On the Unprofitability of Buyer Groups When Sellers Compete

Doh-Shin Jeon* and Domenico Menicucci†

September 20, 2018

Abstract

We study how the formation of a buyer group affects buyer power when sellers compete and buyers operate in separate markets. Previous research (Inderst and Shaffer, 2007, and Dana, 2012) has considered a buyer group that can commit to an exclusive purchase and has found that the formation of a buyer group strictly increases buyer power unless buyers have identical preferences. In contrast, we assume that no commitment to exclusive purchases is possible. We find that the formation of a buyer group has no effect if each seller's cost function is concave. If it is strictly convex, the buyer group strictly reduces the buyers' total payoff as long as the Pareto-dominant equilibrium for sellers is played when a buyer group is formed.

Keywords: Buyer Group, Buyer Power, Competition in Nonlinear Tariffs, Discriminatory Offers, Common Agency.

JEL numbers: D4, K21, L41, L82

*Toulouse School of Economics, University of Toulouse Capitole (IDEI) and CEPR. dohshin.jeon@gmail.com

†Corresponding author: Università degli Studi di Firenze, Dipartimento di Scienze per l'Economia e l'Impresa, Via delle Pandette 9, I-50127 Firenze, Italy; phone: +39-055-2759666, email: domenico.menicucci@unifi.it

1 Introduction

Does the formation of a buyer group lead to buyer power? Do large buyers obtain size-related discounts from their suppliers? Examples of buyer groups abound in retailing (Inderst and Shaffer, 2007, Caprice and Rey, 2015), healthcare (Marvel and Yang, 2008), cable TV (Chipty and Snyder, 1999), and academic journals (Jeon and Menicucci, 2017), for example. Understanding how the formation of a buyer group or the size of a buyer affects buyer power is very important as policy makers in Europe and the U.S. are concerned about buyer power due to increasing buyer market concentration (European Commission, 1999 and OECD, 2008).[1]

There is a large body of literature addressing the above questions, both theoretically and empirically, but it provides nuanced answers such that large-buyer discounts do not arise under all circumstances but only under certain conditions. For instance, a strand of the theoretical literature considers bargaining between a monopolist seller and each of multiple buyers: Chipty and Snyder (1999) and Inderst and Wey (2007) find that the formation of a buyer group increases (reduces) the total payoff of the group members if the seller's cost function is convex (concave).[2] Normann, Ruffle, and Snyder (2007) confirm this finding in a laboratory experiment.

Our paper is more closely related to another strand of the theoretical literature that studies the formation of a buyer group when sellers compete (Inderst and Shaffer, 2007, Marvel and Yang, 2008, Dana, 2012, Chen and Li, 2013). In particular, Inderst and Shaffer (2007) and Dana (2012) consider a buyer group that can commit to an exclusive purchase and find that the formation of a buyer group never decreases the total payoff of its members and strictly increases it unless the members have identical preferences. On the empirical side, Ellison and Snyder (2010) find that supplier competition is a necessary condition for large-buyer discounts in the U.S. wholesale pharmaceutical industry.

This paper fills a gap in the theoretical literature about buyer groups by analyzing the case in which sellers producing substitute products compete, and buyers (including a buyer group) cannot commit to exclusive purchases. We find that the formation of a buyer group has no effect on the buyers' payoffs when the sellers' cost function is concave and strictly reduces them when the cost function is convex. Therefore, combining our result with those of Inderst and Shaffer (2007) and Dana (2012) indicates that the formation of a buyer group increases the total payoff of its members only if the group can precommit to limit its purchases to a subset of sellers.

[1] In the U.S., in 2000, the Federal Trade Commission organized a workshop regarding slotting allowances, a major buyer power issue in grocery retailing. See Chen (2007) for a survey of the literature regarding buyer power and antitrust policy implications.

[2] Inderst and Wey (2003) find the same result in a two-seller-two-buyer game, which results in each agent earning his Shapley value. More broadly, in cooperative game theory, there is a body of literature on the joint bargaining paradox (Harsanyi, 1977) or collusion neutrality (van den Brink, 2012), which studies when a coalition formation increases or reduces the joint payoff of its members from bargaining.

This prediction is consistent with the empirical findings of Ellison and Snyder (2010), the Competition Commission of U.K. (2008) and Sorensen (2003). Their common findings are that buyer size alone does not explain discounts, but rather, it is a buyer's credible threat to exclude certain products from purchase that explains discounts. For instance, Ellison and Snyder (2010) find that large buyers (chain drugstores) receive either very small or no statistically significant discounts relative to small buyers (independent drugstores) for off-patent antibiotics that have one or more generic substitutes available but that hospitals and health-maintenance organizations (HMOs) receive substantial discounts relative to drugstores. They explain this finding by the fact that chain drugstores and independent drugstores do not differ much in terms of substitution opportunities, whereas hospitals and HMOs can and do commit to limit their purchases to certain drugs by issuing restrictive formularies, as they can control which drugs their affiliated doctors prescribe.[3]

In our setting, sellers producing substitutes compete by offering personalized nonlinear tariffs. We do not consider competition among buyers: each buyer operates in a separate market, as in Chipty and Snyder (1999) and Inderst and Wey (2003, 2007). After showing that all equilibria are efficient, regardless of whether the buyers form a group (Proposition 1), we consider a baseline model with two symmetric sellers and two symmetric buyers. We characterize the set of equilibria that arise when there is a buyer group (Section 3) and the set of equilibria that arise without a buyer group (Section 4), and then, we compare these sets (Section 5). All equilibria can be ranked according to the payoff of each buyer, and we find that the interval of each buyer's equilibrium payoffs without a buyer group is a strict subset of the interval with a buyer group if each seller's cost function is strictly convex, whereas the two intervals are identical if the cost function is concave.

This finding suggests that a buyer group has no effect on the buyers' payoff when the sellers' cost function is concave. When the cost function is strictly convex and there is a buyer group, we select the equilibrium that is Pareto-dominant in terms of the sellers' payoffs, as in Bernheim and Whinston (1986, 1998). This equilibrium is called a "sell-out equilibrium" by Bernheim and Whinston (1998).[4] Under this equilibrium selection, we find that the formation of a buyer group strictly reduces the buyers' payoff regardless of the equilibrium played without a buyer group. Therefore, we can conclude that the

[3]Similarly, the study of the Competition Commission of the U.K. (2008) regarding the grocery industry finds significant buyer-size discounts for nonprimary- brand goods (for which the grocer can freely substitute among different suppliers) but not for primary-brand goods (for which grocers have limited substitution opportunities). Sorensen (2003) also finds that size alone cannot explain why some insurers obtain much better deal from hospitals than other insurers and that an insurer's ability to channel its patients to selected hospitals can explain why small managed-care organizations are often able to extract deeper discounts from hospitals than very large indemnity insurers.

[4]In the sell-out equilibrium, each seller j uses a sell-out strategy such that the payment of the buyer group for purchasing quantity q_j takes the form of $F_j + C_j(q_j)$, where F_j is a fixed fee and C_j is j's cost function.

formation of a buyer group does not increase the buyers' payoff, regardless of whether the sellers' cost function is convex or concave.

In Section 6, we generalize the result obtained from the baseline model with convex costs to a setting with two asymmetric sellers and $n(\geq 2)$ asymmetric buyers. The group formed by all n buyers is called the grand coalition. First, we show that as long as a seller's cost function is strictly convex, in any equilibrium with no buyer group, the buyers obtain a strictly higher total payoff than in the sell-out equilibrium with the grand coalition (Proposition 5). Second, using this result, we show that the grand coalition is the worst group structure in terms of the buyers' total payoff, as in any equilibrium of any group structure different from the grand coalition, the buyers obtain a strictly higher total payoff than in the sell-out equilibrium with the grand coalition. Finally, to address the incentive to form a subgroup, we consider an example of three symmetric buyers and show that the formation of a group of two buyers reduces their payoff and does not affect the payoff of the buyer outside of the group, compared to the payoff without any group. Hence, two buyers have no incentive to form a subgroup. We conjecture that this result holds more generally, that is, that buyers have no incentive to form any group, regardless of its size.

Below, we provide the intuition for our result in the baseline model regarding the equilibrium that is Pareto-dominant in terms of the sellers' payoffs. It is well known from Bernheim and Whinston (1998) that in the equilibrium with a buyer group, each seller j is indifferent between inducing the group to buy from both sellers (as in the equilibrium) and inducing the group to buy exclusively from seller j. The latter strategy is equivalent to the strategy, without the group, that induces both buyers to buy exclusively from seller j. However, when there is no group, seller j can also deviate by inducing only one buyer to buy exclusively from himself while inducing the other buyer to keep buying the equilibrium quantity from the rival seller. Not forming a buyer group reduces the sellers' (best) payoff (and hence increases the buyers' (worst) payoff)[5] if and only if the deviation inducing both buyers to buy exclusively is less powerful than the deviation inducing only one buyer to buy exclusively. Note that inducing both buyers to buy exclusively requires a larger increase in output of j than inducing only one buyer to buy exclusively. Therefore, when the cost function is strictly convex, because of increasing marginal costs, if the deviation inducing only one buyer to buy exclusively is not profitable, the deviation inducing both buyers to buy exclusively is not profitable either. However, the reverse does not hold, and if a seller is indifferent between no deviation and the deviation inducing both buyers to buy exclusively, the deviation inducing only one buyer to buy exclusively becomes profitable since the marginal cost increases less due to smaller output expansion. A similar reasoning implies that when the cost function is concave, if the deviation inducing both buyers to

[5]Since all equilibria are efficient, a decrease in the sellers' payoff implies an increase in the buyers' payoff.

buy exclusively is not profitable, the deviation inducing only one buyer to buy exclusively is not profitable. Therefore, the formation of a buyer group reduces the buyers' (worst) payoff if the cost function is strictly convex, whereas it does not affect the payoff if the cost function is concave.

Our analysis of the symmetric setting in Sections 3 and 4 also has some technical contribution. We show that any symmetric equilibrium can be replicated by an equilibrium obtained by restricting each seller to offer a tariff with only two pairs of quantity and price. Under this approach, characterizing the whole set of the equilibrium payoffs becomes a relatively simple problem, which we solve depending on whether the buyers formed a group.[6]

In Section 7, we apply our insights to a situation in which one seller's entry is endogenous and the buyers decide whether to form a group before the entry decision.[7] We assume that the sellers' cost functions are strictly convex. The entrant has to incur a fixed cost of entry, which is randomly drawn from a (commonly known) distribution, as in Aghion and Bolton (1987), Innes and Sexton (1994) and Chen and Shaffer (2014). Upon entry, both sellers simultaneously offer nonlinear tariffs, and we assume that they play the Pareto-dominant equilibrium. Therefore, when the buyers decide whether to form a group, they face a trade-off: forming a buyer group increases the probability of entry but reduces the total payoff of the buyers conditional on entry. In particular, conditional on the group formation, the private entry decision coincides with the socially optimal one, and as in the sell-out equilibrium, each firm's profit is equal to its social marginal contribution. This result implies that no formation of a group leads to a socially suboptimal entry. The existing literature regarding naked exclusion explains suboptimal entry by the incumbent's taking advantage of coordination failure among buyers (Rasmusen, Ramseyer and Wiley 1991, Segal and Whinston, 2000, Fumagalli and Motta, 2006, 2008, Chen and Shaffer, 2014). Whereas the literature typically assumes that the incumbent makes offers to buyers before the entry, Fumagalli and Motta (2008) show that the coordination failure survives even if both sellers make simultaneous offers. However, in these papers, buyers have no reason not to form a group (at least among those operating in separate markets), as this would remove the coordination failure. Our application shows that buyers may choose to not form a group even if this leads to a suboptimal entry.

[6]We follow this approach partly because the sell-out equilibrium, which exists with a buyer group, does not exist without a group if the costs are strictly convex or strictly concave. Hence, even if we want to focus on the Pareto-dominant equilibrium, our approach is useful for the analysis of no group.

[7]For instance, state-wide (or multi-state) large pharmaceutical purchasing alliances (Ellison and Snyder, 2010) can affect the entry decisions of generic drug producers, or a buying alliance among large chains of supermarkets (Caprice and Rey, 2015) can have an impact on entry of national brand manufacturers.

1.1 Literature review

The literature has used various methods to generate size-related discounts. Katz (1987), Scheffman and Spiller (1992) and Inderst and Valletti (2011) model buyer power as downstream firms' ability to integrate backwards by paying a fixed cost. The formation of a buyer group makes this outside option stronger because the group can share the fixed cost among its members, which in turn allows the group to obtain a larger discount from a seller. The papers that do not consider such size-related outside options can be regrouped into two different categories. On the one hand, there are papers studying buyer groups among competing downstream firms. Hence, the formation of a buyer group not only allows them to joins force as buyers but also eliminates competition between them in the downstream market: see von Ungern-Sternberg (1996), Dobson and Waterson (1997), Chen (2003), Erutku (2005) and Gaudin (2017). Caprice and Rey (2015) is an exception because they consider a buyer group among retailers that maintain downstream competition. On the other hand, there are papers that focus on "pure" buyer power in the sense that group members only interact on the buying side, as they do not compete. This second branch of literature can be further distinguished depending on whether they consider a monopoly seller (Chipty and Snyder, 1999, and Inderst and Wey, 2007) or competing sellers (Inderst and Shaffer, 2007, Marvel and Yang, 2008, Dana, 2012, Chen and Li, 2013). As we consider upstream competition but no downstream competition, our paper is close to the last subcategory, in particular, to Inderst and Shaffer (2007) and Dana (2012), who assume that a buyer group can commit to an exclusive purchase. We fill the gap in the literature by considering the case in which buyers (including a buyer group) cannot commit to exclusive purchase. However, in terms of prediction based on the curvature of the sellers' cost function, our paper is also related to Chipty and Snyder (1999) and Inderst and Wey (2003, 2007). Namely, they predict that the formation of a buyer group increases (decreases) the total payoff of the group members if the seller's cost function is convex (concave). This result occurs because buyers are assumed to have some bargaining power, and the incremental cost to serve the group is smaller (larger) than the sum of the incremental cost to serve each buyer if the sellers' costs are convex (concave).[8] In contrast, we predict that the formation of a buyer group reduces (does not affect) the total payoff of the group members for strictly convex (concave) cost functions.[9]

[8] In our setting, using the Shapley value to compute the buyers' payoffs with and without a buyer group yields the same results as in Inderst and Wey (2003).

[9] The literature regarding buyer groups (or buyer power) is vast. Snyder (1996, 1998) applies the idea of Rotemberg and Saloner (1986) to explain why large buyers obtain discounts. Namely, he considers a model of tacit collusion among sellers and shows that sellers charge lower prices to large buyers while charging monopoly prices to small buyers. Ruffle (2013) experimentally tests the theory and finds support for it. Chae and Heidhues (2004) (DeGraba, 2005) generate large-buyer discounts from buyers' risk aversion (sellers' risk aversion). Loertscher and Marx (2017) study the competitive effects of an upstream merger in a market with buyer power.

To the best of our knowledge, we are the first to analyze the formation of a buyer group in a framework that extends common agency (Bernheim and Whinston, 1986, 1998) to multiple buyers. For the setting without a buyer group, Prat and Rustichini (2003) analyze a general setup with multiple sellers and multiple buyers in which buyers' utility functions are concave and sellers' cost functions are convex. These authors allow sellers to use more complex tariffs than ours: seller j can make buyer i's payment depend on the whole vector of i's purchases, that is, on what buyer i buys from seller j and on what i buys from the other sellers. They prove the existence of an efficient equilibrium but do not specify the equilibrium strategies and do not compare the case of a buyer group with the case of no group. We consider tariffs such that the payment of each buyer i to each seller j depends only on the quantity that buyer i buys from seller j,[10] and compare the case of a buyer group with the case of no group.[11]

Our paper is distantly related to our companion paper (Jeon and Menicucci, 2017), which studies the formation of a library consortium in the market for academic journals. Each seller (i.e., publisher) is a monopolist of his journals and sells a bundle of his electronic academic journals at a personalized price(s). However, we assume that each seller's marginal cost is zero such that without a library consortium, each library's market can be studied in isolation. Competition among sellers occurs because of the budget constraint of each buyer. We find that depending on the sign and degree of correlation between each member library's preferences, building a library consortium can increase or reduce the total payoff of the buyers.[12]

2 The model and a preliminary result

There are two sellers (A and B) and two buyers (1 and 2). Let $q_j^i \geq 0$ represent the quantity of the product that buyer i $(= 1, 2)$ buys from seller j $(= A, B)$. For each buyer i, the gross utility from consuming (q_A^i, q_B^i) is given by $U^i(q_A^i, q_B^i)$, with U^i strictly increasing and strictly concave in (q_A^i, q_B^i). Precisely, we let $U_j^i = \partial U^i / \partial q_j^i$, $U_{jk}^i = \partial U^i / \partial q_j^i \partial q_k^i$ and

[10] There are two reasons for restricting attention to such tariffs. First, a seller j may not observe the quantity that a buyer buys from the other seller. Second, market-share contracts that provide price rebates conditional on buying a large share of quantity from the seller making the offer are often prohibited by antitrust authorities when practiced by dominant firms. For instance, on May 13, 2009, the European Commission imposed a fine of 1.06 billion euros on Intel for such behavior.

[11] Jeon and Menicucci (2012) extend the common agency to competition between portfolios in the presence of a buyer's slot constraint and provide conditions to make the "sell-out equilibrium" the unique equilibrium. Contrary to what happens under a slot constraint, Jeon and Menicucci (2006) show that in the presence of the buyer's budget constraint, the well-known result in the common agency literature that competition between sellers achieves the outcome that maximizes all parties' joint payoff fails to hold.

