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In many real-life house allocation problems, rents are bounded from above by price ceilings imposed by the government or a local administration. This is known as rent control. Because some price equilibria may be disqualified given such restrictions, a weaker equilibrium concept is suggested. Given the weaker notion, this paper defines an allocation mechanism, tailored to capture the specific features of housing markets with rent control, which always selects a weak price equilibrium. The main results demonstrate the existence of a weak price equilibrium and that the introduced allocation mechanism is efficient and non-manipulable for any given price ceiling. In its two bounding cases, the mechanism reduces to the weak version of the serial dictatorship mechanism (Svensson, 1994) and the competitive price mechanism (Demange and Gale, 1985), respectively. In this sense, the housing market with rent control, investigated in this paper, integrates two of the predominant models in the two-sided matching literature into a more general framework.

Keywords: House allocation, rent control, rationing, weak price equilibrium, priority efficiency, non-manipulability.

1. INTRODUCTION

House allocation is a classical problem in the mechanism design literature. The aim is to allocate a number of houses (or some other indivisible items such as jobs or tasks) among a group of agents given that each agent is interested in renting, or buying, at most one house. In an early paper, Hylland and Zeckhauser (1979) proposed the serial dictatorship mechanism as a solution to this problem for the case when rents are exogenously given and in the absence of existing tenants[1]. This deterministic rule assigns each agent a priority according to some criteria (random, queue, etc.) and the agents have to choose a house

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[1]The problem with existing tenants is not considered in this paper. It was first investigated by Shapley and Scarf (1974). See also Abdulkadiroğlu and Sönmez (1998), Ma (1994), Roth and Postlewaite (1977), Roth (1982), and Svensson (1999) among others. Also, the "kidney exchange problem" resembles a housing market with existing tenants, see Roth et al. (2004) for a detailed discussion. The house allocation model with both existing tenants and new applicants was first considered by Abdulkadiroğlu and Sönmez (1999).

from the set of "remaining houses" when the agents with higher priorities have made their choices. Svensson (1994) demonstrated that a weaker version of this mechanism (that also allows for indifference relations) is Pareto efficient and non-manipulable.[2]

Even if the assumption of exogenously given and fixed rents is restrictive, it is satisfied in many real-life applications. Examples include on-campus housing and public housing. However, a priority based fixed price mechanism cannot take into consideration the existence of agents who are willing to accept higher rents to receive some other house in the allocation process. In this sense, there are potential welfare gains by allowing for more flexible rents. The obvious solution to this problem is to adopt a competitive price mechanism for the allocation procedure. This idea has been advocated by Crawford and Knoer (1981), Demange and Gale (1985), Demange et al. (1986), Leonard (1983), and Shapley and Shubik (1972) among others. The type of housing markets were such mechanisms are applicable are common in metropolitan areas. Also here, Pareto efficient and non-manipulable allocation rules exist. More precisely, because the set of equilibrium prices forms a complete lattice (Demange and Gale, 1985; Shapley and Shubik, 1972), the existence of a unique minimal equilibrium price vector is guaranteed. Then by using this price vector as a direct mechanism for allocating the houses, a Pareto efficient outcome where no agent has any incentive to misrepresent his preferences is assured as demonstrated by Demange and Gale (1985). See also Andersson and Svensson (2008), Leonard (1983), Sun and Yang (2003), and Svensson (2009).

The serial dictatorship mechanism and the competitive price mechanism can be regarded as two polar cases for allocating houses when there are no existing tenants as rents are totally non-flexible in the former and fully flexible in the latter. A third and intermediate practice for allocating houses is that the government or the local administration, by laws or ordinances, imposes a price ceiling and thereby allows for a limited flexibility in the rents. Even if there has been a widespread agreement among economists that this type of rent control generates a mismatch between houses and tenants, discourage new construction, retard maintenance etc. (see e.g. Arnott, 1995; Turner and Malpezzi, 2003, for a detailed discussion), this practice is widely used. As of year 2011, legislated rent control existed in approximately 40 countries around the world including, e.g., Canada, Denmark, Egypt, France, Germany, India, Italy, Netherlands, Spain, Sweden, Turkey, United Kingdom, and United States.

A general problem on housing markets with rent control is that there typically will be an unbalanced relationship between supply and demand (this is obviously also a problem for housing markets where fixed price mechanisms are used). Thus, the phenomenon of a price rigidity arises and a rationing mechanism is normally needed to facilitate the distribution of houses among agents in additional to the rent leverage. This situation is studied in the classical paper by Drèze (1975) where a variant of a competitive equilibrium based on rationing was introduced.[3] To model such a housing market, the rents (or prices) must be bounded to belong to an exogenously given set that specifies the price ceilings as well as

[2]See e.g Ehlers (2002), Pápai (2000), and Zhou (1990) for additional results and characterizations.

[3]See also Cox (1980), Dehez and Drèze (1984), Kurz (1982), and van der Laan (1980) among others.

the minimal acceptable rent (price) for the landlord (the "reservation rent"). This is also the type of housing market considered in this paper.[4]

The two exogenously given components at a housing market with rent control are (a) the set that defines upper and lower bounds on rents, denoted by Ω, and (b) the rationing mechanism. Here, it is assumed that the rationing mechanism is given by a priority-order (again, it may be based on a queue, a random mechanism, needs, etc.) as in the case when the serial dictatorship mechanism is adopted. In the bounding case when Ω only contains a single vector of rents, our framework collapses to the generalization in Svensson (1994) of the classical housing market considered by Hylland and Zeckhauser (1979). In the other extreme case, when the set Ω not is bounded from above by price ceilings, any rents that are weakly higher than the "reservation rents" of the landlord are allowed. Consequently, the housing market reduces to the classical market with competitive rents (e.g. Demange and Gale, 1985; Shapley and Shubik, 1972). In such event, the rationing mechanism will not play any role on the housing market. Hence, two of the predominant house allocation models reduce to special cases of the housing market with rent control considered in this paper.

Because the set of price equilibria that respects the price ceilings may be empty for some preference profiles, a weaker version of the concept of a price equilibrium is suggested. In this weakening, the rationing mechanism will play a role. It is, however, desirable to marginalize the significance of the priority-order to respect the core meaning of a price equilibrium. Noting that it is the price ceilings that may destroy the existence of an equilibrium, the weakening of the price equilibrium concept, considered in this paper, can roughly be described as follows. A price vector defines a weak price equilibrium if the assignment of the houses to the agents is "efficient"[5], and each agent is assigned a house from his demand-set (i.e. rationing is not needed) except in the limiting case when (i) the price of each house in the demand-set equals the price ceiling and (ii) each house in the demand-set is assigned to some other agent with a higher priority. In this sense, the priority-order will only be effective in the special case when the rent (price) that is attached to a house equals its exogenously given price ceiling.

We show the existence of a weak equilibrium price vector, and that the set of such vectors has a unique minimal price vector. Using this insight, a rule where the minimal weak equilibrium price vector is used as a direct mechanism for allocating the houses among the agents is defined. This rule is called the "minimal WPE mechanism", and it is demon-

[4]If the indivisible items are jobs or positions, the interpretation of the model is that wages are bounded from below by legislated minimum wages (corresponding to the price ceiling in the rent control model). In this case, the upper bounds on wages (corresponding to the "reservation rent" of the landlord in the rent control model) is given by the employers maximum willingness to pay for a worker.

[5]Note that a Pareto improvement, as traditionally defined, need not respect a given priority-order. Because the priority-order may be effective when considering a housing market with rent control, this paper adopts an efficiency concept called "priority efficiency". This definition essentially states that the assignment of the houses to the agents is "efficient" if the priority-order is respected, and if it is not possible to make a Pareto improvement by reallocating the houses among the agents and at the same time respect the priority-order (see Definition 6). In the Introduction, however, we say that an assignment of houses is "efficient" instead of "priority efficient".

strated to be efficient and non-manipulable.[6] In the limiting cases where rents are fixed and where rents are fully flexible it reduces to the weak version of the serial dictatorship mechanism (Svensson, 1994) and the competitive price mechanism (Demange and Gale, 1985), respectively.

To the best of our knowledge, this paper is the first to define an efficient and non-manipulable allocation rule for a housing market with rent control. This does, of course, not mean that this type of market has not been considered earlier. As already explained in the above, many classical papers consider such a market (see e.g. Drèze, 1975, or footnote 3) but the main focus in those papers is the weakening of the price equilibrium concept in exchange economies to handle the case with price rigidities when items are perfectly divisible. The case with indivisibilities on housing markets with price rigidities has recently been considered by Talman and Yang (2008) and Zhu and Zhang (2011) where price adjustment processes that converge to a "constrained price equilibrium" are considered. However, their rationing mechanisms are very different from the one considered in this paper as agents ex ante are disqualified from being assigned specific houses. In our rationing mechanism, each agent can ex ante be assigned any house, and weather rationing is needed or not is determined endogenously by the reported preferences of the agents. In addition, it is not clear if their mechanisms are non-manipulable, and their considered models do not integrate the two bounding models in the literature described in the above.

