Why truck distance charges are contagious and drive fuel taxes to the bottom

Svante Mandell\textsuperscript{a} and Stef Proost\textsuperscript{b}

Abstract
This paper analyzes how countries with international and local truck traffic decide to switch from a fuel tax system only to a dual system of fuel taxes and kilometer charging. We show what drives one country to switch and how this affects the level of fuel taxes and the incentives for the other countries to also opt for the dual system. The model is able to partially explain the gradual extension of the kilometer charging in Europe. The model also shows how, in the absence of diesel cars, the gradual introduction of kilometer charges will make fuel taxation for trucks virtually disappear and lead to a system where truck use is mainly taxed by distance charges only but is taxed too heavily. When the fuel tax also has to serve as externality tax on diesel cars, the introduction of distance charges for trucks will give rise to diesel taxes that will be lower than the external cost of diesel cars. In the case of trucks this leads to a sum of diesel taxes and distance charges that will be higher than the external cost of trucks.

Keywords
Diesel taxes, fuel taxes, kilometer charges, tax competition, pricing of trucks

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1. Introduction

This paper addresses how international competition affects the choice of tax instruments on road freight transport. The main tax instrument used in most countries is still an excise tax on diesel fuel. As international trucks can decide in which country to refuel, this has given rise to international fuel tax competition. This is the case in Europe where member states kept almost full authority on excise taxes while the international road freight haulage has expanded strongly over the last 25 years as a result of the EU trade integration. Over the last 15 years, the diesel excise tax has been supplemented in several EU countries by a distance charge. Many other federal countries where member states can add state gasoline taxes face the same issue (US, India, Australia, ...).

Using a simple analytical setting with two countries, we analyze how the addition of a kilometer charge for trucks changes the tax competition game. The main contribution of this paper is to provide a deeper understanding about the dynamics of the tax competition game when a kilometer charge becomes available. Distance charges change the rules of the game as they have to be paid when a truck uses the roads of a country. Consequently, any country that introduces the distance charge can reduce its fuel taxes and threaten the tax base of the neighbors that do not yet have a distance charge. As a response, the neighboring countries will also implement distance charging and the result will be the spreading out of high distance charging and very low fuel charges.

More precisely, we offer the following results. Consider first the case where there are tax instruments other than fuel excises available to tax diesel cars. Then if only diesel taxes are available to tax trucks, in the Nash equilibrium, the diesel taxes can be lower or higher than the external and infrastructure costs of trucks. The taxes will typically be low in countries of equal size when there is intensive tax competition. When countries differ in size, the tax in the small country will be lower than in the larger country. This confirms results from the literature (Kanbur & Keen, 1993). When also distance charges are available and their implementation costs are low, all countries will adopt distance charges for trucks and fuel taxes are driven to the bottom. The distance charges will all be higher than the external cost and the margin will be highest in the smaller countries. Moving from diesel taxes to distance charges can therefore be welfare decreasing.

As diesel cars are a good substitute for heavily taxed gasoline cars, it is important to take these side effects into account. We offer therefore results for the case where there are no other instruments to tax diesel cars. Thus, consider the case where fuel taxes are used to tax not only trucks but also diesel cars. If there are no distance charges, the fuel tax will have to balance the optimal taxation of diesel cars and trucks. As only one instrument is used, it is impossible to set the diesel fuel tax equal to the external costs
of both types of vehicles and the tax will be a weighted average of external costs of diesel cars, trucks and margins on international trucking. Again the tax competition effects on the fuel market for international trucks can increase or decrease the tax but diesel use by cars is typically less vulnerable to tax competition. The result will be that the diesel tax in one country reacts less strongly to tax changes in a neighboring country. Introduce now distance charges for trucks. Both countries will use distance charges and fuel excises. The sum of distance charges and fuel excises will be higher than the external cost for trucks and the diesel tax will be lower than the external costs of diesel cars. Again there is no guarantee that the introduction of distance charges improves pricing from a welfare perspective.

The paper is organized as follows. In section 2 we illustrate some of the recent truck charging developments. In section 3 we review the literature. In section 4 we set up the model. Section 5 is devoted to the analysis of the game with fuel taxes as the only instrument but where diesel cars are unimportant or can be taxed using other instruments. Section 6 analyses the effects of the introduction of a distance charge. Section 7 introduces diesel cars that also need to be taxed using a fuel tax. Section 8 concludes with caveats.

2. Charging trucks for road use

Almost all countries charge excises for diesel fuel used by trucks. Because trucks can cover 1000 to 2000 km with a single tank, countries or regions engage in fuel tax competition. In the US, state diesel excise taxes represent 50% or more of the total diesel excise. Also in India, states are responsible for an important part of the total diesel excise. Within the EU, some smaller countries chose a strategy of low excise taxes and this has brought the EU to negotiate a minimum tax level. In 2012, Germany charged an excise of 0.589 $/liter while Luxemburg, a tiny neighbor, charged only the EU minimum that is 0.343 $/liter (IEA, 2013). I addition to the diesel excise taxes, the EU member states were allowed to charge additional fees for the use of the road under the form of a vignette (annual, monthly, or daily fixed payment per vehicle). This Eurovignette had to be non-discriminatory and had to be based on the actual infrastructure costs2 (see Vierth & Schleusser, 2012). Over time it was extended such that it can also charge for environmental costs.

Technological progress in charging techniques made that several countries with a lot of transit wanted to introduce distance based charging. In 2001, Switzerland (not an EU member) replaced its vignette system by a km charging system that charged trucks much more than before. The neighboring countries followed: Austria (a transit country parallel to Switzerland) in 2004, Germany in 2005 (although it wanted to start earlier), Czech Republic in 2007, Slovakia in 2010 and Poland in 2011. Other countries (Belgium,..) are

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2 See directive 1999/62/EC followed by directives 2006/38/EC and 206/103/EC.
preparing its introduction. Some other EU countries had already a tolling system for most of its motorways (France, Italy, Spain). In most countries this serves to cover the infrastructure costs with the restriction that an untolled alternative (national roads) had to be available. Appendix A represents the different charging systems in place in Europe in 2012. This shows a clear pattern, as the introduction of distance charges was geographically strongly correlated. The member states in the center of Europe tend to use distance based charges. This in spite of the transaction costs associated to a distance based system vary between 10 and 20% of the revenues (to be checked, see Hamilton & Eliasson, 2013), probably much larger than the fuel excise transaction costs. States further from the center use vignettes or no charges at all apart from fuel taxes. These observations are our main motivation for analyzing why one country moves to the more costly dual system, how this may force other countries to follow this example and what are the tax levels this may generate.

