Optimal Financial Structure and Asset Prices

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Abstract

I study the welfare properties of competitive equilibria in an economy with financial frictions. In the model, entrepreneurs raise funds to set up a firm, then they exert effort, and finally they trade assets. Private financial contracts do not internalize their impact on asset prices. On the one hand, lower prices depress firms' liquidation value, which reduces the pledgeable income – the *collateral effect*. On the other hand, lower prices boost entrepreneurs' incentives to achieve good performance, which alleviates the moral hazard problem and raises the pledgeable income – the *incentive effect*. I show that the latter effect outweighs the former, implying that a *decrease* in asset prices improves welfare. Even though, from a first-best perspective the competitive equilibrium displays too low asset prices, from a second-best point of view prices are too high.

1 Introduction

Both developed and emerging countries have experienced severe financial and banking crises since at least the 1980s, as illustrated by the ongoing turmoil. These episodes are characterized by a collapse in asset prices, credit and investment. The propagation mechanisms through which plummeting asset prices weaken firms' balance sheet and worsen credit rationing, have received extensive attention.[1] The perceived harm of these episodes has raised the issue of policy intervention, either preventive (e.g., prudential regulation, monetary policy) or curative (e.g., public bailouts, lending of last resort).[2] However, policy actions that reduce the losses of poor performers are criticized on the ground that they create moral hazard. Any public intervention should therefore tradeoff the benefits of sustaining the value of collateral against the adverse impact on incentives. Besides, any rationale for policy action has to include a theory of why private decisions can lead to socially inefficient outcomes.

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[1]The idea of focusing on the general equilibrium feed-back between financial distress and asset prices goes back to Shleifer and Vishny (1992) and Kiyotaki and Moore (1997). See also the subsequent literature that I discuss later on.

[2]See, for instance, Kashyap, Rajan and Stein (2008) for a discussion on capital regulation, Bordo and Jeanne (2002) for arguments in favor of a proactive monetary policy, Freixas, Parigi and Rochet (2004) and Acharya and Yorulmazer (2008) for models of lending of last resort and of bank bailouts in the presence of moral hazard.

This paper investigates the tradeoff between collateral and incentives, and exhibit a pecuniary externality that operates through the price of assets. I analyze constrained efficiency by considering a social planner who faces the same constraints as the private economy, and asking whether a modification of financial contracts can lead to a welfare improvement. My main result is that asset prices are too high in the equilibrium. Even though, from a first-best perspective the competitive equilibrium displays too low asset prices, from a second-best point of view prices are too high.

First, I extend Innes (1990)' model of financing under moral hazard with continuous action and continuous payoff, to the case of a continuous and variable size of the firm, and, more importantly, I relax the assumption that the density of the payoff conditional on the level of effort has the Monotonic Likelihood Ratio Property (MLRP). This assumption means that more effort increases the probability of high payoffs, and all the more so for the highest payoffs. Innes establishes that a debt contract is optimal in that case. I show that this remains true when the scale of investment can be chosen before effort is put in, and the firm can be resized once the quality of the firm is known.

When the MLRP is satisfied, the moral hazard problem is an effort problem. This rules out, for instance, the possibility of risk-shifting. I relax the MLRP assumption and show that the optimal financial contract specifies an investors' payoff function that is piecewise linear with a slope switching between 0 and 1. The debt contract is one special case of this optimal contract, which arises under a slightly weaker assumption than required by Innes (1990). When there is risk-shifting, the optimal contract can be implemented with a combination of outside senior debt and equity, and inside junior debt. As suggested by Jensen and Meckling (1976) and documented by Kaplan and Strömberg (2003) in the venture capital industry, this can also be done with convertible securities.

Then, I plug this optimal contracting setting in a general equilibrium framework to show that the competitive economy is not constrained efficient. There is a continuum of ex ante identical firms, which are subject to idiosyncratic shocks. Once firms learn their quality, they can adjust their scale of operation by trading assets with each others on a spot market. Since firms are atomistic, they do not take into account the general equilibrium effect of their financial structure on prices, which opens the door to a pecuniary externality.

Lower prices reduce firms' liquidation value. The flip side of the coin is that they make the expansion of successful firms cheaper. Besides, the optimal financial contract tends to allocate cash flow rights to investors in case of bad performance and liquidation, and to managers in case of good performance and firm expansion.[3] Therefore, on the one hand, a low price depresses the investors' revenue in case of liquidation; this *collateral effect* tends to reduce the pledgeable income. On the other hand, a low price boosts the manager's incentives to achieve good performance and purchase additional assets; this *incentive effect* relaxes the moral hazard problem inside the firm. I show that the incentive effect dominates the collateral effect and that a *decrease* in asset prices can improve welfare. Since firms are ex ante identical,

[3] Although not exactly when there is risk-shifting, since the manager should not be rewarded too much in case of very good performance.

this is also a Pareto improvement from an ex ante point of view.

That a lower asset price improves welfare comes from the optimality of the financial contract. Because the compensation of the entrepreneur cannot grow faster than the firm value (Innes, 1990), the power of incentives given to the entrepreneur is limited. When the asset price decreases, good outcomes become even better because acquisition opportunities are cheaper, and bad outcomes are even worse since liquidation values are lower. Then, it becomes possible to give more high-powered incentives to the entrepreneur at a given cost for the investor. Although lower prices reduce the investor's revenue and improve incentives before the optimal contract adjusts, it is possible to adjust the contract so that the investor's revenue is unaffected and the entrepreneur's incentives are improved.

My point differs from the usual moral hazard argument against saving poor performers. The usual argument, which applies in a frictionless economy, is that any price distortion moves privately optimal decisions away from their socially optimal level. However, in the presence of financial frictions, firms cannot carry out their preferred strategy, but have to implement a constrained strategy. When one increases the asset price, a double moral hazard problem arises: first, the preferred strategy deviates from the socially optimal one, as in the usual moral hazard argument; second, the moral hazard problem inside the firm worsens and the constrained strategy that is actually implemented goes further from the preferred one. In principal-agent terminology, not only does the rescue of poor performers distorts the objective function of the principal, it also makes it more difficult to incentivize the agent.

This paper contributes to several strands of literature. First, it contributes to the literature on optimal capital structure. Innes (1990) shows that when the probability density function of the payoff conditional on effort has the MLRP, the optimal contract is a debt contract. Biais and Casamatta (1999) build a double moral hazard model including an effort problem and a risk-shifting problem, with discrete action and payoff. They extrapolate the (continuous) capital structure from a discrete set of payoffs, and show that the optimal contract can be implemented with a combination of debt and equity when risk-shifting is the main source of moral hazard. Hellwig (2007) derives similar results in a more continuous setting, but in his model the adverse effect of risk-shifting is to increase the probability of the bad realization of a binary outcome. Palomino and Prat (2003) determine the optimal contract for a money manager who chooses the riskiness of its portfolio in a continuum. They show that a bonus contract in which the agent is paid a fixed sum if the portfolio return is above a threshold, and nothing otherwise, is optimal. My analysis and results differ because I impose the additional constraint that contracts must be monotonic as in Innes (1990), which rules out bonus contracts.

I also contribute to the literature that focuses on the interactions between asset prices and financial frictions. Kiyotaki and Moore (1997) exhibit mechanisms through which price drops weaken firms' balance sheet when they need collateral the most, leading to the amplification of macroeconomic shocks. Shleifer and Vishny (1997) make a similar point in the money management industry to account for limits of arbitrage. Krishnamurthy (2003) shows that these results are not robust to the use of state-contingent contracts, but can be restored if hedging is limited by the limited liability of financial intermediaries.

This work is closely related to the papers that study welfare from a second-best perspective and identify an externality of firms' financial decisions. The idea that the competitive equilibrium in economies with financial frictions can be constrained inefficient goes back to Geanakoplos and Polemarchakis (1986) and Kehoe and Levine (1993). More recently, Caballero and Krishnamurthy (2001, 2003) start from the insight that the interest rate is lower than the marginal product of capital in the presence of financial frictions. They show that this provides firms with incentives to have a weak balance sheet, which results in the underprovision of liquidity, or, equivalently in their model, to excessive foreign debt. Relatedly, Biais and Mariotti (2008) exhibit that investment exerts a pecuniary externality on wages, making it sometimes welfare improving to strengthen collateral constraint. Gromb and Vayanos (2002) develop a model with financially constrained arbitrageurs and unconstrained investors, making it welfare improving to transfer wealth from the latter towards the former. They show that, if arbitrageurs are expected to reduce their asset holdings, then they are hurt by price drops and the asset price is socially too low, while the opposite holds if they are expected to increase their positions. In their model, the pecuniary externality through the asset price can thus work in either direction, implying that arbitrageurs may take on too much or not enough risk. In Lorenzoni (2008), firms in a financially constrained sector sell assets to firms in an unconstrained sector during fire sale episodes, which results in too low asset prices and too much debt. Note that although these papers include aggregate uncertainty, it can be shown that the pecuniary externality they exhibit would persist with idiosyncratic risk only.[4] To clarify this point, I write my model with no aggregate uncertainty. Holmström and Tirole (2008) present a model in which firms engage in a costly arm race for liquidity hoarding to overbid each other on the asset market. They show that if asset sellers do not value liquidity, there is overprovision of liquidity in the competitive economy.

My contribution is to endogeneize the fundamental value of assets, whereas the above mentioned papers focus on the incentives to build up liquidity reserves and take as given the fundamental value of assets. In terms of modeling strategy, I assume that moral hazard takes place before the shock, instead of after the shock. Besides, I neutralize liquidity issues by assuming that credit markets are frictionless once uncertainty is resolved. Endogenous riskiness of assets can also be found in the banking literature, as in Freixas, Parigi and Rochet (2004). However, they assume that banks are run by a their owners and not by managers, which rules out agency issues. In Rochet and Tirole (1996) and Holmström and Tirole (2000), there is moral hazard inside the bank that determines the quality of assets, and thus the decision to liquidate the bank's portfolio, but there is no asset market and liquidated assets are lost.