[12] Therefore, in terms of the results, the companion paper is situated between Inderst and Shaffer (2007) and Dana (2012), on the one hand, and the current paper, on the other hand.

assume that for each buyer i, (i) $U_j^i > 0$, but $U_j^i \to 0$ as $q_j^i \to +\infty$; (ii) the Hessian matrix of U^i is negative-definite, and hence, U^i is strictly concave; and (iii) $U_{AB}^i < 0$, and hence, the goods are strict substitutes. For each seller j, the cost of serving the buyers is $C_j(q_j^1 + q_j^2)$. We assume that $C_j(0) = 0$ and C_j is strictly increasing, but C_j can be convex or concave. However, if C_j is concave, then we assume that U^1, U^2 are sufficiently concave and that social welfare $U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) - C_A(q_A^1 + q_A^2) - C_B(q_B^1 + q_B^2)$ is a concave function.

Each seller offers a nonlinear tariff to each buyer (or to a buyer group). In particular, we consider tariffs such that buyer i's payment to seller j depends on only the quantity that buyer i purchases from seller j and not on the quantity that she purchases from the other seller. When there is no buyer group, we allow for each seller to offer personalized tariffs: the tariff offered by seller j to buyer i is denoted by T_j^i (with $T_j^i(0) = 0$) and can be different from the tariff seller j offers to buyer h ($\neq i$). After seeing the tariffs (T_A^i, T_B^i), buyer i chooses (q_A^i, q_B^i) in order to maximize $U^i(q_A^i, q_B^i) - T_A^i(q_A^i) - T_B^i(q_B^i)$. However, for some tariffs (T_A^i, T_B^i), this maximization problem has no solution, that is, no optimal purchase for buyer i exists.[13] In order to avoid this problem, we restrict each seller to offer a finite set of price/quantity pairs, that is, for $i = 1, 2$ and for $j = A, B$, we require that the tariff T_j^i satisfies the following property: there exists a finite (possibly very large) set Q_j^i of quantities such that $T_j^i(q)$ is very high if $q > 0$ and $q \notin Q_j^i$. Therefore, for buyer i, it is unprofitable to buy quantity q from seller j if $q \notin Q_j^i$, and in practice, the quantities in Q_j^i are the only quantities that buyer i may buy from seller j. If a tariff satisfies this property, then we say it is a *finite tariff*. When T_A^i and T_B^i are finite, the problem

$$\max_{q_A^i \geq 0, q_B^i \geq 0} U^i(q_A^i, q_B^i) - T_A^i(q_A^i) - T_B^i(q_B^i) \tag{1}$$

necessarily has a solution because it is equivalent to $\max_{q_A^i \in Q_A^i, q_B^i \in Q_B^i} U^i(q_A^i, q_B^i) - T_A^i(q_A^i) - T_B^i(q_B^i)$, in which the feasible set is a finite set.

When the buyers form a buyer group, the sellers compete to serve the group. Let G denote the buyer group, q_j^G, the quantity G buys from seller j, and T_j^G, the nonlinear tariff that seller j offers to G (with $T_j^G(0) = 0$). We define $U^G(q_A^G, q_B^G)$ as follows:

$$U^G(q_A^G, q_B^G) \equiv \max_{q_A^1, q_B^1, q_A^2, q_B^2} U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) \tag{2}$$

$$\text{subject to} \qquad q_A^1 + q_A^2 = q_A^G, \qquad q_B^1 + q_B^2 = q_B^G. \tag{3}$$

Thus, $U^G(q_A^G, q_B^G)$ is the group's gross utility from buying $\left(q_A^G, q_B^G\right)$, as it results from the optimal allocation of $\left(q_A^G, q_B^G\right)$ between the two buyers.[14] Hence, after seeing the tariffs

[13] For instance, there exists no optimal (q_A^i, q_B^i) for buyer i if T_A^i and T_B^i are such that $T_A^i(q) = 0$ for $q \in [0, 3)$ and $T_A^i(q)$ is very high if $q \geq 3$; $T_B^i(q) = 0$ for each $q \geq 0$. Since U^i is strictly increasing in q_A^i and q_B^i, buyer i would like to select q_A^i equal to the greatest number in $[0, 3)$, which does not exist, and q_B^i equal to the greatest number in $[0, +\infty)$, which does not exist. Hence, no optimal (q_A^i, q_B^i) exists.

[14] In principle, the constraints in (3) should be written as $q_A^1 + q_A^2 \leq q_A^G$, $q_B^1 + q_B^2 \leq q_B^G$, but since U^1, U^2 are strictly increasing, equality holds in the optimum.

$\left(T_A^G, T_B^G\right)$, the group chooses $\left(q_A^G, q_B^G\right)$ in order to maximize $U^G(q_A^G, q_B^G) - T_A^G(q_A^G) - T_B^G(q_B^G)$. In the case of a buyer group, we restrict, as in the case of no buyer group, seller j to offer a finite tariff T_j^G: there exists a finite set Q_j^G of quantities such that $T_j^G(q)$ is very high if $q > 0$ and $q \notin Q_j^G$. As a consequence, G considers buying from seller j only a quantity in Q_j^G. This result implies that there exists a solution to the problem

$$\max_{q_A^G \geq 0, q_B^G \geq 0} U^G(q_A^G, q_B^G) - T_A^G(q_A^G) - T_B^G(q_B^G). \tag{4}$$

We consider a game with the following timing:

- Stage zero: The buyers decide whether they will form a group.
- Stage one: When there is no buyer group, each seller j $(= A, B)$ simultaneously chooses a finite tariff T_j^i, for $i = 1, 2$. When the buyer group is formed, each seller j $(= A, B)$ simultaneously chooses a finite tariff T_j^G.
- Stage two: Each buyer i, or the buyer group, makes purchase decisions.

As we are mainly interested in comparing the outcome of competition with a buyer group with the one without a buyer group, we mainly consider the game that begins at stage one, both for the case in which a group has been formed and for the case in which no group has been formed. To this game, we apply the notion of subgame-perfect Nash equilibrium, which implies that in stage two, without a group, buyer i purchases (q_A^i, q_B^i) that is a solution to problem (1), for $i = 1, 2$; when buyers have formed a group, G purchases (q_A^G, q_B^G) that is a solution to problem (4). Since subgame-perfect Nash equilibrium is the only equilibrium notion we use, in the rest of the paper, we will simply call it *equilibrium*.

Define $\mathbf{q}^* \equiv (q_A^{1*}, q_B^{1*}, q_A^{2*}, q_B^{2*})$ as the unique allocation vector that maximizes social welfare,[15] and V_{AB}^G as the social welfare in the first-best allocation $\mathbf{q}^*$:

$$\mathbf{q}^* \equiv \arg \max_{q_A^1, q_B^1, q_A^2, q_B^2} U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) - C_A(q_A^1 + q_A^2) - C_B(q_B^1 + q_B^2), \tag{5}$$

$$V_{AB}^G \equiv U^1(q_A^{1*}, q_B^{1*}) + U^2(q_A^{2*}, q_B^{2*}) - C_A(q_A^{1*} + q_A^{2*}) - C_B(q_B^{1*} + q_B^{2*}). \tag{6}$$

From Proposition 2 on, we assume that each buyer buys a positive quantity from each seller in the first best allocation: $q_j^{i*} > 0$ for $i = 1, 2$ and $j = A, B$.[16]

We say that an equilibrium is *efficient* if the equilibrium allocation is $\mathbf{q}^*$. Our first result proves that each equilibrium is efficient both in the case of a buyer group and in the case of no group.

[15] Our assumptions imply that for each maximization problem below, a maximizer exists and is unique since the objective function is strictly concave.

[16] This assumption is not needed in Proposition 1.

Proposition 1 *(efficiency) Any equilibrium is efficient regardless of whether the buyers form a group.*

This result implies that the quantities the buyers consume in equilibrium do not depend on whether they form a group. In other words, the total payoff of the buyers is completely determined by the payments they make to the sellers. Hence, if the formation of a buyer group increases (reduces) the total payoff of the buyers, it is because the buyers pay less (more) for the same total quantity. This reasoning creates a natural connection to the literature that studies buyer power defined as size-related discounts.

The result for the case of a buyer group is a consequence of the following property (see the proof of Proposition 1): in any equilibrium with a buyer group, the equilibrium quantities q_A^{Ge}, q_B^{Ge} satisfy

$$q_A^{Ge} \in \arg\max_{q_A^G} \left(U^G(q_A^G, q_B^{Ge}) - C_A(q_A^G)\right), \qquad q_B^{Ge} \in \arg\max_{q_B^G} \left(U^G(q_A^{Ge}, q_B^G) - C_B(q_B^G)\right). \quad (7)$$

This result means that q_A^{Ge} maximizes the sum of G's payoff and firm A's profit, given $q_B^G = q_B^{Ge}$, and an analogous property holds for q_B^{Ge}. Therefore,

$$\begin{cases} U_A^G(q_A^{Ge}, q_B^{Ge}) \le C_A'(q_A^{Ge}), & \text{with equality if } q_A^{Ge} > 0; \\ U_B^G(q_A^{Ge}, q_B^{Ge}) \le C_B'(q_B^{Ge}), & \text{with equality if } q_B^{Ge} > 0. \end{cases} \quad (8)$$

Since U^1 and U^2 are concave, it follows that U^G is concave and that social welfare $U^G(q_A^G, q_B^G) - C_A(q_A^G) - C_B(q_B^G)$ is a concave function of $\left(q_A^G, q_B^G\right)$, as we assumed above. The conditions in (8) are the necessary first-order conditions for maximization of social welfare, and moreover, they are sufficient since social welfare is concave. Hence, $\left(q_A^{Ge}, q_B^{Ge}\right)$ maximizes social welfare, that is, $q_A^{Ge} = q_A^{1*} + q_A^{2*}$, $q_B^{Ge} = q_B^{1*} + q_B^{2*}$.[17]

A similar principle applies also to the case of no buyer group: in the proof of Proposition 1, we show that in any equilibrium, the equilibrium quantities $(q_A^{1e}, q_B^{1e}, q_A^{2e}, q_B^{2e})$ are such that

$$\begin{cases} (q_A^{1e}, q_A^{2e}) \in \arg\max_{q_A^1, q_A^2} \left(U^1(q_A^1, q_B^{1e}) + U^2(q_A^2, q_B^{2e}) - C_A(q_A^1 + q_A^2)\right); \\ (q_B^{1e}, q_B^{2e}) \in \arg\max_{q_B^1, q_B^2} \left(U^1(q_A^{1e}, q_B^1) + U^2(q_A^{2e}, q_B^2) - C_B(q_B^1 + q_B^2)\right). \end{cases} \quad (9)$$

This result means that (q_A^{1e}, q_A^{2e}) maximizes the sum of the buyers' total payoff and firm A's profit, given $(q_B^1, q_B^2) = (q_B^{1e}, q_B^{2e})$, and an analogous property holds for (q_B^{1e}, q_B^{2e}). Since social welfare $U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) - C_A(q_A^1 + q_A^2) - C_B(q_B^1 + q_B^2)$ is a concave function, it follows from (9) that $(q_A^{1e}, q_B^{1e}, q_A^{2e}, q_B^{2e})$ maximizes social welfare, that is, $(q_A^{1e}, q_B^{1e}, q_A^{2e}, q_B^{2e}) = (q_A^{1*}, q_B^{1*}, q_A^{2*}, q_B^{2*})$.

[17] Property (7) is established also by Proposition 1 in O'Brien and Shaffer (1997). Moreover, O'Brien and Shaffer (1997) exhibit a setting in which an inefficient equilibrium exists but that setting has discontinuous cost functions for sellers (because of a fixed cost that each seller bears for any positive quantity produced). Therefore, the social welfare function is not concave.

In the next three sections, we consider a setting in which both buyers and sellers are symmetric, that is, $U^1(\cdot) = U^2(\cdot) \equiv U(\cdot)$, $U(q_A, q_B) = U(q_B, q_A)$, and $C_A(\cdot) = C_B(\cdot) \equiv C(\cdot)$. In this setting, $q_A^{1*} = q_B^{1*} = q_A^{2*} = q_B^{2*} \equiv q^*$ such that

$$U_j(q^*, q^*) = C'(2q^*) \qquad \text{for} \quad j = A, B \tag{10}$$

and

$$U^G(q_A^G, q_B^G) = 2U(\frac{1}{2}q_A^G, \frac{1}{2}q_B^G) \tag{11}$$

We call this the *symmetric setting.*

In the symmetric setting, we will illustrate our results through the following example:

$$U(q_A, q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2} \quad \text{and} \quad C(q) = \frac{1}{2}q^2.$$

The example implies $U_A(q_A, q_B) = \frac{1}{4q_A^{1/2}(q_A^{1/2}+q_B^{1/2})^{1/2}}$, hence q^* solves

$$\frac{1}{4q^{1/2}(q^{1/2} + q^{1/2})^{1/2}} = 2q$$

Therefore, $q^* = \frac{1}{4}$, $U(q^*, q^*) = 1$, $C(2q^*) = \frac{1}{8}$, and $V_{AB}^G = \frac{7}{4}$. From (11), we have

$$U^G(q_A^G, q_B^G) = 2^{3/4}((q_A^G)^{1/2} + (q_B^G)^{1/2})^{1/2}.$$

3 Equilibria with a buyer group

In this section, we consider the symmetric setting and study the case in which buyers have formed a group G. Given the symmetric setting, we focus on symmetric equilibria such that each seller offers to G the same finite tariff $T_A^G = T_B^G \equiv T^G$. We determine the set of the equilibrium payoffs of the buyer group, and in particular, we describe how the minimum and maximum of this set are determined by the constraints that equilibrium imposes on the group's payoff. We conclude the section by illustrating our results in the context of the example that we introduced at the end of Section 2.

Here, we provide some intuitive explanation of how we characterize the set of the equilibrium payoffs of the buyer group. Notice first that in any equilibrium, each seller sells the efficient quantity $2q^*$ to the group by charging a price $T^G(2q^*)$ that makes the group indifferent between trading with both sellers and trading only with the other seller. Let $(\hat{q}, T^G(\hat{q}))$ represent the quantity the group buys and the price it pays when trading only with a single seller. We show that all the equilibrium outcomes can be obtained by restricting attention to a simple class of tariffs in which each seller offers only two contracts, $(2q^*, T^G(2q^*))$ and $(\hat{q}, T^G(\hat{q}))$. Obviously, the two contracts constitute an equilibrium as long as no seller has an incentive to deviate. The best deviation of a seller differs depending on the quantity that the deviating seller induces the group to buy from the nondeviating seller, which can be equal to zero or $\hat{q}$. Consider an extreme case in which the payoff of G is very low, say, approximately zero. Then, the sum of the payoff of G and that of a seller j is approximately half of the total surplus V_{AB}^G. Hence, the deviation of seller j that induces G to buy exclusively from seller j is profitable, as the surplus G and seller j generate by trading without the other seller is greater than $\frac{1}{2}V_{AB}^G$ (because the products are substitutes). This reasoning helps us to identify a lower bound for G's payoff. Consider the opposite extreme case in which the payoff of G is sufficiently high, say, equal to V_{AB}^G. Then, the profit of seller j is 0, and a profitable deviation induces G to buy $\hat{q}$ from seller $k \neq j$ because it yields to seller j his marginal contribution to social surplus given $q_k^G = \hat{q}$, which cannot be negative. This reasoning allows us to determine an upper bound for G's payoff. In the rest of the section, we develop these arguments in more detail.

Since each equilibrium is efficient (Proposition 1), G buys quantity $2q^*$ [see (10)] from each seller in any equilibrium, and G's payoff is $u^G \equiv U^G(2q^*, 2q^*) - 2T^G(2q^*)$. In fact, u^G is also equal to $\max_{q\geq 0}[U^G(0,q) - T^G(q)]$, that is, the highest payoff G can obtain by trading only with one seller. This equality holds because if $u^G < \max_{q\geq 0}[U^G(0,q) - T^G(q)]$, then buying quantity $2q^*$ from each seller is not G's best choice. If $u^G > \max_{q\geq 0}[U^G(0,q) - T^G(q)]$, then seller A (to fix the ideas) for instance can increase his profit as follows: he makes a take-it-or-leave-it offer to G with quantity $2q^*$ and payment $T^G(2q^*) + \varepsilon$ (with $\varepsilon > 0$ and small), and if G buys $2q^*$ from each seller, then his payoff is $U^G(2q^*, 2q^*) - 2T^G(2q^*) - \varepsilon = u^G - \varepsilon$, which is still greater than the highest payoff G can make by trading only with seller B.

For future reference, we let

$$\hat{q} \in \arg\max_{q\geq 0}[U^G(0,q) - T^G(q)], \qquad \hat{t}^G \equiv T^G(\hat{q}), \qquad t^{G*} \equiv T^G(2q^*). \tag{12}$$

Hence,

$$u^G = U^G(2q^*, 2q^*) - 2t^{G*} = U^G(0, \hat{q}) - \hat{t}^G. \tag{13}$$

Note that $\hat{q} \geq 2q^*$ should hold since $2q^* \in \arg\max_{q\geq 0}[U^G(2q^*, q) - T^G(q)]$, and the products are substitutes. In fact, we can prove that $\hat{q} > 2q^*$ in each equilibrium: see the proof of Lemma 1.

Lemma 1 below proves that each equilibrium is characterized by $(\hat{q}, \hat{t}^G)$, that is, G's best choice when trading only with a seller, such that there exists an equivalent equilibrium (in terms of outcome) in which each seller offers only two contracts, $(2q^*, t^{G*})$ and $(\hat{q}, \hat{t}^G)$.[18] Note that once $(\hat{q}, \hat{t}^G)$ is determined, t^{G*} is determined from (13). In this way, we can characterize all the equilibrium outcomes by restricting our attention to a simple class of tariffs.

[18]This means that each seller chooses a tariff T^G such that $T^G(q)$ is very high for each $q > 0$, $q \neq 2q^*$, $q \neq \hat{q}$.