There are three interesting transition features to note. First, all distance based charging generated a lot more revenues than the vignettes they replaced. In Germany distance charge revenues were 6.5 times larger than the previous Eurovignette revenues. This is curious as both systems are supposed to be limited by the sum of the actual costs of road building and maintenance as well as environmental costs. The second point we note is that the distance charging schemes discriminate much more in function of conventional air pollution than do the Eurovignette systems. Finally, if one compares the distance charges in Table 1, one finds that Switzerland charges 10 times more per kilometer than the EU countries and also Austria charges significantly more than the others. Of course, infrastructure costs may be higher in these countries but the main reason is the strategic position of Switzerland as a transit country. Austria is also a transit country but it is slightly less interesting route and is moreover bounded by the EU cap on truck charges while Switzerland is not.
Table 1 Distance charges in different countries (source: Vierth & Schleussner, 2012)

<table>
<thead>
<tr>
<th>Country</th>
<th>Year of introduction</th>
<th>Charged weight class</th>
<th>Average toll rate (Euro/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>2001</td>
<td>&gt; 3.5 tonnes</td>
<td>2.23</td>
</tr>
<tr>
<td>Austria</td>
<td>2004</td>
<td>&gt; 3.5 tonnes</td>
<td>0.269</td>
</tr>
<tr>
<td>Germany</td>
<td>2005</td>
<td>&gt; 12 tonnes</td>
<td>0.16</td>
</tr>
<tr>
<td>Czech republic</td>
<td>2007</td>
<td>&gt; 12 tonnes</td>
<td>0.07</td>
</tr>
<tr>
<td>Slovakia</td>
<td>2010</td>
<td>3.5 &gt; 12 tonnes</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; 12 tonnes</td>
<td>0.175</td>
</tr>
<tr>
<td>Poland</td>
<td>2011</td>
<td>3.5 &gt; 12 tonnes</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; 12 tonnes</td>
<td>0.096</td>
</tr>
</tbody>
</table>

3. Literature review

We will rely on three types of literature. There is the more general public finance literature on tax competition. There is the literature on policy competition and pricing in transportation networks. Finally, there is the empirical literature on tax competition for diesel fuel.

The major insight we will use from the theoretical literature (Mintz & Tulkens, 1986) and surveyed in Wilson (1999) and Zodrow (2003) come from Kanbur and Keen (1993). They show how the tax setting behavior of Leviathan governments depends on the relative size of a country. Smaller countries can gain by undercutting their neighbors as they have more to gain from cross-border shopping than from domestic tax revenues.

The literature on policy competition between governments in the transport sector has been reviewed in De Borger & Proost (2012). More in particular, they review the competition of private or public agents that can each control part of a transport network. The simplest model setups are the parallel network and the serial set up. In the parallel setup (De Borger, Proost, Van Dender, 2005), international trucks can choose between two links (countries) and the countries compete in tolls (and in infrastructure capacity), taking into account that there is also domestic traffic on the network. Each country will charge more than the external cost but the margin will be restricted by the competition for transit traffic and by the deadweight loss on domestic traffic. We will use more intensely, a serial network type set up (De Borger, Dunkerley, Proost, 2007) where a truck has to go through at least two countries to complete its trip. In this case each country charges a monopoly margin that is limited by the deadweight loss on domestic traffic. Overall, there is a risk of double marginalization and thus of overcharging the use of the road network. In these two papers, trucks are charged per trip and they cannot escape the charge if they use the road network of a given country. In our paper we drop this assumption and include two tax instruments in the analysis. We start with the fuel excise taxes that can be escaped by fuelling abroad and give countries the option to add...
We will mainly deal with pricing the use of existing capacity and do not discuss the relation between kilometer charges, congestion levels and infrastructure supply. The European regulation capped distance charges to the average infrastructure charges and this can be important to force member countries to introduce efficient charges (see Van der Loo & Proost (2013) and Grahn-Voorneveld (forthcoming)).

The competition on excise taxes for diesel fuel in the EU has been studied intensively. Evers, de Mooij and Vollebergh (2004) studied a panel data set for 17 countries (1978-2001) and estimated Nash reaction functions for diesel excises. They found strong evidence of tax competition. When neighbouring countries increase their fuel excise tax by 10%, an average EU country increases its tax by 2 to 3%. They also found that the imposition of minimum tax rates has increased overall excise levels but the intensity of the tax competition has not decreased. Rietveld & van Woudenberg (2005) analyzed the setting of gasoline and diesel taxes and found strong tax competition effects for diesel excise taxes in Europe. Paizs (2010) confirms the evidence on diesel excise competition. He also finds that larger countries react more aggressively to changes in their neighbors’ tax rates and that smaller countries tend to charge lower fuel excise taxes as predicted by Kanbur and Keen. The focus in our paper is not on empirical validation but on understanding the transition to another charging system for road use by trucks than fuel excises.

4. Assumptions and model elements

Assumptions

We use four simplifying assumptions.

First we assume that the fuel efficiency of trucks is fixed. This may be a minor assumption as trucks are designed to be used in several countries and their fuel efficiency will be a function of the expected fuel taxes and fuel efficiency regulations in the different countries where the truck is used.

The second assumption is that the trucks are homogeneous and that the external cost of trucks differs among countries but is constant per kilometre and independent of the volume of total truck use in a given region. We consider three types of external costs: wear & tear of infrastructure, local air pollution and congestion. The homogeneity assumption is problematic for road damage that depends on the design and loading of the truck. In principle congestion depends also on volume of truck use and is therefore not fixed. But we have two lines of defence for this assumption. Consider first the short term. As trucks are only 5 to 20% of total volume of road use, small variations in the total truck volume due to truck tax variations may justify somewhat the constant external cost assumption. Consider next the long term with variable road capacity, if we have constant returns to scale in infrastructure extension, the external
congestion cost becomes a constant. For this reason we will use throughout this text external congestion cost and infrastructure cost as synonyms. Trucks also differ in the emission of traditional air pollutants. This is more or less a transition problem as regulations are only imposed on new trucks.

The third assumption is that we use a model with only two countries; A and B. We assume that both countries take the behaviour of the other country as given. This is more easily justified in the case of many countries. As long as all countries are identical, we can easily generalize the model to \( n \) countries that interact. For instance we could consider a Löshian model with hexagonal countries where every country has always six neighbours that generate international traffic and set fuel taxes and distance charges. We could also consider a setting with one big country, surrounded by \( n \) smaller neighbours.

The fourth assumption is that we can isolate the pricing of trucks and of cars. This can be justified by assuming that either the share of diesel cars is negligible or that governments use other types of car taxes (registration or purchase taxes) to align the taxation of diesel cars to the taxation of gasoline as it is in their interest to separate both users of diesel fuel. We will relax this assumption later.

