leq S + a + \varepsilon$ , from (A-16) and the fact that  $a + \varepsilon > \bar{a}$ , firm  $a$ 's payoff is bounded above by

$$(S + a + \varepsilon) \frac{S - s_1(a) + \bar{a}}{S - s_1(a) + a + \varepsilon} < S + a + \varepsilon,$$

which again is a contradiction. Hence  $P_1(s_1(a)) > S + a + \varepsilon > S + \bar{a}$ . It then follows from

(4) that there must exist  $s_2$  such that  $S - s_1(a) - s_2 \geq I$  and

$$E[S + a + b \mathbf{1}_{S - s_1 - s_2 \geq I} | s_1 = s_1(a), s_2] \geq P_1(s_1(a)). \quad (\text{A-17})$$

On the one hand, the equilibrium payoff of firm  $a \in A$  is—using (A-16), together with  $P_1(s_1(a)) > S + \bar{a}$ —bounded above by

$$P_1(s_1(a)) \frac{S - s_1(a) + \bar{a}}{P_1(s_1(a)) - s_1(a)} \leq P_1(s_1(a)) \frac{I + \bar{a}}{P_1(s_1(a)) + I - S}.$$

On the other hand, the payoff to firm  $a$  to instead deviating and using strategy  $(s_1(a), s_2)$ , where  $s_2$  is as above, is bounded below by

$$\frac{S - s_1(a) - s_2 + a + b}{1 - \frac{s_1(a) + s_2}{P_1(s_1(a))}} \geq \min \left\{ P_1(s_1(a)), P_1(s_1(a)) \frac{I + a + b}{P_1(s_1(a)) + I - S} \right\}.$$

Since this is strictly greater than the upper bound on firm  $a$ 's equilibrium payoff, firm  $a$  has the incentive to deviate. This contradicts the equilibrium condition, and completes the proof.

**Proof of Proposition 5:**

*Perturbation (I), exogenous probability of no capital market transaction at date 1:*

The proof is by construction. Take the equilibrium actions stated in the proof of Proposition 3. If Case 2 of the proof of Proposition 3 applies, set  $A_1 = [\underline{a}, a_1]$  and  $A_0 = \emptyset$ , so that no firm does nothing at date 1 and then issues at date 2.

Relative to the proof of Proposition 3, the new step entails the handling of off-equilibrium beliefs to ensure that NDOC is satisfied. Write  $(\tilde{s}_1, \tilde{s}_2)$  for an arbitrary off-equilibrium action.

Off-equilibrium beliefs are as follows. Date 2 repurchases  $\tilde{s}_2 > 0$  are associated with the best firm  $\bar{a}$  and issues  $\tilde{s}_2 < 0$  are associated with the worst firm  $\underline{a}$ . At date 1, repurchases  $\tilde{s}_1 > 0$  are associated with the best firm  $\bar{a}$  with probability  $1 - \varepsilon$  and the worst firm with probability  $\varepsilon$ ; while issues  $\tilde{s}_1 < 0$  are associated with the best firm  $\bar{a}$  with probability  $\varepsilon$  and



the worst firm  $\underline{a}$  with probability  $1 - \varepsilon$ . Note that these date 1 beliefs, together with the fact that with probability  $\alpha > 0$  all firm types do nothing at date 1, mean that the specification of date 2 beliefs satisfies NDOC.

Write  $\tilde{P}_1$  and  $\tilde{P}_2$  for the associated off-equilibrium prices. Given the stated off-equilibrium beliefs, there exists some  $\kappa > 0$  such that

$$\begin{aligned} \tilde{P}_1 &\geq S + \bar{a} - \varepsilon\kappa && \text{if } \tilde{s}_1 > 0 \\ &\leq S + \underline{a} + b + \varepsilon\kappa && \text{if } \tilde{s}_1 < 0 \end{aligned} \quad (\text{A-18})$$

Moreover,

$$\tilde{P}_2 = \begin{cases} \frac{S - \tilde{s}_1 + \bar{a} + b \mathbf{1}_{S - \tilde{s}_1 - \tilde{s}_2 \geq I}}{1 - \frac{\tilde{s}_1}{\tilde{P}_1}} & \text{if } \tilde{s}_2 > 0 \\ \frac{S - \tilde{s}_1 + \underline{a} + b \mathbf{1}_{S - \tilde{s}_1 - \tilde{s}_2 \geq I}}{1 - \frac{\tilde{s}_1}{\tilde{P}_1}} & \text{if } \tilde{s}_2 < 0 \end{cases} \quad (\text{A-19})$$