Lemma 1 *Consider the symmetric setting with a buyer group. Given any symmetric equilibrium in which each seller offers the same tariff T^G, there exists an equilibrium in which each seller offers only two contracts, $(2q^*, t^{G*})$ and $(\hat{q}, \hat{t}^G)$, and of which the outcome is the same as that of the original equilibrium.*

Therefore, from now on, we consider equilibria in which each seller offers only the two contracts $(2q^*, t^{G*})$ and $(\hat{q}, \hat{t}^G)$. Suppose that seller B offers the contracts $(2q^*, t^{G*})$ and $(\hat{q}, \hat{t}^G)$ but that seller A considers deviating with a take-it-or-leave-it offer (q_A, t_A) to G. Then, G accepts A's offer if and only if $U^G(q_A, q_B) - t_A - T^G(q_B) \geq u^G$ for at least one $q_B \in \{0, 2q^*, \hat{q}\}$. Therefore, $U^G(q_A, q_B) - T^G(q_B) - u^G$ is the highest payment that seller A can obtain from G, and given $q_B \in \{0, 2q^*, \hat{q}\}$, seller A chooses $q_A \in \arg\max_{x \geq 0}[U^G(x, q_B) - T^G(q_B) - u^G - C(x)]$. For this reason, for each $b \geq 0$, we define

$$f^G(b) \equiv \max_{x \geq 0} \left(U^G(x, b) - C(x)\right). \tag{14}$$

Conditional on seller A inducing G to buy $q_B = b$ from seller B, the deviation generates a joint payoff of $f^G(b) - T^G(b)$ for G and seller A, and $f^G(b) - T^G(b) - u^G$ is the profit of seller A.

The next proposition identifies the set of equilibrium payoffs for G.

Proposition 2 *(Equilibria with a Buyer Group) Consider the symmetric setting and suppose that the buyers formed a group. There exists a symmetric equilibrium in which the group's payoff is u^G if and only if*

$$2f^G(0) - V_{AB}^G \leq u^G \leq V_{AB}^G - 2 \lim_{b \to +\infty} [f^G(b) - U^G(0, b)]. \tag{15}$$

Precisely, given u^G that satisfies (15), an equilibrium in which the group's payoff is u^G is such that
(i) in stage one, each seller offers the two contracts $(2q^*, t^{G*})$ and $(\hat{q}, \hat{t}^G)$ with $t^{G*} = \frac{1}{2}(U^G(2q^*, 2q^*) - u^G)$, $\hat{t}^G = U^G(0, \hat{q}) - u^G$ and $\hat{q}$ such that $f^G(\hat{q}) - U^G(0, \hat{q}) \leq \frac{1}{2}(V_{AB}^G - u^G)$;
(ii) in stage two, given any finite tariffs T_A^G, T_B^G, the group buys (q_A^G, q_B^G) that solves (4).

We below explain how the left-hand and right-hand sides in (15) are obtained. First, we notice that $f^G(0) = \max_{x \geq 0} \left(U^G(x, 0) - C(x)\right)$ is the maximal social welfare when G trades only with seller A, and in any equilibrium, the sum of G's payoff u^G and A's profit π^A is not less than $f^G(0)$. This is so because if $u^G + \pi^A < f^G(0)$, then seller A can deviate by offering to G a suitable trade in which G buys only from A – in what follows, we call this the *deviation with* $q_B^G = 0$ – and their joint payoff increases to $f^G(0)$. Therefore, in each equilibrium, the profit of seller B, π^B, is not greater than the maximal social welfare V_{AB}^G (when G trades with both sellers, defined in (6)) minus $f^G(0)$, that is, $\pi^B \leq V_{AB}^G - f^G(0)$. Likewise, $\pi^A \leq V_{AB}^G - f^G(0)$, and these upper bounds for π^A, π^B yield a lower bound for u^G equal to $V_{AB}^G - \left(V_{AB}^G - f^G(0)\right) - \left(V_{AB}^G - f^G(0)\right)$, which is the left-hand side in (15). Precisely, the lower bound $u^G = 2f^G(0) - V_{AB}^G$ is achieved if $\hat{q} = \arg\max_{x\geq 0}[U^G(0,x) - C(x)]$, $\hat{t}^G = C(\hat{q}) + V_{AB}^G - f^G(0)$, and $t^{G*} = C(2q^*) + V_{AB}^G - f^G(0)$. The agents' payoffs in this equilibrium are the same as in the sell-out equilibrium (Bernheim and Whinston, 1998).

In fact, there is another possible deviation for seller A, which induces G to buy quantity $\hat{q}$ ($> 2q^*$) from seller B and q from A, with q suitably chosen – in the following, we call this the *deviation with* $q_B^G = \hat{q}$. This deviation generates a total payoff of $f^G(\hat{q}) - \hat{t}^G$ for G and seller A; hence, $u^G + \pi^A \geq f^G(\hat{q}) - \hat{t}^G$ must hold in any equilibrium, which implies that $\pi^B \leq V_{AB}^G - (f^G(\hat{q}) - \hat{t}^G)$ must hold. However, the latter is necessarily a weaker restriction on π^B than $\pi^B \leq V_{AB}^G - f^G(0)$; otherwise, we would conclude that in any equilibrium, u^G is strictly greater than $2f^G(0) - V_{AB}^G$, which contradicts the existence of the sell-out equilibrium. Therefore, the lower bound for u^G is determined by the deviation with $q_B^G = 0$.

In regard to the right-hand side in (15), the deviation of seller A with $q_B^G = \hat{q}$ yields A the profit $f^G(\hat{q}) - \hat{t}^G - u^G = f^G(\hat{q}) - U^G(0,\hat{q})$, where the equality is from $\hat{t}^G = U^G(0,\hat{q}) - u^G$ in (13); hence, $\pi^A \geq f^G(\hat{q}) - U^G(0,\hat{q})$ and $\pi^B \geq f^G(\hat{q}) - U^G(0,\hat{q})$. Since $\pi^A + \pi^B + u^G = V_{AB}^G$, we obtain $u^G \leq V_{AB}^G - 2[f^G(\hat{q}) - U^G(0,\hat{q})]$, an upper bound for u^G. Moreover, substitute goods imply that $f^G(\hat{q}) - U^G(0,\hat{q})$ is decreasing in $\hat{q}$: as $\hat{q}$ (bought from seller B) increases, the goods offered by seller A are less valuable to G, and A's profit from the deviation is reduced.[19] Hence, for this upper bound, the supremum is obtained by letting $\hat{q}$ tend to infinity. In fact, seller A may deviate also with $q_B^G = 0$ and earn the profit $f^G(0) - u^G$, but this profit is less than $f^G(\hat{q}) - U^G(0,\hat{q})$ when $u^G = V_{AB}^G - 2[f^G(\hat{q}) - U^G(0,\hat{q})]$, that is,

$$f^G(0) - V_{AB}^G + 2[f^G(\hat{q}) - U^G(0,\hat{q})] < f^G(\hat{q}) - U^G(0,\hat{q}). \tag{16}$$

Therefore, the upper bound for u^G is determined by the deviation with $q_B^G = \hat{q}$, as $\hat{q} \to +\infty$.[20]

Remark It is interesting to notice that even if we consider competition between differentiated products, it may result in the outcome of zero profit for the sellers. Namely, in some cases, we find that

$$\lim_{b\to+\infty}[f^G(b) - U^G(0,b)] = 0 \tag{17}$$

[19]Formally, $U^G(x,b) - U^G(0,b) - C(x) = \int_0^x (U_A^G(z,b) - C'(z))dz$ is decreasing in b, given that goods are substitutes. Hence, $\max_{x\geq 0}[U^G(x,b) - U^G(0,b) - C(x)]$ is decreasing in b.

[20]It is simple to verify that (16) is equivalent to $f^G(\hat{q}) + f^G(0) < V_{AB}^G + U^G(0,\hat{q})$, which coincides with $\max_{x,y}[U^G(x,\hat{q}) + U^G(0,y) - C(x) - C(y)] < \max_{x,y}[U^G(x,y) + U^G(0,\hat{q}) - C(x) - C(y)]$. The latter inequality holds as long as $\hat{q}$ is greater than $\arg\max_y[U^G(0,y) - C(y)]$, since then, strict substitutes imply $U^G(x,\hat{q}) - U^G(0,\hat{q}) < U^G(x,y) - U^G(0,y)$, that is, $U^G(x,\hat{q}) + U^G(0,y) < U^G(x,y) + U^G(0,\hat{q})$.

and then, for each $\varepsilon > 0$, there exists an equilibrium such that each seller's profit is less than ε, and G's payoff is greater than $V_{AB}^G - 2\varepsilon$. For instance, if

$$\lim_{b\to+\infty} U_A^G(0,b) < \inf_{x\geq 0} C'(x) \tag{18}$$

then the effect of substitute goods is sufficiently strong to make the optimal x in the maximization problem in (14) equal to 0 for large values of b. Therefore, $f^G(b) = U^G(0,b)$ for each large b, and (17) holds; hence, the right-hand side in (15) is equal to V_{AB}^G.

Example Consider the example of $U(q_A, q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2}$ and $C(q) = \frac{1}{2}q^2$, with $q^* = \frac{1}{4}$, $U(q^*, q^*) = 1$, $V_{AB}^G = \frac{7}{4} = 1.75$. Since $U^G(q_A^G, q_B^G) = 2^{3/4}((q_A^G)^{1/2} + (q_B^G)^{1/2})^{1/2}$, we have $f^G(0) = \max_{x\geq 0}(2^{3/4}x^{1/4} - \frac{1}{2}x^2) = 1.300245$; hence, the lower bound for u^G is $2f^G(0) - V_{AB}^G = 0.8505$. Regarding the upper bound, we find

$$f^G(b)-U^G(0,b) = \max_{x\geq 0}[2^{3/4}(x^{1/2}+b^{1/2})^{1/2}-\frac{1}{2}x^2-2^{3/4}b^{1/4}] = \max_{x\geq 0}[\frac{2^{3/4}x^{1/2}}{(x^{1/2}+b^{1/2})^{1/2}+b^{1/4}}-\frac{1}{2}x^2]$$

For each fixed $x \geq 0$, we have that $\frac{2^{3/4}x^{1/2}}{(x^{1/2}+b^{1/2})^{1/2}+b^{1/4}}$ tends to 0 as $b \to +\infty$, and we can use this to prove that $\lim_{b\to+\infty}\left(f^G(b) - U^G(0,b)\right) = 0$. Therefore, the interval for u^G is

$$[0.8505, 1.75).^{21}$$

For each u^G in this interval, we find t^{G*} from $2 - 2t^{G*} = u^G$, that is, $t^{G*} = 1 - u^G/2$, and $\pi^G = \left[V_{AB}^G - u^G\right]/2 = 7/8 - u^G/2$. For the contract $(\hat{q}, \hat{t}^G)$, we need to determine $\hat{q}$ such that $f^G(\hat{q}) - U^G(0,\hat{q}) \leq \pi^G$ in order to make unprofitable a seller's deviation that induces G to buy $\hat{q}$ from the other seller.[22] Such a $\hat{q}$ exists since $\pi^G > 0$ and $\lim_{b\to+\infty}\left(f^G(b) - U^G(0,b)\right) = 0$. For instance, if $u^G = 1.2$, then $t^{G*} = 0.4$, $\pi^G = 0.275$, and a suitable $\hat{q}$ is $\hat{q} = 4$, since $f^G(4) - U^G(0,4) = 0.2703 < 0.275$.[23] Hence, $\hat{t}^G = U^G(0,4) - 1.2 = 1.1784$ and the pair of strategies in which each seller offers the two contracts $(2q^*, t^{G*}) = (\frac{1}{2}, 0.4)$ and $(\hat{q}, \hat{t}^G) = (4, 1.1784)$ is an equilibrium in which G's payoff is equal to 1.2.

4 Equilibria without a buyer group

In this section, we keep considering the symmetric setting and study the case in which buyers have not formed any group. We still focus on symmetric equilibria, in which each seller offers the same finite tariff T to each buyer.

[21] In Proposition 2, we have written $u^G \leq V_{AB}^G - 2\lim_{b\to+\infty}[f^G(b) - U^G(0,b)]$, with a weak inequality, because for a fixed b, $V_{AB}^G - 2[f^G(b) - U^G(0,b)]$ is the highest possible payoff for G. However, in the example, $\lim_{b\to+\infty}\left(V_{AB}^G - 2[f^G(b) - U^G(0,b)]\right) = 1.75$ is greater than $V_{AB}^G - 2[f^G(b) - U^G(0,b)]$ for each b. Hence, 1.75 is not an equilibrium value for u^G.

[22] The deviation of a seller that induces G to not buy from the other seller yields a profit of $f^G(0) - u^G = 1.300245 - u^G$, which is less than $\frac{7}{8} - \frac{1}{2}u^G$ since $u^G \geq 0.8505$.

[23] Any $\hat{q}$ greater than 4 is also suitable since $f^G(\hat{q}) - U^G(0,\hat{q})$ is decreasing in $\hat{q}$.

Before we characterize the set of equilibria, we note an important difference between the case of a buyer group and the case of no group. With a buyer group, every quantity that G buys from either seller is split equally between the buyers: see (11). In contrast, when there is no group, seller A can make different offers to different buyers; we call them *discriminatory offers*. These offers may induce buyer 1 to buy from seller B a quantity that is different from the quantity that buyer 2 buys from seller B. At an extreme, discriminatory offers of seller A may induce buyer 1 to buy exclusively from seller A and buyer 2 to buy a positive quantity from seller B. In equilibrium, no seller makes any discriminatory offer, but the possibility to deviate with discriminatory offers is the main difference with respect to the setting in which the buyers formed a group. We prove below that this affects the set of equilibria without a buyer group if the cost functions are strictly convex but not if the cost functions are concave (which includes the case of linear costs).

Since each equilibrium is efficient (see Proposition 1), we know that each buyer buys quantity q^* [defined in (10)] from each seller in equilibrium, and the payoff of each buyer is $u \equiv U(q^*, q^*) - 2T(q^*)$. In fact, we can argue like in the previous section to prove that u is also equal to $\max_{q\geq 0}[U(0,q) - T(q)]$, the highest payoff a buyer can make when trading only with one seller. For future reference, we let

$$\tilde{q} \in \arg\max_{q\geq 0}[U(0,q) - T(q)], \qquad \tilde{t} \equiv T(\tilde{q}), \qquad t^* \equiv T(q^*). \tag{19}$$

Hence, we have

$$u = U(q^*, q^*) - 2t^* = U(0, \tilde{q}) - \tilde{t} \tag{20}$$

Note that $\tilde{q} \geq q^*$ should hold since $q^* \in \arg\max_{q\geq 0}[U(q^*, q) - T(q)]$, and the products are substitutes. In fact, we can prove that $\tilde{q} > q^*$ must hold in any equilibrium. As in the case of a buyer group, it is useful to show that without loss of generality, we can restrict our attention to equilibria in which each seller offers only two contracts to each buyer. This allows us to focus on a class of simple tariffs.

Lemma 2 *Consider the symmetric setting without a buyer group. Given any symmetric equilibrium in which each seller offers the same tariff T to each buyer, there exists an equilibrium in which each seller offers only two contracts, (q^*, t^*) and $(\tilde{q}, \tilde{t})$, to each buyer and that generates the same outcome as the original equilibrium.*

Suppose now that seller B offers two contracts (q^*, t^*) and $(\tilde{q}, \tilde{t})$ to each buyer, but seller A considers a deviation that consists of making a take-it-or-leave-it offer (q_A^i, t_A^i) to buyer i, for $i = 1, 2$. Then, we can argue as in the case of a buyer group to show that given that seller A induces buyer 1 to buy $q_B^1 = b^1 \in \{0, q^*, \tilde{q}\}$ and buyer 2 to buy $q_B^2 = b^2 \in \{0, q^*, \tilde{q}\}$ from seller B, seller A chooses (q_A^1, q_A^2) in $\arg\max_{x^1\geq 0, x^2\geq 0}[U(x^1, b^1) + U(x^2, b^2) - C(x^1 + x^2)]$. For this reason, for each $b^1 \geq 0$, $b^2 \geq 0$, we define

$$f(b^1, b^2) = \max_{x^1\geq 0, x^2\geq 0}[U(x^1, b^1) + U(x^2, b^2) - C(x^1 + x^2)] \tag{21}$$

and $f(b^1,b^2)-T(b^1)-T(b^2)-2u$ is the profit of seller A given that he induces the buyers to buy $(q_B^1,q_B^2)=(b^1,b^2)$ from seller B. Note that f is symmetric in its arguments, that is, $f(b^1,b^2)=f(b^2,b^1)$, and it satisfies the following property.

Lemma 3 *Given $\underline{b},\bar{b}$ such that $0\leq\underline{b}<\bar{b}$, inequality (22) holds if C is strictly convex, whereas inequality (23) holds if C is concave:*

$$f(\bar{b},\bar{b})-f(\bar{b},\underline{b}) \quad < \quad f(\underline{b},\bar{b})-f(\underline{b},\underline{b}); \tag{22}$$

$$f(\bar{b},\bar{b})-f(\bar{b},\underline{b}) \quad \geq \quad f(\underline{b},\bar{b})-f(\underline{b},\underline{b}). \tag{23}$$

In order to explain Lemma 3, we first focus on the inequality (22), which is established in the case of strictly convex C. When b^2 increases from $\underline{b}$ to $\bar{b}$, the value of f increases, but the magnitude of the increase depends on whether $b^1=\bar{b}$ or $b^1=\underline{b}$. Precisely, (22) reveals that the increase is less if $b^1=\bar{b}$ than if $b^1=\underline{b}$, that is, the return from increasing b^2 is smaller the greater b^1 is. This result is determined by the marginal utility of good B for buyer 2, as we now illustrate.

