Online Appendix of "The World Income Distribution: The Effects of International Unbundling of Production"

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A Alternative Measures of Capital-Intensity Dispersion

Table A.1: Ratio of Dispersion Measures of Intermediates to Varieties

	1990	2000
St. Deviation	1.41	1.53
Interquartile Range	1.37	1.98
Range	1.79	1.99

Notes: We define variety as a 3-digit NAICS and intermediate as a 6-digit NAICS (the highest level of disaggregation available). We use the 1997 direct requirement U.S. Input-Output tables from the BEA to impute the weight of each intermediate in the production of each variety. We compute capital-intensity as the cost share of capital as in the NBER CES Manufacturing database. In particular, the cost share of capital is computed as $\alpha_k = 1 - \sum_{i \in \mathcal{I}} \alpha_i$, where $\mathcal{I}$ denotes production workers, non-production workers, energy and materials. Std. Dev. for varieties refers to the standard deviation of the average capital-intensity of each variety. Std. Dev. for intermediates refers to the average of the standard deviation computed at six-digit level conditional on belonging to a given variety. Analogous definitions are used to compute the range and the interquartile range (Q3-Q1).

B Equilibrium Refinement for the Symmetry Breaking

In this section, we formalize the arguments stated in Section 3.1 and introduce our equilibrium refinement concept. The refinement is based on using arbitrarily small perturbations to productivity across countries. In our view, this refinement is natural, as the key economic message from symmetry breaking is that arbitrary small differences are magnified to non-arbitrary differences.

Let $\delta > 0$ and $\mathcal{J} = \{1, \ldots, J\}$. Consider the following perturbation operator $\Pi : \mathcal{J} \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$, such that $\Pi(j, \theta) = \theta + \frac{J-j}{J}\delta \equiv \tilde{\theta}_j$. In words, the perturbation operator increases the productivity of a country as a function of its rank j. Note that this operator introduces a strict ranking of productivities, as $\tilde{\theta}_j > \tilde{\theta}_{j+1}$ for all $j \in \mathcal{J}$.

Consider the steady-state equilibrium in the perturbed productivities. From our analysis in Section 2.2.1, the steady-state equilibrium features the threshold property whereby country j specializes in $z \in (z_{j-1}, z_j]$. Denote by $\varepsilon = \sup_{i,j} |\tilde{\theta}_i - \tilde{\theta}_j|$ for $i, j \in \mathcal{J}$ with $i \neq j$. The maximum difference in perturbed productivities is obtained when comparing θ_1 and θ_J as the perturbation is a monotonically decreasing function. Thus, $\varepsilon \leq \delta$.

The first order approximation to the recursive assignment equation (19) for two adjacent countries $\tilde{\theta}_j$ and $\tilde{\theta}_j + \delta$ around $\delta = 0$ is given by

$$1 + \frac{\delta}{\theta(1 - z_j)} + O(\delta)^2 = \frac{\Delta_j}{\Delta_{j+1}} \qquad \text{for} \quad j = 1, \ldots, J-1. \tag{B.1}$$

Note that the right hand side of (B.1) is independent of δ, while the left hand side depends on δ. Moreover, as $\theta > 0$ and $1 - z_j \in (0, 1)$, $\theta(1 - z_j)$ is bounded. Thus, taking the limit of (B.1) for δ going to zero

$$\lim_{\delta \to 0} 1 + \frac{\delta}{\theta(1 - z_j)} + O(\delta)^2 = 1 = \frac{\Delta_j}{\Delta_{j+1}} \qquad \text{for} \quad j = 1, \ldots, J-1.$$

The resulting equation coincides with the one analyzed in Section 3.1.2. We have shown in Proposition 2 that the solution to this equation is unique and given by (22). Thus, as by taking the limit $\delta \to 0$, $\varepsilon \to 0$, i.e., countries become identical, we have that the perturbation operator Π generates a unique solution.