From the proof of Proposition 3, the equilibrium payoff of any firm  $a \in [\underline{a}, \bar{a}]$  strictly exceeds the payoff from direct issue under investor beliefs  $\underline{a}$ , namely  $\frac{I+a+b}{1+\frac{I-S}{S+\underline{a}+b}}$ . Moreover, for firms  $a$  sufficiently close to  $\bar{a}$ , the equilibrium payoff also strictly exceeds the payoff from doing nothing, namely  $S + a$ . (Of course, this relation holds weakly for *all* firms.) Hence it is possible to choose  $\varepsilon > 0$  such that, for all firms  $a \in [\underline{a}, \bar{a}]$ ,

$$\max \left\{ \frac{I + a + b}{1 + \frac{I - S - \varepsilon\kappa}{S + \underline{a} + b + \varepsilon\kappa}}, a \frac{S + \bar{a} - \varepsilon\kappa}{\bar{a} - \varepsilon\kappa} \right\} < \text{equilibrium payoff of firm } a. \quad (\text{A-20})$$

Moreover, and using  $b > 0$  and inequality (7), choose  $\varepsilon > 0$  sufficiently small such that, in addition to inequality (A-20), the following pair of inequalities holds:

$$\frac{a}{\underline{a} + b} \leq \frac{I + a + b}{I + \underline{a} + b} \quad \text{if } a \in [\underline{a} + b, \underline{a} + b + \varepsilon\kappa], \quad (\text{A-21})$$

$$\underline{a} + b + \varepsilon\kappa \leq \bar{a} - \varepsilon\kappa. \quad (\text{A-22})$$

Firm  $a$ 's payoff from an arbitrary off-equilibrium strategy  $(\tilde{s}_1, \tilde{s}_2)$  is

$$\frac{S - \tilde{s}_1 - \tilde{s}_2 + a + b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2 \geq I}}{1 - \frac{\tilde{s}_1}{\tilde{P}_1} - \frac{\tilde{s}_2}{\tilde{P}_2}}.$$

First, observe that

$$-\frac{\tilde{s}_2}{\tilde{P}_2} \geq -\frac{\tilde{s}_2}{S - \tilde{s}_1 + \underline{a} + b} \left(1 - \frac{\tilde{s}_1}{\tilde{P}_1}\right).$$

This follows directly from (A-19) if  $\tilde{s}_2 < 0$ , and from (A-19) together with (7) if  $\tilde{s}_2 > 0$ .

Second, observe that

$$-\frac{\tilde{s}_1}{\tilde{P}_1} \geq -\frac{\tilde{s}_1}{S + \underline{a} + b + \varepsilon\kappa}.$$

This follows directly from (A-18) if  $\tilde{s}_1 < 0$ , and from (A-18) together with (A-22) if  $\tilde{s}_1 > 0$ .

Consequently, firm  $a$ 's payoff is bounded above by

$$\frac{S - \tilde{s}_1 - \tilde{s}_2 + a + b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2 \geq I}}{\left(1 - \frac{\tilde{s}_1}{S + \underline{a} + b + \varepsilon\kappa}\right) \left(1 - \frac{\tilde{s}_2}{S - \tilde{s}_1 + \underline{a} + b}\right)} = \frac{S - \tilde{s}_1 - \tilde{s}_2 + a + b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2 \geq I}}{S - \tilde{s}_1 - \tilde{s}_2 + \underline{a} + b} \frac{S - \tilde{s}_1 + \underline{a} + b}{S - \tilde{s}_1 + \underline{a} + b + \varepsilon\kappa} (S + \underline{a} + b + \varepsilon\kappa). \quad (\text{A-23})$$

To complete the proof, by (A-20) it is sufficient to show that expression (A-23) is bounded above by either the LHS of (A-20), or by  $S + a$ . There are four cases:

If  $S - \tilde{s}_1 - \tilde{s}_2 \geq I$  it is immediate that (A-23) is bounded above by  $\frac{I+a+b}{I+\underline{a}+b} (S + \underline{a} + b + \varepsilon\kappa)$ , which is the first term in the LHS of (A-20).

If  $S - \tilde{s}_1 - \tilde{s}_2 < I$  and  $a \leq \underline{a} + b$  then (A-23) is bounded above by  $(S + \underline{a} + b + \varepsilon\kappa)$ .