The allocation rule considered in this paper is also one of few in the literature that integrates existing efficient and non-manipulable rules in a more general framework. A famous example is the You-Request-My-House I-Get-Your-Turn mechanism, introduced by Abdulkadiroğlu and Sönmez (1999), which integrates the serial dictatorship mechanism with strict preferences (Hylland and Zeckhauser, 1979) and the top-trading cycles mechanism (Shapley and Scarf, 1974). Another example is the Kidney Exchange mechanism, first investigated by Roth et al. (2004), where a generalized version of the top-trading cycles mechanism (Shapley and Scarf, 1974) that also takes trading-chains into account is considered. A third example is Pápai (2000) where a large class of mechanisms that solve the house allocation problem is proposed. These rules are called hierarchical exchange rules, and they can be regarded as a generalization of the top-trading cycles mechanism (Shapley and Scarf, 1974) even if no initial ownership is assumed in the model. An important difference between these mechanisms and the one considered in this paper is that the former mechanisms only work in the absence of monetary transfers whereas the rule considered here can be applied to housing markets independently of if monetary transfers can be carried out or not. To the best of our knowledge, the investigated mechanism is the first efficient and non-manipulable rule for allocating houses that achieves this task.

The paper is outlined as follows. Section 2 introduces the house allocation model with rent control. Section 3 defines a concept of efficiency that respects any given priority-order, and so-called isolated sets of houses. The concept of a weak price equilibrium is

[6]Strictly speaking, this rule is demonstrated to be non-manipulable for "almost all" utility profiles. For the remaining profiles, it is an open question whether or not the mechanism is non-manipulable. See the discussion following Definition 10 and Remark 1.

defined and analyzed in Section 4. In particular, the set of weak equilibrium price vectors is demonstrated to be nonempty. Finally, in Section 5, the "minimal WPE mechanism" is defined and proved to be non-manipulable.

2. THE HOUSING MARKET WITH RENT CONTROL

There is a finite set of *houses* and a finite set of *agents* denoted by $H = \{1, \ldots, m\}$ and $N = \{1, \ldots, n\}$, respectively. The houses in H do not have any existing tenants by assumption. At some housing markets, it is natural to assume that there are several owners of the houses but since our results do not require this, it is, for notational simplicity, assumed that there is a single owner of all houses (our arguments extend with only a few modifications to the case with multiple owners). Agents wish to buy, or rent, at most one house and have an option not to buy, or rent, a house at all. This outside option is formally represented by a *null house*, denoted by 0, which has an unlimited supply.

An *assignment* is a mapping $\mu : N \to H \cup \{0\}$ such that $\mu_i = \mu_{i'}$ for $i \neq i'$ only if $\mu_i = 0$. Hence, two distinct agents can not be assigned the same house in H. Denote by μ_0 the set of houses that not is assigned to any agent at assignment μ, i.e.:

$$\mu_0 = \{h \in H; \mu_i \neq h \text{ for all } i \in N\} \cup \{0\}.$$

Note that the null house always is included in μ_0, by construction, since its supply is unlimited.

Let $p \in \mathbb{R}^{m+1}$ be a *price vector*. A coordinate in p is denoted by p_h and it represents the price, or rent, of house $h \in H \cup \{0\}$. The price of the null house is, without loss of generality, always supposed to equal zero, i.e. $p_0 = 0$. Price vectors are assumed to be restricted by exogenously given lower and upper bounds denoted by $\underline{p} \in \mathbb{R}^{m+1}$ and $\overline{p} \in \mathbb{R}^{m+1}$, respectively, where $\underline{p} \leq \overline{p}$. The lower bounds can be thought of as the sellers reservation prices or rents, and the upper bounds as a legislated rent control. Note also that because p_0 is always zero, it is clear that $\underline{p}_0 = \overline{p}_0 = 0$. The *price space* is given by:

$$\Omega = \{p \in \mathbb{R}^{m+1}; \underline{p}_h \leq p_h \leq \overline{p}_h \text{ for } h \in H \cup \{0\}\}.$$

Each agent $i \in N$ has *preferences* on pairs of houses and prices. Denote by R_i agent i's preference relation on the set of houses and prices $(H \cup \{0\}) \times \mathbb{R}$. The corresponding strict preference and indifference relations are denoted by P_i and I_i, respectively. The following notation will be used: if $h, h' \in H \cup \{0\}$ and $p_h R_i p'_{h'}$, then agent $i \in N$ weakly prefers house h at price p_h to house h' at price $p'_{h'}$.[7] When the price vector is fixed during the analysis (i.e. $p = p'$), the simplified notation $h R_i h'$ is employed. Preferences are assumed to be *rational* and *monotonic* for all agents $i \in N$, i.e., R_i is a complete and transitive binary relation on $(H \cup \{0\}) \times \mathbb{R}$ and $p_h P_i p'_h$ if $p_h < p'_h$. Further, preferences are assumed to be *continuous*, i.e., the sets $\{p_h \in \mathbb{R};\ p_h R_i p'_{h'}\}$ and $\{p_h \in \mathbb{R};\ p'_{h'} R_i p_h\}$ are closed for each

[7] Note that p_h has a dual meaning. It represents the real number p_h as well as the pair (h, p_h). This will not cause any confusion as p_h is equivalent to the pair (h, p_h) if and only if it is written in connection to a preference relation.

$i \in N$ and all $h, h' \in H \cup \{0\}$ and all $p'_{h'} \in \mathbb{R}$. A preference profile, or for short a *profile*, is a list $R = (R_i)_{i \in N}$ of the agents' preferences. The set of profiles where preferences are rational, monotonic and continuous is denoted by $\mathcal{R}$.

A *state* is a pair (p, μ) consisting of a price vector and an assignment. The entire net trade between any two states can be decomposed into a number of unique trading cycles as explained in the following definition.[8]

DEFINITION 1 Let (p, μ) and (p', μ') be two states. A group $G \subseteq N$ of agents constitutes a *trading cycle* if G is a sequence $(i_1, \ldots, i_t)$ of distinct agents such that $\mu_{i_j} \in H$ for $2 \leq j \leq t$, $\mu'_{i_j} = \mu_{i_{j+1}}$ for $1 \leq j < t$, and either:
(i) $\mu'_{i_t} = \mu_{i_1}$ and $\mu_{i_1} \in H$ (*closed trading cycle*), or;
(ii) $\mu'_{i_t} \in \mu_0$ and $\mu_{i_1} \in \mu'_0$ (*open trading cycle*).

For a given profile R, a state is a price equilibrium (PE, henceforth) if all prices are weakly higher than the reservation price of the landlord (equal if the house is not assigned to any agent) and each agent is assigned his weakly most preferred house at the given prices. Formally:

DEFINITION 2 For a given profile $R \in \mathcal{R}$, a state (p, μ) is a *price equilibrium* if:
(i) $p_h \geq \underline{p}_h$ for all $h \in H$,
(ii) $p_{\mu_i} R_i p_h$ for all $i \in N$ and all $h \in H \cup \{0\}$, and;
(iii) $p_h = \underline{p}_h$ for all $h \in \mu_0$.

On a housing market without price ceilings (i.e. when Ω not is bounded from above), it is well-known that a PE exists under very general conditions, see e.g. Demange and Gale (1985), Shapley and Shubik (1972), or Svensson (1983). If prices are bounded to belong to Ω, an allowable PE does not exist for all profiles $R \in \mathcal{R}$. Hence, a weakening of the concept PE is needed to analyze housing markets with rent control (such a weakening is provided in Section 4). This weakening must contain some kind of rationing mechanism as prices alone cannot solve the allocation problem due to the price ceilings. Here, it is assumed that the rationing mechanism is based on an exogenously given *priority-order* π. Formally, $\pi : N \to N$ is a bijection where the highest ranked agent $i \in N$ is the agent with $\pi_i = 1$, the second highest ranked agent i' has $\pi_{i'} = 2$, and so on.

3. PRIORITY EFFICIENT ASSIGNMENTS AND ISOLATED SETS

Given that the priority-order may play a role in the allocation process, as explained in Section 2, and the obvious observation that Pareto improvements not necessarily take

[8]We remark that an alternative presentation of Definition 1 is to describe the net trade by a number of directed arcs from agents to houses, and from houses to agents. In this case, the description of closed and open trading cycles would be more closely related to the description of *top-trading cycles* (Shapley and Scarf, 1974) and *w-chains* (Roth et al., 2004), respectively. Note, however, that top-trading cycles as well as w-chains are mechanisms used to reallocate items among the agents in the case of initial ownership whereas trading cycles, as defined in this paper, only are used as tools to describe allocative distinctions between any two given states. Also, initial ownership is not assumed in this paper.

the priority-order into account, the main objective of this section is to provide a notion of efficiency that respects any given priority-order. This notion will play a key role when weakening the concept of price equilibrium (Section 4). In addition, the existence of so-called isolated sets of houses is established. These sets will be useful when proving the non-emptiness of the set of weak price equilibria as well as the existence of an efficient and non-manipulable allocation mechanism tailored for housing markets with rent control (Section 5). Throughout this section, prices are assumed to be fixed. In this case, we recall, from Section 2, that the simplified notation hR_ih' will be employed instead of $p_hR_ip_{h'}$.

