Speculative Betas*

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Abstract

We provide a theory and evidence for when the Capital Asset Pricing Model fails. When investors disagree about the common factor of cash-flows, high beta assets are more sensitive to this aggregate disagreement than low beta ones and hence experience a greater divergence-of-opinion about their cash-flows. Costly short-selling then results in high beta assets experiencing binding short-sales constraints and being over-priced. When aggregate disagreement is low, the Security Market Line is upward sloping due to risk-sharing. But when it is large, the Security Market Line is initially increasing and then decreases with beta. At the same time, high beta assets in a dynamic setting also have greater share turnover, and especially so when aggregate disagreement is high. Using the dispersion of stock analysts' earnings forecasts to measure speculative disagreement, we find strong support for these predictions.

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1. Introduction

In this paper, we provide a theory and evidence for when Sharpe (1964)'s Capital Asset Pricing Model (CAPM) fails. Over the last twenty years, financial economists have developed a large and impressive body of findings on the excess predictability of cross-sectional asset returns. These studies reject the CAPM beta, the sensitivity of an asset's return to the market portfolio return, as being a sufficient statistic for explaining the variation in asset returns. These findings such as the value-growth effect, stocks with low price-to-fundamental ratios out-performing those with high ones, or the momentum effect, recent winning stocks out-performing recent losing ones, have led to a search for multiple factor models, whether they be dynamic extensions of the CAPM or liquidity- or behavioral-based explanations.[1]

The debate over how to interpret these famous asset pricing patterns left ignored the behavior of the Security Market Line (SML) – that the plot of expected excess returns on beta ought to be upward sloping. The presumption is that the SML is flat to perhaps mildly upward sloping and has little explanatory power because of these other factors or idiosyncratic risks. Indeed many of the well-known recent deviations from the CAPM all predict a positive relationship between risk and expected returns: in these models, assets with high beta or high price volatility ought to have, if there is any association at all, high expected returns.[2]

But there is suggestive evidence that the risk and return relationship is not only not strong, but is frequently going the wrong way. Baker, Bradley, and Wurgler (2011) point out that a portfolio long low-beta stocks and short high-beta stocks (appropriately adjusted to be beta neutral) in the US between 1968-2008 produce a Sharpe ratio comparable to or larger than the much more famous anomalies of value-growth or momentum effects. In the most systematic study of this phenomenon to date, Frazzini and Pedersen (2010) show that

[1]For surveys of this evidence, see Barberis and Thaler (2003), Hirshleifer (2001), and Hong and Stein (2007).

[2]In Merton (1987)'s segmented CAPM due to clientele effects, idiosyncratic volatility attracts higher returns. In Delong, Shleifer, Summers, and Waldmann (1990), high noise trader risk yields high return. In Campbell, Grossman, and Wang (1993), high liquidity risk yields high expected return.

this low risk, high return puzzle is present in stocks of all market capitalizations and also in international markets and across asset classes. A strategy of buying low-price volatility and shorting high-price volatility assets produces similar results and is more well-known from the influential work of Ang, Hodrick, Xing, and Zhang (2006). Earlier studies, which have not received due attention, anticipate this body of evidence (see, Black (1972), Haugen and Heins (1975), Blitz and Vliet (2007), Cohen, Polk, and Vuolteenaho (2005)).

In this paper, we provide a theory of this low risk, high expected return puzzle by incorporating speculative disagreement and costly short-selling into the CAPM. Mean-variance investors have a speculative motive for trade due to disagreement about the market or common component of fundamentals or cash flows in addition to their usual risk-sharing motive. The aggregate disagreement might come from overconfidence or heterogeneous priors and beliefs and is likely to be slowly mean-reverting.[3] Assets which load more on this market factor, i.e. higher cash-flow beta assets and as a result also higher market beta assets, are thus more subject to potential investor disagreement about their cash-flows. Investors also face a quadratic short-selling cost.[4][5]

We show that the expected return of an asset in our model is determined by two forces. When disagreement is low, all investors are long and short-sales constraints do not bind in that no investor takes a short position. The traditional risk-sharing motive leads high beta assets to attract a lower price or higher expected return. The CAPM holds and the Security Market Line (SML) is upward sloping. But when aggregate disagreement increases, high beta assets experience large enough divergence-of-opinion so that there is short-selling for these high-beta assets. Because short-selling is costly, there is over-pricing as prices only partially reflect the pessimists' beliefs. This is the multi-asset, costly short-selling extension

[3] There is now plentiful evidence from individual portfolio choice in support of this behavior (see, e.g. Odean (1999)).

[4] Using a quadratic cost function facilitates the exposition of the results, but our analysis does not depend on this assumption. The important assumption is that this shorting cost function is convex.

[5] The evidence for short-sales constraints comes in many forms. Most institutions investors such as mutual funds are simply prohibited form shorting by charter in the US. Short-sales are literally banned in many countries or are banned during times of crises when short-sellers are apt to make the most profits. And micro-evidence looking at the mechanisms of shorting are also confirming.

of the one-asset, prohibitive short-selling result in Miller (1977) and Chen, Hong, and Stein (2002). The CAPM then does not hold – there is no longer an increasing relationship between expected returns and betas.

Instead, the shape of the SML depends on the magnitude of the aggregate disagreement. For disagreement of moderate size, the relationship between risk and returns is kink-shaped. For assets with a beta below a certain cut-off, expected returns are increasing in beta as there is little disagreement about these stock's cash-flows and no short-selling. In other words, for these assets, the risk-sharing motive for trading dominates the speculative motive so that investors – even pessimist ones – want to be long these assets. For assets with a beta above the cut-off, disagreement about the dividend becomes sufficiently large that the pessimist investors want to short these assets. Because of costly short-sales, these assets are overpriced and exhibit lower expected returns than in the standard CAPM framework. As a consequence, expected returns are increasing with beta but at a much lower pace than for assets with a beta below this cut-off. As disagreement increases, the cut-off level for beta below which there is no shorting falls and the SML takes on an inverted-U shape: expected return initially rises with beta but then falls with beta above this cut-off. When aggregate disagreement is so large that pessimists short-sell all assets[6], the SML can even become entirely downward sloping.

In a dynamic extension of this model, investors anticipate that high beta assets are more likely to experience binding short-sales constraints and hence have a higher resale option than low beta ones (Harrison and Kreps (1978), Scheinkman and Xiong (2003) and Hong, Scheinkman, and Xiong (2006)). Since a high resale option leads to high turnover, high beta assets end up having more share turnover than low beta assets. As disagreement increases, the share turnover gap between high and low beta assets also increases, provided that we look at assets with sufficiently different betas.

We then take our model to the data and test its key predictions using analysts earnings'

[6]We assume that all assets in our model have a strictly positive loading on the aggregate factor. Thus, it is always possible that pessimists want to be short an asset, provided disagreement is large enough.

forecasts as a measure of aggregate disagreement. Disagreement for a stock's cash-flow is simply the standard deviation of its analysts earnings forecasts as in Diether, Malloy, and Scherbina (2002). The *aggregate* disagreement measure is typically a value-weighted average of analyst earnings forecast dispersion for all stocks as in Yu (2010). But the aggregate measure suggested from our model is that each stock's disagreement is weighted by the stock value *and* its beta. After all, stocks with very low betas have by definition almost no sensitivity to aggregate disagreement – so that the disagreement on these stocks will reflect idiosyncratic disagreement, which, as we argue in our model, should not be relevant for the predictions that we test. We use this key insight from our model to derive our measure of aggregate disagreement.

Our predictions are the following: (1) when our value*beta weighted aggregate disagreement measure is low, SML is increasing, but when it is high, SML is initially increasing and then decreases with beta and the slope is overall smaller; (2) when our aggregate disagreement is high, high beta stocks also have higher disagreement about its cash-flows, higher short interest and higher turnover than low beta stocks. We find strong support for these predictions.

The consideration of a general disagreement structure about both means and covariances of asset returns with short-sales restrictions in a CAPM setting is developed in Jarrow (1980). Jarrow (1980) shows that short-sales restrictions in one asset might increase the prices of others. Our modeling novelty is to focus on a one-factor disagreement structure about common cash-flows and to derive implications for asset pricing by beta. Our insight that high beta assets are more speculative builds on Hong and Sraer (2011)'s analysis of credit bubbles. They point out how debt, with a bounded upside, is less disagreement sensitive than equity and hence less prone to speculative overpricings.

Other researchers have attempted to explain the low-risk anomaly. These other explanations can be categorized along the following lines. The first is about benchmarks, indexing and fund manager incentives. Baker, Bradley, and Wurgler (2011) explains why this low risk, high return mispricing is not easy for arbitrageurs to correct. If investors are benchmarked against the market because of agency issues, fund managers have a desire to own high beta stocks that track the market and hence have limited incentives to engage in shorting high beta stocks if and when they are over-priced. Karceski (2002) makes a similar point that if managers care more about outperforming during bull markets than underperforming during bear markets due to investor preferences for trend chasing, then they would increase their demand for high-beta stocks, reducing the required returns on these stocks. These limits of arbitrage explanations, however, take mispricings as given. We provide an explanation for why high beta assets are more prone to overpricing due to speculation and short-sales constraints.

The second set of explanations is money illusion or behavioral factors that lead to a negative equity risk premium. Cohen, Polk, and Vuolteenaho (2005) provide evidence for the mechanism in which because of money, investors fail to account for illusion in their portfolio choice, leading to lower equity risk premia in times of high inflation. In their setting, the CAPM still holds exactly but the slope of SML is negative when there is high inflation. Our empirical analysis indicates that the failure of the CAPM is not simply due to a negative equity risk premium or high inflation rates. Indeed, the inverted-U shaped SML indicates that its really only high beta assets that are mispriced, not low beta ones.

The third is borrowing constraints. Frazzini and Pedersen (2010) develop a dynamic analysis of Black (1972)'s restricted borrowing CAPM, in which investors who are prohibited from accessing leverage will tilt their portfolio toward high beta assets, thereby bidding up the price of these assets and leading to a flat SML. They show that this flat SML provides an opportunity for investors who can borrow to trade against those who are prohibited. They show that a strategy of short high beta and long low beta stocks appropriately levered is profitable, especially when margin constraints are tight, which in the data is captured by high TED spreads. Both their and our mechanism emphasize the importance of market frictions: in their case leverage constraints, in ours short-sales constraints.

Our model delivers additional predictions beyond a flat SML. Because of the interaction of speculative disagreement and short-sales constraints, we get potentially more varied shapes for the SML, including an inverted-U shaped or an entirely downward sloping SML. We verify distinct predictions on how aggregate dispersion of opinion affects disagreement at the individual stock level, short interest and share turnover across stocks of different betas. We also directly show that speculative disagreement about the aggregate market has strong explanatory power for the failure of the CAPM even controlling for the factors, suggested by these related papers, that might moderate the slope of the SML, including the inflation rate, the TED spread and market volatility.

More generally, we show how a speculation motive generates radically different predictions from a risk-sharing or liquidity motive for the pricing of assets in a CAPM setting. We also show how these predictions actually match the facts about risk and returns in financial markets over the last thirty years.[7] Moreover, our analysis yields a unified perspective on long swings in price-to-fundamental ratios or mispricings such as the dot-com and housing bubbles. This stands in contrast to more piecemeal approaches in asset pricing, which focus on either cross-sectional implications or try to examine time-series implications.

The usefulness of our approach is that it links directly to the centrality of market beta in the analysis of financial markets. The power of the CAPM lies in its simplicity and in the measurability of market beta. In other words, CAPM gives strong predictions linking asset prices to underlying technologies of firms. Earlier attempts at deviations of the CAPM by and large lack this strong technological connection and do not address the centrality of market beta. Market beta is fundamental to financial economics and any model hoping to address the limited explanatory power of CAPM needs to address beta. Our theory provides such a natural connection. High beta assets are speculative. This can be tested and used in much the same way as the CAPM. Our model is a natural extension of the CAPM to

[7]Our behavioral model can be juxtaposed to two different but interesting approaches using mental accounting as in Barberis and Huang (2001) and overconfident investors who take un-diversified positions as in Daniel, Hirshleifer, and Subrahmanyam (2001).

address a world where speculation is as if not more important than risk-sharing as a motive for trade.

Our paper proceeds as follows. We present the model in Section 2. We describe the empirical findings in Section 3. We conclude in Section 4. All proofs and extensions are in the Appendix Section 5.

2. A CAPM with heterogenous beliefs and short-sales constraints

2.1. Static Setting

We consider an economy populated with a continuum of investors of mass 1. There are two periods, $t = 0, 1$. There are N risky assets and the risk-free rate is r and is exogenous. Risky asset i delivers a dividend $\tilde{d}_i$ at date 1. We decompose the dividend processes into systematic and idiosyncratic components:

$$\forall i \in \{1, \ldots, N\}, \quad \tilde{d}_i = w_i \tilde{z} + \tilde{\epsilon}_i,$$

where $\tilde{z} \sim \mathcal{N}(\bar{z}, \sigma_z^2)$, $\tilde{\epsilon}_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$, and $\mathrm{Cov}(\tilde{z}, \tilde{\epsilon}_i) = 0 \ \forall i \in \{1, \ldots, N\}$. w_i is the cashflow beta of asset i and is assumed to be non-negative.

Each asset $i \in \{1, \ldots, N\}$ has a supply $s_i > 0$ and we assume w.l.o.g. that:

$$\frac{w_1}{s_1} < \frac{w_2}{s_2} < \cdots < \frac{w_N}{s_N}.$$

That is, we index assets in the economy by their cashflow betas, with these betas increasing in i. Asset N has the highest cashflow beta while asset 1 has the lowest. We assume that the value-weighted average w in the economy is 1 ($\sum_{i=1}^N w_i s_i = 1$).[8]

[8]This is just to ensure that as markets become complete ($N \to \infty$), the economy remains bounded.

