Monopoly Behaviour with Speculative Storage

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Abstract

We analyze the effects of competitive storage when the production of the good is controlled by a monopolist. The existence of competitive storers serves to reduce the monopolist’s effective demand when speculators are selling and to increase it when they are buying. This results in the monopolist manipulating the frequency of stockouts, and hence, the price-smoothing effects of competitive storage. We find that competitive storage affects both the level and volatility of price under monopoly. The average price level is higher with storage due to the monopolist’s desire to induce stockouts by occasionally keeping the price just at the level that induces a stockout. Although storage does reduce the volatility of prices under monopoly production, prices are more volatile than they would be under perfectly competitive production, even though stockouts occur less frequently under monopoly. These results are demonstrated through closed-form solutions of the two-period version of the model and computational solutions to the infinite horizon of the model.
1 Introduction

Competitive storage has the potential to be an important influence on the behaviour of firms with market power. For many storable commodities, production is undertaken in concentrated industries. Examples include petroleum, natural gas, nickel, tin\(^1\), and aluminum. Although the storage technology differs across these commodities, they generally share the feature that storage by intermediaries is not precluded. For the non-ferrous metals traded on the London Metal Exchange, it is relatively simple to store the commodity as the exchange organizes storage facilities. In effect, any individual can purchase and store these metals. Storage of commodities like natural gas is more complicated, although deregulation in the U.S.A. over the past couple of decades has resulted in entry of independent storage firms.\(^2\) For all of these commodities, it is natural to ask how speculative storage will affect the use of market power. We analyze this question by examining the effects that competitive storage has on the behaviour of a monopolist/cartel in order to highlight any incentives that firms with market power have to influence speculative activity, and consequently affect the distribution of prices. Speculation can have two different effects on the residual demand faced by firms. When they are selling their stocks, speculators are competing with producers, reducing residual demand. Conversely, when speculators are accumulating inventory, they increase residual demand. Firms with market power then have an incentive to lower price in order to induce speculative purchases, but this comes at the cost of increased speculative sales in the future. In this paper, we demonstrate that these incentives do result in changing the way in which speculative activity affects the distribution of prices.

One can view production and storage as sequential stages of activity for which agents that store the good need not be the same as those that produce it, in this way allowing for different degrees of market power at different points in the supply chain. Much of the work that has been done on the effects of speculative storage imposes perfect competition at both the production and storage stages. The theory under these conditions has been well established by Samuelson [13], Newbery and Stiglitz [12], Scheinkman and Schechtman [14], Williams and Wright [16], and Deaton and Laroque [2]. As this body of work demonstrates, the constraint that inventories be non-negative causes the distribution of price to have two regimes: one corresponding to positive inventories being held, and one corresponding to stock-outs.

The effects of market power in the storage activity has been examined by Newbery [11], Williams and Wright [16] (Chapter 11), and McLaren [8]. Newbery [11] shows that a firm with monopoly power over storage will smooth harvest fluctuations more than a competitive storage sector would. In an extension to his basic model, he allows the monopolist

\(^{1}\)The International Tin Agreement collapse in 1985 was partly attributable to speculators’ behaviour (Anderson and Gilbert [1])

\(^{2}\)As of 2005, independent storage operators accounted for 13% of storage capacity in the U.S.A. (Energy Information Association [4]). This represented a substantial increase over just a few years prior.
to control the harvest and demonstrates that the monopolist’s residual demand is kinked when the price causes competitive storage to occur, although he does not solve this version of model. The model we develop in this paper also generates this type of residual demand for the monopolist. McLaren [8] examines an oligopoly in the storage activity and finds that in a Markov-perfect equilibrium, the oligopoly smooths price less than perfectly competitive firms do. In contrast to these papers that examine market power in storage with competitive production, we examine market power in production with competitive storage.

The usual approach in models of competitive storage has random production as the source of uncertainty. We wish to consider endogenous production, so in our model the source of uncertainty is a random component of demand. The problem of a monopolist facing stochastic demand for its output was first addressed by Mills [9]. The timing assumed by Mills is quite different from ours, in particular, Mills has demand uncertainty resolved after production decisions are made. This implies that production is not adjusted in response to demand conditions and conditions of excess demand can occur. A market in which both production and storage occur under imperfect competition is examined in Thille [15]. Cournot duopolists are able to store their output, giving them a strategic motive to use inventories to reduce marginal production costs. It is shown that firms with market power make less use of inventories than is efficient, resulting in more volatile prices in the face of demand fluctuations and less volatile prices in the face of cost fluctuations. The logic underlying this result is also present in our model. Imperfectly competitive firms face altered incentives to vary sales and production in response to random events due to the fact that they adjust quantities in order to maintain the equality of marginal revenue and marginal cost. To the extent that the slope of marginal revenue differs from that of demand, the adjustment of sales and production differs from that of a perfectly competitive industry. We examine how the introduction of competitive storage affects the application of this logic.

In order to analyse the effects of competitive speculation on a firm with market power, we separate production and storage completely by examining a monopolist that is unable to store the good, so that any storage that occurs must be done by the competitive storers. However, the monopolist is able to induce storage by manipulating price to induce speculative purchases or sales. There are two broad questions that we ask. How does speculation affect the behaviour of a monopolist? How does the frequency of stock-outs and the distribution of price differ under monopoly as opposed to competitive production? We find that competitive storage results in a price slightly higher, but less volatile than occurs in the absence of storage. The price level is higher due to an incentive to raise price to induce a stockout, resulting in no stocks carried into the following period. Compared to perfectly competitive production, we find that stockouts occur less frequently under the monopolist even though prices are more volatile. In what follows, we first present the general model. We then examine a two period model for which we can get a closed form solution, after which we analyze an infinite horizon model using numerical techniques.
2 The Model

We consider a discrete time economy with a horizon $T$ which may be infinite. In every period $t = 1, ..., T$, consumers have a demand $D_t$ for a homogeneous and non-perishable product they can buy on a spot market. Consumers' demand in period $t$, $D_t$, is a decreasing function of the spot price $p_t$, and is an increasing function of a random state $a_t$ which represents consumers' maximum willingness to pay for the product in this period. We assume that consumers' demand is a linear function of $p_t$ and $a_t$, given by

$$D_t = \max\{a_t - p_t, 0\},$$

(1)

where the random state $a_t$ is drawn by Nature at the beginning of period $t$ and known to every participant of the spot market before decisions are made. All market participants have rational expectations over future demand conditions, but these conditions are not known before they are realized: only the distribution of the future random states is known. We assume that random states $\{a_t\}_{t=1,...,T}$ are independently and identically distributed according to a time-invariant cumulative distribution function $F$.

It is important to note that our assumption of linear demand is matters for the results concerning price volatility. It was pointed out in Newbery [11] that the implications of market power for price volatility is sensitive to the demand specification. We maintain the linear demand specification largely because it allows us to compute closed-form solutions in the two-period case. Linear demand has the additional simplification that when combined with an additive random term, it is inconsequential whether demand or its inverse is used (see Mills [9]).

A comment is in order concerning the relationship of our model to the standard commodity storage model\(^3\). The standard model is usually concerned with how random production is allocated across periods, which is a sensible assumption given the type of commodity envisaged: an agricultural one with competitive producers facing uncertainty due to weather. For these models, a production decision corresponds to planting a crop and it is natural to consider a random output following the planting decision as the dominant source of uncertainty. For commodities such at metals and natural gas, although supply disruptions occur, demand fluctuations are also important. Since we want to focus on a situation in which production is endogenous, we put the stochastic element of the model into demand.\(^4\)

In every period $t$ a fringe of independent storers, the competitive speculators, are able to buy or sell on the spot market, and are able to store the product. We treat speculators as separate agents from consumers in order to simplify the exposition. What is important is that aggregate consumption and storage decisions can be treated separately from each other. In particular, consumption is not directly affected by the level of stocks, past prices,

\(^3\)See Chapter 2 of Williams and Wright [16] for an exposition of the standard commodity storage model.

