Imperfect Eco-labeling Signal in a Bertrand Duopoly

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Abstract

In a Bertrand duopoly model, we study firms’ eco-labeling behavior when certification process imperfectly signals environmental product quality to consumers. The test is noisy in the sense that brown products may be labeled while green products may not be. We study how strategic interaction shapes firms’ incentives to get certified, equilibrium demand, prices and social welfare. We find that the eco-labeling policy is welfare enhancing for all parameters values. Nevertheless, the separating testing equilibrium may be too costly to sustain when the green firm probability to pass the test is small. Moreover, if the certification technology is lenient, meaning that both brown and green units are awarded the label with high probability, it is easier to sustain a separating equilibrium. This is a consequence of price strategic interaction between firms that gives firms incentives to coordinate on a separating equilibrium.

JEL classification: C72, D21, D60, D82, L15, Q50.

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1 Introduction

The emergence of conscious consumers ready to pay a price premium for products providing better environmental and/or social performance has put responsible consumption at the forefront of the environmental debate (See The final report of the Steering Committee of the State-of-Knowledge Assessment of Standards and Certification, 2012). Producers may want to respond to this green demand to catch the willingness to pay of these responsible consumers. However, as credence attributes, environmental characteristics (organic agriculture, green production techniques, . . . ) are not readily apparent even after consumption. If producers cannot differentiate their product to obtain a price premium and cover their investment in environmental friendly technique, no green product will be sold. To avoid the disappearance of the market for those goods (Akerlof, 1970) information on quality should be revealed. Traditional information revelation mechanisms as prices, advertisement, reputation, warranties, etc. suitable for experience goods are not able to solve the informational problem raised by credence goods. Eco-labels relying on third party certification may be one of solutions to solve this informational problem (Cason and Gangadharan, 2002). Certification is a process where a third party verifies the fulfillments of a firm to certain criteria or standards. Information on product quality can be provided by setting up voluntary codes of conduct and then providing eco-labels to firms that comply. Eco-labels are present in many markets as coffee, forest products, cereals, fishes, clothes etc. and can be administrated by governments or private agents (industry, Non-Governmental-Organization (NGO), consumers associations). In 2013, the eco-label index tracks 493 eco-labels in 197 countries and 25 sectors. Examples of eco-labels are organic products labels provided by USDA or the European Union delivered to products meeting their label criteria or the Fresh Produce Audit Verification voluntary program developed by FDA and USDA delivering a certification mark to firms implementing Good Agricultural practices and so on.

Eco-labeling is a multi-faceted theme generating substantial theoretical literature that covers a huge set of questions. The major part of the literature on labels assumes perfect certification (Bottega and De Freitas, 2009; De Freitas and Bottega, 2009; Ibanez and Stenger, 2000; Auriol and Schilizzi, 2003), that is, the certifier is able to assess the quality of the product with certainty, therefore the label is a perfect signal for consumers. In

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1The final report of the Steering Committee of the State-of-Knowledge Assessment of Standards and Certification, 2012 proposes a very comprehensive survey on the role and limitation of certification.


3For more information on the Fresh Produce Audit Verification program, see http://www.ams.usda.gov/AMSv1.0/GAPGHPAuditVerificationProgram
Ibanez and Grolleau (2008) certification is not perfect, but deterministic, in the sense that firms would be awarded the label for sure if they ask for certification, yet brown firms have to bear a higher certification cost. Incentives to apply for the label are generally studied on previous literature, of particular interest is the trade-off between product information revelation and the possible deadweight loss that emerges as the label allows firms to differentiate, and therefore to earn extra profits through price increase. Kiesel et al (2005) for instance, studied fluid milk and whether there is a price premium for organic milk. They found that organic milk accounted for about 13 percent of the market and sold for 63 to 90 percent more than conventional milk. Another example is given by Casadesus-Masanell et al (2009). They show that customers were willing to pay a price premium for organic cotton (compared to conventional cotton shirts) with the average willingness to pay being 14.6% higher than increase in production cost.

However, certification may be imperfect as the certifier may fail in the attribution of a label, labeling low quality products (false positive) while rejecting high quality products (false negative). This seems to be a more realistic approach. Mason (2011) lists various sources of testing errors as insufficient frequency of the test (random monitoring) or imperfect correlation of standards with environmental friendliness. One may also think of sampling errors if the test is based on limited sample size or certifiers corruption, etc. Producers of low quality may have an incentive to claim their product is of high quality and earn extra profit without bearing the entire cost of a high quality product. For instance, in 2011, the Tageszeitung newspaper reported that “Italian authorities said they had busted a scheme in which more than 700,000 tonnes of non-organic food was sold as organic – and sold at the higher prices commanded by such products”.