Finally, let Σ denote a permutation operator that defines a bijection $\Sigma : \mathcal{J} \longrightarrow \mathcal{J}$, such that $\Sigma(j) = i$, for $i, j \in \mathcal{J}$. It is clear that making any permutation of this type to the ordering of countries does not affect the unique equilibrium other than by changing the labels of which country falls in which place of the assignment. In other words, for any permutation operation Σ, the equilibrium income levels generated by the perturbation operator with the permutation on country ordering Σ, $\Pi\left(\Sigma\left(\mathcal{J}\right), \theta\right)$, are identical to the original $\Pi\left(\mathcal{J}, \theta\right)$. Thus, this analysis makes precise our claim in the paper that the equilibrium is unique up to permutations in the country labeling.

C Endogenous number of varieties

In this section we provide an exact microfoundation to the exogenous number of varieties that we postulated in the baseline model of the main text.

We assume that there exists an innovation sector that produces new varieties. The innovators sell the patents to the producer of varieties. Inventors extract all the surplus of the producer of varieties, who has monopoly rights on the production of the variety.

The final good is needed to produce innovation. In particular, we follow Jones (1995) and assume that the production function of ideas perceived by an innovator is

$$\mu_j = \phi_j i_j,$$

where $\phi_j = (\widetilde{\kappa}\theta_j)^{1-\lambda}(i_j)^{\lambda-1}$ and i_j is the amount of final good devoted to innovation. One can think of ϕ_j as the probability of finding a new idea, which is increasing with the productivity in the country and decreasing with the number of innovators looking for a new idea.

The innovator sells the blueprint to the producer of a variety who has monopoly rights on the production of the variety. However, we assume that there exists a competitive fringe that can copy the variety at a marginal cost $(1+\sigma)$ higher than the blueprint's marginal cost. This imposes a constraint on the price that the monopoly producer of any variety v can charge,

$$p_j(v) = (1+\sigma)MC_j(v),$$

where $MC_j(v)$ denotes the marginal cost of production using the blueprint. The demand of variety v at this price is

$$x_j(v) = \frac{1}{(1+\sigma)MC_j(v)} \frac{\sum_{i=1}^{J} p_i Y_i}{N}.$$

Thus, the profits of the producer of a variety are total revenues less variable costs and the price of purchasing the idea p_j^R,

$$\pi_j(v) = \frac{(1+\sigma)MC_j(v)}{(1+\sigma)MC_j(v)} \frac{\sum_{i=1}^{J} p_i Y_i}{N} - \frac{MC_j(v)}{(1+\sigma)MC_j(v)} \frac{\sum_{i=1}^{J} p_i Y_i}{N} - p_j^R,$$

which simplifies to

$$\pi_j(v) = \frac{\sigma}{1+\sigma} \frac{\sum_{i=1}^{J} p_i Y_i}{N} - p_j^R.$$

Profit maximization in the innovation sector implies that

$$p_j = \phi_j p_j^R.$$

Given that innovators extract all the rents of the producer of varieties, it follows that $p_j^R = \frac{\sigma}{1+\sigma} \frac{\sum_{i=1}^{J} p_i Y_i}{N}$.

Note that we can set $p_j = 1$ and find that the amount of final good used in the innovation sector in country j is

$$i_j = p^{R^{\frac{1}{1-\lambda}}} \widetilde{\kappa}\theta_j.$$

Finally, we can use this equation and the production function of varieties to find that the number of varieties in country j is

$$\mu_j = \kappa \theta_j,$$

with $\kappa = \widetilde{\kappa}^{1-\lambda} \left(\frac{\sigma}{1+\sigma} \frac{\sum_{i=1}^{J} Y_i}{\sum \theta_j} \right)^{\lambda}$.

D General Production Function of Intermediates

We now consider an intermediate case in which only a fraction α of each intermediate z can traded (denoted by T) and the reminder $1-\alpha$ is not traded (denoted NT). Note that this is an intermediate case between the trade regime without unbundling ($\alpha = 0$) and the trade regime with unbundling ($\alpha = 1$). In this case, the production function of varieties is given by the next expression

$$x_j(v, t) = \exp \left[\int_0^1 dz \left(\alpha \ln a_j^T(z) + (1-\alpha) \ln a_j^{NT}(z) \right) \right],$$

where $a_j^i(z)$ denotes the amount of intermediates z with $i = \{NT, T\}$.