For a given assignment μ, the set of unenvied agents F and the set of envied agents E are defined as:

$$\begin{aligned} F &= \{i \in N;\ \mu_{i'}R_{i'}\mu_i \text{ for all } i' \in N\}, \\ E &= N \setminus F. \end{aligned}$$

An assignment μ is *envy-free* if E is empty and all agents in N prefer their assigned house to any unassigned house (Foley, 1967). Note also that if (p, μ) is a PE, then μ is envy-free by Definition 2(ii).

Suppose now that E is nonempty, say $i \in E$. In this case, it will be convenient to single out the agent with the highest priority that envies agent i (such an agent always exists, by definition, whenever E is nonempty). This agent is said to have priority to envy agent i.

Definition 3 Agent $i' \in N$ has *priority to envy* agent $i \in E$ at assignment μ if:
(i) $\mu_iP_{i'}\mu_{i'}$, and;
(ii) for all $i'' \in N$ and $i'' \neq i'$, $\mu_iP_{i''}\mu_{i''}$ only if $\pi_{i'} < \pi_{i''}$.

The following definition introduces the concept of a priority respecting assignment.[9] At such assignment, a higher ranked agent can never envy a lower ranked agent and, furthermore, all agents weakly prefer their assigned house to any unassigned house.

Definition 4 Assignment μ is *priority respecting* if:
(i) $\mu_{i'}P_i\mu_i$ only if $\pi_{i'} < \pi_i$ for $i, i' \in N$, and;
(ii) μ_iR_ih if $h \in \mu_0$.

Note that if the assignment μ is envy-free, then it is priority respecting by definition. We next remark that by the definition of envy-freeness, it follows that an envy-free assignment cannot be Pareto dominated by a reallocation of the houses. However, when considering priority respecting assignments, the set of envied agents may be nonempty and in this case it may be possible to find a Pareto improving reallocation of the houses that respects the given priority-order.[10] Thus, a weaker concept than Pareto efficiency is obtained if reallocations are restricted to priority respecting improvements.

[9] This concept is closely related to the concept of *justified envy* introduced by Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003).

[10] To see this, suppose that $N = \{1, 2\}$, $H = \{a, b\}$, $\pi_1 < \pi_2$, aI_1bP_10 and aP_2bP_20. In this case, the assignment $(\mu_1, \mu_2) = (a, b)$ is priority respecting and $E = \{1\}$. However, the reallocation where $(\mu'_1, \mu'_2) = (b, a)$ makes agent 2 strictly better off without harming agent 1.

DEFINITION 5 Assignment μ' is a *priority respecting improvement* of assignment μ if there is a sequence $G = (i_1, \ldots, i_t)$ of distinct agents such that:
(1i) G is a closed or open trading cycle,
(1ii) $\mu'_i = \mu_i$ for $i \in N \setminus \{i_1, \ldots, i_t\}$,
(2i) $\mu'_i R_i \mu_i$ for all $i \in N$ and $\mu'_i P_i \mu_i$ for some $i \in N$, and;
(2ii) if $\mu'_{i_j} P_i \mu'_i$ for some $i \in N \setminus \{i_j\}$ and some $1 \leq j \leq t$, then $\pi_{i_j} < \pi_i$.

Conditions (1i) and (1ii) state that μ' is obtained from μ by *one* cyclical trade, and conditions (2i) and (2ii) mean that the priority-order is respected in the Pareto improving reallocation of the houses. Consequently, if assignment μ is priority respecting, so is assignment μ'.

DEFINITION 6 An assignment μ is *priority efficient* if it is priority respecting and if there are no priority respecting improvements of μ.

Note that if an assignment is envy-free, then it is also priority efficient, because it is priority respecting (since E is empty) and it is not possible to find a priority respecting improvement.

LEMMA 1 If assignment μ is priority efficient, then the set of unenvied agents F is nonempty.

PROOF: Suppose that μ is priority efficient. To obtain a contradiction, assume that $F = \emptyset$. But then, $E = N$ by construction. First note that if $\mu_i = 0$ for some $i \in N$, then $i \in F$. Hence, $\mu_i \neq 0$ for all $i \in N$. Because $E = N$, let $i_1 \in E$ and define recursively a sequence $(i_1, \ldots, i_t)$ of distinct agents such that i_{j+1} has priority to envy i_j for $1 \leq j < t$. Since $F = \emptyset$, there are indices t and k such that i_t has priority to envy i_k and $k < t$. But then there is a priority respecting improvement of μ, denoted by μ', defined by a closed trading cycle where:
(i) $\mu'_{i_j} = \mu_{i_{j+1}}$ for $k \leq j \leq t-1$,
(ii) $\mu'_{i_t} = \mu_{i_k}$, and;
(iii) $\mu'_{i_j} = \mu_{i_j}$ for the remaining agents.
But this contradicts the assumption that μ is priority efficient. Hence, $F \neq \emptyset$. *Q.E.D.*

We end this section by introducing the notion of an isolated set. Informally, a set S of houses is isolated, at a given assignment, if the assignment is envy-free among the agents that are assigned a house in S, and all agents that not are assigned a house in S strictly prefer their assigned house to any house in the set S.

DEFINITION 7 A set of houses $S \subset H$ ($S \neq H$) is *isolated* at assignment μ if:
(i) $\mu_i R_i h$ for all $i \in N$ and all $h \in S$, and;
(ii) $\mu_i P_i h$ for all $i \in N$ with $\mu_i \notin S$ and all $h \in S$.

THEOREM 1 If the set of envied agents E is nonempty and if the assignment μ is priority efficient, then there is an isolated set $S \subset H$ of houses. In addition, $S \setminus \mu_0 \neq \emptyset$.

PROOF: See the Appendix. *Q.E.D.*

4. WEAK PRICE EQUILIBRIUM

This section provides a weakening of the concept of a price equilibrium. To motivate this weakening, it is first observed that standard textbook arguments state that it is reasonable that prices on houses that are overdemanded[11] should increase. However, because prices are bounded from above by the price vector $\overline{p}$, it may not be possible to increase the prices in such fashion that all overdemanded sets of houses are eliminated which is necessary for obtaining a PE (see e.g. Demange et al., 1986; Hall, 1935; Mishra and Talman, 2010). In other words, it is the price ceiling that may destroy the possibility to reach a PE. In this case, some kind of rationing mechanism is needed. This rationing mechanism is here given by the priority-order π as already explained in Section 2. It is, however, desirable to marginalize the significance of π to respect the core meaning of a PE. By the above arguments, this means that the priority-order should only be effective when the price of one or several houses equal the upper price bound. This naturally leads to the following requirement: if agent i strictly prefers the house assigned to agent i' to his own at assignment μ, then the price of the house assigned to agent i' must equal the upper price bound and agent i' must have a higher priority than agent i. Formally:

$$p_{\mu_{i'}} P_i p_{\mu_i} \text{ only if } \pi_{i'} < \pi_i \text{ and } p_{\mu_{i'}} = \overline{p}_{\mu_{i'}}.$$

Note first that the former requirement always is satisfied when μ is priority respecting (Definition 4) or priority efficient (Definition 6). Recall next, from Section 3, that there exists no Pareto improving trading cycle at a PE but that the above condition alone, in general, not necessarily is a guarantee for that (see, for instance, footnote 10). For this reason, priority efficiency must be embedded in, instead of being a consequence of, the weaker version of a PE. This leads to the following definition of a weak price equilibrium (WPE, henceforth).

DEFINITION 8 For a given profile $R \in \mathcal{R}$ and a given priority-order π, a state (p, μ) is a *weak price equilibrium* if:

(i) $p \in \Omega$,
(ii) μ is priority efficient,
(iii) $p_{\mu_{i'}} P_i p_{\mu_i}$ only if $p_{\mu_{i'}} = \overline{p}_{\mu_{i'}}$, and;
(iv) if $h \in \mu_0$ then $p_h = \underline{p}_h$.

Note that the outcome of the weak version of the serial dictatorship mechanism (Svensson,

[11] That is when the number of agents that only demand houses from some set $H' \subset H$ is strictly larger than the number of elements in the set H', see e.g. Demange et al. (1986) or Mishra and Talman (2010).

1994)[12] is a WPE. This follows directly as it is a fixed price mechanism (i.e. $\underline{p}_h = \overline{p}_h$ for all $h \in H \cup \{0\}$) that always generates a priority efficient assignment (Svensson, 1994, Theorem 1). Moreover, any PE is also a WPE. This observation is an immediate consequence of Definition 2, the observation that any PE is envy-free (as previously remarked in Section 3), and the fact that Pareto efficiency is implied by envy-freeness (Svensson, 1983).