**Objective function of the governments**

We assume that country governments maximize the sum of the consumer surplus of domestic trucking plus part of the consumer surplus of international truck transport plus the total tax revenues minus the total external costs within the country (infrastructure costs plus external congestion costs on local car use and local environmental costs). As all international trips correspond to a transaction where both countries gain, we assume that they both share equally in the gains.

Using the sum of consumer surplus plus tax revenues minus external costs is a rather normative approach for government behavior. A popular alternative among economists is the Leviathan assumption (Kanbur & Keen, 1993) where governments simply aim to maximize total tax revenues. The Leviathan assumption is also at the basis of the empirical work on diesel excises cited above but the Leviathan assumption itself is not tested empirically in that empirical work. There is however some factual evidence for our less extreme assumption. First, whenever there is a proposal to raise the level of fuel excises, local truck drivers lobby strongly against and governments often give in so the consumer surplus of trucks counts to some extent. Second, the distance charges are strongly differentiated in function of emission rates of trucks (Vierth & Schleusser, 2012). This implies that environmental benefits also matter.
Behaviour of domestic and international trucks

In each country there are domestic truck trips and there are international truck trips. The whole trucking industry is competitive. The domestic truck traffic uses only the local road network and buys fuel locally\(^3\). The international trips use the road network at home and the road network in the other country. The international hauler buys more fuel in the cheapest country. Figure 1 presents the model set up.

![Diagram](image)

**Figure 1. Model set up**

Total length of the road running through both countries equals 2. All countries have the same spatial density. Whenever countries differ in size, we always take country A as the bigger country. Country A has size \(2\gamma\) and country B has size \(2(1-\gamma)\), where \(\gamma\in[0,1]\). On average, domestic trips will cover a distance \(\gamma\) in country A. In country B, trips will be of length \((1-\gamma)\). International trips will be of length 1 with a part \(\gamma\) in country A and a part \((1-\gamma)\) in country B. The trip length is fixed but the number of trips will be variable\(^4\).

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\(^3\) In some tax competition models (Kanbur&Keen, 1993 and others), tax competition takes the form of shopping abroad, so incurring transport costs to buy goods abroad. This makes more sense for consumer goods (tobacco, alcohol). In the case of trucks the additional costs of the driver and truck makes it unlikely for domestic truck trips to make this pure shopping trip.

\(^4\) Charging more for fuel or per mile may make trips shorter and may substitute international trips by shorter domestic trips. This is not considered in this paper.
The number of international trips will, by construction, be the same in both countries. We allow for that the number of domestic trips in country \( A \) may be proportionally larger than in country \( B \) because a large country offers proportionally more internal trade opportunities. If \( 1/\zeta_A \) represents the relative number of trips in country \( A \), we have that there are relatively more domestic trips in the larger country \( A \):

\[
\frac{\zeta_B}{\zeta_A} \leq \frac{\gamma}{1-\gamma}
\]  

(1)

Trucks face three types of costs (all expressed per unit length): costs before taxes \( c \), fuel tax \( t_i \) \((i=A,B)\) and a kilometre charge \( T_i \) \((i=A,B)\). The total cost is denoted by the generalised cost \( g \) for local trucks and \( G \) for international truck traffic.

Local demand functions (number of trips) equal:

\[
d_A = a - \zeta_A b \ g_A \\
d_B = a - \zeta_B b \ g_B
\]

(2)

Where \( a \) and \( b \) are non-negative parameters. International demand function (number of trips) is:

\[
D = \alpha - \beta G
\]

(3)

Where \( \alpha \) and \( \beta \) are non-negative parameters. However, for ease of presentation we will later sometimes use the assumption that international truck demand is totally inelastic.

The generalised costs of a domestic trip take into account its relative distance\(^5\):

\[
g_A = \gamma(c + t_A + T_A) \\
g_B = (1 - \gamma)(c + t_B + T_B)
\]

(4)

And the generalised cost of international trips equals a sum of costs in both countries:

\[
G = c + \gamma T_A + (1 - \gamma)T_B + \sigma(t_A, t_B) \ t_A + (1 - \sigma(t_A, t_B)) \ t_B \\
\sigma(t_A, t_B) = \gamma - \rho(t_A - t_B) \\
\sigma(t_A, t_B) = 1 \text{ when } t_B - t_A \geq \chi_B \\
\sigma(t_A, t_B) = 0 \text{ when } t_A - t_B \geq \chi_A
\]

(5)

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\(^5\) An alternative set up would be that all domestic trips have the same length irrespective of the country.
International haulers minimize their fuel costs by buying more fuel in the cheaper country they drive through. We use a simple reduced form formulation for this cost minimization process of the haulers. (5) represents the refueling choices of the haulers by use of a simple linear formulation for the share of fuel \( \sigma(t_A, t_B) \) bought in country A. When the difference in taxes is larger than a given constant \( \chi \), a hauler buys only fuel in the cheapest country. The parameter \( \rho \) is a measure of the intensity of tax competition: a small \( \rho \) means that an increase in the fuel tax difference does not affect strongly the market share \( \sigma \) in the international trucking fuel market.

When fuel taxes are equal in both countries, the fuel purchases are proportional to the size of the country. One could also use more complex formulations like the logistic function but results do not really depend on this. The fuel market share function implies that the generalized cost of the international haulers is a non-decreasing function of the fuel taxes in both countries:

\[
\frac{\partial G}{\partial t_A} \geq 0, \quad \frac{\partial G}{\partial t_B} \geq 0
\]

\[
\frac{\partial G}{\partial t_A} = \sigma - \rho(t_A - t_B)
\]

\[
\frac{\partial G}{\partial t_B} = (1 - \sigma) + \rho(t_A - t_B)
\]

5. Fuel Tax setting behaviour of governments in the absence of diesel cars

We have now all the elements to study the behavior of governments. We first concentrate on the fuel tax as the only instrument and study the Nash equilibrium in fuel taxes. In the next section we will add distance charging as tax instrument. Throughout sections 5 and 6 we will assume that there are either no diesel cars or that they can be taxed using additional instruments.

The structure of the first order condition

Each country government maximizes the sum of consumer surplus of local trips \((cs)\) plus half of the consumer surplus \((CS)\) of international trips. As all international trips correspond to a transaction where both countries gain, we assume that they both share equally in the gains. In addition, governments take into account their own tax revenues minus the external costs minus the implementation costs\(^6\). As we always have fuel taxes, we only consider the implementation costs \((IT)\) of kilometer charges. External costs can differ among countries.