If  $S - \tilde{s}_1 - \tilde{s}_2 < I$  and  $a \in [\underline{a} + b, \underline{a} + b + \varepsilon\kappa]$  then (A-23) is bounded above by  $\frac{a}{\underline{a}+b} (S + \underline{a} + b + \varepsilon\kappa)$ , and the result then follows from (A-21).

Finally, consider the case  $S - \tilde{s}_1 - \tilde{s}_2 < I$  and  $a > \underline{a} + b + \varepsilon\kappa$ . Note first that since  $S - \tilde{s}_1 - \tilde{s}_2 < I$ , the off-equilibrium beliefs imply that the firm weakly loses money on its

date 2 transactions, so that its payoff is bounded above by

$$\frac{S - \tilde{s}_1 + a}{1 - \frac{\tilde{s}_1}{\tilde{P}_1}} = \tilde{P}_1 \frac{S - \tilde{s}_1 + a}{\tilde{P}_1 - \tilde{s}_1}.$$

If  $\tilde{s}_1 > 0$ , this expression is bounded above by  $\max \left\{ S + a, \frac{a\tilde{P}_1}{\tilde{P}_1 - S} \right\}$ , which by (A-18) is bounded above by  $\max \left\{ S + a, a \frac{S + \underline{a} - \varepsilon\kappa}{\underline{a} - \varepsilon\kappa} \right\}$ . If instead  $\tilde{s}_1 < 0$  this expression is bounded above by  $\max \left\{ S + a, \tilde{P}_1 \right\}$ , which by (A-18) is bounded above by  $\max \{ S + a, S + \underline{a} + b + \varepsilon\kappa \} = S + a$ . This completes the proof of Part (I).

*Perturbation (II), exogenous upper bound  $\bar{S}$  on repurchase size:*

When the equilibrium of the proof of Proposition 3 falls in Case 1, off-equilibrium beliefs are defined in an identical way to Part (I) above, and the proof is identical.

For the remainder of the proof suppose that the equilibrium of the proof of Proposition 3 falls in Case 2. As a preliminary step, recall that the proof of Proposition 3 entails choosing  $s_1$  to lie below some bound (defined in the proof). Here, choose  $\bar{S}$  to lie below this same bound. Then set  $s_1 = \bar{S}$ .

Choose the sets  $A_0$  and  $A_1$  so that  $A_1$  contains  $\underline{a}$  and  $A_0$  contains a point  $\underline{a}^+$  that is close to  $\underline{a}$ . Such a choice is always possible. (It does not matter here whether  $\gamma_0 > 0$  or  $\gamma_0 = 0$ .) Off-equilibrium beliefs are identical to Part (I), with the exception of off-equilibrium beliefs following  $s_1 = 0$ : now, these beliefs put probability 1 on type  $a_2$  if  $\tilde{s}_2 > 0$ , and put probability 1 on type  $\underline{a}^+$  if  $\tilde{s}_2 < 0$ . Note that these beliefs satisfy NDOC.

Given these beliefs, a firm's payoff from deviating to  $(s_1 = 0, \tilde{s}_2)$ , where  $\tilde{s}_2 > 0$ , is

$$\frac{S - \tilde{s}_2 + a}{1 - \frac{\tilde{s}_2}{S + a_2}}.$$

For  $a \leq a_2$  this expression is below the do-nothing payoff of  $S + a$ . If instead  $a > a_2$ , this expression is below  $\frac{S - \bar{S} + a}{1 - \frac{\bar{S}}{S + a_2}} = \frac{S - s_1 + a}{1 - \frac{s_1}{S + a_2}}$ , which is the payoff from following the equilibrium strategy  $(s_1, 0)$  (recall the repurchase price is  $S + a_2$ ). Hence no deviation of this type strictly improves a firm's payoff relative to the equilibrium payoff.

Finally, a parallel proof to Part (I) establishes that provided  $\underline{a}^+$  is chosen sufficiently close to  $\underline{a}$ , no deviation of the type  $(s_1 = 0, \tilde{s}_2)$  with  $\tilde{s}_2 < 0$  strictly improves a firm's payoff relative to the equilibrium payoff.

All other deviations are handled exactly as in Part (I), completing the proof.