The population of investors is divided into two groups, A and B, which hold heterogenous beliefs about the mean value of the aggregate shock $\tilde{z}$. Agents in group A believe that $\mathbb{E}[\tilde{z}] = \bar{z} + \lambda$ while agents in group B believe that $\mathbb{E}[\tilde{z}] = \bar{z} - \lambda$. We assume w.l.o.g. that $\lambda > 0$ so that group A is the optimistic group and group B the pessimistic one. Investors maximize their date-1 wealth and have mean variance preferences:[9]

$$U(\tilde{W}_{j,1}) = \mathbb{E}[\tilde{W}_{j,1}] - \frac{1}{2\gamma} Var(\tilde{W}_{j,1})$$

where $j \in \{A, B\}$ and γ is the investors' risk tolerance.

We assume that agents can take any long position but have to pay a cost $\frac{c}{2}\mu^2$ if they hold a short position μ in an asset.[10] We use the term binding short-sales constraints in an asset when one of the groups takes a non-positive position in that asset.

The following theorem characterizes the equilibrium.

Theorem 1. *Let* $\theta = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \frac{\gamma c}{2}}$ *and let* $(u)_{i \in [0,N+1]}$ *be a sequence such that* $u_{N+1} = 0$, $u_i = \frac{s_i}{w_i}\left(\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{j<i} w_j^2 + \theta \sum_{j \geq i} w_j^2\right)\right) + \sigma_z^2 (1-\theta)\left(\sum_{j \geq i} w_j s_j\right)$ *for* $i \in [1, N]$ *and* $u_0 = \infty$. *u is a strictly decreasing sequence.*

Suppose that disagreement λ *is such that for some* $\bar{i} \in [1, N]$*:*

$$u_{\bar{i}} > \lambda\gamma > u_{\bar{i}-1} \tag{1}$$

Then, group B agents are long assets $i < \bar{i} - 1$ *and short assets* $i \geq \bar{i}$. *Group A agents are*

[9] In our static model, investors could also simply be endowed with CARA preferences. In our dynamic extension, however, the mean-variance preferences are more tractable as the interim realizations of prices are not gaussians – but troncated gaussians.

[10] The results in this paper carry through to the more a generic cost function $C(\mu)$ – provided this cost function $C()$ is strictly convex. Derivations are available from the authors upon request.

long all assets. Equilibrium asset prices are given by:

$$P_i(1+r) = \begin{cases} \bar{z}w_i - \frac{1}{\gamma}\left(w_i\sigma_z^2 + s_i\sigma_\epsilon^2\right) & \textit{for } i < \bar{i} \\ \bar{z}w_i - \frac{1}{\gamma}\left(w_i\sigma_z^2 + s_i\sigma_\epsilon^2\right) + \underbrace{\frac{1}{\gamma}\left(1-\theta\right)\sigma_\epsilon^2\left(w_i\left(\frac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) - s_i\right)}_{\textit{speculative premium}} & \textit{for } i \geq \bar{i} \end{cases} \tag{2}$$

The equilibrium positions for Group B investors are given by:

$$\mu_i^B = \begin{cases} s_i + w_i\left(\frac{\sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right) - \lambda\gamma}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) > 0 & \textit{for } i < \bar{i} \\ \theta\left[s_i + w_i\left(\frac{\sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right) - \lambda\gamma}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right)\right] < 0 & \textit{for } i \geq \bar{i} \end{cases} \tag{3}$$

Assets with high cashflow betas, i.e. $i \geq \bar{i}$, are over-priced (relative to the benchmark with no short-sales constraints) and the degree of over-pricing is increasing with the cost of shorting c and with disagreement λ.

Proof. See Appendix. □

Holding fixed the other parameters, high w_i assets are more sensitive to aggregate disagreement (λ) and hence experience a greater divergence of opinion than low w_i assets. When disagreement grows, agents in the pessimist group become more pessimistic on high w_i stocks first and then on low w_i stocks. Thus, their desired holdings of risky securities decreases and especially more so for high w_i stocks, until it becomes negative. At this point, agents in group B becomes short-sales constrained on high w_i stocks (i.e. they effectively are short and have to pay the shorting costs). This creates a speculative premium or over-pricing on these stocks. The larger the disagreement, the more stocks experience short-sales constraints and thus the greater the over-valuation.

Observe that $\theta < 1$ and θ decreases with c. From Equation (3), the short position for the Group B investors for assets $i \geq \bar{i}$ increases with θ or decreases with c. When short-selling is costless, $c = 0$ and $\theta = 1$ and the equilibrium prices from Equation (2) do not depend on λ. Since disagreement is symmetric[11], the pessimists are as pessimistic as the optimists, and hence costless shorting leads to an averaging out of sentiment from prices. But when $c > 0$ and short-selling is costly, the equilibrium prices of high w_i assets, i.e. those assets $i \geq \bar{i}$, depend on λ and hence over-pricing since the costly shorting limits the amount of short-selling on the part of group B investors.

The third term of the equilibrium prices for assets $i \geq \bar{i}$, given by

$$\frac{\sigma_\epsilon^2}{\gamma}(1-\theta)\left(w_i\left(\frac{\lambda\gamma - \sigma_z^2(1-\theta)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) - s_i\right),$$

captures the degree of over-pricing due to costly short-selling. This is the difference between the equilibrium prices and the price that would prevail in the absence of heterogenous beliefs and costly shorting. This term decreases with θ, i.e. increases with the cost of shorting c. For a fixed w_i, the larger is the divergence of opinion λ, the greater the over-pricing. The lower is the supply s_i and the greater is the risk absorption capacity $\frac{\sigma_\epsilon^2}{\gamma}$ the greater is the over-pricing. When there is more supply and less risk absorption capacity, even in the presence of disagreement, investors require a lower price to hold the asset and everyone, even the pessimists, are long the asset. Note that by taking $c \to \infty$, we obtain the model with prohibitive short-sales constraints as discussed in the single asset setting in Miller (1977) and Chen, Hong, and Stein (2002).

It is interesting to consider more deeply the intuition for why the high beta assets hit short-sales constraints first. The simplest way to see this is to consider the first order condition for the pessimists on a short-sales constrained asset:

$$(\bar{z}-\lambda)w_i - P_i(1+r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} w_k\mu_k^B\right)w_i\sigma_z^2 + \mu_i^B\left(\sigma_\epsilon^2 + \gamma c\right)\right)$$

We then replace the equilibrium excess return with the risk premium and the speculative

[11]The assumption that disagreement is symmetric is made to ensure that agents in the model hold, on average, the right belief about the aggregate factor $\tilde{z}$. Hong and Sraer (2011) introduces sentiment in a pricing model with heterogenous beliefs and short-sales constraints.

term:

$$-\lambda w_i + \frac{1}{\gamma}\left(w_i\sigma_z^2 + s_i\sigma_\epsilon^2\right) = \frac{1}{\gamma}\left(Sw_i\sigma_z^2 + \mu_i^B\left(\sigma_\epsilon^2 + \frac{\gamma c}{2}\right)\right)$$

This equation says that the marginal benefit of buying an additional unit of asset i is equal to $-\lambda w_i$ due the distorted belief, $\frac{\sigma_z^2}{\gamma}w_i$ which is the compensation for the systematic risk in asset i and $\frac{\sigma_\epsilon^2}{\gamma}s_i$ which is the compensation for the idiosyncratic risk. This marginal benefit has to be equal to the marginal cost, which is equal to $S\frac{\sigma_z^2}{\gamma}w_i$ or the additional covariance in the investor's portfolio from holding the additional unit of asset i and $\frac{\mu_i^B}{\gamma}\left(\sigma_\epsilon^2 + \frac{\gamma c}{2}\right)$, which is the equilibrium "cost" coming from decreased idiosyncratic variance and decreased short-selling cost. (Note that $c/2$ of the shorting cost is already priced so that the actual "cost" is only $c/2$).

This condition can then be rewritten as:

$$\left[-\lambda\gamma + \sigma_z^2(1-S)\right]w_i = \mu_i^B\left(\sigma_\epsilon^2 + \frac{\gamma c}{2}\right) - s_i\sigma_\epsilon^2 < 0$$

Now let $N \geq l > k > 0$, i.e. $\frac{w_l}{s_l} > \frac{w_k}{s_k}$. Assume asset k is shorted at equilibrium but not asset l, we can write:

$$\left(\sigma_\epsilon^2\frac{\mu_2}{s_2} - \left(\sigma_\epsilon^2 + \frac{\gamma c}{2}\right)\frac{\mu_1}{s_1}\right) = \left[-\lambda\gamma + \frac{1}{\gamma}w_i\sigma_z^2(1-S)\right]\left(\frac{w_2}{s_2} - \frac{w_1}{s_1}\right) < 0$$

so that $\frac{\mu_2}{s_2} < \frac{\sigma_\epsilon^2 + \frac{\gamma c}{2}}{\sigma_\epsilon^2}\frac{\mu_1}{s_1} < 0$ which is a contradiction. Thus, large w assets are shorted first.

Intuitively, if one asset is shorted at equilibrium, this means that for this asset, the marginal "costs" coming from the aggregate factor (i.e. assets has low subjective returns, covariance terms minus compensation for aggregate risk) are equal to the marginal benefits coming from an increase in the holdings in terms of idiosyncratic risk (compensation for idiosyncratic risk and decreased idiosyncratic risk in the portfolio as holding is < 0). Because these latter benefits are > 0, this means the former costs are > 0. In particular, these costs are all proportional to w, so that as w increase, the costs increase, so the benefits have to increase. But the marginal benefits are decreasing with μ (as the shorting cost is strictly convex) for $\mu < 0$, which means that it needs to be the case that μ is lower, i.e. as w increases, the asset is more shorted.

We can also restate the equilibrium prices as expected returns and relate them to the familiar β_i from the CAPM. Because stocks with large w's naturally have larger β's, we show here that higher disagreement will flatten the Security Market Line when there is costly short-selling.

Corollary 1. *Let* $\beta_i = \frac{\mathrm{Cov}(\tilde{R}_i, \tilde{R_M})}{Var(\tilde{R_M})}$ *and* $\bar{i}$ *is such that Group B investors are long assets* $i < \bar{i}$ *and short assets* $i \geq \bar{i}$. *Define* $\kappa(\theta, \lambda) = \frac{\lambda\gamma - \sigma_z^2(1-\theta)\left(\sum_{i\geq\bar{i}} \frac{w_i}{N}\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta \sum_{i\geq\bar{i}} w_i^2\right)} > 0$. *The expected returns are given by:*

$$\mathbb{E}[\tilde{R}_i] = \begin{cases} \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} & \text{for } i < \bar{i} \\ \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} \left(1 - \dfrac{\sigma_\epsilon^2}{\sigma_z^2}(1-\theta)\kappa(\theta,\lambda)\right) + \dfrac{\sigma_\epsilon^2}{\gamma N}(1-\theta)\left(1 + \dfrac{\sigma_\epsilon^2}{\sigma_z^2}\kappa(\theta,\lambda)\right) & \text{for } i \geq \bar{i} \end{cases}$$

The intuition for this corollary is the following. Notice that for $c = 0$ ($\theta = 1$), λ does not affect the expected returns of the assets and the standard CAPM formula holds. The expected returns depend on the covariance of the asset with the market return, and the risk premium is simply determined by the ratio of the variance of the market return (which is close to the variance of the aggregate factor σ_z^2 when N is large) to the risk tolerance of investors γ.

However, when short-selling is costly and $\theta < 1$, the expected returns for the assets $i \geq \bar{i}$ depend on the disagreement parameter λ. Higher *beta* stocks have a larger disagreement and will have more binding short-sales constraints, higher prices and hence lower expected returns. The CAPM does not hold and the Security Market Line is kink-shaped. For assets with a beta above some cut-off ($\bar{i}$ is determined endogenously and depends itself on λ), the expected return is increasing with beta but at a lower pace than for assets with a beta below this cut-off (this is the $-\frac{\sigma_\epsilon^2}{\sigma_z^2}(1-\theta)\kappa(\theta,\lambda) < 0$ term above). This is due to high beta assets being over-priced and having lower expected returns due to costly short-sales constraints.

The return/β relationship is not linear in the model. However, it is easily seen that provided N is large enough, the relation between expected excess returns and β will be close to linear. For instance, Figure 1 shows the actual relationship between excess returns for the following parameterization (which is not a calibration of the model but a simple illustrative numerical example): $N = 100$, $\sigma_z^2 = 1$, $\sigma_\epsilon^2 = 50$, $\gamma = .5$ and $\theta = .05$. Three cases are displayed on this figure: $\lambda = .5$ (no shorting at equilibrium), $\lambda = 4.8$ (100 assets are shorted at equilibrium) and $\lambda = 8$ (170 assets are shorted at equilibrium). Notice that for λ small, SML is upward sloping with beta. As λ rises, SML rises with beta up to some point and then its slope flattens. For large λ, SML is initially increasing with beta and then actually decreases with beta.

In our empirical analysis below, we look for the inflection point (if any) as suggested in this figure. In addition, rather than relying fully on the structure of the model, we take a simpler approach and compute directly the slope of the security market line delivered by the model, i.e. the coefficient estimate of an OLS regression of realized excess returns on β. We show that this coefficient strictly decreases with disagreement.