A price vector p is a WPE vector if there is an assignment μ such that the state (p, μ) is a WPE. For a given profile $R \in \mathcal{R}$, the set of WPE is denoted by Σ_R, and the corresponding set of WPE vectors is denoted by Π_R, i.e.:

$$\Pi_R = \{p \in \Omega; (p, \mu) \in \Sigma_R \text{ for some assignment } \mu\}.$$

For purely technical reasons, it will sometimes be convenient to analyze a weaker version of Definition 8 where only conditions (i)–(iii) are satisfied. In this case, the state is called a *weak quasi equilibrium* (WQE, henceforth). As in the above, a price vector p is a WQE vector if there is an assignment μ such that the state (p, μ) is a WQE. For a given profile $R \in \mathcal{R}$, the set of WQE is denoted by $\overline{\Sigma}_R$, and the set of WQE vectors is denoted by $\overline{\Pi}_R$. Clearly, $\Sigma_R \subset \overline{\Sigma}_R$ and $\Pi_R \subset \overline{\Pi}_R$.

PROPOSITION 1 For any profile $R \in \mathcal{R}$, the set $\overline{\Pi}_R$ is nonempty, bounded and closed.

PROOF: $\overline{\Pi}_R$ is bounded and nonempty because Ω is bounded and $\overline{p} \in \overline{\Pi}_R$, respectively. To prove that $\overline{\Pi}_R$ is closed, suppose that $R \in \mathcal{R}$ and let $(p^j)_{j=1}^{\infty}$ be a convergent sequence of price vectors such that $p^j \in \overline{\Pi}_R$ and $p^j \to p$ as $j \to \infty$. Let $(p^j, \mu^j) \in \overline{\Sigma}_R$. Since there is only a finite number of distinct assignments, it is, without loss of generality, assumed that $\mu^j = \mu$ for all j. To prove the result, we need to check that $p \in \overline{\Pi}_R$, i.e., that conditions (i)–(iii) of Definition 8 are satisfied. Condition (i) is trivially fulfilled as Ω is closed. For the other two conditions:

Condition (ii). If μ is envy-free, then μ is priority efficient as remarked in Section 3. Suppose instead μ is not priority efficient and that $p_{\mu_{i'}} P_i p_{\mu_i}$ for some $i, i' \in N$ where $i \neq i'$. Then $p^j_{\mu_{i'}} P_i p^j_{\mu_i}$ for j "sufficiently large" by continuity of the preferences. Hence, $\pi_{i'} < \pi_i$ as $(p^j, \mu) \in \overline{\Sigma}_R$. Consequently, μ is priority respecting. It is now clear that there exists a priority efficient assignment μ', because if μ is not priority efficient then a priority efficient assignment can be obtained by a sequence of priority respecting improvements. During this sequence, all obtained assignments will be priority respecting as μ is priority respecting (see Definition 5, and the discussion following it).

[12] In a formal description of this mechanism, let, without loss of generality, $\pi_i = i$ for all $i \in N$, and $\tau_i(H') = \{h \in H'; hR_ih' \text{ for all } h' \in H'\}$ for $H' \subset H$ and any $R \in \mathcal{R}$. Consider profile $R \in \mathcal{R}$, and define recursively a decreasing sequence $\{C_i\}_{i \in N}$ of maximal choice sets according to: $C_1 = H$, and $C_{i+1} = \{h \in H'; \exists \mu \text{ such that } \mu_{i'} \in \tau_{i'}(C_{i'}) \text{ for } i' \leq i \text{ and } \mu_{i+1} = h\}$ for all $i \in N \setminus \{n\}$. The outcome of the weak version of the serial dictatorship mechanism is given by the following set of individually utility equivalent assignments: $g(R) = \{\mu; \mu_i \in \tau_i(C_i) \text{ for all } i \in N\}$. Note that if no monetary transfers are feasible and indifference relations not are allowed, this mechanism reduces to the serial dictatorship mechanism of Hylland and Zeckhauser (1979).

Condition (iii). Suppose that $p_{\mu_{i'}} P_i p_{\mu_i}$ for some $i, i' \in N$ where $i \neq i'$. Then $p^j_{\mu_{i'}} P_i p^j_{\mu_i}$ for all j "sufficiently large" by continuity of the preferences. But then $p^j_{\mu_{i'}} = \overline{p}_{\mu_{i'}}$ for all j "sufficiently large". Hence, $p_{\mu_{i'}} = \overline{p}_{\mu_{i'}}$. *Q.E.D.*

We conclude this section by proving the existence of a weak price equilibrium. Denote by p^* a *minimal price vector* in $\overline{\Pi}_R$, i.e., a vector $p^* \in \overline{\Pi}_R$ such that there is no $p \in \overline{\Pi}_R$ where $p \neq p^*$ and $p \leq p^*$. Such a vector exist since $\overline{\Pi}_R$ is nonempty, bounded and closed by Proposition 1. The following lemma which is due to Alkan et al. (1991, Perturbation Lemma) will be useful in the proof of the result.

Lemma 2 If the state (p, μ) satisfies condition (ii) of Definition 2 and $p_h > \underline{p}_h$ for all $h \in H$, then for each $\varepsilon > 0$ there exists another state (p', μ') satisfying conditions (i) and (ii) of Definition 2 where $p_h - \varepsilon < p'_h < p_h$ for all $h \in H$.

Theorem 2 For any profile $R \in \mathcal{R}$, if p^* is a minimal price vector in $\overline{\Pi}_R$, then $p^* \in \Pi_R$.

Proof: Let $(p^*, \mu) \in \overline{\Sigma}_R$, and suppose that p^* is a minimal price vector in $\overline{\Pi}_R$ and that the assignment μ is chosen so that the number $\#\{h \in \mu_0;\ p^*_h > \underline{p}_h\} = \nu$ is minimal. To prove the statement, we need to establish that $\nu = 0$ because then $p^* \in \Pi_R$ by Definition 8(iv). To obtain a contradiction, suppose that $h' \in \mu_0$ and $p^*_{h'} > \underline{p}_{h'}$, i.e., $\nu > 0$.

Note first that $p^*_{\mu_j} R_j p^*_{h'}$ for all $j \in N$ by Definition 4(ii) since $(p^*, \mu) \in \overline{\Sigma}_R$. Note next that $p^*_{\mu_i} I_i p_{h'}$ for some $i \in N$ because if this not is the case, then it is possible to decrease $p^*_{h'}$ by a small $\varepsilon > 0$ which contradicts that p^* is minimal.

It is next demonstrated that $p^*_{\mu_i} > \underline{p}_{\mu_i}$ whenever $p^*_{\mu_i} I_i p^*_{h'}$. To see this, let the assignment μ' be defined by $\mu'_i = h'$ and $\mu'_j = \mu_j$ for all $j \in N \setminus \{i\}$. But then $(p^*, \mu') \in \overline{\Sigma}_R$ by identical arguments as in the above. Hence, $\nu > \#\{h \in \mu'_0;\ p^*_h > \underline{p}_h\}$ if $p^*_{\mu_i} = \underline{p}_{\mu_i}$ because $h' \notin \mu'_0$ and $\mu_i \in \mu'_0$. But this contradicts that μ is chosen so that ν is minimal. Thus, $p^*_{\mu_i} > \underline{p}_{\mu_i}$.

It is now proved that $i \in F$ whenever $p^*_{\mu_i} I_i p^*_{h'}$. Suppose, to obtain a contradiction, that $i \in E$. Let $j \in N$ be the agent with priority to envy agent i, and define assignment μ' as $\mu'_i = h'$, $\mu'_j = \mu_i$, and $\mu'_k = \mu_k$ for the remaining agents. Again, $(p^*, \mu') \in \overline{\Sigma}_R$ which contradicts that (p^*, μ) is priority efficient. Hence, $i \in F$.

Let now $F' = \{i \in F; p^*_{\mu_i} > \underline{p}_{\mu_i}\}$. This set is nonempty by the above conclusions. Define next the set $F'' \subset F'$ in the following way: agent $i \in F''$ if and only if there is a sequence $(i_1, \ldots, i_t)$ of distinct agents $i_j \in F'$ for $1 \leq j \leq t$ such that $i_1 = i$, $p^*_{\mu_{i_j}} I_{i_j} p^*_{\mu_{i_{j+1}}}$ for $1 \leq j < t$, and $p^*_{\mu_{i_t}} I_{i_t} p^*_{h'}$. F'' is nonempty by the above conclusions. Let also:

$$S = \{h \in H; h = \mu_i \text{ for some } i \in F''\} \cup \{h'\}.$$

Note first that if S is isolated, then it is possible to decrease p^*_h for each $h \in S$ and generate a new WQE, according to Lemma 2, contradicting that p^* is minimal. For this reason, it is, in the remaining part of the proof, assumed that S not is isolated. In this case, there are agents $k \in N - F'$ and $l \in F''$ such that $p^*_{\mu_k} I_k p^*_{\mu_l}$. We need to consider two cases:

Case (i) $k \in F - F'$. Note first that $k \in F - F'$ means that $p^*_{\mu_k} = \underline{p}_{\mu_k}$. Moreover, there is an assignment μ' and an open trading cycle $(i_1, \ldots, i_t)$ such that $i_1 = k$, $i_2 = l$ and $i_j \in F''$

for $1 < j \leq t$. Moreover, $\mu'_{i_j} = \mu_{i_{j+1}}$ for $1 \leq j < t$, $\mu'_{i_t} = h'$ and $\mu'_i = \mu_i$ for the remaining agents. Hence, $(p^*, \mu') \in \overline{\Sigma}_R$ and $\nu > \#\{h \in \mu'_0;\ p^*_h > \underline{p}_h\}$ because $h' \notin \mu'_0$ and $\mu_k \in \mu'_0$. This is a contradiction to ν being minimal.

