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A communication game on electoral platforms

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Abstract

This paper proposes a game to study strategic communication on platforms by parties. Parties' platforms have been chosen in a multidimensional policy space, but are imperfectly known by voters. Parties strategically decide the emphasis they put on the various issues, and thus the precision of the information they convey to voters on their position on each issue. The questions we address are the following: what are the equilibria of this communication game? How many issues will they address? Will parties talk about the same issues or not? Will they talk on issues that they "own" or not?

JEL: C70, D72

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1 Introduction

There is a long tradition in political science on ignorant ill-informed voters (e.g. Campbell *et al.* 1960), emphasizing that voters have little incentive to invest by themselves time and effort to gather all the relevant information about the stakes of elections. In this perspective, electoral campaigns are perceived as important, since they may provide voters with the opportunity to learn at low cost about the candidates' personal characteristics, the parties' platforms, and the stakes of the election. But since campaigns are orchestrated by parties, it suggests that how much voters learn about parties platforms is partly determined by parties themselves. Even if the candidates do not 'lie', they may have incentives to make some of this information hard to obtain for voters, by making extremely vague and ambiguous statements for instance (Page 1978) or by avoiding to address some issues.

So far, the key feature of the electoral campaign that has been the most studied in formal models is the 'where to stand' question, as in the standard spatial model of Downs (1957) or Hotelling (1929) (see however the literature referred to below). But if parties strategically decide the emphasis they put on the various issues and the precision of the information they convey

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on each issue, another key question is: 'what will they talk about?' And this may prove to be extremely important when voters mainly learn about the platforms through the campaign. By deciding which issues they want to emphasize, parties will determine the quality of voters' information, the dividing lines in the electorate and the issues on which the election will eventually depend upon. The objective of this paper is precisely to analyze these decisions, in particular to put forward some trade-offs faced by the parties when they convey information on their platforms.

We develop a game that parties - or candidates - may play once platforms have been chosen. Since platforms have already been chosen, parties only care about their probability of winning the election (or alternatively about the vote share they get) and choose their strategies accordingly. These strategies are how much time candidates will spend during the campaign explaining their position on each issue, possibly subject to some time/resource constraints. How these explanations modify voters' information is described below. At the end of the electoral campaign, voters vote for the party they prefer.

The idea we want to capture here is that in the couple of weeks before an election, it may be impossible for a candidate to adjust his platform the way he wished he could. For instance, this platform may have been decided by the party and officially written in a manifesto. Due to poorly informed voters, even though platforms are chosen, they is still a lot of room for the candidates to be strategic, regarding the features of their platform they want to put special emphasis on. When invited on a TV show, a candidate may want to speak mainly about law and order issues, or mainly about economic issues, or on the contrary, avoid as much as possible such issues. We assume that voters have *a priori* beliefs regarding where parties stand on the various issues. They are ready to update these beliefs when they get new information from the campaign. The more a candidate talks about an issue, the better-informed voters will be regarding his position on this issue.

Our model captures what we believe are the three most important effects of speeches. Two bear on voters. The third one bears on parties. First, a speech conveys information on where a candidate truly stands, which may or may not be beneficial to a party Indeed, it it depends on where the party stands relative to the average voter in the electorate. Second, a speech reduces voters' uncertainty about the party's platform, which is unambiguously favorable to a party, as soon as voters are risk-averse. This effect may explain why parties may both want to address a same issue. The third effect is some 'strategic risk' that a candidate undertakes when he tries to explain his position. Indeed, speeches may however be understood differently by voters, which introduces a source of uncertainty for parties explaining their positions. This strategic risk is maximal when a party speaks on an issue but too little an amount of time to make clear enough where it stands. Due to this effect, a party may refrain from speaking or, at the opposite, increase his speech.

The questions we want to answer are the following: what are the equilibria of this communication game? How much information is transmitted to voters through the campaign? Will parties talk about the same issues or not? Will they talk about issues on which they are close to the average voter or not? Because of the interactions of the three effects of a speech described above, various types of equilibrium strategies can be obtained. We think that this new model shed some light on previous puzzles. One such puzzle concerns "issue convergence". Petrocik (1996) argues that each candidate enjoys an *a priori* advantage ('ownership') on some issues. Viewing the election as a two person zero sum game, no issue can work to the advantage of both candidates, and opponents should address different 'orthogonal' issues (Austen-Smith 1993, Simon 2002). And thus one should observe no issue convergence (defined by the fact that both parties address the same issues). This prediction is at odd with some empirical evidence (Sigelman and Buell 2004). We propose a modified version on the this ownership theory that is consistent with those empirical findings.

Related literature. The literature, both theoretical and empirical, on issue convergence will be surveyed and discussed in section 6.4 in the light of our results. Besides, our paper is related to several strands of literature.

Other models have studied the allocation of campaign resources in the presence of uninformed voters. But those models make very different assumptions on the way campaign spending influence voters. In Brams and Morton (1974) or Baron (1994) for example, the probability that an uninformed voter votes for the democratic party is the ratio between the amount of spending by the democratic party over the total amount of spending by both parties. In Snyder (1989), some asymmetry of the efficiency of spending across parties (and across districts) is introduced, but in an *ad hoc* form that remains silent on the mechanisms which may generate this asymmetry. In this paper on the contrary, we model a channel through which campaign spending affect uninformed voters' decisions, that is, through the information that is conveyed to voters about the parties' platforms.

Our paper also relates to the literature that models an electoral campaign as a manipulation game. As here parties have 'true' platforms, which are imperfectly known by voters. A party's platform may be interpreted as its preferred policy, the one it will implement once in office. Announcements serve to 'manipulate' voters' beliefs, and may be more or less effective in transmitting information depending on voters' reactions. No information is transmitted if the game is pure cheap talk zero-sum game with strategic voters. Introducing some cost born by the winning candidate not only makes communication possible but also induces a multiplicity of equilibria (Banks 1990). Our game is not a cheap talk game since speaking always conveys information. With respect to that literature, some distinctive modeling assumptions should be made precise. The first feature concerns parties' sincerity. During the campaign, candidates are assumed to be truthful although voters may wrongly interpret their speeches. Specifically, parties' speeches are interpreted as noisy but unbiased signals about the parties' true positions. A second important feature, related to the previous one, is about commitment. Voters vote according to their assessment about parties' platforms. Hence, as in the standard spatial electoral competition game (Downs 1957, Hotelling 1929), it is implicit that platforms will be implemented (or that deviations are due to unforeseen circumstances). Third, voters, although Bayesian, are not perfectly rational. The sensitivity of our results to those modelling assumptions will be discussed in subsection 6.2.

Section 2 introduces the model, Section 3 carefully analyzes the impact of the electoral campaign on voters' beliefs. Sections 4 and 5 are devoted to the analysis of equilibria, first in the case where candidates face a time constraint per issue (section 4), and then when then face a global time constraint (section 5). Section 6 provides a number of discussions about the model. First, a counter-intuitive example shows that although only unbiased information is transmitted to voters in the course of the campaign, campaign might be welfare reducing for voters.(subsection 6.1). The assumptions of naïve voters and faithful parties are discussed (subsection 6.2). We briefly introduce and discuss an alternative model of information transmission, where when addressing an issue, a candidate not only conveys information to voters about where he stands on the issue, but also information about where his opponent stands on the same issue (subsection 6.3).The link with the theoretical and empirical literature on issue convergence is made in more detail (subsection 6.4). A last section concludes.

2 The electoral campaign model

We consider a multidimensional policy space, $X = R^K$, where K is the number of *a priori* relevant issues.

Voters and parties. Each voter is characterized by a bliss point $x = (x^1, x^2, ..., x^K)$ in X, and by a vector of weights $\alpha = (\alpha^1, \alpha^2, ..., \alpha^K) \in R_+^K$ describing how she weights the various issues at stake. Her utility is represented by (the opposite of) a weighted distance to her bliss point: if policy $z = (z^1, z^2, ..., z^K) \in X$ is implemented, a voter with a weight vector α and bliss point $x \in X$ gets the utility:

$$u(z; \alpha, x) = -\sum_k \alpha^k \left(z^k - x^k\right)^2.$$

The parameters (α, x) are distributed with density $g(\alpha, x)$.

There are two parties, party A and party B. Parties have fixed platforms in the policy space X. Denote by $x_A = (x_A^1, x_A^2, ..., x_A^K)$ party A's platform and similarly for B. Those are the platforms that parties will implement if elected. We take those platforms as fixed; for example, they are contained in a written manifesto on which members of the party have reached a consensus. At this electoral campaign stage, candidates are trying to get as many votes as possible, even though they may not be purely office-motivated. Each party knows its platform, as well as that of its opponent. But voters do not know any of those platforms with certainty. Before the electoral campaign starts, voters share the same *a priori* beliefs on these platforms. Voters' *a priori* on parties' position on the various issues are independent across parties and across issues. Voter's *a priori* on x_J^k follows a normal distribution $\mathcal{N}\left(m_J^k, \left(s_J^k\right)^2\right)$. Those beliefs may come from past campaigns or from observing the policies chosen by the party in charge during the previous legislatures, or from what they have heard during the party congress. Acquiring a perfect information about parties' platforms would be prohibitively costly. Yet, voters have some opportunity to learn about these platforms during the electoral campaign, and to update their beliefs as to where parties stand.

The campaign will be analyzed according to the following timing (see Figure 1).

(1) Candidates choose their emphasis strategy (or communication strategy).

(2) Voters are receptive to the campaign. There may however be some variation in the speeches they listen to, the meetings they attend to, and how they interpret them. This may result in their receiving different "signals" on parties' platforms (y_A, y_B). Furthermore, voters may be affected by idiosyncratic bias σ as defined below.

(3) Voters vote and the party getting the highest number of votes is elected. The winning party implements its platform.

We now describe in more details each of these stages.

***** INSERT FIGURE1 ABOUT HERE *****

The electoral campaign stage *Strategy sets.* Each party decides how much time it wants to devote to each issue. It is represented by a non negative vector $t = (t^1, t^2, ..., t^K)$. We consider two types of time constraint that will be interpreted in due course.

In Section 4, up to a normalization, the maximal amount of time for each issue is one unit (this normalization can be made issue by issue and for each party, as can be checked later on), and the strategy space for a party writes as $T = [0, 1]^K$.

In Section 5, parties are limited by a global time constraint. Up to a normalization the strategy space for a party writes as: $T = \left\{ t \in [0, 1]^K \text{ s.t. } \sum_k t^k \leq 1 \right\}$.

Signals. Signals on a party's position on a particular issue are noisy, unbiased, with a precision that depends on the time spent by the party on that issue. Signals are normally distributed, independent across parties and across issues. Specifically, when party J spends time t_J^k speaking on issue k, each voter receives an imperfect signal on party J's true position on this issue, y_J^k with distribution $\mathcal{N}\left(x_J^k, \left(\sigma_J^k\right)^2 \left(t_J^k\right)\right)$ where function σ_J^k is defined over $[0, 1]$, and satisfies : $\sigma_J^k(0) = +\infty$ (pure noise if there is no speech) and $\left(\sigma_J^k\right)'(t) < 0$ for all $t \in [0, 1[$ (additional speech always makes signals more precise). We make no specific assumption regarding the correlations of signals across voters. They can be independently distributed (conditional on x_A, x_B) or correlated.