Regarding $f(\underline{b},\underline{b})$ on the right-hand side of (22), notice that when $(b^1,b^2)=(\underline{b},\underline{b})$, the optimal x^1 and x^2 in the maximization problem (21) have the same value, denoted by $\underline{x}$. Regarding $f(\bar{b},\underline{b})$ on the left-hand side of (22), notice that when $(b^1,b^2)=(\bar{b},\underline{b})$, the marginal utility of good A for buyer 1 is lower than when $(b^1,b^2)=(\underline{b},\underline{b})$, given substitutes. This finding implies that A sells a lower quantity to buyer 1, and this output contraction together with C strictly convex implies that the marginal cost for the choice of x^2 becomes lower; hence, the optimal x^1,x^2, denoted by $\bar{x}^1,\bar{x}^2$, are such that $\bar{x}^1<\underline{x}<\bar{x}^2$. From $\underline{x}<\bar{x}^2$, it follows that the marginal utility of good B for buyer 2 is smaller when $(b^1,b^2)=(\bar{b},\underline{b})$ than when $(b^1,b^2)=(\underline{b},\underline{b})$; hence, (22) is proven by applying the Envelope Theorem.

A similar argument applies when C is concave because, then, the output contraction relative to x^1 together with C concave implies that the marginal cost for the choice of x^2 becomes (at least weakly) higher, which results in $\bar{x}^1<\underline{x}$, $\bar{x}^2\leq\underline{x}$. Therefore, the marginal utility of good B for buyer 2 when $(b^1,b^2)=(\bar{b},\underline{b})$ is at least as large as when $(b^1,b^2)=(\underline{b},\underline{b})$, which implies (23).

Lemma 4 and 5 below rely on Lemma 3 to prove that some deviation is less powerful or more powerful than other deviations, depending on the curvature of C. It is also useful to know that (22) is equivalent to

$$f(\bar{b},\bar{b})-f(\underline{b},\underline{b})<2\left[f(\underline{b},\bar{b})-f(\underline{b},\underline{b})\right],$$

and (23) is equivalent to

$$f(\bar{b},\bar{b})-f(\underline{b},\underline{b})\geq 2\left[f(\underline{b},\bar{b})-f(\underline{b},\underline{b})\right].$$

Given u, each seller's profit is $2t^*-C(2q^*)$, which is equal to $f(q^*,q^*)-U(q^*,q^*)-u$ [from (20) and (21)]. In equilibrium, this profit should be higher than any deviation profit. Let (IC (b^1, b^2)) represent the incentive constraint that makes it unprofitable for seller j $(= A, B)$ to deviate by inducing the buyers to buy $(q_k^1, q_k^2) = (b^1, b^2)$ from seller k $(= A, B)$ with $j \neq k$, $b^1 \in \{0, q^*, \tilde{q}\}$, $b^2 \in \{0, q^*, \tilde{q}\}$ and $(b^1, b^2) \neq (q^*, q^*)$:

$$\text{(IC } \left(b^1, b^2\right)) \qquad f(q^*, q^*) - U(q^*, q^*) - u \geq f(b^1, b^2) - T(b^1) - T(b^2) - 2u.$$

We start by considering two deviations that determine a lower bound of u. First, the deviation that induces both buyers to buy exclusively from seller j yields a profit of $f(0,0) - 2u$. Therefore, $f(q^*, q^*) - U(q^*, q^*) - u \geq f(0,0) - 2u$ must be satisfied, which is equivalent to

$$\text{(IC } (0,0)) \qquad f(0,0) - f(q^*, q^*) + U(q^*, q^*) \leq u. \tag{24}$$

Second, the deviation that induces one buyer to buy exclusively from j and the other buyer to buy q^* from k yields j a profit of $f(0, q^*) - 2u - t^*$. Therefore, $f(q^*, q^*) - U(q^*, q^*) - u \geq f(0, q^*) - 2u - t^*$ must be satisfied, which is equivalent to

$$\text{(IC } (0, q^*)) \qquad 2f(0, q^*) - 2f(q^*, q^*) + U(q^*, q^*) \leq u. \tag{25}$$

The second deviation involves a discriminatory offer, and we now inquire when this strategy has a bite in the sense that it is stronger than the deviation inducing both buyers to buy exclusively from j. For this purpose, we use Lemma 3 with $\underline{b} = 0 < \overline{b} = q^*$:

Lemma 4 *Consider the symmetric setting without a buyer group.*

(i) Suppose that C is strictly convex. Then, if (IC $(0, q^*)$) is satisfied, (IC $(0,0)$) is satisfied as well: $f(q^*, q^*) - f(0,0) < 2\,[f(0, q^*) - f(0,0)]$

(ii) Suppose that C is concave. Then, if (IC $(0,0)$) is satisfied, (IC $(0, q^*)$) is satisfied as well: $f(q^*, q^*) - f(0,0) \geq 2\,[f(0, q^*) - f(0,0)]$.

Let us consider seller A's deviations. Both deviations we are focusing on require seller A to produce more output than without the deviations, but the deviation that induces both buyers to make exclusive purchases from A requires a greater increase in output than the other deviation. When costs are convex (concave), the marginal cost is increasing (decreasing), and hence, inducing a first buyer to buy exclusively involves a smaller (larger) increase in cost per output than inducing a second buyer to buy exclusively. Therefore, when costs are strictly convex, inducing both buyers to make an exclusive purchase is not profitable if inducing only one buyer to make an exclusive purchase is not profitable. Symmetrically, when costs are concave, inducing only one buyer to make an exclusive purchase is not profitable if inducing both buyers to make an exclusive purchase is not profitable.

Lemma 4 implies the following lower bound on u, depending on whether C is convex or concave:

$$\begin{array}{ll} 2f(0, q^*) - 2f(q^*, q^*) + U(q^*, q^*) \leq u & \text{if } C \text{ is strictly convex;} \\ f(0,0) - f(q^*, q^*) + U(q^*, q^*) \leq u & \text{if } C \text{ is concave.} \end{array} \tag{26}$$

Now, we consider two different deviations that generate an upper bound for u. First, the deviation that induces buyer 1 to buy q^* and buyer 2 to buy $\tilde{q}(> q^*)$ from B yields seller A the profit $f(q^*, \tilde{q}) - t^* - \tilde{t} - 2u$. Therefore, $f(q^*, q^*) - U(q^*, q^*) - u \geq f(q^*, \tilde{q}) - t^* - \tilde{t} - 2u$ must be satisfied, which is equivalent to

$$\text{(IC } (q^*, \tilde{q})) \qquad u \leq 2f(q^*, q^*) - U(q^*, q^*) - 2[f(q^*, \tilde{q}) - U(0, \tilde{q})].$$

Second, the deviation that induces each buyer to buy $\tilde{q}$ from B yields seller A profit $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u$. Therefore, $f(q^*, q^*) - U(q^*, q^*) - u \geq f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u$ must be satisfied, which is equivalent to

$$\text{(IC } (\tilde{q}, \tilde{q})) \qquad u \leq f(q^*, q^*) - U(q^*, q^*) - [f(\tilde{q}, \tilde{q}) - 2U(0, \tilde{q})].$$

In order to inquire which deviation is stronger, we again use Lemma 3, this time with $\underline{b} = q^* < \bar{b} = \tilde{q}$:

Lemma 5 *Consider the symmetric setting without a buyer group.*

(i) Suppose that C is strictly convex. Then, if (IC $(q^*, \tilde{q})$) is satisfied, (IC $(\tilde{q}, \tilde{q})$) is satisfied as well: $f(\tilde{q}, \tilde{q}) - f(q^*, q^*) < 2\,[f(q^*, \tilde{q}) - f(q^*, q^*)]$.

(ii) Suppose that C is concave. Then, if (IC $(\tilde{q}, \tilde{q})$) is satisfied, (IC $(q^*, \tilde{q})$) is satisfied as well: $f(\tilde{q}, \tilde{q}) - f(q^*, q^*) \geq 2\,[f(q^*, \tilde{q}) - f(q^*, q^*)]$.

Both deviations we are considering require seller A to produce less output than without the deviations, and the deviation that induces each buyer to buy quantity $\tilde{q}$ from seller B is the deviation involving a higher reduction in output than the other deviation. When costs are convex (concave), the marginal savings in cost from output contraction decreases (increases) as the amount of output reduction increases. Therefore, when costs are convex, the deviation inducing $(q_B^1, q_B^2) = (\tilde{q}, \tilde{q})$ generates lower savings per reduced output than the deviation inducing $(q_B^1, q_B^2) = (q^*, \tilde{q})$, which implies that if the latter deviation is unprofitable, then the former deviation is unprofitable as well. A symmetric argument applies when costs are concave, which implies that if the deviation inducing $(q_B^1, q_B^2) = (\tilde{q}, \tilde{q})$ is unprofitable, the deviation inducing $(q_B^1, q_B^2) = (q^*, \tilde{q})$ is unprofitable.

Lemma 5 implies the following upper bound on u, depending on whether C is convex or concave:

$$\begin{array}{ll} u \leq 2f(q^*, q^*) - U(q^*, q^*) - 2[f(q^*, \tilde{q}) - U(0, \tilde{q})] & \text{if } C \text{ is strictly convex;} \\ u \leq f(q^*, q^*) - U(q^*, q^*) - [f(\tilde{q}, \tilde{q}) - 2U(0, \tilde{q})] & \text{if } C \text{ is concave.} \end{array} \tag{27}$$

From (26) and (27), we derive an interval of values for u that depends on the curvature of C. In fact, before we can conclude that this interval is the set of equilibrium values for each buyer's payoff, we need to consider also the deviation that induces buyer 1 to buy exclusively from seller A and buyer 2 to buy quantity $\tilde{q}$ from seller B. Given $(t^*, \hat{t})$ satisfying (20), such deviation is unprofitable if and only if

$$f(0,\tilde{q}) - U(0,\tilde{q}) \leq f(q^*,q^*) - U(q^*,q^*) \tag{28}$$

The proof of the next proposition shows that each of the two right-hand sides in (27) is increasing in $\tilde{q}$ and that (28) is satisfied if $\tilde{q}$ is sufficiently large.

Proposition 3 *(Equilibrium without a buyer group) Consider the symmetric setting without a buyer group.*

(a) Suppose that C is strictly convex. There exists a symmetric equilibrium in which each buyer's payoff is u if and only if

$$U(q^*,q^*) - 2f(q^*,q^*) + 2f(0,q^*) \leq u \leq 2f(q^*,q^*) - U(q^*,q^*) - 2\lim_{b\to+\infty}[f(q^*,b) - U(0,b)] \tag{29}$$

Precisely, given u that satisfies (29), an equilibrium in which each buyer's payoff is u is such that

(i) in stage one, each seller offers the contracts (q^*,t^*) and $(\tilde{q},\tilde{t})$ with $t^* = \frac{1}{2}(U(q^*,q^*)-u)$, $\tilde{t} = U(0,\tilde{q}) - u$ and $\tilde{q}$, satisfying (28) and $f(q^*,\tilde{q}) - U(0,\tilde{q}) \leq f(q^*,q^*) - \frac{1}{2}(u + U(q^*,q^*))$;

(ii) in stage two, given any finite tariffs T_A^i, T_B^i, buyer i buys (q_A^i, q_B^i) that solves (1), for $i = 1, 2$.

(b) Suppose that C is concave. There exists a symmetric equilibrium in which each buyer's payoff is u if and only if

$$f(0,0) - f(q^*,q^*) + U(q^*,q^*) \leq u \leq f(q^*,q^*) - U(q^*,q^*) - \lim_{b\to+\infty}[f(b,b) - 2U(0,b)] \tag{30}$$

Precisely, given u that satisfies (30), an equilibrium in which each buyer's payoff is u is such that

(i) in stage one, each seller offers the contracts (q^*,t^*) and $(\tilde{q},\tilde{t})$ with $t^* = \frac{1}{2}(U(q^*,q^*)-u)$, $\tilde{t} = U(0,\tilde{q}) - u$ and $\tilde{q}$, satisfying (28) and $f(\tilde{q},\tilde{q}) - 2U(0,\tilde{q}) \leq f(q^*,q^*) - U(q^*,q^*) - u$;

(ii) in stage two, given any finite tariffs T_A^i, T_B^i, buyer i buys (q_A^i, q_B^i) that solves (1), for $i = 1, 2$.

Example Consider again the example of $U(q_A,q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2}$ and $C(q) = \frac{1}{2}q^2$, with $U(q^*,q^*) = 1$, $C(2q^*) = \frac{1}{8}$. Since C is strictly convex, the interval of values for u is (29). Since $f(q^*,q^*) = \frac{15}{8}$ and $f(0,q^*) = \max_{x\geq 0, y\geq 0}[x^{1/4} + (y^{1/2} + \frac{1}{2})^{1/2} - \frac{1}{2}(x+y)^2] = 1.5913$, the lower bound for u is 0.4326.

Regarding the upper bound for u, we find that

$$\begin{aligned} f(q^*,b) - U(0,b) &= \max_{x\geq 0, y\geq 0}[(x^{1/2} + \frac{1}{2})^{1/2} + (y^{1/2} + b^{1/2})^{1/2} - \frac{1}{2}(x+y)^2 - b^{1/4}] \\ &= \max_{x\geq 0, y\geq 0}[(x^{1/2} + \frac{1}{2})^{1/2} - \frac{1}{2}(x+y)^2 + \frac{y^{1/2}}{(y^{1/2} + b^{1/2})^{1/2} + b^{1/4}}] \end{aligned} \tag{31}$$

and $\lim_{b\to+\infty}(f(q^*,b)-U(0,b)) = \max_{x\geq 0}[(x^{1/2}+\frac{1}{2})^{1/2}-\frac{1}{2}x^2] = 0.98442$.

Hence, the interval for u is

$$[0.4326, 0.78116)$$

5 Comparison: a buyer group vs. no group

In this section, we still consider the symmetric setting and compare the equilibria with a buyer group and those without a buyer group. In the case of a buyer group, if u^G is the group's payoff, then each buyer's payoff is $u^G/2$; hence, the bounds in (15) for the case of a buyer group can be expressed in terms of u, as in (32) below, where we exploit the equalities $f^G(0) = f(0,0)$ and $\lim_{b\to+\infty}[f^G(b)-U^G(0,b)] = \lim_{b\to+\infty}[f(b,b)-2U(0,b)]$:

$$f(0,0)-\frac{1}{2}V_{AB}^G \leq u \leq \frac{1}{2}V_{AB}^G - \lim_{b\to+\infty}[f(b,b)-2U(0,b)]. \tag{32}$$

Since $\frac{1}{2}V_{AB}^G = U(q^*,q^*)-C(2q^*) = f(q^*,q^*)-U(q^*,q^*)$, we see that the interval of payoffs in (32) is the same as the interval without a buyer group that we previously identified in (30). Therefore, when C is concave (which includes the case of linear C), the set of equilibrium outcomes is unaffected by the formation of the buyer group. The situation is different when C is strictly convex, as the next proposition establishes.

Proposition 4 *Consider the symmetric setting.*

(a) If C is concave, then the interval of equilibrium payoffs for each buyer is the same regardless of whether the buyers form a group.

(b) If C is strictly convex, then the interval of equilibrium payoffs for each buyer without a buyer group is a strict subset of the interval of equilibrium payoffs for each buyer with a buyer group. Formally,

$$f(0,0)-\frac{1}{2}V_{AB}^G < U(q^*,q^*)-2f(q^*,q^*)+2f(0,q^*) \tag{33}$$

$$2f(q^*,q^*)-U(q^*,q^*)-2\lim_{b\to+\infty}[f(q^*,b)-U(0,b)] < \frac{1}{2}V_{AB}^G - \lim_{b\to+\infty}[f(b,b)-2U(0,b)] \tag{34}$$

The formation of the buyer group has no effect on the interval of the equilibrium payoffs for the buyers when C is concave, but if C is strictly convex, from the buyers' point of view, the payoff of the worst equilibrium without a buyer group is higher than the payoff of the worst equilibrium with a buyer group, whereas the payoff of the best equilibrium without a buyer group is lower than the payoff of the best equilibrium with a buyer group.

In order to see why these differences arise, recall from Section 3 that the lower bound for u^G is determined by seller A's deviation with $q_B^G = 0$, which is equivalent to the deviation of seller A with $q_B^1 = 0, q_B^2 = 0$ when there is no buyer group. The latter deviation implies $2u + \pi^A \geq f(0,0)$ in any equilibrium; hence, $\pi^B \leq V_{AB}^G - f(0,0)$. However, without a buyer group, seller A can also deviate with $q_B^1 = 0$, $q_B^2 = q^*$. This deviation generates a total payoff for buyers and seller A equal to $f(0,q^*) - t^*$; hence, $2u + \pi^A \geq f(0,q^*) - t^*$. When $u = f(0,0) - \frac{1}{2}V_{AB}^G$, we find that $f(0,0) \geq f(0,q^*) - t^*$ if C is concave, but $f(0,q^*) - t^* > f(0,0)$ if C is strictly convex. Therefore, for concave C, the lower bound in (30) is determined by the deviation that induces both buyers to make an exclusive purchase from the deviating seller, as with a buyer group. Conversely, when C is strictly convex, the inequality $f(0,q^*) - t^* > f(0,0)$ implies for π^B (and for π^A) that $\pi^B \leq V_{AB}^G - (f(0,q^*) - t^*) < V_{AB}^G - f(0,0)$; therefore, the lower bound in (29) is greater than the lower bound in (32), as (33) establishes.

