“Voting on Road Congestion Policy”

Antonio Russo
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ANTONIO RUSSO†

Abstract

We study the political economy of urban traffic policy. We consider a city and its suburbs. The city decides, by majority voting, on a parking charge in the Central Business District (CBD) and restrictions on road space dedicated to cars. City and suburbs vote on road pricing in the CBD. Results include the following. When the majority of city voters prefers cars to public transport sufficiently more than the average voter, car charges and space restrictions are smaller than optimal. If the suburbs’ voters have stronger preferences for cars than the city’s, road pricing has the lowest political support among the instruments we consider. Tax exporting and imperfect government coordination may inflate total charges. This is welfare enhancing if it compensates for voters’ opposition to car restraining policies. Earmarking charge revenues for public transport is welfare enhancing only if they are topped up by extra funds from a national government.

JEL classification: R41, D78, H77, H23

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†Tolouse School of Economics. E-mail: antonio.russo@tse-fr.eu.
1 Introduction

Road congestion in major urban areas is an increasingly significant problem. Yet, fearing voters’ opposition, policy makers often prefer to avoid traffic restraining policies. In spite of economists’ recommendations, plans to introduce road pricing have been recently abandoned in Edinburgh, Birmingham, Manchester, Paris and New York. They never entered the political agenda in many other cities.

Automobiles are an important part of modern lifestyles. This is linked to cities becoming sprawled and suburbanized, lack of viable alternative travel options, but also to the comfort, independence and flexibility of travel that cars provide. Building a model that captures all of these factors may be a too ambitious task. It seems however intuitive that a negative relation may exist between voters’ reliance on cars and their willingness to accept traffic restraining policies. In most of the cities that recently implemented road pricing, the majority of peak-hour travellers were not drivers (at the time of the schemes’ introduction). For instance, in London around 12% of trips to the charge zone were made by car (TfL (2003)). In Stockholm, only a third of commuters travelled by car (Armelius and Hulkrantz (2006)). On the contrary, most cities in the U.S. and Australia depend overwhelmingly on car travel. Few local governments have shown determination to restrict it. A notable exception is New York City. There, however, road pricing was blocked due to a decision by the New York State Assembly. The latter represents the city as well as suburban areas, where most commuters are car drivers (unlike people living in the city itself). On the other hand, suburban opposition was less of an obstacle to the introduction of stricter parking regulation, which can be an

1Schlag (1995, p.8) claims that “the car serves at the same time as a status-symbol, pleasure time activity and an article of daily use. Most people regard freedom of choice on when and where to travel as a basic right”. Commenting on a survey of commuters in Stuttgart he notes that “95% of participants agreed with the statement ’The car guarantees my independence’ and that 75% with ’Driving a car is fun’”.

2A good number of cities in which travellers strongly depend on cars exist also in Europe. An example is Dublin. More than 60% of people in the Greater Dublin area use cars to get to work: Irish Transport Minister Dempsey stated in 2008 that “congestion pricing would be ruled out for at least eight more years”. See http://www.independent.ie/national-news/congestion-levies-ruled-out-for-eight-years-by-dempsey-12986005.html

3For example, in 2009 (the same year in which plans for road pricing were abandoned), the city increased hourly parking meter rates by 50%, at peak periods, in various central areas (Litman (2010)).
alternative to road pricing in discouraging car use.

The New York example suggests that, on top of voters’ preferences for travel modes, the institutional setup may also influence traffic policy. Parking pricing and space management are generally controlled by city governments. Instead, road pricing often involves governments (or pressure groups) representing wider portions of the population. New York City’s D.o.T. controls parking policy, while, as mentioned, the decision on road pricing was ultimately taken by the State Assembly. A further example is Paris, where a “London-style” road pricing scheme was hard to sell politically because mainly affecting the car-dependent suburbs. The city chose instead to increase parking charges in the centre and restrict the amount of road space available to cars (Preud’homme and Kopp (2008)).

The objective of this article is to study how voters’ preferences for travel modes and the institutional setup may shape traffic congestion policy. In our setup, individuals differ in the utility they get from traveling by car relative to public transport (their default option). All travel is to and from a city’s Central Business District (CBD). We consider two distinct populations: the first lives in the city and the second in its hinterland. We assume, consistently with the examples provided above, that a city government controls a parking charge in the CBD. Some cities have tried to stimulate modal shift by restricting road space dedicated to cars and reallocating it to public transport. We let the city government decide on this as well. We also include a regional government that decides on road pricing, in the form of a cordon toll paid to drive to the CBD. This government represents people in the city as well as the hinterland. Both decide on policy through majority voting.

We begin our study by looking at the case in which revenues from parking charges and

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4We can provide other examples: in Manchester, parking policy is under the responsibility of the Manchester City Council, while the (rejected) congestion toll scheme was to be managed by the Association of Greater Manchester Authorities, representing the whole metropolitan area. In Edinburgh, the City Council is in charge of parking policy in the central city, while the decision on road pricing (ultimately taken by referendum only by the same constituency) saw the involvement of outer councils. It seems likely that the latters’ strong opposition played at least some part in its final rejection. In Stockholm, parking policy is managed by the Stockholm Municipality, while road pricing was introduced by the national government. Although approval in a referendum by the city was decisive for the project to go ahead, consultative polls were held in several hinterland municipalities. All voted against it.
road pricing (net of the costs of road space reallocation) are rebated to voters in the form of lump-sum transfers. A simple result emerges: when the majority of city voters has sufficiently stronger (resp. weaker) preferences for cars than the average voter, charges and space reallocation are smaller (larger) than optimal. Therefore, if the majority strongly values cars over public transportation, while only a minority of the population does not, charges and space reallocation are suboptimal in equilibrium (and viceversa). However, incentives for voters of city and hinterland are not the same. We assume the share of population preferring cars to public transport to be larger in the hinterland than in the city. Also, the city government can exploit tax-exporting possibilities, imposing a (parking) tax on drivers from the suburbs, who have no say in it. By its nature, the regional government cannot do so. As a consequence, among the instruments we study, road pricing gathers the smallest political support. To continue, since the two governments are assumed not to coordinate, charges and space reallocation are at least as high as in a single-government scenario. In the presence of a "downward" bias on traffic charges coming from voters’ preferences, the "upward" bias produced by imperfect governmental coordination may partially correct the first one. Interestingly, social welfare in the urban area may thus be higher with two non-coordinating governments than if a single one controlled the whole policy.\(^5\) This contrasts previous results suggesting that social welfare is diminished in the presence of several non-coordinating governments involved in traffic policy (e.g., De Borger et al. (2007)).

Finally, we focus our attention on the role of improvements to public transportation. Recent experience indicates that they may be a lever to reduce voters’ opposition to car taxes. We find that they can, indeed, induce voters to choose a policy closer to the optimum. However, a necessary condition is that the money raised by the city is topped up by extra funds (for instance, from a national government). This is true both when improvements to public transport are financed via lump-sum taxes and by earmarking revenues from car

\(^5\)A similar reasoning suggests that the possibility for the local government to exploit tax-exporting opportunities may actually be welfare enhancing. King et al. (2007) make, in a non-technical paper, a similar argument for road pricing.
charges. This provides an additional justification for subsidies from national governments to local public transport systems.

The rest of the paper is organized as follows: Section 2 relates this work to existing literature. Section 3 presents the model. Section 4 studies majority voting on traffic policy. Proofs of all propositions and lemmas are provided in an Appendix. Section 5 presents a numerical illustration of the results. Section 6 concludes.

2 Related literature

There is a large body of literature studying road congestion policy from a normative perspective (see Small and Verhoef (2007)). Political acceptability is one of the main issues holding back traffic restraining measures. Yet, there are, quite surprisingly, very few papers looking at traffic policy from a positive perspective. To the best of my knowledge, only De Borger and Proost (2010) and Marcucci et al. (2005) study voting on road pricing. De Borger and Proost use a majority voting setup to study the role of voters’ uncertainty on the cost of switching travel mode. Differently from them, we consider the presence of multiple governments and taxes. We also look at decisions on road space, while we neglect uncertainty. Marcucci et al. use a citizen-candidate game to model the political decision process on road pricing. A common finding of both papers is that using charge revenues to subsidize public transport can improve the acceptability of the optimal charge. We obtain results in line with this prediction, although we also point out the important role for extra funding from external governments. Another related paper is Dunkerley et al. (2010), studying the political economy of fuel taxes. They find that when aggregate income is high enough that drivers constitute the majority of the population, voting results in too low fuel charges and

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6 Most of the literature focuses on road pricing and infrastructure. There is also a part of this literature looking at parking issues (e.g. Arnott and Inci (2006)). However, these papers take a purely normative perspective, also neglecting the presence of multiple governments involved in congestion policy.

7 Parry (2002) provides a normative analysis comparing the effects of congestion charges and public transport subsidies.
vice versa. The opposite happens for road capacity. While this result is similar to that of our paper, we consider individuals that are heterogeneous in preferences for alternative transport modes. We also study interactions between overlapping local governments.

There is a growing literature that focuses on governmental competition in pricing and capacity of road networks. This literature does not use a political economy approach and considers governments as (local) welfare maximizers. De Borger et al. (2007) study the interaction of different governments in setting traffic policy on parallel and serial networks. They find that imperfect coordination can lead to significant deviations from the optimal pricing and investment scheme. As mentioned above, this is not necessarily the case in our model. In fact, imperfect coordination among the two governments may actually increase social welfare. Ubbels and Verhoef (2008) study the choice of pricing and capacity investments by a city and a hinterland government, each controlling one part of a two link road network leading to the city’s Central Business District. Horizontal tax competition and tax exporting lead to higher tolls on the city than on the hinterland section of the network. These results, however, do not explain why car charges in city centers may face strong political opposition. Our paper provides a possible explanation.

3 The model

Spatial structure. There is a “large” population (whose size is normalized to 1) living in an urban area. A first group of individuals, a fraction \( \lambda \in (0, 1) \) of the total, is assumed to live within the boundaries of a city’s jurisdiction. A second group (the remaining \( 1 - \lambda \) fraction) lives in the city’s hinterland. Residential locations are assumed fixed. The city also includes a Central Business District (CBD). We model the three areas as point sized islands, denoted \( CBD, C \) and \( H \). See Figure 1.

Individuals. There are three goods in the economy: a consumption good \( N \) (whose price is normalized to one) and transport trips to \( CBD \), on two alternative modes. We have trips
by car, whose quantity is denoted by \( q \), and trips by public transport, whose quantity is denoted as \( b \). Both are assumed to be non-negative continuous variables. For all individuals, travel demand is fixed at a positive quantity \( Y \), so \( b + q = Y \). This assumption fits well commuting travel at peak hours. We set the marginal utility of a trip by public transport to zero. The marginal utility of a car trip is individual specific and increasing in the parameter \( r \geq 0 \). This captures how much the individual values traveling by car compared to public transportation. For instance, the car generally allows greater flexibility and independence of movement. Its value can depend on the physical structure of the city, the place where the individual resides (see below) and personal habits. We assume individuals have the following utility function\(^8\)

\[
U(q, N; r) = 2 (qr)^{\frac{1}{2}} + N
\]

The parameter \( r \) is exogenously distributed, in each population \( C \) and \( H \), according to a CDF \( F_C(r) \) for \( C \) and \( F_H(r) \) for \( H \), with the same support \( \sigma = [0, r_u] \). We assume also that \( F_H(r) \leq F_C(r) \), for any \( r \in \sigma \). Denoting \( \hat{r}_C \) (resp. \( \hat{r}_H \)) as the median individual in \( C \) (resp. \( H \)), so \( F_C(\hat{r}_C) = F_H(\hat{r}_H) = \frac{1}{2} \), this implies that \( \hat{r}_C \leq \hat{r}_H \). The idea is that, all else equal, an individual living in \( H \) is more likely than an individual living in \( C \) to find car travel more attractive compared public transport. This may be due to people more accustomed to car travel choosing to live farther from central city or to smaller quality of the public transport network in suburban areas. Also, the greater flexibility provided by cars may be more valuable to people living farther from central areas. We denote the average value of \( r \), for the entire population as \( \bar{r} = \lambda \int_0^{r_u} r dF_C(r) + (1 - \lambda) \int_0^{r_u} r dF_H(r) \). We also denote

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\(^8\)This functional form is convenient because of its tractability, allowing smooth aggregation of preferences. Linearity in consumption and monetary and time costs of travel is however a common assumption in the literature on road pricing (see, e.g., De Borger and Proost (2010), de Palma et al. (2010), Arnott et al. (1993)).
\[ \hat{r} = \lambda \hat{r}_C + (1 - \lambda) \hat{r}_H. \]

**Travel options, costs and budgets.** Consider individuals living in \( C \). The monetary cost of a car trip from \( C \) to \( CBD \) and back is \( p = t + d \), where \( t \) are taxes and \( d \) is an exogenous resource cost. \( T \) is the (monetary equivalent of) time cost of such a trip. We denote by \( A \) the generalized cost of a public transport trip. The governments can also decide to reduce it by an amount \( s \). This can be through monetary subsidies reducing the fare or improvements in the quality of public transport networks.\(^9\) For an individual living in \( C \), the budget constraint is

\[ M + L \geq N + (p + T)q + (A - s) b \]

where \( M \) is undifferentiated (exogenous) income and \( L \) a lump sum transfer paid by the government(s).

A car trip consists of two complementary activities: driving to \( CBD \) and parking the car once there. We assume \( T \) is a linear function of traffic volume \( Q \). It is also a function of \( K \), which represents the amount of road space intially dedicated to cars (for cruising or parking) that is reallocated to public transport (for instance creating bus lanes or pedestrian space that makes walking to and from bus stops more enjoyable). Therefore

\[ T(Q, K) = (\gamma + K)Q \quad \text{and} \quad A = \delta - K \]

where \( \gamma, \delta > 0 \) and \( \delta \) is large enough for \( A \) to be positive. We have

\[ T_Q = (\gamma + K) \quad \frac{dT}{dK} = T_K Q + T_Q \frac{dQ}{dK} \quad \text{and} \quad T_{QK} = -A_K = 1 \]

When deciding on the number of car trips to make, the individual takes \( T \) as given.

We assume that, for trips by car and public transport to \( CBD \), individuals living in \( H \) have to sustain an additional (per-trip) cost of \( x \), with respect to people in \( C \). They have to\(^9\)We assume that, if \( s = 0 \), the marginal cost of providing public transportation trips is equal to the fare.
travel longer distances.\footnote{We assume a single bottleneck placed at the edge of CBD. Thus, $x$ is independent of traffic volumes. We also assume that the generalized cost of a public transport trip from $H$ to $C$ is higher than that of a trip by car. This is often the case in reality, public transport being underdeveloped in suburban areas. Individuals wanting to reach the CBD by public transport thus opt to drive from $H$ to a park-and-ride facility in $C$. This justifies $x$ being undifferentiated across the two modes, since the cost of the trip from $H$ to $C$ is the same as if the whole travel happened by car. However, due to the inconvenience of switching modes, the attractiveness of public transport trips for individuals in $H$ is likely to be smaller than for individuals in $C$, compared to simply driving the whole way. This is captured by systematic differences in $r$ for the two populations.} For an individual in $H$, we have the following budget constraint:

$$M + L \geq N + (p + T + x)\,q + (A - s + x)\,b$$

**Individual's behavior.** Suppose $t$, $K$ and $s$ are set and denote $\pi = (t, K; s)$ the vector of traffic policy variables. Individuals maximize their utility choosing the amount of trips $q$ and $b$ as well as consumption $N$ (after receiving transfers from the governments). This leads to the demand function

$$q(\pi; r) = \frac{r}{(p + s + T - A)^2}$$

If $r = 0$, the individual obtains the same utility from a car and a public transport trip. Since we assume $p + T > A - s$ always holds, she will not travel by car. Recall that $b + q = Y$, with total travel demand $Y$ being fixed, so $b(\pi; r) = Y - q(\pi; r)$. By the linearity of $U(.)$ in $N$, trip quantities are independent of $M$ and $L$. Substituting $q(\pi; r)$ and $b(\pi; r)$ into $U(.)$ and using the individuals’ budget constraint, we get, after simplification, the indirect utility functions

$$V_C(\pi; r) = \frac{r}{(p + s + T - A)} + M + L + (s - A)\,Y$$

$$V_H(\pi; r) = \frac{r}{(p + s + T - A)} + M + L + (s - A - x)\,Y$$

for individuals living, respectively, in $C$ and $H$. Note that $p$ and $s$ play the same role in providing incentives for the use of transport modes. This is due to the assumptions of fixed transport demand and that trip costs enter utility linearly. However, their impact on local governments’ budgets may not be the same.