Remark. One may consider that x_J is a consensus reached within party J. Signals are noisy because they are conveyed by different party's members. Depending on the electoral system (and the union within the party), the noise in the signals will be more or less important. An alternative interpretation is that x_J is the candidate's platform instead of the party's platform. This may be a more sensible interpretation in some elections where the candidate is quite independent from the party, such as the US presidential election.[1]

[1] Having in mind those two interpretations, we indifferently use the term "party" or "candidate" to refer to this player.

Voters' treatment of information. What are the voter's posterior beliefs regarding the parties' platforms after the reception of these signals? Using the signals received during the campaign, voters update their beliefs regarding the parties' platforms. Each voter receives signals from the candidates, $y_A = (y_A^1, y_A^2, ..., y_A^K)$ from candidate A and $y_B = (y_B^1, y_B^2, ..., y_B^K)$ from candidate B, and she also perceives the time spent by both candidates $(t_A, t_B) \in T \times T$ on the various issues.

Consider a party, say A. Consider a voter who perceived a vector of signals y_A on party A's platform, with an emphasis vector $t_A \in T$. The conditional distribution on party A's position on issue k follows $\mathcal{N}\left(\widehat{x_A^k}\left(y_A^k, t_A^k\right)\left(\widehat{s_A^k}\right)^2\left(t_A^k\right)\right)$, where:

$$\frac{1}{\left(\widehat{s_A^k}\right)^2\left(t_A^k\right)} = \frac{1}{\left(s_A^k\right)^2} + \frac{1}{\left(\sigma_A^k\right)^2\left(t_A^k\right)}, \tag{1}$$

$$\widehat{x_A^k}\left(y_A^k, t_A^k\right) = \left[\frac{\widehat{s_A^k(t_A^k)}}{s_A^k}\right]^2 m_A^k + \left[\frac{\widehat{s_A^k}(t_A^k)}{\sigma_A^k\left(t_A^k\right)}\right]^2 y_A^k. \tag{2}$$

Such a voter with bliss point x and weights α gets the expected utility if A is elected:

$$\widehat{u_A}(y_A, t_A; \alpha, x) = -\sum_k \alpha^k \left[\left(\widehat{x_A^k}\left(y_A^k, t_A^k\right) - x\right)^2 + \left(\widehat{s_A^k}\right)^2\left(t_A^k\right)\right]. \tag{3}$$

The level $\widehat{u_B}(y_B, t_B; \alpha, x)$ is similarly defined for party B.

Note that voters are assumed to be naive (although they are Bayesian) in the sense that they take at face values the messages sent by parties. They do not interpret the messages as stemming from parties' strategies. For example, when party A does not speak on issue k $\left(t_A^k = 0\right)$, the voter's *a posteriori* beliefs regarding party A's position on this issue coincide with her *a priori* beliefs. She does not interpret the fact that if a candidate does not talk about an issue, it might be because he has no incentive to do so. Subsection 6.2 considers the case of more sophisticated voters.

The vote stage. We model voters' behavior using a "probabilistic voting" model. Candidates do not only differ with respect to the policy platforms they put forward, they also differ in some other dimension, unrelated to the policy issues at stake, which parties do not influence through the campaign stage. It may involve some other attributes of the candidates, such as personal characteristics (gender, race, age, ...), on which voters also have preferences. Assume that a voter with parameters (α, x) votes for party A upon receiving signals y_A, y_B and given parties' emphasis t_A, t_B iff $\widehat{u_A}(y_A, t_A; \alpha, x) - \widehat{u_B}(y_B, t_B; \alpha, x) > \sigma$, where σ is an indiviual-specific bias in favor of candidate B. Individual biases are taken to be i.i.d, with a uniform distribution on $\left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right]$. Parties know the distribution of these biases but they do not know their realized values for each individual at the time they have to choose their emphasis strategies.[2]

[2] This "probabilistic voting" model, considering individuals' shocks on preferences which are independent of preferences on platforms, has been introduced and first used by Coughlin (1983) and Lindbeck and Weibull (1993) *inter alia*. The noise ensures the existence of an equilibrium in the standard model where purely office motivated

The probability that such a voter votes for A is:

$$\frac{1}{2} + \phi \left[\widehat{u_A}(y_A, t_A; \alpha, x) - \widehat{u_B}(y_B, t_B; \alpha, x) \right].^{3} \tag{4}$$

Now, the expected vote share for a party only depends on the expectation of the probability of votes over the electorate, given the time spent t_A and t_B. This is computed in two steps. First by taking the expectation of the probability (4) that a given voter votes for A conditional on t_A and t_B before the reception of the signals, and second by taking the average over the electorate.

Let $E[\widehat{u_J}(\tilde{y}_J, t_J; \alpha, x)|t_J, x_J]$ denote the expected value for a voter with characteristics α and x that J is elected conditional on t_J before the reception of the signals (the ˜ over a variable, here y_J, denotes that the value is random). This yields the expected vote share for party A as:

$$\pi_A(t_A, t_B; x_A, x_B) = \frac{1}{2} + \phi \left[\overline{U_A}(t_A; x_A) - \overline{U_B}(t_B; x_B) \right], \tag{5}$$

where

$$\overline{U_J}(t_J; x_J) = \int_\alpha \int_x E[\widehat{u_J}(\tilde{y}_J, t_J; \alpha, x)|t_J, x_J] g(\alpha, x) d\alpha dx \tag{6}$$

is the average expected utility for party J in the electorate when party J chooses emphasis strategy t_J, given its true position x_J.[4] The expected vote share for party B is the complement to 1:

$$\pi_B(t_A, t_B; x_A, x_B) = 1 - \pi_A(t_A, t_B; x_A, x_B). \tag{7}$$

An alternative interpretation of the model. There is one single issue, and each k represents a distinct electorate body -the electorate in a geographical area or an ethnic or social group for instance. Under this interpretation, t^k represents the time spent speaking to group k, through the local media or the 'ethnic' TV. A discussion of the results under this interpretation is provided in subsection 5.4.

3 Impact of the campaign on votes

Before going to the equilibrium analysis, we first examine in some detail how the uncertainty in the emphasis strategy impacts the party's expected shares. In particular, we isolate the three

parties choose their platforms. Here, choosing some specific assumptions about the noise (additive and uniformly distributed) allows for a simple analysis. Indeed, it yields a very simple form for the parties' objectives (see Persson and Tabellini (2000) who popularized these assumptions).

[3] More precisely, the expression holds true when $\widehat{u_A}(y_A, t_A; \alpha, x) - \widehat{u_B}(y_B, t_B; \alpha, x) \in \left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right]$. We take ϕ to be large enough, so that we can neglect the cases where it does not hold.

[4] In our probabilistic model, what matters for a candidate is the estimation of the number of votes. Hence expression (5) is identical whether signals are identical or conditionally independent across voters (or more generally correlated). This is not true in general, as can be seen in a deterministic model. Without individual bias. an individual with characteristics (α, x) votes for A upon receiving signals y_A, y_B if $\widehat{u_A}(y_A, t_A; \alpha, x) - \widehat{u_B}(y_B, t_B; \alpha, x) > 0$. If signals are independent (and independent of characteristics) the number of votes is independent of the sample of the signals received by voters assuming a law of large numbers. Hence the impact of the speech is deterministic. If instead signals are identical, the number of votes depends on the common signal received hence the impact of a speech is random.

channels we have hinted to in the introduction, and we derive an explicit form for the parties vote shares.

Note first that given the expected vote share for party A (in (5)) and for party B (in (7)), each party controls the utility expected by the electorate if it becomes elected. That is, J controls $\overline{U_J}$, and the game is degenerate, in the sense that party J's best response does not depend upon its opponent's choice.

Consider A for example. Given its position x_A, it solves the program:

$$Max_{t_A \in T}\overline{U_A}(t_A; x_A).$$

We analyze how the campaign conducted by party A affects the average expected utility in the electorate if A is elected $\overline{U_A}$. Observe that $\overline{U_A}$ is separable across issues. To save on notation, we first present the analysis in the single-issue case ($K = 1$), before proceeding to the multi-dimensional case.

The single issue case. Our objective here is to analyze the various effects of strategy t_A. For that purpose, we derive some explicit formulation for $\overline{U_A}$ as given by (6). We proceed in three steps, first by computing the conditional utility $\widehat{u_A}$ for a voter given the signals received (hence when she votes), second by taking the point of view of party A which does not know the received signals, and third by taking the average over the population

Impact of the campaign at the voter's level. Consider first expression (3), which gives the expected utility of a voter with characteristics (α, x) if candidate A is elected, upon the receipt on signals y_A :

$$\widehat{u_A}(y_A, t_A; \alpha, x) = -\alpha\left[(\widehat{x_A}\,(y_A, t_A) - x)^2 + \widehat{s_A}^2(t_A)\right]. \tag{8}$$

Note that without campaign, this voter achieves the expected utility $-\alpha[(m_A - x)^2 + s_A^2]$ if A is elected. The precision of information on A's position has two effects on the expected utility of A being elected. A first effect is a change in the perception on A's position from the prior m_A to the posterior[5] $\widehat{x_A}(y_A, t_A)$, which is a combination of the signal y_A and the prior m_A, as can be seen from (2). A second effect is to reduce the voter's uncertainty on the party's platform, from ${s_A}^2$ to $\widehat{s_A}^2(t_A)$, which is unambiguously favorable to A.

Impact on a voter conditional on t_A. Since candidate A does not know the signal that will be received by the voter, it views the impact of speech on the voter's posterior belief, $\widehat{x_A}(y_A, t_A)$, as random. The expectation of this posterior belief is $E(\widehat{x_A}|t_A, x_A) = E(\widehat{x_A}(\tilde{y}_A, t_A)|t_A, x_A)$. (Since signals are unbiased this expectation is the posterior obtained for a signal equal to the true position, $\widehat{x_A}\,(x_A, t_A)$, by linearity of the posterior with respect to the signal.) By contrast, the variance of the posterior $\widehat{x_A}(\tilde{y}_A, t_A)$ given t_A is independent of the position x_A. We denote it by $var(\widehat{x_A}|t_A)$ and call it the *strategic risk.* This risk is null when no information is conveyed ($t_A = 0$), in which

[5] In the sequel, we shall call m_J the prior belief on J's position, or simply prior, and $\widehat{x_J}$ the posterior belief or simply posterior (although beliefs commonly design the entire distribution, we do not think it can create any confusion).

case the party knows that the best guess of the voter on the party's position is m_A (and), or when full information is conveyed, in which case the party knows that the voter knows the true position x_A. Indeed,

$$var(\widehat{x_A}|t_A) = \left[1 - \frac{\widehat{s_A}^2(t_A)}{s_A^2}\right]\widehat{s_A}^2(t_A).$$

This yields the conditional utility of a voter with characteristics (α, x) for A:

$$E[\widehat{u_A}(\tilde{y}_A, t_A; \alpha, x)|t_A, x_A] = -\alpha\left[(E(\widehat{x_A}|t_A, x_A) - x)^2 + \widehat{s_A}^2(t_A) + var(\widehat{x_A}|t_A)\right].$$

Impact on the electorate. The party's objective $\overline{U_A}(t_A; x_A)$ is obtained by taking the average of the above expression over the electorate. It is more convenient, and equivalent from a strategic point of view, to state the objective in terms of the change due to the campaign.