Taking account of possible failure in certification, we study firms’ strategic behaviour in prices and certifying decisions in a duopolistic market. We also investigate the welfare implications of labeling in this framework. Literature on labeling under imperfect certification (De and Nabar, 1991; Giannakas, 2002; Liang and Jensen, 2007; Mason and Sterbenz, 1994; Mason, 2009, 2011; Gu, 2008) mostly assumes competitive markets. Notably Mason (2011) studies firms’ labeling strategy when the test is noisy and results either in false positives or false negatives as in our case. He discusses the conditions for the emergence of pooling and separating equilibrium and the attendant welfare effects. Contrary to Mason, we allow for strategic price setting as in Ibanez and Grolleau (2008). They also study labeling and price decision. However, we differ from their approach by considering a noisy test and presenting the welfare effect of implementing an eco-labeling scheme in this imperfect

4Source: http://www.thelocal.de/society/20111208-39391.html
certification context.

On the one hand, it is harder to reach a separating equilibrium under a noisy certification test as firms may not be awarded the label and earn zero profits with positive probability. In [De and Nabar (1991)] only a pooling equilibrium exists, [Strausz (2010)] also finds a separating equilibrium where the high quality firm certifies while the low quality firm does not certify. Nevertheless Strausz’s result is a consequence of buyers’ rationality whereas our is due to strategic interaction (imperfect competition) among firms. On the other hand, in our setting, if competition was perfect, nobody would be willing to certify. Strategic interaction among firms facilitates the emergence of a separating equilibrium as firms are able to differentiate. [Mason (2011)] also finds a separating equilibrium when the cost of certification is sufficiently large, as in our case, though, in our paper when the probability to pass the test is sufficiently large a separating testing equilibrium exists even for a fee equal to zero.

Concerning social welfare, we find that the eco-labeling policy is welfare enhancing for all parameters values. Nevertheless, the separating testing equilibrium may be too costly to sustain when firm $g$ probability to pass the test is small. We find the surprising result that if the certification technology is lenient, meaning that both brown and green units are awarded the label with high probability, a separating equilibrium could be sustained for a fee equal to zero. This is a consequence of the price strategic interaction between firms that gives firms incentives to coordinate on a separating equilibrium.

When firms can choose to be green or brown (environmental quality is no longer exogenous) we find that two green firms coexisting can not be an equilibrium. We describe the conditions supporting a separating testing equilibrium with one firm choosing to be brown and one firm choosing to be green. We find, by means of numerical examples, that the certification fee must be large enough to prevent the brown firm from deviating and become green.

The rest of the paper is organized as follows: section 2 introduces the model, in section 3 we study firms’ pricing strategies. In section 4, firms’ testing decisions are presented with the conditions for the emergence of a pooling and a separating equilibrium. In section 5 a welfare analysis is conducted. In section 6 we relax the assumption of exogenous quality. Main results are presented using a numerical example. Section 7 concludes.
2 The model

Consider a vertical differentiation duopoly model with firms exogenously characterized by their product quality $q \in \{q_b, q_g\}$ with $q_g > q_b \geq 0$. We refer to firm $g$ (for green or good) as a firm producing the high quality variant $q_g$ and to firm $b$ (for brown or bad) as a firm producing the low quality variant $q_b$. We normalize the lowest quality to zero: $q_b = 0$.

The supply side consists of one firm $g$ and one firm $b$ and this is common knowledge. We relax this assumption in section 6 by endogenizing quality choice. In this alternative scenario, the supply side may be composed of two firms $g$ or two firms $b$. Quality reflects environmental friendliness but it may take several other forms that would fit our framework: social conditions, health considerations, etc.

Both firms face a constant and common marginal production cost $c$. Firm $g$ bears an additional fixed sunk cost $C_g$ to reflect the investment undertaken for the production of a high quality variant.

Each producer knows the true quality level of its product and that of its rival. However, environmental quality as a credence attribute is observable to consumers neither before nor after purchase and use. Producers can rely on a voluntary eco-labeling scheme to signal their greeness and differentiate their product to obtain a price premium that would cover their investment in quality. The certifier ignores the type of the firm applying for certification. To infer firms’ product quality the certifier performs a test and charges a fixed fee denoted by $F$. The fixed fee is sunk and covers the cost associated to the certification process. As in Ibanez and Stenger (2000) information transmission is done solely through labels, but in our paper monitoring is imperfect in the sense that a low quality product could be erroneously identified as one of high quality and viceversa. Thus the test may result in false positives or false negatives. As a consequence of test imperfection, firm $b$ may have an incentive to mimic firm $g$ to obtain the green premium without bearing the cost of producing a high quality variant.

While the test is noisy, still firm $g$ has a higher probability to pass the test. Passing probabilities, conditional on taking a test, are independent and exogenously given. The probability that firm $b$ passes the test is $\phi_b$ while the probability that firm $g$ passes the test equals $\phi_g$ with $1 \geq \phi_g > \phi_b > 0$. The certifying agency is not strategic and awards the label to firms passing the test. A firm remains unlabeled whenever she fails to pass the test or does not take it.

5These fixed costs are sunk meaning that there are paid whether or not the firm produces. This does not influence short run decisions as in our framework but may influence firms’ decisions in the long run. Indeed, $C_g$ plays a crucial role in section 6, when firms choose their type.
Consumers neither observe firms’ product quality nor firms’ testing decisions, but they know the distribution of types and test passing probabilities. Consumers learn whether the good has been awarded a label and infer its quality considering all the information available in the market. Note that before observing any labeling, consumers’ prior belief that a product is green equals \( \frac{1}{2} \) for all consumers and all products.