To obtain the aggregate demand for cars $Q(\pi)$, integrate $q(\pi; r)$ over $\sigma$, for both $C$ and $H$,
to get \( Q(\pi) = q(\pi; \bar{r}) = \frac{\bar{r}}{(p + s + T - A)^2} \). Thus, the aggregate amount of driving in the economy coincides with that of the “average” individual, of type \( r = \bar{r} \).\(^{11}\) It is easy to show that \( Q \) is strictly decreasing in \( p \) and \( s \) (even after accounting for feedback effects due to the reduction in \( T \)). We also have

\[
\frac{dQ}{dp} = \frac{dQ}{ds} \quad \text{and} \quad \frac{dQ}{dK} = (T_{QK}Q - AK) \frac{dQ}{dp} = (Q + 1) \frac{dQ}{dp}
\]

**Governments and policy instruments.** We assume the existence of three governments: \( G_W, G_C \) and \( G_R \). The first is a welfare-maximizing government, caring for the entire population. Its powers are limited. It cannot directly impose traffic policy to local governments.

It can, nevertheless, try to influence their choice by using other instruments. Its role may be thought of as that of a national government trying to shape traffic policy by its local counterparts.\(^{12}\) First of all, \( G_W \) chooses the rebate rule for (net) car charge revenues \( \rho = \{l, e\} \), where \( \rho = l \) means lump-sum redistribution and \( \rho = e \) earmarking of revenues to finance \( s \).

It also decides on the share \( 0 < \alpha \leq 1 \) of such expenditures that should be financed directly by the city government \( G_C \). \( G_W \)’s budget will cover the remaining share, with money raised from general taxation (assumed to put a negligible burden on the urban area’s population, being collected nationwide). Finally, in regime \( \rho = l \), \( G_W \) decides, prior to voting (i.e., at Stage 1 below), the amount of \( s \).

\( G_C \) is a city government representing the population in \( C \). It controls a non-negative monetary charge \( t_C \) paid per each car trip by all drivers.\(^{13}\) \( G_C \) also controls (and pays for)

\(^{11}\)From now on, we will denote, in order to save on notation, \( q(\pi; r) \) simply as \( q(r) \). Similarly, \( Q(\pi) \) will be simply denoted by \( Q \)

\(^{12}\)For example, in the UK, the government set up the Transport Innovation Fund to provide money for local governments with the aim of “supporting packages which combine road pricing, modal shift, and better bus services” (UK DfT (2004), Chpt. 5). Before granting such funds, however, the central government had a significant say in the plans they were to be used for. In Stockholm, the national government was heavily involved in the design of the road pricing scheme and in financing it at early stages. This included the expansion of the public transport system (Armelius and Hulkrantz (2006)).

\(^{13}\)In many cities, drivers who pay for parking are relatively small fraction of the total. This may be due to the limited powers of local governments (Bonsall and Young (2010)), but it may also be due to lack of political will. It seems therefore appropriate to study the behavior of local governments allowing them to price parking for every trip to the CBD.
reallocating road space in the central city $K$, whose cost is described by the function $c(K)$, assumed to be increasing and convex. Thus, $c_K, c_{KK} > 0$. $G_C$ also finances a portion $\alpha$ of expenditures for $s$, equal to $s(Y - Q)$.$^{14}$

$G_R$ is a regional government representing both people living in $C$ and in $H$. It is assumed to control a non-negative per-car-trip charge $t_R$, paid by all drivers. To be consistent with what is often observed in reality (see the Introduction), we identify $t_C$ as a parking charge and $t_R$ as a cordon toll around the city’s CBD. The total charge paid for a car trip to CBD is thus $t_C + t_R$. There are no other taxes on car trips, therefore

$$t = t_C + t_R$$

Both $G_C$ and $G_R$ decide on policy through majority voting. As mentioned, we will consider two rebate rules for the (net) revenues generated by traffic charges, denoted $\rho = \{l, e\}$. Rule $\rho = l$ is lump-sum redistribution: the local governments $G_C$ and $G_R$ fully rebate to each individual in their respective populations an equal share of the (net) charge revenues, using undifferentiated lump-sum transfers $L_C$ and $L_R$. The budget constraints are thus

$$\lambda L_C = t_C Q - c(K) - \alpha s(Y - Q) \quad \text{and} \quad L_R = t_R Q$$

for $G_C$ and $G_R$ respectively. Recall that $G_C$ represents only a fraction $\lambda \epsilon(0,1)$ of the total population. They are the only ones who are entitled to transfer $L_C$. An individual in $C$ will receive $L = L_C + L_R$, while an individual in $H$ will only receive $L = L_R$. Rebate rule $\rho = e$ involves instead earmarking of charge revenues to finance public transport. This implies that $L_R = 0$ and $\lambda L_C = -c(K)$, while $tQ = \alpha s(Y - Q)$.

**Timing.** The sequence of events is as follows

1. $G_W$ chooses rebate rule $\rho$ and, if $\rho = l$, interventions on public transport $s$. Moreover,

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$^{14}$We could have let the city vote on $s$ as well as $t_C$. We discuss this in Section 4.
it decides the share of their cost that falls on the city’s budget $\alpha$.

2. Individuals in $C$ and $H$ vote on traffic policy variables $t_C, K, t_R$, fully anticipating their utility at the following stage.

3. Taking policies as given, individuals decide the amount of trips $q, b$ and consumption $N$, in order to maximize $U(.)$.

**Social Welfare.** We assume $G_W$’s objective is to maximize the utilitarian social welfare function $W(\pi)$. This is obtained by integrating (1) and (2) over $\sigma$, for both $C$ and $H$, and subtracting the cost of $s$ which is not covered by $G_C$’s budget, $(1 - \alpha) s (Y - Q)$. We have thus

$$W(\pi) = \lambda \int_0^{r_u} V_C(\pi, T, A; r) dF_C(r) + (1 - \lambda) \int_0^{r_u} V_H(\pi, T, A; r) dF_H(r) - (1 - \alpha) s (Y - Q)$$

$$= \frac{\bar{r}}{(p + s + T - A)} + M + \lambda L_C + L_R + (s - A) Y - (1 - \alpha) s (Y - Q) - xY (1 - \lambda)$$

which, replacing for $\lambda L_C$ and $L_R$ gives (irrespectively of $\rho$)

$$W(\pi) = \frac{\bar{r}}{(p + s + T - A)} + M - AY + (t + s) Q - c(K) - xY (1 - \lambda) \quad (3)$$

From a pure welfare maximization perspective, only the sum $t + s$ matters: $t_C, t_R$ and $s$ are perfectly equivalent instruments. Importantly, this is not the case from voters’ perspective, given that they affect governments’ transfers in different ways.

**Benchmark.** As a benchmark, consider here the case in which $G_W$ is able to set directly $t, s$ and $K$, maximizing (3). We obtain the following.

**Lemma:** Denote $Q^{FB} \equiv Q(\pi^{FB})$ and $T_Q^{FB} \equiv \gamma + K^{FB}$. A welfare-maximizing policy vector $\pi^{FB} = (t^{FB}, K^{FB}; s^{FB})$ is such that $t+s$ is equal to the marginal external cost of a car trip: $(t + s)^{FB} = Q^{FB} T_Q^{FB}$. Road space reallocation $K^{FB}$ is such that $c_K + Q^{FB^2} = Y - Q^{FB}$
Not surprisingly, given quasilinear utility and a utilitarian social welfare function, the first-best combination of car taxes and improvements (or subsidies) to public transport \((t + s)^{FB}\) is equal to the marginal external cost of a car trip. Note that this is computed at the optimal amount of (reallocated) road space \(K^{FB}\). This amount is such that the marginal benefit in terms of reduced travel time on public transportation \(-A_K(Y - Q) = Y - Q\) is equal to the marginal cost of reallocation \(c_K\) plus the increased time cost (on aggregate) of car trips \(T_QK \cdot Q^2 = Q^2\).\(^{15}\) Note, also, that \((t + s)^{FB}\) and \(K^{FB}\) are unique as long as the cost function \(c(K)\) is sufficiently convex. We assume this condition holds throughout the paper.

4 Voting on traffic policy

We now introduce majority voting as the social choice process that determines parking charge \(t_C\), road toll \(t_R\), as well as road space restrictions for cars \(K\) at Stage 2. In order to capture imperfect coordination between city and regional governments, we study voting using a Shepsle Procedure (Shepsle (1979)). The outcome is a Nash equilibrium, in which each government chooses (by majority voting) its policy as the best response to that chosen by the counterpart (given the conditions set by \(G_W\) at Stage 1). In order to avoid the complications of multidimensional voting, we assume that the vote for the city policy takes place with the same “separate” procedure on the two dimensions \(t_C\) and \(K\).\(^{16}\)

\(^{15}\)When \(\rho = t\), there is actually an infinite set of welfare-maximizing vectors, described by the infinite combinations of \(t\) and \(s\) such that their sum is \((t + s)^{FB}\). This is due to the assumption of fixed travel demand. However, if \(\rho = e\), only one of them would be feasible: the one satisfying condition \(tQ = \alpha s(Y - Q)\). We assume that this vector always exists and is such that a marginal increase in \(t\) produces an increase in revenues, allowing to marginally increase \(s\).

\(^{16}\)City voters’ preferences satisfy the Single Crossing property (as proven in the Appendix) and the same individual is decisive on both dimensions. Under these conditions, De Donder et al. (2010, Proposition 3) proved that the equilibrium obtained in this way coincides with that of simultaneous two-dimensional majority voting, as long as the latter exists. This seems, therefore, a reasonable simplification.
4.1 Voting on traffic policy under lump-sum revenue redistribution

We consider here the case of lump-sum redistribution $\rho = l$, with $\alpha$ and $s$ having been set at Stage 1. We first describe the vote for city and regional policies separately. Then, we describe the equilibrium of the full voting procedure. Finally, we study the comparison between equilibrium and social optimum.

4.1.1 Voting by the city: parking charge $t_C$ and car space restrictions $K$

We now describe preferences over parking charge $t_C$ and reallocated road space $K$ for individuals living in $C$. We start from the indirect utility function (1) for a generic individual of type $r$ (written after replacing $L_C$ and $L_R$):

$$V_C(\pi; r) = \frac{r}{(p+T-A+s)} + M + (s-A)Y + \frac{t_CQ - c(K) - \alpha s(Y-Q)}{\lambda} + t_RQ$$  (4)

To find the most-preferred parking charge $t^*_C(K,t_R; s,r)$ and space $K^*(t_C,t_R; s,r)$ by the type-$r$ individual, we maximize (4) with respect to $t_C$ and $K$. The first-order derivatives write as

$$\frac{\partial V_C(\pi; r)}{\partial t_C} = -q(r) \cdot \left(1 + T_Q \frac{dQ}{dp} + \frac{Q + (t_C + \alpha s) \frac{dQ}{dp}}{\lambda} + t_R \frac{dQ}{dp} \right)$$  (5)

$$\frac{\partial V_C(\pi; r)}{\partial K} = -q(r) \cdot \left(T_Q \frac{dQ}{dK} + T_QK \right) + (Y - q(r)) + \frac{(t_C + \alpha s) \frac{dQ}{dK} - c_K}{\lambda} + t_R \frac{dQ}{dK}$$  (6)

A marginal increase in $t_C$ affects $V_C(\pi; r)$ in two ways: it raises (since $0 < 1 + T_Q \frac{dQ}{dp} < 1$) the generalized price of a car trip $p+T$. This affects individuals depending on the amount of driving $q(r)$. Secondly, it changes the amount of revenues from car charges $t_C$ and $t_R$, as well as expenditures to finance the public transport $s$. As for reallocated road space $K$, increasing its amount increases the time costs of a car trip, by making the road more congestible (note that $0 < T_Q \frac{dQ}{dK} + T_QK$), but also reduces the cost of public transport trips. Moreover, it has a negative impact on net revenues since $c_K > 0$ and the car tax base $Q$.
is reduced. The relevance of effects on revenues is greater the smaller the size of the city population relative to the entire urban area’s $\lambda$. This encourages city voters to raise $t_C$, due to a tax-exporting phenomenon (i.e. shifting the tax burden mostly on individuals who are not entitled to revenues). On the contrary, it discourages $K$ due to greater per-capita cost of space reallocation. The most-preferred policy vector by an individual of type $r$, denoted $\pi^*_C(t_R; s, r) = (t^*_C(K^*, t_R; s, r), K^*(t^*_C, t_R; s, r))$, is such that both (5) and (6) are equal to zero. Combining the two, we can rewrite (6) as

$$\lambda Y - Q = T_{QK}Q^2 + c_K$$

This implies that any individual would follow the same rule in setting road space, irrespectively of her type. This rule differs from the “first best” rule (see the benchmark of Section 3) because of $\lambda$, which has a discouraging role. However, $K^*(t^*_C, t_R; s, r)$ still varies with the individual’s type $r$, since it also depends on $t^*_C$. Indeed, individuals characterized by a stronger preferences for cars will choose smaller values of both $t_C$ and $K$.

Individuals’ preferences on $t_C$ satisfy the Single Crossing property, for any $K$ and $t_R$. The same is true for preferences for $K$, for any $t_C$ and $t_R$. This is the basis to establish the following

**LEMMA 1:** When the city votes on the parking charge $t_C$ and road space reallocation $K$, for given road toll $t_R$ and public transport improvement $s$, there exists a unique majority voting equilibrium $\pi_C(t_R; s) = (t_C(K, t_R; s), K(t_C, t_R; s))$. It coincides with the most-preferred policy vector (given $t_R$ and $s$) for the median voter in the city population, $\pi^*_C(t_R; s, \hat{r}_C)$. This vector is such that $\frac{\partial t_C}{\partial r_C} > 0, \frac{\partial K}{\partial r_C} > 0$ and $\frac{\partial K}{\partial \lambda} > 0$. Moreover, $-1 < \frac{\partial t_C}{\partial t_R} \leq 0$ and $\frac{\partial K}{\partial t_R} > 0$.

The intuition is simple: the stronger her preferences for cars (represented by $\hat{r}_C$) the more she will suffer from higher parking charges and road space restrictions. Smaller city size $\lambda$ (relative to the total population) makes increasing the parking charge more interesting but will, on the other hand, discourage space reallocation. Lemma 1 also describes how the city
government responds to a marginal increase in the road toll $t_R$. When facing an increased toll, the parking charge is reduced less than proportionally. This depends on the fact that a higher toll shrinks the tax base for both charges. However, city voters only partially internalize the effect on the latter.\footnote{From the perspective of the city government, parking and congestion charge are strategic substitutes. Although this depends on the structure of our model, it is not an uncommon finding in the literature. There is also anecdotal evidence that the introduction of congestion charges has led to a reduction in parking charges (see “Congestion charge brings an unlikely benefit – parking in Central London at 20p an hour”, http://www.timesonline.co.uk/tol/news/politics/article4144284.ece).} Consequently, the combined car tax $t$ goes up and the volume of traffic is reduced. This, in turn, reduces the costs of shrinking road space. Hence, $\frac{\partial K}{\partial t_R} > 0$.

### 4.1.2 Voting by the region: the cordon toll $t_R$

To describe the vote for the regional policy, it is important to distinguish between individuals in the city $C$ and the hinterland $H$. The former will choose their most-preferred $t_R$ (with $t_C$, $s$ and $K$ given) maximizing (4). An individual living in $H$ will instead maximize

$$V_H(\pi; r) = \frac{r}{(p + T - A + s)} + M + (s - A - x)Y + t_RQ$$

(7)

and, unlike individuals living in $C$, neglect the fact that a higher road toll $t_R$ reduces the tax base for the city government. Moreover, $t_R$ revenues are redistributed by the regional government $G_R$ to the entire urban area: there is no tax-exporting motive for road tolls. On the other hand, since $\hat{r}_C \leq \hat{r}_R$, we can expect the majority of the population in the hinterland to drive more often, to get to CBD, than that in the city. This makes them more reluctant to accept the toll $t_R$ than city-dwellers.