In a recent paper, Acharya, Shin and Yorulmazer (2008) build a model with a risk-shifting problem inside the bank and a secondary market on which banks trade assets with each others.

[4]It would nevertheless introduce one subtle difference in Gromb and Vayanos (2002) and Lorenzoni (2008). With aggregate uncertainty, ex ante Pareto improvements are possible through transfers across states of nature, whereas only utilitarian welfare improvements exist in a deterministic economy. This distinction does not arise in Caballero and Krishnamurthy (2001, 2003) nor in the present paper, since firms are ex ante identical.

They assume that banks use debt (deposit) contracts, although they are not optimal in the presence of risk-shifting. They show that banks take excessive risk, which results in socially too low asset prices.[5] My analysis unveils that their result is overturned if, instead, banks use optimal contracts.

The rest of the paper is organized as follows. Section 2 presents the model. I solve for the optimal financial structure in Section 3 and for the asset market equilibrium in Section 4. I offer several robustness checks in Section 5 and conclude in Section 6.

2 Model

There is a mass one continuum of identical risk-neutral entrepreneurs. Each entrepreneur has a project and has initial wealth A. There are also competitive risk-neutral investors with a sufficiently large amount of wealth. There are three periods $t = 1, 2, 3$.

At date 1, each entrepreneur raises funds from investors to set up his project. The scale of the project, $k > 0$, is a continuous variable that can be freely selected. The technology has constant returns to scale with respect to the initial size of the project. The entrepreneur then exerts an effort, e, that is also a continuous real variable with support $[\underline{e}, \overline{e}]$. The cost of providing effort e is proportional to the firm size: ek.

At date 2, the quality of the project, θ, is revealed. θ is distributed on $[\underline{\theta}, \overline{\theta}]$, $\underline{\theta} > 0$, with cumulative distribution $F(\theta|e)$ and twice continuously differentiable density $f(\theta|e)$. A higher level of effort improves the quality of the firm in a sense that I shall precise at the end of the section. Besides, θ is independent across firms, therefore there is idiosyncratic risk only. I assume diminishing returns to effort: $F_{ee}(\theta|e) > 0$ for any $\theta \in (\underline{\theta}, \overline{\theta})$ and $e \in [\underline{e}, \overline{e}]$. I also rule out a knife-edge set of distribution functions by assuming that $\{\theta : (-F_e/(1-F))(\theta|e) = x\}$ is of measure zero for any $e \in [\underline{e}, \overline{e}]$ and $x > 0$.

Then, the firm can be rescaled to size $k' \geq 0$ by trading assets with the other firms at the (endogenous) asset price p. Returns to scale with respect to the size expansion between date 1 and date 2 are decreasing. Formally, the project pays off $\theta v(k'/k)k$ at date 3, with $v(0) = 0$, $v' > 0$, $v'' < 0$, $v'(0) = +\infty$ and $v'(+\infty) = 0$. Normalizing the discount rate to zero, the net present value of the project, net of the effort cost, is equal to

$$[-1 + \theta v(s) - (s-1)p - e]k,$$

where $s = k'/k$ is the resizing factor.

The contract between the entrepreneur and the investor specifies the initial investment, k, as well as the resizing strategy, $s(\theta)$, and the entrepreneur's wage, $w(\theta)$, as functions of the quality of the project. The entrepreneur is limited liable, thus

$$w(\theta) \geq 0 \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]. \tag{1}$$

[5] They also show that, in other circumstances, banks may take less risk than is socially optimal, because of an inefficient arm race for liquidity hoarding similar to the one in Holmström and Tirole (2008).

Then, following Innes (1990), I assume two key contractual frictions. First, the entrepreneur's effort is not verifiable. Second, I restrict my attention to monotonic contracts in which the entrepreneur's payoff and the investor's payoff are nondecreasing in θ

$$w(\theta_1) \leq w(\theta_2) \qquad \forall\, \theta_1 < \theta_2, \tag{2}$$

$$w(\theta_2) \leq w(\theta_1) + [\theta_2 v(s(\theta_2)) - s(\theta_2)p]k - [\theta_1 v(s(\theta_1)) - s(\theta_1)p]k \qquad \forall\, \theta_1 < \theta_2. \tag{3}$$

A rationale for constraint (2) is that, at date 3, the entrepreneur can sabotage the firm and decrease the final payoff. If θ and s are not verifiable and the contract is contingent on the final payoff only, then the entrepreneur is never induced to sabotage the firm if, and only if, (2) is satisfied. Symmetrically, constraint (3) must hold if the investor can sabotage the firm.[6]

Finally, I look for renegotiation-proof contracts. Therefore, for all θ, $s(\theta)$ must maximize $[\theta v(s) - sp]k$. The Inada conditions on $v(.)$ ensure that the solution, which I shall also denote by $s(\theta)$ to save notation, is interior and satisfies the first order condition

$$v'(s(\theta)) = \frac{p}{\theta} \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}].$$

To conclude that section, I specify in which sense the effort improves θ. The expected net present value, net of the effort cost, of a project implemented with effort e is linear in k. I denote it by $V(e)k$, where[7]

$$V(e) = -1 + E[\theta v(s(\theta)) - (s(\theta) - 1)p|e] - e.$$

Noticing that $s(.)$ is an increasing function of θ, the following assumption states that a higher effort improves the firm's quality.

Assumption 1. *(a) $V(e)$ is strictly quasiconcave with an interior maximum e^{FB}, and there exists $e_{\inf} < e^{FB}$ such that $V(e_{\inf}) = 0$. (b) $E[s(\theta)|e]$ is strictly increasing in e.*

Assumption 1-(a) means that a higher effort increases the expected NPV from 0 when the level of effort is equal to $e_{\inf}$, to its maximum level when the first-best level of effort e^{FB} is reached. When the level of effort increases above e^{FB}, the marginal cost of effort outweighs the marginal improvement in the quality of the project. Assumption 1-(b) states that effort increases the expected date 2-resizing factor.

3 Optimal Financial Structure

3.1 Optimal Contract

The optimal contract between the entrepreneur and the investor consists in an initial investment, a level of effort, and a compensation scheme for the entrepreneur. Since firms

[6] Another rationale for (3) is that the entrepreneur may secretely borrow at date 3 to reveal an apparent profit higher than the true profit. If (3) is not satisfied, the entrepreneur would borrow in any decreasing segment of the investor's payoff function. These possibilities are discussed at greater length in Innes (1990). Relatedly, Holmström and Milgrom (1991) emphasize that the possibility to manipulate the performance measure can be understood as a multitask problem.

[7] $s(\theta)$ and $V(e)$ are also functions of p. However, since firms take the asset price as given, I omit for the moment p as an argument.

are atomistic, they take the asset price as given. The optimal contract solves the following program, denoted by $\mathcal{P}$:

$$\max_{k,\,e,\,w(.)} \quad \left[-1+\int_{\underline{\theta}}^{\overline{\theta}}[\theta v(s(\theta))-(s(\theta)-1)p]f(\theta|e)d\theta-e\right]k, \tag{4}$$

$$\text{s.t.} \quad \left[-1+\int_{\underline{\theta}}^{\overline{\theta}}[\theta v(s(\theta))-(s(\theta)-1)p]f(\theta|e)d\theta\right]k-\int_{\underline{\theta}}^{\overline{\theta}}w(\theta)f(\theta|e)d\theta+A\geq 0, \tag{5}$$

$$e=\arg\max_{\tilde{e}}\int_{\underline{\theta}}^{\overline{\theta}}w(\theta)f(\theta|\tilde{e})d\theta-\tilde{e}k, \tag{6}$$

limited liability (1) and contract monotonicity (2) and (3).

The objective function is the expected NPV, $V(e)k$. The break-even condition constraint (5) imposes that the investor's profit is nonnegative. The incentive compatibility constraint (6) states that the entrepreneur's effort is determined by its compensation scheme. It can be handled by a first order approach, since, as formally shown in Appendix A.1, the expected revenue of the entrepreneur net of the effort cost is strictly concave in e by $F_{ee}>0$. The incentive compatibility condition (6) then rewrites as

$$\int_{\underline{\theta}}^{\overline{\theta}}w(\theta)f_e(\theta|e)d\theta-k=0. \tag{7}$$

In the optimal contract, the wage function is chosen to implement a given level of effort at the lowest cost for the investor. I define $w^{e,k}(.)$ as the wage function $w(.)$ that minimizes $E[w(\theta)|e]$ while satisfying (1), (2), (3), and implementing the effort level e when the initial investment is k.[8] Noting that $w^{e,k}(.)$ is linear in k since the incentive compatibility constraint is linear in k, I define $z^e(.)=w^{e,k}(.)/k$. Besides, the power of incentives given to the manager is limited by the constraint that the contract is monotonic. This imposes a maximum level of effort that can be implemented.

Lemma 1. *There exists a maximal level of effort $e_{\sup}\in[e^{FB},\overline{e}]$ that can be implemented.*

Proof. See Appendix A.1. □

I assume that, for any level of effort $e\in[e_{\inf},e_{\sup}]$, the break-even condition prevents an arbitrarily large investment scale: the expected wage by unit of investment required to induce the effort e exceeds the pledgeable value of a unit of investment $E[z^e(\theta)|e]>-1+E[\theta v(s(\theta))-(s(\theta)-1)p|e]$. The maximal level of investment that can be funded with an effort level e is thus given by

$$K(e)=\frac{A}{1-E[\theta v(s(\theta))-(s(\theta)-1)p+z^e(\theta)|e]}.$$

I also assume that F is sufficiently convex in e, which ensures

[8] $w^{e,k}(.)$ is unique up to its values on a set of measure zero.