The objective function of the country government A becomes:

\(^6\) Our formulation neglects the general equilibrium effect of road taxes and fuel taxes on the price of products. A competitive international haulage sector will pass on the large part of its additional taxes onto consumers and producers. This may be a motive for a government to restrict taxes on road freight to the marginal external costs.
Note the difference between the tax revenue per trip and the external cost. The external cost is fixed per trip (or per mile) and the only way to reduce the external costs is to reduce the volume of trips via the fuel tax or via the distance charge.

In order to study the Nash equilibrium in fuel taxes, we derive the first order condition with respect to fuel tax $t_A$ and set the distance charges $T_A, T_B$ to zero. Take country A and differentiate objective function (7), with respect to $t_A$ to get the following optimality condition:

$$
-c_A(g_A{.}) + d_A{.}(\gamma t_A + \gamma T_A - \gamma e c_A) + 0.5 CS(G{.}) + D{.}(\sigma t_A + \gamma T_A - \gamma e c_A) - IT_A
$$

The left hand side consists of seven terms. The first captures the change in consumer surplus on the market for domestic transportation. This is entirely cancelled by the tax revenues from domestic transportation in the third term because this is a transfer within the country. Remaining effects on the domestic market are captured by the second term, which multiplies the change in domestic transport demand (measured as length $\gamma$ times the number of trips) with the taxes net of external costs. The last four terms deal with international transports. First, note that the change in consumer surplus (fourth term) and the change in tax revenues (last term) do not cancel in a way corresponding to what we see on the domestic market. This is not surprising since part of the tax is paid by haulers from country B. The tax effect consists of three parts. The first, in term five, follows from the change in demand due to the change in tax. The second, in term six, follows from the change in tax that influences the choice of where to fuel. The last effect, in term seven, is the direct tax revenue effect capturing the share of the total international demand that will fuel in country A.

**Fuel tax equilibrium in the symmetric case**

More clear cut results are available in the symmetric case where both countries are of equal size and have the same external costs.

Using the first order condition and the symmetry, we obtain the following result:

**PROP 1** In the absence of diesel car considerations, the symmetric Nash equilibrium if only diesel fuel taxes are available is:

$$
t_A = t_B = \frac{\beta \gamma^2 + by \zeta_A}{\beta \gamma^2 + by \zeta_A + \rho D} ec_A + \frac{\gamma^2}{\beta \gamma^2 + by \zeta_A + \rho D} D
$$
To understand the level of the fuel taxes consider first the extreme case where there is no international traffic at all ($D=0$), then (9) reduces to charging the external cost. This is expected as the fuel efficiency is fixed so the external cost is strictly proportional to the distance driven and the best a policy maker can do is to charge the marginal external cost. This is in line with efficiency results for the production sector when there are externalities (Diamond & Mirrlees, 1971, Mayeres & Proost, 1997).

Whenever there is international truck transport, the equilibrium fuel tax will consist of two elements: an external cost component (first term with $e c_A$) and a tax revenue component (second term with $D$). We see that the first component is always smaller than the external cost and that the second component is always positive. This means that the fuel tax can be larger or smaller than the external cost. We have four elements playing a role in the size of the fuel tax compared to the external cost:

- Raising the fuel tax above the external cost distorts the allocation of domestic transport so the more important is domestic transport and its fuel price sensitivity $b \zeta$, the closer the fuel tax will stay to the external cost.
- A larger price-sensitivity of international traffic $\beta$, restricts the possibilities to make an extra margin on the fuel tax and this restricts the fuel tax.
- The more important is the international truck volume $D$, the larger will be the revenue raising component as long as the tax competition intensity $\rho$ is sufficiently small.
- When the fuel tax competition becomes more intense ($\rho$ increases because of smaller country sizes), and international transport becomes larger ($D$ increases), both fuel tax components become smaller and the fuel tax decreases and may become smaller than the external cost – when tax competition becomes less important, both countries set a tax rate above external cost. The tax is higher than the external cost because tax revenue from international transport is worth more than the lost consumer surplus of an international trip.

In a symmetric equilibrium, and given our assumptions, any fuel tax that is different from the external cost is inefficient for both countries. The driver of the inefficiency is the tax competition incentive. The following two figures help to convey the intuition for the symmetric case.
Figure 2, *Equilibrium with low intensity of fuel tax competition*

To see the intuition of Figure 2, assume first that country B is charging the external cost, then country A will find it profitable to have a fuel tax somewhat larger than the external cost because tax competition is relatively weak and the foreign haulers pay extra taxes while the cost to local haulers is of a second order (the deadweight loss). The fuel tax will be above the external cost but the tax competition, even if it is weak, and the deadweight loss to the locals will ultimately limit the fuel tax.

We can also have a regime with very strong tax competition. This could be the case of smaller countries where there is relatively more international traffic and where the international fuel market share is very price sensitive ($\rho$ large in (5)). This is illustrated in Figure 3. To see the intuition assume again that country B charges the external cost. Then it becomes profitable for country A to undercut country B as it can make international trucks take fuel in country A. With a fuel tax lower than the external cost, the cost for country A is the additional volume of local truck activity in country A not paying the full external cost while the gain are the international haulers taking fuel for their trips in country A AND in country B. For the additional trips in B, country A does not bear the external costs.
Figure 3, equilibrium with high intensity of fuel tax competition.

Thus, the fuel tax equilibrium can generate too high as well as too low fuel taxes compared to the external costs. Both cases thus involve inefficient pricing due to tax competition.

**Fuel tax equilibrium in the non-symmetric case**

Thus far, we have focused on the symmetric case where country A and B are identical. In particular, they are of the same size and have the same external costs. We now turn to the non-symmetric case. It is more difficult to reach easily interpretable results for this case. To facilitate the presentation we therefore add an assumption that international transport is price inelastic. We have then the following result:

**PROP 2** If there are other tax instruments for diesel cars, a non-symmetric Nash equilibrium if only fuel taxes are available is:

Either \( t_A >> t_B \) and \( t_A = ec_A \)

Or \( t_A << t_B \) and \( t_A = ec_A - \frac{0.5 D}{\sigma_{d,t_A}} > ec_A \)

Or \( t_A = \frac{1}{\beta^y + 1} ec_A + \frac{2}{\beta^y D} + \frac{1}{1 + \beta^y Dp} t_B \) \hspace{1cm} (10)

There are several equilibriums possible when countries differ in size and external costs. When would country A (by assumption larger in size than B) go for a particular equilibrium?
Charging the external cost \( (t_A \gg t_B \text{ and } t_A = ec_A) \) can be a Nash Equilibrium when the external cost in \( A \) is much higher than in country \( B \) and there is also no strong tax competition. When the tax in \( B \) is sufficiently lower than in \( A \), all international traffic will fuel in \( B \) and any marginal changes in \( t_A \) will not influence this. In that case, it is, for country \( A \), not interesting to go for much lower fuel taxes as it would distort strongly its domestic traffic for no tax revenue gain.