Existence of a voting equilibrium for the road toll $t_R(\pi_C; s)$, for any policy $\pi_C$ chosen by the city and improvement to public transport $s$, is ensured by the fact that voters’ preferences on $t_R$ are single-peaked (this is shown in the Appendix). Yet, since individuals in city and suburbs face different budget constraints, the identity of the pivotal voter depends on $\pi_C$. As this vector is endogenous, identification of the pivotal voter for the road toll is problematic in equilibrium. We thus limit ourselves to proving that $t_R(\pi_C; s)$ belongs to the interval spanned
by the most-preferred values \( t^C_R^*(\pi_C; s, \hat{r}_C) \) and \( t^H_R^*(\pi_C; s, \hat{r}_H) \) of the median individuals in, respectively, city and hinterland. Albeit partial, this will be useful information to characterize the full equilibrium \( \pi^E \) below.

**Lemma 2:** For any policy chosen by the city \( \pi_C \) and improvements to public transport \( s \), denote \( t^C_R^*(\pi_C; s, \hat{r}_C) \) as the most-preferred road toll by the median individual in \( C \). Denote \( t^H_R^*(\pi_C; s, \hat{r}_H) \) as the most-preferred road toll by the median individual in \( H \). When the entire urban area’s population votes on the road toll, a majority voting equilibrium \( t_R(\pi_C; s) \) exists and belongs to \( I = [t^C_R^*(\pi_C; s, \hat{r}_C), t^H_R^*(\pi_C; s, \hat{r}_H)] \).

### 4.1.3 The voting equilibrium without prior improvements to public transport

We denote as \( \pi^E = (t^E_C, t^E_R, K^E; s) \) the equilibrium policy vector resulting from the full voting procedure. In order to provide a description of \( \pi^E \), it is useful to begin by identifying intervals containing its components \( t^E_C, t^E_R \) and \( K^E \), for any \( s \).

**Lemma 3:** Define \( \bar{\pi} = (\bar{t}_C, \bar{t}_R, K; s) \) where \( \bar{t}_R = t^H_R(\bar{t}_C, K; s, \hat{r}_H) \), \( \bar{t}_C = t_C(\bar{t}_R, K; s) \) and \( K = K(\bar{t}_C, \bar{t}_R; s) \). Define also \( \underline{\pi} = (\underline{t}_C, \underline{t}_R, K; s) \) where \( \underline{t}_R = t^C_R^*(\underline{t}_C, \hat{r}_C; s, \hat{r}_C) \), \( \underline{t}_C = t_C(\underline{t}_R, K; s) \) and \( K = K(\underline{t}_C, \underline{t}_R; s) \). The equilibrium policy vector \( \pi^E \) is such that \( 0 \leq \underline{t}_C \leq t^E_C \leq \bar{t}_C \), \( 0 = \underline{t}_R \leq t^E_R \leq \bar{t}_R \) and \( \bar{K} \leq K^E \leq K \). Moreover, \( \bar{t}_C \leq t^E \leq \bar{t}_C + \bar{t}_R \).

Figure 2 provides an illustration of intervals in which \( t^E_C, t^E_R, t^E \) and \( K^E \) lie. Vector \( \bar{\pi} \) (that would obtain if \( t^E_R = t^C_R^*(t_C, K; s, \hat{r}_C) \), i.e. the median city voter \( \hat{r}_C \) were decisive at both the city and the regional votes) is such that the road toll is zero, i.e. \( t_R = 0 \). If she could decide on the policy chosen by both \( G_C \) and \( G_R \), the decisive voter in the city would make use of only the parking charge \( t_C \). This guarantees her the largest share of revenues. To continue, since \(-1 < \frac{\partial t_C}{\partial t_R} \leq 0 \) and \( \frac{\partial K}{\partial t_R} > 0 \) (by Lemma 1), components of \( \bar{\pi} \) mark the lower bound for road toll \( t^E_R \) and reallocated road space \( K^E \), as well as the upper bound for the parking charge \( t^E_C \). For the same reason, \( \bar{t}_C \) also represents the lower bound on the combined car tax \( t^E = t^E_C + t^E_R \) in equilibrium. The opposite extremes are marked by vector \( \pi \), which we would
obtain if the median individual in the hinterland population $H$ were decisive in the vote on the road toll $t_R$, i.e. $t^E = t^H_R(t_C, K; s, \hat{r}_C)$. Components of $\pi$ also identify the upper bounds for $K^E$ and $t^E$.

We are now in a position to compare the equilibrium $\pi^E$ and the welfare-maximizing policy vector $\pi^{FB}$. For the moment, we are going to proceed under the assumption that local public transport receives no additional funds prior to the voting stage, so $s = 0$. We consider below how a positive $s$ may modify $\pi^E$ and $\pi^{FB}$. We being by assuming lump-sum redistribution of revenues generated by car taxes, i.e. $\rho = l$. Earmarking will be studied further below. Since $\pi^{FB}$ is such that only the sum of car taxes $t$ is defined (for any $s$), we limit the comparison to $t$ and reallocated space $K$.

Consider, to begin, the simplified case in which the regional government $G_R$ is not involved.
in traffic policy. There is no regional tax, so $t_R = 0$. In this scenario, the charge set by the city $t_C$ can be thought of either as a parking charge with zero road toll or as a generalized charge (sum of parking and road pricing) that the city would set if it were in charge of the whole package of traffic policy instruments. The result is $\pi^E = \pi_C (0; s)$, which we compare to $\pi^{FB}$ in the following

**PROPOSITION 1:** Suppose that the city government controls the entire traffic policy and redistributes revenues from car charges to city voters (net of space reallocation costs) via a lump-sum transfer ($\rho = 1$). Suppose also that voting took place with no prior improvements to public transport (i.e. $t_R = s = 0$). Then:

- If the majority of the city population prefers car travel, compared to public transport, sufficiently more than the average individual, then the charge on car trips $t_C^E$ and road space reallocated from cars to public transportation $K^E$ are smaller than optimal. That is, if $\hat{r}_C \geq \frac{\xi}{\lambda}$, then $\pi^E = \pi_C (0; 0)$ is such that $t_C^E < t^{FB}$ and $K^E < K^{FB}$. The opposite (i.e. $t_C^E \geq t^{FB}$ and $K^E \geq K^{FB}$) can take place if and only if the city majority does not prefer car travel, compared to public transport, sufficiently more than average, i.e. $\hat{r}_C < r^a$, with $r^a < \frac{\xi}{\lambda}$.

- The car charge $t_C^E$ is decreasing with the relative size of the city $\lambda$, while the amount of space reallocation $K^E$ is increasing in it.

As mentioned in the Introduction, casual observation suggests a negative relation between the extent to which a city relies on cars as a travel mode and its willingness to accept traffic restraining policies. A scenario in which the majority of the city population is “car dependent” (with high $r$) and finds driving significantly more attractive than public transport, while only a minority (low $r$) is not, is consistent with a left-skewed distribution of $r$, where $\hat{r}_C \geq \frac{\xi}{\lambda}$. This, according to the Proposition above, will undermine political support for traffic restraining instruments. This also depends on the ability by the local government to “export” car taxes.
That is, make drivers that come from outside its jurisdiction (and do not vote) pay for them. Such possibility is, intuitively, more interesting for the city government the smaller the relative size of its population, compared to the total population of the urban area. This is because the volume of revenues that can be extracted from suburban commuters is larger. Interestingly, however, if voters do not support car taxes, the tax exporting possibility may actually be welfare-enhancing. On the other hand, smaller $\lambda$ also reduces the incentives to spend in space reallocation. Indeed, even if $\hat{r}_C = \frac{r}{\lambda}$ and the car tax is equal to the marginal external cost of a trip (i.e. $t_C^E = T_QQ^E$), the equilibrium would still entail too little space allocated to cars, i.e. $K^E < K^{FB}$. Consequently, $t_C^E < t^{FB}$, since the marginal external cost of a trip would be lower than if space dedicated to cars was optimally restrained. Note also that condition $\hat{r}_C \geq \frac{r}{\lambda}$ is possible only if the relative size of the city population is not “too small”, i.e. if $\lambda \geq \frac{1}{2}$. This is because $\hat{r}_i \in [0, 2\bar{r}]$ $i = C, H$. Therefore, from now on, we are going to restrict attention only to such case.

Let us now move on to consider the equilibrium in which both $G_C$ and $G_R$ are allowed to intervene in traffic policy. Before introducing the results, we need to define $r^+ \equiv \bar{r} \left(1 + \frac{1 - \lambda}{1 + T_Q \frac{dQ}{dp}(\pi^x)}\right)$, where $\pi^x = (t^x, K^x)$ is such that $t^x = T_Q^x Q^x$ and $\lambda Y - Q^x = T_Q K^x Q^x - c_K$ (where $Q^x \equiv Q(\pi^x)$ and $T_Q^x = \gamma + K^x$).\(^{18}\) Note that $r^+ > \bar{r}$. We have the following

**PROPOSITION 2:** Suppose the city government controls the parking charge $t_C$ and road space reallocation $K$ while the regional government controls the road toll $t_R$. Suppose revenues from car charges (net of space reallocation costs) are rebated via lump-sum transfers ($\rho = 1$) and voting takes place with no prior improvements to public transport being made ($s = 0$). The equilibrium vector $\pi^E = (t_C^E, t_R^E, K^E; 0)$ is such that

- If the majority of voters in the city and in the entire urban area (including the hinterland) prefer car travel, compared to public transport, sufficiently more than the average

\(^{18}\)There is a unique $\pi^x$, given $\bar{r}$ and $\lambda$. This is also true for $r^+$. Moreover $\frac{dQ}{dp}(\pi^x)$ is a function of $\pi^x$ and $\bar{r}$. There is therefore no endogeneity in the conditions presented below, as they come down to comparing a linear combination of $\hat{r}_C$ and $\hat{r}_H$ to a function of $\bar{r}$ and $\lambda$. All are exogenous parameters.
If a voter (i.e. \( \hat{r}_C \geq \frac{\tilde{r}}{\lambda} \) and \( \hat{r} \geq r^+ \)), then total charges on car trips and road space reallocated from cars to public transportation are lower than optimal: 
\[ t_C^E + t_R^E < t^{FB} \text{ and } K^E < K^{FB}. \]

- The opposite, i.e. 
\[ t_C^E + t_R^E \geq t^{FB} \text{ and } K^E \geq K^{FB}, \]
  happens if the majority of the city population does not prefer car travel, compared to public transport, sufficiently more than average (i.e. \( \hat{r}_C < r^a \), where \( r^a < \frac{\tilde{r}}{\lambda} \)). In that case, however, if the entire urban area’s population (including the hinterland) values car travel strongly enough (i.e. \( \hat{r} > r^+ \)), there will be no cordon toll: \( t_R^E = 0. \)

- Total charges and reallocated road space are always at least as high as if the city government were the only one to intervene in traffic policy.

Lemma 3 provided us with some bounds on the policy variables in equilibrium. In particular, we obtained that 
\[ t_C^E + t_R^E \leq t_C^E + t_R^E < t^{FB} \text{ and } K \leq K^E \leq \bar{K}. \] In the Appendix, we prove that, if \( \hat{r} \geq r^+ \) and \( \hat{r}_C \geq \frac{\tilde{r}}{\lambda}, \) then \( t_C^E + t_R^E < t^{FB} \text{ and } K < K^{FB}. \) Therefore, 
\[ t_C^E + t_R^E < t^{FB} \text{ and } K^E < K^{FB}. \]

We also prove that, if \( \hat{r}_C < r^a \) (where \( r^a \) is a threshold strictly smaller than \( \frac{\tilde{r}}{\lambda} \)), we have 
\[ t_C^E + t_R^E \geq t^{FB} \text{ and } K \geq K^{FB}. \] Thus, 
\[ t_C^E + t_R^E \geq t^{FB} \text{ and } K^E \geq K^{FB}. \]

Finally, we prove that, when \( \hat{r} > r^+ \), it is either the case that 
\[ t_C^E + t_R^E > 0, \] 
\[ t_C^E + t_R^E < t^{FB} \text{ and } K < K^{FB}, \] 
\[ \text{or } t_R^E = 0 \text{ and } \bar{\pi}^E = \bar{\pi} = \bar{\pi}. \] Given the bounds derived in Lemma 3, we can conclude that, when both 
\( \hat{r}_C < r^a < \frac{\tilde{r}}{\lambda} \) and \( \hat{r} > r^+ \) are verified, there will be no cordon toll, while parking charge \( t_C^E \) and space reallocation \( K^E \) will be higher than optimal. Figure 3 provides an illustration.\(^{19}\)

\(^{19}\)Unfortunately, there is also a set of parameter values (when \( \hat{r}_C \geq \frac{\tilde{r}}{\lambda} \) but \( \hat{r} < r^+ \)) such that equilibria with higher-than-optimal as well as lower-than-optimal charges and space restrictions are possible. This is the price to pay for not being able to identify a pivotal individual when voting involves the regional government \( G_R. \)
Figure 3: Bounds for $t_C^E$, $t_R^E$ (left graph) and $K^E$ (right), plotted against $\hat{r}$ and $\hat{r}_C$. In this example, we set $\hat{r}_C = \hat{r}_H = \hat{r}$ and use $\bar{r} = 10000$, $\lambda = 0.66$. Also, $\frac{\bar{r}}{\lambda} = 13333$ and $r^+ = 11206$.

These findings suggest quite a negative perspective for the implementation of road pricing. Suppose the majority of the city population cares for car driving significantly more than the average, so $\hat{r}_C > \frac{\bar{r}}{\lambda}$ holds. The city is, then, unwilling to support parking charge, road toll and space restrictions. The hinterland population is also likely to do so. Suppose instead the majority in the city cares little more for car driving than for public transport (i.e., $\hat{r}_C < \frac{\bar{r}}{\lambda}$ holds) and supports taxes and restrictions. If the majority of the hinterland population cares for driving to a sufficiently large extent (so that $\hat{r} \geq r^+$), then the road toll $t_R$ will collapse to zero. There will be no road pricing even if the city population is, on the whole, favorable to traffic restraining taxes. The reason is that the city will prefer to use parking charges.

The results are interesting in light of recent experiences in Paris and New York. In both cities the introduction of road pricing turned out to be unfeasible because of significant political opposition. Such opposition was expressed mostly by representatives of car-dependent suburban areas. Yet, the municipalities raised parking charges in central areas. Moreover, in Paris, road and parking spaces were significantly reduced. This is in accordance with our results: as long as the suburban population is more likely to value car travel than the city population (and to raise stronger opposition to car charges than the city), the cordon toll ends up being below the optimum. When parking charges are high enough, it actually collapses to zero. On the other hand, when the city population does not care for driving, it will more favorably adopt space restrictions.
Let us now focus on the last point in Proposition 2. The reason for it is the imperfect coordination between the two local governments. Previous literature has argued that such a phenomenon may lead to significant reductions in social welfare (De Borger et al. (2007), Ubbels and Verhoef (2008)). In our setup, this is not necessarily true. Even if the two governments were perfectly coordinating (as if a single one were in charge of all policy), they may, responding to voters’ preferences, choose to implement lower than optimal charges and space reallocation. When it is the case, the presence of governmental competition (as was the case for tax-exporting, as discussed in Proposition 1) could act as a compensation for the “political” policy bias.

COROLLARY: Consider the case where the majority in both city and hinterland values car travel, compared to public transport, sufficiently more than average (i.e. \( \hat{r}_C \geq \frac{r}{\lambda} \) and \( \hat{r} \geq r^+ \)). Then, social welfare when the non coordinating city and regional government are involved in traffic policy is at least as high as when the city government controls the whole policy.

4.1.4 Effect of improvements to public transportation prior to the voting stage

We here study voting on car charges and space reallocation in the presence of a reduction to the generalized cost of public transport trips, that is \( s > 0 \). This is financed via (lump-sum) general taxation. We will look at earmarking of charge revenues below. A positive \( s \) may here be interpreted as the result of interventions, coordinated by the welfare-maximizing government \( G_W \), to improve public transportation in the city. This has the objective of influencing the choice of road congestion policy by the local government. As an example, one may think of the Transport Innovation Fund created by the UK government in the early 2000s (UK DfT, 2004).

In order to point out the results in the simplest way, we assume all traffic policy to be under the control of the city government, as in Proposition 1. We set, therefore, the charge controlled by the regional government \( t_R \) to zero. As a consequence, \( t = t_C \). The car tax set
by the city can be interpreted as a combination of parking and road pricing. Provision of public transport services is often advocated to weaken voters’ opposition to car restraining policies. This is why we focus our attention on the case where, when \( s = 0 \), city voters choose a policy \( \pi^E \) such that charges and reallocated road space are below the welfare maximizing values.