The demand side consists of a continuum of consumers with heterogeneous preferences over quality indexed by \( \theta \). Parameter \( \theta \) represents consumers’ willingness to pay for quality and is uniformly distributed over \([0, 1]\). A consumer of type \( \theta \) has the following indirect utility:

\[
V(\theta) = \begin{cases} 
    y + \theta E[q|l] - p_l & \text{if he buys a labeled product at price } p_l \\
    y + \theta E[q|u] - p_u & \text{if he buys an unlabeled product at price } p_u \\
    0 & \text{If he doesn’t buy at all}
\end{cases}
\]  

(1)

where \( E[q|l] \) (Resp. \( E[q|u] \)) denotes the expected quality of a product given it is labeled (Resp. unlabeled). The market is fully covered, therefore \( y \) denotes consumers’ reservation price for an uncertified good. It is sufficiently large so that consumers buy either one unit of the labeled product or one unit of the unlabeled product. Consumers make buying decisions so as to maximize their utility.

The timing of the game we consider is as follows:

**Stage 1**: Firms decide simultaneously whether to test the quality of their product (\( t \)) or not (\( n \)) in order to be awarded the label.

**Stage 2**: The certifier monitors product quality and awards the label to firms passing the test.

**Stage 3**: Firms compete à la Bertrand in the third stage. Consumers observe if the product is labeled (\( l \)) or not (\( u \)), update their beliefs following Bayes’ rule and make their purchase decisions. We solve the model backwards. We begin with price competition and end up with firms’ testing decisions.

## 3 Price competition

The label provides information on the environmental characteristics of the product but this signal relying on a noisy test is imperfect. Consumers knowing the test passing probabilities

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\(^6\)For instance, it is the case for \( y \geq q_g + c \), as the market will be fully covered for any price resulting from firms’ price competition if the previous condition is satisfied.
will use this information to update their beliefs on the quality of the product. Note that consumers can not distinguish between a firm that have taken the test and failed and one that didn’t apply, as both rest unlabeled.\footnote{Here we thus consider that there is no information disclosure on unsuccessful attempts. On this topic, see for instance Farhi et al. (Forthcoming).}

Firms choose their price simultaneously given consumers’ purchase decision, which are determined according to the presence (or not) of an eco-label on the sold products. Consequently, two cases are to be considered: Both or none of the firms sell labeled products (first case) or firms sell differently labeled products (second case). In the first case firms sell identically labeled products. As consumers cannot distinguish products, Bertrand competition implies marginal-cost pricing.

Let’s now turn to the second case where firms sell differently labeled products. For we need to compute the demand for labeled and for unlabeled variants.

The indifferent consumer between buying the labeled and unlabeled product is equal to,

\[ \tilde{\theta} = \frac{p_l - p_u}{E[q|l] - E[q|u]} \]  

(2)

If firm \( g \) takes the test with positive probability, the demand for labeled and unlabeled products are respectively, \( x_l = 1 - \tilde{\theta} \), and \( x_u = \tilde{\theta} \).

As sunk fixed costs do not influence pricing decisions, we focus on variable profits defined as the difference between total revenue and variable cost. Each firm chooses the price maximizing its variable profit given the price chosen by its rival.

The variable profit of the labeled firm is,

\[ \left( \frac{E[q|l] - E[q|u] - (p_l - p_u)}{E[q|l] - E[q|u]} \right) (p_l - c) \]  

(3)

The variable profit of the unlabeled firm is,

\[ \left( \frac{p_l - p_u}{E[q|l] - E[q|u]} \right) (p_u - c) \]  

(4)

We obtain the following equilibrium prices when products are differently labeled:

\[ p_l = \frac{2}{3} \Delta E + c \]
\[ p_u = \frac{1}{3} \Delta E + c \]
where $\Delta E = E[q|l] - E[q|u]$. Replacing equilibrium prices in (3) and (4) we obtain total (variable) profit for the labeled and unlabeled firm: $\frac{1}{2} \Delta E$ and $\frac{1}{2} \Delta E$, respectively. The indifferent consumer is characterized by $\hat{\theta} = \frac{1}{3}$. As a benchmark, under full information, equilibrium prices for the high and low quality variant are, respectively, $\frac{2}{3}q_g + c$ and $\frac{1}{3}q_g + c$.

4 Testing decisions

We now turn to firms’ testing choices given firms’ pricing decisions described in the previous section. Firms’ testing strategies depend on the way consumers revise their beliefs when they observe the type of products they face: labeled or unlabeled.

Let $\mu_i$ be the probability that firm $i$, $i = b, g$, asks for certification. So, firm $i$ does not take the test with probability $1 - \mu_i$. A pair $(\mu^*_b, \mu^*_g)$ forms a Nash equilibrium of the certification game if $\mu^*_b$ is firm $b$’s best response to firm $g$ strategy $\mu^*_g$, and $\mu^*_g$ is firm $g$’s best response to firm $b$ strategy $\mu^*_b$.