Preliminary proposition *Given the true position x_A, the change in the average expected utility for A due to time t_A, which we denote by $\Delta\overline{U_A}(t_A; x_A) = \overline{U_A}(t_A; x_A) - \overline{U_A}(0; x_A)$, is:*

$$\overline{\alpha}\left[(m_A - \overline{x})^2 - (E(\widehat{x_A}|t_A, x_A) - \overline{x})^2\right] + \overline{\alpha}\left[s_A^2 - \widehat{s_A}^2(t_A)\right] - \overline{\alpha}var(\widehat{x_A}|t_A), \tag{9}$$

where $\overline{\alpha}$ the average weight in the electorate and $\overline{x}$ is the average weighted bliss point:

$$\overline{\alpha} = \int_\alpha\int_x \alpha g(\alpha, x)d\alpha dx, \overline{x} = \int_\alpha\int_x \frac{\alpha x}{\overline{\alpha}} g(\alpha, x)d\alpha dx.$$

The effect of speech can be decomposed into an effect on voters (the two first terms in (9)) and the strategic risk (the third term). Specifically the sum of the first two terms is the change in the expected utility for a "representative voter" if A gets elected due to the campaign. The bliss point of this representative voter is the average position $\overline{x}$, his perception is the average perception over the electorate, and the variance is the same as that of each voter.

Let us examine in more detail each term. The first term in (9) results from the change in the voters' expected assessments regarding party A's true position. Let us label it *the expected posterior effect.* This term is maximal when t_A is such that $E(\widehat{x_A}|t_A, x_A)$ is made as close as possible to the average weighted bliss point in the electorate. The second term in (9) results from the decrease voters 'uncertainty regarding party A's position, and is unambiguously favorable (because $\widehat{s_A}^2(t_A)$ is decreasing in t_A from (1)). The third term results from the strategic risk borne by parties, which may be non monotone, as we have seen.

It will prove useful in the sequel to work with a measure of the reduction in the uncertainty on a party's position due to the campaign and to introduce normalized variables. Given time spent on the issue by party A, $t_A \in [0, 1]$, let us define:

$$h_A(t_A) = 1 - \left(\frac{\widehat{s_A}(t_A)}{s_A}\right)^2. \tag{10}$$

Note that for all t_A, $h_A(t_A) \in [0, 1]$, $h_A(0) = 0$, and $h_A(t_A)$ is strictly increasing in t_A. As for normalization, let us consider the deviation from the true position to the prior in terms of the standard error on A position $e_A(x_A)$ and the deviation from the the average bliss point to the prior in terms of the standard error on A position d_A:

$$e_A(x_A) = \frac{x_A - m_A}{s_A},\ d_A = \frac{\overline{x} - m_A}{s_A}. \tag{11}$$

With this notation, the expected posterior effect writes as $\overline{\alpha} s_A^2 \left[2 d_A e_A(x_A) h_A(t_A) - e_A^2(x_A) h_A^2(t_A)\right]$, the reduced voters' uncertainty effect as $\overline{\alpha} s_A^2 h_A(t_A)$, and the strategic risk as $-\overline{\alpha} s_A^2 h_A(t_A)(1 - h_A(t_A))$. Note that there are respectively, constant and increasing marginal benefit of precision. Adding the last two effects, one finds $\overline{\alpha} s_A^2 h_A^2(t_A)$, which is unambiguously increasing in the precision, with increasing marginal benefit. The sum of the last two terms sums up the total effect on uncertainly (borne both by voters and by the party through the strategic risk). We label it the *reduced variance effect*. Adding the expected posterior effect and this reduced variance effect, the marginal benefit of precision may be increasing or decreasing, depending on which effect matters more. Indeed, the variation $\Delta\overline{U_A}$ writes as:

$$\Delta\overline{U_A}(t_A; x_A) = \overline{\alpha} s_A^2 \left[\left(1 - e_A^2(x_A)\right) h_A^2(t_A) + 2 d_A e_A(x_A) h_A(t_A)\right], \tag{12}$$

which is a second degree polynom in h_A.
The marginal benefit of precision at precision $h_A(t_A)$ is:

$$2\overline{\alpha} s_A^2 \left[\left(1 - e_A^2(x_A)\right) h_A(t_A) + d_A e_A(x_A)\right]. \tag{13}$$

There are increasing marginal benefits from precision when the party's position is less than a standard error from the prior, $e_A^2(x_A) < 1$, in which case the issue is said to be *standard*. In the opposite case of a *non standard* issue, there are decreasing marginal benefits from precision.
The marginal benefit of precision at zero is positive for $d_A e_A(x_A) > 0$, or equivalently for x_A and $\overline{x}$ located on the same side of m_A, in which case the party's position x_A on the issue is said to be *favorable* (and *non favorable* in the opposite case).

Multiple issues case Let h_A^k denote for each issue k the precision of party A on issue k:

$$h_A^k(t_A^k) = 1 - \left(\frac{\widehat{s_A^k(t_A^k)}}{s_A^k}\right)^2,\ t_A^k \in [0,1].$$

The change in the average expected utility for A being elected induced by the campaign depends on the time party A spends discussing each issue $t_A = (t_A^k) \in T$ and on its position on each one $x_A = (x_A^k)$. It is given by:

$$\Delta\overline{U_A}(t_A; x_A) = \sum_k \overline{\alpha^k} \left(s_A^k\right)^2 \left[\left(1 - \left(e_A^k(x_A^k)\right)^2\right)\left(h_A^k(t_A^k)\right)^2 + 2 d_A^k e_A^k(x_A^k) h_A^k(t_A^k)\right],$$

where

$$\overline{\alpha^k} = \int_\alpha \int_x \alpha^k g(\alpha, x) d\alpha dx, \overline{x^k} = \int_\alpha \int_x \frac{\alpha^k x^k}{\overline{\alpha^k}} g(\alpha, x) d\alpha dx, e_A^k(x_A^k) = \frac{x_A^k - m_A^k}{s_A^k}, d_A^k = \frac{\overline{x^k} - m_A^k}{s_A^k}.$$

4 The case of a time constraint per issue

4.1 Optimal strategies

We have already observed that $\overline{U_A}$ is separable across issues. This immediately implies that in the absence of a global time constraint, that is, when the strategy space for a party writes as $T = [0,1]^K$, the game can be analyzed issue by issue. This assumption about the strategy space is certainly the right assumption to make if we interpret the strategies of the parties in terms of choice of ambiguity. In that case, we may assume that a party decides the precision it wishes to reach on each issue, with no global constraint. To save on notation in that case, we drop the subscript k pertaining to the issue under study in this subsection. We will come back to the full notation later in the text.

Party A chooses the time it spends explaining the issue, $t_A \in [0,1]$, knowing its position x_A. Proposition 1 describes the optimal amount of time to be spent on the issue, as a function of x_A. Equivalently, party A actually chooses the precision h_A reached on the issue. Denote by $\overline{h_A}$, $\overline{h_A} \in]0,1]$, the *maximal reachable precision* that candidate A can reach when he talks full time ($\overline{h_A} = h_A(1)$). When $\overline{h_A}$ is smaller than 1, *full precision* ($h_A = 1$) is not reachable. Party A can choose any precision h_A in the interval $\left[0, \overline{h_A}\right]$. Results are presented in the case where the prior on A 's position (m_A) is on the left of the electorate bliss point, and straightforwardly adapt to the opposite case.

Proposition 1 *Let $m_A \leq \overline{x}$, or equivalently $d_A \geq 0$. The optimal strategy is characterized by two thresholds $\underline{e_A} < \overline{e_A}$, such that:*

(i) for $e_A(x_A) < \underline{e_A}$, party A does not talk,

(ii) for $\underline{e_A} < e_A(x_A) < \overline{e_A}$, party A uses the maximal amount of time,

(iii) for $e_A(x_A) > \overline{e_A}$, two cases are to be considered. If $d_A = 0$, party A does not talk. If $d_A > 0$, party A uses a positive but less than maximal amount of time, which is decreasing with x_A.

The thresholds are given by

$$\underline{e_A} = \frac{d_A}{\overline{h_A}} - \sqrt{1 + \left(\frac{d_A}{\overline{h_A}}\right)^2} < 0, \; \overline{e_A} = \frac{1}{2}\frac{d_A}{\overline{h_A}} + \sqrt{1 + \left(\frac{1}{2}\frac{d_A}{\overline{h_A}}\right)^2}.$$

The proof of Proposition 1 follows from straightforward computation provided in the appendix. The optimal strategy is illustrated on Figure 2.

***** INSERT FIGURE 2 ABOUT HERE *****

Comments. The optimal strategy solves the trade-off (if any) between the expected posterior effect and the reduced variance effect, as defined in the previous section. It can be summarized as follows: a party speaks when its position is favorable ($e_A(x_A) > 0$) or when its position, although not favorable, is close enough to the prior to allow for a reduction in voters' uncertainty without too much negative impact on the expected posterior (for $d_A > 0$). Let us discuss this result in more detail, assuming that full precision is reachable ($\overline{h_A} = 1$) so as to keep the comments simpler.

Let us consider the case $m_A < \overline{x}$, i.e. $d_A > 0$. The case $d_A < 0$ is symmetric, and the case $d_A = 0$ will be discussed at the end of this sub-section.