PROPOSITION 3: Denote as \( \pi_{E0}^E = (t_E^0, K_{E0}^E; 0) \) the voting equilibrium with lump-sum revenue redistribution and conditional on no charge set by the regional government and no prior interventions on public transport \( (t_R = s = 0) \). Suppose that, in such a case, there is insufficient political support for car charges and space reallocation, that is \( t_{FB}^E > t_{E0}^E \) and \( K_{FB}^E > K_{E0}^E \). Then, if prior improvements to the public transport system are decided, the voting equilibrium \( \pi^E = (t^E, K^E; s) \) is closer to the welfare-optimum. That is, with \( s > 0 \), \( (t + s)^{FB} > t^E + s > t_{E0}^E \) and \( K_{FB}^E > K^E > K_{E0}^E \). However, a necessary condition is that such improvements are funded, at least partially, from the welfare-maximizing government (i.e. \( \alpha < 1 \)).

From a welfare-maximization perspective, car charges \( t \) and improvements to public transport \( s \) are perfectly substitutable instruments (see expression (3) and the benchmark of Section 3). This is not the case for local voters. A marginal increase in \( s \) induces travellers to shift away from cars and towards public transport. This reduces, for each voter, the extra (private) expenditures generated by a car tax hike. On the other hand, raising \( s \) reduces the tax base \( tQ \) and increases expenditures \( s(Y - Q) \). Nonetheless, as expenditures are only partially financed by the city (if \( \alpha < 1 \)), local voters do not fully internalize the impact of making more significant improvements to public transport. This is why a marginally higher \( s \) leads to a reduction in the charge \( t \) that is less than proportional. Thus, the sum of \( t \) and

\(^{20}\)One may wonder what would happen if we had let the city vote on \( s \) as well as \( t_C \). As long as \( \alpha = 1 \), this would result in \( s = 0 \) and, consequently, no change in \( \pi^E \). This is because, to control the volume of traffic, city voters would rather make use of a car tax that produces extra revenues (being paid also by the people from \( H \)), than finance a subsidy to public transport that is used also by people in \( H \). In order to have \( C \) choose a positive value of \( s \), it would be necessary to have \( \alpha < 1 \). Our qualitative results would, therefore, not change.
s in equilibrium gets closer to the welfare-optimal level \((t + s)^{FB}\). This mechanism breaks down when \(\alpha = 1\). When the city pays in full for improvements to public transport, local voters fully internalize their budgetary impact. Then, they would consider them as perfect substitutes for taxes on car trips.

Road pricing is usually proposed as part of “policy packages” that include upgrades to public transport. For example, in Stockholm a significant expansion of public transport services was put in place in preparation for the road pricing trial. Our results suggest that the improvement should not be financed entirely by the local government that introduces it. External funding can be seen as a second-best instrument. By choosing suboptimal road taxes and space reallocation, (the majority of) city voters impose a negative externality on the rest of the population. In the absence of a direct instrument that aligns their interest with society’s, a financial contribution from a higher government level can help correct the “political” externality.

### 4.2 Earmarking of car charge revenues to finance public transportation

The introduction of car charges is often proposed on condition that revenues be earmarked to finance public transportation. Up to now, we have disregarded this possibility. We now compare voting equilibria under lump-sum redistribution \((\rho = l)\) and earmarking \((\rho = e)\).

We again simplify the setup assuming all car charges to be entirely under the control of the city government. The charge set by the city can then be interpreted as a combination of parking charge and road pricing, and we simply call it \(t\). The voting equilibrium under \(\rho = e\) is denoted \(\pi^E_e = (t^E_e, K^E_e, s^E_e)\). We have argued in Section 3 that a first-best policy vector \(\pi^{FB}\) is such that the optimal combination of car charges and improvements to public transport is equal to the marginal external cost of a car trip, i.e. \((t + s)^{FB} = Q^{FB}T_Q^{FB}\). Since there is an infinity of couples \(t, s\) whose sum gives \((t + s)^{FB}\), we have an infinite set of \(\pi^{FB}\). However, in the earmarking regime \(\rho = e\), the first-best vector \(\pi^{FB}_e\) will be unique, since, in
addition, it has to satisfy the budgetary condition $t^{FB}_{e} Q^{FB}_{e} = \alpha s^{FB}_{e} (Y - Q^{FB}_{e})$.

**Proposition 4:** Denote as $\pi^{E}_0 = (t^{E}_0, K^{E}_0, 0)$ the voting equilibrium with lump-sum revenue redistribution and conditional on no charge set by the regional government and no prior interventions on public transport ($t_R = s = 0$). Suppose that, in such case, there is insufficient political support for car charges and space reallocation, that is $t^{FB} > t^{E}_0$ and $K^{FB} > K^{E}_0$. Then, earmarking revenues from $t$ to finance $s$ leads to an equilibrium $\pi^{E}_e = (t^{E}_e, s^{E}_e, K^{E}_e)$ that is closer to the welfare-optimum. That is, $(t + s)^{FB} \geq t^{E}_e + s^{E}_e > t^{E}_0$ and $K^{FB} \geq K^{E}_e > K^{E}_0$. However, a necessary condition is that such interventions are funded, at least partially, from the welfare-maximizing government (i.e. $\alpha < 1$).

Car charges and improvements to public transport are, from voters’ perspective, imperfect substitutes if and only if the city does not have to finance the latter entirely on her own, i.e. $\alpha < 1$. Under earmarking, the introduction of the car charge $t$ has to be combined with an improvement to public transport $s$. Therefore, $(t + s)^{FB} - t^{E}_0$ is necessarily larger than $(t + s)^{FB} - (t^{E}_e + s^{E}_e)$. However, this holds only if there is an “external” contribution from an upper level of government (i.e. $\alpha < 1$). Otherwise, $s$ and $t$ are perfect substitutes from voters’ perspective, regardless of the rebate rule adopted. Earmarking revenues for public transportation can help soften voters’ opposition to car charges. This is in line with the findings of De Borger and Proost (2010, Proposition 4). A less intuitive implication is that, in order to be welfare-enhancing, these funds have to be topped up by an external government. In reality, subsidies to local public transport systems, partially financed by national governments, are a common practice. Our result may provide an additional justification for such practice.

5 A numerical example

We here provide a numerical illustration of the results. We consider four scenarios, each characterized by distributions $F_C(r)$ and $F_H(r)$. These are very stylized, with a discrete support
including only 3 values: 0, 5000 and 10000. Since suburban car dependence characterizes most cities, we chose to consider a population in $H$ that is, for its majority, strongly attached to cars. $F_H(r)$ is such that 70% of the population has strong preferences for cars ($r = 10000$), while 30% does not care for driving ($r = 0$). The scenarios differ in $F_C(r)$, as we vary the fraction of the city population characterized by $r = 10000$. The distributions are depicted below.\footnote{The remaining parameter values are as follows: $\lambda = 0.75$, $\alpha = 0.9$, $d = 20$, $Y = 100$, $\gamma = 1$, $\delta = 10$. We also use $c(K) = K^2$ as the cost function for space reallocation.}

In Scenario 1, $F_C(r)$ and $F_H(r)$ coincide. Focusing first on the $\rho = l$ and $s = 0$ regime, we can see that conditions ensuring too small car charge and space reallocation ($t^E < t^{FB}$ and $K^E < K^{FB}$), as seen in Proposition 2 are verified. Moreover, the portion of individuals with high valuation for cars in $H$ is sufficiently large that there is no support at all for the cordon toll, $t^E_R = 0$. When improvements to public transport are introduced (we consider $s = 0.1$) we obtain a reduction in the equilibrium car tax $t^E$ smaller than the value of $s$, while reallocated road space $K^E$ goes up. This indicates that the local economy gets closer to the social optimum with respect to the case in which there are no interventions on public transport. A similar phenomenon takes place when shifting to the earmarking ($\rho = e$) regime.

Scenario 2 and 3 present distributions of preferences in $C$ such that the fraction of individuals strongly attached to cars is smaller than in Scenario 1. It is, nevertheless, still dominant. Thus, while the majority of the population is still made of frequent drivers, the total volume of car trips is smaller. Quite interestingly, this brings to charges and space reallocation which are even smaller, compared to the first best, than in Scenario 1. The reason for this is that the volume of funds that can be rebated to the population is smaller: the frequently-driving majority is thus even more penalized by the introduction of traffic restraint measures. In both scenarios there is no cordon toll and, in Scenario 3, the parking charge is also equal to zero. The role of $s$ is the same as in Scenario 1. Switching to $\rho = e$ in Scenario 3 does not bring to any change in the equilibrium: this is due to the fact that, with a zero tax being the optimal choice of the city population, no funds for $s$ are available.
even when earmarking is introduced.

Finally, in Scenario 4, people with strong valuation for cars do not represent the majority in the city anymore, while they still do in the hinterland. The result we obtain is that of a parking charge that is above the optimum. Space reallocation, while larger than in all the other 3 scenarios, it is still lower than optimal. The reason being the local government’s sub-optimal incentives, for given traffic volumes, to sustain the necessary expenditures. Finally, the equilibrium is still such that there is no cordon toll.
6 Concluding remarks

We have studied how voters’ preferences and the institutional setup may influence traffic congestion policy. We have looked at the case involving two (partially) overlapping governments. Previous literature has studied governmental competition in traffic policy, but neglected the fact that governments respond to heterogeneous voters. Our paper embeds the effects of imperfect governmental coordination in a majority voting framework. Road tolls may face stronger political opposition than parking charges and restrictions of road space for cars. We have also seen that when the majority of voters prefers the car to public transportation and only a minority does not, lack of support for traffic restraining measures can be expected. Moreover, imperfect coordination among local governments can be welfare-enhancing, contrary to common wisdom. Finally, we have found a beneficial role for public transport subsidies, but only if partially financed by higher levels of government.

The results obtained rest on some important assumptions. We have assumed that all individuals have to pay both charges and did not consider the possibility of “resident discounts” schemes, nor the issue of parking spaces provided by employers or shopping centres. Moreover, we have ignored the role of voters uncertainty in the face of reforms and concerning politicians’ quality. One may also consider another type of uncertainty. We assumed that the local government is able to hold on to the revenues from the charges it levies. While it is generally the case for at least a share of such revenues, they may have to be relinquished to higher levels of government in the face of budget crises. This possibility may change the incentives for raising car taxes. These would all be interesting points to extend our research for the future.
Appendix

Initial remark In the following proofs, we will often appeal to “sufficient” convexity of the cost function for space reallocation \( c(K) \) (i.e. require that the second derivative \( c_{KK} \) is sufficiently large). This is a sufficient condition for uniqueness of multi-dimensional voting equilibria that we will implicitly characterize using first-order conditions. Such a (technical) condition allows tractability of the model. We consider it to be a reasonable simplification.

**Lemma A:** When the C population votes on \( t_C \) and \( K \), and when the C and H population vote on \( t_R \), for every voting variable (taking the others as given) voters’ preferences satisfy the Single Crossing Property

**Proof:** When the C population votes on \( t_C \) and \( K \), given \( t_R \) and \( s \), define

\[
MRS_{tCLC}^C(\pi; r) \equiv \frac{\partial V^C(\pi; r)}{\partial t_C} \quad MRS_{tCLC}^K(\pi; r) \equiv \frac{\partial V^C(\pi; r)}{\partial K}
\]

and when the C and H population vote on \( t_R \), given \( t_C \), \( K \) and \( s \), define

\[
MRS_{tRLR}^C(\pi; r) \equiv \frac{\partial V^C(\pi; r)}{\partial t_R} \quad MRS_{tRLR}^H(\pi; r) \equiv \frac{\partial V^H(\pi; r)}{\partial t_R}
\]

Now \( MRS_{tCLC}^C = MRS_{tRLR}^C = MRS_{tRLR}^H = -q(r) \cdot \left( 1 + T_Q \frac{dQ}{dp} \right) \), for any \( \pi \) and \( r \). Therefore \( \partial MRS/\partial r = -\frac{\partial q(r)}{\partial r} \cdot \left( 1 + T_Q \frac{dQ}{dp} \right) < 0 \), since \( \frac{\partial q}{\partial r} > 0 \) and \( 1 + T_Q \frac{dQ}{dp} > 0 \) for any \( \pi \) and \( r \). We also have \( MRS_{tCLC}^K = -q(r) \cdot \left( T_Q \frac{dQ}{dr} + T_KKQ \right) \), so \( \partial MRS/\partial r = -\frac{\partial q(r)}{\partial r} \cdot \left( T_Q \frac{dQ}{dr} + T_KKQ \right) > 0 \). Using the results of Gans and Smart (1996), the Single Crossing condition holds. \( \blacksquare \)
Proof of the benchmark policy

\(G_W\)'s objective is

\[
\max_{\{t,s,K\}} W(\pi)
\]

The first order conditions are

\[
\frac{\partial W}{\partial t} = \frac{\partial W}{\partial s} = -QT_Q \frac{dQ}{dp} + (t + s) \frac{dQ}{dp} = 0 \Rightarrow t + s = QT_Q
\]

\[
\frac{\partial W}{\partial K} = -Q \cdot (1 + T_Q \frac{dQ}{dp}) \cdot (Q + 1) + Y + (t + s) \frac{dQ}{dp} (Q + 1) - c_K = 0
\]

(Recall that \(\frac{dp}{dt} = 1\) and \(A_K = T_{QK} = -1\)). Using the first of these expressions we can rewrite the second as \(Y - Q = Q^2 + c_K\). Let \(\Pi^{FB}\) be the set of stationary points of \(W(\pi)\). We now verify that all elements of \(\Pi^{FB}\) are characterized by the same values \(K^{FB}\) and \((t + s)^{FB}\) (which are therefore unique), under sufficient convexity of \(c(K)\). The hessian matrix \(h\) is

\[
\begin{bmatrix}
\frac{\partial^2 W}{\partial t^2} & \frac{\partial W}{\partial t \partial K} \\
\frac{\partial W}{\partial t \partial K} & \frac{\partial^2 W}{\partial K^2}
\end{bmatrix}
\]

We have

\[
\frac{\partial^2 W}{\partial t^2} = -\left(\frac{dQ}{dp}\right)^2 T_Q - QT_Q \frac{d^2 Q}{dp^2} + \frac{dQ}{dp} + (t + s) \frac{d^2 Q}{dp^2}
\]

\[
\frac{\partial W}{\partial t \partial K} = -\frac{dQ}{dK} T_Q \frac{dQ}{dp} - QT_{QK} \frac{dQ}{dp} - QT_Q \frac{d^2 Q}{dpdK} + (t + s) \frac{d^2 Q}{dpdK}
\]

In the neighborhood of any element of \(\Pi^{FB}\) (for which the first order conditions above hold), these expressions simplify to \(\frac{\partial^2 W}{\partial t^2} = \frac{dQ}{dp} - \left(\frac{dQ}{dp}\right)^2 T_Q < 0\) and \(\frac{\partial W}{\partial t \partial K} = -\frac{dQ}{dp} (Q + 1) \left(1 + T_Q \frac{dQ}{dp}\right) > 0\). Finally

\[
\frac{\partial^2 W}{\partial K^2} = -\frac{dQ}{dK} \left(1 + T_Q \frac{dQ}{dp}\right) (Q + 1) - Q \left(T_Q \frac{d^2 Q}{dpdK} + \frac{dQ}{dp}\right) (Q + 1) +
\]

\[
-Q \left(1 + T_Q \frac{dQ}{dp}\right) \frac{dQ}{dK} + (t + s) \frac{d^2 Q}{dpdK} (Q + 1) + (t + s) \frac{dQ}{dp} \frac{dQ}{dK} - c_{KK}
\]
which, in the neighborhood of any element of \(\Pi^{FB}\), simplifies to

\[
\frac{\partial^2 W}{\partial K^2} = -\frac{dQ}{dK} \left(1 + T_Q \frac{dQ}{dp}\right)(Q + 1) - 2Q \frac{dQ}{dK} - c_{KK}
\]

We thus get

\[
\det h = \left(-\frac{dQ}{dK} \left(1 + T_Q \frac{dQ}{dp}\right)(Q + 1) - 2Q \frac{dQ}{dK} - c_{KK}\right) \cdot \left(\frac{dQ}{dp} - \frac{dQ^2}{dp} T_Q\right) + \\
- \left(\frac{dQ}{dp} (Q + 1) \left(1 + T_Q \frac{dQ}{dp}\right)\right)^2
\]

The sign of \(\frac{\partial^2 W}{\partial K^2}\) is ambiguous. We need to proceed under the assumption that \(c(K)\) is sufficiently convex (i.e. \(c_{KK}\) is large enough) to ensure \(\det h > 0\). If so, \(h\) is negative definite in the neighborhood of any stationary point in \(\Pi^{FB}\). Thus, given that \(W(\pi)\) is a continuously differentiable function of \(\pi\), under sufficient convexity of \(c(K)\) there can exist a unique value of \(K^{FB}\) and \((t + s)^{FB}\). Also, \(W(\pi)\) is a strictly concave function of \(\pi\).