Corollary 2. *Let $\hat{\mu}$ be the coefficient estimate of a cross-sectional regression of realized returns $\tilde{R}_i$ on β_i (and assuming there is a constant term in the regression). The coefficient $\hat{\mu}$ decreases with λ the aggregate disagreement. This effect is larger for larger shorting costs c.*

In the absence of costly shorting ($c = 0$ – i.e. shorting is costless – or $\lambda < \frac{\sigma_\epsilon^2 s_N}{\gamma w_N}$ – i.e. disagreement is low so no investors want to be short), the slope of the security market line is simply $\hat{\mu} = \frac{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}{\gamma}$. When at least one asset is being constrained (i.e. when $\lambda \geq \frac{\sigma_\epsilon^2 s_N}{\gamma w_N}$), $\hat{\mu}$, the slope of the security market line (as estimated from a regression of excess returns on β), is strictly decreasing with λ, the aggregate disagreement parameter. In particular it is direct that $\hat{\mu}$ will be strictly negative, provided that λ is large enough relative to γ (i.e. that the speculation motive for trading is large relative to the risk-sharing motive for trading). Furthermore, the role of aggregate disagreement is magnified by the shorting costs: in an economy with high shorting costs (i.e. short-sales constraints), an increase in λ leads to a much larger decrease in the estimated slope of the security market line than in an economy with low shorting costs.

Thus our proposition states that: (1) the slope of the SML ($\hat{\mu}$) is strictly lower when short-sales constraints are binding ($\lambda\gamma > u_N$) than in the absence of binding short-sales constraints ($\lambda\gamma < u_N$) and (2) the slope of the security market line is strictly decreasing with λ as soon as $\lambda\gamma > u_N$. In particular, provided $\gamma\lambda$ is high enough, the estimated slope of the security market line $\hat{\mu}$ becomes negative.

2.2. Limiting Cases

In this section, we consider the role disagreement may play in a market with an infinite number of assets but where shorting is costly.

We first consider the case where investors can short, i.e. $c < \infty$ and we let $N \to \infty$. Consider the equation defining u_i, the threshold above which assets are shorted at equilibrium. If $\theta > 0$, then, $\sigma_z^2 \left(\sum_{i<j} w_i^2 + \theta \sum_{i \geq j} w_i^2\right) > \theta\sigma_z^2 \left(\sum_{i<j} w_i^2 + \sum_{i \geq j} w_i^2\right) \to \infty$ when $N \to \infty$. Furthermore, $|\sum_{i \geq j} w_i s_i| < 1$ so that $u_i \to \infty$ when $N \to \infty$ for all $i > 0$. In other words, when the number of assets go to infinity (markets become complete) and short-selling costs are finite, then there are no assets being shorted at equilibrium and thus the CAPM applies (i.e. expected returns are a strictly increasing function of βs).

Consider now the case where $c \to \infty$ ($\theta = 0$), i.e. the short-sales constraint case. To simplify the exposition, consider the following parameterization:[12] for all $i > 0$, $s_i = \frac{1}{N+1}$ and $w_i = \frac{2i}{N}$. Then, $w_i \in]0, 2]$ and $\sum_{i=1}^N w_i = 1$. The next proposition characterizes the security market line in this limiting case:

Proposition 1. *When $c = 0$ and $N \to \infty$, three cases arise:*

[12] This is without loss of generality. All results carry through in the general case but are harder to expose.

1. *When $\lambda\gamma < \sigma_z^2$, expected returns follow the standard CAPM formula $i > 0$:*

$$\mathbb{E}[\tilde{R}_i] = \beta_i \frac{\sigma_z^2}{\gamma}$$

2. *When $\lambda\gamma > \sigma_z^2 + \frac{\sigma_\epsilon^2}{2}$, the speculative premium applies on all assets' prices so that expected returns can be written as:*

$$\mathbb{E}[\tilde{R}_i] = \beta_i \left(\frac{2\sigma_z^2}{\gamma} - \lambda \right)$$

3. *Finally, when $\sigma_z^2 < \lambda\gamma < \sigma_z^2 + \frac{\sigma_\epsilon^2}{2}$, there is a finite number of assets $\bar{i} - 1 < \infty$ that are not shorted at equilibrium and expected returns have the following expression:*

$$\mathbb{E}[\tilde{R}_i] = \begin{cases} \beta_i \dfrac{\sigma_z^2}{\gamma} & \text{if } i < \bar{i} \\ \beta_i \left(\dfrac{2\sigma_z^2}{\gamma} - \lambda \right) & \text{if } i \geq \bar{i} \end{cases} \tag{4}$$

$\bar{i}$ increases weakly with λ.

Proof. See Appendix. □

As the number of assets goes to infinity, four cases happen. When disagreement is high ($\lambda\gamma > \sigma_z^2 + \frac{\sigma_\epsilon^2}{2}$), all assets are being shorted. There is a speculative premium in the price of each asset and conversely a speculative discount on their returns. When disagreement is low, ($\lambda\gamma < \frac{2}{3}\sigma_z^2$), there are no assets shorted at equilibrium and therefore prices are set according to the standard CAPM formula with $N \to \infty$. For low intermediate disagreement, ($\sigma_z^2 > \lambda\gamma > \frac{2}{3}\sigma_z^2$), there is a fraction $1 - x > 0$ of assets that are shorted at equilibrium. However, this has no impact on prices at the limit as the number of assets being shorted ($\bar{i} = xN$) goes to infinity and the speculative premium thus goes down to 0 (as $\sum_{j<\bar{i}} w_j^2 \to \infty$). Therefore, the CAPM pricing formula eventually holds for all $\lambda\gamma < \sigma_z^2$. Finally, for high intermediate disagreement, ($\sigma_z^2 + \frac{\sigma_\epsilon^2}{2} > \lambda\gamma > \sigma_z^2$), there is a finite number of assets that are not shorted at equilibrium. For the assets where even pessimists are long, the standard CAPM formula applies. For the assets that are being shorted, the speculative premium applies, and in particular the expression for the speculative premium is the same as the one obtained in the case where all assets are shorted at equilibrium.

A final remark from this section is that the speculative premium survives at equilibrium in the context of complete markets, only where there are strict short-selling constraints as opposed to short-selling costs. Intuitively, what explains this result is that because of the quadratic cost, the marginal cost to short at 0 is 0, so that pessimists will always short a little bit whatever c. Thus, when markets become complete, pessimists can build a short portfolio with no idiosyncratic risk and make the price goes down.

2.3. Extension to a Dynamic Setting

We now develop a simple dynamic version of the previous model. We make the simplifying assumption that agents are myopic: they only care about the next date's dividends and prices. There are now three periods: $t = 0, 1, 2$ but still two groups of agents (A and B) as before. At date 0, agents have homogenous beliefs about the dividend process. At date 1, agents' beliefs diverge: agents in group A (respectively B) believe the average aggregate factor is $\bar{z} + \tilde{\lambda}$ (respectively $\bar{z} - \tilde{\lambda}$), where $\tilde{\lambda} = \lambda$ with probability 1/2 and $\tilde{\lambda} = -\lambda$ with probability 1/2. The asset delivers a dividend at date 1 (before the shock to belief is realized) given by the vector $\tilde{\mathbf{d}}^1$ and then another dividend at date 2 given by the vector $\tilde{\mathbf{d}}^2$. We assume that the pay-off structures are similar to the static setting and the dividend payoffs are i.i.d. over time. Agents face similar shorting costs as in the static setting.

At date 1, after the belief shock is realized, the equilibrium is similar to the static model's equilibrium.

Corollary 3. *Stock prices at date 1 are given by* $(1 + r)P_i(\tilde{\lambda}) = m_i + \psi^i(\tilde{\lambda})$ *where* $m_i =$

$\bar{z}w_i - \frac{1}{\gamma}\left(w_i\sigma_z^2 + s_i\sigma_\epsilon^2\right)$ *and*

$$\psi^i(\tilde{\lambda}) = \begin{cases} 0 \quad if \;\; |\tilde{\lambda}|\gamma < u_i \\ \dfrac{1}{\gamma}\left(1-\theta\right)\sigma_\epsilon^2 \left(w_i \left(\dfrac{|\tilde{\lambda}|\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)} \right) - s_j \right) \quad if \;\; |\tilde{\lambda}|\gamma > u_i. \end{cases} \tag{5}$$

The only difference from the static setting is that either group can be pessimist depending on the realization of λ. However, the equilibrium price depends only on the belief of the optimist group and hence on the absolute value of $\tilde{\lambda}$. Otherwise the equilibrium prices are identical to those in Theorem 1.

At date 0, the maximization problem of agents in both groups is simply given by (with the myopic assumption and homogeneous priors at $t = 0$):

$$\max_{\mu} \mu' \left(\bar{z}\mathbf{w} + \mathbb{E}_0[\mathbf{P^1}(\tilde{\lambda})] - (1+r)\mathbf{P}^0\right) - \frac{1}{2\gamma}\mu'\Omega\mu \tag{6}$$

where μ is the vector of shares, w is the vector of cashflow betas, $\mathbf{P}^1$ is the vector of time-1 prices, $\mathbf{P}^0$ is the vector of time 0 prices, and $\mathbf{\Omega}$ is the variance-covariance matrix of $\tilde{\mathbf{d}}^1 + \mathbf{P}^1(\tilde{\lambda})$.

Because agents have homogenous beliefs at date 0, both types of agents will be long at date 0 so that no assets are shorted at date 0. Moreover, because $\tilde{\lambda}$ is binomial and symmetric, there is no resale price risk – i.e. the date-1 prices depends only on $|\lambda|$ which is certain:

$$\mathbb{E}_0[P_j(\tilde{\lambda})] = P_j(\lambda) = P_j(-\lambda) = P_j(|\lambda|) \tag{7}$$

Moreover, Ω is then simply equal to $\mathbf{\Sigma} = \sigma_z^2\mathbf{w}\mathbf{w'} + \sigma_\epsilon^2\mathbb{I}$.

Thus, the date-0 prices are simply given by the first order conditions:

$$\gamma\left(\bar{z}\mathbf{w} + \mathbf{P}^1(|\lambda|) - (1+r)\mathbf{P}^0\right) = \mathbf{\Sigma}\mu \tag{8}$$

Setting supply equal to demand then gives the equilibrium prices.

Theorem 2. *The equilibrium prices at date 0 are given by*

$$(1+r)\mathbf{P}^0 = \mathbf{P}^1(|\lambda|) + \bar{z}\mathbf{w} - \frac{\sigma_z^2\mathbf{w} + \sigma_\epsilon^2\mathbf{s}}{\gamma}. \tag{9}$$

Because of the homogenous priors at date 0 assumption (and the consequent absence of shorting), the standard CAPM formula applies from date 0 to 1. The expected return vector is given by:

$$\mathbb{E}[\tilde{\mathbf{R}}^1] = \beta\mathbb{E}[R_M^1] = \beta\frac{s'\Sigma s}{\gamma} = \beta\frac{\sigma_z^2 + \sigma_\epsilon^2\sum_{i=1}^N s_i^2}{\gamma}, \tag{10}$$

where β is the vector of betas. At date 0, agents hold the market portfolio:

$$\mu_A^0 = \mu_B^0 = \frac{1}{2}\mathbf{s}. \tag{11}$$

As a result, from date 0 to 1, the SML is upward sloping and independent of $|\lambda|$. From Corollary 2, we know that the SML is flattened or even downward sloping when disagreement is large enough. As a result, when disagreement as measured by $|\lambda|$ increases, the average of the slopes of the security market line across the two periods decreases with disagreement.

We formally state this in the following corollary.

Corollary 4. *Let $\hat{\mu}^{0\to1}$ ($\hat{\mu}^{1\to2}$) be the coefficient estimate of a cross-sectional regression of realized returns $\tilde{R}_i^1$ between dates 0 and 1 ($\tilde{R}_i^2$ between dates 1 and 2) on β_i (and assuming there is a constant term in the regression).*

$$\hat{\mu}^{0\to1} = \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma}\left(1 + \frac{\gamma}{\sigma_z^2}m_1\right). \tag{12}$$

where m_1 is the realized aggregate market return at $t=1$. And let $\hat{\mu} = \frac{1}{2}\hat{\mu}^{0\to1} + \frac{1}{2}\hat{\mu}^{1\to2}$ denote the average slope of the security market line across the two periods. $\hat{\mu}$ is decreasing with the interim disagreement $|\lambda|$. Moreover, the effect of $|\lambda|$ on the estimated slope of the security *market line $\hat{\mu}$ is larger for larger shorting cost c.*

It is obvious that the slope of the SML is decreasing with $|\lambda|$, the ex interim disagreement, as $\hat{\mu}^{0\to1}$ is independent of λ and we have shown in the previous section that $\hat{\mu}^{1\to2}$ is decreasing with λ (and strictly decreasing over $[u_N, \infty]$). It is also a direct consequence of the static model that as λ becomes large enough, the estimated slope of the SML becomes negative.

In this dynamic setting, high beta assets, which are more sensitive to aggregate disagreement, will also have higher turnover than low beta assets. And under certain conditions, when aggregate disagreement increases, the share turnover gap between high and low beta assets will also increase.

Corollary 5. *Let $\mathbb{T}$ denote the expected share turnover or the number of shares exchanged between date 0 and 1.*

$$\mathbb{T}_j = \begin{cases} w_j \kappa(|\lambda|, \theta) & \text{if } |\lambda| < u_j \\ (1-\theta)s_j + w_j \theta \kappa(|\lambda|, \theta) & \text{if } |\lambda| > u_j. \end{cases} \tag{13}$$

High β assets have higher turnover than low β assets. Furthermore, an increase in $|\lambda|$ will lead to a relative increase in the turnover of high β stocks relative to the turnover of low β stocks if and only if:

$$w_j \theta > w_k \tag{14}$$

An increase in $|\lambda|$ leads to an increase in turnover of $w_j \Delta(\kappa(|\lambda|, \theta))$ for an asset j with no binding short-sales constraints (i.e. $|\lambda| < u_j$). A similar increase leads to an increase in turnover of $w_k \theta \Delta(\kappa(|\lambda|, \theta)$ for an asset k with binding short-sales constraints (i.e. $|\lambda| > u_k$). This differential turnover of high vs. low beta stocks thus reacts ambiguously to an increase in λ (as $\theta < 1$).