Charging much less than the other country \( (t_A \ll t_B \text{ and } t_A = ec_A - \frac{0.5 D}{\gamma^{\frac{1}{2}}t_A} > ec_A) \) can be an equilibrium when the external cost in \( A \) is much lower than in country \( B \), or when for some other reason \( t_B \) is very high relative to \( t_A \). We then have the opposite situation as above, i.e., all international transport will fuel only in \( A \). In that case, \( A \) will increase the tax above its external cost in order to charge a monopoly margin with as only restraint the distortion on the domestic trucking market.

Finally, the equilibrium may consist of taxes that are not too different so that both countries have a positive share in the fuel sales to international trucks. Then taxes can be higher or lower than the external cost. One of the elements that matter is size. Figure 4 presents the equilibrium with fuel taxes only. Because \( B \) is a smaller country it can gain lots of revenues from international haulage by setting a low fuel tax. The bigger country has much more to lose from using low fuel taxes as the local trucking is proportionally much more important. From Kanbur and Keen (1993), we know this will be a Nash equilibrium where the smaller country undercuts the large country. The reason is that undercutting gives the small country access to a much larger tax base, while for the big country the home market is proportionally more important. Note however that we have a different setting than Kanbur and Keen because in our model, governments are not Leviathan. In our setting, not only the size of the country but also the relative share of the home traffic and the international traffic count.
Making the government a Leviathan

Throughout the paper, the government is not only concerned with its tax revenues but also with the consumer surplus of its own inhabitants and the external cost. For comparison, it is interesting to derive the result under an assumption of the government being a Leviathan. Consider again the symmetric case, and assume that only government tax revenues count. In the symmetrical Nash equilibrium, the fuel tax becomes:

\[
t_A = t_B = \frac{y_D + d}{\beta y^2 + by\gamma_A + \rho D}
\]

The level of the tax is again determined by price sensitivity of local (\(b\)) and of international transport demand (\(\beta\)) but also by the level of international tax competition: a high \(\rho\) value limits the overall fuel tax rate that can be charged. So taxes are not necessarily higher than the external and infrastructure cost and this result holds for different objective functions of the government.

We may compare the leviathan outcome in the symmetrical case with the outcome represented in equation (9). Starting by assuming that the external costs, and thus also the first term in (9), are zero, we find that the leviathan fuel tax is strictly higher than the one in (9). This is driven by two effects. First, in (9) the fuel tax on domestic transport is just a transfer and so it is of no concern to the government. A leviathan government – that does not care about the consumer surplus for domestic transport – will implement a higher fuel tax and thereby also gain additional tax revenues from domestic transport. Second, also the tax on international transport will be higher under the leviathan assumption. We see this
from that $D$ is multiplied by $\gamma$ in the expression above, but by $\gamma^2$ in equation (9), remember that $\gamma$ is between 0 and 1.

However, when the external cost is larger than zero, the first term in (9) is also larger than zero. Thus, for large enough external costs, the first term will outweigh the two effects above and the leviathan tax will be lower than the tax in (9).

6 Adding the distance charge as policy instrument in the absence of diesel cars

Thus far, we have addressed the implications from strategic interaction between governments on fuel taxes. However, the subject of main interest in this paper regards how countries react when a distance charge becomes possible. In particular, we are interested in how the introduction of distance charges changes the way the governments’ choice of fuel taxes and how the underlying mechanisms work.

To examine this, we start again from the objective function (7) of the government of country A and derive first order conditions for $t_A$ and $T_A$. Note that it is possible not to use both instruments. The first order condition for the fuel tax in $A$, conditional on its distance charge, then becomes:

$$-\gamma d_A + \gamma \frac{\partial d_A}{\partial t_A} (t_A + T_A - ec_A) + \gamma d_A - 0.5 D \frac{\partial G_A}{\partial t_A} + \frac{\partial D}{\partial t_A} (\sigma t_A + \gamma T_A - \gamma ec_A) + \frac{\partial \sigma}{\partial t_A} D t_A + \sigma D = 0$$

Similarly, the first order condition for the distance charge, $T_A$, is given by:

$$-\gamma d_A + \frac{\partial d_A}{\partial T_A} (t_A + T_A - ec_A) + \gamma d_A - 0.5 D \frac{\partial G_A}{\partial T_A} + \frac{\partial D}{\partial T_A} (\sigma t_A + \gamma T_A - \gamma ec_A) + \gamma D = 0$$

PROP 3 Assume distance charges can be used and that there are other tax instruments for diesel cars, then a Nash equilibrium cannot have diesel fuel taxes in both countries and $0 < \sigma < 1$. The equilibrium distance charges are larger than the marginal external cost:

$t_A = t_B = 0$

$T_A - ec_A = \frac{0.5D}{\gamma(b + \beta)} > 0$

$T_B - ec_B = \frac{0.5D}{(1-\gamma)(b + \beta)} > 0$
Before showing the intuition behind this result, consider the equilibrium values of the distance charges in the absence of fuel taxes. The distance charges will be larger than the external costs, the mark-up on top of the external cost will be larger in small countries as they have more revenue to gain from international trips. Finally the mark up in the distance charges will be smaller when domestic truck trips are very price sensitive and/or more important.

We can show the intuition behind the dominance result for distance charges using a thought experiment. Consider the effects of the introduction of the distance tax when a fuel tax is already present. Taking our departure from Figure 2, we construct Figure 5. Starting from the initial equilibrium with fuel taxes only, we consider the gradual substitution of the initial equilibrium with only a fuel tax $t_A^0$ by a combination of a smaller $t_A$ and a small positive $T_A$ but keeping the same sum $t_A + T_A = t_A^0$ and keeping the same $t_B^0$. This substitution will not affect local traffic as this is only influenced by the sum of the two taxes. The main effect will be that country A will now have larger tax revenues. The consumer surplus for international transport will also decrease, but by a smaller amount. What will make the difference is the larger market share on the fuel market: country A will have its revenues increased. We did not alter the reaction function of country B, this can change slightly as it is also a function of the level of $T_A$. The initial fuel tax in country B ($t_B$) is no longer the best reply of country B and we will move to a new equilibrium with lower and lower fuel taxes in both countries.