Case (ii) $k \in E$. In this case, there is a unique sequence $(i_1, \ldots, i_t)$ of distinct agents such that $i_1 \in F$, $i_j \in E$ for $1 < j \leq t$, $i_t = k$, and i_j has priority to envy i_{j+1} for $1 \leq j < t$. Moreover, there is a sequence $(i_{t+1}, \ldots, i_r)$ of distinct agents such that $i_{t+1} = l$, $i_j \in F''$ for $t < j \leq r$, $p^*_{\mu_{i_j}} I_{i_j} p^*_{\mu_{i_{j+1}}}$ for $t < j < r$, and $p^*_{\mu_{i_r}} I_{i_r} p^*_{h'}$. Now if $i_{t'} = i_1$ for some $t' > t$ there is a priority respecting improvement of μ where agents $(i_1, \ldots, i_t, \ldots, i_{t'-1})$ constitute a closed trading cycle. On the other hand, if $i_j \neq i_1$ for all $j > t$ then there is also a priority respecting improvement of μ where agents $(i_1, \ldots, i_r)$ constitute an open trading cycle. Hence, both situations is a contradiction to μ being priority efficient.

In summary, if $\nu > 0$ then $(p^*, \mu) \notin \overline{\Sigma}_R$, which contradicts our assumption. Hence, $\nu = 0$, the desired conclusion. *Q.E.D.*

5. NON-MANIPULABILITY

In this section, a priority efficient and non-manipulable allocation mechanism that implements a WPE state is defined. Formally, an allocation mechanism is a function f with domain $\mathcal{R}$ that selects a WPE state as an outcome, i.e., $f(R) \in \Sigma_R$. This paper employs the following definition of non-manipulability.

DEFINITION 9 An allocation mechanism f is manipulable at a profile $R \in \mathcal{R}$ by an agent $i \in N$ if there is a profile $R' = (R'_i, R_{-i}) \in \mathcal{R}$, and two states $f(R) = (p, \mu)$ and $f(R') = (p', \mu')$ such that $p'_{\mu'_i} P_i p_{\mu_i}$. If the mechanism f not is manipulable by any agent $i \in N$ at any profile $R \in \mathcal{R}$, it is non-manipulable.

The analysis of non-manipulability will be done in a subset $\tilde{\mathcal{R}} \subset \mathcal{R}$ where the set $\tilde{\mathcal{R}}$ is the set of profiles such that no two houses are "connected by indifference" at any price vector $p \in \Omega$.

DEFINITION 10 For a given profile $R \in \mathcal{R}$, houses h_1 and h_t in $H \cup \{0\}$ are *connected by indifference* at price vector $p \in \Omega$ if there is a sequence $(i_1, \ldots, i_t)$ of distinct agents and a sequence $(h_1, \ldots, h_t)$ of distinct houses such that:
(i) $p_{h_1} \in \{\underline{p}_{h_1}, \overline{p}_{h_1}\}$ and $p_{h_t} \in \{\underline{p}_{h_t}, \overline{p}_{h_t}\}$, and;
(ii) $p_{h_j} I_{i_j} p_{h_{j+1}}$ for $1 \leq j \leq t-1$.

We next argue that the restriction of profiles to $\tilde{\mathcal{R}}$ is a mild assumption. More precisely, for any profile $R \in \tilde{\mathcal{R}}$, any sequence $(i_1, \ldots, i_t)$ of distinct agents, any sequence $(h_1, \ldots, h_t)$ of distinct houses and any choice of the price vector $\overline{p}_{h_1}$, the prices p_{h_j} may be chosen so that $p_{h_1} = \overline{p}_{h_1}$ and $p_{h_j} I_{i_j} p_{h_{j+1}}$ for $1 \leq j \leq t-1$. In that case, p_{h_t} will be uniquely determined by continuity and monotonicity of the preferences. However, if preferences are chosen randomly, the probability is zero that $p_{h_t} = \overline{p}_{h_t}$ since $\overline{p}$ is given exogenously. Consequently, very few profiles are excluded in $\tilde{\mathcal{R}}$ compared to $\mathcal{R}$, i.e., "almost all" profiles are included in $\tilde{R}$. We also remark that a common assumption in the house allocation literature when no monetary transfers are allowed is that preferences are strict. Also this is a mild assumption if preferences are chosen randomly since the probability of an indifference is zero, i.e., very few preference profiles are excluded if only strict preferences are considered. Thus, one can argue that the restriction of profiles to $\tilde{\mathcal{R}} \subset \mathcal{R}$ is an assumption of the same character as assuming strict preferences in the absence of monetary transfers.

Proposition 2 below is a consequence of the choice $R \in \tilde{\mathcal{R}}$ of profiles, and it shows that given two WPE states the trading cycles connecting the states are such that if one agent in a trading cycle is strictly better of by the trade, then all agents in that trading cycle are weakly better of. This result will be useful in the strategic analysis, so the proposition is formulated somewhat more general than necessary at this point. The proof is preceded by some notation and two lemmas.

Notation 1 Consider the states $(p, \mu) \in \Sigma_R$ and $(p', \mu') \in \Sigma_{R'}$ where $R, R' \in \tilde{\mathcal{R}}$, and $R' = (R'_q, R_{-q})$ for some agent $q \in N$ with $p'_{\mu'_q} P_q p_{\mu_q}$. Then let: $S_1 = \{h \in H;\ p'_h < p_h\}$, $S_2 = \{h \in H;\ p'_h = p_h\} \cup \{0\}$, and $S_3 = \{h \in H;\ p'_h > p_h\}$.[13]

Lemma 3 Consider the states, $(p, \mu) \in \Sigma_R$ and $(p', \mu') \in \Sigma_{R'}$ where $R, R' \in \tilde{\mathcal{R}}$, and $R' = (R'_q, R_{-q})$ for some agent $q \in N$ with $p'_{\mu'_q} P_q p_{\mu_q}$. Let $(i_1, \ldots, i_t)$ be a sequence of distinct agents such that:

$$\mu_{i_j} \in S_2 \text{ for } 1 < j < t \text{ and } \mu'_{i_j} = \mu_{i_{j+1}} \text{ for } 1 \leq j < t.$$

If $p'_{\mu'_{i_1}} P_{i_1} p_{\mu_{i_1}}$, then $p'_{\mu'_{i_j}} R_{i_j} p_{\mu_{i_j}}$ for $1 < j < t$.

Proof: Let, without loss of generality, $i_j = j$. With this notation, $p'_{\mu_2} P_1 p_{\mu_1}$ by assumption. Then since $(p, \mu) \in \Sigma_R$ and $\mu_2 \in S_2$, it is clear that $p'_{\mu_2} = p_{\mu_2} = \overline{p}_{\mu_2}$ and $\pi_2 < \pi_1$. Now, if $p'_{\mu_{j+1}} R_j p_{\mu_j}$ for $1 < j < t$ the proof is complete. To obtain a contradiction, let k be the agent with the lowest index $1 < k < t$ such that $p_{\mu_k} P_k p'_{\mu_{k+1}}$. Note that this also means that $k \neq q$. Then because $(p, \mu) \in \Sigma_R$ and $\mu_k \in S_2$, by assumption, it follows that $p'_{\mu_k} = p_{\mu_k} = \overline{p}_{\mu_k}$ and $\pi_{k-1} < \pi_k$. Hence, $k > 2$ as $\pi_2 < \pi_1$. Moreover, by the above conclusions about agent 1, there must exist an agent l with a highest index $1 \leq l < k$ such that $p'_{\mu_{l+1}} P_l p_{\mu_l}$, $p'_{\mu_{l+1}} = p_{\mu_{l+1}} = \overline{p}_{\mu_{l+1}}$ and $\pi_{l+1} < \pi_l$. Now, $p'_{\mu_{i+1}} I_i p_{\mu_i}$ for $l < i < k$ by definition of agents l and k (again, $i \neq q$). But then houses μ_{l+1} and μ_k are connected by indifference, which is a contradiction to $R \in \tilde{\mathcal{R}}$. Hence, $p'_{\mu_{j+1}} R_j p_{\mu_j}$ for $1 < j < t$ must be the case. *Q.E.D.*

Lemma 4 Consider the states, $(p, \mu) \in \Sigma_R$ and $(p', \mu') \in \Sigma_{R'}$ where $R, R' \in \tilde{\mathcal{R}}$, and $R' = (R'_q, R_{-q})$ for some agent $q \in N$ with $p'_{\mu'_q} P_q p_{\mu_q}$.