There are two effects at play. While an increase in $|\lambda|$ leads to an increase in demand for both types of assets, the supply of high beta assets can only imperfectly respond to this increased demand as shorting is costly: this tends to make the turnover of high beta assets respond less to an increase in λ than the turnover of low beta assets. But the increase in demand following an increase in $|\lambda|$ is much larger for high beta assets, leading to a larger willingness to short the stock and thus a larger increase in turnover.

When the condition given in Equation (14) holds, the second effect dominates the first and the share turnover gap between high and low beta assets increase with disagreement. Empirically, this condition says that provided that we keep only the most extreme stocks in terms of betas, we should expect to see a positive correlation between the relative turnover of high vs. low beta stocks and disagreement.

Note that when all stocks have binding short sales constraint ($|\lambda| > u_1$), then an increase in $|\lambda|$ leads to a strictly larger increase in turnover for high β stocks (i.e. $\frac{\partial^2 \mathbb{T}}{\partial w \partial |\lambda|} > 0$). If there are no binding short-sales constraints ($\lambda < u_N$), then we have similarly that an increase in $|\lambda|$ leads to a strictly larger increase in turnover for high β stocks (i.e. $\frac{\partial^2 \mathbb{T}}{\partial w \partial |\lambda|} > 0$).

2.4. Discussions

In our framework, we only allow investors to disagree on the expectation of the aggregate factor, $\tilde{z}$. A more general framework would allow investors to also disagree on the idiosyncratic component of stocks dividend $\tilde{\epsilon}_i$. However, introducing this idiosyncratic disagreement would not significantly modify our analysis. Think first about the case where this idiosyncratic disagreement is independent of the β of the stock (more precisely independent of w_i). Then, the potential over-pricing generated by the idiosyncratic disagreement would also be independent of β so that our conclusions in terms of the slope of the security market line would be left unaffected. If there is any correlation between this idiosyncratic disagreement and the β of the stock, one would expect it to be positive. Loosely speaking, stocks with high β (small stocks, dot-com stocks, etc.) are also stocks with more idiosyncratic uncertainty so that there is "more" to disagree about. Empirically, as we show below, the overall disagreement on a stock – as measured by the dispersion of analyst earnings forecasts on this stock – is indeed strongly positively correlated with the stock β. As a consequence, the additional overpricing created by idiosyncratic disagreement would be positively correlated with β making the slope of the SML even less upward-sloping and reinforcing our result. Thus, we believe that the most parsimonious way to introduce the effect of speculation on the pricing of the cross-section of stocks is to focus on investors heterogeneous priors about the aggregate factor.

3. Empirical Findings

In this section, we provide evidence for the following main predictions of our model. First, periods in which this aggregate disagreement measure is high is when high beta stocks are likely to under-perform low beta stocks. Second, during these periods, high beta stocks also have higher disagreement about its cash-flows, higher short interest and higher turnover than low beta stocks.

Table 1 summarizes the variables in our empirical analysis. We follow the literature in constructing beta portfolios in the follow manner. Each month, we use the past twelve months of daily returns to estimate the market beta of each stock in that cross-section. We sort stocks into beta decile portfolios. We then take the raw difference between the value-weighted average of the next 3-months, 6-months and 12-months returns of these portfolios.

We derive two measures of the slope of the SML from these beta decile portfolios. The first is the long top decile beta and short bottom decile beta portfolio. When this long-short portfolio has a negative return, this implies a non-upward sloping SML. Here we do not make this long-short portfolio beta neutral in contrast to the literature since this will involve additional assumptions. We prefer the reader to see the simple returns excess of the risk-free rate. In Table 1, the first three rows report the summary statistics for the 3-months, 6-months and 12-months forward excess returns of this long top beta decile and short bottom beta decile portfolio. These are three monthly time series. The 3-months long-short portfolio return has a mean of 1.37% with a standard deviation of 12.87%. The 6-months long-short portfolio return has a mean of 2.07 with a standard deviation of 18.28%. The 12-months long-short portfolio return has a mean of 4.14% with a standard deviation of 27.31%.

As we explain below, our preferred benchmark is the 12-months long-short portfolio return since investor aggregate disagreement is a persistent variable and hence mispricings generated by these disagreements are likely to mean revert slowly. As a result, we ought to look at longer horizon returns to measure our effect. With this in mind, we derive a second measure of the slope of the SML, which we call SML slope, which is simply the coefficient estimate of a monthly OLS regression of the 10 beta-portfolio 12-months forward returns on each portfolio beta. The mean of SML slope is 1.99 with a standard deviation of 13.72.

We will run these two dependent variables of interest on the following independent variables. The first is our value*beta weighted measure of the investor disagreement of aggregate cash-flows motivated from our model. We construct this monthly common disagreement measure by averaging Diether, Malloy, and Scherbina (2002)'s dispersion of analysts' earnings forecasts across all firms in the US stock market during the period of 1980-2010, using as weight the product of market capitalization and β. The reason why we also weight this stock-level disagreement measure by β comes directly from the model: for low β stocks, disagreement will come mostly from idiosyncratic disagreement, which, as argued in section 2.4, is irrelevant for the slope of the security market line. Thus, we want to give more weight to the disagreement measure when it emanates from a high β stock. We show below that this aggregate disagreement measure indeed forecasts stock market returns consistent with the predictions of the theory of opinion divergence and short-sales constraints applied to the stock market as a whole. [13]. Our variable, Agg. Dis., has a mean of 3.73 and a standard deviation of 0.79. Our coefficient of interest is a regression of these two dependent variables on Agg. Dis. or various transformations of it.

As covariates for potential alternative explanations, we also include Agg. For. which corresponds to the value*β-weighted average of the mean earnings forecasts for all firms.

[13] Our empirical findings also hold when directly using Yu (2010)'s value-weighted measure but we get stronger statistical significance when using our value*β weighted measure

Price/Earnings is the value-weighted average of the price-to-earnings ratios of firms in the market. Div/Price is the dividend yield of the market. SMB is the monthly return to a portfolio of long small and short big stocks. HML is a the monthly return of a portfolio long low price-to-book stocks and short high price-to-book stocks. TED is the TED spread. Inflation is the yearly inflation rate. VIX is the implied volatility index derived from options on the stock market.

In Figure 2, we plot the time series of the 12-months forward returns to our long top beta decile minus low beta decile portfolio and the time series of the aggregate disagreement. It is easy to see that many of the monthly returns are decidedly negative, consistent with a downward sloping security market line (SML) for a significant fraction of the sample period. There is an remarkable negative correlation between these two time series. A simple correlation of these two series yield a coefficient of -0.4. There are a few highlights from this series. Notice that during the Internet Bubble period of 1996-1999, the low-beta portfolio under-performs the high-beta portfolio since Internet stocks were high beta. But this pattern reverses post-bubble. The aggregate disagreement measure is quite persistent. As such, our empirical analysis ought to look at longer-horizon expected returns. However, the downward sloping SML is not due simply to the observations during this period. It is a more systemic feature of the data. And the correlation between aggregate disagreement and the slope of the SML is also robust to the Internet period as we show below.[14]

We show in Table 2 the estimated relationship of the returns to long the high beta decile and short the low beta decile portfolio on the aggregate disagreement measure. In column (1), we regress the 1-month return on our value*beta weighted aggregate disagreement measure. The coefficient is -0.88 with a t-statistic of -1.3. As we suggested above, we expect our excess return predictability effects to be stronger at longer horizons. But it is comforting to note that the coefficient has the right sign. In column (2), we regress the 3-months forward

[14]For instance, our results are nicely monotonic across quartiles of disagreement, indicating that our effects are not only coming from the Internet bubble months (which supposedly should all be in the top quartile of aggregate disagreement).

returns of our high minus low beta portfolio on the lagged aggregate disagreement measure. The coefficient is -3.6 with a t-statistic of -1.78. Given that the disagreement series is fairly persistent, it is likely we will find stronger results when we look at longer-horizon returns. We see that this is indeed the case in column (3) when we regress the 6-months return on this disagreement measure. It attracts a coefficient of -8.6 with a t-statistic of -3. In column (4), we examine the 12-months return of the low minus high beta portfolio. The coefficient is -18 with a t-statistic of -3.7. The implied economic significance is large. A standard deviation increase in the aggregate dispersion measure (0.79) leads a decrease in the left-hand side variable (-14) that is nearly 52% of a standard deviation of the dependent variable (-14/27). All the standard errors computed in this Table use the Newey-West correction for auto-correlation, allowing for respectively 2, 5 and 11 lags in column (2), (3) and (4).

To see the negative correlation between aggregate disagreement and the 12-months forward returns of our long high beta short low beta portfolio more clearly, in Figure 3, we plot the monthly returns of the high minus low beta portfolio against the monthly observations of the market disagreement measure. One sees again a downward sloping relationship. We also draw the fitted value from the regression model in column (3). This scatter plot suggests that our regression result in column (3) is not being driven by outlier observations.

In columns (5)-(11) of Table 2, we then add in various well-known predictors of market returns as covariates to gauge the robustness of the estimated relationship. In column (5), we control for the value-weighted mean of the analysts' forecasts. This mean does not come in significantly but does reduce the economic coefficient in front of aggregate disagreement to -15, though it is still highly significant. In column (6), we add in Price/Earning and Dividend/Price of the market and find that they do not change our coefficient of interest. Neither do adding in SMB and HML in column (7) though both of these factors do have significant forecasting power for our long-short portfolio's returns.

In columns (8)-(10), we add in variables motivated by earlier work that also might explain the slope of the SML. The idea here is to run a horse race of our disagreement measure against these others and see if our disagreement measure remains significant once we account empirically for these additional explanations. In column (8), we add in TED, which is motivated from the work of Frazzini and Pedersen (2010) who use TED to explain the returns to their BAB factor. Note that our dependent variable of interest is different from their BAB factor but the economics of their analysis captured by TED might also explain the returns to our long-short portfolio. It turns out that adding TED does not change our coefficient of interest. In column (9), we add in the inflation rate as suggested in the analysis of Cohen, Polk, and Vuolteenaho (2005). If anything, it increases the significance of our coefficient of interest. In column (10), we add in the VIX to see if our disagreement factor is simply capturing perhaps time varying risk-aversion or other risks in the market. Again, our coefficient of interest remains unchanged. Note in particular that the number of observations in column (10) drops to 240 as VIX is available only after 1988.

In column (11), rather than fitting aggregate disagreement linearly, we add in dummy variables for quartiles of disagreement to gauge for potential non-linearities in the relationship between our long-short returns and disagreement. The results from column (11) suggest that the linear specification is not a bad one. Each of the quartile dummies attracts a significant coefficient with the magnitude rising proportionally as we increase disagreement. This is particularly reassuring in a time-series test like this one where the results could be mostly driven by one episode, for instance the dot-com bubble. The results in column (11) – and particularly the fact that months in the third quartile of disagreement also experience significantly less upward-sloping security market lines – suggests that this is not the case.

In Table 3, we present the analogous regression results to Table 2 except that our dependent variable of interest is now SML slope. In column (1), the coefficient of interest is -8.9 with a t-statistic of -3.8. A one standard deviation move in aggregate disagreement then decreases the SML slope by -6 or roughly half of a standard deviation of the left hand side variable. The economic magnitude here is comparable to that of Table 2 and provides comfort that our results there are robust. Without belaboring the point, the results in columns

(2)-(8) all point toward the same conclusion as that drawn in Table 2.

In Figure 4, we plot the average 3-months, 6-months and 12-months value-weighted excess of the risk-free returns for stocks in each beta decile for months when the aggregate disagreement is low versus months when the aggregate disagreement is high. The low and high aggregate disagreement month cut-offs are defined as below the sample and above the sample median of aggregate disagreement realizations. The blue dots indicate the low disagreement periods, while the red dots indicate the high disagreement periods. When disagreement is low, the blue dot SML line is upward sloping, consistent with the risk-sharing motive in the CAPM. But when disagreement is high, the SML is hump-shaped, initially increasing and then decreasing with beta consistent with our model.

In Table 4, we show that there is indeed a statistically monotonic upward slope in the SML for low disagreement months. But in high disagreement months, the slope is initially upward sloping but then decreasing. What we are most interested in is the difference between the low and high disagreement months, which is what we documented above. High beta stocks really underperform, both economically and statistically, low beta stocks in high disagreement periods.

In Figure 5, we calculate for the same beta deciles and low and high disagreement periods the average dispersion in analyst earnings forecasts, short interest ratio and share turnover. Note that this dispersion of forecasts is for individual stocks or individual stock disagreement as opposed to aggregate disagreement. The premise of our mechanism is that when aggregate disagreement is high, then dispersion of opinion regarding high beta stocks' cash-flows will be high compared to low beta stocks. Consistent with our model, we find that high beta stocks have higher average dispersion than low beta stocks and even more so in high disagreement periods. They also have a higher short interest ratio and again more so during high disagreement periods. A similar conclusion holds for share turnover. Note however that our analysis of share turnover excludes the years from 2005 and after. The reason is that share turnover of NASDAQ stocks jump significantly during this latter period due to high-frequency trading which makes comparisons of share turnover on beta difficult (see, e.g, Chordia, Roll, and Subrahmanyam (2011))

In Table 5, we examine the results in Figure 5 further by regressing these beta portfolio characteristics (disagreement, short interest, and turnover) onto the beta of the beta portfolios, the beta interacted with low/high disagreement as well as size (the log of the equity value of stocks in each beta portfolio) and size interacted with high and low disagreement. This shows for instance that our variables of interest not only increase with beta (which could be explained by omitted variables such as size), but also that their increase is significantly stronger for months in which disagreement is high. While endogeneity is always a potential concern, it is not clear here that this *relative* differential between low and high disagreement months could be explained by omitted variables. The regressions in Table 5 also control directly for size so that the effects shown in Figure 4 are not driven by size.