**Lemma A-2** *If an equilibrium features capital transactions  $(s'_1, s'_2)$  and  $(s''_1, s''_2)$  with  $S - s'_1 - s'_2 = S - s''_1 - s''_2$ , then the associated transaction prices  $P'_1, P'_2, P''_1, P''_2$  are such that*

$$1 - \frac{s'_1}{P'_1} - \frac{s'_2}{P'_2} = 1 - \frac{s''_1}{P''_1} - \frac{s''_2}{P''_2}. \quad (\text{A-24})$$

**Proof of Lemma A-2:** Let  $a'$  and  $a''$  be firms that play  $(s'_1, s'_2)$  and  $(s''_1, s''_2)$  respectively. The equilibrium conditions for firm  $a'$  include

$$\frac{S - s'_1 - s'_2 + a' + b\mathbf{1}_{S-s'_1-s'_2 \geq I}}{1 - \frac{s'_1}{P'_1} - \frac{s'_2}{P'_2}} \geq \frac{S - s''_1 - s''_2 + a' + b\mathbf{1}_{S-s''_1-s''_2 \geq I}}{1 - \frac{s''_1}{P''_1} - \frac{s''_2}{P''_2}},$$

which simplifies to  $1 - \frac{s''_1}{P''_1} - \frac{s''_2}{P''_2} \geq 1 - \frac{s'_1}{P'_1} - \frac{s'_2}{P'_2}$ . The symmetric equilibrium condition for a firm  $a''$  playing  $(s''_1, s''_2)$  then implies (A-24). QED

**Proof of Proposition 6:**

*Part (A):* Firms that repurchase  $s_1$  at date 1 are, at date 2, in exactly the situation characterized by Proposition 2. Consequently, at date 2 a positive-measure subset of these firms must issue an amount  $s_2$  such that investment is possible, i.e.,  $S - s_1 - s_2 \geq I$  at date 2. If almost all firms that repurchase  $s_1$  also issue  $s_2$ , then  $P_1 = P_2$ , and the proof is complete. Otherwise, let  $A_1^{s_1}$  denote the set of firms that repurchase  $s_1$  at date 1. From Proposition 2, there exists  $a^*$  such that almost all firms in  $A_1^{s_1} \cap [a^*, \bar{a}]$  choose not to issue  $s_2$  at date 2. The equilibrium condition for any firm  $a \in A_1^{s_1} \cap [a^*, \bar{a}]$  in this non-issuing set is

$$\frac{S - s_1 + a}{1 - \frac{s_1}{P_1}} \geq \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}}.$$

Hence

$$E \left[ \frac{S - s_1 + a}{1 - \frac{s_1}{P_1}} \mid a \in A_1^{s_1} \cap [a^*, \bar{a}] \right] > E \left[ \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}} \mid a \in A_1^{s_1} \cap [\underline{a}, a^*] \right],$$

so that the date 2 share price of non-issuing firms strictly exceeds the date 2 share price of issuing firms, i.e.,  $P_2$ . Since the date 1 share price equals the conditional expectation of the date 2 share price, it follows that  $P_2 < P_1$ .

*Part (B):* First, suppose a positive measure of firms issue  $s'_1 < 0$ . If  $S - s'_1 \geq I$ , then by the argument of Proposition 2, almost all firms play  $s'_2 = 0$ . If instead  $S - s'_1 < I$ , then by the argument of Proposition 2, there exists  $s'_2$  such that  $S - s'_1 - s'_2 \geq I$  and such that a positive measure of firms play  $(s'_1, s'_2)$ , and almost all the remainder play  $(s'_1, 0)$ . Moreover,  $\Pr(s'_2 | s'_1) = 1$ , as follows. Suppose to the contrary that  $\Pr(s'_2 | s'_1) < 1$ . The equilibrium condition for a firm  $a$  that plays  $(s'_1, 0)$  is

$$\frac{S - s'_1 + a}{1 - \frac{s'_1}{P_1(s'_1)}} \geq \frac{S - s'_1 - s'_2 + a + b}{\left(1 - \frac{s'_1}{P_1(s'_1)}\right) \left(1 - \frac{s'_2}{E[S - s_1 + a + b | s'_1, s'_2]}\right)},$$

which simplifies (using  $s'_2 < 0$ ) to

$$\frac{S - s'_1 + a}{E[S - s'_1 + a + b | s'_1, s'_2]} \geq 1 - \frac{b}{s'_2}.$$

Hence any firm  $a$  that plays  $(s'_1, 0)$  must satisfy  $a > E[a + b | s'_1, s'_2]$ . By Lemma 1, firms that play  $(s'_1, 0)$  are better than firms that play  $(s'_1, s'_2)$ . Hence  $P_1(s'_1) < S + \sup\{a : a \text{ plays } s'_1\}$ ; and almost all firms sufficiently close to  $\sup\{a : a \text{ plays } s'_1\}$  play  $(s'_1, 0)$ , and would obtain a higher payoff by doing nothing, a contradiction. This establishes that  $P_2(s'_1, s'_2) = P_1(s'_1)$ .