Assumption 2. *(a) $K(e)$ is strictly decreasing on $e \in [e_{\text{inf}}, e_{\text{sup}}]$, with $K(e_{\text{inf}}) > 0$. (b) $V(e)K(e)$ is strictly concave on $e \in [e_{\text{inf}}, e_{\text{sup}}]$.*

The problem $\mathcal{P}$ finally consists in maximizing $V(e)K(e)$ on $e \in [e_{\text{inf}}, e_{\text{sup}}]$. On $[e_{\text{inf}}, e^{FB}]$, $V(.)$ is strictly increasing and $K(.)$ is strictly decreasing. This reflects a tradeoff between quality and size. On the one hand, a higher level of effort increases the average quality of the project. On the other hand, it requires a higher compensation to the entrepreneur, which lowers the investor's profit by unit of investment and reduces the scale of the project. The optimal level of effort never rises above e^{FB}, since it would reduce the NPV and consume additional pledgeable income.

Proposition 1. *$\mathcal{P}$ has a unique solution, with effort $e^* \in (e_{\text{inf}}, e^{FB})$, investment $k^* = K(e^*)$, and wage function*

$$w^*(\theta) = \int_{\underline{\theta}}^{\theta} 1_{\{\frac{-F_e}{1-F}(t|e^*) > \frac{\lambda}{\mu}\}} v(s(t))k^* dt \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}], \tag{8}$$

where $\lambda > 0$ and $\mu > 0$ are the shadow prices of, respectively, the break-even constraint (5) and the incentive compatibility constraint (7).

Proof. See Appendix A.1. □

The intuition for the form of the wage function is the following. First, $w(\underline{\theta})$ is optimally set to 0, since only the slope of the wage function matters for incentives. Indeed, if $w(\underline{\theta}) > 0$, lowering all the $w(\theta)$ by $w(\underline{\theta})$ relaxes the break-even constraint without affecting the incentive compatibility constraint.

Then, let me inspect the optimal slope of $w(.)$ for a given θ: What is the effect of increasing the slope in θ, or, equivalently, of increasing $w(t)$ by an infinitesimal amount δw for all the $t \in [\theta, \overline{\theta}]$? On the one hand, it lowers the expected revenue of the investor by

$$(1 - F(\theta|e^*))\delta w.$$

On the other hand, it increases the marginal return of effort for the entrepreneur by

$$-F_e(\theta|e^*)\delta w.$$

Starting from the optimal contract, given the shadow prices, increasing $w(t)$ by a small amount for all the $t \in [\theta, \overline{\theta}]$ improves the objective function if

$$-\lambda(1 - F(\theta|e^*)) - \mu F_e(\theta|e^*) > 0. \tag{9}$$

Conversely, decreasing $w(t)$ by a small amount for all the $t \in [\theta, \overline{\theta}]$ improves the objective function if inequality (9) is reversed. Besides, the monotonicity of the entrepreneur's and investor's payoffs, represented by conditions (2) and (3), implies that the slope of $w(.)$ in θ is comprised between 0 and the slope of the firm's value, $v(s(\theta))k^*$. Therefore, in the optimal contract, the slope of $w(.)$ in θ is equal to $v(s(\theta))k^*$ when $(-F_e/(1-F))(\theta|e^*) > \lambda/\mu$, and to 0 when $(-F_e/(1-F))(\theta|e^*) < \lambda/\mu$.[9] The ratio $(-F_e/(1-F))(\theta|e^*)$ is the (percentage) increase of the likelihood of a quality better than θ when the level of effort increases.

3.2 Implementation

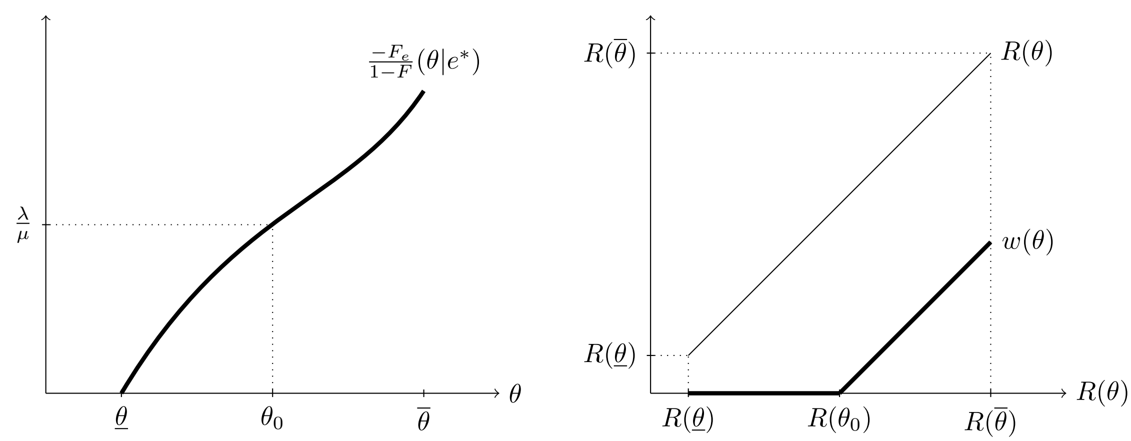
Proposition 1 can be restated in terms of the delta of the entrepreneur, defined as the derivative of the wage $w(\theta)$ with respect to the firm's value $R(\theta) = [\theta v(s(\theta)) - (s(\theta) - 1)p]k^*$:

$$\Delta(R(\theta)) = \frac{dw(\theta)}{dR(\theta)} = \begin{cases} 1 & \text{if} \quad (-F_e/(1-F))(\theta|e^*) > \lambda/\mu, \\ 0 & \text{if} \quad (-F_e/(1-F))(\theta|e^*) < \lambda/\mu. \end{cases} \tag{10}$$

The delta of the entrepreneur is equal to 1 when the benefits of effort outweigh the costs of increasing the expected wage, which is the case when the ratio $-F_e/(1-F)$ is large. I now turn to the implementation of the optimal contract with standard financial instruments. The key determinant of the financial structure is the shape of the function $(-F_e/(1-F))(.|e^*)$.

Consider first that the ratio $-F_e/(1-F)$ is increasing in θ, as in Figure 1. This means that a higher effort increases (multiplicatively) the probability of a quality better than θ by more when θ is higher. In that case, the optimal wage function is flat as long as the firm's value is below a threshold θ_0, and steep with the same slope than the firm's value above that threshold. This a debt contract with face value $R(\theta_0)$. Therefore, a sufficient condition for the optimal contract to be a debt contract is that $(1-F)(.|e^*)$ has the Monotone Likelihood Ratio Property (MLRP), where a family of functions $\{\varphi(\theta|e)\}$ is said to have the MLRP if φ_e/φ is increasing in θ (Milgrom, 1981).

Figure 1: When $1-F$ has the MLRP, a debt contract is optimal.



Corollary 1. *When $(1-F)(.|e^*)$ has the MLRP, the optimal contract is implemented with outside debt.*

[9]Remind that equation (9) holds with an equality for a set of θs of measure zero. This implies that the slope of $w(.)$ in these points does not matter.

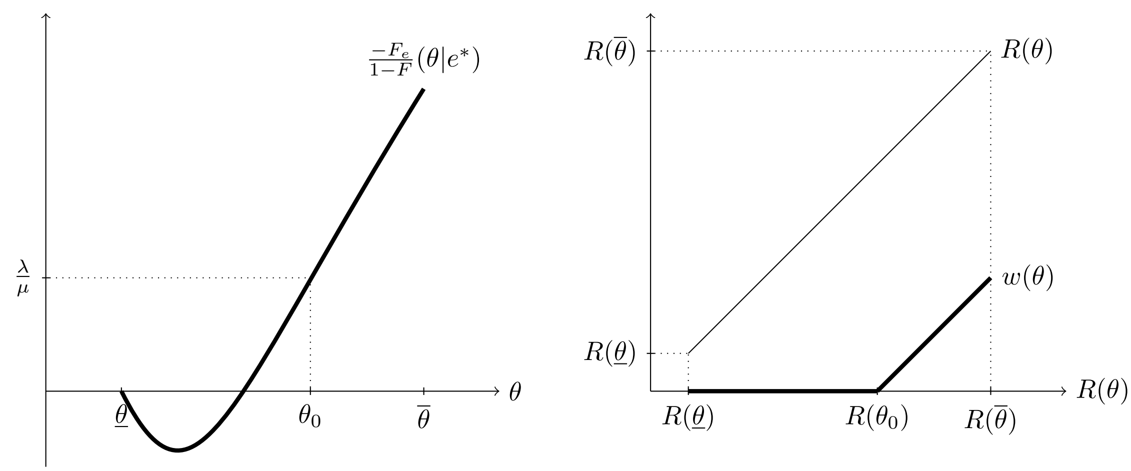
Innes (1990) establishes that f having the MLRP is a sufficient condition for the optimality of debt. This is a stronger condition than $1-F$ satisfying the MLRP, since

$$\frac{\partial}{\partial\theta}\frac{-F_e(\theta|e)}{1-F(\theta|e)} = \frac{f(\theta|e)}{(1-F(\theta|e))^2}\int_\theta^{\overline{\theta}}\left(\frac{f_e(t|e)}{f(t|e)} - \frac{f_e(\theta|e)}{f(\theta|e)}\right)f(t|e)dt$$

is always positive if $(f_e/f)(.|e)$ is increasing, but the reverse is not true.