\(^4\)We could implement uncertainty in cost instead of demand, although even this does not exactly capture the notion of random production. See Thille [15] for a comparison of demand and cost uncertainty.
or expected future prices. This point is made for the standard competitive storage model in Williams and Wright [16] (pp 28-29). Let $x_t$ denote the position of speculators on the spot market of period $t$: if $x_t$ is positive, then speculators are selling the product, while if $x_t$ is negative speculators are buying the product. Speculators are able to store the product at a unit cost of $w$ per period and we denote $h_t$ the amount of available stocks at the beginning of period $t$. Inventories do not depreciate. The transition equations for inventories is then

$$h_{t+1} = h_t - x_t.$$  

(2)

Negative inventories are not allowed and an aggregate storage capacity of $\bar{h}$ that cannot be overcome is available. This aggregate storage capacity constraint does not appear in the standard commodity storage model. We include it for two reasons. First, for some commodities such as natural gas and oil, storage facilities are specialized to the commodity and so cannot be quickly increased. Second, we make this assumption to allow us to consider situations in which speculators are “small” in aggregate. This allows a simpler presentation of the two-period model below. The possibility that $\bar{h}$ is so large that it does not represent a binding constraint is considered in the infinite horizon version of the model presented below. Consequently, in every period aggregate speculative sales must satisfy

$$x_t \in [h_t - \bar{h}, h_t].$$  

(3)

Finally, we assume that final inventories, $h_{T+1}$, can be destroyed at no cost. Let the discount factor be $\delta \leq 1$, and let $E_t$ denote the expectation operator conditional on the information available in period $t$, the payoffs to competitive speculators from the sequence of aggregate sales are equal to

$$\Pi^S_0 = E_0 \sum_{t=0}^{T} \delta^t (p_t x_t - wh_t).$$  

(4)

We assume that the monopolist’s choice variable is price. After observing the current state of demand and speculative stocks, it sets a price and then is committed to produce the quantity that is demanded of it. $^5$ The monopolist produces output $q_t$ using a decreasing returns to scale technology conforming to the assumed convex cost function

$$C(q_t) = \frac{c}{2}q_t^2,$$  

(5)

where the rate of increase of the marginal cost, $c$, is constant. The monopolist cannot store its output, but due to the convex production costs is interested in storage to smooth production costs. At the price it chooses, the monopolist has to serve the demand addressed

$^5$We get the same solutions if the monopolist chooses output and then price is set by an auctioneer. We assume the monopolist chooses price as this allows a somewhat less complicated presentation.
to him. Its payoff is the discounted sum of expected profits

$$\Pi_0^m = E_0 \sum_{t=0}^T \delta^t \pi_t^m$$  \hspace{1cm} (6)

where $$\pi_t^m = (p_t q_t - \xi q_t^2)$$ and the quantity produced, $$q_t$$, must equal the total quantity demanded,

$$q_t = -x_t + D_t.$$  \hspace{1cm} (7)

To summarize the timing of the events within a period we have that first the available stocks, $$h_t$$ and current realization of the demand shock, $$a_t$$, are observed by all parties. The monopolist then chooses a price, $$p_t$$. Speculators observe the price and decide how much to buy or sell, $$x_t$$ and consumers observe the price and decide how much to purchase, $$D_t$$. Finally, the monopolist produces the quantity $$q_t$$ equal to the sum of consumption and speculative demand.

3 Analysis

As is well known (see for example Williams and Wright [16]), in most cases it is not possible to obtain closed form expressions for the equilibrium outcomes due to the dependence of the solution on the expected future price. In this section, we discuss the general form of the solution, which forms the base underlying our solutions to the two-period and infinite horizon cases.

3.1 The behaviour of speculators

As in standard commodity storage models, speculators behaviour is driven by the relationship between current and expected future prices. In particular, if the current price equals the discounted future price less storage costs, $$p_t = \delta E_t[p_{t+1}|h_{t+1}] - \delta w$$, speculators are indifferent to any action available to them. We say the market is in a smoothing regime.

If the current spot price is strictly higher than the discounted expected future price minus the discounted storage cost, $$p_t > \delta E_t[p_{t+1}|h_{t+1}] - \delta w$$, then speculators wish to sell as much as possible. As they increase their sales, $$h_{t+1}$$ falls, changing $$E_t[p_{t+1}|h_{t+1}]$$. In the standard commodity storage model, $$p_t$$ would also fall with speculative sales, but as $$p_t$$ is chosen by the monopolist here all the adjustment occurs through $$E_t[p_{t+1}|h_{t+1}]$$. If it does not fall enough to remove the incentive to sell stocks a stockout occurs. In this case, equilibrium prices and speculative sales satisfy the following complementarity condition:

$$p_t - \delta E_t[p_{t+1}|0] + \delta w > 0 \quad \Rightarrow \quad x_t = h_t$$  \hspace{1cm} (8)

In this case we will say that the market is in a stock-out regime.

\(^6\)We expect that $$E_t[p_{t+1}|h_{t+1}]$$ is decreasing in $$h_{t+1}$.  \hspace{1cm} (8)
If the current spot price is strictly lower than the discounted expected price of next period minus the discounted storage cost, \( p_t < \delta E_t[p_{t+1}|h_{t+1}] - \delta w \), speculators wish to store as much as possible. Again, as \( p_t \) is fixed by the monopolist, all adjustment occurs through the expected future price. If insufficient capacity is available to restore the smoothed price relationship, speculators store up to their capacity and equilibrium prices and speculative sales satisfy the following complementarity condition:

\[
p_t - \delta E_t[p_{t+1}|\bar{h}] + \delta w < 0 \quad \Rightarrow \quad x_t = h_t - \bar{h}.
\] (9)

In this case we will say that the market is in a capacity regime. This regime is absent in the standard commodity storage model, but note that our approach is equivalent to the standard one for \( \bar{h} \) sufficiently large, a case that we include in section 5. The price relationship in (9) is consistent with free entry of speculators who must bid to use the available capacity. We do not model the resulting equilibrium price of capacity explicitly, but note that in the capacity regime, the price to use storage facilities would then rise above \( w \) to reflect the shadow price of the storage constraint, which is equal to \( \delta E_t[p_{t+1}|h_{t+1}] - p_t - \delta w \).

In summary, speculative sales are given by

\[
x_t = \begin{cases} 
  h_t - \bar{h} & \text{if } p_t < \delta E_t[p_{t+1}|\bar{h}] - \delta w \\
  \in [h_t - \bar{h}, h_t] & \text{if } p_t = \delta E_t[p_{t+1}|h_{t+1}] - \delta w \\
  h_t & \text{if } p_t > \delta E_t[p_{t+1}|0] - \delta w
\end{cases}
\] (10)

3.2 The behaviour of the monopolist

The monopolist must know the precise level of speculative sales for any price it chooses in order for it to determine its residual demand (see (7)). In order to do this we assume the existence of a unique equilibrium in which \( E_t[p_{t+1}|h_{t+1}] \) is a smooth, monotonically decreasing function of \( h_{t+1} \), which we denote \( g(h_{t+1}) \). Under this assumption the monopolist’s problem can be clearly defined.

To determine speculative sales in the smoothing regime, we use \( p_t = \delta g(h_{t+1}) - \delta w \) to find

\[
h_{t+1} = g^{-1}(p_t/\delta + w) \equiv \psi(p_t),
\] (11)

i.e. \( \psi(p_t) \) is the level of stocks that need to be carried to the next period in order to not induce a stockout or capacity storage regime. Under the assumption that \( g() \) is smooth and monotonic, \( \psi() \) is smooth and monotonic.

\[\text{Our assumption of IID demand shocks means that } a_t \text{ does not affect this expectation.}\]
We can now write speculative sales more precisely as

\[
x_t = \begin{cases} 
    h_t - \bar{h} & \text{if } p_t < p_\ell \\
    h_t - \psi(p_t) & \text{if } p_\ell \leq p_t \leq p_u \\
    h_t & \text{if } p_t > p_u 
\end{cases}
\]

(12)

where \(p_\ell\) is the price threshold that triggers the capacity regime and \(p_u\) is the price threshold that triggers the stockout regime. Continuity implies that

\[
p_\ell = \delta g(\bar{h}) - \delta w, \tag{13}
\]

and

\[
p_u = \delta g(0) - \delta w. \tag{14}
\]

We can now express the residual demand faced by the monopolist as

\[
D^R(p_t; a_t, h_t) = \begin{cases} 
    a_t - p_t - h_t + \bar{h} & \text{if } p_t < p_\ell \\
    a_t - p_t - h_t + \psi(p_t) & \text{if } p_\ell \leq p_t \leq p_u \\
    a_t - q_t - h_t & \text{if } p_t > p_u 
\end{cases}
\]

(15)

Given \(g(h_{t+1}) = E_t[p_{t+1}|h_{t+1}]\), we can solve for the monopolist’s optimal price in period \(t\). Even though the monopolist has no direct control over the future state, it still faces a dynamic optimization problem since it can influence the level of inventories that speculators carry forward into the next period via its choice of price. The Bellman equation for the monopolist’s problem is

\[
V^m(a_t, h_t) = \max_{p_t} \left\{ p_t D^R(p_t; a_t, h_t) - \frac{c}{2} [D^R(p_t; a_t, h_t)]^2 + \delta E_t V^m(a_{t+1}, h_{t+1}) \right\} \tag{16}
\]

subject to \(h_{t+1} = h_t - x_t\) and \(x_t\) given by (12). There are five cases to be analyzed corresponding to the interiors of the three cases in (15) and the two boundaries.