---

7 If we allow for the tax influencing international transport demand, the substitution will generate a small increase in international haulage as haulers will benefit from lower fuel taxes in country A and B and this compensates the increase in $T_A$. There will be an additional external cost in both countries but this is certainly covered in country A by the sum of the two taxes.
Total tax for the use of the road in country A may increase strongly as the kilometer tax allows A to better charge the external cost and to raise revenue on international traffic.

Let us start from another extreme equilibrium where both countries initially only use a kilometer charge. This equilibrium is an equilibrium of the serial network type where both countries tax the same tax base (international traffic) and this gives an equilibrium with distance charges larger than the external cost (De Borger, Dunkerley, Proost, 2007). The reason is that part of the tax base of each country is foreign and this will always be taxed above the external cost. If we allow for elastic international transport demand, the reaction function $T_A(T_B)$ will be downward sloping because every increase of the kilometer charge in the other country will decrease the international haulage volume and for this reason, the optimal distance charge will also decrease. Now, will one of the countries gain by re-introducing a fuel tax?

Consider Figure 6 where we start from an equilibrium $(T^o_A, T^o_B)$ where only distance charges can be used and the fuel tax instrument is not used. This is illustrated by the intersection of the two solid lines. Consider now a substitution of part of $T_A$ by a fuel tax in country A. This corresponds to the dotted line parallel with the reaction function of country A. Can country A gain from such a substitution? This time the answer is very clear: NO, country A cannot gain. For local traffic it will not make a difference. But international traffic will buy fuel in country B where the fuel is not taxed, so country A will always lose tax revenues.

**Figure 5**, *equilibrium when country A also disposes on kilometer tax but country B has only the fuel tax.*
Thus in equilibrium, when both countries can freely choose fuel tax levels and distance charges, they will both prefer a distance charge that will be above the external cost while the fuel tax is driven to 0.

**PROP 4 (conjecture)** *If there are no diesel cars or if there are other tax instruments for diesel cars, the option to introduce distance charges can be welfare decreasing in the symmetric equilibrium*

When we move from a fuel tax only system to a distance charge system, there are two differences. First there is the difference in implementation costs, where one can reckon that the distance charges are more costly than fuel taxes. Second, the distance charges are always larger than the external cost while the deviation between the external cost and the fuel tax can be lower or higher than in the case of the distance charge. Of course when the distance charge becomes smarter and becomes time and place dependent, the charge becomes much more efficient but that is not the type of distance charge we consider here.

It is obvious that the large country can benefit from introducing a kilometer charge to escape the downward pressure of the small country on its fuel tax level. For the small country, the costs and benefits of the kilometer charge are less obvious as it can in both systems extract revenue from foreign international traffic.
Consider the case where only kilometer charges are used by countries that are different in size. This is represented in Figure 6. The solid lines in the figure illustrate the symmetric case, where the countries are of equal size. Consequently, the Nash equilibrium entails \( t_A = t_B \) and thus lies on the 45 degree line. Here, we have chosen country A to be the smaller country. The reaction function of the smaller country A lies above the corresponding reaction function in the symmetric case. This follows from that country A has proportionally more foreign traffic on its roads and can benefit by charging more per kilometer. This is the reverse of the case with fuel taxes, where the smaller country undercuts the fuel tax of its large neighbor.

**Figure 6, equilibrium with kilometer charges only and when A is smaller country**

7 Introducing diesel cars

**Completing the model by adding diesel cars**

When we assume that the gasoline fuel tax on cars is fixed and there is no other tax on diesel cars other than the diesel fuel excise we need to adapt the model. This is necessary to take into account the consumer surplus of diesel car users and the possible side effects of diesel taxes on gasoline car market. We assume that cars do not fuel abroad and that the gasoline tax is fixed. In order to complete the model we need to add a demand function for diesel cars and for gasoline cars. Let the demand functions for diesel and gasoline cars be:
In this formulation, superscript \( d \) and \( g \) denote diesel cars and gasoline cars respectively, \( x \) represents the mileage of cars and \( g \) represents the generalized cost per trip. The generalized driving costs are equal to

\[
\begin{align*}
g^d_A &= \gamma (c^d + \theta^d t_A) \\
g^g_A &= \gamma (c^g + t^g_A)
\end{align*}
\]

Where \( \theta^d \) represents the relative consumption of fuel per mile of a diesel car compared to a diesel truck. Remember that \( t_A \) represents fuel tax paid per mile by a truck. So \( \theta^d \) is typically \( \frac{1}{4} \) or less. This matters as the same fuel tax has to price external costs of a truck and external costs of a car. When \( t_A = ec \) there is no guarantee to have \( \theta^d t_A = ec^d \).

Next we need to add three terms to the objective function (7): the consumer surplus of the local diesel car user \( (cs^d) \) as well as the fuel tax revenues and external costs for diesel and for gasoline cars. Let \( ec^d \) and \( ec^g \) be the external costs per mile of diesel and of gasoline cars:

\[
\begin{align*}
cs_A \{ g_A \} &= d_A \{ \cdot \} (\gamma T_A + \sigma T_A - \gamma ec_A) + 0.5 CS \{ G \} + D \{ \cdot \}(\sigma T_A + \gamma T_A - \gamma ec_A) - IT_A + \\
cs^d \{ g^d_A, g^g_A \} &= x^d_A (\gamma d t_A - \gamma ec^d_A) + x^g_A \{ \gamma d t_A - \gamma ec^g_A \}
\end{align*}
\]

\[
(11)
\]

**When the fuel tax is the only instrument**

In a symmetric Nash equilibrium, we obtain the following fuel tax:

\[
\begin{align*}
t_A = t_B &= \frac{\beta \gamma^2 + b \gamma \zeta_A}{\varphi} ec_A + \frac{\gamma d b^d}{\varphi} D + \frac{\gamma^2}{\varphi} ec_A + \frac{\partial x^g_A}{\partial t^g_A} ( \gamma t_A^g - ec^g_A )
\end{align*}
\]

Where \( \varphi = \beta \gamma^2 + b \gamma \zeta_A + \rho D + \theta^d b^d \)

\[
(12)
\]

We see that the diesel tax takes now on board four elements: the external and infrastructure cost for trucks (1st term) and the revenue raising term for international truck transport (3rd term) were already discussed before. The new terms are the 2nd term that represents the external cost of diesel car use and the 4th term that represents the effect of the diesel tax changes on the distortion on the gasoline market. If the gasoline tax can be set optimally the last term disappears. The diesel tax is then one instrument that is used to correct external costs of two very different vehicles (cars and trucks) as well as to try to raise revenues.
from international transport. The externality correction objectives receive relatively more weight when the domestic car and truck demand become more price sensitive. The revenue raising objective receives less weight when competition for international fuel sales is fierce (high $\rho$).