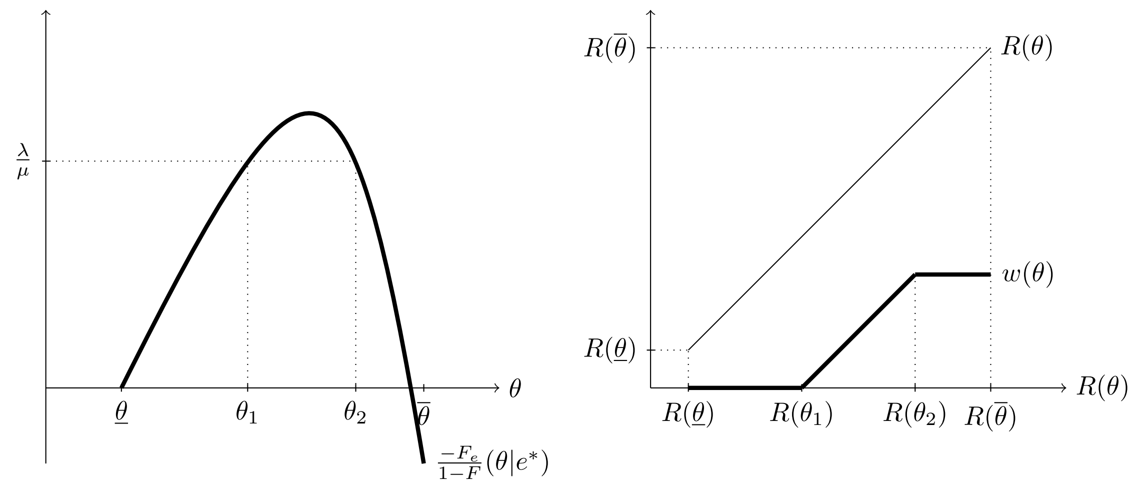
I claim that a debt contract is still optimal when exerting effort consists in taking on more risk, in which case $(1-F)(.|e^*)$ does not have the MLRP. Indeed, in the reasoning leading to Corollary 1, it suffices that $-F_e/(1-F)$ be increasing in θ in the neighborhood of the θs such that $(-F_e/(1-F))(\theta|e^*) > 0$. For instance, if $-F_e/(1-F)$ is negative for θ below a threshold, and positive and increasing above that threshold, as in Figure 2, then a debt contract is still optimal. This comes from the fact that the wage function is flat when $F_e > 0$, since rewarding the manager when $F_e \leq 0$ would reduce its incentives and consume pledgeable income. Hence, the monotonicity of $-F_e/(1-F)$ matters only when it is positive. An interpretation of the ratio $-F_e/(1-F)$ represented in Figure 2 is that exerting effort consists in taking on more risk. In that case, a debt contract is appropriate since it provides the manager with a convex payoff function.

Figure 2: When effort involves risk-taking, a debt contract is optimal.



Consider now that a low effort consists in risk-shifting. In that case, the ratio $-F_e/(1-F)$ is hump-shaped in θ as represented in Figure 3. Then, the optimal wage function is flat for θ below a threshold θ_1 and above another threshold $\theta_2 > \theta_1$, and steep between θ_1 and θ_2. Such a financial structure can be implemented with outside debt with face value $R(\theta_1)$ and outside equity, together with inside junior debt with face value $R(\theta_2) - R(\theta_1)$. A common form of inside debt are pensions. Yermack and Sundaram (2007) document that pensions represent a significant component of CEOs' compensation in large U.S. companies. As suggested by Jensen and Meckling (1976), the optimal contract can also be implemented by giving convertible debt to the investor, with face value $R(\theta_1)$ and a conversion price of $R(\theta_2) - R(\theta_1)$ for the totality of the shares. Kaplan and Strömberg (2003) document that convertible preferred stock is extensively used in venture capital contracts.

Figure 3: When moral hazard involves risk-shifting ($-F_e/(1-F)$ hump-shaped), a combination of outside debt and equity and inside junior debt, or the use of convertible securities, is optimal.



Corollary 2. *When $(-F_e/(1-F))(.|e^*)$ is hump-shaped, the optimal contract is implemented with a combination of outside debt and equity and inside junior debt, or equivalently with convertible securities.*

Note that if $-F_e/(1-F)$ is hump-shaped, but $(-F_e/(1-F))(\overline{\theta}|e^*) > \lambda/\mu$, as in Figure 4, then a debt contract is still optimal. Such a distribution function does not, however, exactly fits a risk-shifting problem. Indeed, effort increases the probability of a type higher than θ for high θs, although less than for intermediate θs.

Finally, one can complexify the shape of the ratio $(-F_e/(1-F))(.|e^*)$. In the general case, the optimal payoff of the entrepreneur, or, by complementarity, of the investor, is continuous piecewise linear with a slope switching between 0 and 1, as illustrated in Figure 5. Besides, the wage of the entrepreneur is always equal to zero in the neighborhood of $\underline{\theta}$ since $(-F_e/(1-F))(\underline{\theta}|e^*) = 0$.

4 Capital Market Equilibrium

4.1 Competitive Equilibrium

I now solve for the competitive equilibrium of the economy. I define a competitive equilibrium as an asset price and firms' financial contracts, such that financial contracts are optimal and the asset market clears. Since firms are identical, a competitive equilibrium of the economy

Figure 4: When $-F_e/(1-F)$ is hump-shaped and $(-F_e/(1-F))(\overline{\theta}|e^*) > 0$, (a) debt or (b) convertible securities can be optimal.

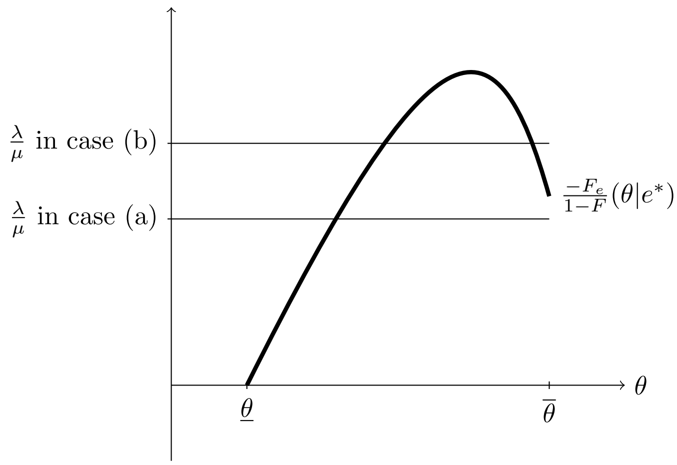
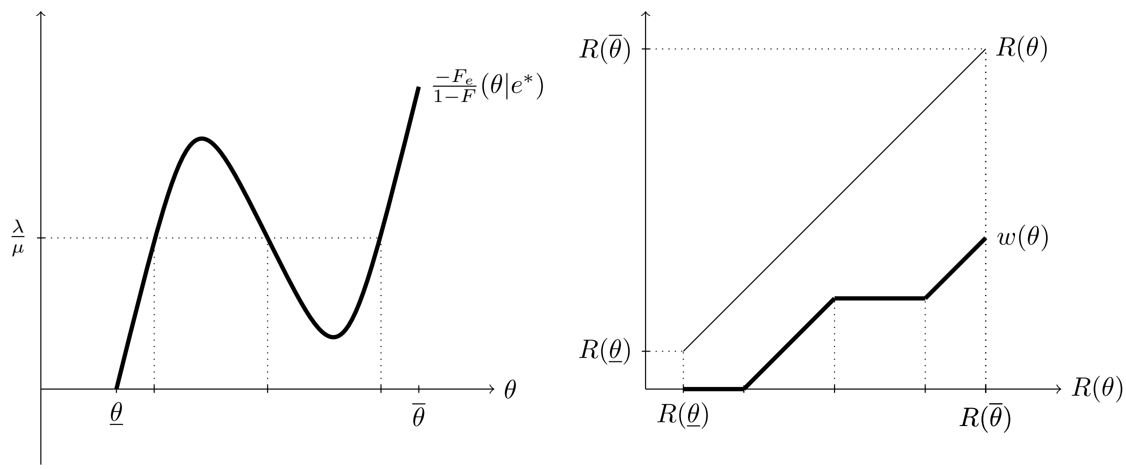


Figure 5: In the general case, the wage function is piecewise linear with a slope switching between 0 and 1.



is a triplet (k^{CE}, e^{CE}, p^{CE}) which satisfies $k^{CE} = k^*(p^{CE})$, $e^{CE} = e^*(p^{CE})$ and $p^{CE} = p^*(e^*(p^{CE}))$.

In Section 3, I have determined the optimal contract for any given asset price. I now add p as an argument of all the variables that were implicitly defined as functions of p: the first-best effort level $e^{FB}(p)$, the variables in the optimal contract $e^*(p)$, $k^*(p)$, $s(\theta, p)$, $w^*(\theta, p)$, the expected NPV $V(e,p)$ and the maximal investment $K(e,p)$.[10]

The optimal investment $k^*(p)$ and effort level $e^*(p)$ depend *a priori* on the asset price in an ambiguous way. The direct effect of a higher price is to modify the tradeoff between quality and size emphasized in Section 3.1 in favor of size. Indeed, being of low quality is a

[10] As well as $e_{\inf}(p)$ and $e_{\sup}(p)$. The shadow prices λ and μ are also functions of p, but I continue to omit the asset price as an argument to allege the notations.

less adverse outcome when the resale value of assets increases, and being of high quality is a less desirable outcome when firm expansion is more expensive. Thus, by the direct effect, e^* tends to be decreasing and k^* increasing in p. However, as shown later on, the asset price also affects the firm's pledgeable value, which modifies the optimal contract in an ambiguous way.[11] These indirect effects could potentially make e^* locally increasing in p. As will appear shortly, this opens the door to multiple equilibria on the asset market. Although multiple equilibria would not modify the essence of my results, I assume that the direct effect outweighs the indirect ones to streamline the analysis.