1. **Stock-out regime, \(p_t^* > p_u\)**: Here, speculators sell their entire stock in \(t\), so \(h_{t+1} = 0\) and the monopolist’s choice of output has no influence on next period’s state. If the solution is interior to this region, the optimal price is that which maximizes its static profit with \(x_t = h_t\):

\[
p_t^* = \frac{1 + c}{2 + c} (a_t - h_t) \tag{17}
\]

and output will be

\[
q_t^* = \frac{a_t - h_t}{2 + c} \tag{18}
\]
resulting in the value for the monopolist of

\[ v_1(a_t, h_t) = \frac{(a_t - h_t)^2}{2(2 + c)} + \delta E_t V^m(a_{t+1}, 0) \] (19)

2. **Stock-out boundary,** \( p^*_t = p_u \): It is possible that a solution occurs on the boundary of the stock-out region in which case we have

\[ p^*_t = p_u, \quad q^*_t = a_t - h_t - p_u \] (20)

and the value is

\[ v_2(a_t, h_t) = p_u q^*_t - \frac{c}{2} [q^*_t]^2 + \delta E_t V^m(a_{t+1}, 0) \] (21)

3. **Smoothing regime,** \( p^*_t \in (p_L, p_u) \): In this case we have

\[ q_t = a_t - p_t - h_t + \psi(p_t) \]

so \( p^*_t \) solves

\[ \max_{p_t} \{p_t[a_t - p_t - h_t + \psi(p_t)] - \frac{c}{2} [a_t - p_t - h_t + \psi(p_t)]^2 + \delta E_t V^m(a_{t+1}, \psi(p_t))\} \]. (22)

For an interior optimum in this region, given the assumed differentiability of \( g() \) and hence \( \psi() \), the following necessary condition must hold\(^8\):

\[ (\psi'(p_t) - 1)(p_t - c(a_t - p_t - h_t + \psi(p_t))) + a_t - p_t - h_t + \psi(p_t) + \delta \frac{\partial E_t V^m(a_{t+1}, \psi(p_t))}{\partial p_t} = 0 \] (23)

Given a solution, \( p^*_t \) to (23) and corresponding \( q^*_t \), we have \( v_3(a_t, h_t) \) as the corresponding value in this region.

4. **Capacity boundary,** \( p^*_t = p_L \): If the solution occurs on the boundary of the capacity region we have

\[ p^*_t = p_L, \quad q^*_t = a_t - (h_t - \bar{h}) - p_L \] (24)

and the value is

\[ v_4(a_t, h_t) = p_L q^*_t - \frac{c}{2} [q^*_t]^2 + \delta E_t V^m(a_{t+1}, \bar{h}) \] (25)

5. **Capacity regime,** \( p^*_t < p_L \): In this case, speculators purchase and carry the maximal amount of stocks into the next period: \( h_{t+1} = \bar{h} \). Since this means that next period’s speculative stocks are unaffected by changes in the monopolist’s output, the optimal output for the monopolist is that which maximizes its static profit, given \( x_t = h_t - \bar{h} \). If the solution is interior to this region we have

\[ p^*_t = \left( \frac{1 + c}{2 + c} \right) (a_t - (h_t - \bar{h})) \] (26)

\(^8\)We are also assuming here that the expectation of next period’s value function is differentiable with respect to stocks.
\[ q^*_t = \frac{a_t - (h_t - \bar{h})}{2 + c}, \tag{27} \]

and

\[ v_5(a_t, h_t) = \frac{(a_t - (h_t - \bar{h}))^2}{2(2 + c)} + \delta E_t V^m(a_{t+1}, \bar{h}). \tag{28} \]

To summarize this analysis, the monopolist’s price and output in \( t \) will be determined by which of the regimes provides the highest value. Consequently

\[ V^m(a_t, h_t) = \max[v_1(a_t, h_t), v_2(a_t, h_t), v_3(a_t, h_t), v_4(a_t, h_t), v_5(a_t, h_t)]. \tag{29} \]

Note that the functional form of \( v_1, v_2, v_4 \) and \( v_5 \) are known. They are quadratic functions of \( a_t \) and \( h_t \) since \( \{a_t\}_{t=1}^{T} \) are independently distributed. The solution in the smoothing region is more complex. In all cases, the solution requires knowledge of the unknown functions \( V^m \) and \( g \) (as it determines \( p_t, p_u, \) and \( \psi() \)). In the next section, we restrict the time horizon to two periods which simplifies the analysis sufficiently for us to obtain a closed-form solution. Following that, we examine numerical solutions to the infinite horizon version of the model.

4 Two period horizon

As a first step in our analysis of the model described in the previous section we examine the equilibrium when there are only two periods. Although restrictive, this exercise has the advantage that a closed-form solution can be obtained. We first solve the problem with monopoly production and then compare it to the case in which production is perfectly competitive. We assume throughout this section that the distribution of the demand intercept has support bounded away from zero. Define \( \underline{a} \) to be the lower bound of the support. We maintain the following assumption:

Assumption 1 Speculators are small: \( \bar{h} < \underline{a} \).

Assuming that speculators storage capacity is limited seems realistic in practice for some commodities and is made to ensure that selling the maximal amount of inventories in the second period will not drive the price to zero, in which case the monopolist would produce nothing.

In the second period speculators have inventories of \( h_2 \) available. Since the storage cost is sunk at this point and they are price-takers, they will sell their entire stock as long as the market price for the product is non-negative. Under Assumption 1, speculative inventories can never be such that a stock-out in the second period causes the spot price to be zero. Profits to speculators in period two are thus \((p_2 - w)h_2\). In the first period, speculators have \( h_1 \) stocks available and choose sales as described by (12).
4.1 Monopolistic production, competitive speculation

The objective of the monopoly is to maximize its expected total profit taking into account the impact of its choice on future inventories. The Bellman equation for the monopolist’s problem is

\[ V^m_1(a_1, h_1) = \max_{p_1} \left\{ p_1 D^R(p_1; a_1, h_1) - \frac{c}{2} (D^R(p_1; a_1, h_1))^2 + \delta E_1 V^m_2(a_2, h_2) \right\} \] (30)

The price and quantity sold in second period result from the maximization of the second period monopoly profit \( \pi^m_2 = p_2 (a_2 - h - p_2) - c (a_2 - h - p_2)^2 / 2 \) with respect to \( p_2 \), giving

\[ p^m_2 = \beta^m (a_2 - h_2), \] (31)

and

\[ V^m_2(a_2, h_2) = \frac{(1 - \beta^m)}{2} (a_2 - h_2)^2, \] (32)

where \( \beta^m = (1 + c)/(2 + c) \).

Given (31), we have \( g(h_2) = E_1(p_2|h_2) = \beta^m (E(a) - h_2) \), from which we derive the price thresholds (13) and (14):

\[ p^m_\ell = \delta g(\bar{h}) - \delta w = \delta \beta^m (E(a) - \bar{h}) - \delta w \] (33)

and

\[ p^m_u = \delta g(0) - \delta w = \delta \beta^m E(a) - \delta w. \] (34)

For \( p_1 \in [p^m_\ell, p^m_u] \) speculative sales must be sufficient to ensure that \( p_1 = \delta g(h_2) - \delta w \). Using the above expression for \( g(h_2) \) we have

\[ h_2 = \psi(p_1) = E(a) - \left( w + \frac{P_1}{\delta} \right) / \beta^m. \] (35)

We can now write the residual demand faced by the monopolist:

\[ D^R(p_1; a_1, h_1) = \begin{cases} 
  a_1 - h_1 - p_1 + \bar{h} & \text{if } p_1 < p^m_\ell \\
  a_1 - h_1 - p_1 + E(a) - (w + \frac{P_1}{\delta}) / \beta^m & \text{if } p_1 \in [p^m_\ell, p^m_u] \\
  a_1 - h_1 - p_1 & \text{if } p_1 > p^m_u 
\end{cases} \] (36)

The first period value function can now be constructed from (29). We present the solution to the monopolist’s problem in Proposition 1 below, but first note that the objective function for the monopolist’s problem is continuous but non-differentiable at prices \( p^m_\ell \) and \( p^m_u \). It is important to note that both the residual demand, (36), and stocks carried into the second period (and hence second period profit) exhibit kinks at these prices. More precisely marginal profit jumps jumps up at price \( p^m_\ell \) and down at price \( p^m_u \). The reason is
the following: when moving from prices below to prices above \( p^m_1 \), the market moves from a capacity to a smoothing regime. Although the second period residual demand is unaffected by a marginal change in the first period price as long as speculators store up to capacity, it positive in the smoothing region, as fewer stocks held result in higher second period profit. Consequently, while first period marginal profit jumps down at \( p_1^m \), second period marginal profit jumps up. We show in the proof to the following proposition that the second period effect dominates. The upward jump in the marginal payoff at \( p_1^m \) means that there can be two local maxima, however, we show that the global maximum in this case occurs at the price in the smoothing regime. These situation is reversed at the threshold price \( p^m_u \), where the marginal payoff to the monopolist exhibits a downward jump. This downward jump in marginal payoff implies that there is a set of \( a_1 \) and \( h_1 \) values for which the monopolist charges the same price, \( p_u \).