We need first to describe the way consumers update their belief and form their expectations. Remember that consumers’ prior belief on the probability that any good in the market is of high quality is $\frac{1}{2}$, that is the probability of buying from firm $g$. When consumers face two labeled or unlabeled products, it is equivalent to the no information case; the expected quality is therefore of $\frac{1}{2}q_g$ and due to Bertrand competition, prices equate marginal cost.

When consumers face differently labeled products, they use Bayes’ rule to update their beliefs as follows. Let $\beta(q_g, l)$ be consumers’ updated belief on the probability that the good is of high quality provided that it is labeled, and $\beta(q_g, u)$ be consumers’ belief that the good is of high quality given that it is unlabeled. At equilibrium,

$$
\beta(q_g, l) = \frac{\phi_g \mu^*_g (1 - \mu^*_b) + \phi_b (1 - \phi_b) \mu^*_g \mu^*_b}{\phi_g \mu^*_g (1 - \mu^*_b) + \phi_b \mu^*_b (1 - \mu^*_g) + (\phi_g (1 - \phi_b) + \phi_b (1 - \phi_g)) \mu^*_g \mu^*_b} \tag{5}
$$

The expected quality of the labeled variant equals,

$$
E[q|l] = \beta(q_g, l) q_g \tag{6}
$$

While expected quality of the unlabeled variant is,

$$
E[q|u] = \beta(q_g, u) q_g = (1 - \beta(q_g, l)) q_g \tag{7}
$$

Given the belief revision process described previously, we are now able to look at firms’
decision to test or not their product in order to be awarded the label. Each firm has two strategies: to test \((t)\) or not to test \((n)\). Firms’ expected payoffs given firm \(b\)’s and firm \(g\)’s testing strategies are depicted in table 1.

Each cell represents the payoffs of firm \(b\) and firm \(g\). For instance, the upper rightmost cell says that if firm \(b\) plays \(t\) and firm \(g\) plays \(n\), they get \(\Pi_b(t, n)\) and \(\Pi_g(t, n)\), respectively.

<table>
<thead>
<tr>
<th>Firm (b) (\backslash) Firm (g)</th>
<th>(t)</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
<td>((\Pi_b(t, t), \Pi_g(t, t)))</td>
<td>((\Pi_b(t, n), \Pi_g(t, n)))</td>
</tr>
<tr>
<td>(n)</td>
<td>((\Pi_b(n, t), \Pi_g(n, t)))</td>
<td>((\Pi_b(n, n), \Pi_g(n, n)))</td>
</tr>
</tbody>
</table>

Provided firms’ variable profits computed in the price competition stage explored above, expected payoffs corresponding to consumers’ expectations and firms’ probabilities to get a label are,

\[
\Pi_b(t, t) = \frac{4\phi_b(1 - \phi_g) + \phi_g(1 - \phi_b)}{9} \Delta E - F 
\]

\[
\Pi_b(t, n) = \frac{4}{9} \phi_b \Delta E - F 
\]

\[
\Pi_b(n, t) = \frac{1}{9} \phi_g \Delta E 
\]

\[
\Pi_b(n, n) = 0 
\]

\[
\Pi_g(t, t) = \frac{\phi_b(1 - \phi_g) + 4\phi_g(1 - \phi_b)}{9} \Delta E - F - C_g 
\]

\[
\Pi_g(t, n) = \frac{1}{9} \phi_b \Delta E - C_g 
\]

\[
\Pi_g(n, t) = \frac{4}{9} \phi_g \Delta E - F - C_g 
\]

\[
\Pi_g(n, n) = -C_g 
\]

We focus on pure strategy Nash equilibrium. To begin with, we study under which circumstances the separating equilibrium \((\mu_b^s = 0 \text{ and } \mu_g^s = 1)\) where only firm \(g\) asks for certification emerges. This equilibrium is denoted by \((b, g) = (n, t)\). Then we explore under which conditions the pooling equilibrium \((\mu_b^p = 1 \text{ and } \mu_g^p = 1)\) emerges. This equilibrium is denotes by \((b, g) = (t, t)\)
4.1 Separating testing equilibrium \((b,g)=(n,t)\): \(\mu_b^* = 0\) and \(\mu_g^* = 1\)

For \(\mu_b^* = 0\) and \(\mu_g^* = 1\), consumers’ expected quality of the labeled and unlabeled variant are \(E[q|l] = q_g\) and \(E[q|u] = 0\), respectively. The pair \(\mu_b^* = 0\) and \(\mu_g^* = 1\) is a Nash equilibrium of the testing game if \(\Pi_b(n,t) > \Pi_b(t,t)\) and \(\Pi_g(n,t) > \Pi_g(n,n)\).

We next show in Proposition 1 that a separating equilibrium \((n,t)\) always exists for appropriate \(F\).