D.1 Equilibrium Derivation

Note that the problem of the consumer and final good producer is the same as in the cases studied before. In particular, the problem of the final-good producer is

$$\max p_j Y_j - \int_0^N p(v) x(v) dv.$$

Therefore, given that varieties are traded, the aggregate demand for variety v is

$$p(v)x(v) = \frac{Y^{world}}{N},$$

where $Y^{world} = \sum_j p_j Y_j$.

The problem of the producer of variety v in country j is

$$\max p_j(v) x_j(v) - \int_0^1 p^T(z) a^T(z) dz - \int_0^1 p^{NT}(z) a^{NT}(z) dz.$$

It follows that

$$\begin{aligned} p^{NT}(z) a^{NT}(z) &= (1-\alpha) p(v) x(v), \\ p^T(z) a^T(z) &= \alpha p(v) x(v). \end{aligned}$$

The demand for non-tradeable intermediates only comes from the variety producers in country j and the demand for traded intermediate z in country j comes from all producers of varieties in the world, thus

$$p^{NT}(z)a^{NT}(z) = (1-\alpha)\frac{\mu_j}{N}Y^{world},$$
$$p^{T}(z)a^{T}(z) = \alpha Y^{world}.$$

Finally, the problem of intermediate producer z in country j is

$$\max p_j^i(z)a_j^i(z) - r_j k_j^i(z) - w_j l_j^i(z).$$

It follows that

$$r_j k_j^i(z) = z p_j^i(z)a_j^i(z),$$
$$r_j l_j^i(z) = (1-z)p_j(z)a_j^i(z).$$

Each country needs to produces all the non-traded intermediates and we assume that it specializes in the production of a subset Z_j of traded intermediates. Therefore, the capital market clearing conditions are given by

$$1 = l_j^{NT} + l_j^T = \int_0^1 (1-z)\frac{p_j^{NT}(z)a_j^{NT}(z)}{w_j}dz + \int_{z\in Z_j}(1-z)\frac{p_j^T(z)a_j^T(z)}{w_j}dz = \left[\frac{1}{2}(1-\alpha)\frac{\mu_j}{N} + \alpha\int_{z\in Z_j}(1-z)dz\right]\frac{Y^{world}}{w_j}.$$
$$r_j k_j = r_j k_j^{NT} + r_j k_j^{NT} = \int_0^1 z p_j^{NT}(z)a_j^{NT}(z)dz + \int_{z\in Z_j} z p_j^T(z)a_j^T(z)dz = \left[\frac{1}{2}(1-\alpha)\frac{\mu_j}{N} + \alpha\int_{z\in Z_j} zdz\right]Y^{world}.$$

Finally, we need to compute the trade balance, which is given by the next equation.[1]

$$\frac{\mu_j}{N}\left[Y^{world} - P_jY_j\right] + \alpha Z_j\frac{N-\mu_j}{N}Y^{world} = \frac{N-\mu_j}{N}P_jY_j + \alpha(1-Z_j)\frac{\mu_j}{N}Y^{world}.$$

Rearranging the above expression, it follows that the world income share of country j is

$$s_j = \frac{P_jY_j}{Y^{world}} = (1-\alpha)\frac{\mu_j}{N} + \alpha Z_j = (1-\alpha)\frac{\theta_j}{\sum_j \theta_j} + \alpha Z,$$

where the last equality follows from the assumption that $\mu_j = \kappa\theta_j$. Note that when $\alpha = 1$,

[1]Each country produces μ_j varieties, therefore, the value of the exported varieties is $\mu_j\left[\frac{Y^{world}}{N} - \frac{P_jY_j}{N}\right]$. The country imports the rest of varieties, therefore, the value of imported varieties is $(N-\mu_j)\frac{P_jY_j}{N}$. The country produces the traded intermediates in the set Z. The exports of intermediates are the foreign demand for these intermediates, that is, the demand of foreign producers of varieties. A given variety producer demands $\frac{1}{p_{j(z)}}\frac{\alpha Y^{world}}{N}$ of each intermediate z, therefore, the value of all exported intermediates in country j is $\alpha Z_j\frac{N-\mu_j}{N}Y^{world}$. Similarly, the domestic producers of varieties need to import all the intermediates which are not produced at home. Therefore, the value of imported intermediates is $\alpha(1-Z_j)\frac{\mu_j}{N}Y^{world}$.

the income share is only determined by trade in intermediates (unbundling equilibrium) and when $\alpha = 0$ we have that the income share only depends on the share of varieties (without unbundling equilibrium).