(i) if $\mu_i \in S_1$, then $p'_{\mu'_i} P_i p_{\mu_i}$, and;
(ii) if $\mu'_i \in S_3$, then $p_{\mu_i} P_i p'_{\mu'_i}$.

[13] Note that the null house cannot belong to S_1 or S_3 as $p'_0 = p_0 = \underline{p}_0 = \overline{p}_0 = 0$ by assumption.

Proof: Note that part (i) holds by definition if $i = q$. Suppose now that $i \neq q$. In this case, $p'_{\mu'_i} R_i p'_{\mu_i}$ by Definition 8(iii) since $(p', \mu') \in \Sigma_{R'}$ and $\mu_i \in S_1$ (i.e. $p'_{\mu_i} < \overline{p}_{\mu_i}$). But then $p'_{\mu_i} P_i p_{\mu_i}$ by monotonicity as $p'_{\mu_i} < p_{\mu_i}$.

To prove part (ii), note that $p_{\mu_i} R_i p_{\mu'_i}$ by Definition 8(iii) since $(p, \mu) \in \Sigma_R$ and $\mu'_i \in S_3$ (i.e. $p_{\mu'_i} < \overline{p}_{\mu'_i}$). But then $p_{\mu'_i} P_i p'_{\mu'_i}$ by monotonicity since $p_{\mu'_i} < p'_{\mu'_i}$. Finally, we remark that these arguments are also valid if $i = q$ because at the state (p, μ), the preference relation of agent q is given by R_q. *Q.E.D.*

Proposition 2 Consider the states, $(p, \mu) \in \Sigma_R$ and $(p', \mu') \in \Sigma_{R'}$ where $R, R' \in \tilde{\mathcal{R}}$, and $R' = (R'_q, R_{-q})$ for some agent $q \in N$. Let also a sequence $(i_1, \ldots, i_t)$ of distinct agents be a trading cycle such that $p'_{\mu'_{i_l}} P_{i_l} p_{\mu_{i_l}}$ for some i_l in the trading cycle. Then:

$$p'_{\mu'_{i_j}} R_{i_j} p_{\mu_{i_j}} \text{ and } \mu'_{i_j} \in S_1 \cup S_2 \text{ for } 1 \leq j \leq t.$$

Proof: Without loss of generality, let the trading cycle be given by $(1, \ldots, t)$. Suppose first that the trading cycle is closed and assume, without loss of generality, that $l = 1$. Because $p'_{\mu'_1} P_1 p_{\mu_1}$, by assumption, it follows from Lemma 4(ii) that $\mu_2 = \mu'_1 \in S_1 \cup S_2$. If $\mu_2 \in S_1$, it is clear from Lemma 4(i) that $p'_{\mu'_2} P_2 p_{\mu_2}$, and if $\mu_2 \in S_2$ it follows from Lemma 3 that $p'_{\mu'_2} R_2 p_{\mu_2}$. In any case, $\mu_3 = \mu'_2 \in S_1 \cup S_2$ by Lemma 4(ii). By repeating exactly the same arguments as in the case when $j = 2$ for all indices $j = 3, \ldots, t$, we get $p'_{\mu'_j} R_j p_{\mu_j}$ and $\mu'_j \in S_1 \cup S_2$ for $1 \leq j \leq t$.

Suppose now that the trading cycle is open. In this case, $\mu_1 \in \mu'_0$ by Definition 1(ii). Note first that $\mu_1 \in S_1 \cup S_2$. This follows as $\mu_1 \in S_3$ means that $p'_{\mu_1} > p_{\mu_1}$ so $(p', \mu') \notin \Sigma_{R'}$ by Definition 8(iv) and $\mu_1 \in \mu'_0$. But this contradicts our assumptions. Hence, $\mu_1 \in S_1 \cup S_2$. Let now l be the lowest index in the open trading cycle such that $p'_{\mu'_l} P_l p_{\mu_l}$. Obviously, identical arguments as in the case of a closed trading cycle can be adopted to demonstrate that $p'_{\mu'_j} R_j p_{\mu_j}$ and $\mu'_j \in S_1 \cup S_2$ for $l \leq j \leq t$. Thus, if $l = 1$, the desired conclusion is obtained. This is the case if $\mu_1 \in S_1$ by Lemma 4(i). Suppose therefore that $l \neq 1$ and $\mu_1 \notin S_1$ (so $\mu_1 \in S_2$, by the above conclusion). If $\mu_1 \in S_2$, then $p'_{\mu_1} = p_{\mu_1} = \underline{p}_{\mu_1}$ by Definition 8(iv) since $\mu_1 \in \mu'_0$ and $(p', \mu') \in \Sigma_{R'}$. Note also that $p'_{\mu'_1} I_1 p_{\mu_1}$ by Definition 8(iii) as $(p', \mu') \in \Sigma_{R'}$ and $l \neq 1$. To obtain a contradiction, let k be the lowest index $1 < k < l$ in the open trading cycle such that $p_{\mu_k} P_k p'_{\mu'_k}$. In this case, $p'_{\mu'_{k-1}} = \overline{p}_{\mu'_{k-1}}$ since $(p', \mu') \in \Sigma_{R'}$. Hence, μ_1 and μ'_{k-1} are connected by indifference which contradicts that $R, R' \in \tilde{\mathcal{R}}$. Thus, there is no such agent k and, therefore $p'_{\mu'_j} R_j p_{\mu_j}$ also for $1 \leq j < l$. This conclusion together with Lemma 4(ii) give $\mu'_j \in S_1 \cup S_2$ for $1 \leq j < l$. *Q.E.D.*

Proposition 2 shows that when comparing two WPE states, there are two types of trading cycles. In the first, at least one agent receives a strictly higher utility in the change from (p, μ) to (p', μ') and, as a consequence, all agents in the trading cycle get a weakly higher utility. In the second, all agents receive a weakly lower utility. This is formalized using the following notation.

NOTATION 2 Denote by $N^+ \subset N$ the set of agents belonging to a trading cycle where at least one agent receives a strictly higher utility in the change from (p, μ) to (p', μ'). Let N^- represent the remaining agents, i.e., $N^- = N \setminus N^+$.

The following theorem establishes the uniqueness of a minimal price vector in Π_R.

THEOREM 3 Let $R \in \tilde{R}$ and let $p', p'' \in \overline{\Pi}_R$ be any two minimal price vectors. Then $p \in \Pi_R$, where $p_h = \min\{p'_h, p''_h\}$ for all $h \in H$.

PROOF: Since $p', p'' \in \overline{\Pi}_R$, there are assignments μ' and μ'' such that $(p', \mu') \in \overline{\Sigma}_R$ and $(p'', \mu'') \in \overline{\Sigma}_R$. Consider now the trading cycles for states (p', μ') and (p'', μ''), and define N^+ and N^-. Note that:

$$p_{\mu_i} = p''_{\mu_i} \text{ if } i \in N^+, \text{ and } p_{\mu_i} = p'_{\mu_i} \text{ if } i \in N^-,$$

by construction of p and Proposition 2.

We first demonstrate that $p \in \overline{\Pi}_R$, i.e, that there is some assignment $\tilde{\mu}$ such that $(p, \tilde{\mu}) \in \overline{\Sigma}_R$. We need to check that conditions (i)–(iii) of Definition 8 are satisfied. We note that (i) follows trivially since $p', p'' \in \Omega$. Let now the assignment μ be defined as:

$$\mu_i = \mu''_i \text{ if } i \in N^+, \text{ and } \mu_i = \mu'_i \text{ if } i \in N^-.$$

Consider agents $i, l \in N$ and suppose that $p_{\mu_i} P_l p_{\mu_l}$. We start by establishing:
(a) $\pi_i < \pi_l$, and;
(b) $p_{\mu_i} = \overline{p}_{\mu_i}$.
This conditions are satisfied by Definition 8 if $i, l \in N^+$ or $i, l \in N^-$ as $(p'', \mu'') \in \overline{\Sigma}_R$ and $(p', \mu') \in \overline{\Sigma}_R$, respectively. Suppose instead that $i \in N^+$ and $l \in N^-$. But then $p''_{\mu''_i} P_l p'_{\mu'_l}$ by the assumption $p_{\mu_i} P_l p_{\mu_l}$, and the construction of p and μ. Since $l \in N^-$ it follows from Proposition 2 that $p'_{\mu'_l} R_l p''_{\mu''_l}$. Thus, $p''_{\mu''_i} P_l p''_{\mu''_l}$ by monotonicity. Consequently, conditions (a) and (b) must hold by Definition 8 as $(p'', \mu'') \in \overline{\Sigma}_R$. The same arguments will do in the case when $l \in N^+$ and $i \in N^-$.

