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Web Appendix for “Inference on Vertical Contracts between Manufacturers and Retailers Allowing for Nonlinear Pricing and Resale Price Maintenance”

CÉLINE BONNET AND PIERRE DUBOIS

Ecole d’économie de Toulouse (TSE) - Manufacture des Tabacs
Aile Jean-Jacques Laffont - 21, allée de Brienne - 31000 TOULOUSE
Tél : +33 (0) 5 61 12 85 89 - Fax : +33 (0) 5 61 12 86 37 - **www.TSE-fr.eu** - contact@TSE-fr.eu

Toulouse School of Economics

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Céline Bonnet*, Pierre Dubois†

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1 Matrix Expression of First Order conditions

1.1 Linear Tariffs and Double marginalization

Let I_r define the $(J \times J)$ ownership matrix of the retailer r that is diagonal and whose element $I_r(j,j)$ is equal to one if retailer r sells product j and zero otherwise. Let S_p be the market share response matrix to retailer prices, containing the first derivatives of all market shares with respect to all retail prices, i.e.

$$S_p \equiv \begin{pmatrix} \frac{\partial s_1}{\partial p_1} & \cdots & \frac{\partial s_J}{\partial p_1} \\ \vdots & & \vdots \\ \frac{\partial s_1}{\partial p_J} & \cdots & \frac{\partial s_J}{\partial p_J} \end{pmatrix}$$

In vector notation, previous first order conditions (**??**) implies that the vector γ of retailer r's margins, i.e. the retail price p minus the wholesale price w minus the marginal cost of distribution c, is[1]

$$\gamma \equiv p - w - c = -\left(I_r S_p I_r\right)^{-1} I_r s(p). \tag{1}$$

*Toulouse School of Economics (GREMAQ, INRA), cbonnet@toulouse.inra.fr
†Toulouse School of Economics (GREMAQ, INRA, IDEI), dubois@toulouse.inra.fr

[1] Note that in all the following, when the inverse of non invertible matrices is used, it means the matrix of generalized inverse is considered, meaning that for example $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 0 \end{bmatrix}$.

Consider I_f the ownership matrix of manufacturer f that is diagonal and whose element $I_f(j,j)$ is equal to one if j is produced by the manufacturer f and zero otherwise. P_w the $(J \times J)$ matrix of retail prices responses to wholesale prices, containing the first derivatives of the J retail prices p with respect to the J' wholesale prices w, is introduced.

$$P_w \equiv \begin{pmatrix} \frac{\partial p_1}{\partial w_1} & .. & \frac{\partial p_{J'}}{\partial w_1} & .. & \frac{\partial p_J}{\partial w_1} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial p_1}{\partial w_{J'}} & .. & \frac{\partial p_{J'}}{\partial w_{J'}} & .. & \frac{\partial p_J}{\partial w_{J'}} \\ 0 & .. & 0 & .. & 0 \\ 0 & .. & 0 & .. & 0 \end{pmatrix}$$

Note that the last $J - J'$ lines of this matrix are zero because they correspond to private label products for which wholesale prices have no meaning.

The first order conditions (**??**) can then be expressed in matrix form and the vector of manufacturer's margins is[2]

$$\Gamma \equiv w - \mu = -(I_f P_w S_p I_f)^{-1} I_f s(p). \tag{2}$$

P_w can be deduced from the total differentiation of the retailer's first order conditions (**??**) with respect to wholesale price. Defining $S_p^{p_j}$ the $(J \times J)$ matrix of the second derivatives of the market shares with respect to retail prices whose element (l,k) is $\frac{\partial^2 s_k}{\partial p_j \partial p_l}$, we can write equation (**??**) in matrix form[3]:

$$P_w = I_r S_p (I_r - \widetilde{I}_r) \left[S_p I_r + I_r S_p' I_r + (S_p^{p_1} I_r \gamma | ... | S_p^{p_J} I_r \gamma) I_r \right]^{-1}. \tag{3}$$

Equation (3) shows that one can express the manufacturer's price cost margins vector $\Gamma = w - \mu$ as depending on the function $s(p)$ by substituting the expression (3) for P_w in (2).

As mentionned in section 3, like Sudhir(2001) we can consider only the direct effect of wholesale price on retail price. In this case, the matrix P_w has to be equal to the diagonal matrix, with one on the diagonal except for columns corresponding to private labels where it must be zero.

[2] Rows of this vector that correspond to private labels are zero.

[3] We use the notation $(a|b)$ for horizontal concatenation of a and b.