These statements are presented more formally in Proposition 1, but first we introduce an additional assumption that speculators’ capacity is not too small, which ensures that there is a non-negligible smoothing region:

\textbf{Assumption 2}

\[
\frac{\delta (1 - \beta^m)^2 w}{\beta^m(\delta \beta^m)^2 + 2 \beta^m - 1} \leq (1 + \delta) \bar{h}.
\]

We can now characterize the equilibrium under monopolistic production:

\textbf{Proposition 1} Under Assumptions 1 and 2, and for a given \((a_1, h_1)\), defining the coefficient \( K \) as \( K \equiv \delta \beta^m(1 + \delta)/(\delta \beta^m^2 + 2 \beta^m - 1) \), the equilibrium under monopoly is characterized as follows:

1. (Stock-out) If \( a_1 - h_1 - \delta E(a) + \delta w/\beta^m \geq 0 \) then
   \[
   p^*_1 = \beta^m(a_1 - h_1) \quad \text{and} \quad p^*_2 = \beta^m a_2
   \]

2. (Limit) If \( a_1 - h_1 - \delta E(a) + \delta w/\beta^m < 0 \leq a_1 - h_1 - \delta E(a) + Kw \) then
   \[
   p^*_1 = p^m_u = \delta \beta^m E(a) - \delta w \quad \text{and} \quad p^*_2 = \beta^m a_2
   \]

3. (Smoothing) If \( a_1 - h_1 - \delta E(a) + Kw < 0 \leq a_1 - h_1 - \delta E(a) + (1 + \delta)\bar{h} + Kw \) then
   \[
   p^*_1 = \frac{\delta \beta^m(a_1 - h_1 + E(a))}{1 + \delta} + \frac{\delta(1 - 2 \beta^m) w}{\delta \beta^m^2 + 2 \beta^m - 1}
   \]
   and
   \[
   p^*_2 = \beta^m a_2 + \frac{\beta^m(a_1 - h_1 - \delta E(a))}{1 + \delta} + \frac{\delta \beta^m^2 w}{\delta (\beta^m)^2 + 2 \beta^m - 1}
   \]
4. (Capacity) Finally, if \( a_1 - h_1 - \delta E(a) + (1 + \delta)\bar{h} + Kw < 0 \) then

\[
p_1^* = \beta^m(a_1 - h_1 + \bar{h}) \quad \text{and} \quad p_2^* = \beta^m(a_2 - \bar{h}).
\]

where \( \beta^m = (1 + c)/(2 + c) \).

**Proof:** See appendix.||

The regime labeled “limit” is called this because of its relation to the idea of limit-pricing. The monopolist keeps out “entry” of speculators into period 2 by keeping price high. The high price causes existing stocks to be sold and the monopolist to have no competition from speculators in the second period. We plot the areas of \((a_1, h_1)\) where the various regimes of Proposition 1 occur in the right panel of Figure 1.

It is interesting to compare the level of prices charged by a monopoly which faces speculation with the level of prices charged by a static monopoly, which is equal to \( \beta^m a_t \) in any period. Because of the speculative activity, prices are often higher compared to the static monopoly in one period and lower in the other. There is however an interesting case: when a stock-out occurs at price \( p_m^l \), the price may be higher than what a static monopoly would charge in the first period, and equal to the static monopoly price in second period (case 2 of Proposition 1). This implies that the average price is increased by speculative activity. The region where this occurs is represented by the shaded part of the limit region in Figure 1.

We now turn to a discussion of the implications of these results for price volatility. We state two caveats about what the two period model can say about price volatility. First, the level of \( h_1 \) is exogenous in the two period model, so all moments are conditional on the value of \( h_1 \). Second, the volatility of the second period price is not as interesting that of the first period price since all inventories are sold in the second period. Therefore we focus on the volatility of the first period price only. As the first period price is a linear function of \( a_1 \), we can discuss volatility (conditional on \( h_1 \)) as determined by the derivative of price with respect to \( a_1 \). We say that price is more responsive to demand shocks in a region if the derivative is larger. No matter the distribution, the variance of first period price is lower when storage is possible. This can be seen by comparing the responsiveness of prices in the two cases: either it is identical in both cases, or the responsiveness when the monopoly faces speculators is lower.

### 4.2 Perfect competition

We wish to present for comparison to the monopoly case the equilibrium to an equivalent perfectly competitive market. Since the cost function for the monopolist exhibits decreasing returns to scale, the number of competitive firms is not irrelevant. We assume that

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\(^9\) We are grateful to an anonymous referee for suggesting this interpretation.
production is undertaken by a fixed number of perfectly competitive firms whose aggregate marginal cost function is the same as the monopolist’s, equal to \( cq \). In this situation, the competitive supply function each period is equal to \( S_t(p_t) = p_t/c \). As speculators sell \( h_2 \) on the second period market, demand in the second period is \( D_2(p_2; a_2, h_2) = a_2 - p_2 - h_2 \). Under Assumption 1 non-negative prices exist for which demand is positive. The intersection between the competitive demand and the competitive supply gives

\[
p_{c2} = \beta c(a_2 - h_2),
\]

where \( \beta c = c/(1 + c) \). Given this expression for the second period price, it is immediate that \( g^c(h_2) = E_1(p_2|h_2) = \beta c(E(a) - h_2) \). From this, we have the price thresholds (13) and (14):

\[
p_c^l = \delta g^c(h) - \delta w = \delta \beta c(E(a) - \bar{h}) - \delta w
\]

and

\[
p_c^u = \delta g^c(0) - \delta w = \delta \beta c E(a) - \delta w.
\]

We now need to determine the level of speculative sales, \( x_t \), undertaken when we are in a smoothing regime: \( p_1 \in [p_c^l, p_c^u] \). Since \( p_1 = \delta (g^c(h_2) - w) \), using our expression for \( g^c(h_2) \), we can solve for

\[
h_2 = E(a) - \left( w + \frac{p_1}{\delta} \right) / \beta c.
\]

\(^{10}\)It is equivalent to think of this as being the outcome of a social planner operating a single firm with a marginal cost equal to \( cq \).
Using \( h_2 = h_1 - x_1 \), we have

\[
x_1 = h_1 - E(a) + \left( w + \frac{p_1}{\beta} \right) / \beta^c.
\]

(41)

Demand net of speculation is consequently equal to

\[
D^R(p_1; a_1, h_1) = \begin{cases} 
  a_1 - h_1 - p_1 + \bar{h} & \text{if } p_1 < p_1^c \\
  a_1 - h_1 - p_1 + E(a) - \left( w + \frac{p_2}{\delta} \right) / \beta^c & \text{if } p_1 \in [p_1^c, p_u^c] \\
  a_1 - h_1 - p_1 & \text{if } p_1 > p_u^c \end{cases}
\]

(42)

The competitive equilibrium can be obtained from the intersection of this demand with competitive supply, which we summarize as\(^{11}\):

**Proposition 2.** Under Assumption 1, and for a given \((a_1, h_1)\), the equilibrium under perfect competition is characterized as follows:

1. **(Stock-out)** If \( a_1 - h_1 - \delta E(a) + \delta w / \beta^c \geq 0 \), then
   
   \[
   p_1^c = \beta^c(a_1 - h_1) \quad \text{and} \quad p_2^c = \beta^c a_2
   \]

2. **(Smoothing)** If \( a_1 - h_1 - \delta E(a) + \delta w / \beta^c < 0 \leq a_1 - h_1 - \delta E(a) + (1 + \delta)\bar{h} + \delta w / \beta^c \), then
   
   \[
   p_1^c = \frac{\delta \beta^c (a_1 - h_1 + E(a)) - \delta w}{1 + \delta} \quad \text{and} \quad p_2^c = \beta^c a_2 + \frac{\beta^c (a_1 - h_1 - \delta E(a)) + \delta w}{1 + \delta}
   \]

3. **(Capacity)** If \( a_1 - h_1 - \delta E(a) + (1 + \delta)\bar{h} + \delta w / \beta^c < 0 \), then
   
   \[
   p_1^c = \beta^c(a_1 - h_1 + \bar{h}) \quad \text{and} \quad p_2^c = \beta^c(a_2 - \bar{h})
   \]

where \( \beta^c = c / (1 + c) \).

The values of \((a_1, h_1)\) for which each regime of Proposition 2 obtains are depicted in the left panel of Figure 1.

\(^{11}\)The proof of proposition 2 is straightforward and available upon request.
4.3 Comparison between perfect competition and monopoly

The likelihood of the different price regimes as a function of the state \((a_1, h_1)\) varies by market structure. To investigate this issue, let us rank the bounds of all the regions presented in propositions 1 and 2. From the proofs of these two propositions, we know that belonging to each of the regions depends on the comparison between \(a_1 - h_1 - \delta E(a)\) and a function of \(c, \delta, w\) and \(h\). Ranking the different bounds is straightforward and we demonstrate the ranking graphically in Figure 1.