**Proposition 1.** The separating equilibrium where firm \(g\) asks for certification and firm \(b\) doesn’t is a Perfect Bayesian Equilibrium of the game if \(E_{nt} < F < F_{nt}\), where \(E_{nt} = \frac{\phi_b(4-5\phi_g)q_g}{g}\) and \(F_{nt} = \frac{4\phi_g q_g}{g}\). Moreover \(F_{nt} > E_{nt}\) for all \(0 < \phi_b \leq 1\) and \(0 < \phi_g \leq 1\).

**Proof.** Note that \(\Pi_b(n,t) - \Pi_b(t,t) = \frac{\phi_b(4-5\phi_g)q_g}{g} + F\) and \(\Pi_g(n,t) - \Pi_g(n,n) = \frac{4\phi_g q_g}{g} - F\).

It is easy to show that for \(E_{nt} < F < F_{nt}\) firm \(g\) and firm \(b\) have no incentive to deviate from equilibrium \((n,t)\). Moreover \(F_{nt} > E_{nt}\) for \(\phi_b\) and \(\phi_g\) positive and lower than one: \(F_{nt} - E_{nt} = \frac{4(4\phi_g - 5\phi_b)q_g}{g} + 5\phi_g \phi_b q_g > 0\). Then, the separating equilibrium \((n,t)\) always exists for \(F \in (E_{nt}, F_{nt})\).

From the above Proposition we find that the lowest certification fee at which the separating equilibrium \((b,g) = (n,t)\) exists is given by \(\max\{E_{nt}, 0\}\). The certification fee needs to be sufficiently large to dissuade firm \(b\) from testing. For \(\phi_g\) sufficiently large (\(\phi_g \geq \frac{4}{5}\)), the lowest fee guaranteeing the existence (not uniqueness) of a \((b,g) = (n,t)\) equilibrium equals zero. When firm \(g\)’s probability to obtain the label is high, \(F\) does not need to be very important as this is sufficient to dissuade firm \(b\) from testing. The underlying idea is the following: profit is higher for firm \(b\) when products are differentiated. For \(\phi_g\) above \(4/5\), firm \(g\) is almost certain to obtain the label when testing its product, firm \(b\) prefers to differentiate almost surely by not testing its product.

We do not consider the separating equilibrium \((t,n)\) as it would lead to the same outcome as \((n,t)\) but with lower probability. Therefore \((n,t)\) Pareto dominates \((t,n)\). If consumers believe the label has been awarded to firm \(b\), the label will be a negative signal of quality. Still, the label would allow firms to differentiate but consumers would not pay a price premium for the labeled good but for the unlabeled one. As firm \(b\)’s probability to obtain the label is smaller, \(\phi_b > \phi_g\), the probability to reach an outcome where firms differentiate is smaller under the separating testing equilibrium \((t,n)\). In expected terms, then, profits are higher for both firms under the separating equilibrium \((n,t)\).
4.2 Pooling testing equilibrium \((b,g)=(t,t)\): \(\mu^*_b = 1\) and \(\mu^*_g = 1\)

We explore condition on \(F\) for a pooling equilibrium where both firms certify: \((b,g) = (t,t)\).

For \(\mu^*_b = \mu^*_g = 1\), in case we observe a labeled and an unlabeled variant, the expected quality of the labeled variant is strictly lower than \(q_g\):

\[
E[q/l] = \frac{\phi_g (1 - \phi_b)}{(\phi_g (1 - \phi_b) + \phi_b (1 - \phi_g))} q_g < q_g.
\]

As both firms must have incentives to take the test, \(F\) has to be sufficiently low. When both firms ask for certification, there is a positive probability to end-up in a situation where differentiation is not possible and profits are eroded due to price competition.

**Proposition 2.** The pooling Equilibrium where both firms ask for certification is a Perfect Bayesian Equilibrium for:

\[
F < \frac{\phi_b (\phi_g - \phi_b)(4 - 5\phi_b)}{9(\phi_g (1 - \phi_b) + \phi_b (1 - \phi_g))} q_g = F_{tt}
\]

**Proof.** The pair \(\mu^*_b = \mu^*_g = 1\) is a Perfect Bayesian Equilibrium of the testing game if \(\Pi_g(t,t) > \Pi_g(t,n)\) and \(\Pi_b(t,t) > \Pi_b(n,t)\). Replacing \(\mu^*_b = \mu^*_g = 1\) in \(\beta(q_g, l)\) and \(\beta(q_g, u)\) and then replacing the last expressions in \(E[q/l]\) and \(E[q/u]\) give us consumers’ expected quality of the labeled and unlabeled variant, respectively. As a result \(\Delta E = \frac{\phi_g - \phi_b}{(\phi_g (1 - \phi_b) + \phi_b (1 - \phi_g))} q_g\). Remember that total variable profit equals \(\frac{4}{9} \Delta E\) for the labeled variant and \(\frac{1}{9} \Delta E\) for the unlabeled variant. Therefore,

\[
\Pi_g(t,t) - \Pi_g(t,n) = \frac{\phi_g (\phi_g - \phi_b)(4 - 5\phi_b)}{9(\phi_g (1 - \phi_b) + \phi_b (1 - \phi_g))} q_g - F.
\]

\[
\Pi_b(t,t) - \Pi_b(n,t) = \frac{\phi_b (\phi_g - \phi_b)(4 - 5\phi_g)}{9(\phi_g (1 - \phi_b) + \phi_b (1 - \phi_g))} q_g - F.
\]