**Introducing distance charges in the symmetric equilibrium**

In the absence of diesel cars, introducing distance charges leads to an equilibrium where fuel taxes are driven to 0. When diesel cars can only be taxed using fuel taxes, zero fuel taxes are not a candidate equilibrium as they do not have to pay a distance charge. So we need a complex balancing of the fuel tax as externality tax for diesel cars and as a revenue raising tax on international trucking. We proceed in two steps. We explore first intuitively the symmetric equilibrium and discuss next possible generalisations of the intuitive results. The easiest situation to start with is where the fuel taxes are set equal to the external cost of a diesel car, the gasoline tax equals the external cost of a gasoline car and where the distance charges serve to extract revenues from international traffic. This initial equilibrium is represented in Figure 7 as $\theta^d t^o_A = ec^d$ and $T^o_A + t^o_A > ec$. This cannot be an equilibrium. The reason is that the international trucks can still decide where to buy fuel. Whenever there is a diesel tax, there are opportunities for each government to increase their fuel tax revenues by decreasing the fuel tax slightly below the external cost of diesel cars. So the equilibrium $\theta^d t^* A < ec^d$ and $T^* A + t^* A > ec$ will have a fuel tax below the external cost of diesel cars but the sum of the distance charge and the fuel tax will be larger than the external costs of trucks. It is difficult to compare the equilibrium $t^{**}$ when only fuel taxes were available as instrument with the new equilibrium. What we know is that $t^*$ can be above or below the external cost of cars and trucks and that it will be lower than $t+T^*$. When the international fuel competition is strong, $t^{**}$ will be low and when the international fuel tax competition is weak, the fuel tax will be high. The sum of distance charges and fuel taxes will, however, always be higher than in the fuel tax only solution.
Figure 7, *Equilibrium with distance charges and fuel taxes in the presence of diesel cars*

We can discuss the introduction of distance charges in the presence of diesel cars more formally. Assuming that the gasoline tax equals the external cost of gasoline cars, we need to analyze two possible initial equilibria when only fuel taxes can be used.

In the first case the diesel tax charged is larger than the external cost of a diesel car:

\[ \gamma \Theta^d t_A^* - \gamma e c_A^d > 0 \]

this will be the case when diesel cars are rather clean and there is not such a strong tax competition for truck fuel. In this case, a substitution of gasoline taxes by distance charges \( dT_A = -dt_A > 0 \), starting from \( t_A, t_B > 0 \) and \( 0 < \sigma < 1 \), is even more beneficial for country A than in the absence of diesel cars as one decreases the pricing distortion on diesel cars on top of the net revenue gain for country. So whenever \( \gamma \Theta^d t_A - \gamma e c_A^d > 0 \) this cannot be a Nash equilibrium.
Consider next the case where \( \gamma \Theta^{d} r_{A}^{o} - \gamma e C_{A}^{d} < 0 \). In the absence of distance charges for trucks, this can be an equilibrium when there is strong fuel tax competition for trucks and when the external cost of diesel cars is high. Country A can still benefit from a substitution \( dT_{A} = -dt_{A} > 0 \) but the total benefit will be smaller as this has to balance the increasing distortion for diesel cars.

**Some special cases**

Switzerland is a transit traffic country that has a strategic position in between Germany and Italy. Our two country model is not designed to study the case of transit countries. However we can easily reformulate the government objective function (7) so that it comes close to the Swiss case. When international traffic is mainly transit traffic like in Switzerland, a country is not interested in the consumer surplus of the international traffic. When we assume a very low excise tax, we find that the optimal distance charge is simply the revenue maximizing charge that is only mitigated by the deadweight loss on the domestic traffic:

\[
T_{A} - e C_{A} = \frac{-D}{\partial D_{A} / \partial T_{A}}
\]

As Switzerland is a small country but in a strategic corridor that is difficult to avoid, it can generate much more revenue from high distance charges than from undercutting diesel excise taxes. This can explain why it was among the first to introduce distance charges.

The best alternative on the route from Germany to Italy is to pass through Austria. Austria was the first (together with Germany) to follow the example of Switzerland and also to implement a distance charging scheme. As Austria is part of the EU, it has to observe the cap on distance charges: charges cannot exceed the average infrastructure and air pollution costs. But there is a provision in the directive that allows mountainous areas to charge more. The result is that the Austrian charge is higher than in the other EU countries but only one tenth of the Swiss charge.

For Luxemburg it is the other way around: a truck can easily avoid passing through Luxemburg, therefore it is not interested in distance charges, instead, it has used the strategy of undercutting the fuel taxes of its neighbor but this strategy is now at risk. Paradoxically, Luxemburg may soon plead for minimum fuel excises for diesel to protect its revenue base.
Generalising the model to several countries

Model international transport along a line (2 neighbors maximum) or using hexagons following the Lösch model (always 6 neighbours). Symmetric results will have the same flavor as long as there is no competition between different destinations and only two countries are involved in an international trip.

Introducing congestion, infrastructure costs and network capacity

Introducing congestion on the network of the two countries makes the model more complex because there is a feedback from congestion on the volume of demand. In the absence of diesel cars we can conjecture some results. If there are only fuel taxes, the Nash equilibrium in fuel taxes will be higher or lower than the marginal external cost (including congestion) depending on the strength of the tax competition forces. Starting from this equilibrium without distance charges, this cannot be a Nash equilibrium as each country can gain by substituting its fuel tax by a kilometer charge. So the Nash equilibrium is likely to contain only distance charges.

Whenever we introduce congestion, we introduce another policy instrument: road capacity. We know from a serial network type set up (De Borger, Dunkerley, Proost, 2007) that, whenever international trips dominate, the reaction functions for distance charges are independent of capacity because the distance charges tax the same good. Distance charges of the two countries are complements. But the capacities are complements. Whenever also local trips are introduced, results are less clear cut.

A difficulty of introducing capacity, is that capacity has also to serve passenger car demand.

Dropping the homogeneity assumption for trucks

This offers options for price discrimination certainly when countries are allowed to subsidize their own greener trucks.

9. Concluding remarks

We have shown that when the only policy option is using a fuel tax for trucks, there is a large risk that countries will set the tax at an inefficient level due to tax competition. The tax may be set above or below the external costs depending on the characteristics of the countries and the market. Small countries may prefer low fuel taxes as this allows them to have more international trucks fuelling in their home country.