**Proof of Lemma 1**

By Single Crossing, proved in Lemma A, we have existence of majority voting equilibria \(t_C(K, t_R; s) = t^*_C(K, t_R; s, \hat{r}_C)\) and \(K(t_C, t_R; s) = K^*(t_C, t_R; s, \hat{r}_C)\) (this follows from a result in Gans and Smart (1996)). We proceed assuming that \(t^*_C(K, t_R; s, \hat{r}_C)\) is an interior maximizer of \(V_C(\pi; \hat{r}_C)\), for given \(K, t_R\) and \(s\). This is always the case in equilibrium. We verify in Technical Appendix A that (for given \(K, t_R\) and \(s\)) any \(t_C\) satisfying the first order condition \(\frac{\partial V_C(\pi; \hat{r}_C)}{\partial t_C} = 0\) also satisfies the second order condition. Thus, since \(V_C(\pi; \hat{r}_C)\) is a continuously differentiable function of \(t_C\), \(t^*_C(K, t_R; s, \hat{r}_C)\) must be unique.

As for \(K^*(t_C, t_R; s, \hat{r}_C)\), as long as \(c(K)\) is sufficiently convex (i.e. \(c_{KK}\) large enough), any \(K\) satisfying the first order condition \(\frac{\partial V_C(\pi; \hat{r}_C)}{\partial K} = 0\), for given \(t_C, t_R\) and \(s\), also verifies the second order condition. Therefore, since \(V_C(\pi; \hat{r}_C)\) is a continuously differentiable function of \(K\), \(K^*(t_C, t_R; s, \hat{r}_C)\) is unique.
We now prove uniqueness of $\pi_C(t_R; s)$. This vector is such that the following first order conditions hold simultaneously

$$F_1 : \quad \frac{\partial V_C(\pi; \hat{c})}{\partial t_C} = -q(\hat{c}) \cdot \left(1 + T_Q \frac{dQ}{dp}\right) + \frac{(t_C + \alpha s) dQ}{\lambda} + t_R \frac{dQ}{dp} = 0$$

$$F_2 : \quad \frac{\partial V_C(\pi; \hat{c})}{\partial K} = -q(\hat{c}) \cdot (T_Q \frac{dQ}{dK} + T_K Q - A_K) + Y + \frac{(t_C + \alpha s) dQ}{\lambda} - c_K + t_R \frac{dQ}{dK} = 0$$

$F_2$ can be rearranged as

$$\left(-q(\hat{c}) \cdot \left(1 + T_Q \frac{dQ}{dp}\right) + \frac{(t_C + \alpha s) dQ}{\lambda} + t_R \frac{dQ}{dp}\right) \cdot (Q + 1) + Y - \frac{c_K}{\lambda} = 0$$

and, using $F_1$, rewritten as

$$F_2' : \lambda Y - Q - T_K Q^2 - c_K = 0$$

in the neighborhood of $\pi_C(t_R; s)$. By the Implicit Function Theorem, $F_1$ implicitly defines $t_C$ as a continuously differentiable function of $K$ (as well as of other policy parameters that we here treat as given). Similarly, $F_2$ implicitly defines $K$ as a continuously differentiable function of $t_C$ (and of other policy parameters). A vector $\pi_C(t_R; s)$ is a crossing of such functions on the $(t_C, K)$ plane. If we can show that one always crosses the other from the same direction, then they must cross only once. If so, $\pi_C(t_R; s)$ must be unique. We have, by the Implicit Function Theorem

$$\frac{\partial t_C}{\partial K_{F1}} = -\frac{\partial F_1}{\partial t_C} \frac{\partial t_C}{\partial K_{F2}} = -\frac{-\frac{dq(\hat{c})}{dp} \cdot \left(1 + T_Q \frac{dQ}{dp}\right) - q(\hat{c}) \cdot \left(\frac{dQ}{dp} + T_Q \frac{d^2Q}{dpdK}\right) + \frac{(t_C + \alpha s) \frac{d^2Q}{dpdK} + \frac{dQ}{dK}}{\lambda} + t_R \frac{dQ}{dpdK}}{-\frac{dq(\hat{c})}{dp} \cdot \left(1 + T_Q \frac{dQ}{dp}\right) - q(\hat{c}) \cdot \left(\frac{dQ}{dp} + T_Q \frac{d^2Q}{dpdK}\right) + \frac{(t_C + \alpha s) \frac{d^2Q}{dpdK} + \frac{dQ}{dK}}{\lambda} + t_R \frac{dQ}{dpdK}}$$

$$\frac{\partial t_C}{\partial K_{F2'}} = -\frac{\partial F_2'}{\partial t_C} = -\frac{-\frac{dQ}{dp} - 2 \frac{dQ}{dK} Q - c_{KK}}{-\frac{dQ}{dp} - 2 \frac{dQ}{dK} Q}$$

Note that we work with $F_2'$ since its form is simpler than that of $F_2$. Anyway, since we focus our attention on neighborhoods of $\pi_C(t_R; s)$, $F_2'$ and $F_2$ are equivalent. Let us examine the sign of these terms in the neighborhood of $\pi_C(t_R; s)$. $\frac{\partial F_1}{\partial t_C} = \frac{\partial^2 V_C}{\partial t_C^2} < 0$, as we prove in Technical Appendix A. We proceed under the assumption of a sufficiently convex $c(K)$ (i.e. sufficiently large $c_{KK}$) to ensure that $-\frac{dQ}{dK} - 2 \frac{dQ}{dK} Q - c_{KK} < 0$, so $\frac{\partial F_2'}{\partial K} = \frac{\partial^2 V_C}{\partial K^2} < 0$. It is
easy to see that $\frac{\partial F_2'}{\partial t_C} > 0$. The sign of $\frac{\partial F_1}{\partial K}$ is ambiguous. Yet, on condition that $c_{KK}$ is large enough, we have $0 < \frac{\partial t_C}{\partial K} F_1 < \frac{\partial t_C}{\partial K} F_2'$ in the neighborhood of $\pi_C(t_R; s)$. Under such a condition, we can be sure $F_1$ and $F_2'$ cross only once. Hence, $\pi_C(t_R; s)$ is unique.

Let us now prove the comparative statics. In a neighborhood of $\pi_C(t_R; s)$, we can express $t_C$ and $K$ as functions of $\hat{t}_C, \lambda, t_R$ and $s$, using the Implicit Function Theorem. We have

$$\frac{\partial t_C}{\partial t} = - \frac{\text{det} \begin{bmatrix} \frac{\partial F_1}{\partial t_C} & \frac{\partial F_1}{\partial K} \\ \frac{\partial F_2'}{\partial t_C} & \frac{\partial F_2'}{\partial K} \end{bmatrix}}{\text{det} \begin{bmatrix} \frac{\partial F_1}{\partial \lambda} & \frac{\partial F_1}{\partial K} \\ \frac{\partial F_2'}{\partial \lambda} & \frac{\partial F_2'}{\partial K} \end{bmatrix}}$$

The denominator of all three expressions is $\frac{\partial F_1}{\partial t_C} - \frac{\partial F_2'}{\partial t_C}$. It is positive as long as $c(K)$ is sufficiently convex to ensure that $0 < \frac{\partial t_C}{\partial K} F_1 < \frac{\partial t_C}{\partial K} F_2'$ holds (this necessarily holds as long as $\pi_C(t_R; s)$ is unique). Let us look at determinants at the numerator of $\frac{\partial t_C}{\partial t_C}$ and $\frac{\partial t_C}{\partial \lambda}$, we have

$$\text{det} \begin{bmatrix} \frac{\partial F_1}{\partial t_C} & \frac{\partial F_1}{\partial K} \\ \frac{\partial F_2'}{\partial t_C} & \frac{\partial F_2'}{\partial K} \end{bmatrix} = \frac{\partial F_1}{\partial \hat{t}_C} \cdot \frac{\partial F_2'}{\partial K}$$

and

$$\text{det} \begin{bmatrix} \frac{\partial F_1}{\partial \lambda} & \frac{\partial F_1}{\partial K} \\ \frac{\partial F_2'}{\partial \lambda} & \frac{\partial F_2'}{\partial K} \end{bmatrix} = \frac{\partial F_1}{\partial \lambda} \cdot \frac{\partial F_2'}{\partial K} - Y \frac{\partial F_1}{\partial K}$$

since $\frac{\partial F_2'}{\partial t_C} = 0$ and $\frac{\partial F_2'}{\partial \lambda} = Y$. Now $\frac{\partial F_1}{\partial \hat{t}_C} \cdot \frac{\partial F_2'}{\partial K} > 0$ since $\frac{\partial F_1}{\partial \hat{t}_C} < 0$ and $\frac{\partial F_2'}{\partial K} < 0$, $\frac{\partial F_1}{\partial \lambda} \cdot \frac{\partial F_2'}{\partial K} > 0$ since $\frac{\partial F_1}{\partial \lambda} < 0$. The sign of $\frac{\partial F_1}{\partial K}$ is ambiguous. However, once again, assuming a large enough $c_{KK}$ (i.e. that $c(K)$ is sufficiently convex), $\frac{\partial F_2'}{\partial K}$ will be large enough to ensure both determinants are positive. Then $\frac{\partial t_C}{\partial t_C} < 0$ and $\frac{\partial t_C}{\partial \lambda} < 0$, as stated in the text.

Let us now focus on $\frac{\partial t_C}{\partial t_R}$. We have $\frac{\partial F_2'}{\partial t_C} = \frac{\partial F_2'}{\partial t_C} > 0$. We prove in the Technical Appendix A that $\frac{\partial F_1}{\partial t_R} = \frac{\partial F_2'}{\partial t_C} t_C < 0$ in the neighborhood of $\pi_C(t_R, s)$. Therefore, $\frac{\partial F_1}{\partial t_R} \frac{\partial F_2'}{\partial t_R} - \frac{\partial F_2'}{\partial t_R} \frac{\partial F_1}{\partial t_R} > 0$.

---

$^{22}$It is reasonable to assume that $t_C \frac{dQ}{dt} + Q \geq 0$. There is no reason, whatever the preferences of the decisive city voter, to choose a $t_C$ such that the marginal revenues are negative.
assuming, still, large enough $c_{KK}$. As a consequence, $\frac{\partial c}{\partial t_R} > -1$ if and only if

$$\frac{\partial F_1 \partial F_2'}{\partial t_R \partial K} - \frac{\partial F_2' \partial F_1}{\partial t_R \partial K} < \frac{\partial F_1 \partial F_2'}{\partial t_C \partial K} - \frac{\partial F_2' \partial F_1}{\partial t_C \partial K}$$

Now, this is true if and only if $\frac{\partial F_1}{\partial t_R} > \frac{\partial F_1}{\partial t_C}$ given that $\frac{\partial F_2'}{\partial t_R} = \frac{\partial F_2'}{\partial t_C}$. Since $\frac{\partial F_1}{\partial t_R} + \frac{\partial Q(\frac{1}{\lambda} - 1)}{\partial p} = \frac{\partial F_1}{\partial t_C}$ and $\frac{\partial Q(\frac{1}{\lambda} - 1)}{\partial p} < 0$, then indeed $\frac{\partial F_1}{\partial t_R} > \frac{\partial F_1}{\partial t_C}$. Thus, $0 > \frac{\partial c}{\partial t_R} > -1$.

As for comparative statics involving $K$, we have

$$\frac{\partial K}{\partial t_C} = - \det \begin{bmatrix} \frac{\partial F_1}{\partial t_C} & \frac{\partial F_1}{\partial t_C} \\ \frac{\partial F_2'}{\partial t_C} & \frac{\partial F_2'}{\partial t_C} \end{bmatrix}, \quad \frac{\partial K}{\partial t_R} = - \det \begin{bmatrix} \frac{\partial F_1}{\partial t_R} & \frac{\partial F_1}{\partial t_R} \\ \frac{\partial F_2'}{\partial t_R} & \frac{\partial F_2'}{\partial t_R} \end{bmatrix}$$

for all three expressions, the denominator is positive under conditions discussed above (sufficient convexity of $c(K)$ is still required). As for the numerators, we have

$$\det \begin{bmatrix} \frac{\partial F_1}{\partial t_C} & \frac{\partial F_1}{\partial t_C} \\ \frac{\partial F_2'}{\partial t_C} & \frac{\partial F_2'}{\partial t_C} \end{bmatrix} = - \frac{\partial F_1}{\partial t_C} \cdot \frac{\partial F_2'}{\partial t_C} \cdot \det \begin{bmatrix} \frac{\partial F_1}{\partial t_C} & \frac{\partial F_1}{\partial t_C} \\ \frac{\partial F_2'}{\partial t_C} & \frac{\partial F_2'}{\partial t_C} \end{bmatrix} = Y \frac{\partial F_1}{\partial t_C} - \left( \frac{\partial F_1}{\partial t_C} \cdot \frac{\partial F_2'}{\partial t_C} \right)$$

because $\frac{\partial F_2'}{\partial t_C} = 0$ and $\frac{\partial F_2'}{\partial t_C} = Y$. Now $-\frac{\partial F_1}{\partial t_C} \cdot \frac{\partial F_2'}{\partial t_C} > 0$. To continue, $\frac{\partial F_1}{\partial t_C} \cdot \frac{\partial F_2'}{\partial t_C} > 0$ since $\frac{\partial F_1}{\partial t_C} < 0$ (as argued above) and $\frac{\partial F_2'}{\partial t_C} < 0$, with a sufficiently convex $c(K)$. We also have $Y \frac{\partial F_1}{\partial t_C} - \left( \frac{\partial F_1}{\partial t_C} \cdot \frac{\partial F_2'}{\partial t_C} \right) < 0$, since $\frac{\partial F_1}{\partial t_C} < 0$, as proven in Technical Appendix A. Therefore, $\frac{\partial K}{\partial t_C} < 0$ and $\frac{\partial K}{\partial t_R} > 0$ as stated in the text. Focus, to conclude, on $\frac{\partial K}{\partial t_R}$. The numerator is

$$\frac{\partial F_1}{\partial t_C} \cdot \frac{\partial F_2'}{\partial t_R} - \frac{\partial F_2'}{\partial t_C} \cdot \frac{\partial F_1}{\partial t_R} > 0,$$

since $\frac{\partial F_2'}{\partial t_R} = \frac{\partial F_2'}{\partial t_C} < 0$ and $\frac{\partial F_1}{\partial t_R} - \frac{\partial F_1}{\partial t_C} > 0$. Therefore $\frac{\partial K}{\partial t_R} > 0$. 