We next establish the price comparison with firms that issue after previously repurchasing, i.e.,  $P_2(s_1, s_2)$ . Given the first step, we handle the two cases in the proposition together: let  $(s'_1, s'_2)$  be a strategy with  $S - s'_1 - s'_2 \geq I$  and  $s'_1, s'_2 \leq 0$ . At any date with strictly

positive issue, the price is  $P_2(s'_1, s'_2) = E[S + a + b | s'_1, s'_2]$ . We first show that

$$S - s'_1 - s'_2 \geq S - s_1 - s_2. \quad (\text{A-25})$$

The proof is by contradiction: suppose instead that  $S - s'_1 - s'_2 < S - s_1 - s_2$ . So by Lemma 1,  $E[a | s'_1, s'_2] > E[a | s_1, s_2]$ . By Part (A),  $P_1(s_1) \geq P_2(s_1, s_2) = \frac{S - s_1 + E[a | s_1, s_2] + b}{1 - \frac{s_1}{P_1(s_1)}}$ , and so  $P_1(s_1) \geq S + E[a | s_1, s_2] + b$ . Hence

$$\left(1 - \frac{s_1}{P_1(s_1)}\right) \left(\frac{1}{S - s_1 + E[a | s_1, s_2] + b}\right) \geq \frac{1}{S + E[a | s_1, s_2] + b}.$$

So the payoff to firm  $a$  from  $(s_1, s_2)$  is

$$\frac{S - s_1 - s_2 + a + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right) \left(1 - \frac{s_2}{S - s_1 + E[a | s_1, s_2] + b}\right)} \leq \frac{S - s_1 - s_2 + a + b}{\frac{S - s_1 - s_2 + E[a | s_1, s_2] + b}{S + E[a | s_1, s_2] + b}}.$$

Fix a firm playing  $(s_1, s_2)$  with  $a > E[a | s_1, s_2]$ . By the supposition  $S - s'_1 - s'_2 < S - s_1 - s_2$ , the payoff from  $(s_1, s_2)$  for firm  $a$  is strictly less than

$$\frac{S - s'_1 - s'_2 + a + b}{\frac{S - s'_1 - s'_2 + E[a | s_1, s_2] + b}{S + E[a | s_1, s_2] + b}},$$

which since  $E[a | s'_1, s'_2] > E[a | s_1, s_2]$  is in turn strictly less than

$$\frac{S - s'_1 - s'_2 + a + b}{\frac{S - s'_1 - s'_2 + E[a | s'_1, s'_2] + b}{S + E[a | s'_1, s'_2] + b}} = \frac{S - s'_1 - s'_2 + a + b}{1 - \frac{s'_1 + s'_2}{S + E[a | s'_1, s'_2] + b}}.$$

But this contradicts the equilibrium condition, since the RHS is firm  $a$ 's payoff from deviating and playing  $(s'_1, s'_2)$ , and establishes inequality (A-25).

To complete the proof of Part (B), we consider in turn the cases in which (A-25) holds with equality, and in which it holds strictly. First, if (A-25) holds with equality, Lemma A-2

implies

$$-\frac{s_1}{P_1(s_1)} - \frac{s_2}{P_2(s_1, s_2)} = -\frac{s'_1 + s'_2}{P_2(s'_1, s'_2)}.$$

From Part (A),  $P_2(s_1, s_2) \leq P_1(s_1)$ , and since  $s_1 \geq 0$ , this implies

$$-\frac{s_1 + s_2}{P_2(s_1, s_2)} \leq -\frac{s'_1 + s'_2}{P_2(s'_1, s'_2)},$$

which since (A-25) holds with equality, implies  $P_2(s_1, s_2) \geq P_2(s'_1, s'_2)$ .

Second, if instead (A-25) holds strictly, taking the expectation over the equilibrium condition for all firms  $a$  playing  $(s_1, s_2)$ , together with the implication of Lemma 1 that  $E[a|s_1, s_2] > E[a|s'_1, s'_2]$ , yields

$$\frac{S - s_1 - s_2 + E[a|s_1, s_2] + b}{1 - \frac{s_1}{P_1(s_1)} - \frac{s_2}{P_2(s_1, s_2)}} \geq \frac{S - s'_1 - s'_2 + E[a|s_1, s_2] + b}{1 - \frac{s'_1}{P_1(s'_1)} - \frac{s'_2}{P_2(s'_1, s'_2)}} > \frac{S - s'_1 - s'_2 + E[a|s'_1, s'_2] + b}{1 - \frac{s'_1}{P_1(s'_1)} - \frac{s'_2}{P_2(s'_1, s'_2)}}.$$

Since the first and last terms in this inequality are simply  $P_2(s_1, s_2)$  and  $P_2(s'_1, s'_2)$  respectively, this establishes  $P_2(s_1, s_2) > P_2(s'_1, s'_2)$ .