4. Conclusion

We show that incorporating the speculative motive for trade into asset pricing models yields strikingly different results from the risk-sharing or liquidity motives. High beta assets are more speculative since they are more sensitive to disagreement about common cash-flows. Hence they experience greater divergence of opinion and in the presence of costly short-selling, they end up being over-priced relative to low beta assets. When the disagreement is low, the SML is upward sloping. As opinions diverge, the slope can be initially positive and then negative for high betas. Empirical tests using security analyst disagreement measures confirm these predictions. We also verify the premise of our speculative mechanism by finding that high beta stocks have much higher individual disagreement about cash-flows, higher shorting and higher share turnover than low beta ones and that these gaps grow with aggregate disagreement. The empirical tests showing how high market beta assets can actually be more speculative and hence yield lower expected returns cuts directly at the heart of what we teach finance students in terms of how to price risk and have potentially important implications for capital budgeting decisions.

5. Appendix

Proof of Theorem 1:

Proof. Consider an equilibrium where group B investors are long on assets $i < \bar{i}$ and hold a short position for assets $i \geq \bar{i}$. The first order conditions for group A agents are simply:

$$\forall i \in [1, N]: \quad (\bar{z} + \lambda)w_i - P_i(1+r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} w_k \mu_k^A\right) w_i \sigma_z^2 + \mu_i^A \sigma_\epsilon^2\right)$$

The first order condition for agents in group B however depends on the assets:

$$\begin{cases} (\bar{z} - \lambda)w_i - P_i(1+r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} w_k \mu_k^B\right) w_i \sigma_z^2 + \mu_i^B \sigma_\epsilon^2\right) & \text{for } i < \bar{i} \\ (\bar{z} - \lambda)w_i - P_i(1+r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} w_k \mu_k^B\right) w_i \sigma_z^2 + \mu_i^B \left(\sigma_\epsilon^2 + \gamma c\right)\right) & \text{for } i \geq \bar{i} \end{cases}$$

We use the market clearing condition ($\frac{\mu_i^A + \mu_i^B}{2} = s_i$) and sum the first-order conditions of agents in group A and B for asset i to obtain:

$$\begin{cases} \bar{z}w_i - P_i(1+r) = \frac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2\right) & \text{for } i < \bar{i} \\ \bar{z}w_i - P_i(1+r) = \frac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2 + \frac{\gamma c}{2}\mu_i^B\right) & \text{for } i \geq \bar{i} \end{cases}$$

Call $S = \sum_{i=1}^{N} w_i \mu_i^B$. We can plug the previous equations into the first order conditions to get:

$$\begin{cases} -\lambda w_i + \frac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2\right) = \frac{1}{\gamma}\left(S w_i \sigma_z^2 + \mu_i^B \sigma_\epsilon^2\right) & \text{for } i < \bar{i} \\ -\lambda w_i + \frac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2 + \frac{\gamma c}{2}\mu_i^B\right) = \frac{1}{\gamma}\left(S w_i \sigma_z^2 + \mu_i^B \left(\sigma_\epsilon^2 + \gamma c\right)\right) & \text{for } i \geq \bar{i} \end{cases}$$

Which can be rewritten as:

$$\begin{cases} -\lambda w_i + \frac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2\right) = \frac{1}{\gamma}\left(S w_i \sigma_z^2 + \mu_i^B \sigma_\epsilon^2\right) & \text{for } i < \bar{i} \\ -\lambda w_i + \frac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2\right) = \frac{1}{\gamma}\left(S w_i \sigma_z^2 + \mu_i^B \left(\sigma_\epsilon^2 + \frac{\gamma c}{2}\right)\right) & \text{for } i \geq \bar{i} \end{cases} \tag{15}$$

Call $\sigma_c^2 = \sigma_\epsilon^2 + \frac{\gamma c}{2}$.From the previous expression, we can compute S:

$$S = 1 - \frac{\left(1 - \frac{\sigma_\epsilon^2}{\sigma_c^2}\right)\left(\sum_{i \geq \bar{i}} w_i s_i\right) + \lambda\gamma\left(\sum_{i<\bar{i}} \frac{w_i^2}{\sigma_\epsilon^2} + \sum_{i \geq \bar{i}} \frac{w_i^2}{\sigma_c^2}\right)}{1 + \sigma_z^2\left(\sum_{i<\bar{i}} \frac{w_i^2}{\sigma_\epsilon^2} + \sum_{i \geq \bar{i}} \frac{w_i^2}{\sigma_c^2}\right)}$$

We can use equations 15 to get an expression for the holdings of agents in group B.

$$\mu_i^B = \begin{cases} s_i + w_i \left(\dfrac{\sigma_z^2 \left(1 - \frac{\sigma_\epsilon^2}{\sigma_c^2}\right) \left(\sum_{i \geq \bar{i}} w_i s_i\right) - \lambda\gamma}{\sigma_\epsilon^2 + \sigma_z^2 \left(\sum_{i<\bar{i}} w_i^2 + \frac{\sigma_\epsilon^2}{\sigma_c^2} \sum_{i\geq \bar{i}} w_i^2\right)} \right) & \text{for } i < \bar{i} \\ \dfrac{\sigma_\epsilon^2}{\sigma_c^2} s_i + \dfrac{\sigma_\epsilon^2}{\sigma_c^2} w_i \left(\dfrac{\sigma_z^2 \left(1 - \frac{\sigma_\epsilon^2}{\sigma_c^2}\right) \left(\sum_{i \geq \bar{i}} w_i s_i\right) - \lambda\gamma}{\sigma_\epsilon^2 + \sigma_z^2 \left(\sum_{i<\bar{i}} w_i^2 + \frac{\sigma_\epsilon^2}{\sigma_c^2} \sum_{i\geq \bar{i}} w_i^2\right)} \right) & \text{for } i \geq \bar{i} \end{cases}$$

Finally, the price of assets depend on whether they are sold short or not:

$$P_i(1+r) = \begin{cases} \bar{z} w_i - \dfrac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2\right) & \text{for } i < \bar{i} \\ \bar{z} w_i - \dfrac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2\right) - \dfrac{1}{\gamma} \dfrac{\sigma_c^2 - \sigma_\epsilon^2}{\sigma_c^2} \sigma_\epsilon^2 \left(s_i + w_i \left(\dfrac{\sigma_z^2 \left(1 - \frac{\sigma_\epsilon^2}{\sigma_c^2}\right) \left(\sum_{i \geq \bar{i}} w_i s_i\right) - \lambda\gamma}{\sigma_\epsilon^2 + \sigma_z^2 \left(\sum_{i<\bar{i}} w_i^2 + \frac{\sigma_\epsilon^2}{\sigma_c^2} \sum_{i\geq \bar{i}} w_i^2\right)} \right)\right) & \text{for } i \geq \bar{i} \end{cases}$$

Introduce $\theta = \frac{\sigma_\epsilon^2}{\sigma_c^2}$. θ decreases with the cost of shorting from 1 when there is no shorting cost to 0 when the shorting cost is infinite. The previous expression can be rewritten as:

$$P_i(1+r) = \begin{cases} \bar{z} w_i - \dfrac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2\right) & \text{for } i < \bar{i} \\ \bar{z} w_i - \dfrac{1}{\gamma}\left(w_i \sigma_z^2 + s_i \sigma_\epsilon^2\right) - \dfrac{1}{\gamma} (1-\theta) \sigma_\epsilon^2 \left(s_i + w_i \left(\dfrac{\sigma_z^2 (1-\theta) \left(\sum_{i \geq \bar{i}} w_i s_i\right) - \lambda\gamma}{\sigma_\epsilon^2 + \sigma_z^2 \left(\sum_{i<\bar{i}} w_i^2 + \theta \sum_{i\geq \bar{i}} w_i^2\right)} \right)\right) & \text{for } i \geq \bar{i} \end{cases}$$

The marginal asset is asset $\bar{i}$ if and only if $\mu_{\bar{i}}^B < 0$ and $\mu_{\bar{i}-1}^B > 0$. These conditions can be expressed in the following manner:

$$\begin{aligned} & \tfrac{s_{\bar{i}}}{w_{\bar{i}}} \left(\sigma_\epsilon^2 + \sigma_z^2 \left(\sum_{i<\bar{i}} w_i^2 + \theta \sum_{i\geq\bar{i}} w_i^2\right)\right) + \sigma_z^2 (1-\theta) \left(\sum_{i\geq\bar{i}} w_i s_i\right) < \lambda\gamma \\ \text{and} \quad & \lambda\gamma < \tfrac{s_{\bar{i}-1}}{w_{\bar{i}-1}} \left(\sigma_\epsilon^2 + \sigma_z^2 \left(\sum_{i<\bar{i}} w_i^2 + \theta \sum_{i\geq\bar{i}} w_i^2\right)\right) + \sigma_z^2 (1-\theta) \left(\sum_{i\geq\bar{i}} w_i s_i\right) \end{aligned}$$

Call $u_k = \frac{s_k}{w_k} \left(\sigma_\epsilon^2 + \sigma_z^2 \left(\sum_{i<\bar{i}} w_i^2 + \theta \sum_{i\geq\bar{i}} w_i^2\right)\right) + \sigma_z^2 (1-\theta) \left(\sum_{i\geq\bar{i}} w_i s_i\right)$. Clearly, u_k is a strictly decreasing sequence:

$$u_k - u_{k-1} = \underbrace{\left(\frac{s_k}{w_k} - \frac{s_{k-1}}{w_{k-1}}\right)}_{<0} \left(\sigma_\epsilon^2 + \sigma_z^2 \left(\sum_{i<\bar{i}} w_i^2 + \theta \sum_{i\geq\bar{i}} w_i^2\right)\right) < 0$$

Define $u_0 = +\infty$ and $u_{N+1} = 0$. Then the sequence $(u_i)_{i\in[0,N+1]}$ spans $\mathbb{R}^+$ and the marginal asset is asset $\bar{i}$ provided that:

$$u_{\bar{i}} < \lambda\gamma < u_{\bar{i}-1}$$

Overpricing for assets $i \geq \bar{i}$ is simply defined as the difference between the equilibrium price and the price that would prevail in the absence of heterogenous beliefs and short sales costs:

$$M = \text{mispricing} = \frac{1}{\gamma}\left(1-\theta\right)\sigma_\epsilon^2\left(w_i\left(\frac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) - s_i\right)$$

That mispricing is positive comes directly from the fact that $\lambda\gamma > u_{\bar{i}} > u_i$ for $i \geq \bar{i}$. We now show that mispricing is increasing with c.

$$\begin{aligned}
\frac{\gamma}{\sigma_\epsilon^2}\frac{\partial \mathrm{M}}{\partial\theta} &= -w_i\left(\frac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) + s_i + \\
&\quad (1-\theta)w_i\frac{\sigma_z^2\left(\sum_{i\geq\bar{i}} w_i s_i\right)\left(\sigma_\epsilon^2 + \sigma_z^2\sum_{i<\bar{i}} w_i^2\right) - \sigma_z^2\left(\sum_{i\geq\bar{i}} w_i^2\right)\left(\lambda\gamma - \sigma_z^2\sum_{i\geq\bar{i}} w_i s_i\right)}{\left(\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)\right)^2} \\
&= s_i + \frac{\sigma_\epsilon^2 + \sigma_z^2\sum_{i=1}^N w_i^2}{\left(\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)\right)^2} w_i\left(-\lambda\gamma + (1-\theta)\sigma_z^2\sum_{i\geq\bar{i}} w_i s_i\right) + \frac{(1-\theta)\sigma_z^2\sum_{i\geq\bar{i}} w_i s_i}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)} w_i
\end{aligned}$$

We know by definition of the equilibrium that $\lambda\gamma > u_{\bar{i}}$. This implies that:

$$\begin{aligned}
\frac{\gamma}{\sigma_\epsilon^2}\frac{\partial \mathrm{M}}{\partial\theta} &< s_i - w_i\frac{s_{\bar{i}}}{w_{\bar{i}}}\frac{\sigma_\epsilon^2 + \sigma_z^2\sum_{i=1}^N w_i^2 - (1-\theta)\sigma_z^2\frac{w_{\bar{i}}}{s_{\bar{i}}}\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)} \\
&= s_i - w_i\frac{s_{\bar{i}}}{w_{\bar{i}}}\left(1 + \sigma_z^2(1-\theta)\frac{\sum_{i\geq\bar{i}} w_i^2 - \frac{w_{\bar{i}}}{s_{\bar{i}}}\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right)
\end{aligned}$$

We know that $\forall i \geq \bar{i}$, $w_i > \frac{w_{\bar{i}}}{s_{\bar{i}}} s_i$, so that: $\sum_{i\geq\bar{i}} w_i^2 > \frac{w_{\bar{i}}}{s_{\bar{i}}}\left(\sum_{i\geq\bar{i}} w_i s_i\right)$. Thus:

$$1 + \sigma_z^2(1-\theta)\frac{\sum_{i\geq\bar{i}} w_i^2 - \frac{w_{\bar{i}}}{s_{\bar{i}}}\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)} > 1$$

And:

$$\frac{\gamma}{\sigma_\epsilon^2}\frac{\partial \mathrm{M}}{\partial\theta} < s_i - w_i\frac{s_{\bar{i}}}{w_{\bar{i}}} < 0 \quad \text{as } i \geq \bar{i}$$

Because θ decreases with c, this proves that mispricing increases with c, the cost of shorting. Finally, that mispricing increases with λ is evident from the above definition of mispricing. □

Proof of Corollary 1:

Proof. Note that $\beta_i = \frac{w_i\sigma_z^2 + s_i\sigma_\epsilon^2}{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}$, so that $w_i = \beta_i\frac{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}{\sigma_z^2} - s_i\frac{\sigma_\epsilon^2}{\sigma_z^2}$. We can rewrite the pricing equations in terms of expected returns:

$$\begin{cases} \mathbb{E}[R_i] = \dfrac{w_i\sigma_z^2 + s_i\sigma_\epsilon^2}{\gamma} \quad \text{for } i < \bar{i} \\ \mathbb{E}[R_i] = \dfrac{w_i\sigma_z^2 + s_i\sigma_\epsilon^2}{\gamma} - \dfrac{\sigma_\epsilon^2}{\gamma}(1-\theta)\left[\dfrac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)} w_i - s_i\right] \quad \text{for } i \geq \bar{i} \end{cases}$$

$$\begin{cases} \mathbb{E}[R_i] = \dfrac{w_i\sigma_z^2 + s_i\sigma_\epsilon^2}{\gamma} \quad \text{for } i < \bar{i} \\ \mathbb{E}[R_i] = \dfrac{w_i\sigma_z^2}{\gamma}\left(1 - \dfrac{\sigma_\epsilon^2}{\sigma_z^2}(1-\theta)\dfrac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) + \dfrac{s_i\sigma_\epsilon^2}{\gamma}(2-\theta) \quad \text{for } i \geq \bar{i} \end{cases}$$

Using the definition of β_i, this can be rewritten as:

$$\begin{cases} \mathbb{E}[R_i] = \beta_i \dfrac{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}{\gamma} \quad \text{for } i < \bar{i} \\ \mathbb{E}[R_i] = \beta_i \dfrac{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}{\gamma}\left(1 - \dfrac{\sigma_\epsilon^2}{\sigma_z^2}(1-\theta)\dfrac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) + s_i\dfrac{\sigma_\epsilon^2}{\gamma}(1-\theta)\left(1 + \dfrac{\sigma_\epsilon^2}{\sigma_z^2}\dfrac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) \quad \text{for } i \geq \bar{i} \end{cases}$$

$\square$

Proof of Corollary 2:

Proof. We can write the actual returns as:

$$\tilde{R}_i = \begin{cases} \beta_i \dfrac{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}{\gamma} + \tilde{\eta_i} \quad \text{for } i < \bar{i} \\ \beta_i \dfrac{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}{\gamma}\left(1 - \dfrac{\sigma_\epsilon^2}{\sigma_z^2}(1-\theta)\dfrac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) + s_i\dfrac{\sigma_\epsilon^2}{\gamma}(1-\theta)\left(1 + \dfrac{\sigma_\epsilon^2}{\sigma_z^2}\dfrac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) + \tilde{\eta_i} \quad \text{for } i \geq \bar{i} \end{cases}$$

where $\tilde{\eta_i} = w_i\tilde{u} + \tilde{\epsilon_i}$ and $\tilde{u} = \tilde{z} - \bar{z}$.

$$\tilde{R}_i = \begin{cases} \beta_i \dfrac{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}{\gamma}\left(1 + \dfrac{\gamma}{\sigma_z^2}\tilde{u}\right) - s_i\dfrac{\sigma_\epsilon^2}{\sigma_z^2}\tilde{u} + \tilde{\epsilon_i} \quad \text{for } i < \bar{i} \\ \beta_i \dfrac{\sigma_z^2 + \left(\sum_{i=1}^N s_i^2\right)\sigma_\epsilon^2}{\gamma}\left(1 - \dfrac{\sigma_\epsilon^2}{\sigma_z^2}(1-\theta)\dfrac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)} + \dfrac{\gamma}{\sigma_z^2}\tilde{u}\right) + s_i\dfrac{\sigma_\epsilon^2}{\gamma}(1-\theta)\left(1 + \dfrac{\sigma_\epsilon^2}{\sigma_z^2}\dfrac{\lambda\gamma - \sigma_z^2\left(1-\theta\right)\left(\sum_{i\geq\bar{i}} w_i s_i\right)}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta\sum_{i\geq\bar{i}} w_i^2\right)}\right) - s_i\dfrac{\sigma_\epsilon^2}{\sigma_z^2}\tilde{u} + \tilde{\epsilon_i} \quad \text{f} \end{cases}$$

From now on, assume to simplify the notations that $s_i = \frac{1}{N}$. Note that assuming here that $s_i = \frac{1}{N}$ is without loss of generality. A similar proof could be obtained with general s_i. Our assumption that $\sum_{i=1}^N \beta_i s_i = 1$ becomes $\sum_{i=1}^N \beta_i = N$.

Call $\kappa(\theta,\lambda)=\frac{\lambda\gamma-\sigma_z^2(1-\theta)\left(\sum_{i\geq\bar{i}}\frac{w_i}{N}\right)}{\sigma_\epsilon^2+\sigma_z^2\left(\sum_{i<\bar{i}}w_i^2+\theta\sum_{i\geq\bar{i}}w_i^2\right)}$. The realized returns can be expressed as follows:

$$\tilde{R}_i=\begin{cases}\beta_i\dfrac{\sigma_z^2+\frac{\sigma_\epsilon^2}{N}}{\gamma}\left(1+\dfrac{\gamma}{\sigma_z^2}\tilde{u}\right)-\dfrac{\sigma_\epsilon^2}{\sigma_z^2}\dfrac{\tilde{u}}{N}+\tilde{\epsilon_i}\quad\text{for } i<\bar{i}\\ \beta_i\dfrac{\sigma_z^2+\frac{\sigma_\epsilon^2}{N}}{\gamma}\left(1+\dfrac{\gamma}{\sigma_z^2}\tilde{u}-\dfrac{\sigma_\epsilon^2}{\sigma_z^2}(1-\theta)\kappa(\theta,\lambda)\right)+\dfrac{\sigma_\epsilon^2}{\gamma N}(1-\theta)\left(1+\dfrac{\sigma_\epsilon^2}{\sigma_z^2}\kappa(\theta,\lambda)\right)-\dfrac{\sigma_\epsilon^2}{\sigma_z^2}\dfrac{\tilde{u}}{N}+\tilde{\epsilon_i}\quad\text{for } i\geq\bar{i}\end{cases}$$

A cross-sectional regression of realized returns $\tilde{R}_i$ on β_i and a constant would deliver the following coefficient estimate:

$$\begin{aligned}\hat{\mu}&=\frac{\sum_{i=1}^N\beta_i\tilde{R}_i-\sum_{i=1}^N\tilde{R}_i}{\sum_{i=1}^N\beta_i^2-N}\\&=\frac{\sigma_z^2+\frac{\sigma_\epsilon^2}{N}}{\gamma}\left(1+\frac{\gamma}{\sigma_z^2}\tilde{u}-\left(\frac{\sum_{i\geq\bar{i}}\beta_i^2-\sum_{i\geq\bar{i}}\beta_i}{\sum_{i=1}^N\beta_i^2-N}\right)\frac{\sigma_\epsilon^2}{\sigma_z^2}(1-\theta)\kappa(\theta,\lambda)\right)\\&\quad+\left(\frac{\frac{1}{N}\sum_{i\geq\bar{i}}\beta_i-\frac{N-\bar{i}+1}{N}}{\sum_{i=1}^N\beta_i^2-N}\right)\frac{\sigma_\epsilon^2}{\gamma}(1-\theta)\left(1+\frac{\sigma_\epsilon^2}{\sigma_z^2}\kappa(\theta,\lambda)\right)\end{aligned}$$

Let $\frac{u_{\bar{i}-1}}{\gamma}>\lambda_1>\lambda_2>\frac{u_{\bar{i}}}{\gamma}$. Call $\bar{i}_1$ ($\bar{i}_2$) the threshold associated with disagreement λ_1 (resp. λ_2). We have that $\bar{i}_1=\bar{i_2}=\bar{i}$. Then:

$$\begin{aligned}\hat{\mu}(\lambda_1)-\hat{\mu}(\lambda_2)&=\frac{(1-\theta)\sigma_\epsilon^2(\lambda_1-\lambda_2)}{\sigma_\epsilon^2+\sigma_z^2\left(\sum_{i<\bar{i}}w_i^2+\theta\sum_{i\geq\bar{i}}w_i^2\right)}\left(-\frac{\sigma_z^2+\frac{\sigma_\epsilon^2}{N}}{\sigma_z^2}\frac{\sum_{i\geq\bar{i}}\beta_i^2-\sum_{i\geq\bar{i}}\beta_i}{\sum_{i=1}^N\beta_i^2-N}+\frac{\sigma_\epsilon^2}{\sigma_z^2}\left(\frac{\frac{1}{N}\sum_{i\geq\bar{i}}\beta_i-\frac{N-\bar{i}+1}{N}}{\sum_{i=1}^N\beta_i^2-N}\right)\right)\\&=\frac{(1-\theta)\sigma_\epsilon^2(\lambda_1-\lambda_2)}{\sigma_z^2\left(\sigma_\epsilon^2+\sigma_z^2\left(\sum_{i<\bar{i}}w_i^2+\theta\sum_{i\geq\bar{i}}w_i^2\right)\right)\left(\sum_{i=1}^N\beta_i^2-N\right)}\left(-\left(\sigma_z^2+\frac{\sigma_\epsilon^2}{N}\right)\left(\sum_{i\geq\bar{i}}^N\beta_i^2-\sum_{i\geq\bar{i}}^N\beta_i\right)+\sigma_\epsilon^2\left(\frac{1}{N}\sum_{i\geq\bar{i}}^N\beta_i-\frac{N-\bar{i}+1}{N}\right)\right)\\&=-\frac{(1-\theta)\sigma_\epsilon^2(\lambda_1-\lambda_2)}{\sigma_z^2\left(\sigma_\epsilon^2+\sigma_z^2\left(\sum_{i<\bar{i}}w_i^2+\theta\sum_{i\geq\bar{i}}w_i^2\right)\right)\left(\sum_{i=1}^N\beta_i^2-N\right)}\left(\sigma_z^2\left(\sum_{i\geq\bar{i}}^N\beta_i^2-\sum_{i\geq\bar{i}}^N\beta_i\right)+\frac{\sigma_\epsilon^2}{N}\sum_{i\geq\bar{i}}^N(\beta_i-1)^2\right)\end{aligned}$$

We show that $\sum_{i\geq\bar{i}}\beta_i^2\geq\sum_{i\geq\bar{i}}\beta_i$. Call $\beta_i=1+y_i$ with y_i such that $\sum y_i=0$. Then: $\sum_{i=1}^N\beta_i^2=N+2\underbrace{\sum_{i=1}^N y_i}_{=0}+\sum_{i=1}^N y^2>N=\sum_{i=1}^N\beta_i$. Thus, the relationship is true for $\bar{i}=0$. Now assume it is true for $\bar{i}=k$. We have: $\sum_{i\geq k+1}\beta_i^2-\sum_{i\geq k+1}\beta_i=\sum_{i\geq k}\beta_i^2-\sum_{i\geq k}\beta_i+\beta_k-\beta_k^2$. Either $\beta_k>1$ in which case it is evident that $\sum_{i\geq k+1}\beta_i^2-\sum_{i\geq k+1}\beta_i>0$ as $\beta_i>1$ for $i\geq k$. Or $\beta_k\leq 1$ in which case $\beta_k-\beta_k^2>0$ and using the recurrence assumption, $\sum_{i\geq k+1}\beta_i^2-\sum_{i\geq k+1}\beta_i>0$. Thus, $\hat{\mu}(\lambda_1)-\hat{\mu}(\lambda_2)<0$.

Moreover, we show now that $\hat{\mu}(\lambda)$ is continuous at u_i, for all $i\in[1,N]$. When $\lambda=\left(\frac{u_i}{\gamma}\right)^+$, we have $\bar{i}=i$.

When $\lambda = \left(\frac{u_i}{\gamma}\right)^-$, we have $\bar{i} = i+1$. Thus:

$$\begin{aligned}\hat{\mu}\left(\left(\frac{u_i}{\gamma}\right)^+\right) - \hat{\mu}\left(\left(\frac{u_i}{\gamma}\right)^-\right) &= (\beta_{\bar{i}} - 1)\frac{\frac{\sigma_\epsilon^2}{\gamma}(1-\theta)}{\sum_{i=1}^N \beta_i^2 - N}\left(\frac{1}{N}\left(1 + \frac{\sigma_\epsilon^2}{\sigma_z^2}\kappa(\theta, u_{\bar{i}})\right) - \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\sigma_z^2}\kappa(\theta, u_{\bar{i}})\beta_{\bar{i}}\right) \\ &= (\beta_{\bar{i}} - 1)\frac{\frac{\sigma_\epsilon^2}{\gamma}(1-\theta)}{\sum_{i=1}^N \beta_i^2 - N}\left(\frac{1}{N} - w_{\bar{i}}\kappa(\theta, u_{\bar{i}})\right)\end{aligned}$$

But: $\kappa(\theta, u_{\bar{i}}) = \frac{1}{N w_{\bar{i}}}$ so that $\hat{\mu}\left(\left(\frac{u_i}{\gamma}\right)^+\right) = \hat{\mu}\left(\left(\frac{u_i}{\gamma}\right)^-\right)$. Thus $\hat{\mu}$ is strictly decreasing with λ, the aggregate disagreement.

Going back to the expression for $\hat{\mu}(\lambda_1) - \hat{\mu}(\lambda_2)$, we see that this difference can be expressed as $-C \times \frac{1-\theta}{\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{i<\bar{i}} w_i^2 + \theta \sum_{i \geq \bar{i}} w_i^2\right)}$ with $C > 0$, so that it is clearly increasing with θ. Thus, when c increases, θ decreases, and the difference between $\hat{\mu}(\lambda_1)$ and $\hat{\mu}(\lambda_2)$ decreases. Because this difference is strictly negative, this means that the gap between the two slopes becomes wider.