Assumption 3. *$e^*(p)$ is strictly decreasing in p in the neighborhood of an equilibrium value of p.*

Conversely, for given financial contracts that implement investment k and effort e, I can aggregate individual behaviors on the secondary asset market and determine the equilibrium price. Firms with a low θ liquidate assets, while firms with a high θ purchase assets. The asset market clears at the price $p^*(e)$, such that

$$\int_{\underline{\theta}}^{\overline{\theta}} (s(\theta, p^*(e)) - 1) f(\theta|e) d\theta = 0. \tag{11}$$

Note that the equilibrium asset price does not depend on k because of constant returns to scale. Equation (11) indicates that the equilibrium price $p^*(e)$ strictly increases with effort. Indeed, more effort increases the number of firms that purchase assets, by Assumption 1-(b), which requires a higher asset price to restore the equilibrium, since $\partial s/\partial p < 0$. This ensures, together with Assumption 3, that

Lemma 2. *There exists a unique competitive equilibrium.*

Proof. See Appendix A.2. □

4.2 Welfare

I now study the optimality of the competitive equilibrium. I define welfare as the sum of all the entrepreneurs' and investors' profits. In the competitive equilibrium, since investors earn zero profit and there is a mass one of identical firms, welfare is equal to a firm's NPV.

Consider a social planner who, at date 1, chooses the financial contracts $(k, e, w(.))$. She faces the same constraints as the private economy: investors must break even, entrepreneurs choose the effort that maximizes their revenue, there is limited liability, contracts are monotonic and renegotiation-proof. The only difference with the individual entrepreneur's problem

[11] Precisely, a higher price reduces the firms' pledgeable value, which has two effects on the optimal contract. First, it tends to reduce both k^* and e^* to restore the investor's break-even constraint. Second, since the break-even constraint is more binding (the shadow price increases), it becomes more interesting to relax that constraint, which is done by reducing k^* and increasing e^*.

is that the social planner takes into account the impact of financial contracts on the equilibrium price on the asset market. The problem of the social planner is therefore

$$\max_{e}\ V(e,p^*(e))K(e,p^*(e)).$$

The fact that the second-best allocation is symmetric comes from the strict concavity of $V(e,p)K(e,p)$ in e: implementing simultaneously low effort projects and high effort projects is inefficient. Besides, in order to ensure that the social planner's problem has a unique solution, which I denote by (k^{SB}, e^{SB}, p^{SB}), I assume that

Assumption 4. $V(e,p^*(e))K(e,p^*(e))$ *is strictly quasiconcave in* e.

The maximization program of the social planner makes clear that the asset price is endogenous. There is a pecuniary externality in the competitive equilibrium, since each individual firm neglects the impact of its individual behavior on the asset price. Before stating formally the difference between the competition outcome and the second-best, let me show heuristically how the asset prices in these two allocations compare.

Starting from the competitive equilibrium, what can the social planner do to increase welfare, or, equivalently, firms' NPV? She can change slightly the asset price by modifying slightly financial contracts. Since we start from the competitive equilibrium, financial contracts are privately optimal. The envelope theorem implies that an infinitesimal variation of the financial contract has no first order effect on a firm's NPV. Only the variation of the asset price itself has a first order effect. The social planner should therefore modify slightly financial contracts in order to increase the asset price if the (total) derivative of firm's NPV with respect to p is strictly positive, and decrease p if it is strictly negative. To determine the sign of that derivative, remind that the NPV when the asset price is p, which I denote by $NPV(p)$, is the maximum of

$$\left[-1+\int_{\underline{\theta}}^{\overline{\theta}}[\theta v(s(\theta,p))-(s(\theta,p)-1)p]f(\theta|e)d\theta-e\right]k, \tag{12}$$

subject to the break-even constraint of the investor, with shadow price λ, and the incentive compatibility of the entrepreneur, with shadow price μ. After integrating the expected wage by parts, the break-even condition rewrites as

$$\begin{aligned}&\left[-1+\int_{\underline{\theta}}^{\overline{\theta}}[\theta v(s(\theta,p))-(s(\theta,p)-1)p]f(\theta|e)d\theta\right]k\\ &\qquad -\int_{\underline{\theta}}^{\overline{\theta}}1_{\{\frac{-F_e(\theta|e)}{1-F(\theta|e)}>\frac{\lambda}{\mu}\}}v(s(\theta,p))k(1-F(\theta|e))d\theta+A\geq 0.\end{aligned} \tag{13}$$

Similarly, an integration by parts of the incentive compatibility constraint gives

$$\int_{\underline{\theta}}^{\overline{\theta}}1_{\{\frac{-F_e(\theta|e)}{1-F(\theta|e)}>\frac{\lambda}{\mu}\}}v(s(\theta,p))k(-F_e(\theta|e))d\theta-k=0. \tag{14}$$

The total derivative of the firm's NPV with respect to the asset price around the competitive equilibrium is

$$\begin{aligned}
\frac{dNPV}{dp}(p^{CE}) &= \frac{\partial(12)}{\partial p} + \lambda\frac{\partial(13)}{\partial p} + \mu\frac{\partial(14)}{\partial p} \\
&= (1+\lambda)\int_{\underline{\theta}}^{\overline{\theta}} (1 - s(\theta, p^{CE}))k^{CE} f(\theta|e^{CE})d\theta \\
&\quad +\lambda \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e^{CE})>\frac{\lambda}{\mu}\}}\sigma(\theta, p^{CE})k^{CE}(1 - F(\theta|e^{CE}))d\theta \\
&\quad -\mu \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e^{CE})>\frac{\lambda}{\mu}\}}\sigma(\theta, p^{CE})k^{CE}(-F_e(\theta|e^{CE}))d\theta,
\end{aligned}$$

with $\sigma(\theta, p) = -(\partial s/\partial\theta)(\theta, p) = v'(s(\theta, p))/(\theta|v''(s(\theta, p))|) > 0$. The first term is the direct effect of the asset price on the firm's NPV. Since firms are identical and the asset market clears, each firm sells on average as many assets as it buys, implying that this term is equal to zero. The second term is the effect of the asset price on the break-even constraint. The entrepreneur is mostly rewarded when θ is high. Symmetrically, the investor essentially gets the liquidation value of the firm, therefore a higher price relaxes the break-even constraint. This is the *collateral effect*. The third term is the effect of the asset price on the incentive compatibility constraint. When the asset price increases, the firm's value for high θs is lower, which makes it more difficult to incentivize the entrepreneur. Therefore, a higher price strengthens the incentive compatibility constraint through an *incentive effect*. Although the collateral effect and incentive effect work in opposite directions, the overall effect can always be signed:

$$\frac{dNPV}{dp}(p^{CE}) = \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e^{CE})>\frac{\lambda}{\mu}\}}\sigma(\theta, p^{CE})k^{CE}[\lambda(1 - F(\theta|e^{CE})) - \mu(-F_e(\theta|e^{CE}))]d\theta < 0,$$

since the integrand is strictly negative. The incentive effect always dominates the collateral effect. This implies that a small decrease in the asset price below its competitive level improves welfare.

The intuition why a lower asset price improves welfare is the following. The power of incentives given to the entrepreneur is limited by the constraint that its wage cannot grow faster than the firm value limits. When p decreases, the firm value improves for high θ and worsens for low θ: the firm value becomes a steeper function of θ. Then, it becomes possible to give more high-powered incentives to the entrepreneur *at a given cost for the investor*, that is, for a given expected wage. Formally, the maximum slope of the wage function, $v(s(\theta, p))$, decreases with p.

To complete the analysis, I compare the competitive equilibrium and the second-best with the first-best allocation. The first-best corresponds to an economy with no financial constraint. For given price p and investment k, the first-best level of effort is $e^{FB}(p)$, as defined in Assumption 1-(a). However, with no break-even constraint, the optimal level of investment goes to infinity. In order to have a well-defined benchmark, I put aside the determination of k and define the first-best asset price by $p^{FB} = p^*(e^{FB}(p^{FB}))$. It is necessarily unique as shown in Appendix A.3. $e^{FB}(.) > e^*(.)$ implies $p^{FB} > p^{CE}$ since $p^*(.)$ is increasing.

Proposition 2. *The asset price in the competitive equilibrium is strictly lower than in the first-best, but strictly larger than in the second-best*

$$p^{SB} < p^{CE} < p^{FB}.$$

Proof. See Appendix A.3. □

In the competitive equilibrium, firms do not take into account the impact of their effort on the asset price. They implement a socially too high level of effort, which increases the asset price. This makes more difficult for the other firms to incentivize the entrepreneur. Even though it also raises the value of their collateral, the incentive effect dominates. Therefore, the decentralized level of effort is too high. Equivalently, the decentralized level of investment is too low.

Proposition 3. *In the competitive equilibrium, firms do not invest enough and provide too much effort*

$$k^{SB} > k^{CE},$$
$$e^{SB} < e^{CE} < e^{FB}.$$

Proof. Immediate given that $p^*(e)$ is strictly increasing in e and $K(e,p)$ is strictly decreasing in e and p. □

The second-best allocation improves on the competitive allocation in the sense of utilitarian welfare. Since firms are ex ante identical, this is also a Pareto improvement from an ex ante point of view.

4.3 Example: Debt Contract

More intuition can be grasped from an example. Consider that $1-F$ has the strict MLRP, i.e., $(-F_e/(1-F))(\theta|e)$ is strictly increasing in θ, so that a debt contract is optimal. The face value is defined as in Figure 1 by a threshold θ_0. The optimal face value comes from the following tradeoff: increasing the face value by 1 raises the expected revenue of the investor by $1-F(\theta_0|e)$ and reduces the marginal return of effort for the entrepreneur by $-F_e(\theta_0|e)$. Hence

$$\lambda(1-F(\theta_0|e)) = \mu(-F_e(\theta_0|e)), \tag{15}$$

and the MLRP implies that $\lambda(1-F(\theta|e)) > \mu(-F_e(\theta|e))$ for $\theta < \theta_0$ and $\lambda(1-F(\theta|e)) < \mu(-F_e(\theta|e))$ for $\theta > \theta_0$.