*Part (C):* By the argument of Proposition 2, either almost all firms that play 0 at date 1 also play 0 at date 2; or there exists  $s'_2$  such that  $S - s'_2 \geq I$  and almost all firms play either 0 or  $s'_2$  at date 2. The date 1 price for do-nothing firms satisfies

$$P_1(0) = E[S + a|(0, 0)] \Pr(0|0) + P_2(0, s'_2) \Pr(s'_2|0).$$

From Parts (A) and (B), we know  $P_2(0, s'_2) \leq P_2(s_1, s_2) \leq P_1(s_1)$ . From the equilibrium condition, and firm  $a$  that plays  $(0, 0)$  satisfies  $S + a \leq P_1(s_1)$ , since otherwise firm  $a$  would be strictly better off playing  $(s_1, 0)$ . The result then follows, completing the proof.

**Proof of Proposition 7:** *Part (A):* By the equilibrium condition for firm  $a''$ ,

$$\frac{S - s'' + a''}{1 - \frac{s''}{P''}} \geq \frac{S - s' + a''}{1 - \frac{s'}{P'}}. \quad (\text{A-26})$$

Since  $s'' > s'$ , it is immediate that  $s''/P'' > s'/P'$ , establishing (ii). By the equilibrium condition for firm  $a'$ ,

$$\frac{S - s' + a'}{1 - \frac{s'}{P'}} \geq \frac{S - s'' + a'}{1 - \frac{s''}{P''}}. \quad (\text{A-27})$$

Multiplying (A-27) by  $-1$  and combining with (A-26) yields

$$\frac{a'' - a'}{1 - \frac{s''}{P''}} \geq \frac{a'' - a'}{1 - \frac{s'}{P'}}.$$

If  $a' > a''$  then this inequality contradicts (ii); hence  $a'' \geq a'$ , which (since  $a'' \neq a'$ ) establishes (iii).

Firm  $a'$  also has the choice of doing nothing, and so the equilibrium condition implies  $S + a' \geq P'$ , i.e., firm  $a'$  pays weakly less than its stock is worth. Consequently,

$$\frac{S - s'' + a'}{1 - \frac{s''}{P''}} \geq \frac{S - s' + a'}{1 - \frac{s'}{P'}},$$

i.e., if firm  $a'$  were able to repurchase more stock at the constant price  $P'$ , it would weakly prefer to do so. Combined with (A-27), it then follows that  $P'' \geq P'$ , establishing (i), and completing the proof of Part (A).

*Part (B):* The proof is exactly the same as the final paragraph of the proof of Part (B) of Proposition 6.

**Proof of Proposition 8:** By hypothesis, there are only a finite number of strategies played in equilibrium. Throughout the proof, we ignore any firm that plays a strategy that is played by only a measure zero set of firms. Partition the remaining firms so that if two firms share the same  $s_1 + s_2$  and make the same investment decision, then they lie in the same partition element. Let  $A^1, \dots, A^M$  be the partition elements in which firms invest. Let  $A^0$  be the set of non-investing firms. Without loss, order the sets  $A^1, \dots, A^M$  so that  $i > j$  is equivalent to  $S - s_1 - s_2$  being smaller for firms in  $A^i$  than  $A^j$ . By Lemma 1, it follows that  $A^i$  are intervals, with  $A^i > A^j$  if  $i > j$ . By Corollary 1,  $\inf A^1 = \underline{a}$ . Define  $s^i = s_1 + s_2$  for all firms



in  $A^i$ , and by Lemma A-2, and define  $N^i = 1 - \frac{s_1(a)}{P_1(s_1(a))} - \frac{s_2(a)}{P_2(s_1(a), s_2(a))}$  for all firms  $a \in A^i$ .

If  $s_1 \leq 0$  for some firm in  $A^i$ , an easy adaption of the arguments of Propositions 1 and 2 implies that all firms that use this action at date 1 take the same date 2 action,  $s_2$ . Moreover, by the definition of  $A^i$ , all such firms invest. So for these firms, the date 1 and 2 transaction prices coincide, and by (4), both equal  $E[S + a + b | s_1(a), s_2(a)]$ . Hence in this case  $N^i = 1 - \frac{s^i}{E[S + a + b | s_1(a), s_2(a)]}$ .