When the party's true position is not favorable, that is, where the position x_A and the average bliss point $\overline{x}$ are on opposite sides of the prior m_A ($e(x_A) < 0$), whenever the party talks, the expected posterior is further away from the average bliss point than the prior is. Thus the expected posterior effect is unambiguously negative. The optimal strategy for the party is to remain silent, except if the reduced variance effect is dominant. Note that it requires the marginal incentives to speak to be increasing (standard position), in which case if a party talks, it talks full time. Indeed, the marginal benefit of precision on an issue which is non favorable and non standard is always negative (see (13)). The condition of indifference between no speech and full time speech gives the first threshold value $\underline{e_A}$. Note that if the party's payoff is positive when it speaks full time for some *a priori* deviation from the representative voter $d_A > 0$, then its payoff is also positive when it speaks full time for some *a priori* deviation $0 < d'_A < d_A$, that is, when the party is *a priori* better aligned with the representative voter' interests. This fact explains why the threshold $\underline{e_A}$ increases with d_A.[6]

When the party's true position x_A is favorable, here when $e(x_A) > 0$, the optimal strategy for the party is to speak. It speaks full time and reveals its true position when the position is moderate enough, (that is when $e\left(x_A\right)$ is below a second threshold $\overline{e_A}$ which is larger than 0). This is clearly optimal when $e(x_A) \leq d_A$, since in that case the expected posterior position effect and the reduced variance effect both play in the same direction. It is also clearly optimal when $e(x_A) \leq 1$, since in that case there are increasing marginal returns of precision, and the marginal benefit of precision is always positive (see (13)). Now, one should concentrate on cases where $e(x_A) > \max[d_A, 1]$. In that case, if the party was only concerned with the expected posterior position effect, it would adjust its time so that the expected posterior beliefs about its position exactly matches the average ideal position in the electorate $\overline{x}$: it would not speak full time (but choose an amount of speech $d_A/e(x_A) < 1$). Now, the reduced variance effect induces it to speak full time instead when the position effect is not too detrimental that is when $e(x_A)$ is smaller than the second threshold $\overline{e_A}$. This second threshold is obtained by the condition that the marginal benefit of precision at full precision is zero. Note that there is some "overshooting", in the sense that during the campaign, the party moves from a prior value below the representative voter's position ($m_A \leq \overline{x}$) to a posterior above the average bliss point ($E(\widehat{x_A}) > \overline{x}$). When the party's true position is large enough ($e(x_A)$ above the threshold $\overline{e_A}$), the reduced variance effect induces to speak more than necessary to match the representative voter's position (there is still some overshooting), but not to a point where full time speech is optimal. The optimal speech time is

[6] A similar argument explains why the threshold $\underline{e_A}$ decreases with the maximal reachable precision $\overline{h_A}$.

decreasing with $e(x_A)$ and tends to zero as the position gets infinitely extreme.

The situation where a party's prior coincides with the average bliss point ($d_A = 0$) is rather special since talking about the issue can only deteriorate the expected posterior and the only motive is to reduce voters' uncertainty. The party speaks full time when its position is less than one standard error from the prior and does not speak otherwise.

4.2 *Ex ante* properties of the electoral campaign

In the previous subsection, we have commented upon the parties' optimal strategies, as a function of their true position. We now consider the *ex ante* properties of the electoral campaign, that is, before knowing the candidates' true positions. In what follows, N denotes the cumulative distribution of a centered standard normal variable.

Issues can always be defined in such a way that A is *a priori* on the left to the average bliss point, that is $d_A^k \geq 0$. To simplify the discussion, we shall always do that and assume that on all issues, A is *a priori* perceived as lying on the left-hand side of the average voter (but it could be the case that B is not always on the right-hand side)

Which issues are the most likely to be addressed by a party? If for issue k, $d_A^k = 0$, by Proposition 1, the party speaks full time when its position is standard and does not speak otherwise. Party A's position is standard on issue k whenever $\left(e_A^k\right)^2 \leq 1$. This happens with probability 0.68. Therefore, the *ex ante* probability that a party addresses such an issue is 0.68.

If for issue k, $d_A^k > 0$, a party speaks as soon as $e_A^k\left(x_A^k\right) \geq \underline{e_A^k}$, where $\underline{e_A^k}$ is the threshold defined in Proposition 1. Hence, for $d_A^k > 0$, the probability that a party talks is equal to $1 - N\left[\underline{e_A^k}\right]$. This probability decreases with $\left(\frac{d_A^k}{h_A^k}\right)$ from 0.84 to 1/2.[7] Thus, the chances for a party to talk about an issue are lower the more extreme it is a priori (the larger $\left|\overline{x^k} - m_A^k\right|$), the smaller the uncertainty on its position (the smaller s_A^k)[8]. and the lower the maximal reachable precision.

How many issues are addressed by a party? The discussion above shows that the probability that a party addresses a given issue is at least 1/2 and at most 0.84. Therefore, with a large number of issues, we expect the party to address at least one half of the issues, and even more if the priors on its platforms are close to the average bliss points.

It is very important to note that the probability that the party addresses an issue is at least 1/2, no matter what the maximal reachable precision is, or alternatively, no matter what the available amounts of time on the various issues are (here, we have normalized the available amount of time on each issue to 1). In particular, it remains true if the maximal amount of time per issue is $1/K$,

[7] Since for $d_A^k = 0$, the probability that a party talks is 0.68, there is a jump upward to 0.84 because a large set of positions becomes favorable when the prior does not exactly coincide with the average bliss point.

[8] Large *a priori* uncertainty are more likely to be observed for challengers (whose positions are unknown) or for new issues at stake.

which will make comparison with the globally constrained case (where we assume that the total available time to be allocated across issues is 1) more meaningful.

Are the parties likely to address the same issues or not? In our analysis, it is clearly possible that both parties speak on the same issue, a situation referred to as "issue convergence" or by "dialogue" by Simon (2002). They do so as soon as speaking is favorable to both - which is perfectly possible since the condition which defines whether a position is favorable or not only relates to the party's prior and true positions - relative to the average bliss point). Note that it is also possible that no party talks about an issue.

From the computation above, the probability that both parties engage in dialogue on a specific issue is at least 1/4, and no more than 0.70. This probability is close to its maximum when both parties are *a priori* extremely close to the average bliss point. It is close to its minimum when both parties are *ex ante* very extreme. A natural illustration of such a situation is when the prior values m_A and m_B are each on one side of, and each far apart from, the average bliss point. Another possible case is that both parties are on the same side and far away from the average bliss point: parties agree between themselves but disagree with the electorate. This was the case for example for the European Union issue in the 2007 French presidential election. (See more on issue convergence in subsection 6.4).

4.3 Conclusions for the case without global time constraint

1. A party speaks about an issue when its position on this issue is favorable or when its position, although not favorable, is close enough to the prior to allow for a reduction in voters' uncertainty without too much impact on the posterior.

2. The *ex ante* probability that a party addresses an issue (before knowing its position on that issue) is minimal and equal to 1/2 when it is *a priori* extreme, and it is maximal (close to 0.84) when it is *a priori* close to the average bliss point.

3. Both parties may address the same issues. The chances that both parties address a given issue are at least 1/4, and can go up to 0.70 for issues where both parties are *a priori* close to the average bliss point.

5 The case of a global time constraint

We now turn to the situation in which parties face a global time constraint and have to allocate their overall time across issues: $T = \left\{(t^1, t^2, ..., t^K) \in [0,1]^K \text{ and } \sum_k t^k \leq 1\right\}$, up to a normalization. There is one unit of time that has to be allocated between the issues.

Party A maximizes:

$$\Delta\overline{U_A}(t_A; x_A) = \sum_k \overline{\alpha^k}\left(s_A^k\right)^2\left[\left(1 - \left(e_A^k\right)^2(x_A^k)\right)\left(h_A^k\right)^2(t_A^k) + 2d_A^k e_A^k(x_A^k)h_A^k(t_A^k)\right] \text{ s. t. } t_A \in T.$$

The optimal solution is not only determined by the values of $e_A^k(x_A^k)$ and d_A^k, as in the case where there is no global time constraint, but also by the sensitivity of the precision indices with respect to the strategies. The marginal benefit derived by a marginal increase in addressing issue k writes as:

$$\frac{\partial \overline{U_A}}{\partial t_A^k}(t_A; x_A) = 2\overline{\alpha^k}\left(s_A^k\right)^2\left(h_A^k\right)'(t_A^k)\left[\left(1-\left(e_A^k\right)^2(x_A^k)\right)h_A^k(t_A^k) + d_A^k e_A^k(x_A^k)\right]. \tag{14}$$

To go further, the analysis is simplified by assuming each precision index h_A^k to be linear with respect to t_A^k. If $h_A^k(t_A^k) = \overline{h_A^k} t_A^k$, with $0 < \overline{h_A^k} \leq 1$:

$$\begin{aligned}
\frac{\partial \overline{U_A}}{\partial t_A^k}(t_A; x_A) &= 2\overline{\alpha^k}\left(s_A^k\right)^2\overline{h_A^k}\left[\left(1-\left(e_A^k\right)^2(x_A^k)\right)\overline{h_A^k}t_A^k + d_A^k e_A^k(x_A^k)\right], \\
\frac{\partial^2 \overline{U_A}}{\partial\left(t_A^k\right)^2}(t_A; x_A) &= 2\overline{\alpha^k}\left(s_A^k\overline{h_A^k}\right)^2\left(1-\left(e_A^k\right)^2(x_A^k)\right).
\end{aligned}$$

Under this assumption, the marginal benefit from an additional speech on an issue is either increasing or decreasing, depending on whether the issue is in standard position or not.

5.1 Optimal strategies

Proposition 2 *1. A party speaks on an issue where its position is non standard only if it is favorable.*

2. Assume the linearity of the precision measures h_J^k for party J, and for each issue. A party speaks on one issue in standard position at most.

Comments. Point *1* simply states that speaking on an issue for which both terms $d_A^k e_A^k$ and $1-\left(e_A^k\right)^2$ are non positive is strictly *dominated* for A (neglecting the case where both terms are null). Dominance holds because the marginal benefit from speaking on issue k is always negative as can be seen from (14). This is also true in the case where there is no global time constraint.

As for point *2*, we present the basic intuition (the formal proof is in the appendix). Let the party speak about two issues in standard position, say k and ℓ. Under the global time constraint assumption, the marginal benefits are equalized between these issues and are both positive. The objective is convex with respect to the time spent on one issue. Hence increasing the time spent on one issue at the expense of the other is surely beneficial, which gives the contradiction.

A simple consequence of Proposition 2 can be drawn if all issues are in standard position. Then, the party concentrates on a single issue. If there are both kinds of issues, the time spent on those with decreasing marginal benefit is allocated by equalizing the marginal benefit on these issues and the remaining time is concentrated on a single issue in standard position.

5.2 The ex ante properties of the electoral campaign

How many issues are addressed by a party? The probability for an issue to be in standard position is equal to 0.68, and the probability for a position to be favorable is 0.50. Note that those probabilities are independent of the issue parameters. With a large number of issues, the proposition gives an upper bound of 0.16 per cent on the number of issues that are addressed on average: 0.68 % of the issues are not addressed (those in standard position but one) and half of those in non standard position are not addressed (because their position is non favorable).

Which issues are the most likely to be addressed by a party? The proposition is silent about which issues are *ex ante* addressed. We shall argue that when there are many potential issues, the probability of addressing an issue is larger the more *a priori* extreme the party is on that issue (prior far from the average bliss point).

Consider an issue k, and make a party, say A, *a priori* more extreme on this issue by increasing d_A^k, for $d_A^k > 0$, other issues being unchanged. The question we want to address is whether issue k has now more chances to be addressed by party A. It is important to note that the law of the standardized deviation to the prior $e_A^k = \frac{x_A^k - m_A^k}{s_A^k}$ is independent of d_A^k. Consider a *fixed* vector of deviations $(e_A^1, e_A^2, ..., e_A^K)$ for which the issue is addressed by A with the initial value of $d_A^k > 0$, and let us show that under this same vector of deviations, party A will most certainly still address the issue when d_A^k increases.