In Section 3, we have also explained that with a buyer group, the best equilibrium for buyers is determined by seller A's deviation with $q_B^G = \hat{q}$ (for $\hat{q} \to +\infty$). This is analogous to seller A's deviation with $q_B^1 = \tilde{q}$, $q_B^2 = \tilde{q}$ (for $\tilde{q} \to +\infty$) when there is no buyer group, which yields seller A the profit $f(\tilde{q},\tilde{q}) - 2\tilde{t} - 2u$. However, without a buyer group, seller A can also deviate with $q_B^1 = \tilde{q}$, $q_B^2 = q^*$, which yields A the profit $f(\tilde{q},q^*) - \tilde{t} - t^* - 2u$. When $u = \frac{1}{2}V_{AB}^G - [f(\tilde{q},\tilde{q}) - 2U(0,\tilde{q})]$, we have that $f(\tilde{q},\tilde{q}) - 2\tilde{t} - 2u \geq f(\tilde{q},q^*) - \tilde{t} - t^* - 2u$ if C is concave, but $f(\tilde{q},\tilde{q}) - 2\tilde{t} - 2u < f(\tilde{q},q^*) - \tilde{t} - t^* - 2u$ if C is strictly convex. Therefore, for concave C, the upper bound in (30) is determined by the deviation that induces each buyer to buy from the nondeviating seller a quantity higher than the equilibrium quantity, just like in the deviation that identifies the upper bound in (32) with a buyer group. Conversely, when C is strictly convex, the inequality $f(\tilde{q},\tilde{q}) - 2\tilde{t} - 2u < f(\tilde{q},q^*) - \tilde{t} - t^* - 2u$ increases the lower bound for π^A and hence decreases the upper bound for u, as (34) establishes.

As we remarked in Section 4, the difference between a buyer group and no group lies in the possibility for sellers to use discriminatory offers when there is no group. But when C is concave, it has no effect as the deviations that determine the bounds for the equilibrium payoffs do not involve discriminatory offers. Conversely, when C is strictly convex, discriminatory offers have multiple effects: (i) they intensify competition between sellers at their most favorable equilibrium, thus increasing u in the worst equilibrium for buyers, and (ii) they make more effective the deviation which determines the sellers' lowest equilibrium profit, thus reducing the buyers' highest payoff. As a result, without a buyer group, the bounds for u are tighter. Notice that if discriminatory offers were infeasible, then the interval of equilibrium payoffs for each buyer would be given by (32), that is (30), even when C is strictly convex.

Example Consider the example of $U(q_A, q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2}$ and $C(q) = \frac{1}{2}q^2$. With a buyer group, $u^G/2$ belongs to $[0.4252, 0.875)$, whereas without a buyer group, u belongs to $[0.4326, 0.78116)$. Consistent with Proposition 4(b), given that C is strictly convex, the formation of the buyer group reduces the lower bound of each buyer's payoff and increases the upper bound. In terms of the percentage with respect to the payoff without the buyer group, the buyer group reduces the lower bound by 1.7 percent and increases the upper bound by 12 percent.

When C is concave, the formation of a buyer group has no effect on the set of equilibrium payoffs, and hence, we can say that the formation of a buyer group has no effect no matter the equilibrium selection. When C is strictly convex, one natural and widely applied selection rule among multiple equilibria is the one that focuses on Pareto-optimal equilibria for the players. In the game that starts at stage one (after either decision of the buyers about the group formation at stage zero), the players are sellers A and B, and both in the case of a buyer group and in the case of no group, there exists a unique Pareto-optimal equilibrium for the sellers. This is the equilibrium that corresponds to the lowest u^G in (15) and to the lowest u in (29). Under the Pareto-optimal equilibrium selection rule, Proposition 4 implies that forming a group is harmful to the buyers because of (33). In fact, when C is strictly convex, this prediction holds as long as the Pareto-dominant equilibrium is selected when a buyer group is formed, regardless of the equilibrium selected without a buyer group.

6 More than two buyers

In this section, we allow for an arbitrary number $n \geq 2$ of asymmetric buyers and two asymmetric sellers. When $n \geq 3$, we may consider groups that include at least two buyers but less than n. Then, each group structure can be seen as a partition $S_1, ..., S_m$, with $m \leq n$, of the set $\{1, ..., n\}$ of buyers such that for $h = 1, ..., m$, group h consists of the buyers in the set S_h. In particular, if $m = 1$, all buyers belong to the same single group, which we call the *grand coalition*; the opposite extreme is obtained if $m = n$, as then no group is formed and each buyer acts individually. For group h, the utility function $U^{Gh}(q_A^{Gh}, q_B^{Gh})$ is defined as follows:

$$U^{Gh}(q_A^{Gh}, q_B^{Gh}) = \max_{\{q_A^i, q_B^i\}_{i \in S_h}} \sum_{i \in S_h} U^i(q_A^i, q_B^i) \tag{35}$$

$$\text{s.t.} \quad \sum_{i \in S_h} q_A^i = q_A^{Gh} \quad \text{and} \quad \sum_{i \in S_h} q_B^i = q_B^{Gh} \tag{36}$$

as in (2)-(3) for a group of two buyers. We notice that analyzing competition given the group structure described by the partition $S_1, ..., S_m$ is equivalent to studying a setting with m buyers having utility functions $U^{G1}, ..., U^{Gm}$. In the next subsections, we consider strictly convex cost functions and focus on the Pareto-dominant equilibrium for the sellers, which is the worst equilibrium in terms of the aggregate payoff of the buyers.

6.1 The grand coalition vs. no coalition

The next proposition extends the result in Proposition 4(b) that the grand coalition reduces the worst payoff of the buyers with respect to no coalition. Precisely, when the grand coalition is formed, we consider the buyers' total payoff in the sell-out equilibrium, which is equal to $V_A^G + V_B^G - V_{AB}^G$, with

$$\begin{aligned} V_j^G &= \max_{q_j^1,...,q_j^n} \left(\sum_{i=1}^{n} U^i(q_j^i, 0) - C_j(q_j^1 + ... + q_j^n) \right) \quad \text{for } j = A, B \\ V_{AB}^G &= \max_{q_A^1,...,q_A^n, q_B^1,...,q_B^n} \left(\sum_{i=1}^{n} U^i(q_A^i, q_B^i) - C_A(q_A^1 + ... + q_A^n) - C_B(q_B^1 + ... + q_B^n) \right). \end{aligned}$$

We show that every equilibrium with no coalition yields to the buyers a strictly higher total payoff than $V_A^G + V_B^G - V_{AB}^G$, that is, $u^1 + ... + u^n > V_A^G + V_B^G - V_{AB}^G$ for any vector of the buyers' equilibrium utilities $(u^1, ..., u^n)$ when no coalition is formed.

Proposition 5 *Suppose that there are two sellers and $n \geq 2$ buyers; both the sellers and the buyers can be asymmetric. Suppose that the cost function of at least one seller is strictly convex. Then, in any equilibrium with no coalition (i.e., when $m = n$), the buyers obtain a strictly higher total payoff than in the sell-out equilibrium with the grand coalition (i.e., when $m = 1$).*

The proof of Proposition 5 relies on the following generalization of (22) in Lemma 3 to $n \geq 2$ asymmetric buyers and two asymmetric sellers, which is proven in the proof of Lemma 3 in Appendix. Let us consider seller A, for instance. Given $b^1 \geq 0$, $b^2 \geq 0$, ..., $b^n \geq 0$, $f_A(b^1, b^2, ..., b^n)$ is defined as $\max_{x^1,x^2,...,x^n}[U^1(x^1, b^1) + U^2(x^2, b^2) + ... + U^n(x^n, b^n) - C_A(x^1 + x^2 + ... + x^n)]$. Then, given $\underline{b}^1, ..., \underline{b}^n$ and $\bar{b}^1, ..., \bar{b}^n$ such that $\underline{b}^h < \bar{b}^h$ for $h = 1, ..., n$, we prove that

$$f_A(\bar{b}^1, ..., \bar{b}^n) - f_A(\underline{b}^1, ..., \underline{b}^n) < \sum_{i=1}^{n} \left[f_A(\underline{b}^1, ..., \bar{b}^i, ..., \underline{b}^n) - f_A(\underline{b}^1, ..., \underline{b}^n) \right]$$

if C_A is strictly convex. This generalizes the inequality $f(\bar{b}, \bar{b}) - f(\underline{b}, \underline{b}) < 2\left[f(\underline{b}, \bar{b}) - f(\underline{b}, \underline{b})\right]$ in Section 4.

6.2 The grand coalition vs. any other coalition structure

We can also use Proposition 5 to compare the grand coalition to any another coalition structure. Precisely, we have mentioned above that a setting with the group structure described by $S_1, ..., S_m$ is equivalent to a setting with m buyers having utility functions $U^{G1}, ..., U^{Gm}$ defined in (35)-(36). Then, Proposition 5 implies that $u^{G1} + ... + u^{Gm} >$

$V_A^G + V_B^G - V_{AB}^G$ for any vector of the groups' equilibrium utilities $\left(u^{G1}, ..., u^{Gm}\right)$,[24] that is, the grand coalition is the worst group structure for the buyers (in aggregate) given strictly convex costs. Hence, the next proposition is an immediate corollary of Proposition 5.

Proposition 6 *Suppose that there are two sellers and $n \geq 2$ buyers; both the sellers and the buyers can be asymmetric. Suppose that the cost function of at least one seller is strictly convex. Then, given any coalition structure $S_1, ..., S_m$ with $m \geq 2$, in any equilibrium, the buyers obtain a strictly higher total payoff than in the sell-out equilibrium with the grand coalition.*

This proposition establishes that any group structure is preferred by the buyers to the grand coalition but gives no indication about the profitability of coalitions smaller than the grand coalition relative to no coalition. Inquiring about this issue requires first to characterize the worst equilibrium for the buyers given an arbitrary number m of asymmetric buyers with utility functions $U^{G1}, ..., U^{Gm}$ and then to perform a comparison among equilibria. This general analysis is beyond the scope of this paper, as the first step requires addressing a number of incentive constraints that increases exponentially as m increases, and in particular, we find it difficult to deal with deviations that induce a buyer to buy from the nondeviating seller more than the equilibrium quantity and another buyer to buy from the nondeviating seller less than the equilibrium quantity. For this reason, in the next subsection, we examine this issue in a specific setting with three buyers and obtain a clear-cut result.

6.3 An example with $n = 3$: the grand coalition vs. subcoalition vs. no coalition

Consider the setting with $U^i(q_A, q_B) = (q_A^{1/2} + q_B^{1/2})^{1/2}$ for $i = 1, 2, 3$ and $C_j(q) = \frac{1}{2}q^2$ for $j = A, B$; the utility function and the cost function have been introduced in Section 2. Then, we can prove that

- when no coalition is formed, the worst equilibrium for the buyers is such that each buyer's payoff is 0.4107;
- when the coalition that includes only buyers 1 and 2 is formed,[25] the worst equilibrium for the buyers is such that the coalition's payoff is 0.8118, which is smaller than $2 \cdot 0.4107$, and buyer 3's payoff is 0.4107;
- when the grand coalition is formed, the worst equilibrium for the buyers is such that the grand coalition's payoff is 1.2039, which is smaller than $0.8118 + 0.4107$.

[24] Note that V_A^G, V_B^G, V_{AB}^G are not affected by the coalition structure.

[25] Since the buyers are ex ante symmetric, analogous results are obtained if the coalition includes only buyers 1 and 3 (or only buyers 2 and 3).

Therefore, in this example, the buyers' best alternative is to remain separate. In particular, forming a coalition composed of only two buyers reduces the total payoff of the buyers in the group, although it does not affect the payoff of the buyer outside the coalition. Forming the grand coalition reduces the buyers' total payoff even more, as we know from Proposition 6. Regarding the coalition of buyers 1 and 2, it is interesting to note that forming the coalition does not affect the payoff of buyer 3 with respect to the case of no coalition. This result occurs because each buyer's payoff in the worst equilibrium without any coalition is determined by the deviation that induces each of two buyers to buy quantity q^* and the remaining buyer to buy nothing from the nondeviating seller,[26] and buyer 3's payoff in the worst equilibrium given the coalition of buyers 1 and 2 is determined by the deviation that induces the coalition to buy quantity $2q^*$ and buyer 3 to buy nothing from the nondeviating seller. These are essentially the same deviation, and hence, the same worst payoff for buyer 3 is obtained. Finally, the total payoff of the buyers in the coalition is reduced, as in our result for a setting with just two buyers, and for a similar reason. We have described above the deviation that generates the lower bound on each buyer's payoff when no coalition is formed, but such a deviation cannot be reproduced with regard to the coalition of 1 and 2 when it is formed because no seller can make a discriminatory offer that induces buyer 1 (for instance) to buy quantity q^* and buyer 2 to buy quantity 0 from the nondeviating seller while buyer 3 buys quantity q^* from the nondeviating seller.

We conjecture that the result in our example should hold more generally such that whenever a larger coalition including some existing coalitions or individual buyers is formed, the coalition formation reduces the total payoff of the buyers in the coalition and does not affect the payoff of any coalition (or individual buyer) outside the coalition.

7 Application: entry and a buyer group

In this section, we use our main insight to study how the possibility of a seller's entry affects the buyers' decision to form a group. For instance, a buying alliance among large chains of supermarkets can have an impact on the entry of national brand manufacturers.[27] Innes and Sexton (1993, 1994) allow for the possibility for the buyers to form a coalition in order to contract with an outside entrant (or vertically integrate into the upstream market). In their papers, the incumbent has the first-mover advantage and employs divide-and-conquer strategies to disrupt the coalition building. In contrast, we assume that the buyers move first by deciding whether to form a group and also assume that the incumbent and the entrant compete by making simultaneous offers, as in Fumagalli and

[26] This deviation is analogous to the deviation that generates (IC $(0, q^*)$) in (25) when $n = 2$.

[27] Caprice and Rey (2015) describe the buying alliance between Leclerc and Système U, each having 17% and 9%, respectively, of sales in French grocery and daily goods retail markets in 2009.

Motta (2008).[28]

We consider a setting with an incumbent, seller A, and a potential entrant, seller B, assuming that the cost functions are strictly convex and the products are substitutes. The buyers decide whether to form a group before the realization of the value of the fixed cost of entry f of seller B. Let $V_j^G \equiv \max_{x\geq 0}[U^G(x,0) - C_j(x)]$ denote the welfare generated when the group trades only with seller j $(= A, B)$. We assume that f is distributed over $\left[0, \overline{f}\right]$ with $\overline{f} > V_{AB}^G - V_A^G (> 0)$ with a cumulative distribution function H with a strictly positive density h in $\left[0, \overline{f}\right]$. We study the following game:

- Stage 0: The buyers decide whether to form a group.
- Stage 0.5: The value of the fixed cost of entry $f \geq 0$ is realized. The entrant decides whether to incur the cost.
- Stage 1: If the entrant entered, then the incumbent and the entrant compete by simultaneously proposing nonlinear tariffs to each buyer (or to the buyer group). If the entrant did not enter, the incumbent becomes a monopolist.
- Stage 2: Each buyer i, or the buyer group, makes purchase decisions.

If the entrant enters, given the multiplicity of equilibria, we select the equilibrium that is Pareto-dominant in terms of the sellers' payoffs. Hence, the sellers play the sell-out equilibrium if the buyer group is formed, and then, the sellers' payoffs are $V_{AB}^G - V_B^G$ for seller A, $V_{AB}^G - V_A^G$ for seller B (Bernheim and Whinston, 1998). The assumption $V_{AB}^G - V_A^G \in (0, \overline{f})$ implies that when the buyer group is formed, seller B enters with a positive probability $H(V_{AB}^G - V_A^G) \in (0,1)$. Proposition 5 suggests that the buyers face a clear trade-off. Conditional on entry, the total payoff of the buyers is lower with a buyer group than without a buyer group. However, the fact that the entrant's payoff is higher with a buyer group than without a buyer group makes the probability of entry higher with a buyer group than without a buyer group. Furthermore, in the sell-out equilibrium with a buyer group, each seller obtains the social incremental contribution of his product, implying that the entry decision is always socially optimal. This result in turn implies that if buyers remain separate, then the entry is suboptimal. Therefore, Proposition 5 suggests that the buyers may deliberately induce a suboptimal entry by remaining separate in order to benefit from more intense competition upon entry.

Proposition 7 *(entry) Suppose that the cost functions are strictly convex and that the products are strict substitutes. Consider the game of buyer group formation followed by*

[28] Fumagalli and Motta (2008) assume that the entrant can make his offer before incurring a fixed cost of entry, which seems less natural than our timing, that the entrant makes an offer after incurring the fixed cost of entry.

the entry. In case the entrant enters, we select the equilibrium that is Pareto-dominant in terms of the sellers' payoffs.

(i) If the buyers form a group, then the private entry decision is socially efficient. However, if the buyers remain separate, then the entry is suboptimal.

(ii) The buyers may deliberately induce a suboptimal entry by remaining separate in order to benefit from more intense competition upon entry.

The existing literature regarding naked exclusion explains suboptimal entry by the incumbent's taking advantage of coordination failure among buyers (Rasmusen, Ramseyer and Wiley 1991, Segal and Whinston, 2000, Fumagalli and Motta, 2006, 2008, Chen and Shaffer, 2014). Whereas the literature typically assumes that the incumbent makes offers to buyers before the entry, Fumagalli and Motta (2008) show that the coordination failure survives even if both sellers make simultaneous offers. However, in these papers, buyers have no reason not to form a group (at least among those operating in separate markets), as this would remove the coordination failure. We show that buyers may have an incentive not to form a buyer group even if this leads to a suboptimal entry.

8 Conclusion

We considered a buyer group that cannot precommit to an exclusive purchase and found that the formation of a buyer group does not generate any buyer power (i.e., larger discounts than the case without a buyer group), regardless of the curvature of the sellers' cost function. Combining our result with those of Inderst and Shaffer (2007) and Dana (2012) generates a clear message: in the case of competition among multiple sellers producing substitutes, the formation of a buyer group creates buyer power only if the group can precommit to limit its purchase to a subset of sellers. This finding has policy implications, for instance, for large healthcare procurement alliances. It is not the mere size of the alliance but rather the credible threat to limit its purchase to a subset of sellers that increases its buyer power.