If instead  $s_1 > 0$  for some firm in  $a \in A^i$ , then by Proposition 6, the date 1 and 2 transaction prices satisfy  $P_1 \geq P_2$ , and so using  $s_2(a) < 0$ ,  $N^i \geq 1 - \frac{s^i}{P_2(s_1(a), s_2(a))}$ . Since  $P_2(s_1(a), s_2(a)) = E\left[\frac{S - s^i + a + b}{N^i} | s_1(a), s_2(a)\right]$ , we know

$$P_2(s_1(a), s_2(a)) \leq E\left[\frac{S - s^i + a + b}{1 - \frac{s^i}{P_2(s_1(a), s_2(a))}} | s_1(a), s_2(a)\right].$$

and hence

$$P_2(s_1(a), s_2(a)) \leq E[S + a + b | s_1(a), s_2(a)],$$

and so

$$N^i \geq 1 - \frac{s^i}{E[S + a + b | s_1(a), s_2(a)]}.$$

Moreover, by Proposition 6, the inequality is strict whenever  $\Pr(\text{invest} | s_1(a)) < 1$ .

The above observations imply

$$N^i \geq 1 - \frac{s^i}{E[S + a + b | a \in A^i]}, \tag{A-28}$$

with the inequality strict whenever  $\Pr(\text{invest} | s_1 \in s_1(A^i)) < 1$ .

We next show that  $\Pr(\text{invest} | s_1 \in s_1(A^i)) < 1$  for at least some  $i$ . Suppose to the contrary that this is not the case. Then  $\Pr(\text{not invest} | s_1 \in s_1(A^0)) = 1$ . So  $E[P_3 | s_1 \in s_1(A^0)] = E[P_3 | a \in A^0] = S + E[a | a \in A^0]$ . But a straightforward adaption of the proof of Proposition 4 implies that there exists an upper interval of firms who obtain a payoff strictly in excess of  $S + a$ , and by Corollary A-2, this upper interval has a non-null intersection with  $A^0$ . But

then  $E[P_3|a \in A^0] > S + E[a|a \in A^0]$ , a contradiction.

Boundary firms  $a^{i*} \equiv \sup(A^i)$  must be indifferent across two adjacent issue paths  $s^{i-1}$  and  $s^i$ , i.e., for all  $i < M$ ,

$$\frac{1}{N^i}(a^{i*} + S + b - s^i) = \frac{1}{N^{i+1}}(a^{i*} + S + b - s^{i+1}). \quad (\text{A-29})$$

The heart of the proof is to establish that inequality (A-28), with the inequality strict for at least some  $i$ , implies

$$N^M > 1 - \frac{s^M}{E[a + S + b|a \in [\underline{a}, a^{M*}]]} \quad (\text{A-30})$$

We establish (A-30) by showing inductively that for any  $i = 1, \dots, M$ ,

$$N^i \geq 1 - \frac{s^i}{E[a + S + b|a \in [\underline{a}, a^{i*}]]} \quad (\text{A-31})$$

The initial case  $i = 1$  is immediate from (A-28) and the earlier observation that  $\inf A_1 = \underline{a}$ . For the inductive step, suppose (A-31) holds at  $i = K - 1 < M$ . We show that (A-31) also holds at  $i = K$ .

Observe first that inequality (A-31) at  $i = K - 1$  is equivalent to

$$\frac{S + a^{(K-1)*} + b - s^{K-1}}{N^{K-1}} \leq \frac{S + a^{(K-1)*} + b - s^{K-1}}{1 - \frac{s^{K-1}}{E[a+S+b|a \in [\underline{a}, a^{(K-1)*}]]}}$$

Since  $a^{(K-1)*} \geq E[a|a \in [\underline{a}, a^{(K-1)*}]]$ , the RHS of this inequality is increasing in  $s^{K-1}$ , i.e., if the share price is  $E[a + S + b|a \in [\underline{a}, a^{(K-1)*}]]$ , the best firm  $a^{(K-1)*}$  in pool  $[\underline{a}, a^{(K-1)*}]$  would be better off raising fewer funds than  $S - s^{K-1}$ . We know  $S - s^K < S - s^{K-1}$ , and so

$$\frac{S + a^{(K-1)*} + b - s^{K-1}}{N^{K-1}} < \frac{S + a^{(K-1)*} + b - s^K}{1 - \frac{s^K}{E[a+S+b|a \in [\underline{a}, a^{(K-1)*}]]}}$$