Note that the marginal benefit of the first unit of precision on this issue k writes as $\overline{\alpha^k} \left(s_A^k\right)^2 d_A^k e_A^k$. Thus, this marginal benefit for A increases with d_A^k when the position is initially favorable to A, which occurs when e_A^k is positive, and decreases in the opposite situation. Besides, the fact that the party's position is standard or not on issue k only depends on the deviation e_A^k, and not on the prior.

Now, if initially the position on issue k is favorable ($e_A^k \geq 0$), from the remark just above, it is surely still addressed by A as d_A^k increases. When the position on issue k is not favorable and not standard ($e_A^k \leq -1$), the issue is not addressed in the first place (point 1 of Proposition 2). Thus, the only situation where an increase in d_A^k may deter A from addressing the issue occurs when the issue is in standard position but unfavorable ($-1 \leq e_A^k \leq 0$), and party A initially addressed the issue. However, this case occurs with small probability since at most one issue in standard position is addressed by A (point 2 of Proposition 2).

Are the parties likely to address the same issues or not? From the computation above, with a large number of issues, the probability that a party addresses one given issue is at most 16%. Therefore, the upper bound for the probability that both parties talk about the same issue is below 3%.

Discussion. What can be said without assuming the linearity of the precision parameters h_J^k with respect to time? First, if for each issue, the objective is either concave or convex with respect to time, the same properties as stated above hold. This is likely to occur if the convexity or concavity in the precision parameters is moderate, which is likely under our assumption of tight time constraint. Otherwise, the concavity of the precision parameter may be the more plausible assumption. In that case, the objective is more concave with respect to the allocation of time than with respect to precision. In particular it is concave for each non standard issue.

5.3 Conclusions for the case with a global time constraint

Assume the linearity of the precision measures h_J^k for each party, and for each issue.

1. A party speaks on one issue in standard position at most.
2. With a large number of issues, at most 16% of the issues are addressed by a party.
3. The chances that a party addresses an issue increase when the party becomes *a priori* more extreme on that issue.
4. With a large number of issues, at most 3% of the issues are addressed by both parties: parties do not address the same issues.

5.4 An alternative interpretation of the model

An alternative interpretation is the following one. Time can be interpreted as a scarce resource, some given amount of money to be spent on advertising for instance, and issues can be interpreted as a distinct electorate body - the electorate in a geographical area or an ethnic or social group for instance. Under this interpretation, t_A^k represents the time spent by candidate A speaking to the group k, through the local media or the 'ethnic' TV. The analysis carries through under the proviso that individuals in one group do not listen to the speeches addressed to the other groups. In that case, the model describes the optimal strategy by candidates when deciding how to target the money they have across districts when designing a campaign.

Under this interpretation, there are some restrictions on the parameters. First the position in each area is the candidate' position, which requires $x_A^k = x_A$ for each k. Keeping our assumption that individuals share the same prior at the beginning of the campaign, one has $m_A^k = m_A$ and $s_A^k = s_A$ for each k. Therefore, all groups share the same $e_A^k(x_A)$. To simplify the notation, let us simply denote it by e_A. There is *a priori* no further restriction on the average bliss points in those electorates $\overline{x^k}$ (embodied in the d_A^k),on their size $\overline{\alpha}^k$, nor on their capacities to interpret information (the $\sigma_A^k(.), \sigma_B^k(.)$ functions). If we assume that precision in each group is linear in time, with $h_A^k\left(t^k\right) = t^k$, we can use the results previously obtained to derive the optimal campaign strategy.

If a candidate's position is standard ($e_A^2 < 1$), he speaks to at most one group. If for all groups, $(1 - e_A^2) + 2e_A d_A^k < 0$, he does not talk at all; if not, he speaks full time to the group k^*, where:

$$k^* = \arg\max_k \left[\overline{\alpha^k}\left((1 - e_A^2) + 2e_A d_A^k\right)\right].$$

A group is more likely to be addressed when it represents a large fraction of the electorate (large $\overline{\alpha}^k$) or when it is *a priori* extreme but with an average bliss point on the same side of the prior as the true position (large $e_A d_A^k$).

If a candidate's position is non standard ($e_A^2 > 1$), he speaks to a group only if his position in this group is favorable. Provided that this is the case for at least one group, among the groups in which his position is favorable, he speaks to the group with the highest $\overline{\alpha^k} d_A^k$ and possibly to some other favorable groups, organizing speeches in order to equalize marginal benefits of time $2\overline{\alpha^k}\left((1-e_A^2)t^k + e_A d_A^k\right)$ across those groups. In that case again, large, favorable, extreme groups are better targets.

6 Discussion

6.1 Impact of the campaign on welfare

We briefly discuss some consequences of the electoral campaign on voters' welfare. We do not provide a full welfare analysis here, but simply underline some *a priori* counter-intuitive properties of an electoral campaign. A simple example in the single issue case shows that although parties convey unbiased information, electoral campaign may prove to be detrimental to voters, in the sense that voters' welfare would be higher with no information at all, than with the information conveyed at equilibrium during the campaign.

This result challenges the classical measure of the informational quality of the campaign by the total amount of speeches delivered by parties, or the amount of effective information that is transmitted to voters.

Definition of voters' welfare. We use an *ex ante* utilitarist criterion to assess welfare.

Let $p_J(t_A, t_B, x_A, x_B)$ denote the probability that party J wins the election, given true platforms (x_A, x_B), when party A (resp. B) spends the amount of time t_A (resp. t_B) explaining its platform. The average expected utility in the electorate given those (t_A, t_B, x_A, x_B) is:

$$\widetilde{W}(t_A, t_B, x_A, x_B) = p_A(t_A, t_B, x_A, x_B)\overline{u}(x_A) + p_B(t_A, t_B, x_A, x_B)\overline{u}(x_B),$$

where

$$\overline{u}(z) = \int_\alpha \int_x u(z; \alpha, x) g(\alpha, x) d\alpha dx \tag{15}$$

is the average utility in the electorate when the platform z is implemented.

We further assume that the probability that J wins the election is an affine function of party J's expected vote share π_J: $p_J = \frac{1}{2} + \beta\left(\pi_J - \frac{1}{2}\right), 0 < \beta < 1$, where party J's expected vote share is given by (5) and (7).[9] Therefore, the change in voters' welfare induced by emphasis (t_A, t_B) given true platforms (x_A, x_B) is:

$$\Delta\widetilde{W}(t_A, t_B, x_A, x_B) = \beta\phi\left(\Delta\overline{U_A}\,(t_A, x_A) - \Delta\overline{U_B}\,(t_B, x_B)\right)\left(\overline{u}(x_A) - \overline{u}(x_B)\right).$$

This expression shows that the campaign is welfare enhancing iff the campaign increases the probability that the party which is the closest to the average bliss point wins the election. Indeed,

[9] For example, following Persson and Tabellini (2000), we assume that an additive uniformly distributed macro random shock occurs after parties have decided their emphasis strategies.

in the expression above, $\beta\phi\left(\Delta\overline{U_A}\left(t_A, x_A\right) - \Delta\overline{U_B}\left(t_B, x_B\right)\right)$ is the change in the probability that party A wins the election which is induced by emphasis (t_A, t_B) given the true platforms (x_A, x_B).

If one denotes by $T_J : R \to [0,1]$ a strategy for party J, that is the amount of time spent on the issue depending on the true position, the *ex ante* welfare variation induced by the campaign depends on these strategies T_A, T_B, and writes as:

$$\Delta W(T_A, T_B) = E_{x_A, x_B} \Delta\widetilde{W}(T_A(x_A), T_B(x_B), x_A, x_B). \tag{16}$$

The positive value of unconditional information. Consider first as a benchmark the case where the communication strategies are independent of the true positions x_A, x_B, that is, the communication by the parties t_A, t_B are fixed *ex ante*. In that case, $T_J(x_J) = t_J = cst$. Simple computation yields the following formula for the welfare variation:

$$2\beta\phi\overline{\alpha}^2 \sum_{J=A,B} s_J^4 \left[2d_J^2 h_J(t_J) + h_J^2(t_J)\right].^{10} \tag{16}$$

In that case, the welfare is increasing and convex in h_A and h_B. When the precision conveyed is independent from the positions x_A, x_B, more precision is always valuable.

An example of welfare reducing campaign. Now, to assess welfare in the electoral campaign game, one needs to replace in the expression (16) above the emphasis vector $(T_A(x_A), T_B(x_B))$ by the optimal strategies computed in section 4.

We show in a very simple example that an electoral campaign may be detrimental to voters' welfare. Consider the special case where both parties' *a priori* positions coincide with the average bliss point in the electorate ($d_J = 0$). In that case (proposition 1), party J talks full time when $e_J^2(x_J) \leq 1$, and remains silent in all other cases. Hence the optimal strategy is $T_J(x_J) = 1$ if $e_J^2(x_J) \leq 1$ and $T_J(x_J) = 0$ otherwise. Simple computation yield the following formula for the variation of welfare induced by the campaign (assuming that full precision is reachable by both parties):

$$\beta\phi\overline{\alpha}^2\left[2s_A^2 s_B^2\left(C_0 - C_2\right) - \left(s_A^4 + s_B^4\right)\left(C_2 - C_4\right)\right], \text{ where } C_n = \int_{e^2<1} e^n f(e) de.^{11} \tag{16}$$

[10]

$$\text{Indeed, } \Delta\overline{U_J}\left(t_J, x_J\right) = \overline{\alpha} s_J^2\left[-h_J^2\left(t_J\right) e_J^2(x_J) + 2d_J h_J\left(t_J\right) e_J(x_J) + h_J^2\left(t_J\right)\right], \text{ and}$$
$$\overline{u}(x_A) - \overline{u}(x_B) = -\overline{\alpha}\left[s_A^2\left(e_A(x_A) - d_A\right)^2 - s_B^2\left(e_B(x_B) - d_B\right)^2\right].$$

Since $E_{x_J}\Delta\overline{U_J}\left(t_J, x_J\right) = 0$, $\Delta W = \Sigma_J E_{x_J}\left[\Delta\overline{U_J}\left(t_J, x_J\right)\overline{u}(x_J)\right]$, with

$$E_{x_J}\left[\Delta\overline{U_J}\left(t_J, x_J\right)\overline{u}(x_J)\right] = 2\overline{\alpha}^2 s_J^4\left[2d_J^2 h_J(t_J) + h_J^2(t_J)\right].$$

[11]

$$\text{Indeed, } E_{x_A x_B}\left[\left(\Delta\overline{U_A}\left(T_A(x_A), x_A\right) - \Delta\overline{U_B}\left(T_B(x_B), x_B\right)\right)\left(\overline{u}(x_A) - \overline{u}(x_B)\right)\right] =$$
$$E_{e_A^2(x_A)\leq 1}\left[\Delta\overline{U_A}\left(1, x_A\right)\left(\overline{u}(x_A) - E_{x_B}\left[\overline{u}(x_B)\right]\right)\right] - E_{e_B^2(x_B)\leq 1}\left[\Delta\overline{U_B}\left(1, x_B\right)\left(E_{x_A}\left[\overline{u}(x_A)\right] - \overline{u}(x_B)\right)\right],$$

Comments. (1) Note first that an electoral campaign may be detrimental to the voters' welfare. For example, if s_A is small enough (given a value for s_B), voters would be better off with no campaign. It might sound counter-intuitive, since during the campaign, only unbiased information is conveyed to the electorate. The reason is that the candidate who has the more incentives to talk - and benefits the more from the campaign - might not be the one who is the better for the electorate (that is, that whose position is closer to the average voter). This situation is likely to happen when an incumbent candidate faces an unknown challenger.