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Technical Appendix A

The second derivative of $V_C(\pi; \hat{r}_C)$ with respect to $t_C$ is

$$\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C^2} = -\frac{dq(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_C) \left( T_Q \frac{d^2 Q}{dp^2} \right) + \frac{2\frac{dQ}{dp} + (t_C + \alpha s) \frac{d^2 Q}{dp^2}}{\lambda} + t_R \frac{d^2 Q}{dp^2}$$

in the neighborhood of interior (local) maximisers of $V_C$, using (5) (equated to zero), we can replace $-q(\hat{r}_C) T_Q + \frac{(t_C + \alpha s)}{\lambda} + t_R$ and rewrite the above expression as

$$\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C^2} = -\frac{dq(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) + \frac{2\frac{dQ}{dp}}{\lambda} + \left( \frac{q(\hat{r}_C) - Q}{\frac{dQ}{dp}} \right) \frac{d^2 Q}{dp^2}$$

This expression can be simplified using $\frac{dQ}{dp}$, $\frac{dq(r)}{dp}$, and $\frac{d^2 Q}{dp^2}$. We have

$$\frac{dq(r)}{dp} = -\frac{r}{\frac{1}{2} (p + T - A + s)^3 + \bar{r}T_Q}$$

and

$$\frac{d^2 Q}{dp^2} = \frac{3}{4} \frac{(p + T - A + s)^5}{(\frac{1}{2} (p + T - A + s)^3 + \bar{r}T_Q)^3}$$

with $\frac{dQ}{dp} = \frac{dq(r)}{dp}$. Substituting into $\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C^2}$ and rearranging, we have

$$\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C^2} = \frac{1}{2} \left( \frac{\hat{r}_C - 2\bar{r}}{\lambda} \right) \left( p + T - A + s \right)^3 - \frac{2}{\lambda} \bar{r} T_Q - \frac{3}{4} \left( \hat{r}_C - \bar{r} \right) \left( p + T - A + s \right)^3$$

$$\left( \frac{1}{2} (p + T - A + s)^3 + \bar{r}T_Q \right)^2$$

since the denominator is positive, the sign of $\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C^2}$ depends on its numerator. Let us focus on it. One needs, first, to divide it by $(p + T - A + s)^3$ to obtain, using $T = T_Q Q$ and

$$Q = \frac{\bar{r}}{(p + T - A + s)^2},$$

$$\frac{1}{2} \left( \hat{r}_C - 2\bar{r} \right) - \frac{2}{\lambda} \frac{\bar{r}T}{p + T - A + s} - \frac{3}{4} \left( \hat{r}_C - \bar{r} \right)$$

Simple rearrangements allow us to write

$$\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C^2} < 0 \Leftrightarrow \frac{T}{p + T - A + s} > -\frac{1}{8} \left( \frac{\hat{r}_C \lambda}{\bar{r}} + 1 \right)$$

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The last expression being negative, the condition is always verified. This implies that second order condition are verified. Consider now

\[
\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C \partial t_R} = -\frac{dq(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_C) \left( T_Q \frac{d^2 Q}{dp^2} + \frac{dQ}{dp} + (t_C + \alpha s) \frac{d^2 Q}{dp^2} + t_R \frac{d^2 Q}{dp^2} + \frac{dQ}{dp} \right)
\]

in the neighborhood of interior (local) maximisers of \( V_C \), using (5) (equated to zero), we can write it as

\[
\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C \partial t_R} = -\frac{dq(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) + \left( 1 + \frac{1}{\lambda} \right) \frac{dQ}{dp} + \left( \frac{q(\hat{r}_C) - Q}{\lambda} \right) \frac{d^2 Q}{dp^2}
\]

following similar steps as above, we obtain

\[
\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C \partial t_R} < 0 \iff \frac{T}{p + T - A + s} > -\frac{1}{\hat{r}_C} \left( \frac{\lambda}{\lambda + 1} \right) + \left( \frac{3}{4(\lambda + 1)} - \frac{1}{2} \right)
\]

which is always verified, given that \( \hat{r}_C \in [0, 2\bar{r}] \), as long as (but not only if) \( \lambda \geq \frac{1}{2} \).

**Proof of Lemma 2**

**Proof of single-peakedness of tax preferences on** \( t_R \) **(given** \( \pi_C \) **and** \( s \))

The most-preferred \( t_R^C(\pi_C; s, r) \), given \( t_C, K \) and \( s \), for an individual living in \( C \), satisfies the following first order condition

\[
\frac{\partial V_C(\pi; r)}{\partial t_R} = -q(r) \left( 1 + T_Q \frac{dQ}{dp} \right) + \left( t_C + \alpha s \right) \frac{dQ}{dp} + Q + t_R \frac{dQ}{dp} \leq 0
\]

similarly, for an individual living in \( H \), \( t_R^H(\pi_C; s, r) \) satisfies

\[
\frac{\partial V_H(\pi; r)}{\partial t_R} = -q(r) \left( 1 + T_Q \frac{dQ}{dp} \right) + t_R \frac{dQ}{dp} + Q \leq 0
\]
If the latter is negative for all $t_R$, then clearly $t^H_\pi (\pi_C; s, r) = 0$ and $V_H (\pi, r)$ is everywhere decreasing in $t_R$. The same can be said for $t^C_\pi (\pi_C; s, r) = 0$ and $V_C (\pi; r)$. Single-peakedness would immediately follow. Consider the case in which, instead, there is at least one $t_R$ such that the first order conditions above hold at equality (i.e. a stationary point of either $V_C$ or $V_H$, given $\pi_C$ and $s$). We prove in Technical Appendix B that such a point (for any $r$) would also satisfy second order conditions. This implies that any stationary point of $V_C (\pi; r)$ and $V_H (\pi; r)$ is a local maximizer. However, since these are continuously differentiable functions of $t_R$ (given $\pi_C$ and $s$, for any $r$), they can have at most one local maximizer. As a consequence, single-peakedness holds.

**Technical Appendix B**

The first derivative of $V_H (\pi; \hat{r}_H)$ with respect to $t_R$ is

$$\frac{\partial V_H (\pi; \hat{r}_H)}{\partial t_R} = -q(\hat{r}_H) \left( 1 + T_Q \frac{dQ}{dp} \right) + t_R \frac{dQ}{dp} + Q$$

and the second derivative is

$$\frac{\partial^2 V_H (\pi; \hat{r}_H)}{\partial t_R^2} = - \frac{dq(\hat{r}_H)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - (\hat{r}_H) \left( T_Q \frac{d^2Q}{dp^2} \right) + 2 \frac{dQ}{dp} + t_R \frac{d^2Q}{dp^2}$$

The first derivative of $V_C (\pi; \hat{r}_C)$ with respect to $t_R$ is

$$\frac{\partial V_C (\pi; \hat{r}_C)}{\partial t_R} = -q(\hat{r}_C) \left( 1 + T_Q \frac{dQ}{dp} \right) + \frac{(t_C + \alpha s) \frac{dQ}{dp}}{\lambda} + Q + t_R \frac{dQ}{dp}$$

and the second derivative is

$$\frac{\partial^2 V_C (\pi; \hat{r}_C)}{\partial t_R^2} = - \frac{dq(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_C) \left( T_Q \frac{d^2Q}{dp^2} \right) + \frac{(t_C + \alpha s) \frac{d^2Q}{dp^2}}{\lambda} + 2 \frac{dQ}{dp} + t_R \frac{d^2Q}{dp^2}$$

In the neighborhood of interior (local) maximisers of $V_H (\pi; \hat{r}_H)$ and $V_C (\pi; \hat{r}_C)$, using, respectively $\frac{\partial^2 V_H (\pi; \hat{r}_H)}{\partial t_R^2} = 0$ and $\frac{\partial^2 V_C (\pi; \hat{r}_C)}{\partial t_R^2} = 0$, we can write $\frac{\partial^2 V_C (\pi; \hat{r}_C)}{\partial t_R^2}$ and $\frac{\partial^2 V_H (\pi; \hat{r}_H)}{\partial t_R^2}$ as
\[ \frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_R^2} = -\frac{dq(\hat{r}_C)}{dp} \left( 1 + TQ \frac{dQ}{dp} \right) + 2 \frac{dQ}{dp} + \left( q(\hat{r}_C) - Q \right) \frac{d^2 Q}{dp^2} \]

\[ \frac{\partial^2 V_H(\pi; \hat{r}_H)}{\partial t_R^2} = -\frac{dq(\hat{r}_H)}{dp} \left( 1 + TQ \frac{dQ}{dp} \right) + 2 \frac{dQ}{dp} + \left( q(\hat{r}_H) - Q \right) \frac{d^2 Q}{dp^2} \]

Following the steps of Technical Appendix A, rearrangements allow us to write that

\[ \frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_R^2} = \frac{\partial^2 V_H(\pi; \hat{r}_H)}{\partial t_R^2} < 0 \iff \frac{T}{p + T - A + s} > -\frac{1}{8} \left( \frac{\hat{r}_i}{\bar{r}} + 1 \right) \quad i = C, H \]

The last expression being negative, the condition is always verified. This implies that second order conditions are verified.

**Proof that \( t_R(\pi_C; s) \) lies in the interval \( I = [t^*_R(\pi_C; s, \hat{r}_C), t^*_R(\pi_C; s, \hat{r}_H)] \)**

Suppose \( t_R(\pi_C; s) < t^*_R(\pi_C; s, \hat{r}_H) \leq t^*_R(\pi_C; s, \hat{r}_C) \). Given Lemma A, Single-Crossing of preferences for \( t_R \) implies that at least half of the \( H \) population would strictly prefer \( t^*_R(\pi_C; s, \hat{r}_H) \) to \( t_R(\pi_C; s) \). The same has to be true for at least half of the individuals in \( C \), given that \( \hat{r}_C \) also prefers \( t^*_R(\pi_C; s, \hat{r}_H) \) to \( t_R(\pi_C; s) \) by single-peakedness. Therefore, \( t_R(\pi_C; s) < t^*_R(\pi_C; s, \hat{r}_H) \leq t^*_R(\pi_C; s, \hat{r}_C) \) is not possible. It cannot be a Condorcet Winner since at least half of the total population would prefer a \( t_R \) between \( t^*_R(\pi_C; s, \hat{r}_H) \) and \( t^*_R(\pi_C; s, \hat{r}_C) \) to \( t_R(\pi_C; s) \). A similar reasoning shows that \( t^*_R(\pi_C; s, \hat{r}_H) \leq t^*_R(\pi_C; s, \hat{r}_C) < t_R(\pi_C; s) \) is not possible either. The reasoning would be the same had we supposed \( t^*_R(\pi_C; s, \hat{r}_H) \geq t^*_R(\pi_C; s, \hat{r}_C) \).

We now prove that \( t^*_R(\pi_C; s, \hat{r}_H) \geq t^*_R(\pi_C; s, \hat{r}_C) \). Consider any equilibrium vector \( \pi^E \).

Since \( t^E_C = t^*_C(t_R; s, \hat{r}_C) \) , \( \pi^E \) must satisfy the first-roder condition

\[ \frac{\partial V_C(\pi; r)}{\partial t_C} = -q(\hat{r}_C) \left( 1 + TQ \frac{dQ}{dp} \right) + \frac{Q + (t_C + \alpha s) \frac{dQ}{dp}}{\lambda} + t_R \frac{dQ}{dp} \leq 0 \]
which implies that, when evaluated at $\pi^E$,

$$\frac{\partial V_C(\pi; r)}{\partial t_R} = -q(\hat{r}_C) \left(1 + T_Q \frac{dQ}{dp}\right) + \frac{(t_C + \alpha s) \frac{dQ}{dp}}{\lambda} + Q + t_R \frac{dQ}{dp} < 0$$

since $\frac{Q}{\hat{\lambda}} - Q > 0$. We have shown above that voters’ preferences over $t_R$, given $\pi_C$ and $s$, are single-peaked. Thus, both $\frac{\partial V_C(\pi; r)}{\partial t_R}$ and $\frac{\partial V_H(\pi; r)}{\partial t_R}$ are decreasing in $t_R$. Now, when evaluated at $t^*_R(\pi_C; s, \hat{r}_C)$, $\frac{\partial V_C(\pi; r)}{\partial t_R} = 0$. Therefore, if $\frac{\partial V_H(\pi; r)}{\partial t_R} < 0$ when evaluated at $\pi^E$, it must be the case that $t^*_R > t^*_C(\pi_C; s, \hat{r}_C)$. Now consider the first-order derivative for individual $\hat{r}_H$

$$\frac{\partial V_H(\pi; r)}{\partial t_R} = -q(\hat{r}_H) \left(1 + T_Q \frac{dQ}{dp}\right) + Q + t_R \frac{dQ}{dp}$$

still evaluating at $\pi^E$. There are 2 possibilities: if $\frac{\partial V_H(\pi; r)}{\partial t_R} \geq 0$, $t^*_R$ is smaller (or equal) than $t^*_R(\pi_C, s; \hat{r}_H)$ because of single-peakedness. Then, surely $t^*_R(\pi_C, s; \hat{r}_H) \geq t^*_R > t^*_C(\pi_C; s, \hat{r}_C)$. If $\frac{\partial V_H(\pi; r)}{\partial t_R} < 0$, unless $t^*_R = 0$ (in which case $t^*_R(\pi_C, s; \hat{r}_H) = t^*_R(\pi_C; s, \hat{r}_C) = 0$), the $\pi^E$ considered cannot be an equilibrium. This is because we would have

$$\max \left(t^*_R(\pi_C; s, \hat{r}_C); t^*_R(\pi_C; s, \hat{r}_H)\right) < t^*_R$$

which is not possible, as proven above. The consequence of this reasoning is that there is no $\pi^E$ such that $t^*_R(\pi_C; s, \hat{r}_C) > 0$. We must always have that $t^*_R(\pi_C; s, \hat{r}_C) = 0$. This also means that $\pi^E$ must always be such that $t^*_R(\pi_C; s, \hat{r}_H) \geq t^*_R(\pi_C; s, \hat{r}_C)$.

**Proof of Lemma 3**

Let us begin from the case in which the most-preferred $t^*_R(\pi_C; s, \hat{r}_H)$ is an interior maximizer, i.e. such that $\frac{\partial V_H(\pi; r)}{\partial t_R} = 0$. It is unique, by single-peakedness of voters preferences for $t_R$,

---

23This is true unless $t^*_R(\pi_C, s; \hat{r}_C) = 0$ as a corner solution. If it were the case, anyway, we would be sure that $t^*_R(\pi_C; s, \hat{r}_H) \geq t^*_R(\pi_C; s, \hat{r}_C)$. 

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proved in Lemma 2. It then has to satisfy

\[ F : \frac{\partial V^H (\pi; r)}{\partial t_R} = -q(\hat{r}_H) \left( 1 + T_Q \frac{dQ}{dp} \right) + t_R \frac{dQ}{dp} + Q = 0 \]

the Implicit Function Theorem tells us that \( \frac{\partial t^*_R (\pi_C; s, \hat{r}_H)}{\partial t_C} = -\frac{\partial F}{\partial t_C} \). Now, \( \frac{\partial F}{\partial t_R} = \frac{\partial^2 V^H (\pi; r)}{\partial t_R^2} < 0 \) when evaluated at \( t^*_R (\pi_C; s, \hat{r}_H) \). This is proven in Technical Appendix B. Moreover,

\[ \frac{\partial F}{\partial t_C} = -\frac{dq(\hat{r}_H)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_H)T_Q \frac{d^2Q}{dp^2} + t_R \frac{d^2Q}{dp^2} + \frac{dQ}{dp} \]

When evaluated at \( t^*_R (\pi_C; s, \hat{r}_H) \), as we prove in Technical Appendix C, \( \frac{\partial F}{\partial t_C} = \frac{\partial^2 V^H (\pi; r)}{\partial t_R \partial t_C} < 0 \). Therefore, we get \( \frac{\partial t^*_R (\pi_C; s, \hat{r}_H)}{\partial t_C} < 0 \). Focus now on the case in which \( t^*_R (\pi_C; s, \hat{r}_H) = 0 \) as a corner solution. Then necessarily \( \frac{\partial t^*_R (\pi_C; s, \hat{r}_H)}{\partial t_C} = 0 \).

**Proof of uniqueness of \( \bar{\pi} \)** Let us begin from the case in which \( \bar{\pi} \) is such that \( t^*_R (\pi_C; s, \hat{r}_H) = 0 \) as a corner solution. Then \( \bar{\pi} = \pi_C (t_R; s) \), with \( t_R = 0 \). Uniqueness of \( \bar{\pi} \) follows from Lemma 1.