The resizing technology is given by $v(0)=0$, $v'(s)=1$ if $s<1$, $v'(s)=0.5$ is $s \in (1,2)$ and $v'(s)=0$ if $s>2$.[12] Therefore, the optimal date 2-investment satisfies

$$s(\theta,p) = \begin{cases} 0 & \text{if} \quad \theta < p, \\ 1 & \text{if} \quad p < \theta < 2p, \\ 2 & \text{if} \quad \theta > 2p. \end{cases}$$

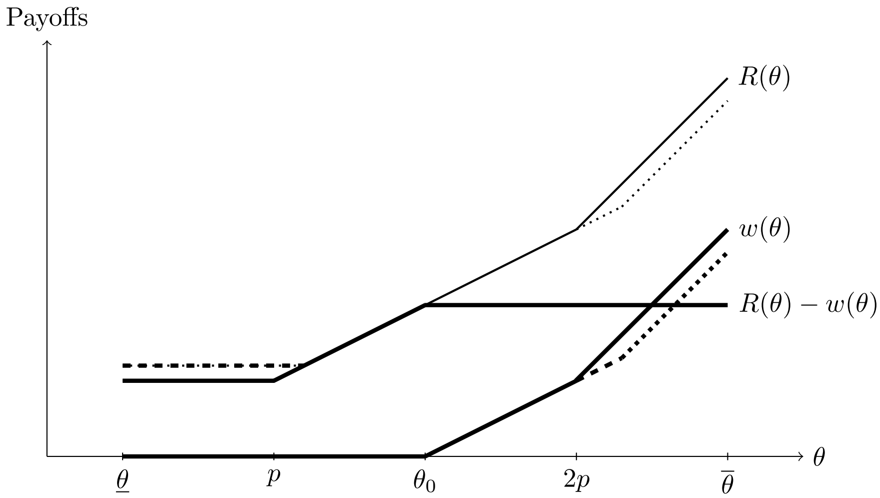
[12] $v(.)$ is not continuously differentiable as assumed in the general model. However, the analysis is extended without difficulty to piecewise linear functions $v(.)$.

The firm's value is then

$$R(\theta, p) = \begin{cases} pk & \text{if} \quad \theta < p, \\ \theta k & \text{if} \quad p < \theta < 2p, \\ (1.5\theta - p)k & \text{if} \quad \theta > 2p. \end{cases}$$

The sharing rule between the entrepreneur and the investor is represented by the solid lines in Figure 6. Finally, the equilibrium asset price satisfies $F(p^{CE}|e^{CE}) = 1 - F(2p^{CE}|e^{CE})$, and I assume that $\theta_0 \in (p^{CE}, 2p^{CE})$.

Figure 6: Debt contract and piecewise linear technology.



When the asset price increases by 1, the expected revenue of the entrepreneur increases by $F(p^{CE}|e^{CE})k^{CE} = (1 - F(2p^{CE}|e^{CE}))k^{CE}$, and the marginal return of effort is reduced by $(-F_e(2p^{CE}|e^{CE}))k^{CE}$. The new payoff functions are represented by the dashed lines in Figure 6. The effect on the NPV is

$$\frac{dNPV}{dp}(p^{CE}) = \lambda(1 - F(2p^{CE}|e^{CE}))k^{CE} - \mu(-F_e(2p^{CE}|e^{CE}))k^{CE} < 0,$$

by (15) the optimality condition of the face value.

That the incentive effect dominates the collateral effect comes from the optimality of the financial structure. To illustrate that point, consider now that there is a risk-shifting problem, but firms still use a (non optimal) debt contract. $-F_e/(1 - F)$ is hump-shaped as in Figure 3. Conditionally on using a debt contract, the optimal face value is still given by condition (15), where θ_0 is the lower of the two values, θ_0 and θ_0', satisfying (15).[13] If $\theta_0' < 2p^{CE}$, then $(dNPV/dp)(p^{CE})$ is strictly positive and the collateral effect dominates the incentive effect. This sheds light on the results in Acharya, Shin and Yorulmazer (2008). Using the terminology

[13]Using that $-F_e/(1 - F)$ is increasing in θ in the neighborhood of θ_0 and decreasing in the neighborhood of θ_0', it is straightforward to show that the second order condition associated with the first order condition (15) is satisfied for θ_0 only.

of the current paper, in their model entrepreneurs are banks which use debt contracts although there is a risk-shifting problem. They show that the collateral effect dominates the incentive effect. My analysis highlights that this result relies crucially on the (non optimal) use of debt contracts, and is overturned if, instead, contracts are optimal.

5 Robustness Checks

The main result of this paper is that the incentive effect dominates the collateral effect. Since this is a clear-cut result, I show that it continues to hold when alternative modeling choices are made. This section also illustrates that the optimal contracting framework developed in this paper is highly tractable and can be easily extended in a number of directions.

Returns to scale. Let me clarify the role of returns to scale in the model. There are decreasing returns to scale with respect to date 2-investment, k'. This assumption is essential, because it prevents that the single firm with the highest θ acquires all the assets in the economy. I have formalized this assumption with a payoff function $\theta v(k'/k)$ that satisfies the Inada condition as regards final size k'. Alternatively, I could have used an adjustment cost by positing a payoff function $\theta v(k'/k) = \theta k'/k - c(k'/k - 1)$, with $c(0) = 0$, $c'(0) = 0$ and $c'' > 0$. In that case, the Inada condition $v'(0) = +\infty$ is not satisfied any more and low θ-firms can be completely shut down. That would not modify the analysis. All what is needed is that $v'(.)$ is decreasing in the neighborhood of $+\infty$, to prevent the $\overline{\theta}$-firms to absorb all the assets.

Then, the assumption of constant returns to scale with respect to initial investment, k, can be relaxed without affecting the results. If the final payoff, $\theta v(s)k$, is replaced by $\theta v(s)u(k)$, with $u(0) = 0$, $u' > 0$ and $u'' < 0$, and the cost of effort ek becomes $ec(k)$, with $c(0) = 0$, $c' > 0$ and $c'' > 0$, it is straightforward to check that all the results still hold. Moreover, if $u'(+\infty) = 0$ or $c'(+\infty) = +\infty$, then the first-best level of investment k^{FB} is well-defined and strictly larger than k^{SB}. In that case, the model would also admit an initial wealth of entrepreneurs A equal to 0.

Aggregate shock. Shocks θ are idiosyncratic so that there is no aggregate uncertainty. The model and the results are easily extended to include an aggregate shock that shifts the distribution of θ, as long as the asset price p is contractible. This is true because contracting on p boils down to contracting on the aggregate state of nature. In that case, there is one compensation scheme by state of nature, and each one satisfies equation (8), where the distribution function is now conditional on the aggregate state of nature. The other results then follow. In particular, the price of the asset is too high in every state of nature.

Multidimensional effort. The model can be extended to the case of a multidimensional effort $e = (e_1, \ldots, e_N)$, as in the previous papers that deal simultaneously with an effort problem and a risk-shifting problem (e.g., Biais and Casamatta, 2001). Consider for simplicity a separable cost of effort, $\sum_{i=1}^{N} e_i k$. I still assume that the N first order conditions corresponding to the incentive compatibility constraint,

$$\int_{\underline{\theta}}^{\overline{\theta}} w(\theta) f_{e_i}(\theta|e) d\theta - k = 0 \qquad \forall\, i = 1, \ldots, N,$$

are sufficient. I denote by μ_i their respective shadow prices. The optimal wage function is derived as in the basic model. Increasing the slope of $w(.)$ in θ reduces the investor's profit by $1 - F(\theta|e)$ and increases the marginal return of each effort e_i by $-F_{e_i}(\theta|e)$. Thus, the optimal wage function is steep if $\sum_{i=1}^{N} \mu_i(-F_{e_i}(\theta|e)) > \lambda(1 - F(\theta|e))$, and flat otherwise:

$$w(\theta) = \int_{\underline{\theta}}^{\theta} 1_{\{\sum_{i=1}^{N} \frac{\mu_i}{\lambda} \frac{-F_{e_i}}{1-F}(t|e) > 1\}} v(s(t,p)) k dt \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}].$$

The competitive asset price is still socially too high, since

$$\frac{dNPV}{dp}(p^{CE}) =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\sum_{i=1}^{N} \frac{\mu_i}{\lambda} \frac{-F_{e_i}}{1-F}(\theta|e^{CE}) > 1\}} \sigma(\theta, p^{CE}) k^{CE} \left[\lambda(1 - F(\theta|e^{CE})) - \sum_{i=1}^{N} \mu_i(-F_{e_i}(\theta|e^{CE})) \right] d\theta < 0.$$

Risk-averse entrepreneur. If one introduces risk-aversion in the model, the entrepreneur and the investor contract not only to fund the project, but also to share risk. I show that if the main motive for contracting remains to raise funds, then the asset price is still too low. By contrast, if risk-aversion is so strong that risk-sharing becomes the main motive for contracting, then the competitive equilibrium is socially optimal.