If there is no priority respecting improvement of μ, then μ is priority efficient and $(p, \mu) \in \overline{\Sigma}_R$ by (a) and (b). Suppose instead that μ not is priority efficient. In this case, there is a priority respecting improvement $\tilde{\mu}$ of μ by Definitions 5 and 6. Note first that (a) always is preserved in a priority respecting improvement (see Definition 5, and the discussion following it). Moreover, if $p_{\tilde{\mu}_i} P_l p_{\tilde{\mu}_l}$, then $p_{\tilde{\mu}_i} P_l p_{\mu_l}$ since $\tilde{\mu}$ is a Pareto improvement of μ. Thus, $p_{\tilde{\mu}_i} = \overline{p}_{\tilde{\mu}_i}$ as (b) is valid for the assignment μ. Hence, conditions (a) and (b) are preserved at the assignment $\tilde{\mu}$. If $\tilde{\mu}$ is not priority efficient, we can find another priority respecting improvement of $\tilde{\mu}$ that respects (a) and (b) by adopting the same arguments as in the above. After a finite number of priority respecting improvements, a priority efficient assignment that in addition satisfies (a) and (b) is obtained. Hence, conditions (i)–(iii) of Definition 8 are satisfied. Consequently, $p \in \overline{\Pi}_R$.

Note, finally, that p' and p'' are minimal price vectors in $\overline{\Pi}_R$, by assumption, and belong to Π_R by Theorem 2. This together with $\Pi_R \subset \overline{\Pi}_R$, the construction $p \leq p'$ and $p \leq p''$ yield $p = p' = p''$. Hence, $p \in \Pi_R$, the desired conclusion. *Q.E.D.*

From Theorem 3, it is clear that the following mechanism is well defined for any $R \in \tilde{\mathcal{R}}$.

DEFINITION 11 *The minimal WPE mechanism* f is defined to be the mapping of profiles $R \in \tilde{\mathcal{R}}$ to a weak price equilibria (p^*, μ), where p^* is the unique minimal price vector in Π_R and μ is any selection of the assignments such that $(p^*, \mu) \in \Sigma_R$.

Note that in the bounding case when Ω contains a single price vector, the minimal WPE mechanism makes the same selection as the weak version of the serial dictatorship mechanism (Svensson, 1994). This follows as prices are fixed and the assignment generated by the latter mechanism is priority efficient as already pointed out in Section 4. Moreover, in the other limiting case when $\overline{p}_h \to \infty$ for all $h \in H$, the set of weak price equilibria coincides with the set of price equilibria (again, see Section 4). Then because the set of price equilibria contains a unique minimum price vector (Shapley and Shubik, 1972; Demange and Gale, 1985), this price vector must also be the minimal vector in Π_R, and it follows that the competitive price mechanism (Demange and Gale, 1985) reduces to a special case of the minimal WPE mechanism. Consequently, the main non-manipulability results in Demange and Gale (1985, Theorem 2) and Svensson (1994, Theorem 1) can be expressed as corollaries to the following theorem.

THEOREM 4 Let f be a minimal WPE mechanism. Then f is cannot be manipulated by any agent at any profile $R \in \tilde{\mathcal{R}}$.

PROOF: Suppose that agent $q \in N$ can manipulate the mechanism at a profile $R \in \tilde{\mathcal{R}}$. Then there is a profile $R' = (R'_q, R_{-q}) \in \tilde{\mathcal{R}}$ and two states $(p, \mu) = f(R)$ and $(p', \mu') = f(R')$ such that $p'_{\mu'_q} P_q p_{\mu_q}$ by Definition 9. This also means that $N^+ \neq \emptyset$ as $q \in N^+$ by assumption. To prove the theorem, it is sufficient to show that there is an isolated set $S \subset S_1$. Then it is possible to decrease p_h for all $h \in S$ by a small amount $\varepsilon > 0$ and obtain a new WPE according to Lemma 2, which contradicts that (p, μ) is selected by the minimal WPE mechanism.

Consider now the restriction μ^+ of μ to N^+, i.e., $\mu^+ : N^+ \to H^+$ and $\mu_i^+ = \mu_i$, where H^+ is the range of μ^+, i.e., $H^+ = \{h \in H \cup \{0\}\,;\ h = \mu_i \text{ for some } i \in N^+\}$. By Proposition 2, $H^+ \subset S_1 \cup S_2$. We next prove that the restriction μ^+ of μ is priority efficient. Suppose that the restriction μ^+ is not priority efficient. Then there is a sequence $(i_1, \dots, i_t)$ of distinct agents $i_j \in N^+$ for $1 \leq j \leq t$, defining a trading cycle, which is a priority respecting improvement of μ^+ that can be "blocked" by an agent $k \in N^-$. This means that there is an agent $i_l \in \{i_1, \dots, i_t\}$ such that $p_{\mu_{i_l}} P_k p_{\mu_k}$ and $\pi_k < \pi_{i_{l-1}}$. But then $p_{\mu_{i_l}} = \overline{p}_{\mu_{i_l}}$ since $(p, \mu) = f(R)$. Note also that agent i_{l-1} is the agent in N^+ with priority to envy agent i_l by definition of a priority respecting improvement. Consider now the state (p', μ'), and note that (a) $\mu'_r = \mu_{i_l}$ for some agent $r \in N^+$ or (b) $\mu_{i_l} \in \mu'_0$. We prove that in either case, a contradiction to the assumption that μ^+ is not priority efficient is obtained.

(a) If $\mu_{i_l} = \mu'_r$ for some $r \in N^+$, then $p_{\mu'_r} P_k p_{\mu_k}$ since $p_{\mu_{i_l}} P_k p_{\mu_k}$ by assumption. Further, because $\mu_{i_l} = \mu'_r \in S_1 \cup S_2$, by Proposition 2, monotonicity yields $p'_{\mu'_r} P_k p_{\mu_k}$. But $k \in N^-$ and, therefore, $p'_{\mu'_r} P_k p'_{\mu'_k}$. Consequently, $\pi_r < \pi_k$ by Definition 8(iii). Because agent i_{l-i} is the agent in N^+ with priority to envy i_l, it is clear that $\pi_{l-1} < \pi_r$. But this together with $\pi_r < \pi_k$ is a contradiction to $\pi_k < \pi_{i_{l-1}}$.

(b) If $\mu_{i_l} \in \mu'_0$, then $p'_{\mu_{i_l}} = \underline{p}_{\mu_{i_l}}$ as $(p', \mu') = f(R')$. Now, $p_{\mu_k} R_k p'_{\mu'_k}$ since $k \in N^-$, and $p'_{\mu'_k} R_k \underline{p}_{\mu_{i_l}}$ because $\mu_{i_l} \in \mu'_0$. Moreover, $\underline{p}_{\mu_{i_l}} R_k p_{\mu_{i_l}}$ by monotonicity. Hence, $p_{\mu_k} R_k p_{\mu_{i_l}}$ which contradicts that $p_{\mu_{i_l}} P_k p_{\mu_k}$.

We conclude that the restriction μ^+ of the assignment μ to the group N^+ is priority efficient. But then there is an isolated set $S \subset S_1 \cup S_2$ such that:

(1) $S \neq \emptyset$ and $S \cap \mu_0 = \emptyset$,
(2) if $G = \{i \in N^+; \mu_i \in S\}$ then $p_{\mu_i} R_i p_{\mu_l}$ for all $i, l \in G$, and;
(3) if $i \in N^+ \setminus G$ then $p_{\mu_i} P_i p_k$ for all $k \in S$.

This follows because if some agent in N^+ is envied by some agent in N^+, then there is an isolated set $S \subset (S_1 \cup S_2) \cap \mu_0$ by Theorem 1, and if no agent in N^+ is envied by an agent in N^+, the set $S = \{h \in S_1 \cup S_2; h = \mu_i \text{ for some } i \in N^+\}$ satisfies points (1)–(3) from the above. Note also that if S is isolated, then the set $S \cap \mu_0$ is isolated.

It is next demonstrated that $S \cap S_2 = \emptyset$. Suppose that $S \cap S_2 \neq \emptyset$. Then, there is an agent i' such that $\mu_{i'} \in S \cap S_2$ where the set S satisfies points (1)–(3) from the above. Let $(i_1, \ldots, i_t)$ be a trading cycle from μ to μ' such that $i_j = i'$ for some $1 \leq j \leq t$. Then $i_j \in N^+$ for all $1 \leq j \leq t$, by Proposition 2, as $i' \in N^+$ is implied by the above construction of S. We next consider two cases and prove that in each case a contradiction is obtained, implying that $S \cap S_2 = \emptyset$.

(i) If $\mu_{i_j} \in S \cap S_2$ for all $1 \leq j \leq t$, then $p_{\mu_{i_j}} I_{i_j} p_{\mu_{i_{j+1}}}$ for all $1 \leq j \leq t$ as S is envy-free. This contradicts that $i_j \in N^+$ because at least one strict preference is required by Notation 2.