The lowest cutoff is \(F = \frac{\phi_b (\phi_g - \phi_b)(4 - 5\phi_g)}{9(\phi_g (1 - \phi_b) + \phi_b (1 - \phi_g))} q_g = F_{tt}\)

The certification fee needs to be sufficiently small for both firms to test their products. Again, for sufficiently high \(\phi_g\) (\(\phi_g > \frac{4}{5}\)), the fee would need to be negative for a pooling equilibrium to exist. As a result, with a sufficiently lenient certification test, firm \(b\) never wants to mimic firm \(g\).
Note that, none firm asking for certification is also a pooling equilibrium. Results would be similar as those presented in the no information case.

5 Welfare Analysis

All agents in our economy (consumers, firms and the certifier) are risk neutral. Social welfare results from the sum of expected consumers surplus, and expected total firms’ profits. Here we explore whether an eco-label policy is welfare enhancing. To this purpose we compute social welfare corresponding to each equilibrium, in particular \((n,n)\), \((n,t)\) and \((t,t)\). As a benchmark, as both firms share the same marginal cost, but the good produced by firm \(g\) is of superior quality, the first best outcome would be the complete information scenario where only firm \(g\) produces and all consumers buy from firm \(g\), as long as the fixed cost of quality is sufficiently small.

Social welfare would be the same in case both firms were awarded the label, no firm certifies, or in the absence of an eco-label as in those cases no information is revealed to consumers. As in the eyes of consumers goods would be homogeneous prices would equate marginal cost (Bertrand paradox). Consumers would, then, buy from either firm if variants were sold at the same price. Market would split equally with expected quality equal to \(\frac{1}{2}q_g\). We refer to this equilibrium as the no information equilibrium and denote it \((n,n)\).

In contrast with the no information equilibrium, the separating testing equilibrium, \((n,t)\), reveals perfectly information on quality to consumers through the eco-label, this occurs with probability \(\phi_g\). As a consequence firms, being able to differentiate, sell at a price above marginal cost. The price of the labeled and unlabeled variant being \(p_l = \left(\frac{2}{3}\right)q_g + c\) and \(p_u = \left(\frac{1}{3}\right)q_g + c\), respectively. More consumers buy from firm \(g\) with total demand of \(\left(\frac{2}{3}\right)\) of the market. The rest of consumers, \(\left(\frac{1}{3}\right)\) of the market, buy the unlabeled variant.

The pooling testing equilibrium, \((t,t)\), lies between the separating and the no information equilibrium in the sense that, thought quality is not fully revealed to consumers, the probability that the labeled variant is of the highest quality \((\phi_g(1 - \phi_b))\) is larger than the probability to buy a wrongly labeled unit \((\phi_b(1 - \phi_g))\), as firm \(g\) has greater probability to pass the certification test. In the case of differently labeled products, both variants would command prices above marginal cost, but prices would be lower than under the separating equilibrium. The price of the labeled and unlabeled variant are respectively, \(p_l = \left(\frac{2}{3}\right)\Delta E + c\) and \(p_u = \left(\frac{1}{3}\right)\Delta E + c\), with \(\Delta E = (\phi_g - \phi_b)/(\phi_g(1 - \phi_b) + \phi_b(1 - \phi_g))q_g < q_g\).

\[8\] With probability \((1 - \phi_g)\) no information is revealed and the outcome of the game is equivalent to the no information equilibrium.
In case both firms rest unlabeled, which occurs with probability \((1 - \phi_g)(1 - \phi_b)\), or both are labeled \((\phi_g, \phi_b)\), the eco-label does not provide useful information to consumers and the outcome is similar to the no information case.

Comparing equilibria \((n,n)\), \((n,t)\) and \((t,t)\), in expected terms, the separating testing equilibrium is closer to the first best outcome, as, in expected terms, more consumers buy from firm \(g\). Nevertheless, as showed in the previous section, we need \(F\) to be sufficiently large to support the separating testing equilibrium. The fee \(E_{nt}\) could be interpreted as the cost of information transmission. It is not obvious at first sight which equilibrium provides the highest social welfare. Next table shows expected consumer surplus (CS), expected total profits (\(\sum \Pi\)) and social welfare (\(W\)). For ease of presentation, we omit parameter \(y\) in the CS and the cost of the green technology, \(C_g\), in \(\sum \Pi\), as they do not affect welfare comparison among equilibrium outcomes. Therefore, we obtain total welfare by adding \(y\) and subtracting \(C_g\) in \(W\) for all cases depicted in Table 2.