Assuming that the entrepreneur is risk-averse and its preferences are additively separable in revenue and effort, the problem writes as

$$\max_{k,\, e,\, w(.)} \quad \int_{\underline{\theta}}^{\overline{\theta}} u(w(\theta)) f(\theta|e) d\theta - ek,$$
$$\text{s.t.} \quad \left[-1 + \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(s(\theta,p)) - (s(\theta,p) - 1)p] f(\theta|e) d\theta - e \right] k - \int_{\underline{\theta}}^{\overline{\theta}} w(\theta) f(\theta|e) d\theta + A \geq 0,$$
$$\int_{\underline{\theta}}^{\overline{\theta}} u(w(\theta)) f_e(\theta|e) d\theta - k = 0,$$

limited liability (1) and contract monotonicity (2) and (3),

where $u' > 0$ and $u'' < 0$. If the constraint (3) that the investor's payoff is nondecreasing is binding on a set of strictly positive measure, then a lower p increases $v(s(\theta, p))$, which relaxes the constraint. In that case, the competitive asset price is higher than in the second-best.

If, instead, (3) is never binding, the competitive equilibrium achieves constrained efficiency. This is the case if, for all $\theta \in [\underline{\theta}, \overline{\theta}]$, increasing or decreasing $w(\theta)$ does not improve the objective function, which provides the first order condition: $u'(w(\theta))f(\theta|e) - \lambda f(\theta|e) + \mu u'(w(\theta)) f_e(\theta|e) = 0$. Differentiating this expression with respect to θ provides

$$w'(\theta) = \frac{u'(w(\theta))(f'(\theta|e) + \mu f'_e(\theta|e)) - \lambda f'_e(\theta|e)}{|u''(w(\theta))|(f(\theta|e) + \mu f_e(\theta|e))},$$

which must be comprised between 0 and $v(s(\theta,p))k$ for all θ. This is the case if $u(.)$ is sufficiently concave, that is, if the entrepreneur is very risk-averse.

Distribution of bargaining power. So far I have used the standard assumption that investors are competitive so that entrepreneurs extract all the surplus of the project. The results are robust to a modification of the distribution of bargaining power. Assume that entrepreneurs are competitive and investors extract all the surplus. The optimal contract then solves

$$\begin{aligned}
\max_{k,\,e,\,w(.)} \quad & \left[-1+\int_{\underline{\theta}}^{\overline{\theta}}[\theta v(s(\theta,p))-(s(\theta,p)-1)p]f(\theta|e)d\theta - e\right]k - \int_{\underline{\theta}}^{\overline{\theta}} w(\theta)f(\theta|e)d\theta, \\
\text{s.t.} \quad & \int_{\underline{\theta}}^{\overline{\theta}} w(\theta)f(\theta|e)d\theta - ek - A \geq R, \\
& \int_{\underline{\theta}}^{\overline{\theta}} w(\theta)f_e(\theta|e)d\theta - k = 0, \\
& \text{limited liability (1) and contract monotonicity (2) and (3),}
\end{aligned}$$

where $R \geq 0$ is the reservation utility of the entrepreneur.

The break-even constraint of the entrepreneur is necessarily binding, otherwise the objective function can be improved by lowering k and $w(.)$ by a same factor. The incentive compatibility constraint is also binding, otherwise one should increase e. Increasing the slope of $w(.)$ in θ reduces the objective function by $1-F(\theta|e)$, relaxes the break-even constraint (with shadow price λ) by $1-F(\theta|e)$ and the incentive compatibility constraint (with shadow price μ) by $-F_e(\theta|e)$. The optimal wage function is steep if, and only if, the weighted sum of these three terms is positive, hence

$$w(\theta) = \int_{\underline{\theta}}^{\theta} 1_{\{\frac{-F_e}{1-F}(t|e) > \frac{1-\lambda}{\mu}\}} v(s(t,p))kdt \qquad \forall\, \theta \in [\underline{\theta},\overline{\theta}].$$

The objective function is the investor's profit, but since the entrepreneur's profit (net of the effort cost) is fixed, the derivative of the NPV is the same as the derivative of investor's profit, and it is strictly negative

$$\frac{dNPV}{dp}(p^{CE}) = \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e^{CE}) > \frac{1-\lambda}{\mu}\}} \sigma(\theta,p^{CE})k^{CE}[(1-\lambda)(1-F(\theta|e^{CE})) - \mu(-F_e(\theta|e^{CE}))]d\theta < 0.$$

Nonmonotonic contracts. The constraints that contracts are monotonic can be weaken by imposing a cost to sabotage the firm equal to $\alpha > 0$ times the magnitude of the manipulation. The constraints (2) and (3) now impose that the delta of the entrepreneur is comprised between $-\alpha$ and $1+\alpha$. Then, the slope of the optimal wage function switches between $1+\alpha$

in the incentive parts, and $-\alpha$ in the non-incentive parts if the wage is not already equal to zero

$$w(\theta) = \int_{\underline{\theta}}^{\theta} \left(-\alpha 1_{\{\frac{-F_e(t|e)}{1-F(t|e)} \leq \frac{\lambda}{\mu}\}} 1_{\{w(t)>0\}} + (1+\alpha) 1_{\{\frac{-F_e(t|e)}{1-F(t|e)} > \frac{\lambda}{\mu}\}} \right) v(s(t,p)) k dt \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}].$$

The asset price is still too low in the private economy since $(dNPV/dp)(p^{CE}) < 0$.

6 Concluding Remarks

In this paper I relax the MLRP assumption in the Innes (1990) model of moral hazard with continuous action and continuous payoff. The solution has a simple closed-form expression which can be intuitively interpreted in terms of shadow prices. This allows me to plug this optimal contracting framework in a general equilibrium setting to study the constrained efficiency of the competitive equilibrium, and to show that the asset price in the private economy is socially too high. I argue that this result is robust to a number of alternative assumptions. The tractability of the optimal contracting framework developed in this paper can also be useful for future research that needs a continuous payoff and a general moral hazard dimension.

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A Appendix

A.1 Proof of Lemma 1 and Proposition 1

The proof proceeds in several steps:

1. The incentive compatibility constraint is equivalent to the first order condition.
2. For $e \geq e_{\inf}$, there exists a real x such that

$$w^{e,k}(\theta) = \int_{\underline{\theta}}^{\theta} 1_{\{\frac{-F_e}{1-F}(t|e)>x\}} v(s(t))kdt \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}].$$

3. Existence of $e_{\sup}$.
4. $x = \lambda/\mu$ in the optimal contract.

Step 1. Let $w(.)$ a wage function that satisfies the constraints (1), (2) and (3). (2) and (3) imply that $w(.)$ is Lipschitz-continuous. Therefore, $w(.)$ is absolutely continuous and there exists a collection $\{\delta(t), t \in [\underline{\theta}, \overline{\theta}]\}$ such that

$$w(\theta) = w(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \delta(t)v(s(t))kdt \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}],$$

and $0 \leq \delta(\theta) \leq 1$, except possibly on a set of measure zero, by the monotonicity of the entrepreneur's and the investor's payoff functions. Besides, limited liability implies $w(\underline{\theta}) \geq 0$.

The second derivative of the expected wage

$$\int_{\underline{\theta}}^{\overline{\theta}} w(\theta) f_{ee}(\theta|e)d\theta = -\int_{\underline{\theta}}^{\overline{\theta}} \delta(\theta)v(s(\theta))kF_{ee}(\theta|e)d\theta$$

is strictly negative, implying that the first order condition is necessary and sufficient.[14]

Step 2. Let $e \geq e_{\inf}$ such that there exists a wage function that satisfies (1), (2), (3), and implements effort e. Notice first that $w^{e,k}(\underline{\theta}) = 0$, otherwise lowering the wage by the same constant for all θ relaxes the break-even constraint without affecting the incentive compatibility constraint. Writing $w^{e,k}(.)$ as in Step 1, let me show that there exists a real x such that

$$\delta(\theta) = 1_{\{\frac{-F_e}{1-F}(\theta|e)>x\}} \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}].$$

By contradiction, assume that there exists y such that the sets $\Theta_0 = \{\theta : \delta(\theta) > 0$ and $(-F_e/(1-F))(\theta|e) < y\}$ and $\Theta_1 = \{\theta : \delta(\theta) < 1$ and $(-F_e/(1-F))(\theta|e) > y\}$ are of strictly positive measure. I can build another wage function $\tilde{w}^{e,k}(.)$ that generates the same level of effort than $w^{e,k}(.)$ with a strictly lower expected wage:

$$\tilde{w}^{e,k}(\theta) = \int_{\underline{\theta}}^{\theta} \tilde{\delta}(t)dt \qquad \tilde{\delta}(\theta) = \begin{cases} 0 & \text{if } \theta \in \tilde{\Theta}_0, \\ 1 & \text{if } \theta \in \tilde{\Theta}_1, \\ \delta(\theta) & \text{otherwise,} \end{cases}$$

[14]If the wage function is constant, then the second derivative is equal to zero, but the first order condition cannot be satisfied.

where the subsets $\tilde{\Theta}_0 \subset \Theta_0$ and $\tilde{\Theta}_1 \subset \Theta_1$ are such that

$$\int_{\tilde{\Theta}_0} \delta(\theta)v(s(\theta))k(-F_e(\theta|e))d\theta = \int_{\tilde{\Theta}_1} (1-\delta(\theta))v(s(\theta))k(-F_e(\theta|e))d\theta > 0.$$

The new wage function $\tilde{w}^{e,k}(.)$ generates the same level of effort than $w^{e,k}(.)$, since

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}} (\tilde{w}^{e,k}(\theta) - w^{e,k}(\theta))f_e(\theta|e)d\theta &= \int_{\underline{\theta}}^{\overline{\theta}} (\tilde{\delta}(\theta) - \delta(\theta))v(s(\theta))k(-F_e(\theta|e))d\theta \\ &= -\int_{\tilde{\Theta}_0} \delta(\theta)v(s(\theta))k(-F_e(\theta|e))d\theta + \int_{\tilde{\Theta}_1} (1-\delta(\theta))v(s(\theta))k(-F_e(\theta|e))d\theta = 0,\end{aligned}$$

and it lowers the expected wage, since

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}} (\tilde{w}^{e,k}(\theta) - w^{e,k}(\theta))f(\theta|e)d\theta &= \int_{\underline{\theta}}^{\overline{\theta}} (\tilde{\delta}(\theta) - \delta(\theta))v(s(\theta))k(1-F(\theta|e))d\theta \\ &= -\int_{\tilde{\Theta}_0} \delta(\theta)v(s(\theta))k(1-F(\theta|e))d\theta + \int_{\tilde{\Theta}_1} (1-\delta(\theta))v(s(\theta))k(1-F(\theta|e))d\theta \\ &< -\int_{\tilde{\Theta}_0} \delta(\theta)v(s(\theta))k\frac{-F_e(\theta|e)}{y}d\theta + \int_{\tilde{\Theta}_1} (1-\delta(\theta))v(s(\theta))k\frac{-F_e(\theta|e)}{y}d\theta = 0,\end{aligned}$$

which contradicts the optimality of the wage function $w^{e,k}(.)$.