If we introduce an additional policy instrument in the form of distance charges, we show that a Nash equilibrium is likely to contain only distance charges. The same tendency will subsist if the fuel tax has also to internalize the externalities of diesel cars: some fuel taxes will remain but they will decrease. The end result of the possibility to introduce distance charges for trucks will be a level of taxation of
international trucking that is strictly larger than the external cost. The tax on the use of diesel cars will however be too small.

Our results are important for policy design. This is particularly true in the EU where we currently see strong tendencies towards implementing distance based charges. Judging from the geographical developments, the implementation seems to follow a sequential pattern: distance charges are contagious. The central EU states already have a distance based charge in operation. Several states bordering to the central ones, are currently working towards implementing such a charge.

Distance charges clearly play an important role in internalizing external costs from freight transportation. However, they are costly to introduce and maintain and, hence, it is not obviously welfare enhancing to implement them. As a consequence, our result that if one country adopts distance charges, the tax competition more or less forces the other country to follow, points towards a risk for inefficiencies in the choice of policy instruments.
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Vierth I., H.Schleusser (2012), Impacts of different environmentally differentiated truck charges on mileage, fleet composition and emissions in Germany and Sweden, CTS working paper 2012:22
Appendix 1, map illustrating the policy mix in the EU.
Appendix 2 Proof of Proposition 3

PROP 3 Assume distance charges can be used and that there are other tax instruments for diesel cars, then a Nash equilibrium cannot have diesel fuel taxes in both countries and $0 < \sigma < 1$. The equilibrium distance charges are larger than the marginal external cost:

$t_A = t_B = 0$

$T_A - ec_A = \frac{0.5D}{\gamma(b + \beta)} > 0$

$T_B - ec_B = \frac{0.5D}{(1 - \gamma)(b + \beta)} > 0$

PROOF

We examine the case where in a potential Nash equilibrium $t_A, t_B > 0$ and $0 < \sigma < 1$ so that the international fuel market is shared between the two countries. For this case we will show that a country can always gain by substituting one Euro of diesel tax per kilometer by one Euro of distance charges per kilometer and that therefore such a potential equilibrium cannot be an equilibrium. Next we discuss briefly what happens when $t_A, t_B > 0$ and $\sigma = 0$ or 1.

1. For $t_A, t_B > 0$ and $0 < \sigma < 1$ we will show that the value of the objective function of country A (the same reasoning holds for B):

$$cs_A\{g_A(\cdot)\} + d_A(\cdot)(\gamma t_A + \gamma T_A - \gamma ec_A) + 0.5 CS\{G(\cdot)\} + D(\cdot)(\sigma t_A + \gamma T_A - \gamma ec_A) - IT_A$$

can be increased by a marginal substitution $dT_A = -dt_A > 0$ when the potential Nash equilibrium has $t_A, t_B > 0$ and $0 < \sigma < 1$.

We need to examine first the effect of this substitution on the generalized costs in the potential Nash equilibrium:

$$g_A = \gamma(c + t_A + T_A)$$
So the generalized cost of local truck trips $g_A$ will not be affected by substituting $t$ and $T$. The generalized cost of international traffic is:

$$ G = c + \gamma T_A + (1 - \gamma)T_B + \sigma(t_A, t_B) t_A + (1 - \sigma(t_A, t_B)) t_B $$

After the substitution we see that it will also not be affected:

$$ dG = \gamma dT_A + (\gamma - \rho(t_A - t_B)) dt_A - \rho t_A dt_A + \rho t_B dt_A = 0 $$

We can now compute the effect of this $(t, T)$ substitution on the value of the objective function of country A. There will be no effect on the two first terms of the objective function as local consumer surplus and net revenues on local trips are unaffected by the substitution. There will also be no effect on the consumer surplus of international traffic. The only effect will be on the revenues of international traffic of country A:

$$ D \left\{ \left[ \gamma dt_A - \rho t_A dt_A + \gamma dt_A \right] \right\} = D \left\{ \left[ \rho t_A dT_A \right] > 0 \right\} $$

So, as long as $t_A > 0$ one can increase the value of the objective function by this substitution and so a Nash equilibrium cannot have $t_A, t_B > 0$ and $0 < \sigma < 1$.

2. Assume that a Nash equilibrium exists with $t_A, t_B = 0$, then we can use the first order optimal conditions for A and B that will characterize the Nash equilibrium:

$$ -\gamma d_A + \gamma \frac{\partial d_A}{\partial T_A} (T_A - ec_A) + \gamma d_A - 0.5 D \frac{\partial G}{\partial T_A} + \frac{\partial D}{\partial T_A} (\gamma T_A - \gamma ec_A) + \gamma D = 0 $$

$$ -(1 - \gamma) d_A + (1 - \gamma) \frac{\partial d_B}{\partial T_B} (T_B - ec_B) + (1 - \gamma) d_B - 0.5 D \frac{\partial G}{\partial T_B} + \frac{\partial D}{\partial T_B} ((1 - \gamma)T_B - (1 - \gamma) ec_B) + (1 - \gamma)D = 0 $$

Which gives:

$$ T_A - ec_A = \frac{-0.5D}{\frac{\partial d_A}{\partial T_A} + \frac{\partial D}{\partial T_A}} = \frac{0.5D}{\gamma(b + \beta)} > 0 $$
And

\[ T_B - e d_B = \frac{-0.5D}{\partial d_B} + \frac{\partial D}{\partial T_B} = \frac{0.5D}{(1-\gamma)(b+\beta)} > 0 \]

So the distance charge is always larger than the external cost and the mark-up on external costs is higher in the smaller country.

3. Consider now the case where \( \sigma = 1 \), then country A has the whole fuel market for international trips. Consider again \( t_A, t_B > 0 \) as a potential Nash equilibrium. Note first that also \( T_A = T_A^0 + t_A^0 \), \( t_A = 0 \) would produce the same potential Nash equilibrium as neither the generalized price for domestic trips nor the price for international trips would be affected as country A had the whole international fuel market (\( \sigma = 1 \)).

Consider next whether country B has an incentive to move to an equilibrium with zero fuel taxes. When we operate the same substitution of \( t_B \) by \( T_B \), the generalized costs for domestic trips does not change but the generalized cost for international trips increases:

\[ dG = (1-\gamma)dT_B > 0 \]

And the net effect on the objective function of country B is not necessarily positive:

\[ \{ -0.5DdT_B + DdT_B - \beta dT_B [T_B - e d_B] \}(1-\gamma) \]
\[ = (1-\gamma) \{ 0.5D - \beta(T_B - ec_B) \} dT_B \]

Still to prove that this is positive..

Consider finally the case \( \sigma = 0 \), this is the reverse of the case \( \sigma = 1 \) and the same type of reasoning holds.