(2) When $s_A^2 = s_B^2 = s^2$, the variation in welfare is:

$$2\beta\phi\overline{\alpha}^2 s^4 \left(C_0 - 2C_2 + C_4\right), \text{ with } C_0 - 2C_2 + C_4 = \int_{e^2<1} (1-e^2) f(y) dy > 0,$$

and in that case the electoral campaign is welfare enhancing. The gain in welfare due to the campaign is increasing with voters' *a priori* uncertainty on parties' positions.

(3) This simple example shows that it might be quite misleading to use as a proxi for the quality of the campaign the amount of time that parties have spent discussing an issue. In this example, whatever the value of s_A, s_B, the probability that a party addresses the issue is $C_0 = 0.68$. In some cases (low uncertainty on one party, high uncertainty on the other), the campaign is welfare reducing, whereas in some other cases (for example symmetric parties), the campaign is welfare improving.

6.2 Information transmission about the opponent's platform

So far, we have assumed that parties only send voters information about *their own platform*. We are going to discuss this assumption here. Indeed, it could well be the case that when a party addresses an issue, it actually sends information to voters both as to where it stands, and as to where its opponent stands, on the issue. It might be because when a candidate chooses to address an issue in a radio or TV show, the journalist might prompt him to clarify his position relative to that of his opponent. It might also be the case that when he chooses to put more weight on an issue, newspapers and radios tend to dig into what the opponent wants to do on the same issue, thus conveying some information on the opponent as well. In those examples, we are talking about "involuntary" transmission of information. Another possibility is that the amount of information conveyed about the opponent is a variable that is fully under the control of the party, and can be independent from the amount of time spent discussing its own position on the issue. In this view, each party would have to allocate its time between describing its own position on the various issues, and those of its opponent.

In this section, we explore the consequences of the former kind of information transmission.

$$\begin{aligned}
\text{with } E_{e_A^2(x_A)\le 1}\left[\Delta\overline{U_A}\left(1,x_A\right)\left(\overline{u}(x_A) - E_{x_B}\left[\overline{u}(x_B)\right]\right)\right] &= \overline{\alpha}^2 s_A^2 E_{e_A^2(x_A)\le 1}\left[\left(e_A^2-1\right)\left(s_A^2 e_A^2 - s_B^2\right)\right] \\
&= \overline{\alpha}^2\left[s_A^2 s_B^2\left(C_0 - C_2\right) - s_A^4\left(C_2 - C_4\right)\right], \\
\text{and } - E_{e_B^2(x_B)\le 1}\left[\Delta\overline{U_B}\left(1,x_B\right)\left(E_{x_A}\left[\overline{u}(x_A)\right] - \overline{u}(x_B)\right)\right] &= \overline{\alpha}^2\left[s_A^2 s_B^2\left(C_0 - C_2\right) - s_B^4\left(C_2 - C_4\right)\right].
\end{aligned}$$

We propose a simple model of involuntary leakage of information. This model will be parametrized by a parameter $\rho \in [0, 1/2]$, capturing the amount of information leakage. One polar case, $\rho = 0$, corresponds to the situation in which each party only conveys information on its own platforms. This is the model we have studied in the previous sections. The other polar case, $\rho = 1/2$, corresponds to the situation where a party is constrained to send the same amount of information on its platform and that of his opponent. We are going to present here some results on this other polar case.

Involuntary information leakage. During the campaign, each party decides how much time to spend discussing each issue, subject to some time constraint. We assume that when party A (resp. party B) spends time t_A^k (resp. t_B^k) on issue k, each voter receives two imperfect signals regarding issue k: one on party A's true position, y_A^k, whose variance depends on some weighted average of the times spent by both parties discussing the issue, $(1-\rho)\, t_A^k + \rho t_B^k$, and one signal on party B's true position y_B^k, whose variance depends on $\rho t_A^k + (1-\rho)\, t_B^k$. The parameter ρ, which is assumed to lie in the interval $\rho \in [0, 1/2]$, describes the information leakage technology. We assume here that a party is always more informative about its own position than about that of its opponent. Signals on parties' positions are assumed to be unbiased, independently (across issues) and normally distributed:

$$y_A^k \sim \mathcal{N}\left(x_A^k, \left(\sigma_A^k\right)^2 \left((1-\rho)\, t_A^k + \rho t_B^k\right)\right), \, y_B^k \sim \mathcal{N}\left(x_B^k, \left(\sigma_B^k\right)^2 \left(\rho t_A^k + (1-\rho)\, t_B^k\right)\right),$$

with $\sigma_J^k(0) = +\infty$, $\left(\sigma_J^k\right)'(t) < 0$ for all $t \in [0, 1[$. We make no specific assumption regarding the correlations of signals across voters. They can be independently distributed (conditional on x_A, x_B) or correlated.

Parties expected vote share. Making the same assumptions about voters' treatment of information, and following the same probabilistic voting model as before, one easily checks that the expected vote share for party A is now simply:

$$\pi_A(t_A, t_B; x_A, x_B) = \frac{1}{2} + \phi \left[\overline{U_A}((1-\rho)\, t_A + \rho t_B; x_A) - \overline{U_B}(\rho t_A + (1-\rho)\, t_B; x_B)\right].$$

Remark. The case $\rho = 0$ corresponds to situations where a party fully controls the utility expected by the electorate if it becomes elected, and has no influence on the utility expected by the electorate if its opponent is elected. When ρ is positive, it is no longer the case, and the game between parties is non degenerate. In particular, the case $\rho = 1/2$ corresponds to situations where all that matters in the total quantity of time that is devoted to an issue by both parties, independently on the identity of the party addressing the issue. The remaining part of the subsection is devoted to presenting some result in that case $\rho = 1/2$.

Issue by issue time constraint. Assume first that parties face an issue by issue time constraint: $T = [0, 1]^K$. As noted above, when $\rho = 1/2$, the impact of speech does not depend on which party is speaking (formally, $\frac{\partial \pi_A}{\partial t_A^k}(t_A, t_B; x_A, x_B) = \frac{\partial \pi_A}{\partial t_B^k}(t_A, t_B; x_A, x_B)$). Hence if a party strictly benefits from an additional quantity of speech on any given issue, its opponent is made strictly worse off by it. Besides, with no global time constraint, it is in the interest of at least one party to address each issue. These properties have strong implications.

Proposition 3 *At an equilibrium in pure strategies (assuming it exists), for each issue, one party talks full time and the other remains silent.*

At such an equilibrium, all issues are addressed, but no issue is simultaneously addressed by both candidates. Yet, an equilibrium in pure strategies may fail to exist. To overcome the possible non existence of equilibria in pure strategies, we consider the game in which parties play sequentially. The party that plays first, say A, is the "first mover". The "follower", B, observes the strategy chosen by A before choosing its own emphasis: its strategy is a reaction function.[12] Note that an equilibrium in pure strategies in the simultaneous version of the game remains an equilibrium in the sequential version. An equilibrium always exists in this sequential version of the game. To say more, assume the linearity of the precision indices h_A^k and h_B^k: $h_J^k(t) = \overline{h_J^k} t$, with $0 < \overline{h_J^k} \leqslant 1$ for $J = A, B$. In that case, it can be proven that it is still the case that each issue is addressed by at least one party at equilibrium.

This contrasts with the result obtained with no leakage, where it could be the case that both parties remained silent on some issues, and where both parties could address the same issues at equilibrium.

Global time constraint. Assume now that parties face a global time constraint:

$$T = \left\{ (t^1, t^2, ..., t^K) \in [0,1]^K \text{ and } \sum_k t^k \leq 1 \right\}.$$

Note that it remains true that at an equilibrium in pure strategies of the simultaneous version of game (if it exists), no issue is addressed by both candidates. When considering the sequential version, some similar results obtain.

Proposition 4 *Let precision indices $h_J^k(t)$ be linear with respect to t for each issue k, each party J. At an equilibrium in the sequential version of game, at most one issue is simultaneously addressed by both candidates.*

The proof is given in the appendix. These results are to be contrasted to those obtained in the no leakage case. With no leakage and a global time constraint, the *ex ante* probability that both parties address an issue is also small. Indeed, the upper bound for the probability that both parties talk about the same issue is below 3%. But the rationale behind the results are quite different. With no leakage, the propensity for one party to address an issue is independent of

[12] There is no bad connotation in the term 'follower'. In political life, the incumbent is likely to be the follower. In fact, recall that, in two players zero-sum game, it is never a (strict) advantage to move first.

that of its opponent. Simply, each party addresses a small number of issues, and consequently, the probability that both parties address the same issues is even smaller. With full leakage, the situation is quite different. Now, the propensity for one party to address an issue depends on the relative strength of the parties on that issue. And if one party benefits from addressing it, its opponent loses from it.

6.3 Truthful parties and naive voters

Our analysis relies on two important assumptions.

The first assumption bears on voters, who may be considered as "naive" (although they are Bayesian). Indeed, consider for example party A's optimal strategy in the single issue case, as a function of the position x_A, as described in Proposition 1. A "sophisticated" or strategic voter who knows this function is able to infer more than what we have assumed so far. In particular, such a voter can infer that a candidate who does not address an issue has a position such that $e(x_A)$ is below the threshold $\underline{e_A}$. Similarly, by observing a positive precision below the maximal precision $\overline{h_A}$, she can infer the true position since this occurs only for positions such that $e(x_A)$ is above $\overline{e_A}$ and A's precision is one-to-one for these positions. This changes the voter's behavior. Knowing this, a candidate also changes his strategy. The impact depends on the assumed number of strategic voters. In the discussion that follows, we shall assumed all voters to be strategic.

The second assumption bears on candidates, who are constrained to send unbiased signals on their true platform. In a sense, they are 'sincere', or in other words, they are somewhat committed to their announcements.

We investigate how equilibria are modified when these assumptions are relaxed. Combining the assumptions naive versus sophisticated voters, and unbiased versus free signals, there are three additional cases to consider. We conduct the analysis in the single issue model so that it is equivalent to argue in terms of precision or time. With unbiased signals, a strategy specifies a precision as a function of the true position (x_A). In the absence of such constraint, a strategy specifies a mean and a precision, μ_A and h_A respectively, for A, as a function of the position. To simplify, we shall assume that full precision ($h_A = 1$) is reachable by the candidate.