Note that our results were obtained under a strong assumption that sellers make take-it-or-leave-it offers independently of the formation of a buyer group. This assumption of take-it-or-leave-it offers makes sense when there is no buyer group. However, if many buyers form a large group, the assumption would not be satisfied, as the group is equally likely to make offers to the sellers. Our model does not consider such changes in bargaining power of buyers.

It would be interesting to extend our framework by considering a linear contract between an upstream firm and a downstream firm instead of a nonlinear contract. The linear contract is prominent in relations between TV channels and cable TV distributors (Crawford and Yurukoglu, 2012), between hospitals and medical device suppliers (Grennan, 2013, 2014), and between book publishers and resellers (Gilbert, 2015). With such a simple but realistic contract, the equilibrium outcome is in general inefficient since each upstream firm adds a positive markup. Therefore, if buyer fragmentation intensifies competition among sellers as in this paper, it will increase welfare by reducing the deadweight loss. In addition, the framework with the linear contract may be sufficiently tractable to allow for incorporating competition among downstream firms.

9 Acknowledgements

We thank the anonymous advisory editor, the two anonymous referees, Stéphane Caprice, Zhijun Chen, Jay Pil Choi, Vincenzo Denicolo, Sjaak Hurkens, Biung-Ghi Ju, Bruno Jullien, In-Uck Park, Patrick Rey and the audiences who attended our presentations at Bristol University, EARIE (2014), EEA-ESEM (2014), Postal Economics Conference on "E-commerce, Digital Economy and Delivery Services" (Toulouse, 2014), Seoul National University, Toulouse School of Economics, and Yonsei University.

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] Aghion, Philippe and Bolton, Patrick (1987). “Contracts as a Barrier to Entry.” *American Economic Review*, 77(3): 388–401.

[2] Bernheim, Douglas and Michael Whinston (1986). “Menu Auctions, Resource Allocation and Economic Influence.” *Quarterly Journal of Economics*, 101, 1-31.

[3] Bernheim, Douglas and Michael Whinston (1990). ”Multi-market Contact and Collusive Behavior.” *The RAND Journal of Economics*, 21(1), 1-26.

[4] Bernheim, Douglas and Michael Whinston (1998). “Exclusive Dealing.” *Journal of Political Economy*, 106, 64-103.

[5] Caprice, Stéphane and Patrick Rey (2015). "Buyer Power From Joint Listing Decisions." Economic Journal,125: 1677-1704

[6] Chae, Suchan and Paul Heidhues (2004). "Buyers’ Alliances for Bargaining Power." *Journal of Economics and Management Strategy*, 13: 731–754

[7] Chen, Yuxin and Xinxin Li (2013). "Group Buying Commitment and Sellers’ Competitive Advantages", *Journal of Economics and Management Strategy*, 22:164-183.

[8] Chen, Zhijun and Greg Shaffer (2014). "Naked Exclusion with Minimum-Share Requirements", *RAND Journal of Economics,* 45: 64-91.

[9] Chen, Zhiqi, (2003). "Dominant Retailers and Countervailing Power Hypothesis", *RAND Journal of Economics*, 34: 612-625.

[10] Chen, Zhiqi (2007). "Buyer Power: Economic Theory and Antitrust Policy", *Research in Law and Economics* 22:17-40.

[11] Chipty, T. and Snyder, C.M. (1999). "The Role of Firm Size in Bilateral Bargaining: a Study of the Cable Television Industry", *Review of Economics and Statistics*, 81: 326-40.

[12] Competition Commission (2008). *The Supply of Groceries in the UK Market Investigation*, U.K. Competition Commission, London

[13] Crawford, Gregory S., and Ali Yurukoglu (2012). "The Welfare Effects of Bundling in Multichannel Television Markets." *American Economic Review*, 102: 643–685

[14] Dana, James D. (2012). "Buyer Group as Strategic Commitments", *Games and Economic Behavior*, 74(2): 470-485.

[15] DeGraba, Patrick J. (2005). "Quantity Discounts from Risk Averse Sellers," U. S. Federal Trade Commission Working Paper no. 276.

[16] Dobson, Paul W. and Michael Waterson, (1997). "Countervailing Power and Consumer Prices," *Economic Journal*, 107: 418–430.

[17] Ellison, Sara Fisher and Christopher M. Snyder (2010). "Countervailing Power in Wholesale Pharmaceuticals", *Journal of Industrial Economics*, 58: 32-53.

[18] Erutku, Can, (2005). "Buying Power and Strategic Interactions," *Canadian Journal of Economics*, 38: 1160–1172.

[19] European Commission (1999). *Buyer power and his impact on competition in the food retail distribution sector of the European Union: Final report.*

[20] Fumagalli, Chiara and Massimo Motta (2006). "Exclusive Dealing and Entry, When Buyers Compete," *American Economic Review*, 96: 785-944.

[21] Fumagalli, Chiara and Massimo Motta (2008). "Buyers' Miscoordination, Entry and Downstream Competition," *Economic Journal*, 118: 1196-1222.

[22] Gaudin, Germain (2017). "Vertical Bargaining and Retail Competition: What Drives Countervailing Power?" *Economic Journal*, forthcoming

[23] Gilbert, Richard J. (2015). "E-Books: A Tale of Digital Disruption." *Journal of Economic Perspectives*, 29: 165–184.

[24] Grennan, Matthew (2013). “Price Discrimination and Bargaining: Empirical Evidence from Medical Devices.” *American Economic Review*, 103: 145–177.

[25] Grennan, Matthew (2014). “Bargaining Ability and Competitive Advantage: Empirical Evidence from Medical Devices.” *Management Science*, 60: 3011–3025.

[26] Harsanyi, J. C. (1977). *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*. Cambridge: Cambridge University Press.

[27] Inderst, R and G. Shaffer (2007). "Retail Mergers, Buyer Power and Product Variety", *The Economic Journal*, 117(516): 45-67.

[28] Inderst, R. and Tommaso Valletti (2011). "Buyer Power and the ‘Waterbed effect’", *Journal of Industrial Economics 59: 1-20*

[29] Inderst, R and C. Wey (2003). "Bargaining, Mergers, and Technology Choice in Bilaterally Oligopolistic Industries", *the RAND Journal of Economics,* 34: 1-19.

[30] Inderst, R and C. Wey (2007). "Buyer Power and Supplier Incentives", *European Economic Review*, 51: 647-667.

[31] Innes, R. and R. J. Sexton (1993). ”Customer Coalitions, Monopoly Price Discrimination and Generic Entry Deterrence.” *European Economic Review*, 37, 1569-1597.

[32] Innes, R. and R. J. Sexton (1994). ”Strategic Buyers and Exclusionary Contracts”, *American Economic Review*, 84: 566-584.

[33] Jeon, Doh-Shin and Domenico Menicucci (2005). “Optimal Second-Degree Price Discrimination and Arbitrage: On the Role of Asymmetric Information among Buyers”, *The RAND Journal of Economics*, 36 (2), pp. 337-360.

[34] Jeon, Doh-Shin and Domenico Menicucci (2006). “Bundling Electronic Journals and Competition among Publishers”, *Journal of the European Economic Association*, September 4(5): 1038-83.

[35] Jeon, Doh-Shin and Domenico Menicucci (2012). “Bundling and Competition for Slots”, *American Economic Review*, 102: 1957-1985.

[36] Jeon, Doh-Shin and Domenico Menicucci (2017). “The Benefits of Diverse Preferences in Library Consortia," *Journal of Industrial Economics* 65: 105-135

[37] Katz, Michael (1987). "The Welfare Effects of Third Degree Price Discrimination in Intermediate Goods Markets," *American Economic Review*, 77: 154–167.

[38] Loertscher, Simon and Leslie M. Marx (2017). "Merger Reviews for Markets with Buyer Power," Mimeo

[39] Marvel, H.P. and H. Yang, (2008). "Group Purchasing, Nonlinear Tariffs, and Oligopoly," *International Journal of Industrial Organization*, 26(5): 1090-1105.

[40] Normann, H.-T., Ruffle, B. and Snyder, C. M., (2007). "Do Buyer-Size Discounts Depend on the Curvature of the Surplus Function? Experimental Tests of Bargaining Models," , *Rand Journal of Economics*, 38: 747–767.

[41] O'Brien, Daniel and Greg Shaffer (1997). "Non-linear Supply Contracts, Exclusive Dealing and Equilibrium Market Foreclosure", *Journal of Economics and Management Strategy* 6: 755-785

[42] OECD (2008). *Monopsony and Buyer Power*, Policy Roundtables.

[43] Prat, Andrea and Aldo Rustichini (2003). "Games Played through Agents", *Econometrica*, 71 (4): 989-1026

[44] Rasmusen, Eric B., J. Mark Ramseyer, and John S. Wiley, Jr. (1991). "Naked Exclusion." *American Economic Review*, 81(5): 1137–45.

[45] Ruffle, Bradley J. (2013). "When Do Large Buyers Pay Less? Experimental Evidence", *Journal of Industrial Economics* 61: 108-137

[46] Scheffman, David. T. and Pablo T. Spiller (1992). "Buyers' Strategies, Entry Barriers and Competition," *Economic Inquiry*, 30: 418–436

[47] Segal, Ilya and Michael Whinston (2000). "Naked Exclusion: Comment", *American Economic Review*, 190: 296-309

[48] Snyder, Christopher M., (1996). "A Dynamic Theory of Countervailing Power," *Rand Journal of Economics*, 27: 747–769.

[49] Snyder, Christopher M., (1998). "Why Do Larger Buyers Pay Lower Prices? Intense Supplier Competition," *Economics Letters*, 58: 205–209.

[50] Sorensen, Alan T., (2003). "Insurer-Hospital Bargaining: Negotiated Discounts in Post-Deregulation Connecticut," *Journal of Industrial Economics,* 51: 471–492.

[51] van den Brink, R. (2012), "Efficiency and Collusion Neutrality in Cooperative Games and Networks", *Games and Economic Behavior*, 76, 344-348.

[52] von Ungern-Sternberg, T. (1996). "Countervailing Power Revisited", *International Journal of Industrial Organization*, 14: 507-19.

10 Appendix

10.1 Proof of Proposition 1

Proof that U^G is concave Consider $(q_A^G, q_B^G) = (\bar{q}_A, \bar{q}_B)$, and suppose that $(\bar{q}_A^1, \bar{q}_B^1, \bar{q}_A^2, \bar{q}_B^2)$ is the optimal allocation (i.e., it maximizes G's payoff[29]) given $(q_A^G, q_B^G) = (\bar{q}_A, \bar{q}_B)$, that is, $U^G(\bar{q}_A, \bar{q}_B) = U^1(\bar{q}_A^1, \bar{q}_B^1) + U^2(\bar{q}_A^2, \bar{q}_B^2)$. Likewise, consider $(q_A^G, q_B^G) = (\ddot{q}_A, \ddot{q}_B)$ and suppose that $(\ddot{q}_A^1, \ddot{q}_B^1, \ddot{q}_A^2, \ddot{q}_B^2)$ is the optimal allocation given $(q_A^G, q_B^G) = (\ddot{q}_A, \ddot{q}_B)$, that is, $U^G(\ddot{q}_A, \ddot{q}_B) = U^1(\ddot{q}_A^1, \ddot{q}_B^1) + U^2(\ddot{q}_A^2, \ddot{q}_B^2)$. Then, consider $(q_A^G, q_B^G) = \lambda(\bar{q}_A, \bar{q}_B) + (1-\lambda)(\ddot{q}_A, \ddot{q}_B)$, and notice that $(\lambda\bar{q}_A^1 + (1-\lambda)\ddot{q}_A^1, \lambda\bar{q}_B^1 + (1-\lambda)\ddot{q}_B^1, \lambda\bar{q}_A^2 + (1-\lambda)\ddot{q}_A^2, \lambda\bar{q}_B^2 + (1-\lambda)\ddot{q}_B^2)$ is a feasible allocation. Hence, we have

$$U^G(\lambda(\bar{q}_A, \bar{q}_B) + (1-\lambda)(\ddot{q}_A, \ddot{q}_B))$$

$$\geq U^1(\lambda\bar{q}_A^1 + (1-\lambda)\ddot{q}_A^1, \lambda\bar{q}_B^1 + (1-\lambda)\ddot{q}_B^1) + U^2(\lambda\bar{q}_A^2 + (1-\lambda)\ddot{q}_A^2, \lambda\bar{q}_B^2 + (1-\lambda)\ddot{q}_B^2)$$

$$\geq \lambda U^1(\bar{q}_A^1, \bar{q}_B^1) + (1-\lambda)U^1(\ddot{q}_A^1, \ddot{q}_B^1) + \lambda U^2(\bar{q}_A^2, \bar{q}_B^2) + (1-\lambda)U^2(\ddot{q}_A^2, \ddot{q}_B^2) = \lambda U^G(\bar{q}_A, \bar{q}_B) + (1-\lambda)U^G(\ddot{q}_A, \ddot{q}_B)$$

in which the first inequality follows from the definition of U^G and the second inequality follows from concavity of U^1 and of U^2. The resulting inequality $U^G(\lambda(\bar{q}_A, \bar{q}_B) + (1-\lambda)(\ddot{q}_A, \ddot{q}_B)) \geq \lambda U^G(\bar{q}_A, \bar{q}_B) + (1-\lambda)U^G(\ddot{q}_A, \ddot{q}_B)$ establishes that U^G is concave.

Proof of (7) and of (9) Here, we provide the proof for an arbitrary number $n \geq 1$ of buyers, which covers the case of $n = 1$ (buyer group, in which case the superscript i should be replaced by G) and of $n \geq 2$ (Proposition 5 provides a result for the case of n buyers). For $i = 1, ..., n$, we use (q_j^{ie}, q_k^{ie}) to denote the equilibrium quantities purchased by buyer i, and in order to shorten notation in this proof, we define $U^{ie} \equiv U^i(q_j^{ie}, q_k^{ie})$, $T_j^{ie} \equiv T_j^i(q_j^{ie})$, and $C_j^e \equiv C_j(\sum_{i=1}^n q_j^{ie})$ for $i = 1, ..., n$ and $j = A, B$.

The equilibrium payoff of buyer i is $u^{ie} \equiv U^{ie} - T_j^{ie} - T_k^{ie}$, whereas if i trades only with seller j, then i's payoff is $u_j^i \equiv \max_{q_j^i \geq 0}(U^i(q_j^i, 0) - T_j^i(q_j^i))$. In any equilibrium, it is necessary that $u^{ie} = u_j^i = u_k^i$; therefore, we can use $U^{ie} - T_j^{ie} - T_k^{ie} = u_j^i$ to obtain

$$T_k^{ie} = U^{ie} - T_j^{ie} - u_j^i = U^{ie} - w_j^i, \quad \text{with} \quad w_j^i \equiv \max_{q_j^i \geq 0}(U^i(q_j^i, 0) - T_j^i(q_j^i)) + T_j^{ie}. \tag{37}$$

Likewise, $T_j^{ie} = U^{ie} - w_k^i$ and buyer i's payoff can be written as

$$u^{ie} = w_j^i + w_k^i - U^{ie}. \tag{38}$$

The profit of seller j is

$$\pi_j^e \equiv \sum_{i=1}^n T_j^{ie} - C_j^e = \sum_{i=1}^n (U^{ie} - w_k^i) - C_j^e. \tag{39}$$

[29]Given (q_A^G, q_B^G), the maximization problem in (2)-(3) has a unique solution because the objective function is strictly concave, and the feasible set is convex.

Now, we consider deviations of seller j in the form of take-it-or-leave-it offers (q_j^i, t_j^i) to each buyer $i = 1, ..., n$ such that

$$U^i(q_j^i, q_k^{ie}) - t_j^i - T_k^{ie} \geq u_k^i, \qquad \text{for } i = 1, ..., n.$$

These inequalities state that buyer i weakly prefers to accept j's take-it-or-leave-it offer and continues to buy q_k^{ie} from seller k rather than to trade only with k. Choosing t_j^i to satisfy the above inequalities as equalities for $i = 1, ..., n$, we infer that from this type of deviation, seller j's profit π_j^d is

$$\pi_j^d(q_j^1, ..., q_j^n) = \sum_{i=1}^{n}(U^i(q_j^i, q_k^{ie}) - T_k^{ie} - u_k^i) - C_j(\sum_{i=1}^{n} q_j^i).$$

Since no profitable deviation exists, we infer that $\pi_j^d(q_j^1, ..., q_j^n) \leq \pi_j^e$ for each $(q_j^1, ..., q_j^n)$. The equality holds if j chooses $q_j^i = q_j^{ie}$ for $i = 1, ..., n$, since $T_k^{ie} + u_k^i = w_k^i$ for each i. Therefore, $(q_j^1, ..., q_j^n) = (q_j^{1e}, ..., q_j^{ne})$ is a maximizer of π_j^d (see (7) for the case of $n = 1$, and see (9) for the case of $n = 2$), and this implies that $\sum_{i=1}^{n} U_j(q_j^{ie}, q_k^{ie}) \leq C_j'(\sum_{i=1}^{n} q_j^{ie})$, with equality if $q_j^{ie} > 0$, for $i = 1, ..., n$. Since social welfare is concave and the conditions above are first-order conditions for maximization of social welfare, we conclude that $(q_j^{ie}, q_k^{ie}) = (q_j^{i*}, q_k^{i*})$ for $i = 1, ..., n$.