Consider now the case in which \( t^*_R (\pi_C; s, \hat{r}_H) \) is an interior maximizer of \( V^H (\pi; r) \). \( \bar{\pi} \) is such that the following conditions hold

\[ F_1 : \quad \frac{\partial V_C (\pi; r)}{\partial t_C} = -q(\hat{r}_C) \left( 1 + T_Q \frac{dQ}{dp} \right) + \frac{Q + (t_C + \alpha s) \frac{dQ}{dp}}{\lambda} + t_R \frac{dQ}{dp} = 0 \]

\[ F_2 : \quad \frac{\partial V_H (\pi; r)}{\partial t_R} = -q(\hat{r}_H) \left( 1 + T_Q \frac{dQ}{dp} \right) + t_R \frac{dQ}{dp} + Q = 0 \]

\[ F_3 : \quad \frac{\partial V_C (\pi; r)}{\partial K} = \left( -q(\hat{r}_C) \left( 1 + T_Q \frac{dQ}{dp} \right) + \left( \frac{t_C + \alpha s}{\lambda} + t_R \right) \frac{dQ}{dp} \right) \cdot (Q + 1) + Y - \frac{c_K}{\lambda} = 0 \]

We can use \( F_1 \) to rewrite \( F_3 \) as

\[ F_3' : \quad Y \lambda - Q - c_K - T_Q Q^2 = 0 \]

Next, substituting \( t_R \frac{dQ}{dp} = q(\hat{r}_H) \cdot \left( 1 + T_Q \frac{dQ}{dp} \right) - Q \) from \( F_2 \) into \( F_1 \), multiplying both sides
of the resulting expression by \( \lambda \), and finally adding it to \( F_2 \) we obtain an equation that can replace \( F_1 \). The result is the following equivalent system

\[
F_4 : \quad -q(\hat{r}) \cdot \left( 1 + T_Q \frac{dQ}{dp} \right) + (t + \alpha s) \frac{dQ}{dp} + (2 - \lambda)Q = 0
\]

\[
F_2 : \quad -q(\hat{r}_H) \cdot \left( 1 + T_Q \frac{dQ}{dp} \right) + t_R \frac{dQ}{dp} + Q = 0
\]

\[
F_3' : \quad Y \lambda - Q - c_K - T_QKQ^2 = 0
\]

where \( \hat{r} = \hat{r}_C \lambda + (1 - \lambda) \hat{r}_H \). Importantly, \( F_4 \) contains terms that are function only of \( t \), not of its components \( t_C, t_R \). Thus, condition \( F_4 \), by the Implicit Function Theorem, implicitly defines \( t \) as a continuously differentiable function of \( K \) (as well as other policy parameters that we here treat as fixed). Similarly, \( F_3' \) implicitly defines \( K \) as a continuously differentiable function of \( t \) (and other policy parameters). Any vector \( \bar{\pi} \) must be such that these functions cross on the \((t, K)\) plane. If we can prove that one crosses the other always from the same direction, we can be sure that the crossing point is unique. Note that we work with \( F_3' \) rather than \( F_3 \) since its form is simpler. Anyway, since we focus our attention on neighborhoods of \( \bar{\pi}, F_3' \) and \( F_3 \) are equivalent. Using the Implicit Function Theorem, we obtain that

\[
\frac{\partial t}{\partial K}_{F_4} = -\frac{\partial F_4}{\partial K} \quad \frac{\partial t}{\partial K}_{F_3'} = -\frac{\partial F_3'}{\partial K}
\]

We show in Technical Appendix D that \( \frac{\partial F_4}{\partial t} < 0 \). As for \( \frac{\partial F_4}{\partial K} \), we have, using the fact that

\[
\frac{dq(r)}{dK} = \frac{dq(r)}{dp} \cdot (Q + 1) \quad \text{and} \quad \frac{d^2Q}{dpdK} = \frac{d^2Q}{dp^2} \cdot (Q + 1),
\]

\[
\frac{\partial F_4}{\partial K} = \left( -\frac{dq(\hat{r})}{dp} \cdot \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r})T_Q \frac{d^2Q}{dp^2} + (t + \alpha s) \frac{dQ}{dp} + (2 - \lambda) \frac{dQ}{dp} \right) \cdot (Q + 1) - q(\hat{r}) \frac{dQ}{dp} T_QK
\]

the sign of which is ambiguous. Next, \( \frac{\partial F_3'}{\partial K} = -\frac{dQ}{dK} - 2Q \frac{dQ}{dK} - c_KK \) and \( \frac{\partial F_3'}{\partial t} = -\frac{dQ}{dp} - 2Q \frac{dQ}{dp} \). Now, \( \frac{\partial F_3'}{\partial t} > 0 \) since \( \frac{dQ}{dp} < 0 \). We proceed assuming a sufficiently convex \( c(K) \) (i.e. \( c_{KK} \) is large enough) to ensure that that both \( \frac{\partial F_3'}{\partial K} < 0 \) and \( 0 < \frac{\partial t}{\partial K}_{F_4} < \frac{\partial t}{\partial K}_{F_3'} \). If so, at any point on the \((t, K)\) plane such that both \( F_4 \) and \( F_3' \) are satisfied, the former crosses the latter
always from above. Thus, there is only a single couple \( \bar{t}, \bar{K} \).

Having proved uniqueness of \( \bar{t} = t_C + t_R \) and \( \bar{K} \), we need to prove uniqueness of \( t_C \) and \( t_R \). There could, a priori, be an infinite number of couples \( t_C, \bar{t}_R \) whose sum results in \( \bar{t} \). If \( t_R^+(\pi_C, s; \hat{r}_H) = 0 \) as a corner solution, then \( \bar{t}_R = t_R = 0 \) and \( \bar{t} = \bar{t}_C \). Uniqueness of \( t_C \) and \( t_R \) follows. Assume instead that \( t_R^+(\pi_C, s; \hat{r}_H) \) is an interior maximizer of \( V^H(\pi; r) \) at \( \bar{\pi} \). This vector satisfies conditions labeled \( F_1 \) and \( F_2 \) above. \( F_1 \) implicitly defines \( t_C \) as a continuously differentiable function of \( t_R \) (as well as other policy parameters here treated as given). Similarly, \( F_2 \) implicitly defines \( t_R \) as a continuously differentiable function of \( t_C \). By the Implicit Function Theorem, we have

\[
\frac{\partial t_R}{\partial t_{C F1}} = -\frac{\frac{\partial F_1}{\partial t_C}}{\frac{\partial F_1}{\partial t_R}} = -\frac{-d_q(\hat{t}_C) \left(1 + T_Q \frac{dQ}{dp}\right) - q(\hat{r}_C)T_Q \frac{d^2Q}{dp^2} \frac{\lambda}{\lambda} + t_R \frac{d^2Q}{dp^2}}{-d_q(\hat{r}_C) \left(1 + T_Q \frac{dQ}{dp}\right) - q(\hat{r}_C)T_Q \frac{d^2Q}{dp^2} \frac{\lambda}{\lambda} + t_R \frac{d^2Q}{dp^2} + \frac{dQ}{dp}}
\]

\[
\frac{\partial t_R}{\partial t_{C F2}} = -\frac{\frac{\partial F_2}{\partial t_C}}{\frac{\partial F_2}{\partial t_R}} = -\frac{-d_q(\hat{r}_H) \left(1 + T_Q \frac{dQ}{dp}\right) - q(\hat{r}_H)T_Q \frac{d^2Q}{dp^2} \frac{\lambda}{\lambda} + t_R \frac{d^2Q}{dp^2} + \frac{dQ}{dp}}{-d_q(\hat{r}_H) \left(1 + T_Q \frac{dQ}{dp}\right) - q(\hat{r}_H)T_Q \frac{d^2Q}{dp^2} \frac{\lambda}{\lambda} + t_R \frac{d^2Q}{dp^2} + \frac{dQ}{dp}}
\]

We have shown in Technical Appendix A that \( \frac{\partial F_1}{\partial t_C} = \frac{\partial^2 V_C}{\partial t_C^2} < 0 \) and \( \frac{\partial F_1}{\partial t_R} = \frac{\partial^2 V_C}{\partial t_C \partial t_R} < 0 \), when evaluated in the neighborhood of \( \bar{\pi} \). Note that \( \frac{\partial F_1}{\partial t_C} > \frac{\partial F_1}{\partial t_R} \), since \( \frac{2 dQ}{dp} < (1 + \frac{1}{\lambda}) \frac{dQ}{dp} \). Let us now look at \( \frac{\partial t_R}{\partial t_{C F2}} \). We prove in Technical Appendix C that both \( \frac{\partial F_2}{\partial t_C} = \frac{\partial^2 V_R}{\partial t_C \partial t_R} < 0 \) and \( \frac{\partial F_2}{\partial t_R} = \frac{\partial^2 V_R}{\partial t_R^2} < 0 \), when evaluated in the neighborhood of \( \bar{\pi} \). Moreover, given that \( 2 \frac{dQ}{dp} < \frac{dQ}{dp} \), we have \( \frac{\partial F_2}{\partial t_C} < \frac{\partial F_2}{\partial t_R} \). Thus, \( F_1 \) and \( F_2 \) define, in a neighborhood of \( \bar{\pi} \), \( t_R \) as a strictly decreasing function of \( t_C \). Now, \( \bar{\pi} \) is necessarily such that both these functions cross on the \((t_R, t_C)\) plane. Since we have (at couples satisfying both \( F_1 \) and \( F_2 \)) that \( \frac{\partial t_R}{\partial t_{C F1}} < \frac{\partial t_R}{\partial t_{C F2}} < 0 \), it has to be the case that the first function crosses the second only from above. Since both are continuous, the crossing is unique. Therefore, \( t_C \) and \( t_R \) have to be unique.

**Proof of uniqueness of** \( \bar{\pi} \)  We have proven in Lemma 2 that \( t_R^+(\pi_C; s, \hat{r}_C) = 0 \) at any \( \pi^E \), including \( \bar{\pi} \). So \( \bar{\pi} \) is such that \( t_R = 0 \), and, therefore, the couple \((\bar{t}_C, \bar{K})\) coincides with \( \pi_C(0; s) \).
This vector is unique, as proven in Lemma 1.

**Characterization of the bounds for** \( t_C^E, t_R^E \) **and** \( K^E \) Recall from Lemma 1 that \(-1 < \frac{\partial t_C}{\partial t_R} < 0\). Suppose that there existed a \( \pi^E \) such that \( t_R^E > \bar{t}_R \) and, consequently, \( t_C^E < t_C \). Then \( t_R^E > t_R^{H^*}(\pi_C^E; s, \hat{r}_H) \). However, \( t_R^E > t_R^{H^*}(\pi_C^E; s, \hat{r}_H) \) is not possible since, as proven in Lemma 2, \( \pi^E \) must be such that \( t_R^{H^*}(\pi_C^E; s, \hat{r}_H) \geq t_R^E \). Similarly, we can prove that an equilibrium where \( t_R^E < \bar{t}_R = 0 \) and \( t_C^E > \bar{t}_C \) is not possible. Finally, since \(-1 < \frac{\partial t_C}{\partial t_R} < 0\), the bounds for \( t_C^E + t_R^E \) must be given by \( \bar{t}_C \leq t_C^E + t_R^E \leq t_C + \bar{t}_R \).

**Technical appendix C**

We have

\[
\frac{\partial^2 V_H (\pi; \hat{r}_H)}{\partial t_R \partial t_C} = -\frac{dq(\hat{r}_H)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_H) T_Q \frac{d^2 Q}{dp^2} + t_R \frac{d^2 Q}{dp^2} + \frac{dQ}{dp}
\]

using the first order condition \( \frac{\partial V_H (\pi; \hat{r}_H)}{\partial \hat{r}_H} = 0 \) (holding by assumption, since we are focusing on interior solutions), we can write it as

\[
\frac{\partial^2 V_H (\pi; \hat{r}_H)}{\partial t_R \partial t_C} = -\frac{dq(\hat{r}_H)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) + \frac{dQ}{dp} + \left( q(\hat{r}_H) - Q \right) \frac{d^2 Q}{dp^2}
\]

Using \( \frac{dQ}{dp} \), \( \frac{dq(r)}{dp} \) and \( \frac{d^2 Q}{dp^2} \), as in Technical Appendix A, similar rearrangements yield

\[
\frac{\partial^2 V_H (\pi; \hat{r}_H)}{\partial t_R \partial t_C} < 0 \iff \frac{T}{p + T - A + s} > -\frac{1}{4} \left( \frac{\hat{r}_H}{2\bar{r}} - 1 \right)
\]

which is always verified since \( \hat{r}_H \in [0, 2\bar{r}] \).

**Technical appendix D**

We intend to prove that condition
\[
F_4 : -q(\hat{r}) \cdot \left(1 + T_Q \frac{dQ}{dp}\right) + (t + \alpha s) \frac{dQ}{dp} + (2 - \lambda)Q = 0
\]

where \( \hat{r} = \hat{r}_C \lambda + (1 - \lambda) \hat{r}_H \) is such that, in the neighborhood of \( \bar{\pi} \), its derivative \( \frac{dF_4}{dt} \) is negative. This derivative is

\[
-\frac{dq(\hat{r})}{dp} \cdot \left(1 + T_Q \frac{dQ}{dp}\right) + (3 - \lambda) \frac{dQ}{dp} + (t + \alpha s - q(\hat{r})T_Q) \frac{d^2Q}{dp^2}
\]

using \( F_4 \), it can be written as

\[
-\frac{dq(\hat{r})}{dp} \cdot \left(1 + T_Q \frac{dQ}{dp}\right) + (3 - \lambda) \frac{dQ}{dp} + \left(\frac{q(\hat{r}) - (2 - \lambda) Q}{dQ/dp}\right) \frac{d^2Q}{dp^2}
\]

now using \( \frac{dQ}{dp}, \frac{dq(\hat{r})}{dp} \) and \( \frac{d^2Q}{dp^2} \), as in Technical Appendix A, similar rearrangements allow us to write that

\[
\frac{\partial F_4}{\partial t} < 0 \iff \frac{T}{p + T - A + s} > -\hat{r} \cdot \frac{3}{4} \left(\frac{2 - \lambda}{3 - \lambda} - \frac{2}{3}\right)
\]

which is always verified, since the right hand side is negative.

**Proof of Proposition 1**

Consider condition (5). When \( s = 0 \) and setting \( t_R = 0 \), this expression is the same as \( \frac{\partial W(\pi)}{\partial t} \) if and only if \( \hat{r}_C = \frac{\bar{r}}{\lambda} \) and \( K = K^{FB} \). It is only in that case that \( t_C^E = T_Q^{FB}Q^E \) (recall that \( T_Q = \gamma + K \) so \( T_Q^{FB} = \gamma + K^{FB} \)). Since \( t_C - T_Q^{FB}Q \) is strictly increasing in \( t_C \), then it is only if \( \hat{r}_C = \frac{\bar{r}}{\lambda} \) and \( K^C = K^{FB} \) that \( t_C^E = t^{FB} \). However, \( K^E \) satisfies \( \lambda Y - Q - T_{QK}Q^2 - c_K = 0 \) (with \( \lambda < 1 \)). Therefore, even if \( t_C^E = t^{FB} \) we would have \( K^E < K^{FB} \). As a consequence, when \( \hat{r}_C = \frac{\bar{r}}{\lambda} \), we have \( t_C^E < t^{FB} \) and \( K^E < K^{FB} \). Therefore, since (by Lemma 1) \( t_C^E \) and \( K^E \) are decreasing in \( \hat{r}_C \), there must exist \( r^a < \frac{\bar{r}}{\lambda} \) such that \( K^E = K^{FB} \) if \( \hat{r}_C = r^a \). The rest of the claim follows from the comparative statics provided in Lemma 1.
Proof of Proposition 2

To begin, let us focus on $\bar{\pi}$. Consider, first, the case in which $\bar{\pi}$ is such that $\bar{t}_C$ and $\bar{t}_R$ are interior maximizers of, respectively, $V_C(\bar{\pi}; r)$ and $V_H(\bar{\pi}; r)$. Conditions described as $F_4$, $F_2$ and $F_3'$ in the proof of Lemma 3 must thus hold at $\bar{\pi}$. Importantly, $F_4$ contains only terms that are function of $\bar{t}$, not of its components $\bar{t}_C$ and $\bar{t}_R$. We can use the Implicit Function Theorem to obtain that

$$
\frac{\partial \bar{t}}{\partial \bar{r}} = -\frac{\det \begin{bmatrix} \frac{\partial F_4}{\partial \bar{r}} & \frac{\partial F_4}{\partial K} \\ \frac{\partial F_3'}{\partial \bar{r}} & \frac{\partial F_3'}{\partial K} \end{bmatrix}}{\det \begin{bmatrix} \frac{\partial F_4}{\partial \bar{r}} & \frac{\partial F_4}{\partial K} \\ \frac{\partial F_3'}{\partial \bar{r}} & \frac{\partial F_3'}{\partial K} \end{bmatrix}}
$$

$$
\frac{\partial \bar{t}}{\partial \bar{\lambda}} = -\frac{\det \begin{bmatrix} \frac{\partial F_4}{\partial \bar{\lambda}} & \frac{\partial F_4}{\partial K} \\ \frac{\partial F_3'}{\partial \bar{\lambda}} & \frac{\partial F_3'}{\partial K} \end{bmatrix}}{\det \begin{bmatrix} \frac{\partial F_4}{\partial \bar{\lambda}} & \frac{\partial F_4}{\partial K} \\ \frac{\partial F_3'}{\partial \bar{\lambda}} & \frac{\partial F_3'}{\partial K} \end{bmatrix}}
$$

the denominator of all these derivatives is positive, as proven in Lemma 3. The numerator of $\frac{\partial \bar{t}}{\partial \bar{r}}$ is $\frac{\partial F_4}{\partial \bar{r}} \frac{\partial F_3'}{\partial K} > 0$, since $\frac{\partial F_4}{\partial \bar{r}} < 0$ and, with a sufficiently convex $c(K)$, $\frac{\partial F_3'}{\partial K} < 0$. We also have $\frac{\partial F_3'}{\partial \bar{r}} = 0$. As for $\frac{\partial \bar{t}}{\partial \bar{\lambda}}$, the numerator is $-\frac{\partial F_4}{\partial \bar{\lambda}} \frac{\partial F_3'}{\partial K} > 0$ since $\frac{\partial F_4}{\partial \bar{\lambda}} < 0$ and $\frac{\partial F_3'}{\partial K} > 0$. We have therefore proven that $\frac{\partial \bar{t}}{\partial \bar{r}} < 0$ and $\frac{\partial \bar{t}}{\partial \bar{\lambda}} < 0$. One can repeat the reasoning using $\lambda$ as the independent variable, instead of $\bar{t}$, and obtain similar results.