Under monopoly production, four different regimes are possible, while there are only 3 regimes under perfect competition. The areas labeled “Capacity” in Figure 1 are the sets of values of \((a_1, h_1)\) such that speculators store the product up to their capacity. Given initial inventories \(h_1\), this regime occurs when \(a_1\) is sufficiently lower than \(E(a)\). Note that the area of this region is larger under monopoly production than it is under competitive production. Hence, given \(h_1\), the likelihood of being in a capacity regime is larger under a monopoly than under perfect competition (when it has non-zero probability).

The central “Smoothing” areas of Figure 1 show the values of \((a_1, h_1)\) such that the market is in a smoothing regime. This occurs for intermediate/small values of \(a_1\) compared to \(E(a)\). The areas of the smoothing regions is the same under both market structures, however, smoothing occurs for higher values of \(a_1\) (given \(h_1\)) under a monopoly compared to perfect competition.

In Figure 1 stockouts occur under perfect competition in the region labeled “Stockout” and under monopoly in the sum of the two regions “Stockout” and “Limit”. The area where stock-outs occur is smaller under monopoly than under perfect competition and occurs for higher values of first period demand. In the “Limit” region, the monopoly keeps price constant for all values of the state at stock-out threshold \((p_m^0)\), ensuring that no stocks are carried into the second period.

Comparing the volatility of the first period price under the two market structures, first note that as \(\beta^c < \beta^m\), when \((a_1, h_1)\) is in the stockout or the capacity region under both market structures, price varies more under monopoly than under perfect competition. For the same reason, if \((a_1, h_1)\) is in the smoothing region under both market structures (if the smoothing regions of Figure 1 overlap) price varies more under monopoly than under perfect competition.

However, when \((a_1, h_1)\) is in the limit region for the monopoly, the monopoly price does not vary, consequently price varies more with \(a_1\) under perfect competition. In addition, if \((a_1, h_1)\) is in the smoothing region for the monopolist, but in the stockout region for the competitive industry, price may be more or less volatile under perfect competition depending on parameter values.\(^{12}\) The overall conclusion is therefore ambiguous: first period price may vary more or less under monopoly depending on the relative probability that the distribution of \(a_1\) assigns to the limit region and the region where we have smoothing under monopoly but stockouts under perfect competition.

\(^{12}\)We compare \(\beta^c\) with \(\delta \beta^m / (1 + \delta)\). The former is larger than the latter unless \(c\) is very small.
In the regions where the first period price is more variable under the monopoly than under perfect competition we have a result similar to that in Thille [15]. The logic is similar: when faced with an increase in demand, the monopolist does not adjust sales as much as a competitive industry would because the marginal revenue curve it faces is steeper than the demand curve under our linear demand assumption. In terms of its pricing strategy, the monopolist increases both its price and its profit margin in response to the demand shock whereas in a competitive industry the profit margin does not change.

So far we have dealt with the case in which $h_1$ is exogenous. This does not allow us to consider the possibility that inventory levels may differ across market structures. In order to address this possibility, we now turn to the analysis of the model with an infinite time horizon.

5 Infinite horizon analysis

We now turn to an analysis of the model in an infinite horizon setting. This will relax two limitations of the two-period model. First, the equilibrium price in the previous section was a function of initial stocks, $h_1$. Since there is no way to distinguish the different levels of stocks expected to be held under monopoly versus perfect competition in the two-period setting, we were not able to fully characterize the price distribution. Second, in the two-period model speculators sell their stocks in the second period as long as price is positive. This likely exaggerates the desire of the monopolist to limit storage.

It is well known that there is no closed-form solution to the infinite horizon storage problem, even in the case of competitive production (Williams and Wright [16]). Consequently, the analysis of this section proceeds by numerical solutions to the problem under the alternative conditions of competitive and monopolistic production. We will describe the solution method that we use for the two market structures next. Following that we provide a comparison of the equilibrium under the two market structures for a particular set of values for the parameters.

5.1 Monopoly production and competitive storage

In the model with monopoly production, there are two unknown functions that need to be determined in order to solve the problem: the monopolist’s value function, $V^m(a_t, h_t)$, and speculators’ price expectations, $E_t[p_{t+1}] = g(h_{t+1})$. Since the value function described in (29) is not expected to be smooth, we choose instead to approximate the expectation of next period’s value function, $E_t[V^m(a_{t+1}, h_{t+1})] = \tilde{V}^m(h_{t+1})$. This has the added advantage that the function being approximated depends on only one variable.

We will proceed by using the collocation method\textsuperscript{13} to compute approximate solutions

\textsuperscript{13}See Judd [6], Chapter 11.
for \( g(h_+) \) and \( \tilde{V}^m(h_+) \). In particular, we use
\[
\tilde{V}^m(h_+) \approx \sum_{i=0}^{n} c_i \phi_i(h_+)
\]  
(43)

where \( c_i \) are coefficients and the \( \phi_i(\cdot) \) are known basis functions. We use Chebyshev polynomials\(^{14}\) for the \( \phi_i(\cdot) \) functions in what follows. Similarly
\[
g(h_+) \approx \sum_{i=0}^{n'} d_i \phi_i(h_+)
\]  
(44)

The collocation method forces these approximations to be exact at the \( n \) and \( n' \) collocation nodes.

**The numerical algorithm**

Three numerical routines are required to solve the model: a routine to compute Chebyshev approximations, a routine to integrate the price and value functions, and a routine to solve the fixed point problem in the smoothing region. We use the `gsl_cheb`, `gsl_integrate_qagp`, and `gsl_root_fsolver_brent` routines from the GNU Scientific Library (Galassi [5]) for these tasks. The algorithm is:

**Step 0.** Choose degrees of approximation, \( n \) and \( n' \), and the convergence criterion. Initialize starting values \( d^0, c^0 \). These are chosen as Chebyshev approximations to decreasing linear functions since both \( g(h_{t+1}) \) and \( \tilde{V}^m(h_{t+1}) \) are expected to be decreasing.

**Step 1.** Update the price expectations equation by finding \( d^1 \) such that
\[
\sum_{i=0}^{n'} d_i^1 \phi_i(h'_{t+1}) = \int p(a, h'_{t+1}) \theta(a) da
\]  
(45)

where \( \theta(a) \) is the density of \( a_t \). Here the new \( d^1 \) are found by forcing this to hold at the \( n' \) collocation nodes and the price is evaluated using the approximations defined by \( d^0 \) and \( c^0 \). The price function on the right-hand side is found by computing the feasible choice for the monopolist that yields the highest value and using the resulting price. In other words, given the current approximation, we solve (17), (20), (23), (24), and (26), find the corresponding value for each and determine the maximum value to get the optimal production and price choice. This optimal price is used in the numerical integration of the right-hand side of (45).

\(^{14}\)Miranda [10] examines a variety of numerical methods for solving the competitive storage model and concludes that Chebyshev collocation performs well.
Step 2. Update the value function expectation by $c^1$ such that
\[
\sum_{i=0}^{n} c_i^1 \phi(h_\pm) = \int V^m(a, h_\pm) \theta(a) da \tag{46}
\]

Here the new $c^1$ are found by forcing this to hold at the $n$ collocation nodes and value function is evaluated using the approximations defined by $d^0$ and $c^0$. The value function on the right-hand side is found by computing the feasible choice for the monopolist that yields the highest value. In the same manner as for the previous step, we solve (17), (20), (23), (24), and (26) using the corresponding value for each in (29) to get the values used in the right-side of (46).

Step 3. Stop if $||d^1 - d^0||$ and $||c^1 - c^0||$ are smaller than the convergence criterion. Otherwise set $d^0 = d^1$, $c^0 = c^1$, and return to Step 1.

5.2 Competitive production and storage

The model with competitive production and storage is the benchmark case that has seen substantial analysis in previous work (for example see Williams and Wright [16]). The only difference between our model with competitive production and the standard treatment is the addition of a capacity constraint.