<table>
<thead>
<tr>
<th>(i = {nn, nt, tt})</th>
<th>((n,n))</th>
<th>((n,t))</th>
<th>((t,t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(CS_i)</td>
<td>(\frac{1}{2}q_g - c)</td>
<td>(\left(\frac{1}{4} - \frac{13}{36} \phi_g\right)q_g - c)</td>
<td>(\left(\frac{1}{4} - \frac{13}{36} (\phi_g - \phi_b)\right)q_g - c)</td>
</tr>
<tr>
<td>(\sum \Pi_i)</td>
<td>0</td>
<td>(\frac{5}{9} \phi_g q_g - F)</td>
<td>(\frac{5}{9} (\phi_g - \phi_b) q_g - 2F)</td>
</tr>
<tr>
<td>(W_i)</td>
<td>(\frac{1}{2}q_g - c)</td>
<td>(\left(\frac{1}{4} + \frac{7}{36} \phi_g\right)q_g - c - F)</td>
<td>(\left(\frac{1}{4} + \frac{7}{36} (\phi_g - \phi_b)\right)q_g - c - 2F)</td>
</tr>
</tbody>
</table>

It can be easily showed, by inspection of Table 2, that for a given \(F\), social welfare under the separating equilibrium is larger than social welfare under the pooling equilibrium. Note that the eco-labeling policy is unambiguously welfare enhancing compared to the situation with no label (the \((n,n)\) equilibrium) as for \(F = 0\): \(W_{nt} > W_{tt} > W_{nn}\). Nevertheless, equilibrium \((n,t)\) may not be achieved for \(F = 0\). In effect, the minimum fee guaranteeing a separating equilibrium equals \(E_{nt}\), which is strictly positive for \(\phi_g < 4/5\). Therefore, the separating equilibrium may be costly to achieve in certain cases (strict label).

If we look for the fee leading to the equilibrium with the largest social welfare, let’s name it the optimal fee, we have to compare \(W_{nt}(F = \max\{0, E_{nt}\})\) with \(W_{nn}\) and \(W_{tt}(F = 0)\).

Next Proposition summarizes the main result.

**Proposition 3.** For \(\phi_g \geq 9/20\) social welfare is higher under the separating equilibrium and the optimal fee equals \(\max\{0, E_{nt}\}\). For \(\phi_g < 9/20\), to achieve the separating equilibrium is too costly \((W_{nt}(F = E_{nt}) < W_{tt}(F = 0) < W_{nn})\) and the optimal fee in this case equals zero.
Proof. We wonder whether it is better a separating testing equilibrium at $E_{nt}$ compared to a pooling equilibrium at $F = 0$, whenever it exists. Remember a pooling testing equilibrium exists for $F = 0$ as long as $F_{tt} > 0$, that is the case for $\phi_g < 4/5$. $W_{nt}(E_{nt}) - W_{tt}(F = 0) = \frac{1}{25} \phi_b (20 \phi_g - 9) q_g$, therefore, $W_{nt}(E_{nt}) > W_{tt}(F = 0)$ for $\phi_g > \frac{9}{25}$. If it is too costly to sustain the separating testing equilibrium with a fee equal to $E_{nt}$, better to set $F = 0$ and support a pooling testing equilibrium, as welfare decreases with $F$ and the pooling equilibrium is superior to $(n, n)$. \hfill \blacksquare

From Proposition 3, the optimal fee is equal to zero for $\phi_g < 9/20$, and both firms certify. For $\phi_g \geq 4/5$ the optimal fee is also equal to zero as $E_{nt} \leq 0$, but in this case we reach the separating equilibrium for $F = 0$ and only firm $g$ certifies. Moreover, the optimal fee is strictly positive for intermediate values of $\phi_g$. These results imply that, when the optimal fee equals zero, it may be optimal to subsidize firms to enter certification, whereas for intermediate values of $\phi_g$, the fee prevents $b$ from testing when it is not optimal, therefore, firms should bear, at least partially, the cost of certification in this case.

Note that this result does not depend on the distance between $\phi_b$ and $\phi_g$, provided that $\phi_g > \phi_b$. This implies that the separating equilibrium could be achieved with a fee equal to zero if the certification technology were lenient, as for $\phi_b, \phi_g$ large enough $E_{nt} \leq 0$. This result is a consequence of the strategic interaction between firms, as in the absence of an eco-labeling policy firms face fierce competition. The eco-label relaxes price competition. Firm $b$ has less incentives to mimic firm $g$, specially for $\phi_g$ large enough, to avoid a situation where both end-up labeled, a situation driving profits to zero.

To illustrate the results of the welfare analysis, we present a numerical example capturing the main intuitions presented above.

5.1 Numerical example

For this numerical example we consider $q_g = 1$ and $c = 0$. We first show what the optimal fee is as a function of $\phi_b$ for two possible values of $\phi_b = \{\frac{1}{5}, \frac{3}{5}\}$. We then explore the conditions on $F$ and $C_g$ that sustain a separating testing equilibrium in case firms can freely choose whether to invest in quality in a first stage.