The model can also be considered where retailers and/or manufacturers collude perfectly simply by modifying ownership matrices. In the case of perfect price collusion between retailers, the price cost margins of the retail industry can be obtained by replacing the ownership matrices I_r in (1) by the identity matrix (the situation being equivalent to a retailer in a monopoly situation). Similarly, the price-cost margins vector for manufacturers can be obtained in the case of perfect collusion by replacing the ownership matrix I_f in (2) by a diagonal matrix where diagonal elements are equal to one except for private label goods.

1.2 *Two-part tariffs*

1.2.1 *With resale price maintenance*

When $w_k^* = \mu_k$, the first order conditions become in matrix notation:

$$I_f S_p(\gamma + \Gamma) + I_f s(p) = 0. \tag{4}$$

In the case of private label products, the first order conditions become in matrix notation: for $r = 1, .., R$

$$(\widetilde{I}_r S_p I_r)(\gamma + \Gamma) + \widetilde{I}_r s(p) = 0 \tag{5}$$

where $\widetilde{I}_r$ is the $(J \times J)$ ownership matrix of private label products of retailer r.

We thus obtain a system of equations with (4) and (5) where $\gamma + \Gamma$ is unknown.

$$\begin{cases} I_f S_p(\gamma + \Gamma) + I_f s(p) = 0 \text{ for } f = 1, .., F \\ (\widetilde{I}_r S_p I_r)(\gamma + \Gamma) + \widetilde{I}_r s(p) = 0 \text{ for } r = 1, .., R \end{cases}$$

After solving the system (see appendix **??**), we obtain the expression for the total price-cost margin of all products as a function of demand parameters and of the structure of the industry:

$$\gamma + \Gamma = -\left(\sum_r I_r S_p' \widetilde{I}_r S_p I_r + \sum_f S_p' I_f S_p\right)^{-1} \left(\sum_r I_r S_p' \widetilde{I}_r + \sum_f S_p' I_f\right) s(p). \tag{6}$$

Note that in the absence of private label products, this expression would simplify to the case where the total profits of the integrated industry are maximized, that is

$$\gamma + \Gamma = -S_p^{-1} s(p) \tag{7}$$

because then $\sum_f I_f = I$.

When wholesale prices w_k^* are such that $p_k^*(w_k^*) - w_k^* - c_k = 0$, the first order conditions become in matrix notation: for all $f = 1, .., F$

$$\gamma_f + \Gamma_f = (p - \mu - c) = -(I_f S_p I_f)^{-1} \left[I_f s(p) + I_f S_p \widetilde{I}(\widetilde{\gamma} + \widetilde{\Gamma}) \right]$$

where $\widetilde{\gamma} + \widetilde{\Gamma}$ is the vector of all private label margins and $\widetilde{I}$ is the ownership matrix for private labels ($\widetilde{I} = \sum_r \widetilde{I}_r$).

In this case, profit maximizing strategic pricing of private labels by retailers is also taken into account by manufacturers when they choose fixed fees and retail prices for their own products in the contract. The prices of private labels chosen by retailers maximize their profit on these private labels and the total price cost margin $\widetilde{\gamma}_r + \widetilde{\Gamma}_r$ for these private labels will be such that

$$\widetilde{\gamma}_r + \widetilde{\Gamma}_r \equiv p - \mu - c = -\left(\widetilde{I}_r S_p \widetilde{I}_r \right)^{-1} \widetilde{I}_r s(p). \tag{8}$$

1.2.2 *Without resale price maintenance*

Then the first order conditions become in matrix notation: for all $i \in G_f$

$$I_f P_w s(p) + I_f P_w S_p I_f \Gamma + I_f P_w S_p (p - w - c) = 0.$$

This implies that the manufacturer price cost margin is:

$$\Gamma = (I_f P_w S_p I_f)^{-1} \left[-I_f P_w s(p) - I_f P_w S_p (p - w - c) \right] \tag{9}$$

that allows for an estimate of the price-cost margins with demand parameters using (1) to replace $(p - w - c)$ and (3) for P_w. Remark again that formula (1) directly provides the total price-cost margin obtained by each retailer on his private label.

2 Detailed resolution of system of equations

Generically we have systems of equations to be solved in the following form

$$\begin{cases} A_f(\gamma + \Gamma) + B_f = 0 \\ \text{for } f = 1, .., G \end{cases}$$

where A_f and B_f are given matrices.

Solving this system amounts to solve the minimization problem

$$\min_{\gamma+\Gamma} \sum\nolimits_{f=1}^{G} \left[A_f(\gamma + \Gamma) + B_f\right]' \left[A_f(\gamma + \Gamma) + B_f\right]$$

which leads to the following expression to be found as solution

$$(\gamma + \Gamma) = \left(\sum\nolimits_{f=1}^{G} A_f' A_f\right)^{-1} \sum\nolimits_{f=1}^{G} A_f' B_f.$$