Next, we prove that there exists a unique value $r^+ \equiv \bar{r} \left(1 + \frac{1-\lambda}{1+T_Q \frac{dQ}{dp}}\right)$, with $\pi^*$ as defined in the main text, such that if $\bar{r} = r^+$, then $\bar{\pi}$ is such that $\bar{t} = T_Q^* Q^*$ and $\bar{K} = K^*$. Take condition $F_4'$ and add $Q T_Q \frac{dQ}{dp}$ to both sides. The equality obtained implies (since $q(r) = \frac{r}{(p + T - A + s)^2}$ and $q(\bar{r}) = Q$) that

$$
\frac{2 - \lambda}{(p + T - A + s)^2} + \left(Q - q(\bar{r})\right) T_Q \frac{dQ}{dp} \geq 0 \Leftrightarrow (t - T_Q Q) \frac{dQ}{dp} \leq 0 \Leftrightarrow t - T_Q Q \geq 0
$$

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We evaluate these expressions at $\bar{\pi}$. Now fix $\bar{K}$ and note that $h(\bar{t}) = \bar{t} - \bar{T}_Q \bar{Q}(\bar{\pi})$ (where $T_Q = \gamma + K$) is a strictly increasing function of $\bar{t}$. Then $h(\bar{t}) = 0$ if and only if $\bar{t} = \bar{T}_Q \bar{Q}(\bar{\pi})$.

Therefore, we must have have

$$\bar{t} = \bar{T}_Q \bar{Q}(\bar{\pi}) \Leftrightarrow \frac{(2 - \lambda)\bar{r} - \hat{\bar{r}}}{(p + T - A + s)^2} + (Q - q(\hat{\bar{r}})) \bar{T}_Q \frac{dQ}{dp} = 0$$

One then needs to rearrange the second equality, using $q(r) = \frac{r}{(p+T-A+s)^2}$ and $q(\hat{\bar{r}}) = Q$, to see that it is verified if and only if $\hat{\bar{r}} = \bar{r} \left(1 + \frac{1-\lambda}{1+T_Q \frac{dQ}{dp}(\bar{\pi})}\right)$. Finally, since $\bar{K}$ satisfies $\lambda Y - Q = T_Q \bar{K} Q^2 - c_K$, we have $\bar{t} = t^x = T_Q^x Q^x$ and $\bar{K} = K^x$ if and only if $\hat{\bar{r}} = r^+$, where $r^+ = \bar{r} \left(1 + \frac{1-\lambda}{1+T_Q \frac{dQ}{dp}(\bar{\pi})}\right)$.

Suppose now that $K^x = K^{FB}$. Then, we would have $t^x = t^{FB}$. However, when $t^x = t^{FB}$, it has to be that $K^x < K^{FB}$ since $K^x$ has to satisfy $\lambda Y - Q = T_Q \bar{K} Q^2 - c_K$ and $\lambda < 1$. Therefore, $t^x < t^{FB}$ and $K^x < K^{FB}$. One then needs to use $\frac{dT}{d\pi} < 0$ and $\frac{dK}{d\pi} < 0$, proven above, to see that $\bar{t} < t^{FB}$ and $\bar{K} < K^{FB}$ if $\hat{\bar{r}} \geq r^+$ and that $\bar{t} \geq t^{FB}$ and $\bar{K} \geq K^{FB}$ are possible only if $\hat{\bar{r}} < r^+$.

To conclude the proof, consider the case in which $\bar{\pi}$ is such that $\bar{t}_R = 0$. In such a case, $\pi^E = \bar{\pi} = \bar{\pi}$. $\bar{\pi}$ coincides with $\pi_C (0; s)$, whose comparison to $\pi^{FB}$ was provided in Proposition 1. By Lemma 3, $\bar{t}_C \leq t_C + \bar{t}_R$ and $\bar{K} \leq \bar{K}$. The last point of Proposition 2 follows. Proposition 1 also established that $\bar{t}_C \leq t^{FB}$ and $\bar{K} \leq K^{FB}$ if $\hat{\bar{r}} < r^a < \frac{\bar{r}}{\bar{X}}$. This is why when $\hat{\bar{r}} > r^+$ and $\hat{\bar{r}}_C < r^a < \frac{\bar{r}}{\bar{X}}$, we can be sure that $\pi^E = \bar{\pi} = \bar{\pi}$, so $t^{FB}_E = 0$.

**Proof of Corollary to Proposition 2** Consider the case $\hat{\bar{r}} > r^+$ and $\hat{\bar{r}}_C > \frac{\bar{r}}{\bar{X}}$. Then $\bar{t}_C \leq t^E < t^{FB}$ and $\bar{K} \leq K^E < K^{FB}$. $W(\pi)$ is a concave function of $t$ and $K$, maximized (conditionally on $s = 0$) at $t = t^{FB}$ and $K = K^{FB}$. The claim follows from the last point in Proposition 2.
Proof of Proposition 3

We prove that $\pi_C(0; s)$ is such that $\frac{\partial t}{\partial s} > -1$. The vector must satisfy conditions $F1$ and $F2'$ provided in the proof of Lemma 1, with $t_R = 0$ and denoting now $t_C$ as $t$. The Implicit Function Theorem tells us that, in a neighborhood of $\pi_C(0; s)$

$$\frac{\partial t}{\partial s} = -\frac{\det \left[ \begin{array}{cc} \frac{\partial F_1}{\partial s} & \frac{\partial F_1}{\partial K} \\ \frac{\partial F_2'}{\partial s} & \frac{\partial F_2'}{\partial K} \end{array} \right]}{\det \left[ \begin{array}{cc} \frac{\partial F_1}{\partial t} & \frac{\partial F_1}{\partial K} \\ \frac{\partial F_2'}{\partial t} & \frac{\partial F_2'}{\partial K} \end{array} \right]}$$

$$\frac{\partial K}{\partial s} = -\frac{\det \left[ \begin{array}{cc} \frac{\partial F_1}{\partial s} & \frac{\partial F_1}{\partial K} \\ \frac{\partial F_2'}{\partial s} & \frac{\partial F_2'}{\partial K} \end{array} \right]}{\det \left[ \begin{array}{cc} \frac{\partial F_1}{\partial t} & \frac{\partial F_1}{\partial K} \\ \frac{\partial F_2'}{\partial t} & \frac{\partial F_2'}{\partial K} \end{array} \right]}$$

where

$$\frac{\partial F_1}{\partial s} = -\frac{d\hat{q}(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_C) T_Q \frac{d^2Q}{dp^2} + \frac{dQ}{dp} + \frac{d^2Q}{dp^2} \frac{1}{\lambda}$$

$$\frac{\partial F_1}{\partial t} = -\frac{d\hat{q}(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_C) T_Q \frac{d^2Q}{dp^2} + \frac{dQ}{dp} + \frac{d^2Q}{dp^2} \frac{1}{\lambda}$$

$$\frac{\partial F_1}{\partial K} = \left( -\frac{d\hat{q}(\hat{r}_C)}{dp} \cdot \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_C) T_Q \frac{d^2Q}{dp^2} + \frac{(1+\alpha) d^2Q}{dp^2} + \frac{dQ}{dp} \right) \cdot (Q + 1) - q(\hat{r}_C) T_Q K \frac{dQ}{dp}$$

$$\frac{\partial F_2'}{\partial K} = -\frac{dQ}{dK} - 2Q \frac{dQ}{dK} - c_{KK}$$

$$\frac{\partial F_2'}{\partial t} = -\frac{dQ}{ds} - 2Q \frac{dQ}{ds}$$

Now, $\frac{\partial F_1}{\partial K} = \frac{\partial F_1}{\partial s} \frac{\partial s}{\partial t} < 0$ in the neighborhood of $\pi_C(0; s)$, as proven in Technical Appendix A. The same is true for $\frac{\partial F_2'}{\partial K}$, as long as $c(K)$ is sufficiently convex. We also have $\frac{\partial F_2'}{\partial t} > 0$ while the sign of $\frac{\partial F_1}{\partial K}$ is ambiguous. Anyway, as long as $c(K)$ is sufficiently convex, the determinant at the denominator of both $\frac{\partial t}{\partial s}$ and $\frac{\partial K}{\partial s}$ is positive (see Lemma 1). We have

$$\frac{\partial t}{\partial s} = -\frac{\frac{\partial F_1}{\partial s} \frac{\partial F_2'}{\partial K} - \frac{\partial F_1}{\partial K} \frac{\partial F_2'}{\partial s}}{\frac{\partial F_1}{\partial s} \frac{\partial F_2'}{\partial K} - \frac{\partial F_1}{\partial K} \frac{\partial F_2'}{\partial t}}$$

$$\frac{\partial K}{\partial s} = -\frac{\frac{\partial F_1}{\partial s} \frac{\partial F_2'}{\partial K} - \frac{\partial F_1}{\partial K} \frac{\partial F_2'}{\partial s}}{\frac{\partial F_1}{\partial s} \frac{\partial F_2'}{\partial K} - \frac{\partial F_1}{\partial K} \frac{\partial F_2'}{\partial t}}$$

It is easily seen that $\frac{\partial F_1}{\partial K} \frac{\partial F_2'}{\partial s} = \frac{\partial F_1}{\partial s} \frac{\partial F_2'}{\partial K}$. Therefore, $\frac{\partial t}{\partial s} > -1$ if and only if $\frac{\partial F_1}{\partial s} \frac{\partial F_2'}{\partial K} < \frac{\partial F_1}{\partial K} \frac{\partial F_2'}{\partial s}$.

That is, $\frac{\partial F_1}{\partial s} > \frac{\partial F_1}{\partial t}$. This condition is always verified in a neighborhood of $\pi_C(0; s)$. This is
because \( \frac{\partial F_1}{\partial t_C} = \frac{\partial^2 V_C}{\partial t_C^2} < 0 \). Moreover, as we prove in Technical Appendix E, \( \frac{\partial F_1}{\partial s} = \frac{\partial^2 V_C}{\partial t_C \partial s} < 0 \) if (though not only if) \( \alpha \geq \frac{1}{2} \). If so, then \( \frac{\partial F_1}{\partial s} > \frac{\partial F_1}{\partial t} \), since \( \frac{\partial F_1}{\partial s} = \frac{\partial F_1}{\partial t} \frac{d^2 Q}{dp^2} + \frac{(1 - \alpha) dQ}{dp} \). If instead \( \frac{\partial F_1}{\partial s} > 0 \), then \( \frac{\partial F_1}{\partial s} > \frac{\partial F_1}{\partial t} \) anyway. A similar reasoning can be used to show that \( \frac{\partial K}{\partial s} > 0 \).

**Technical Appendix E**

Consider

\[
\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C \partial s} = -\frac{dQ(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) - q(\hat{r}_C) \left( T_Q \frac{d^2 Q}{dp^2} \right) \frac{(1 + \alpha) \frac{dQ}{dp} + (t_C + \alpha s) \frac{d^2 Q}{dp^2}}{\lambda} + t_R \frac{d^2 Q}{dp^2}
\]

in the neighborhood of interior (local) maximisers of \( V_C(\pi; \hat{r}_C) \), using (5) (equated to zero), we can write it as

\[
\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C \partial t_R} = -\frac{dQ(\hat{r}_C)}{dp} \left( 1 + T_Q \frac{dQ}{dp} \right) + \frac{(1 + \alpha) \frac{dQ}{dp}}{\lambda} + \left( \frac{q(\hat{r}_C) - \frac{Q}{\lambda}}{\frac{dQ}{dp}} \right) \frac{d^2 Q}{dp^2}
\]

using \( \frac{dQ}{dp} \), \( \frac{dQ(r)}{dp} \) and \( \frac{d^2 Q}{dp^2} \), as in Technical Appendix A and following similar steps we obtain

\[
\frac{\partial^2 V_C(\pi; \hat{r}_C)}{\partial t_C \partial s} < 0 \iff \frac{T}{p + T - A + s} > -\frac{1}{4} \frac{\hat{r}_C}{r} \left( \frac{\lambda}{1 + \alpha} \right) + \left( \frac{3}{4 (1 + \alpha)} - \frac{1}{2} \right)
\]

which is always verified as long as (but not only if) \( \alpha \geq \frac{1}{2} \).

**Proof of Proposition 4**

Our strategy to identify \( \pi^E_e \) is the following. Suppose \( s \) could be set independently of \( t \), as in the \( \rho = l \) regime. Then we would be back to the setup of Lemma 1. We have proved there that, for each value of \( s \), there exists an equilibrium vector \( \pi^E = (t^E, K^E; s) \) (where \( t = t_C \) as \( t_R = 0 \)). Thus, by varying \( s \), we can describe a set of vectors. Let this set be denoted by \( \Sigma \). Among the vectors in \( \Sigma \), \( \pi^E_e = (t^E_e, K^E_e; s^E_e) \) is the unique one satisfying the budgetary rule \( BB : tQ - \alpha s (Y - Q) = 0 \) imposed by earmarking. We proceed assuming that vectors
\( \pi^{FB} \) and \( \pi^{E}_e \) are such that \( BB \) describes \( s \) as an increasing function of \( t \), so \( \frac{\partial s}{\partial t_{BB}} > 0. \)

For a given value of \( s \), \( \pi^{E} = \pi_C(0; s) \) is described by conditions named \( F1 \) and \( F2 \) in the proof of Lemma 1 (with \( t^E_R = 0 \), so \( t = t_C \)). In the proof of Proposition 3, we have argued that \( \pi^{E} = \pi_C(0; s) \) is such that \( \frac{\partial t}{\partial s} > -1 \) and \( \frac{\partial K}{\partial s} > 0 \). When adopting the \( BB \) rebate rule, there are two possibilities: if for \( s = 0 \), \( \pi^{E} \) is such that \( t^E_0 = 0 \), then imposing \( BB \) will leave the equilibrium unchanged. If, instead, when \( s = 0 \), \( \pi^{E} \) is such that \( t^E_0 > 0 \), \( \pi^{E}_e \) must be such that both \( t^E_e \) and \( s^E_e \) are strictly positive. However, any vector in \( \Sigma \) is such that \( \frac{\partial t}{\partial s} > -1 \) and \( \frac{\partial K}{\partial s} > 0 \). Therefore, \( \pi^{E}_e \) must be such that \( t^E_e + s^E_e > t^E_0 \) and \( K^E_e > K^E_0 \). Thus, the gap with the first best vector is reduced. In fact, a priori, we cannot rule out the possibility that \( t^E_e + s^E_e > t^{FB}_e + s^{FB}_e \) and \( K^E_e > K^{FB} \). This will happen if \( \alpha \) is small enough.

However, \( G_W \) can always set \( \alpha \) “large enough” (i.e. close enough to one) to make sure that \( t^{FB}_e + s^{FB}_e > t^E_e + s^E_e > t^E_0 \) and \( K^{FB} > K^E_e > K^E_0 \).

References


\(^{24}\)Using the Implicit Function Theorem, one can show that this is true as long as \( Q + t \frac{dQ}{d\alpha} + \alpha s \frac{dQ}{d\alpha} > 0 \). That is, a marginal increase in the car tax produces an increase in the revenues generated by the tax itself (net of expenditures for \( s \)). Note that, in our setup, the highest possible value of \( t \), in equilibrium, is the most-preferred by the individual for which \( r = 0 \). Since he never drives, this individual will simply choose \( t \) so as to maximise total (net) revenues \( tQ - \alpha s (Y - Q) \). There is no reason for her to pick a tax such that marginal revenues are negative.
November.