Figure 2 summarizes the main results. For $\phi_b = 1/5$, the optimal fee is equal to $\frac{(4-5\phi_g)}{45}$ for intermediate values of $\phi_g$ and zero otherwise. The pooling equilibrium is welfare enhancing compared to the separating equilibrium at $E_{nt}$ for $\phi_g \leq 9/20$. For $\phi_b = 3/5$, $\phi_g$ is always above $9/20$, therefore the optimal fee is equal to $\frac{3(4-5\phi_g)}{45}$ for $\phi_g \leq 4/5$ and zero otherwise, being able to reach a separating equilibrium for any $\phi_g$. Notice that the set of $F$ sus-
taining a separating equilibrium is larger for \( \phi_b = 1/5 \), as \( E_{nt}(\phi_b = 1/5) < E_{nt}(\phi_b = 3/5) \).

Figure 2: \( F_s \) cut-offs and the optimal fee
6 Endogenous types

Previous sections showed the equilibrium testing and price strategies and a welfare analysis in the context of an eco-labeling policy, when there exists a brown and a green firm. Such an analysis corresponds to a short-term situation where firms can not adjust their types. We wonder what would happen in case firms could choose their type on a first stage. This may be the case in the long run. Remember the fixed cost to invest on a green technology is $C_g$, and both firms learn rival’s quality choices before the certification game starts.

Due to price competition, and as long as $C_g > 0$, both firms choosing to be green, $gg$, can not be a Perfect Bayesian Equilibrium (PBE) as one of the firms would deviate to save the cost $C_g$. Moreover, firms could not differentiate and earn positive profits when choosing the same type. The more natural equilibrium would be $bg$, (or analogously $gb$).

Though labeling and pricing strategies were already studied for the $bg$ economy previously, the conditions on $F$ (and also on $C_g$) sustaining a $bg$ (or $gb$) separating testing equilibrium as a Perfect Bayesian Equilibrium are harder to satisfy. For the sake of brevity we show this result by means of numerical examples, where it is plotted the set of $F$ and $C_g$ supporting the $bg$ separating testing equilibrium and the corresponding $E_{nt}$, $F_{nt}$.

Notice that to compute the set of $F$ and $C_g$ sustaining $bg$ separating equilibrium as a PBE we need to include restrictions on firms’ profits such that firms are not willing to deviate by choosing a different type.

Figure 3 shows the set of $F$ and $C_g$ supporting a separating testing equilibrium for different values of $\phi_b$ and $\phi_g$. The optimal fee computed in the previous section would be equal to zero for Figures 3b and 2c while it would equal $E_{nt}$ for Figures 3a and 2d. The horizontal lines represent $E_{nt}$ and $F_{nt}$, corresponding to the exogenous types case studied in previous sections. The lowest $F$ supporting the $bg$ separating equilibrium is strictly larger than $E_{nt}$, therefore, it is harder to sustain a separating equilibrium when types are endogenously determined as $F$ must be large enough to reduce firm’s $b$ incentives to deviate and become green.

An equilibrium $bb$ and $(n,n)$ is also possible for sufficiently high $C_g$ and/or very pessimistic beliefs on an out of equilibrium labeled variant. As for the case $gg$, firms profits would be equal to zero due to price competition. Equilibrium $bg$ (or analogously $gb$) seems to be the most plausible equilibrium.

Throughout this section the computations were made using MATHEMATICA software. Computations details are available upon request to authors.
Figure 3: Endogenous types: $F$ and $C_g$ supporting a $bg$ separating testing equilibrium

(a) ($\phi_b = 1/5, \phi_g = 1/3$)

(b) ($\phi_b = 3/5, \phi_g = 4/5$)

(c) ($\phi_b = 1/5, \phi_g = 4/5$)

(d) ($\phi_b = 1/8, \phi_g = 3/8$)
7 Conclusion

We study what are the implications of a noisy eco-labelling policy when strategic interaction between firms is taken into account.

On the one hand, it is harder to reach a separating equilibrium under a noisy certification test as firms may not be awarded the label and earn zero profits with positive probability. On the other hand, in our setting under perfect competition nobody would be willing to certify. Strategic interaction among firms facilitates the emergence of a separating equilibrium as firms will be able to differentiate. Indeed the minimum fee supporting a separating equilibrium may be equal to zero.

Under the first best outcome only firm $g$ sells to the entire market. In the absence of certification firm $g$ would serve half of the market. The certification test allows firms to signal product quality and firm $g$ would serve more than half of the market even in a pooling testing equilibrium where both firms ask for certification. In this sense, the eco-label is welfare enhancing for all parameters values. For a given fee, the separating testing equilibrium provides the greatest (2nd best) social welfare as firm $g$ sells to $2/3$ of the market. Note that though, under the separating equilibrium, firm $b$ is selling to a smaller portion of the market, firm $b$ increases profits compared to the situation with no eco-label as she prices her units above marginal cost.

In our setting we only account for the quality distortion generated by the lack of information. The eco-label policy succeeds in providing information to consumers on quality, and the price increase due to differentiation end-up being a transfer from consumers to firms with no effect on social welfare. If a partially covered market was assumed, we would account for the quantity distortion emerging from imperfect competition between firms. This is certainly an interesting extension of our present model that we left for future research. We assume that firms can only signal quality through the eco-label. It will be interesting to investigate the implications of the eco-labelling policy when firms also use prices to signal quality.

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