10.2 Proof of Lemma 1

Given any symmetric equilibrium in which each seller offers the same tariff T^G to G, we know from Proposition 1 that G buys quantity $2q^*$ from each seller, that is,

$$(2q^*, 2q^*) \in \arg \max_{q_A^G \geq 0, q_B^G \geq 0} \left(U^G(q_A^G, q_B^G) - T^G(q_A^G) - T^G(q_B^G)\right)$$

This implies that

$$(2q^*, 2q^*) \in \arg \max_{q_A^G \in \{0, 2q^*, \hat{q}\}, q_B^G \in \{0, 2q^*, \hat{q}\}} \left(U^G(q_A^G, q_B^G) - T^G(q_A^G) - T^G(q_B^G)\right)$$

[recall that $\hat{q}$ is defined in (12)]. Since $t^{G*} = T^G(2q^*)$ and $\hat{t}^G = T^G(\hat{q})$, it follows that G still buys $2q^*$ from each seller when each seller offers only the two contracts $(2q^*, t^{G*})$, $(\hat{q}, \hat{t}^G)$, essentially because the set of opportunities for G has shrunk, but the best alternative of the group is still available.

Regarding the existence of profitable deviations, consider seller A (to fix ideas), and notice that when seller B offers the tariff T^G, the profit that seller A can make with a take-it-or-leave-it offer (q^G, t^G) is not greater than $\max_{q^G \geq 0, q_B^G \geq 0} \left(U^G(q^G, q_B^G) - T^G(q_B^G) - u^G - C(q^G)\right)$, in which $u^G = U^G(0, \hat{q}) - \hat{t}^G$. Since no profitable deviation exists, we infer that

$$\max_{q^G \geq 0, q_B^G \geq 0} \left(U^G(q^G, q_B^G) - T^G(q_B^G) - u^G - C(q^G)\right) \leq t^{G*} - C(2q^*)$$

This result implies that

$$\max_{q^G\geq 0, q_B^G\in\{0,2q^*,\hat{q}\}}\left(U^G(q^G,q_B^G)-T^G(q_B^G)-u^G-C(q^G)\right)\leq t^{G*}-C(2q^*)$$

Hence, no deviation is profitable for seller A when B offers only the contracts $(2q^*,t^{G*})$, $(\hat{q},\hat{t}^G)$ because the feasible set for q_B^G has shrunk.

Here, we prove that in any equilibrium with a buyer group, the inequality $\hat{q}>2q^*$ holds. In each equilibrium, $(2q^*,t^{G*})$ and $(\hat{q},\hat{t}^G)$ need to satisfy

$$U^G(2q^*,2q^*)-2t^{G*} \geq U^G(\hat{q},2q^*)-\hat{t}^G-t^{G*} \quad (40)$$

$$U^G(\hat{q},0)-\hat{t}^G \geq U^G(2q^*,0)-t^{G*} \quad (41)$$

Hence,

$$U^G(2q^*,2q^*)-U^G(2q^*,0)\geq U^G(\hat{q},2q^*)-U^G(\hat{q},0)$$

or

$$\int_0^{2q^*}\int_{\hat{q}}^{2q^*}U_{BA}^G(w,z)dwdz\geq 0$$

and this implies $\hat{q}\geq 2q^*$ since $U_{BA}^G<0$. Now, suppose that $\hat{q}=2q^*$. Then, both (40) and (41) are equalities, and $t^{G*}=\hat{t}^G$; hence, from (13), we obtain $U^G(2q^*,2q^*)-2t^{G*}=U^G(2q^*,0)-t^{G*}$. Thus, $t^{G*}=U^G(2q^*,2q^*)-U^G(2q^*,0)$. Then, the payoff for G is $U^G(2q^*,2q^*)-2t^{G*}=2U^G(2q^*,0)-U^G(2q^*,2q^*)\equiv u^G$. Next, suppose that seller A deviates with (q,t) such that $U^G(q,0)-t=u^G$; hence, A's profit is $U^G(q,0)-u^G-C(q)$, and its maximum with respect to q has value $f^G(0)-2U^G(2q^*,0)+U^G(2q^*,2q^*)$. This profit is greater than A's profit without deviation $t^{G*}-C(2q^*)=U^G(2q^*,2q^*)-U^G(2q^*,0)-C(2q^*)$, that is, $f^G(0)>U^G(2q^*,0)-C(2q^*)$, since $2q^*$ is not a solution to the problem $\max_{x\geq 0}[U^G(x,0)-C(x)]$.

10.3 Proof of Proposition 2

Given that each seller offers only the contracts $(2q^*,t^{G*})$ and $(\hat{q},\hat{t}^G)$, the group chooses a pair $(q_A^G,q_B^G)\in\{0,2q^*,\hat{q}\}\times\{0,2q^*,\hat{q}\}$. In order to shorten the notation, we define $U^{G*}\equiv U^G(2q^*,0)$, $\hat{U}^G\equiv U^G(\hat{q},0)$, $U_t^{G*}\equiv U^G(2q^*,2q^*)$ (the subscript t means that G buys quantity $2q^*$ *twice*, that is, from both sellers), $\hat{U}^{G*}\equiv U^G(2q^*,\hat{q})$, $\hat{U}_t^G\equiv U^G(\hat{q},\hat{q})$. Recalling that $t^{G*}=\frac{1}{2}U_t^{G*}-\frac{1}{2}u^G$, $\hat{t}^G=\hat{U}^G-u^G$, we compute G's payoffs from the various purchase alternatives:

$$\left\{\begin{array}{llll}\text{alternatives} & q_A^G=0,q_B^G=2q^* & q_A^G=0,q_B^G=\hat{q} & q_A^G=q_B^G=2q^*\\ \text{payoff} & U^{G*}-\frac{1}{2}U_t^{G*}+\frac{1}{2}u^G & u^G & u^G\\ \text{alternatives} & q_A^G=2q^*,q_B^G=\hat{q} & q_A^G=q_B^G=\hat{q} & \\ \text{payoff} & \hat{U}^{G*}-\hat{U}^G-\frac{1}{2}U_t^{G*}+\frac{3}{2}u^G & \hat{U}_t^G-2\hat{U}^G+2u^G & \end{array}\right. \quad (42)$$

Below, Step 1 determines the conditions under which G buys quantity $2q^*$ from each seller. Step 2 determines the conditions under which no seller wants to deviate. Step 3 proves that the latter conditions are more restrictive than the former and that the whole set of values for u^G is given by (15).

Step 1 Given the alternatives in (42), G buys quantity $2q^*$ from each seller if and only if

$$2U^{G*} - U_t^{G*} \le u^G \le U_t^{G*} + 2\hat{U}^G - 2\hat{U}^{G*} \tag{43}$$

Proof From (42), we see that the following inequalities must hold:

$$u^G \ge U^{G*} - \frac{1}{2}U_t^{G*} + \frac{1}{2}u^G, \qquad u^G \ge \hat{U}^{G*} - \hat{U}^G - \frac{1}{2}U_t^{G*} + \frac{3}{2}u^G, \qquad u^G \ge \hat{U}_t^G - 2\hat{U}^G + 2u^G \tag{44}$$

which are equivalent, respectively, to $2U^{G*} - U_t^{G*} \le u^G$, to $u^G \le U_t^{G*} + 2\hat{U}^G - 2\hat{U}^{G*}$ and to $u^G \le 2\hat{U}^G - \hat{U}_t^G$. Finally, we obtain (43) by proving that $U_t^{G*} + 2\hat{U}^G - 2\hat{U}^{G*} < 2\hat{U}^G - \hat{U}_t^G$. This inequality is equivalent to $\hat{U}_t^G - \hat{U}^{G*} < \hat{U}^{G*} - U_t^{G*}$, that is, $\int_{2q^*}^{\hat{q}} U_A^G(z,\hat{q})dz < \int_{2q^*}^{\hat{q}} U_A^G(z,2q^*)dz$, which holds because $\hat{q} > 2q^*$ and goods are substitutes.

Step 2 No profitable deviation exists for a seller if and only if

$$2f^G(0) + U_t^{G*} - 2f^G(2q^*) \le u^G \le 2f^G(2q^*) - U_t^{G*} - 2[f^G(\hat{q}) - \hat{U}^G] \tag{45}$$

Proof The equilibrium profit for each seller is $t^{G*} - C(2q^*) = f^G(2q^*) - \frac{1}{2}U_t^{G*} - \frac{1}{2}u^G$, and now, we examine the conditions under which no profitable deviation for seller A exists. The deviation that induces G to buy only from A yields seller A profit $f^G(0) - u^G$; hence, $U_t^{G*} - 2f^G(2q^*) + 2f^G(0) \le u^G$ is necessary. The deviation in which G buys $2q^*$ from seller B yields A profit $f^G(2q^*) - t^{G*} - u^G$, which is just equal to $f^G(2q^*) - \frac{1}{2}U_t^{G*} - \frac{1}{2}u^G$. The deviation in which G buys $\hat{q}$ from seller B yields A profit $f^G(\hat{q}) - \hat{t}^G - u^G = f^G(\hat{q}) - \hat{U}^G$; hence, $u^G \le 2f^G(2q^*) - U_t^{G*} - 2[f^G(\hat{q}) - \hat{U}^G]$ is necessary.

Step 3 If u^G satisfies (45), then it satisfies (43), and the whole set of G's equilibrium payoffs is obtained from (45) as $\hat{q} \to +\infty$.

Proof The inequality $2U^{G*} - U_t^{G*} \le U_t^{G*} - 2f^G(2q^*) + 2f^G(0)$ reduces to $U^{G*} - C(2q^*) \le f^G(0)$, which holds by definition of $f^G(0)$. The inequality $2f^G(2q^*) - U_t^{G*} - 2[f^G(\hat{q}) - \hat{U}^G] \le U_t^{G*} + 2\hat{U}^G - 2\hat{U}^{G*}$ reduces to $\hat{U}^{G*} - C(2q^*) \le f^G(\hat{q})$, which holds by definition of $f^G(\hat{q})$. Since the right-hand side in (45) is increasing in $\hat{q}$, it follows that (15) describes the set of values for u^G.

Finally, notice that if u^G and $\tilde{q}$ satisfy (15) and $f^G(\hat{q}) - U^G(0,\hat{q}) \le \frac{1}{2}(V_{AB}^G - u^G)$, then (43) and (45) are satisfied, since $2f^G(2q^*) - U_t^{G*} = V_{AB}^G$. Therefore, Proposition 2(i-ii) identifies indeed an equilibrium that yields payoff u^G to G.

10.4 Proof of Lemma 2

This proof is omitted since it is very similar to the proof of Lemma 1.

10.5 Proof of Lemma 3

Proof of inequality (22) for the case of strictly convex C

Here, we prove a generalization of (22) for the case of $n \geq 2$ asymmetric buyers and two asymmetric sellers. Precisely, in this proof, buyers may have different utility functions, and sellers can be asymmetric.

Given $b^1 \geq 0$, ..., $b^n \geq 0$, we define

$$f_A(b^1, ..., b^n) = \max_{x^1,...,x^n} [U^1(x^1, b^1) + ... + U^n(x^n, b^n) - C_A(x^1 + ... + x^n)]. \tag{46}$$

We prove that if C_A is strictly convex and $\underline{\mathbf{b}} = (\underline{b}^1, ..., \underline{b}^n)$ and $\bar{\mathbf{b}} = (\bar{b}^1, ..., \bar{b}^n)$ are such that $\underline{b}^h < \bar{b}^h$ for $h = 1, ..., n$, then

$$f_A(\bar{b}^1, \bar{b}^2, ..., \bar{b}^n) - f_A(\bar{b}^1, \underline{b}^2, ..., \underline{b}^n) < \sum_{i=2}^{n} \left(f_A(\underline{b}^1, ..., \bar{b}^i, ..., \underline{b}^n) - f_A(\underline{b}^1, ..., \underline{b}^n)\right), \tag{47}$$

which is equivalent to

$$f_A(\bar{b}^1, \bar{b}^2, ..., \bar{b}^n) - f_A(\underline{b}^1, ..., \underline{b}^n) < \sum_{i=1}^{n} \left(f_A(\underline{b}^1, ..., \bar{b}^i, ..., \underline{b}^n) - f_A(\underline{b}^1, ..., \underline{b}^n)\right).$$

In the special case of $n = 2$, with $\underline{\mathbf{b}} = (\underline{b}, \underline{b})$ and $\bar{\mathbf{b}} = (\bar{b}, \bar{b})$, (47) reduces to $f_A(\bar{b}, \bar{b}) - f_A(\bar{b}, \underline{b}) < f_A(\underline{b}, \bar{b}) - f_A(\underline{b}, \underline{b})$, which is (22).

Step 1 Consider $\underline{\mathbf{b}}$ and $\bar{\mathbf{b}}$ such that $\underline{b}^i = \bar{b}^i$ for one i and $\underline{b}^h < \bar{b}^h$ for each $h \neq i$, and denote with $\underline{\mathbf{x}}$ (with $\bar{\mathbf{x}}$) the optimal $(x^1, ..., x^n)$ given $\underline{\mathbf{b}}$ (given $\bar{\mathbf{b}}$) in problem (46). Then, $\bar{x}^i > \underline{x}^i$ and $\sum_{h \neq i} \bar{x}^h < \sum_{h \neq i} \underline{x}^h$.

Proof The first-order conditions for $x^1, ..., x^n$ given $\mathbf{b} = \underline{\mathbf{b}}$ and given $\mathbf{b} = \bar{\mathbf{b}}$ are, respectively,

$$U_A^h(\underline{x}^h, \underline{b}^h) - C_A'(\underline{x}^1 + ... + \underline{x}^n) = 0 \text{ for } h \neq i, \; U_A^i(\underline{x}^i, \underline{b}^i) - C_A'(\underline{x}^1 + ... + \underline{x}^n) = 0 \tag{48}$$
$$U_A^h(\bar{x}^h, \bar{b}^h) - C_A'(\bar{x}^1 + ... + \bar{x}^n) = 0 \text{ for } h \neq i, \; U_A^i(\bar{x}^i, \underline{b}^i) - C_A'(\bar{x}^1 + ... + \bar{x}^n) = 0 \tag{49}$$

First, we prove that $\bar{x}^1 + ... + \bar{x}^n < \underline{x}^1 + ... + \underline{x}^n$. In view of a contradiction, we suppose that $\bar{x}^1 + ... + \bar{x}^n \geq \underline{x}^1 + ... + \underline{x}^n$. Then, $U_A^h(\underline{x}^h, \bar{b}^h) - C_A'(\bar{x}^1 + ... + \bar{x}^n) < U_A^h(\underline{x}^h, \underline{b}^h) - C_A'(\underline{x}^1 + ... + \underline{x}^n) = 0$ for $h \neq i$ (as $\bar{b}^h > \underline{b}^h$ and goods are strict substitutes), which implies $\bar{x}^h < \underline{x}^h$ for $h \neq i$. Moreover, $U_A^i(\underline{x}^i, \underline{b}^i) - C_A'(\bar{x}^1 + ... + \bar{x}^n) \leq 0$, which implies $\bar{x}^i \leq \underline{x}^i$. The inequalities $\bar{x}^h < \underline{x}^h$ for $h \neq i$ and $\bar{x}^i \leq \underline{x}^i$ contradict $\bar{x}^1 + ... + \bar{x}^n \geq \underline{x}^1 + ... + \underline{x}^n$. Using $\bar{x}^1 + ... + \bar{x}^n < \underline{x}^1 + ... + \underline{x}^n$ and the equality for $\bar{x}^i$ in (49), we conclude that $\bar{x}^i > \underline{x}^i$. These two inequalities jointly lead to $\sum_{h \neq i} \bar{x}^h < \sum_{h \neq i} \underline{x}^h$. ■

Step 2 Consider $\underline{\mathbf{b}}$ and $\bar{\mathbf{b}}$ such that $\underline{b}^h < \bar{b}^h$ for $h = 1, ..., n$. Then, (47) holds.

PROOF Define the $n-1$ vectors $\underline{\mathbf{b}}^i(t) = (\underline{b}^1, ..., \underline{b}^i + (\bar{b}^i - \underline{b}^i)t, ..., \underline{b}^n)$ for $i = 2, ..., n$, the vector $\bar{\mathbf{b}}(t) = (\bar{b}^1, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t, ..., \underline{b}^n + (\bar{b}^n - \underline{b}^n)t)$, and the two functions $g(t) = \sum_{i=2}^{n} f_A(\underline{\mathbf{b}}^i(t))$, $h(t) = f_A(\bar{\mathbf{b}}(t))$. Then, (47) is equivalent to

$$g(1) - g(0) > h(1) - h(0)$$

We have that $g'(t) = \sum_{i=2}^{n} f_{Ai}(\underline{\mathbf{b}}^i(t))(\bar{b}^i - \underline{b}^i)$ and $f_{Ai}(\underline{\mathbf{b}}^i(t)) = U_B^i(\underline{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)$, in which $\underline{x}^i(t)$ is the optimal x^i given $\mathbf{b} = \underline{\mathbf{b}}^i(t)$; hence, $g(1) - g(0) = \int_0^1 \sum_{i=2}^{n} U_B^i(\underline{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)(\bar{b}^i - \underline{b}^i)dt$. Likewise, $h'(t) = \sum_{i=2}^{n} f_{Ai}(\bar{\mathbf{b}}(t))(\bar{b}^i - \underline{b}^i)$ and $f_{Ai}(\bar{\mathbf{b}}(t)) = U_B^i(\bar{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)$, in which $\bar{x}^i(t)$ is the optimal x^i given $\mathbf{b} = \bar{\mathbf{b}}(t)$; hence, $h(1) - h(0) = \int_0^1 \sum_{i=2}^{n} U_B^i(\bar{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)(\bar{b}^i - \underline{b}^i)dt$. From Step 1, we know that $\bar{x}^i(t) > \underline{x}^i(t)$ for $i = 2, ..., n$, for each $t \in (0, 1)$.[30] Hence, $U_B^i(\underline{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t) > U_B^i(\bar{x}^i(t), \underline{b}^i + (\bar{b}^i - \underline{b}^i)t)$, which implies $g(1) - g(0) > h(1) - h(0)$. ■

Proof of inequality (23) for the case of concave C

Here, we prove a generalization of (23) for the case of C_A concave and $n = 2$ (but without assumption of symmetric buyers or symmetric sellers). Given $(\underline{b}^1, \bar{b}^1)$, $(\underline{b}^2, \bar{b}^2)$ such that $\underline{b}^1 < \bar{b}^1$, $\underline{b}^2 < \bar{b}^2$, we show that

$$f_A(\bar{b}^1, \bar{b}^2) - f_A(\bar{b}^1, \underline{b}^2) \geq f_A(\underline{b}^1, \bar{b}^2) - f_A(\underline{b}^1, \underline{b}^2). \tag{50}$$

Hence, if $\underline{b}^1 = \underline{b}^2 \equiv \underline{b} < \bar{b}^1 = \bar{b}^2 \equiv \bar{b}$, then (50) reduces to $f_A(\bar{b}, \bar{b}) - f_A(\bar{b}, \underline{b}) \geq f_A(\underline{b}, \bar{b}) - f_A(\underline{b}, \underline{b})$, which is (23).