□

Proof of Proposition 1:

Proof. First, using the new parameterization of the model ($s_i = \frac{1}{N+1}$ and $w_i = \frac{2i}{N}$, and assuming that $c = 0$, i.e. $\theta = 1$, it is direct to show that:

$$\forall i > 0, \quad u_i = \frac{N}{N+1}\frac{\sigma_\epsilon^2}{2i} + \sigma_z^2\left(1 - \frac{i^2 - 1}{3N(N+1)}\right)$$

As in the general case, u_i is clearly a strictly decreasing sequence and $u_N \rightarrow_{N\rightarrow\infty} \frac{2}{3}\sigma_z^2$ and $u_1 \rightarrow_{N\rightarrow\infty} \sigma_z^2 + \frac{\sigma_\epsilon^2}{2}$. Trivially, if $\lambda\gamma < \frac{2}{3}\sigma_z^2$ then there are no assets shorted at equilibrium when $N \rightarrow \infty$ and asset prices can be written using the usual CAPM formula. Symmetrically, if $\lambda\gamma > \sigma_z^2 + \frac{\sigma_\epsilon^2}{2}$, then when $N \rightarrow \infty$ all assets are shorted at equilibrium and the speculative premium applies to all assets. In particular, in this limiting case, $\beta_i = w + i$, $s_i = 0$ and $\bar{j} = 1$ so that the speculative premium has the following expression:

$$premium = \frac{\sigma_\epsilon^2}{\gamma}\beta_i\frac{\lambda\gamma - \sigma_z^2}{\sigma_\epsilon^2} = \frac{\beta_i}{\gamma}\left(\lambda\gamma - \sigma_z^2\right)$$

This leads to the following expression for expected returns:

$$\mathbb{E}[\tilde{R}_i] = \beta_i\left(\frac{2\sigma_z^2}{\gamma} - \lambda\right)$$

Consider now one intermediate case where $\sigma_z^2 < \lambda\gamma < \sigma_z^2 + \frac{\sigma_\epsilon^2}{2}$. Then there exists a finite $\bar{i} > 0$ such that $\lim_{N\rightarrow\infty} u_i < \lambda\gamma < \lim_{N\rightarrow\infty} u_{i-1}$, i.e. there are a finite number of assets that are not shorted at equilibrium. But then, at the limit $\lim_{N\rightarrow\infty} \frac{\bar{i}}{N} = 0$, so that the speculative premium on asset i has the same expression as in the case where all assets are shorted (at the limit, asset prices are the same whether there are a finite number of assets shorted or no assets shorted at all). The equilibrium expected returns are thus again:

$$\mathbb{E}[\tilde{R}_i] = \beta_i \left(\frac{2\sigma_z^2}{\gamma} - \lambda \right)$$

The final intermediate case is when $\frac{2}{3}\sigma_z^2 < \lambda\gamma < \sigma_z^2$. In this case, it needs to be the case that $\bar{i} \to \infty$ as $N \to \infty$ when $N \to \infty$, otherwise we know that for all i $u_i > \sigma_z^2$ at the limit so that $\lambda\gamma$ would have to be $> \sigma_z^2$, which would be a contradiction. Call $x = \lim_{N\to\infty} \frac{2i}{N}$ the fraction of assets that are shorted at equilibrium. It is easily seen that $x = 3\sqrt{\sigma_z^2 - \lambda\gamma}$. More importantly, the speculative premium now goes down to zero. This is because the speculative premium can be written in the limit as:

$premium \approx_{n\to\infty} \frac{\sigma_\epsilon^2}{\gamma}\beta_i \left(\frac{\lambda\gamma - \sigma_z^2(1-x^2)}{\sigma_\epsilon^2 + \sigma_z^2 x^3 N} \right) \to_{N\to\infty} 0$ $\square$

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Table 1: Summary Statistics

	Mean	Median	Std. Dev.	25^{th} p	75^{th} p.	Min	Max	Obs.
SML slope	1.99	2.91	13.72	-7.46	10.00	-40.25	47.11	337
3-months Δ ret.	1.37	1.90	12.87	-5.35	8.27	-37.78	52.06	346
6-months Δ ret.	2.07	3.44	18.28	-7.89	13.40	-60.92	70.70	343
12-months Δ ret.	4.14	6.43	27.31	-12.27	20.51	-82.02	81.12	337
Agg. Dis	3.73	3.52	0.79	3.11	4.14	2.68	6.14	349
Agg. For	14.78	14.04	3.06	13.26	15.54	9.77	27.10	349
Price/Earnings	22.12	19.00	15.44	14.69	25.59	7.48	123.79	349
Div./Price	0.03	0.03	0.01	0.02	0.04	0.01	0.06	349
SMB	0.14	-0.03	3.19	-1.61	1.84	-16.62	22.06	349
HML	0.37	0.34	3.13	-1.38	1.91	-12.87	13.88	349
TED	0.56	0.40	0.56	0.22	0.68	0.03	4.62	348
Inflation	3.07	3.00	1.42	2.23	3.92	-2.10	8.39	348
V IX	20.40	19.49	7.87	14.12	24.44	10.42	59.89	252

Note: SML slope is the coefficient estimate of a monthly OLS regression of the 10 beta-portfolios returns on each portfolio beta. 3-months Δ ret. (resp. 6 and 12) is the 3 months (resp. 6 and 12) forward returns of a value-weighted portfolio long in stocks in the top decile of betas and short in stocks in the bottom decile of beta. Agg. Dis. is the value/β-weighted sum of stock level dispersion measured as the standard deviation of analyst forecasts on the stock. Agg. For. is the value/β-weighted sum of analyst forecast on the stock. TED is the TED spread and Inflation is the yearly inflation rate.

Table 2: Disagreement and the failure of CAPM: portfolio returns

	Returns of value-weighted low vs. high beta portfolio										
	1-month	3-months	6-months	12 months							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Aggregate disagreement	-.88	-3.6**	-8.5***	-18***	-15***	-18***	-18***	-18***	-21***	-21***	
	(-1.3)	(-2.4)	(-3)	(-3.7)	(-2.7)	(-3.3)	(-3.2)	(-3.5)	(-4.3)	(-4.9)	
Disagreement Q2											-12*
											(-1.9)
Disagreement Q3											-18***
											(-3)
Disagreement Q4											-28***
											(-3)
Aggregate Forecast					-.95	-1.6	-1.5	-1.4	-.56	-.94	-2.2
					(-.58)	(-1.1)	(-1)	(-1)	(-.4)	(-.74)	(-1.6)
Price/Earning						-.16	-.075	-.03	-.099	-.87	-.27
						(-.29)	(-.14)	(-.063)	(-.21)	(-1.6)	(-.53)
Dividend/Price						-766*	-697*	-698*	-557	-283	-563
						(-1.9)	(-1.8)	(-1.7)	(-1.4)	(-.44)	(-1.4)
SMB							-.91	-.88	-.79	-.51	-.93*
							(-1.6)	(-1.6)	(-1.6)	(-1.2)	(-1.7)
HML							-1.5**	-1.5**	-1.3***	-1.6***	-1.6***
							(-2.5)	(-2.5)	(-2.6)	(-2.9)	(-2.7)
TED								1.6	4.9	-16*	1.9
								(.36)	(1.1)	(-1.9)	(.46)
Inflation Rate									-4.2	-4.5*	-3.4
									(-1.5)	(-1.9)	(-1.2)
VIX										1.8***	
										(3.9)	
Constant	3.8	15***	34***	70***	75***	121***	113***	112***	120***	111***	83***
	(1.6)	(2.9)	(3.5)	(4.1)	(4.1)	(4.5)	(4.2)	(4.3)	(5.2)	(3.5)	(2.8)
Observations	348	346	343	337	337	337	337	336	336	240	336

Notes: OLS estimation with Newey-West adjusted standard-errors allowing for 3 (column 2), 5 (column 3) and 11 lags (column 4 to 10). t-statistics are in parenthesis. The dependent variable is the 12 month forward value-weighted return of a portfolio that is long in the top decile beta portfolio and short in the bottom decile beta portfolio. Aggregate disagreement is the value/β weighted average of stock-level disagreement. Disagreement Q2, Q3 and Q4 are quartiles of monthly disagreement. Inflation rate is the past year CPI growth rate. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table 3: Disagreement and the failure of CAPM: the slope of the security market line

	Slope of Security Market Line							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Aggregate disagreement	-8.9***	-7.8***	-8.9***	-8.6***	-9.2***	-11***	-11***	
	(-3.8)	(-2.7)	(-3.2)	(-3.1)	(-3.5)	(-4.4)	(-4.4)	
Disagreement Q2								-5.8**
								(-2.2)
Disagreement Q3								-11***
								(-3.6)
Disagreement Q4								-14***
								(-3.1)
Aggregate Forecast		-.41	-.76	-.7	-.61	-.1	-.35	-1.1
		(-.49)	(-.99)	(-.91)	(-.82)	(-.14)	(-.55)	(-1.4)
Price/Earnings			-.27	-.23	-.18	-.22	-.66**	-.31
			(-.9)	(-.78)	(-.71)	(-.88)	(-2.2)	(-1.2)
Dividend/Price			-459**	-427**	-428**	-343*	-322	-371*
			(-2.2)	(-2.1)	(-2.1)	(-1.8)	(-.93)	(-1.8)
SMB				-.47	-.44	-.39	-.28	-.46
				(-1.5)	(-1.4)	(-1.4)	(-1.2)	(-1.5)
HML				-.65**	-.62*	-.51*	-.69**	-.65**
				(-2)	(-1.9)	(-1.9)	(-2.6)	(-2.1)
TED					1.6	3.6	-8.7**	2.1
					(.7)	(1.5)	(-2.5)	(1)
Inflation Rate						-2.6*	-2.6**	-2
						(-1.8)	(-2.1)	(-1.4)
VIX							1***	
							(4.9)	
Constant	35***	37***	65***	61***	60***	65***	62***	48***
	(4.3)	(4.2)	(4.6)	(4.3)	(4.4)	(5.2)	(3.4)	(3)
Observations	337	337	337	337	336	336	240	336

Notes: OLS estimation with Newey-West adjusted standard-errors allowing for 11 lags. t-statistics are in parenthesis. The dependent variable is the coefficient estimate of a monthly regression of the 10 beta portfolio 12 months forward excess returns on the portfolio beta. Aggregate disagreement is the value/β weighted average of stock-level disagreement. Disagreement Q2, Q3 and Q4 are quartiles of monthly disagreement. Inflation rate is the past year CPI growth rate. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table 4: Non-linearity in Mean Returns with Decile β portfolios

	12 months excess return of i^{th} decile β portfolio against bottom decile								
	i=2	i=3	i=4	i=5	i=6	i=7	i=8	i=9	i=10
Low Disagreement Months									
	3.7***	5.2***	5.8***	7.3***	7.5***	9.3***	9.7***	10***	11***
	(4.3)	(5.8)	(5.8)	(7.6)	(7.3)	(8.3)	(8.2)	(8.2)	(7.7)
Observations	175	175	175	175	175	175	175	175	175
High Disagreement Months									
	1.5*	3.6***	2.4**	.62	1.2	.77	-.31	-.84	-3.8
	(1.7)	(4.4)	(2.6)	(.66)	(1)	(.71)	(-.24)	(-.48)	(-1.5)
Observations	162	162	162	162	162	162	162	162	162

Notes: 12-months value-weighted excess returns of a portfolio long the i^th decile β portfolio and short the bottom decile, for i=1 to 10. Aggregate disagreement is the value/β-weighted sum of stock level disagreement. High/Low Disagreement is a dummy equal to 1 for months with above/below median aggregate disagreement. Standard errors in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table 5: β portfolios characteristics and disagreement

	Short Interest	Stock Disagreement	Turnover
	(1)	(2)	(3)
High Disagreement	.04***	-3.3***	-1***
	(13)	(-9.2)	(-9.4)
β	.0078***	1***	.3***
	(16)	(34)	(12)
$\beta \times$ High Disagreement	.012***	1.2***	.14***
	(14)	(19)	(3.7)
Size	.00081***	-.23***	.0054
	(7.3)	(-18)	(.84)
Size $\times$ High Disagreement	-.0024***	.19***	.071***
	(-12)	(8.6)	(9)
Constant	-.0062***	5.7***	.21**
	(-4.5)	(29)	(2.6)
Observations	2,490	3,490	2,890

Notes: OLS estimation of monthly β portfolio characteristics on β, size and β and size interacted with aggregate disagreement. Aggregate disagreement is the value/β-weighted sum of stock level disagreement. High Disagreement is a dummy equal to 1 for months with above median aggregate disagreement. Dependent variables are: the value-weighted average monthly short interest ratio (column 1), the value-weighted average stock-level disagreement (column 2), the value-weighted turnover (before 2006 – column 3). β is the value-weighted average β of each 10-β portfolio. Size is the logarithm of the average equity value of stocks in each of the 10-β portfolio. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

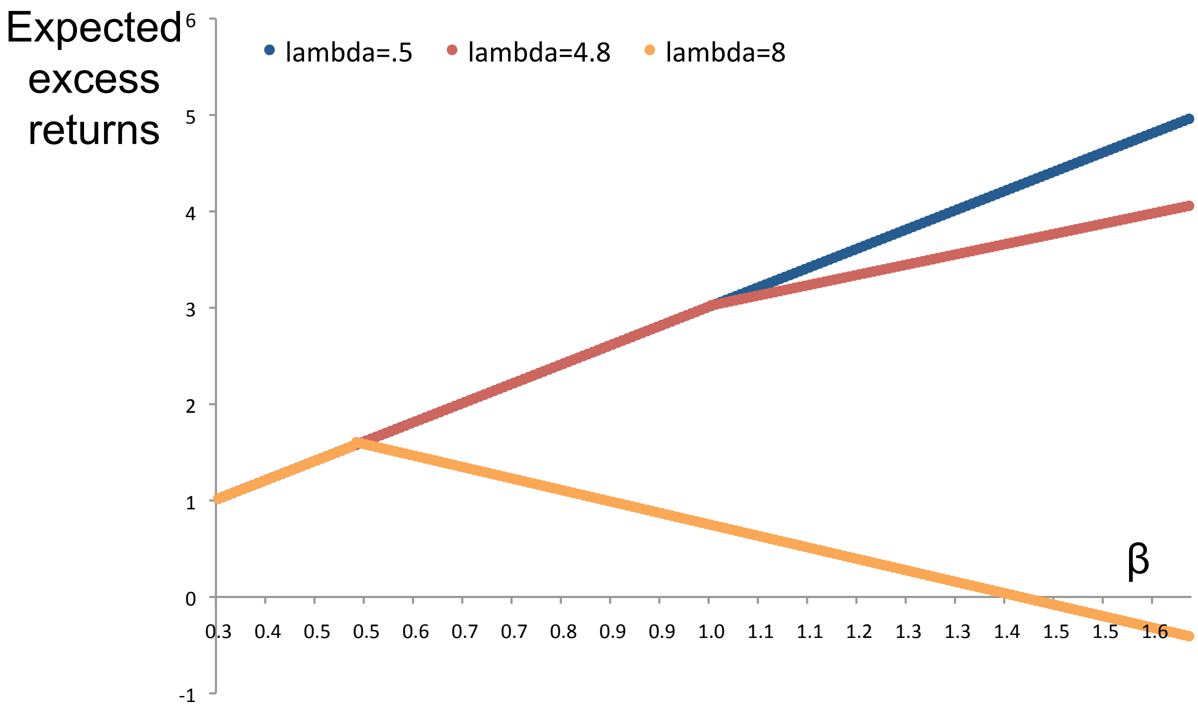


Figure 1: Model simulation.