Naive voters and no constraint on signals. With naive voters, an action (μ_A, h_A) determines the posterior assigned to A position as $E\left(\widehat{x_A}\right) = (1 - h_A)m_A + h_A\mu_A$, and the average expected utility $\overline{U_A}$ for A being elected is given by:

$$\overline{u}\left(E\left(\widehat{x_A}\right)\right) - \overline{\alpha}s_A^2\left(1 - h_A^2\right), \tag{17}$$

where the first term $\left(\overline{u}\left(E\left(\widehat{x_A}\right)\right)\right)$ decreases as the distance between the expected posterior and the average bliss point increases. By choosing the maximal precision (equal to 1) and an announcement that is equal to the average bliss point in the electorate $\overline{x}$,the utility in (17) is null, which is the maximum possible value. Announcing the average bliss point without any ambiguity is an optimal strategy. (Without full precision, the result extends straightforwardly). Of course both candidates will do the same. Hence, with naive voters and free signals, the standard convergence result applies (in announcements): both candidates announce the position of the average voter. No information is transmitted.

Sophisticated voters and no constraint on signals. We will show that in that case, all actions taken by A at equilibrium can only induce the same payoff for A. Assume by contradiction that, at equilibrium, A's strategy is such that both actions (μ_A, h_A) and (μ'_A, h'_A) are played with positive probability, and that voters' equilibrium beliefs are such that the probability that A wins the election is greater for the former action than for the latter. Then when A draws a position such that this strategy specifies that it should play the latter action, it would be strictly better off choosing the former instead. Indeed, when there is no commitment, the action chosen by a party does not convey any direct information to voters on its true position. Sophisticated voters know this. Which contradicts the fact that at equilibrium actions yielding different payoffs can be played with positive probability. Hence, all actions taken by A at an equilibrium strategy have to induce the same payoff to A (note that the argument is valid for pure or mixed strategy as well). No information is transmitted.

Sophisticated voters and unbiased signals. Consider the strategy where both parties always send perfect information on their true position ($h_A = h_B$ identical to 1). It is an equilibrium. To show this, the voters' 'out of equilibrium' behavior must be specified. Assume that when voters observe imprecise messages by one party, they vote for the opponent.[13] With sophisticated voters and commitment all information can be revealed at an equilibrium.

With non-strategic naive voters on the contrary, there are always extreme positions far enough from the average bliss point for which A benefits from being imprecise. For example, by not talking at all, the candidate secures itself the value $\overline{u}(m_A) - \overline{\alpha} s_A^2$ with naive voters.

6.4 Literature on the content of electoral campaigns

We believe that our model helps shed some light on a number of empirical facts or controversies that have recently emerged on the content of electoral campaigns, in particular around the "issue ownership theory" by Petrocik (1996), and the striking differences that have been documented between presidential and senate elections in the US.

According to Petrocik (1996) and the "issue ownership theory", some issues are widely perceived by voters as better handled by one party (say the Republicans), and some other issues by the other party. For example, the Republicans traditionally "own" the crime and national security issues, whereas the Democrats "own" the welfare or environmental issues. According to Petrocik, candidates should tend to campaign on issues on which they have an advantage to prime their

[13] This behavior can be supported by the fact that voters interpret this imprecise message as evidence that the party is so far away from their own bliss point that they are better off voting for the opponent.

salience when voters make their voting decisions: they should emphasize the issues they own and downplay the issues they do not. This predicts that parties should concentrate on few issues, and that there should not address the same issues.

Simon (2002) offers a theoretical model as to why it should be the case[14]. As he puts it (page 64), "As no theme can work to the advantage of both candidates, they will never allocate resources to the same theme. Dialogue is defined as candidates discussing (spending money on) the same dimensions, so, rational candidates should never and will never dialogue." Then, focusing on 49 US Senate races in 1988, 1990 and 1992 and using home-state newspaper coverage of the candidates' statements on 32 issues, he determines how much attention each issue received during the campaigns. His results are partly consistent with the issue ownership theory, in that the Democrats do very little speaking about immigration, and crime, while they instigate a great deal of discussion about welfare, women's issues, environment and education (Table 7.4 page 132). There is less dialogue on the owned issues than on remaining unowned issues (Figure 7.1 page 133).

Yet, other empirical works, notably by Sigelman and Buell (2004), have provided strong evidence against the theory of issue ownership as defined by Simon or Petrocik. They test for "dialogue" in US presidential campaigns, focusing on the statements by campaigners for the two major parties in the eleven presidential campaigns between 1960 and 2000. Those statements were extracted from all the news items published in the New York Times that referred explicitly to the presidential campaigns. They find that both sides generally address the same issues (see table 1 page 655) and conclude from their analysis that dialogue is the rule rather than the exception. Even work by Petrocik *et al.* (2004) offers evidence that can be interpreted as quite contradictory with the issue ownership theory. Their paper presents content analysis of the acceptance addresses and television spot advertisements of presidential candidates from 1952 through 2000. They show that Democrats spend roughly the same amount of time on Democratic and Republican-owned issues, while Republicans spend over two-thirds of their time (67% in TV spots and 69% in nomination addresses) on Republican-owned issues (Table 3 in the appendix page 626).

One reason that may explain those discrepancies might be found in the fact that Simon's study uses senate elections data, whereas the last two afore mentioned papers relate to presidential elections. Indeed, Kaplan *et al.* (2006) provide evidence that the two kind of elections might be quite different, in terms of campaign strategies. They look for issue convergence using data that cover both Senate campaigns (1998, 2000, 2002) and the 2000 presidential election. They divide issues in 50 different issues categories each year. They find much more issue convergence in presidential campaigns than in Senate campaigns. As the authors put it (page 729), "The data suggest, then, that Senate campaigns are qualitatively different than presidential campaigns. On average, there is considerably less issue convergence in Senate races that in the presidential race."

[14] for further analysis of this model, see Amoros and Puy (2007).

Our model may offer a unified framework that could account for those empirical findings. The amount of money available to candidates in presidential and senate campaigns vary widely. One could argue that the Senate races, being strongly financially constrained, are best modelled as campaigns with binding global constraints, whereas the presidential races, that feature at least 10 to 20 times the spending of the average Senate race are much less so. The no global constraint case could be a good approximation of the Presidential campaign, whereas the constrained case would be more relevant to describe Senate elections. We indeed find that there is more issue convergence, and more issues addressed in the former case than in the latter, which is consistent with the empirical findings mentioned above.

On the theoretical part, our paper can be seen as an attempt to provide a somewhat richer version of the issue ownership theory. In Simon (2002), voters know exactly where the parties stand on each issue, and when talking about an issue, candidates only increase the salience of this issue in the voters' eyes. We abandon this assumption of perfect information and assume instead that voters learn about parties' positions through the campaign. In that case, we can define a version of "*ex ante* issue ownership", defining a party as the owner of an issue when it is *a priori* close to the average bliss point on this issue whereas its opponent is *a priori* far away from it on that issue. With this definition, we offer a more nuanced result: even a candidate who is *a priori* far away from the average bliss point on an issue may want to address this issue. Indeed, if the candidate is less extreme that his party is perceived to be on this issue, the candidate will have some incentives to signal to the voter that his position is more congruent with the voters' interests than it was initially thought to be. In our model, all issues are addressed with positive probability. Yet, our results retain some of the flavor of the issue-ownership theory: we indeed find that the probability to address an issue for a party increases with the proximity of the prior to the average voter. If one party is *a priori* very close to the average bliss point, the probability that it talks about this issue is high and close to its maximum (0.84) whereas when it is perceived as *a priori* quite extreme, the probability that it addresses this issue decreases. As Kaplan *et al.* (2006) put it (page 735), "We suspect that issue ownership's failure to account for much variation in issue convergence is due to insufficient attention to the relationship between issue ownership at the party level and issue ownership at the candidate level. Scholars have long noted that candidates may find it in their interest to distance themselves from their party's reputation." We offer a model that precisely addresses this point.

7 Conclusion

We proposed a simple model designed to capture the benefits and the costs that candidates may face when transmitting to voters information about their platforms. The analysis reveals that the equilibrium communication strategies are very different depending on the assumption we make about the time constraint faced by the parties.

With no global time constraint, parties tend to talk on issues on which they are *a priori* congruent with the average voter, and they may address a large number of issues: the fraction of issues that are addressed varies between 50% and 84%. In that case, both parties may address the same issues, they will do so in at least 25% of the issues. On the contrary, when parties face a global time constraint, dialogue will be the exception rather than the rule. At most 16% of the issues are addressed by a party., and at most 3% are addressed by both.

Our results suggest that the communication strategies of parties will be very different depending on the media they use, and on the type of races they are involved in. In a political meeting where they can use their time pretty much as they want, or in an add campaign where they can freely choose the themes they want to advertise, they will tend to focus on a small number of issues. On the contrary, when they cannot freely allocate the time between issues, for example in an interview where a journalist chooses the agenda, they will send relevant information a much larger of issues, and will be more likely to send information on issues on which they are *a priori* congruent with the average voter. Our results also shed light on a number of empirical papers comparing the qualitative properties of presidential and Senate elections campaigns. We provide an explanation as to why more issues are addressed and more dialogue is observed in the former than in the latter.

8 Appendix

Proof of Proposition 1. Party A can choose any precision h_A in $\left[0, \overline{h_A}\right]$, $0 < \overline{h_A} \leq 1$. $\overline{U_A}$ is convex in h_A iff $e_A^2 \leq 1$, that is iff the candidate's position is standard. Let $m_A < \overline{x}$, that is $d_A > 0$.

Consider first the case of a standard position. Since $\overline{U_A}$ is convex, the optimal precision for A is either 0 or $\overline{h_A}$. The precision $h_A = \overline{h_A}$ is optimal iff $(1 - e_A^2(x_A))\overline{h_A} + 2e_A(x_A)d_A \geq 0$, which writes as $e_A(x_A) \geq \underline{e_A}$ and defines threshold $\underline{e_A}$.