We can argue like in the proof of Step 2 above to show that the inequality boils down to $\int_0^1 U_B^2(\underline{x}^2(t), \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t)dt \leq \int_0^1 U_B^2(\bar{x}^2(t), \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t)dt$, in which $\underline{x}^2(t)$ is the optimal x^2 given $(\underline{b}^1, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t)$ and $\bar{x}^2(t)$ is the optimal x^2 given $(\bar{b}^1, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t)$. Thus, it suffices to prove that $\underline{x}^2(t) \geq \bar{x}^2(t)$ for each $t \in [0, 1]$. The first-order conditions for $\underline{x}^1(t)$, $\underline{x}^2(t)$, and for $\bar{x}^1(t)$, $\bar{x}^2(t)$ are as follows (for the sake of brevity, we write $\bar{x}^i$, $\underline{x}^i$ instead of $\bar{x}^i(t)$, $\underline{x}^i(t)$):

$$\begin{aligned} U_A^1(\underline{x}^1, \underline{b}^1) - C_A'(\underline{x}^1 + \underline{x}^2) &= 0 \text{ and } U_A^2(\underline{x}^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C_A'(\underline{x}^1 + \underline{x}^2) = 0 \quad (51) \\ U_A^1(\bar{x}^1, \bar{b}^1) - C_A'(\bar{x}^1 + \bar{x}^2) &= 0 \text{ and } U_A^2(\bar{x}^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C_A'(\bar{x}^1 + \bar{x}^2) = 0 \quad (52) \end{aligned}$$

First, we prove that $\bar{x}^1 < \underline{x}^1$. Consider $F(x^1, x^2) \equiv U^1(x^1, \bar{b}^1) + U^2(x^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C_A(x^1 + x^2)$, which is maximized by $(\bar{x}^1, \bar{x}^2)$. Since F is concave, we have that

$$\begin{aligned} F\left(t\bar{x}^1 + (1-t)\underline{x}^1, t\bar{x}^2 + (1-t)\underline{x}^2\right) &= F\left(\underline{x}^1 + t(\bar{x}^1 - \underline{x}^1), \underline{x}^2 + t(\bar{x}^2 - \underline{x}^2)\right) \quad (53) \\ &\geq tF(\bar{x}^1, \bar{x}^2) + (1-t)F(\underline{x}^1, \underline{x}^2) \qquad \text{for each } t \in [0, 1]. \end{aligned}$$

[30] In Step 1, set $\underline{\mathbf{b}} = \underline{\mathbf{b}}^i(t)$, $\bar{\mathbf{b}} = \bar{\mathbf{b}}(t)$, and notice that $\underline{b}^i = \bar{b}^i$, but $\underline{b}^h < \bar{b}^h$ for each $h \neq i$.

Let $\ell(t)$ denote the left-hand side of (53) and $r(t)$ the right-hand side of (53). From (53), we see that $\ell(t) \geq r(t)$ for each $t \in [0,1]$, and since $\ell(0) = r(0)$, it is necessary that $\ell'(0) \geq r'(0)$. It is immediate that $r'(t) = F(\bar{x}^1, \bar{x}^2) - F(\underline{x}^1, \underline{x}^2) > 0$,[31] and $\ell'(t) = F_1(\underline{x}^1 + t(\bar{x}^1 - \underline{x}^1), \underline{x}^2 + t(\bar{x}^2 - \underline{x}^2))(\bar{x}^1 - \underline{x}^1) + F_2(\underline{x}^1 + t(\bar{x}^1 - \underline{x}^1), \underline{x}^2 + t(\bar{x}^2 - \underline{x}^2))(\bar{x}^2 - \underline{x}^2)$. Thus, $\ell'(0) = F_1(\underline{x}^1, \underline{x}^2)(\bar{x}^1 - \underline{x}^1) + F_2(\underline{x}^1, \underline{x}^2)(\bar{x}^2 - \underline{x}^2)$, and $F_1(\underline{x}^1, \underline{x}^2) = U_A^1(\underline{x}^1, \bar{b}^1) - C_A'(\underline{x}^1 + \underline{x}^2)$ is negative because of (51) and goods are strict substitutes, $F_2(\underline{x}^1, \underline{x}^2) = U_A^2(\underline{x}^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C_A'(\underline{x}^1 + \underline{x}^2)$ is zero because of (51). Therefore, $\bar{x}^1 < \underline{x}^1$ is necessary; otherwise, $\ell'(0) \leq 0 < r'(0)$.
Using $\bar{x}^1 < \underline{x}^1$, we compare $U_A^2(\underline{x}^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C_A'(\underline{x}^1 + \underline{x}^2)$ in (51) with $U_A^2(\bar{x}^2, \underline{b}^2 + (\bar{b}^2 - \underline{b}^2)t) - C_A'(\bar{x}^1 + \bar{x}^2)$ in (52), and since C_A is concave, we conclude that $\bar{x}^2 \leq \underline{x}^2$.

10.6 Proof of Proposition 3

Given that each seller offers only the contracts (q^*, t^*), $(\tilde{q}, \tilde{t})$, each buyer chooses a pair $(q_A, q_B) \in \{0, q^*, \tilde{q}\} \times \{0, q^*, \tilde{q}\}$. In order to shorten notation, we define $U^* \equiv U(q^*, 0)$, $\tilde{U} \equiv U(\tilde{q}, 0)$, $U_t^* \equiv U(q^*, q^*)$ (the subscript t means that the buyer buys quantity q^* *twice*, that is, from both sellers), $\tilde{U}^* \equiv U(q^*, \tilde{q})$, $\tilde{U}_t \equiv U(\tilde{q}, \tilde{q})$. Moreover, recall that $t^* = \frac{1}{2}U_t^* - \frac{1}{2}u$, $\tilde{t} = \tilde{U} - u$.
Step 1 Each buyer buys quantity q^* from each seller if and only if

$$2U^* - U_t^* \leq u \leq U_t^* + 2\tilde{U} - 2\tilde{U}^* \tag{54}$$

Proof We can derive (54) by arguing as in Step 1 of the proof of Proposition 2.
Step 2 No profitable deviation exists for a seller if and only if both (28) and (55) hold:

$$\begin{cases} \max\{U_t^* - 2f(q^*, q^*) + 2f(0, q^*), U_t^* - f(q^*, q^*) + f(0,0)\} \leq u \\ \text{and} \\ u \leq \min\{2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}], f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}]\} \end{cases} \tag{55}$$

Proof For each seller, the equilibrium profit is $\pi^* \equiv 2t^* - C^* = f(q^*, q^*) - U_t^* - u$, in which $C^* \equiv C(2q^*)$. We consider the possible deviations of seller A.
(i) The deviation that induces both buyers to buy only from A yields A the profit $f(0,0) - 2u$, and the inequality $f(0,0) - 2u \leq \pi^*$ reduces to $U_t^* - f(q^*, q^*) + f(0,0) \leq u$.
(ii) The deviation that induces buyer 1 to buy only from A and buyer 2 to buy q^* from B yields A the profit $f(0, q^*) - t^* - 2u$, and the inequality $f(0, q^*) - t^* - 2u \leq \pi^*$ reduces to $U_t^* - 2f(q^*, q^*) + 2f(0, q^*) \leq u$.
(iii) The deviation that induces buyer 1 to buy only from A and buyer 2 to buy $\tilde{q}$ from B yields A the profit $f(0, \tilde{q}) - \tilde{t} - 2u$, and the inequality $f(0, \tilde{q}) - \tilde{t} - 2u \leq \pi^*$ reduces to (28).

[31] Note that since F is strictly concave, $(\bar{x}^1, \bar{x}^2)$ is the *unique* maximum point for F; hence, $F(\bar{x}^1, \bar{x}^2) > F(\underline{x}^1, \underline{x}^2)$.

(iv) The deviation that induces both buyers to buy q^* from B yields A the profit $f(q^*, q^*) - 2t^* - 2u = f(q^*, q^*) - U_t^* - u$, which is just the equilibrium profit.
(v) The deviation that induces buyer 1 to buy q^* from B and buyer 2 to buy $\tilde{q}$ from B yields A the profit $f(q^*, \tilde{q}) - t^* - \tilde{t} - 2u$, and the inequality $f(q^*, \tilde{q}) - t^* - \tilde{t} - 2u \leq \pi^*$ reduces to $u \leq 2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}]$.
(vi) The deviation that induces both buyers to buy $\tilde{q}$ from B yields A the profit $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u$, and the inequality $f(\tilde{q}, \tilde{q}) - 2\tilde{t} - 2u \leq \pi^*$ reduces to $u \leq f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}]$.
The inequalities obtained from (i)-(ii) and (v)-(vi) are summarized by (55).

Step 3 If u satisfies (55), then it satisfies (54).

Proof The inequality $2U^* - U_t^* \leq U_t^* - f(q^*, q^*) + f(0, 0)$ is equivalent to $2U^* - C^* \leq f(0, 0)$, which follows from the definition of $f(0, 0)$. The inequality $f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}] \leq U_t^* + 2\tilde{U} - 2\tilde{U}^*$ is equivalent to $2\tilde{U}^* - C^* \leq f(\tilde{q}, \tilde{q})$, which follows from the definition of $f(\tilde{q}, \tilde{q})$.

Step 4 If C is strictly convex, then no profitable deviation exists for a seller if and only if (28) and (56) both hold, with

$$U_t^* - 2f(q^*, q^*) + 2f(0, q^*) \leq u \leq 2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}] \tag{56}$$

If C is concave, then no profitable deviation exists for a seller if and only if (28) and (57) both hold, with

$$U_t^* - f(q^*, q^*) + f(0, 0) \leq u \leq f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}] \tag{57}$$

Proof We prove that (55) reduces to (56) if C is strictly convex and that (55) reduces to (57) if C is concave.
The inequality $U_t^* - f(q^*, q^*) + f(0, 0) < U_t^* - 2f(q^*, q^*) + 2f(0, q^*)$ is equivalent to $f(q^*, q^*) - f(q^*, 0) < f(0, q^*) - f(0, 0)$, which holds if C is strictly convex, by (22). Conversely, if C is concave, then (23) applies, and it follows that $U_t^* - f(q^*, q^*) + f(0, 0) \geq U_t^* - 2f(q^*, q^*) + 2f(0, q^*)$ holds. The inequality $2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}] < f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}]$ is equivalent to $f(\tilde{q}, \tilde{q}) - f(\tilde{q}, q^*) < f(q^*, \tilde{q}) - f(q^*, q^*)$, which holds if C is strictly convex, by (22). Conversely, if C is concave then (23) applies, and it follows that $2f(q^*, q^*) - U_t^* - 2[f(q^*, \tilde{q}) - \tilde{U}] \geq f(q^*, q^*) - U_t^* - [f(\tilde{q}, \tilde{q}) - 2\tilde{U}]$.

Step 5 Increasing $\tilde{q}$ relaxes (28), (56), and (57).

Proof On the left-hand side of (28), $f(0, \tilde{q}) - \tilde{U} = \max_{x,y}[U(x, 0) + \int_0^y U_A(z, \tilde{q})dz - C(x + y)]$ is decreasing in $\tilde{q}$ since goods are substitutes. The same property implies that the right-hand side of (56), $2\tilde{U} - 2f(q^*, \tilde{q}) = -2\max_{x,y}[U(x, q^*) + \int_0^y U_A(z, \tilde{q})dz - C(x+y)]$, is increasing in $\tilde{q}$, as is the right-hand side of (57), $2\tilde{U} - f(\tilde{q}, \tilde{q}) = -\max_{x,y}[\int_0^x U_A(z, \tilde{q})dz + \int_0^y U_A(z, \tilde{q})dz - C(x + y)]$.

Step 6 Inequality (28) holds if $\tilde{q}$ is sufficiently large, and the whole set of values for u is obtained from (56) and (57) as $\tilde{q} \to +\infty$.

PROOF The left-hand side in (28) is equal to $\max_{x,y}[U(x,0)+\int_0^y U_A(z,\tilde{q})dz - C(x+y)]$, whereas the right-hand side in (28) is equal to $U_t^* - C^* = \max_{x,y}(U(x,y)-C(x+y))$. We prove that $\lim_{\tilde{q}\to+\infty}[U(x,0)+\int_0^y U_A(z,\tilde{q})dz] < U(x,y)$, which implies that (28) holds for a large $\tilde{q}$. Precisely, the previous inequality is equivalent to $\lim_{\tilde{q}\to+\infty}\int_0^y U_A(z,\tilde{q})dz < \int_0^y U_B(x,z)dz = \int_0^y U_A(z,x)dz$ (since U is symmetric), and the latter inequality holds since goods are strict substitutes.

Finally, notice that in the case in which C is strictly convex, if u and $\tilde{q}$ satisfy (29), (28), and $f(q^*,\tilde{q}) - U(0,\tilde{q}) \le f(q^*,q^*) - \frac{1}{2}(u+U(q^*,q^*))$, then (54) and (56) are satisfied. Hence, Proposition 3(a)(i-ii) identifies indeed an equilibrium that yields payoff u to each buyer.

In the case in which C is concave, if u and $\tilde{q}$ satisfy (30), (28), and $f(\tilde{q},\tilde{q}) - 2U(0,\tilde{q}) \le f(q^*,q^*) - U(q^*,q^*) - u$, then (54) and (57) are satisfied. Hence, Proposition 3(b)(i-ii) identifies indeed an equilibrium that yields payoff u to each buyer.

10.7 Proof of Proposition 5

For $j = A, B$, let $V_j^G \equiv \max_{q_j^1,\dots,q_j^n}(U^1(q_j^1,0)+\dots+U^n(q_j^n,0)-C_j(q_j^1+\dots+q_j^n))$. Then, in each equilibrium without any group, we have $\pi^A+u^1+\dots+u^n \ge V_A^G$ because we can argue as when $n=2$ (see the argument just after Proposition 2, and note that this argument does not depend on whether a group is formed): if $\pi^A+u^1+\dots+u^n < V_A^G$, then seller A can deviate by offering to the buyers a suitable trade in which each buyer buys only from A, and their joint payoffs increase to V_A^G.

Hence, we have $\pi^B \le V_{AB}^G - V_A^G$ and $\pi^A \le V_{AB}^G - V_B^G$, which implies $u^1+\dots+u^n \ge V_A^G+V_B^G-V_{AB}^G$, where $V_A^G+V_B^G-V_{AB}^G$ is just the total payoff of the buyers in the sell-out equilibrium with the grand coalition.

Now, we prove that without any group, there exists no equilibrium satisfying $u^1+\dots+u^n = V_A^G+V_B^G-V_{AB}^G$; hence, $u^1+\dots+u^n > V_A^G+V_B^G-V_{AB}^G$ in each equilibrium. We argue by contradiction and suppose that an equilibrium exists such that $u^1+\dots+u^n = V_A^G+V_B^G-V_{AB}^G$. Then, $\pi^A+\pi^B = 2V_{AB}^G - V_A^G - V_B^G$. This result together with $\pi^A \le V_{AB}^G - V_B^G$ and $\pi^B \le V_{AB}^G - V_A^G$ implies $\pi^A = V_{AB}^G - V_B^G$ and $\pi^B = V_{AB}^G - V_A^G$.

Then, we suppose that C_A (to fix the ideas) is strictly convex and consider the deviation of seller A with $b^i = q_B^{i*}$ for one i, $b^h = 0$ for $h \ne i$. This yields A the profit $f_A(0,\dots,q_B^{i*},\dots,0) - t_B^{i*} - (V_A^G+V_B^G-V_{AB}^G)$;[32] hence, this deviation is unprofitable if and only if $f_A(0,\dots,q_B^{i*},\dots,0) - V_A^G \le t_B^{i*}$. The existence of no profitable deviation for A requires that this inequality holds for $i = 1,\dots,n$. Hence, we have

$$\sum_{i=1}^{n} f_A(0,\dots,q_B^{i*},\dots,0) - nV_A^G \le t_B^{1*}+\dots+t_B^{n*}. \tag{58}$$

[32] The function f_A is defined in (46).

Then, notice that $\pi^B = V_{AB}^G - V_A^G$ is equivalent to $t_B^{1*} + ... + t_B^{n*} = V_{AB}^G - V_A^G + C_B(q_B^{1*} + ... + q_B^{n*})$. In addition, we use (58), $V_A^G = f_A(0, ..., 0)$, $V_{AB}^G + C_B(q_B^{1*} + ... + q_B^{n*}) = f_A(q_B^{1*}, ..., q_B^{n*})$ to obtain the inequality

$$\sum_{i=2}^{n} \left(f_A(0, ..., q_B^{i*}, ..., 0) - f_A(0, ..., 0) \right) \leq f_A(q_B^{1*}, ..., q_B^{n*}) - f_A(q_B^{1*}, ..., 0). \tag{59}$$

Finally, we use (47) to show that (59) cannot hold: setting $\underline{b}^i = 0$, $\bar{b}^i = q_B^{i*}$ for $i = 1, ..., n$ in (47) yields the opposite of (59).