Parameterization: $N = 100$, $\sigma_z^2 = 1$, $\sigma_\epsilon^2 = 50$, $\gamma = .5$ and $\theta = .05$

Figure 2: Time series of aggregate disagreement and time-series of 12-months value-weighted returns of low vs. high beta portfolio

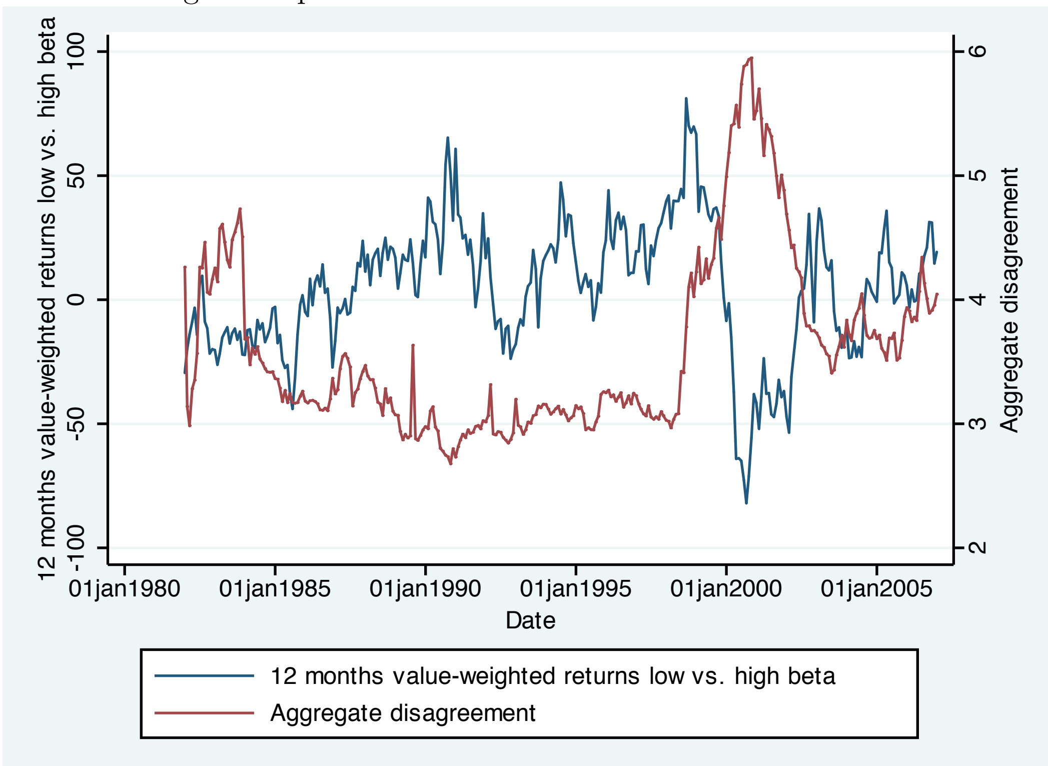


Figure 3: Aggregate disagreement and Failure of CAPM

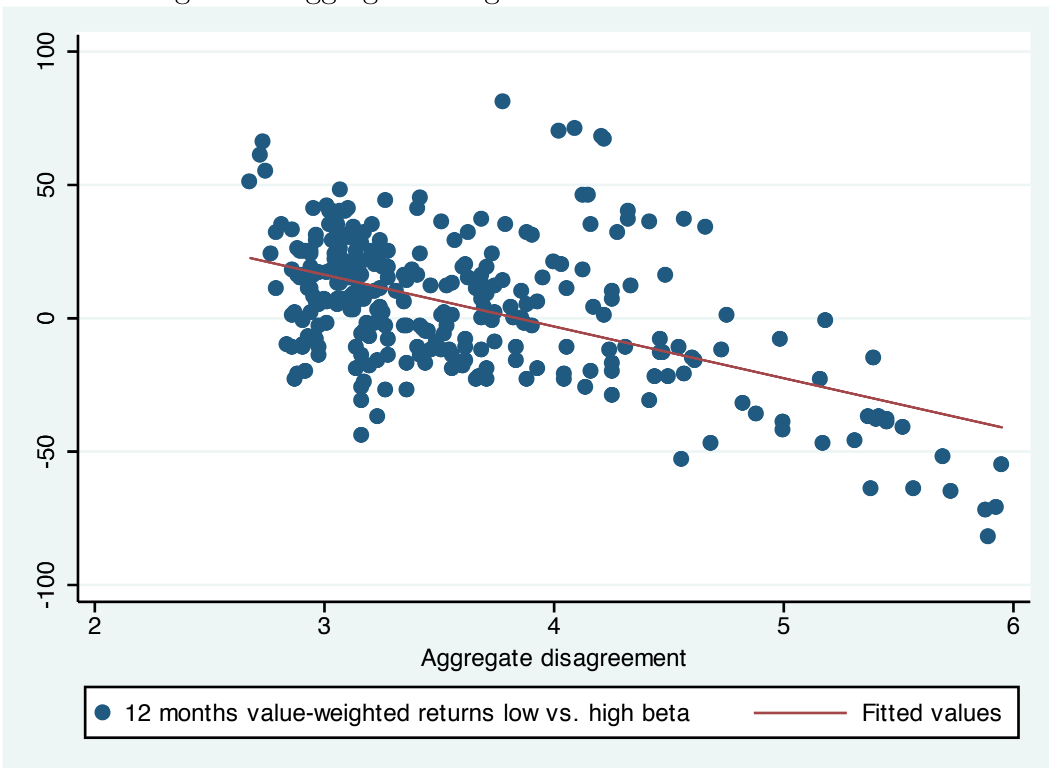
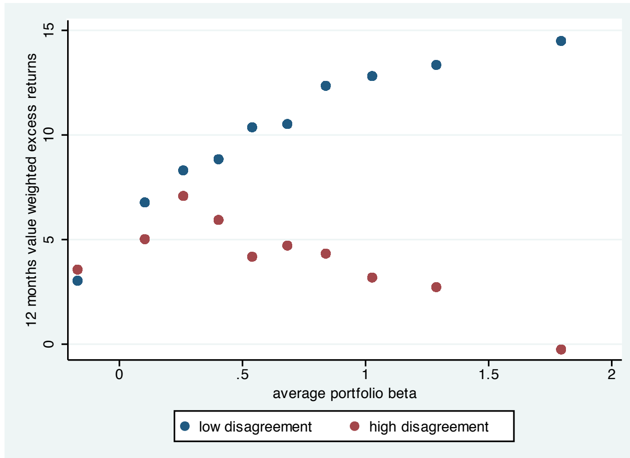
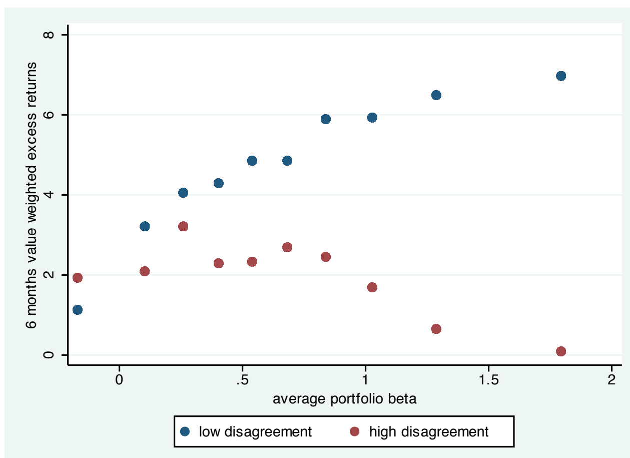


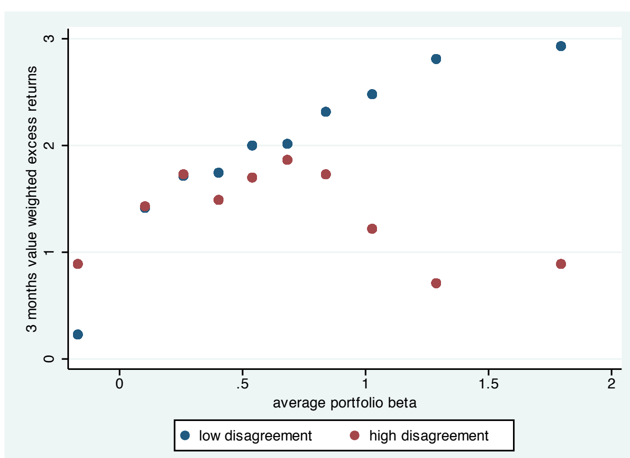
Figure 4: Plot of the average 3-months, 6-months, and 12-months excess of risk-free returns for value-weighted beta decile portfolios during low disagreement and high aggregate disagreement months.



(a) 12 months value-weighted return

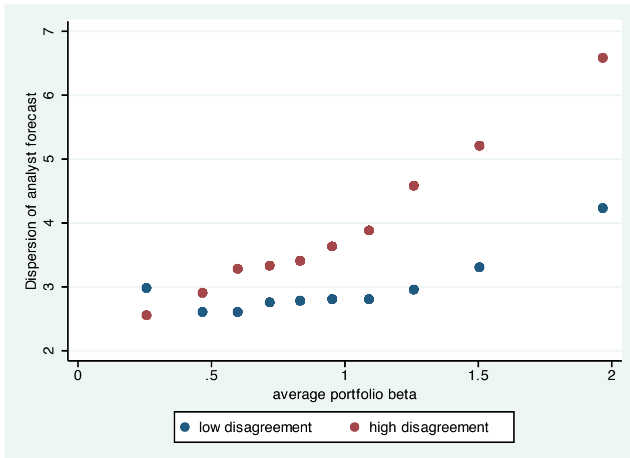


(b) 6 months value-weighted return

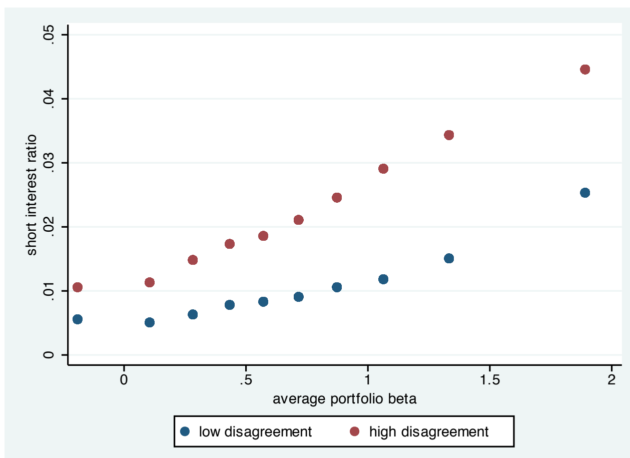


(c) 3 months value-weighted return

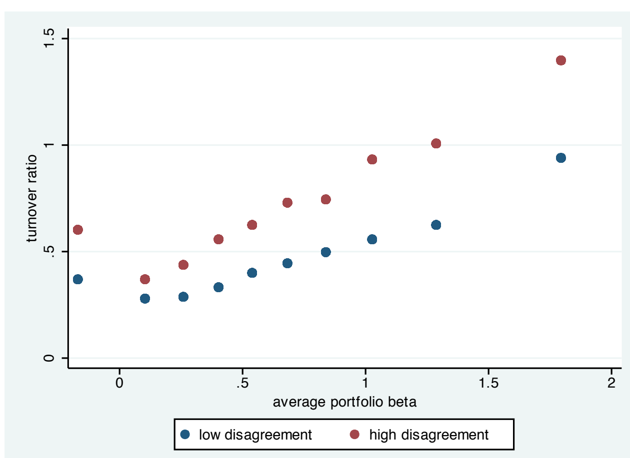
Figure 5: Plot of the value-weighted average of analyst earnings forecasts, short interest ratio and share turnover for stocks by bet deciles during low and high aggregate disagreement months.



(a) Dispersion of analyst earnings forecasts



(b) Short interest ratio



(c) Share turnover