Using the same argument as at the beginning of subsection 4.2 we have competitive supply equal to $p_t/c$. Stock-outs occur if $p_t > \delta (E[p_{t+1}] - w)$, which, using the demand function, reduces to $a_t > h_t + \delta (E[p_{t+1}] - w)(1 + c)/c$. Similarly, storage to capacity occurs if $a_t < h_t - \bar{h} + \delta (E[p_{t+1}] - w)(1 + c)/c$. Hence equilibrium price for any $a_t, h_t$ combination is
\[
p_t = \begin{cases} 
(a_t - h_t + \bar{h})c/(1 + c) & \text{if } a_t < h_t - \bar{h} + (\delta (E[p_{t+1}] - w))(1 + c)/c \\
(a_t - h_t)c/(1 + c) & \text{if } a_t > h_t + (\delta (E[p_{t+1}] - w))(1 + c)/c \\
\delta E_t[p_{t+1}] - \delta w & \text{otherwise}
\end{cases} \tag{47}
\]

This problem is simpler than the one in the case of monopoly production since there is only one unknown function to approximate. We proceed by approximating $E[p_{t+1}|h_{t+1}]$ with a Chebyshev polynomial $\sum_{i=0}^{n} d_i \phi_i(h_{t+1})$ and finding the $d_i$ that result in a close approximation to the expectation of (47) when evaluated at period $t+1$. The main difficulty is to find the value of $h_{t+1}$ to use in the last case of (47). In particular, since $p_t = \delta E_t[p_{t+1}|h_{t+1}] - \delta w$ in this case, we must have
\[
a_t - h_t + h_{t+1} - (\delta \sum_{i=0}^{n} d_i \phi_i(h_{t+1}) - \delta w)/c = \delta \sum_{i=0}^{n} d_i \phi_i(h_{t+1}) - \delta w \tag{48}
\]
which we solve with the root-finding algorithm. The solution algorithm is similar to the one used in the monopoly case, but with Step 2 omitted.
5.3 Example

We present the solution to the problem for parameter values of $\delta = 0.95, c = 0.5, w = 0.1, \bar{h} = 1$. The demand intercept is assumed to be normally distributed with $E[a_t] = 5$ and variance of 1.0. With these parameters, the average price charged by a static monopoly is equal to 3. The orders of the Chebyshev polynomials are $n = 5$ and $n' = 3$ for the expected price and expected value functions in the monopoly case and $n = 7$ in the competitive case. We use 1.0E-10 as the convergence criterion. For the solution to the monopoly problem, the largest residual error for the expected price approximation is smaller than 1.0E-4 and for the expected value approximation is smaller than 1.0E-6. In the perfectly competitive case, the largest residual error for the expect price approximation is smaller than 1.0E-9.

Figure 2 plots the equilibrium price function for both market structures: $p_t$ versus $a_t$ and $h_t$. The upper surface is the monopolist’s price function, while the lower surface is

\[ \text{Figure 2: Equilibrium price: Monopoly (top) and Competitive (bottom)} \]
that for the perfectly competitive market. The relatively flat portion of each plot is where smoothing occurs. Kinks occur at the boundaries to the smoothing region. We see that as was the case in the two-period model, for a given level of inventories, the threshold demand shock at which stock-outs occur is higher for the monopolist than for the perfectly competitive market. This corroborates our findings on the two periods case in the sense that a stock-out is more likely in a perfectly competitive market than a monopoly at a given level of inventories. One final point to make about Figure 2 is that, conditional on the regime, the monopoly price function has a steeper slope as \( a_t \) varies than the competitive one does for any value of \( h_t \). This suggests a higher price variance under monopoly, and consequently a higher return to storage so we would expect to see more inventories carried under monopoly production.

Figure 2 shows price for a given state. However, it is likely that the distribution of inventories will differ under the different market structures. This implies that to know what the actual differences in the price distribution will be, we need to compute the distribution of inventories under the different market structures. In order to see the effects of market structure on the equilibrium distributions of price and inventories, we generate series for \( a_t \) that are 1000 periods long, computing prices and inventories levels for each period, beginning with \( h_0 = 0 \). We construct 500 such samples to generate averages over the 500 samples of some statistics of interest, which are presented in the first two rows of Table 1.

We also present the static case \((\bar{h} = 0)\) as a benchmark along with values for \( \bar{h} = 3.0 \) which is a large enough capacity for the capacity constraint to be effectively non-binding.

Examining the effects of storage on the monopolist’s behaviour (moving from \( \bar{h} = 0 \) to \( \bar{h} = 1 \) or \( 3 \), we find an interesting result. In contrast to the model of Thille [15] and consistent with the results of the two period model, we find effects of storage on the level of price. Comparing the \( \bar{h} = 0 \) and \( \bar{h} > 1.0 \) results, we see that there is a slightly higher average price when the monopolist faces speculators than when it does not. As in the two-period model, there are circumstances in which the monopolist chooses to set the price exactly equal to the stock-out threshold and does so for a range of values of the state variables. This failure to reduce price smoothly with the state results in slightly higher average price when speculators are present. In order to see this more clearly, we plot histograms of prices from a single long simulation (100,000 observations) in Figure 3 for both monopoly and perfect competition. The first column of the figure plots histograms for the parameters discussed above. This figure highlights that the mass point in prices that was found in the two-period model also occurs in the infinite horizon solution. Under monopoly, the price equals the upper bound \((p^u_t)\) about 3% of the time, while under perfect competition it essentially never occurs. Hence the qualitative difference in equilibrium prices that occurred in the two-period model continues to exist in the long-horizon case. The second column plots histograms for a larger value of the storage cost parameter,

\[16\] Of course, the equilibrium level of inventories held under the different market structures will not necessarily be the same.
Table 1: Simulated statistics

<table>
<thead>
<tr>
<th></th>
<th>No Storage</th>
<th>Constrained Storage</th>
<th>Unconstrained Storage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{h} = 0$</td>
<td>$\bar{h} = 1$</td>
<td>$\bar{h} = 3$</td>
</tr>
<tr>
<td></td>
<td>Monopoly</td>
<td>P.C.</td>
<td>Monopoly</td>
</tr>
<tr>
<td>$E[p]$</td>
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<td>1.67</td>
<td>3.02</td>
</tr>
<tr>
<td>$Var[p]$</td>
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<td>0.11</td>
<td>0.22</td>
</tr>
<tr>
<td>$Skew[p]$</td>
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<td>0.00</td>
<td>0.65</td>
</tr>
<tr>
<td>$Cor[p]$</td>
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<td>0.00</td>
<td>0.19</td>
</tr>
<tr>
<td>$E[q]$</td>
<td>2.00</td>
<td>3.33</td>
<td>1.98</td>
</tr>
<tr>
<td>$E[h]$</td>
<td>0.20</td>
<td>0.16</td>
<td>0.24</td>
</tr>
<tr>
<td>% Stock-out</td>
<td>0.52</td>
<td>0.60</td>
<td>0.50</td>
</tr>
<tr>
<td>% Limit</td>
<td>0.03</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>% Capacity</td>
<td>0.05</td>
<td>0.03</td>
<td>0.00</td>
</tr>
</tbody>
</table>

which presents a clearer view of the mass point that occurs under monopoly production. As storage costs become larger, the qualitative differences in the price distribution under the different market structures become accentuated.

Also, prices under monopoly are significantly more volatile than is efficient. Monopoly prices are also slightly more skewed and autocorrelated than competitive ones. The higher price volatility results in storers holding more inventories under a monopolist, which combined with the lower output results in a substantially higher inventory/production ratio. Also, stock-outs occur less frequently and capacity regimes occur more frequently under the monopolist.

The variance of price under monopoly remains higher than that under perfect competition. This occurs in spite of the fact that speculators carry more inventory when production is done by a monopolist. The inventory-production ratio under monopoly production is double what it is under competitive production. The effect of these larger inventory holdings are seen when we compare the reduction in price variance due to storage under the two market structures. Moving from no speculative capacity to $\bar{h} = 1.0$ results in a 27% reduction in price variance under competitive production and a 39% reduction under monopoly production. Hence, speculators have a more dramatic effect on volatility in the monopoly.
Figure 3: Histogram of price

Monopoly (w=0.1)

Monopoly (w=0.5)

Competitive (w=0.1)

Competitive (w=0.5)
Skewness and serial correlation are both higher under monopoly than perfect competition. This is interesting in light of the empirical work examining the ability of the competitive storage model to fit these moments of the price distribution. In particular Deaton and Laroque [2, 3] find that prices exhibit much higher degrees of serial correlation than can be attributed to the competitive storage model. This results here suggest that market power might be another contributing factor to examine.

The final three rows of Table 1, labelled “% Stock-out”, “% Limit” and “% Capacity” report the proportion of periods spent in that regime. We find that, as suggested by the two-period model, stock-outs occur less frequently under monopoly than under perfect competition. There are two effects causing this difference. First, the set of values of \((a_t, h_t)\) for which a stock-out occurs is smaller under monopoly (as was the case in Figure 1). Second, speculators hold more inventories on average under monopoly production. This reduces the likelihood of a stock-out as well. We observe the “limit” regime: 3% of the time under monopoly, price is held at the limit that just induces a stockout. Also consistent with the two-period analysis is the result that capacity regimes are experienced more frequently under monopoly.

To give some idea of the robustness of these results, Figures 4 and 5 plot price variance and inventory levels for a range of values of \(\bar{h}\). The maximum value of \(\bar{h} = 3.0\) represents an effectively non-binding constraint for these parameter values so those values are representative of the model without a capacity constraint. Descriptive statistics for \(\bar{h} = 3.0\) are provided in the last two rows of Table 1. The qualitative results are robust to variations in \(\bar{h}\). The difference between price variance under monopoly versus perfect competition is somewhat reduced as capacity increases, but monopoly price variance remains substantially higher even when capacity no longer binds. Furthermore, Figure 5 suggests that the constraint has more effect on storage under a monopoly, as storers increase average stocks held more quickly as the constraint is relaxed. This later result is again due to the fact that prices are more volatile under the monopoly so speculators have a stronger desire to increase inventories and hence, more likely to be capacity constrained when \(\bar{h}\) is small.