Step 3. To establish the existence of $e_{\sup}$, I first show that $x \geq 0$ in the definition of $w^{e,k}(.)$. To allege the notations, I use the linearity of the problem in k and work by unit of investment. In particular, $w^{e,k}(.) = z^e(.)k$. For any effort e and real y, I define $z(.;e,y)$ by

$$z(\theta;e,y) = \int_{\underline{\theta}}^{\theta} 1_{\{\frac{-F_e}{1-F}(t|e)>y\}} v(s(t))dt.$$

The level of effort implemented by $z(.;e,y)$, which I denote by $E(e,y)$, satisfies

$$0 = \int_{\underline{\theta}}^{\overline{\theta}} z(\theta;e,y)f_e(\theta|E(e,y))d\theta - E(e,y) = \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e)>y\}} v(s(\theta))(-F_e(\theta|E(e,y)))d\theta - E(e,y).$$

Differentiating with respect to y gives

$$\frac{\partial E(e,y)}{\partial y} = \frac{\frac{\partial}{\partial y}\Big|_{E(e,y)} \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e)>y\}} v(s(\theta))(-F_e(\theta|E(e,y)))d\theta}{1 + \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e)>y\}} v(s(\theta))F_{ee}(\theta|E(e,y))d\theta},$$

where the denominator is strictly positive since $F_{ee} > 0$. The numerator is equal to 0 if y is outside the image of $(-F_e/(1-F))(.|e)$. Otherwise, it has the opposite sign than y. Therefore, denoting the image of $(-F_e/(1-F))(.|e)$ by $[y_{\min}, y_{\max}]$, where $y_{\min} \leq 0$ and $y_{\max} > 0$, $E(e,y)$ is constant on $(-\infty, y_{\min}]$, strictly increasing on $[y_{\min}, 0]$ (unless $E(e,y) = \underline{e}$), strictly decreasing on $[0, y_{\max}]$ (unless $E(e,y) = \overline{e}$), and constant on $[y_{\max}, +\infty)$. Besides, $E(e,0)$ is equal to e^{FB} if $y_{\min} = 0$, and strictly larger than e^{FB} if $y_{\min} < 0$, and $E(e, y_{\max}) = \underline{e}$. Therefore, there exists a unique $x \geq 0$, and possibly another $x' < 0$, such that $E(e,x) = e$. The expected wage is strictly smaller with $x \geq 0$.

Finally, a level of effort e can be implemented if, and only if, $E(e,0) \geq e$, or

$$\int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e)>0\}} v(s(\theta))k(-F_e(\theta|e))d\theta - e \geq 0.$$

The LHS of this expression is decreasing in e, and positive in e^{FB}, therefore there exists $e_{\sup} \geq e^{FB}$ such that e is implementable if, and only if, $e \in [\underline{e}, e_{\sup}]$.

Step 4. Given the form of $w^{e,k}(.)$, problem $\mathcal{P}$ rewrites as

$$\max_{k,\,e,\,x} \quad \left[-1+\int_{\underline{\theta}}^{\overline{\theta}}[\theta v(s(\theta))-(s(\theta)-1)p]f(\theta|e)d\theta-e\right]k,$$

$$\text{s.t.} \quad \left[-1+\int_{\underline{\theta}}^{\overline{\theta}}[\theta v(s(\theta))-(s(\theta)-1)p]f(\theta|e)d\theta\right]k-\int_{\underline{\theta}}^{\overline{\theta}}1_{\{\frac{-F_e}{1-F}(\theta|e)>x\}}v(s(\theta))k(1-F(\theta|e))d\theta+A\geq 0,$$

$$\int_{\underline{\theta}}^{\overline{\theta}}1_{\{\frac{-F_e}{1-F}(\theta|e)>x\}}v(s(\theta))k(-F_e(\theta|e))d\theta-k=0,$$

where the expected wage has been integrated by parts. The derivative of the associated Lagrangian with respect to x has the same sign than $(\lambda-\mu x)(1-F(\theta|e))$. The first order condition therefore implies $x=\lambda/\mu$.

A.2 Proof of Lemma 2

The existence of the competitive equilibrium comes from the intermediate value theorem. First, $p^*(e^*(.))$ is continuous. Indeed, $p^*(e)$ is continuous by equation (11). $e^*(p)$ is continuous because it is the argmax of $V(e,p)K(e,p)$, which is strictly concave in e by Assumption 2-(b) and continuous in p. Then, $p^*(e^*(.))$ is comprised between $\underline{\theta}v'(1)$ and $\overline{\theta}v'(1)$, since all firms are on the buy side when $s(\underline{\theta},p)\geq 1$ and on the sell side when $s(\overline{\theta},p)<1$. Therefore $p^*(e^*(.))$ has a fixed point in $[\underline{\theta}v'(1),\overline{\theta}v'(1)]$.

The equilibrium is unique since $p^*(e^*(.))$ is strictly decreasing in the neighborhood a fixed point. $e^*(.)$ is strictly decreasing by Assumption 3. $p^*(.)$ is strictly increasing by equation (11), since $E[s(\theta,p)|e]$ is strictly decreasing in p and strictly increasing in e by Assumption 1-(b).

A.3 Proof of Proposition 2

I first prove the existence and uniqueness of the first-best. I then solve for the second-best.

First-best. Since $p^*(e)$ is strictly decreasing, it suffices to show that $e^{FB}(p)$ is strictly decreasing to obtain the uniqueness of the first-best allocation. For a given p, the first-best level of effort is defined by $(\partial V/\partial e)(e^{FB}(p),p)=0$ and the second order condition is satisfied by Assumption 1-(a). Then

$$\frac{\partial e^{FB}}{\partial p}=\frac{\partial^2 V/\partial e\partial p}{\partial^2 V/\partial e^2}=-\frac{\frac{d}{de}E[s(\theta)|e]}{|\partial^2 V/\partial e^2|}<0$$

by Assumption 1-(b).

The existence is ensured by the intermediate value theorem as in the proof of the existence of the competitive equilibrium.

Second-best. The problem of the social planner consists in maximizing $V(e,p^*(e))K(e,p^*(e))$ on e. Assumption 4 ensures that the solution is given by the first order condition

$$\frac{d(VK)}{de}(e,p^*(e))=\frac{\partial(VK)}{\partial e}(e,p^*(e))+\frac{dp^*}{de}(e)\frac{\partial(VK)}{\partial p}(e,p^*(e))=0.$$

I now show that the solution $e^{SB}\in(e_{\inf}(p^{CE}),e^{CE})$. First, $(d(VK)/de)(e,p^*(e))$ is strictly positive for $e=e_{\inf}(p^{CE})$ since $V=0$ and $\partial(VK)/\partial e>0$ in that point. Second, I claim that it is strictly negative for $e = e^{CE}$. Indeed, $(\partial(VK)/\partial e)(e^{CE}, p^{CE}) = 0$ and $dp^*/de > 0$. It remains to show that $(\partial(VK)/\partial p)(e^{CE}, p^{CE}) < 0$. For a given p, $(VK)(e^*(p), p)$ is given by

$$
\begin{aligned}
\max_{k,\, e,\, x} \quad & \left[-1 + \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(s(\theta,p)) - (s(\theta,p)-1)p] f(\theta|e) d\theta - e\right] k, \\
\text{s.t.} \quad & \left[-1 + \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(s(\theta,p)) - (s(\theta,p)-1)p] f(\theta|e) d\theta\right] k - \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e) > x\}} v(s(\theta,p)) k (1 - F(\theta|e)) d\theta + A \geq 0, \\
& \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e) > x\}} v(s(\theta,p)) k (-F_e(\theta|e)) d\theta - k = 0,
\end{aligned}
$$

with shadow prices λ and μ. Using that $x = \lambda/\mu$, we have

$$
\frac{d(VK)}{dp}(e^{CE}, p^{CE}) = \int_{\underline{\theta}}^{\overline{\theta}} 1_{\{\frac{-F_e}{1-F}(\theta|e^{CE}) > \frac{\lambda}{\mu}\}} \sigma(\theta, p^{CE}) k^{CE} [\lambda(1 - F(\theta|e^{CE})) - \mu(-F_e(\theta|e^{CE}))] d\theta < 0,
$$

with $\sigma(\theta,p) = -(\partial s/\partial\theta)(\theta,p) = v'(s(\theta,p))/(\theta|v''(s(\theta,p))|) > 0$, and where I have used the market clearing condition $\int_{\underline{\theta}}^{\overline{\theta}} (s(\theta, p^{CE}) - 1) f(\theta|e^{CE}) d\theta = 0$. Therefore $e^{SB} \in (e_{\inf}(p^{CE}), e^{CE})$, which implies $p^{SB} < p^{CE}$ since $dp^*/de > 0$.