(ii) If $\mu_{i_j} \notin S \cap S_2$ for some $1 \leq j \leq t$, then there is an agent i_l such that $\mu_{i_l} \notin S \cap S_2$ and $\mu'_{i_l} = \mu_{i_{l+1}} \in S \cap S_2$ (or $\mu'_{i_l} \in \mu_0$). But then, $p_{\mu_{i_l}} R_{i_l} p_{\mu'_{i_l}}$, by Definition 7(i) as S is isolated in $S_1 \cup S_2$. Then $p'_{\mu_{i_l}} P_{i_l} p'_{\mu'_{i_l}}$, by monotonicity, since $\mu'_{i_l} \in S \cap S_2$ and $\mu_{i_l} \in S \setminus S_2$. But if $(p', \mu') = f(R')$, then $p'_{\mu'_{i_l}} R_{i_l} p'_{\mu_{i_l}}$ as $p'_{\mu_{i_l}} < \overline{p}_{\mu_{i_l}}$. Hence, a contradiction.

We conclude that $S \cap S_2 = \emptyset$ and $S \subset S_1$ must be the case. Then for all $h \in S$:

- $p_{\mu_i} R_i p_h$ for all $i \in N^+$ with $\mu_i \in S$ by Definition 7(i),
- $p_{\mu_i} P_i p_h$ for all $i \in N^+$ with $\mu_i \notin S$ by Definition 7(ii),
- $p_{\mu_i} P_i p_h$ for all $i \in N^-$. This follows since $p'_{\mu'_i} R_i p'_h$ by Definition 8(iii) as $p'_h < p_h \leq \bar{p}_h$. Hence, $p'_{\mu'_i} P_i p_h$ by monotonicity. Then once more by monotonicity, $p_{\mu_i} P_i p_h$ since $i \in N^-$.

These above three bullet points demonstrate that $S \subset S_1$ is an isolated group for the entire set H of houses. But then the proof is complete by the arguments in the beginning of the proof. *Q.E.D.*

Remark 1 Finally, we remark that the result in Theorem 4 holds for the restricted preference domain $\tilde{\mathcal{R}}$. In fact, the minimal WPE mechanism is not defined when considering profiles that not are included in $\tilde{\mathcal{R}}$ as its definition requires the existence of a unique minimal WPE vector and Theorem 3 is only proved for the profiles in $\tilde{\mathcal{R}}$. It is an open question how to define the minimal WPE mechanism when considering the full preference domain. However, as argued in the above, the set $\tilde{\mathcal{R}}$ includes "almost all" preference profiles in $\mathcal{R}$.

APPENDIX

DEFINITION 12 Suppose that assignment μ is priority efficient and E is nonempty. The correspondence $\varphi : F \to 2^F$ is then defined as follows. For $i \in F$, $i' \in \varphi(i)$ if there is a sequence $(i_1, \ldots, i_t)$ of distinct agents and an index k, $2 \leq k \leq t-1$, such that:

(i) $i_1 = i$ and $i_t = i'$,
(ii) $i_j \in E$ if and only if $2 \leq j \leq k$,
(iii) i_j has priority to envy i_{j+1} for $1 \leq j < k$,
(iv) $\mu_{i_j} I_{i_j} \mu_{i_{j+1}}$ for $k \leq j < t$.

Note that the definition of φ presupposes that F is nonempty. But this follows from Lemma 1 as μ is priority efficient by assumption. Moreover, $\varphi(i)$ may be empty for some $i \in F$, and of course for all $i \in F$ if $\mu(F)$ is an isolated set.

PROOF OF THEOREM 1: Suppose, as in the theorem, that $E \neq \emptyset$ and that μ is priority efficient. Let, in addition, $\varphi : F \to 2^F$ be the correspondence given by Definition 12. To obtain a contradiction, suppose that there is no isolated set.

First it is demonstrated that $\cup_{i \in F} \varphi(i) = F$. By contradiction, suppose that $i' \in F$ but $i' \notin \cup_{i \in F} \varphi(i)$, and let $F' \subset F$ be defined as: $i \in F'$ if and only if there is a sequence $(i_1, \ldots, i_t)$ of distinct agents $i_j \in F$ such that $i_1 = i$, $i_t = i'$ and $\mu_{i_j} I_{i_j} \mu_{i_{j+1}}$ for $1 \leq j < t$. Since there is no isolated set, by assumption, there is an agent $i'' \in E$ such that $\mu_{i''} I_{i''} \mu_i$ for some $i \in F'$. But then there is also a sequence $(i_q, \ldots, i_0)$ of distinct agents, where $q < 0$, such that:

- $i_q \in F$ and $i_j \in E$ if $j > q$,
- i_j has priority to envy i_{j+1} for $q \leq j \leq -1$,
- $i_0 = i''$.

Note first that the existence of $i_q \in F$ follows as μ is priority efficient, i.e., if always $i_j \in E$ then there would be a priority respecting improvement assignment where only agents in E are trading, contradicting priority efficiency. But now we have a sequence $(i_q, \ldots, i_0, i_1, \ldots, i_t)$ that satisfy properties (i)–(ii) in Definition 12. Hence, $i' = i_t \in \varphi(i_q)$. This shows that $\cup_{i \in F} \varphi(i) = F$ must be the case.

Now let $F^* = \{i \in F; \varphi(i) \neq \emptyset\}$. Then, $F = \cup_{i \in F} \varphi(i) = \cup_{i \in F^*} \varphi(i)$. Hence, for each $i \in F^*$ there is an $i' \in F^*$ such that $i \in \varphi(i')$. Then there are sequences $(i_1, \ldots, i_t)$, $i_j \in F^*$, of distinct agents such that $i_j \in \varphi(i_{j+1})$ for $1 \leq j \leq t-1$. Consider some $i_1 \in F^*$ and choose t as large as possible. This means that $i_t \in \varphi(i_l)$ for some $l < t$. According to the definition of φ, for each $j \leq t$, there are sequences of agents satisfying properties (i)–(iv) of Definition 12 in the following way:

$$(i_{1j}, i_{2j}, \ldots, i_{q_j j}) \text{ with } i_{1j} = i_{j+1} \text{ and } i_{q_j j} = i_j.$$

Thus for $j \geq l$:

$$(i_{1t}, i_{2t}, \ldots, i_{q_t t}) \text{ with } i_{1t} = i_l \text{ and } i_t = i_{q_t t} \in \varphi(i_{1t}),$$
$$(i_{1t-1}, i_{2t-1}, \ldots, i_{q_{t-1} t-1}) \text{ with } i_{1t-1} = i_t \text{ and } i_{t-1} = i_{q_{t-1} t-1} \in \varphi(i_{1t-1}),$$
$$\vdots \qquad \vdots,$$
$$(i_{1l+1}, i_{2l+1}, \ldots, i_{q_{l+1} l+1}) \text{ with } i_{1l+1} = i_{l+2} \text{ and } i_{l+1} = i_{q_{l+1} l+1} \in \varphi(i_{1l+1}),$$
$$(i_{1l}, i_{2l}, \ldots, i_{q_l l}) \text{ with } i_{1l} = i_{l+1} \text{ and } i_l = i_{q_l l} \in \varphi(i_{1l}).$$

Construct now one sequence and rename the agents according to:

$$(i'_1, i'_2, \ldots, i'_{t'}) =$$
$$(i_{1t}, i_{2t}, \ldots, i_{q_t t}, i_{2t-1}, i_{3t-1}, \ldots, i_{q_{t-1} t-1}, \ldots, i_{2l+1}, i_{3l+1}, \ldots, i_{q_{l+1} l+1}, i_{2l}, i_{3l}, \ldots, i_{q_l l}).$$

Here $i'_1 = i_{1t} = i_l$ and $i'_{t'} = i_{q_l l} = i_l$. Then, since $i'_1 = i'_{t'}$, there must be indices k and p such that the subsequence $(i'_k, i'_{k+1}, \ldots, i'_{p-1})$ contains only distinct agents and $i'_k = i'_p$. From the construction of the sequence also follows that some agent in the subsequence belongs to E. But then we can define a priority respecting improvement μ' according to $\mu'_{i'_{j+1}} = \mu'_{i'_j}$ for $j < p$, $\mu'_{i'_1} = \mu'_{i'_p}$ and $\mu'_i = \mu_i$ for the remaining agents. This is a contradiction to μ being priority efficient. Hence, there must be an isolated set.

Finally, if the assignment $\mu : N \to H$ is priority-efficient and the set of envied agents E is nonempty, the same presumptions are valid for the assignment $\tilde{\mu} : N \to H \setminus \mu_0$ where $\tilde{\mu}_i = \mu_i$ for all $i \in H$. Hence, the assignment $\tilde{\mu}$ has an isolated set $\tilde{S}$ where $\tilde{S} \cap \mu_0 = \emptyset$. The set $\tilde{S}$ is obviously also an isolated set for μ. *Q.E.D.*

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