Combined with the indifference condition (A-29) at  $i = K - 1$ , it follows that

$$N^K \geq 1 - \frac{s^K}{E[a + S + b | a \in [\underline{a}, a^{(K-1)*}]]}.$$

Combined with (A-28), it then follows that

$$N^K \geq 1 - \frac{s^K}{E[a + S + b | a \in A^K \cup [\underline{a}, a^{(K-1)*}]]} = 1 - \frac{s^K}{E[a + S + b | a \in [\underline{a}, a^{K*}]]},$$

which establishes the inductive step. Moreover, this inequality must hold strictly for at least one step.

To complete the proof, note that in equilibrium, for all  $a \in A^M$ ,

$$\frac{S - s^M + a + b}{N^M} \geq S + a.$$

So by (A-30)

$$\frac{S - s^M + a^{M*} + b}{1 - \frac{s^M}{E[S+a+b | a \in [\underline{a}, a^{M*}]]}} > S + a^{M*}.$$

Consequently, by continuity together with (6), there exists  $\tilde{a}^* > a^{M*}$  such that

$$\frac{S - s^M + \tilde{a}^* + b}{1 - \frac{s^M}{E[S+a+b | a \in [\underline{a}, \tilde{a}^*]]}} = S + \tilde{a}^*.$$

It is straightforward to show that there is an equilibrium of the one-period benchmark in which firms in  $[\underline{a}, \tilde{a}^*]$  issue shares at a price  $E[S + a + b | a \in [\underline{a}, \tilde{a}^*]]$  to raise funds  $-s^M$  and invest, while firms  $a \in (\tilde{a}^*, \bar{a}]$  do nothing. This completes the proof.

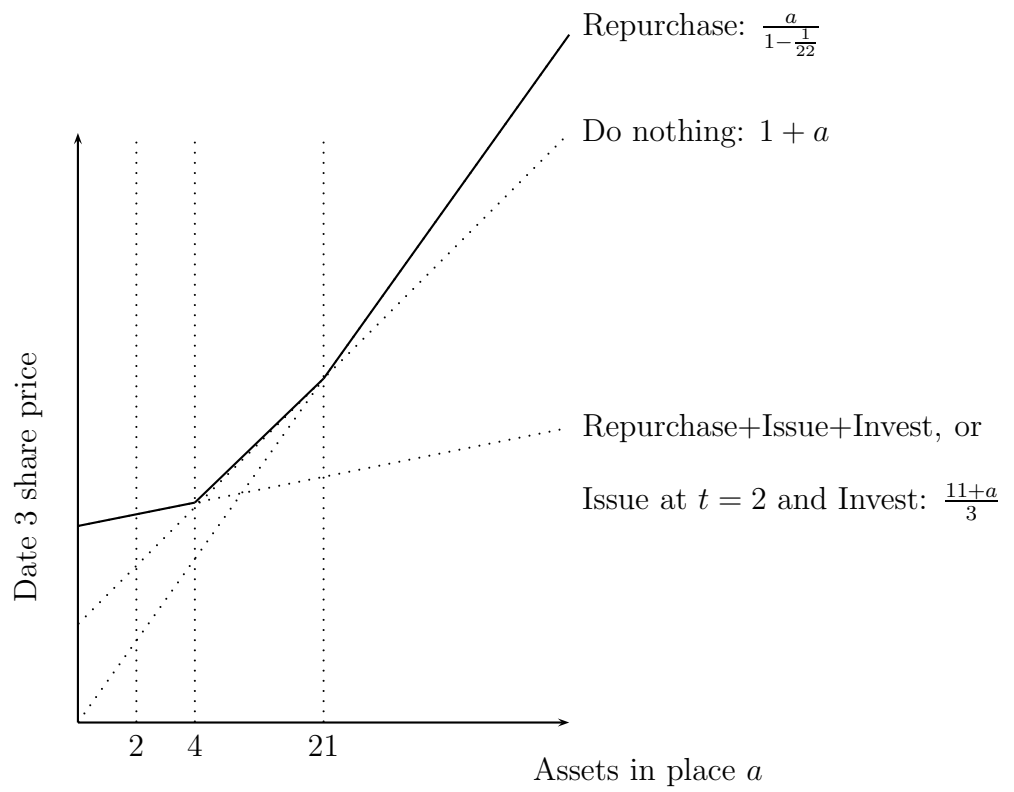


Figure 1: Equilibrium for example described in main text