5.4 The profitability of speculation

Since price volatility differs across market structures, it is natural to ask what the effect of free entry in speculation would be. In particular, we can examine the payoff to speculators (in aggregate) for a given storage capacity. Since prices are more volatile under monopoly production, we would expect speculative returns to be higher for a given level of capacity, and consequently more capacity to appear in a long-run equilibrium.

Rather than finding the speculators’ value function explicitly, we compute the discounted present value of the returns to speculators (equation (4)) over the 1000 periods of the simulated series\(^{17}\) and average over our 500 samples. Speculative returns under the two

\(^{17}\)Given the discount factor we use, the truncation to 1000 periods has no discernible impact on the
Figure 4: Price variance: Competitive vs. Monopoly production

Figure 5: Inventory levels: Competitive vs. Monopoly production
market structures are plotted in Figure 6. Due to the higher price volatility speculation is substantially more profitable under monopoly production. If one were to superimpose a plot of the cost of capacity on this figure (say due to the opportunity cost of space for storage), then the intersection of the two would depict the long-run level of storage capacity. Clearly, for any cost of capacity low enough to allow speculation, a free entry equilibrium will result in more storage capacity under monopoly production than would occur under perfectly competitive production.

We can get some idea of the difference in long-run equilibrium between the monopoly and the perfectly competitive markets by normalizing capacity in the competitive industry to $\bar{h} = 1.0$. From Figure 6, if capacity costs were linear and generated $\bar{h} = 1.0$ for competitive production, zero-profit capacity for the case of monopoly production would occur at approximately $\bar{h} = 1.5$. Facing this capacity, a rough inspection of Figure 4 suggests that equilibrium price variance under the monopolist would be slightly more than 0.20. Comparing this to the variance in the case of $\bar{h} = 1.0$, entry of speculative capacity does narrow the gap in price variance between the two market structures, however price remains significantly more volatile under monopoly, even with the larger amount of inventories being held. In addition, the higher level of inventories held does generate more serial correlation in the monopoly case (0.20 in the case of $\bar{h} = 1.5$), which is interesting given that Deaton and Laroque [3, 2] find that the model with competitive supply does not generate as much serial correlation as is found in the data. However, the increase in serial correlation in prices is not so large as to fully explain the extent of serial correlation that they found in the data.

6 Conclusion

This paper represents a first attempt to analyze the effectiveness of competitive storage in the presence of imperfectly competitive production of a commodity. The fact that competitive storage introduces kinks into the monopolist’s residual demand curve results in a mass point in the price distribution. There exists a non-trivial range of demand states over which the monopolist wishes to hold price constant at the level that just induces a stock-out in order to ensure there are no stocks to compete with the monopolist’s production in the next period. This effect is completely absent under perfectly competitive production and in models in which the monopolist itself stores. The addition of competitive storage has an anticompetitive effect: average price is higher than in the absence of storage.

Our two-period model showed that stock-outs occur less frequently and that storage to capacity occurs more frequently under monopoly than under perfect competition. We confirmed this result using numerical solutions to the infinite horizon version of the model. In addition, even though the long-run equilibrium has more inventories carried under
monopoly production, this extra storage is not enough to remove the significant difference in price volatility under the two market structures.

Appendix

A Proof of proposition 1

The proof proceeds by finding regions of \((a_1, h_1)\) in which solutions interior to the three alternative remimes are feasible. We then examine what happens at the two thresholds to determine the global maximum. It will be useful to introduce notation that represents the monopolists payoff as a function of \(p_1\) for the different regimes. For the stock-out regime we have

\[
\Pi_1^1(p_1) \equiv p_1(a_1 - h_1 - p_1) - \frac{c}{2}(a_1 - h_1 - p_1)^2 + \delta \frac{1 - \beta m}{2} E_1 \left[ a_2^2 \right]. \tag{49}
\]

For the smoothing regime we have

\[
\Pi_1^2(p_1) \equiv p_1 \left( a_1 - h_1 - p_1 + E(a) - \frac{w}{\beta m} - \frac{p_1}{\delta \beta m} \right) - \frac{c}{2} \left( a_1 - h_1 - p_1 + E(a) - \frac{w}{\beta m} - \frac{p_1}{\delta \beta m} \right)^2 + \delta \frac{1 - \beta m}{2} E_1 \left[ E(a) + \frac{w}{\beta m} + \frac{p_1}{\delta \beta m} \right]^2. \tag{50}
\]
Finally, for the capacity regime we have

$$\Pi^3(p_1) \equiv p_1(a_1 - h_1 - p_1 + \bar{h}) - \frac{c}{2}(a_1 - h_1 + \bar{h} - p_1)^2 + \delta \frac{1 - \beta^m}{2} E_1 [(a_2 - \bar{h})^2].$$  \hspace{1cm} (51)

We also define $A \equiv a_1 - h_1 - \delta E(a)$ and recall from the proposition that $K \equiv \delta \beta^m (1 + \delta)/(\delta(\beta^m)^2 + 2\beta^m - 1)$.

A.1 Characterizing interior solutions

We examine the problems of maximizing (49), (50), and (51) in turn:

Stock-out: An interior solution to the problem of maximizing (49) results in $p_1^3 = \beta^m(a_1 - h_1)$. In order for an interior solution to be feasible in this region, $p_1^3 \leq p_m$ must hold, i.e., $\beta^m(a_1 - h_1) \geq \delta \beta^m E(a) - \delta w$, or

$$A + \frac{\delta w}{\beta^m} \geq 0 \hspace{1cm} (52)$$

If condition (52) is satisfied, then there is a local maximum of the profit such that a stock-out occurs in period 1.

Smoothing: An interior solution to the problem of maximizing (50) must satisfy the first order condition:

$$\delta(\beta^m)^2 + 2\beta^m - 1 \frac{(1 - \beta^m)\delta \beta^m}{(1 - \beta^m)^2(\beta^m)^2} \left(1 - \beta^m\right) \delta \beta^m (a_1 - h_1 + E(a)) + \left(1 + \delta\right)(1 - 2\beta^m) \frac{(1 - \beta^m)(\delta(\beta^m)^2 + 2\beta^m - 1)}{(1 - \beta^m)(\beta^m)^2} p_1 = 0$$

so

$$p_1^2 = \frac{\delta \beta^m}{1 + \delta} (a_1 - h_1 + E(a)) + \frac{\delta (1 - 2\beta^m)}{\delta(\beta^m)^2 + 2\beta^m - 1} w.$$ 

This price belongs to $[p_l^m, p_u^m]$ if and only if $p_1^2 \leq p_u^m$ and $p_1^2 \geq p_l^m$ or,

$$A + Kw \leq 0 \hspace{1cm} (53)$$

and

$$A + (1 + \delta)\bar{h} + Kw \geq 0. \hspace{1cm} (54)$$

If conditions (53) and (54) are satisfied, then there is a local maximum of the profit such that smoothing occurs.

Capacity: An interior solution to the problem of maximizing (51) results in $p_1^1 = \beta^m(a_1 - h_1 + \bar{h})$. To be feasible in the capacity regime, $p_1^1 < p_l^m$ must hold, i.e., $\beta^m(a_1 - h_1 + \bar{h}) \leq \delta \beta^m E(a) - \delta \beta^m \bar{h} - \delta w$, or

$$A + (1 + \delta)\bar{h} + \delta \frac{w}{\beta^m} \leq 0. \hspace{1cm} (55)$$

If condition (55) is satisfied, then there is a local maximum of the total profit in which speculators store up to capacity in period 1.
A.2 Finding the global maximum

Due to the continuity of the total profit, and to the linearity of the marginal profit in each of the three regions, finding the global optimum of this profit can be done by inspecting the position of each local optimum with respect to the bounds of its region, $p_l^m$ and $p_u^m$. However due to the discontinuity of the marginal profit, it is possible for $p_l^m$ or $p_u^m$ to be the global optimum of the profit. Assumption 2 rules out some of the potential cases, as we shall now establish. Since $\beta^m \in [1/2, 1]$, it is clear that the denominator of $K$ is strictly positive for any value $\delta \in [0, 1]$. Omitting some trivial intermediate steps,

\[
\frac{\delta \beta^m (1 + \delta)}{\delta (\beta^m)^2 + 2 \beta^m - 1} \geq \frac{\delta}{\beta^m} \iff (\beta^m - 1)^2 \geq 0
\]

which always holds. Therefore $K \geq \frac{\delta}{\beta^m}$ and we obtain the following ranking