Consider now the opposite case where the candidate's position is non standard. $\overline{U_A}$ is (strictly) concave in h_A. Without constraint on h_A, it reaches its maximum at $h_A^* = \frac{d_A e_A(x_A)}{e_A^2(x_A)-1}$. If $h_A^* \leq 0$, the optimal precision for A is 0 ; since the denominator of h_A^* is positive this occurs if $e_A(x_A)d_A < 0$, that is if x_A is not favorable. For x_A favorable. the optimal precision for A is $\overline{h_A}$ if h_A^* is larger than $\overline{h_A}$, which writes as $e_A(x_A) \leq \overline{e_A}$ and defines $\overline{e_A}$. Otherwise, if x_A is favorable and $e_A(x_A) \geq \overline{e_A}$ the optimal precision is $h_A^* < \overline{h_A}$, which is decreasing with $e_A(x_A)$ and goes to zero as $e_A(x_A)$ gets infinitely large. ∎

Proof of proposition 2. Let us write the first order conditions associated with the maximization of $\overline{U_A}$ under a global time constraint. There is λ nonnegative such that

$$\frac{\partial \overline{U_A}}{\partial t_A^k}(t_A; x_A) \leq \lambda \text{ with an equality if } t_A^k > 0.$$

Under linearity, $h_A^k\left(t_A^k\right) = \overline{h_A^k} t_A^k$, $\left(h_A^k\right)'$ is constant and equal to $\overline{h_A^k}$. Point 1 is trivial because speaking on an issue in non standard and non favorable position is dominated. Point 2 follows from the second order condition. At a solution where both t_A^k and t_A^ℓ are positive the second order condition writes as:

$$\left(\overline{h_A^l}\right)^2\left(1-\left(e_A^l\right)^2\right)+\left(\overline{h_A^k}\right)^2\left(1-\left(e_A^k\right)^2\right) \leq 0, \tag{18}$$

which cannot be satisfied if both positions on issues k and ℓ are standard, i.e. if both $\left(e_A^l\right)^2$ and $\left(e_A^k\right)^2$ are smaller than 1. ■

Proof of proposition 3. For any $t \in [0,1]^K$, denote by $g(t)$ the vote share of party A when the average time spent by the two party on issue k is t^k and true positions are x_A, x_B :

$$g(t; x_A, x_B) = \frac{1}{2} + \phi\left[\overline{U_A}(t; x_A) - \overline{U_B}(t; x_B)\right].$$

With this notation,

$$\frac{\partial \pi_A}{\partial t_A^k}(t_A, t_B; x_A, x_B) = \frac{\partial^k \pi_A}{\partial t_B^k}(t_A, t_B; x_A, x_B) = \frac{1}{2}\frac{\partial g}{\partial t^k}\left(\frac{t_A + t_B}{2}; x_A, x_B\right).$$

Proposition 3. states that if (t_A^*, t_B^*) is an equilibrium in pure strategies, necessarily, for each issue k, either $\left(t_A^{*k}, t_B^{*k}\right) = (0,1)$ or $\left(t_A^{*k}, t_B^{*k}\right) = (1,0)$.

Assume by contradiction that (t_A^*, t_B^*) is an equilibrium with $t_A^{*k} \in]0,1[$. In that case, necessarily, $\frac{\partial g}{\partial t^k}\left(\frac{t_A^*+t_B^*}{2}; x_A, x_B\right) = 0$ and $\frac{\partial^2 g}{\partial (t^k)^2}\left(\frac{t_A^*+t_B^*}{2}; x_A, x_B\right) < 0$. But this implies that candidate B would be strictly better off both by spending more time on issue k (if possible) and less time on this issue (if possible). Since at least one of these options (increasing or decreasing the time spent on issue k) is available for candidate B, this contradicts the fact that t_B^* is a best response against t_A^*.

Assume now, by contradiction again, that (t_A^*, t_B^*) is an equilibrium with $\left(t_A^{*k}, t_B^{*k}\right) = (0,0)$. Since not speaking on issue k is a best response for candidate A against candidate B not addressing the issue, it must be the case that there exists some $\varepsilon > 0$ such that

$$\frac{\partial g}{\partial t^k}\left(\frac{t_A^{*1}+t_B^{*1}}{2}, ..., \frac{t_A^{*k-1}+t_B^{*k-1}}{2}, t^k, \frac{t_A^{*k+1}+t_B^{*k+1}}{2}, ..., \frac{t_A^{*K}+t_B^{*K}}{2}; x_A, x_B\right) < 0 \text{ for } t^k \in [0, \varepsilon].$$

But symmetrically, since not speaking on issue k is a best response for candidate B against candidate A not addressing the issue, it must be the case that there exists some $\varepsilon' > 0$ such that

$$\frac{\partial g}{\partial t^k}\left(\frac{t_A^{*1}+t_B^{*1}}{2}, ..., \frac{t_A^{*k-1}+t_B^{*k-1}}{2}, t^k, \frac{t_A^{*k+1}+t_B^{*k+1}}{2}, ..., \frac{t_A^{*K}+t_B^{*K}}{2}; x_A, x_B\right) > 0 \text{ for } t^k \in [0, \varepsilon'].$$

These conditions cannot simultaneously hold. Similarly, it cannot be the case that (t_A^*, t_B^*) is an equilibrium with $\left(t_A^{*k}, t_B^{*k}\right) = (1,1)$. ■

Proof of proposition 4. Assuming linear precision indices, for each issue k, either $\frac{\partial^2 g}{\partial (t^k)^2}(t; x_A, x_B) < 0$ for all $t \in T$, or $\frac{\partial^2 g}{\partial (t^k)^2}(t; x_A, x_B) > 0$ for all $t \in T$. Denote by DR_A (as "Decreasing Relative marginal benefit of precision for party A") the set of issues such that the former condition holds, and by IR_A (as "Increasing Relative marginal benefit of precision for party A") the set of issues such that the latter condition holds.

We prove the following two points, which will together yield the result in proposition 4.

Point 1. Party B addresses at most one issue in DR_A.

Point 2. If party A speaks on one issue in IR_A, then party B does not speak on this issue.

Proof of point 1: Given t_A^*, B chooses its best response so as to minimize π_A Under the global time constraint faced by B, an argument similar to that given in point 2 of proposition 2 shows that B addresses at most one issue for which it has increasing marginal benefit of speech (which are exactly the issues in DR_A).

Proof of point 2: Consider an issue k in IR_A. For some fixed $(t^1, ..., t^{k-1}, t^{k+1}, ..., t^K) \in [0,1]^{K-1}$ and $(x_A, x_B) \in R^2$, denote by $g^k : R \to R, t^k \to g((t^1, ..., t^{k-1}, t^k, t^{k+1}, ..., t^K); x_A, x_B)$. Since issue k is in IR_A, the function g^k is convex. Let $\underline{t^k}$ be the point in R at which the minimum is reached. If $\underline{t^k} \leq 0$, g^k increases on $[0,1]$ and whatever the time spent on the issue by party A, party B does not address issue k. Consider now cases where $\underline{t^k} > 0$. Note that g^k decreases on $]0, \underline{t^k}]$. At equilibrium, it cannot be the case that $t_A^k/2$ lies in the interval $]0, \underline{t^k}]$. Indeed, assume by contradiction that A chooses t_A^k such that $t_A^k/2$ lies in $]0, \underline{t^k}]$. In that case, a best response by party B entails that either B does not speak on issue k or it speaks so as to decrease g^k at most up to the point where $\underline{t^k}$ is reached. But then party A would be strictly better off by not addressing the issue at all. Indeed, whatever the value reached by g^k when B plays a best response against t_A with $t_A^k/2$ in $]0, \underline{t^k}]$, it will "cost" party B more time to reach the same outcome on g^k if A does not address issue k. Since time is valuable, it shows that either A does not speak on issue k, or A chooses a time t_A^k such that $t_A^k/2 \geq \underline{t^k}$. In the case where $t_A^k/2 \geq \underline{t^k}$, B faces an increasing g^k (since g^k is increasing for any t^k larger than larger $\underline{t^k}$) and B is better off not speaking. ∎

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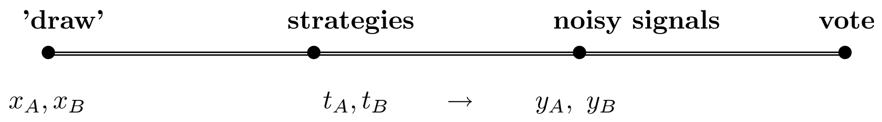

Figure 1: Campaign

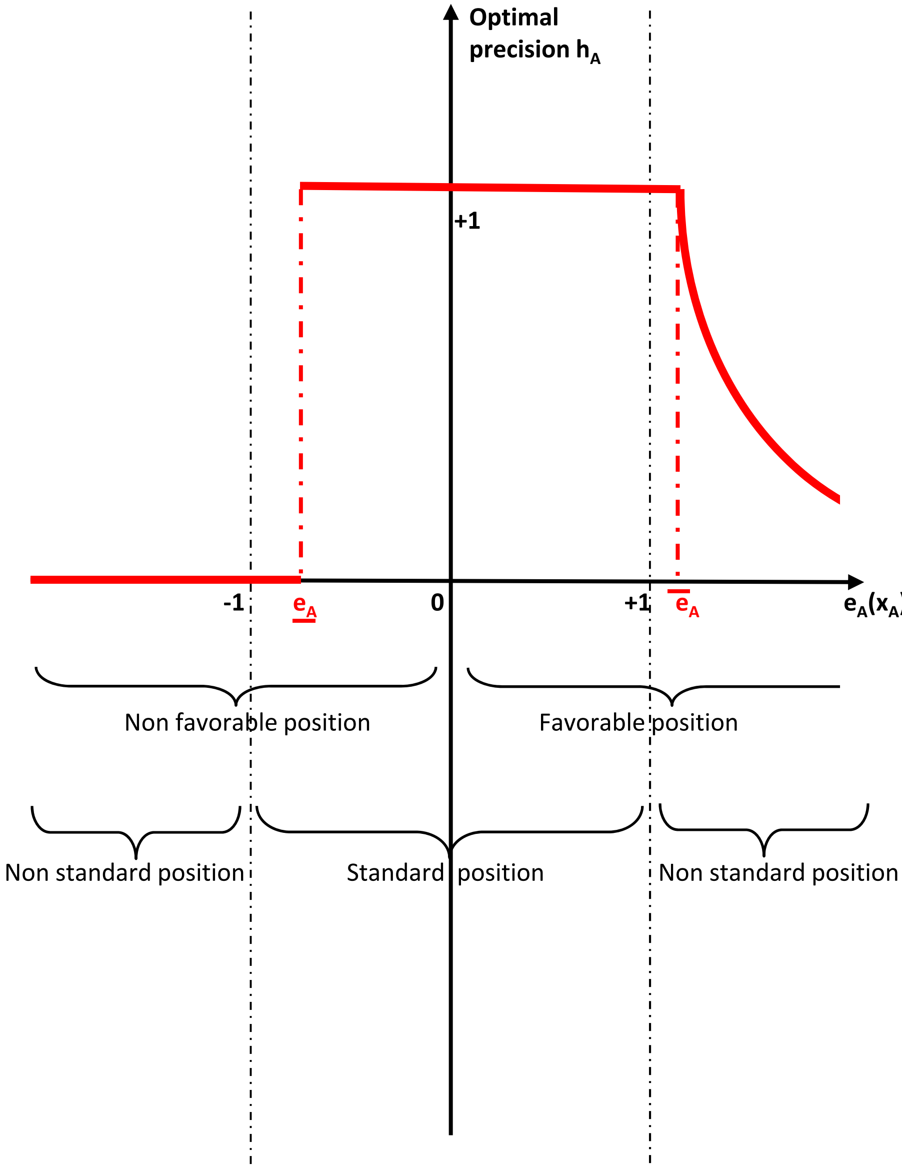

Figure 2: Optimal precision as a function of $e_A(x_A)$

Parameters : d_A=0.3, $\overline{h_A}$=1