(i) $A + \frac{\delta}{\beta^m} w \leq A + Kw \leq A + (1 + \delta)h + Kw$

(ii) $A + \frac{\delta}{\beta^m} w \leq A + (1 + \delta)h + \frac{\delta}{\beta^m} w \leq A + (1 + \delta)h + Kw$

To rank conditions (55) to (52) it remains to compare $A + (1 + \delta)h + \frac{\delta}{\beta^m} w$ with $A + Kw$. We have

\[
A + Kw \leq A + (1 + \delta)h + \frac{\delta}{\beta^m} w \iff \left(\frac{\delta \beta^m (1 + \delta)}{\delta (\beta^m)^2 + 2 \beta^m - 1} - \frac{\delta}{\beta^m}\right) w \leq (1 + \delta)h.
\]

If Assumption 2 holds then

\[
A + \frac{\delta}{\beta^m} w \leq A + Kw \leq A + (1 + \delta)h + \frac{\delta}{\beta^m} w \leq A + (1 + \delta)h + Kw
\]

In this case, we can determine the global maximum as follows:

1. If $0 \leq A + \frac{\delta}{\beta^m} w$ then the profit is strictly increasing for $p_1 \leq p_l^m$, strictly increasing for $p_1 \in [p_l^m, p_u^m]$ and has a local maximum at $p_1 > p_u^m$. By continuity of the profit, the price chosen by the monopoly is $p_1^* = p_1^2 = \beta^m (a_1 - h_1)$, and the condition under which this price is the optimum of the monopolist can be rewritten

\[
0 \leq A + \frac{\delta}{\beta^m} w \iff \beta^m (a_1 - h_1 - \delta E(a)) + \delta w \geq 0 \iff \frac{\partial \Pi_1}{\partial p_1} (p_u^m) \geq 0.
\]

2. If $A + \frac{\delta}{\beta^m} w \leq 0 \leq A + Kw$, then the profit is strictly increasing for $p_1 \leq p_u^m$ and strictly decreasing for $p_1 \geq p_u^m$. The global optimum is therefore $p_1^* = p_u^m = \delta \beta^m E(a) - \delta w$. The first condition under which this price is the optimum is equivalent to $\frac{\partial \Pi_1}{\partial p_1} (p_u^m) < 0$, and it is immediate to verify that the second one is equivalent to $\frac{\partial \Pi_1}{\partial p_1} (p_u^m) \geq 0$. 

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3. If $A + Kw \leq 0 \leq A + (1 + \delta)\bar{h} + \frac{\delta}{\beta w} w$ then the profit is strictly increasing for $p_1 \leq p_{1m}$, strictly decreasing for $p_1 \geq p_{1m}$ and has a global maximum at

$$p_1^* = p_1^2 = \frac{\delta \beta m}{1 + \delta} (a_1 - h_1 + E(a)) + \frac{\delta (1 - 2 \beta m)}{\delta (\beta m)^2 + 2 \beta m - 1} w.$$

The first condition is equivalent to $\frac{\partial \Pi_1^1}{\partial p_1} (p_{1m}) < 0$. We leave the second condition for the next case.

4. If $A + (1 + \delta)\bar{h} + \frac{\delta}{\beta m} w \leq 0 \leq A + (1 + \delta)\bar{h} + Kw$ then two solutions must be compared, namely $p_1^3$ and $p_1^2$. When the price is $p_1^3$, (51) produces

$$\Pi_1^3 (p_1^3) = \beta m \left(1 - \beta m\right)(a_1 - h_1 + \bar{h})^2 - \frac{2 \beta m - 1}{2} \left(1 - \beta m\right)^2 (a_1 - h_1 + \bar{h})^2 + \delta E_1 \left(\frac{1 - \beta m}{2}\right) \left(a_2 - \bar{h}\right)^2 = \frac{1 - \beta m}{2} \left((a_1 - h_1 + \bar{h})^2 + \delta E[a^2] + \delta (E(a) - \bar{h})^2\right).$$

When the price is equal to $p_1^2$, (50) reduces to

$$\Pi_1^2 (p_1^2) = \frac{1 - \beta m}{2} \left(\frac{\delta}{1 + \delta} (a_1 - h_1 + E(a))^2 + \frac{\delta^2}{\delta (\beta m)^2 + 2 \beta m - 1} w^2 + \delta E[a^2]\right).$$

Comparing $\Pi_1^3 (p_1^3)$ and $\Pi_1^2 (p_1^2)$ we have $\Pi_1^3 (p_1^3) - \Pi_1^2 (p_1^2) \geq 0$ if

$$(a_1 - h_1 + \bar{h})^2 + \delta (E(a) - \bar{h})^2 \geq \frac{\delta}{1 + \delta} (a_1 - h_1 + E(a))^2 + \frac{\delta^2}{\delta (\beta m)^2 + 2 \beta m - 1} w^2.$$

Adding and substracting $E(a)$ to $a_1 - h_1 + \bar{h}$ in the left-hand-side produces

$$(a_1 - h_1 + E(a))^2 - 2(a_1 - h_1 + E(a))(E(a) - \bar{h}) + (E(a) - \bar{h})^2 + \delta (E(a) - \bar{h})^2 \geq \frac{\delta}{1 + \delta} (a_1 - h_1 + E(a))^2 + \frac{\delta^2}{\delta (\beta m)^2 + 2 \beta m - 1} w^2$$

which reduces to

$$\left(\frac{a_1 - h_1 + E(a)}{\sqrt{1 + \delta}} - \sqrt{1 + \delta} (E(a) - \bar{h})\right)^2 \geq \frac{\delta^2}{\delta (\beta m)^2 + 2 \beta m - 1} w^2.$$

The difference between $\Pi_1^3 (p_1^3)$ and $\Pi_1^2 (p_1^2)$ is positive if and only if

$$\left(\frac{a_1 - h_1 + E(a)}{\sqrt{1 + \delta}} - \sqrt{1 + \delta} (E(a) - \bar{h}) - \frac{\delta w}{\sqrt{\delta (\beta m)^2 + 2 \beta m - 1}}\right) \times$$

$$\left(\frac{a_1 - h_1 + E(a)}{\sqrt{1 + \delta}} - \sqrt{1 + \delta} (E(a) - \bar{h}) + \frac{\delta w}{\sqrt{\delta (\beta m)^2 + 2 \beta m - 1}}\right) \geq 0.$$
If $a_1 - h_1 + E(a) \leq (1 + \delta)(E(a) - \tilde{h}) - \frac{\delta \sqrt{1 + \delta}}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}w$, then both terms of the product above are negative and the product is positive, if $a_1 - h_1 + E(a) \geq (1 + \delta)(E(a) - \tilde{h}) - \frac{\delta \sqrt{1 + \delta}}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}w$ and $a_1 - h_1 + E(a) \leq (1 + \delta)(E(a) - \tilde{h}) + \frac{\delta \sqrt{1 + \delta}}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}w$, then one term is positive, the other is negative, and the product is negative. Finally if $a_1 - h_1 + E(a) \geq (1 + \delta)(E(a) - \tilde{h}) + \frac{\delta \sqrt{1 + \delta}}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}w$ then both terms of the product are positive and the product is positive. The initial conditions on parameters such that two local maxima co-exist can be rewritten:

$$A + (1 + \delta)\tilde{h} + \frac{\delta}{\beta^m}w \leq 0 \iff a_1 - h_1 + E(a) \leq (1 + \delta)(E(a) - \tilde{h}) - \frac{\delta}{\beta^m}w$$

and

$$0 \leq A + (1 + \delta)\tilde{h} + K(\beta^m, \delta)w \iff a_1 - h_1 + E(a) \geq (1 + \delta)(E(a) - \tilde{h}) - K(\beta^m, \delta)w.$$ 

Since $K(\beta^m, \delta)$ is positive, $(1 + \delta)(E(a) - \tilde{h}) - K(\beta^m, \delta)w < (1 + \delta)(E(a) - \tilde{h}) + \frac{\delta \sqrt{1 + \delta}}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}w$, and it suffices to compare the lower bound of the two inequalities above, $(1 + \delta)(E(a) - \tilde{h}) - \frac{\delta}{\beta^m}w$, with the lowest root of the product $(1 + \delta)(E(a) - \tilde{h}) - \frac{\delta \sqrt{1 + \delta}}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}w$ to end up the characterization of the global maximum. Omitting intermediate steps,

$$-\frac{\delta}{\beta^m}w \geq -\frac{\delta \sqrt{1 + \delta}}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}w \iff (\beta^m)^2 - 2\beta^m + 1 \geq 0$$

which always holds. Therefore the sign of the difference of the profits is always negative in the region considered and the global optimum is $p^2_1$.

5. Finally, if $A + (1 + \delta)\tilde{h} + K(\beta^m, \delta)w \leq 0$ then the profit has a global maximum for $p^3_1 = \beta^m(a_1 - h_1 + \tilde{h})$ and the condition leading to this case is that $\frac{\partial \Pi_1}{\partial p_1}(p^m_1) < 0$.

References