Finally, a note on the equilibrium selection of intermediates. In the steady-state the cost of producing intermediate z in country j is

$$c_j(z) = \theta_j^{-1} w_j^{1-z} \rho^z$$

Therefore, following a similar argument as in the main text, the endogenous selection of intermediates is given by the next difference equation

$$\left(\frac{\theta_j}{\theta_{j+1}}\right)^{\frac{1}{1-z_j}} = \frac{\frac{1}{2}(1-\alpha)\frac{\theta_j}{\sum_{i=1}^J \theta_j} + \alpha\Delta_j}{\frac{1}{2}(1-\alpha)\frac{\theta_{j+1}}{\sum_{i=1}^J \theta_i} + \alpha\Delta_{j+1}}, \tag{D.1}$$

where $\Delta_j = \int_{z_j}^{z_{j-1}} (1-z)dz$ and with terminal conditions $z_0 = 1$ and $z_J = 0$.

D.2 Symmetry Breaking

We next show that the symmetry breaking results extends to this general case. For ex-ante identical countries, we have that $\theta_j = \theta_{j+1}$. This implies that the left hand side of (D.1) is 1. Moreover, as $\theta_j / \sum_{i=1}^J \theta_i$ is equal to $\theta_{j+1} / \sum_{i=1}^J \theta_i$, it has to be the case that $\Delta_j = \Delta_{j+1}$ so that the right hand side of (D.1) is also 1. This shows that the solution thresholds z_j are given by the solution to the recurrence equation given by $\Delta_j = \Delta_{j+1}$ which is the same condition we have for the equilibrium with unbundling, $\alpha = 1$.

E Proofs of Propositions in Section 3

Proof of Proposition 2 Equation (22) can be derived as follows. Denoting by $x_j = z_j - \frac{z_j^2}{2}$, equation (22) implies that $x_j - x_{j+1} = d$ for some $d > 0$. Moreover, $\sum_{j=1}^J x_j = 1/2$. These two conditions imply that $d = 1/2J$. Thus, $x_j = x_{j+1} + d$ and $x_j = (J-j)/2J$. Solving for z_j, we find equation (22).That is, using the definition of x_j in terms of z_j we obtain $-\frac{z_j^2}{2} + z_j - \frac{J-j}{2J} = 0$. The unique solution of this second-order equation that is between zero and one is $z_j = 1 - \sqrt{j/J}$. Note that it satisfies the boundary conditions, $z_J = 0$ and $z_0 = 1$. Alternatively, the same thresholds can be derived by taking the limit for $J \to \infty$ and working with a differential equation, as in Section 3.2.2. In this case, the differential equation governing the assignment is $(1 - z(j))\frac{z''(j)}{z'(j)} - z'(j) = 0$,with terminal conditions $z(0) = 1$ and $z(J) = 0$. The solution to this differential equation is (22).

Proof of Proposition 3 We need to check that $\Delta s_2 = \frac{1}{2}(z^* - \theta_2) < 0$,where z^* is the solution to the threshold z^* that divides the intermediates produced by each country and given by the next expression, $A(\theta, z^*) = B(z^*)$,where $A(\theta, z) = \left(\frac{\theta_1}{\theta_2}\right)^{\frac{1}{1-z}}$ and $B(z) = \frac{\frac{1}{2}-\left(z-\frac{z^2}{2}\right)}{\left(z-\frac{z^2}{2}\right)}$. Note that, on the one hand, $A(\theta, z)$ is increasing in z with $A(\theta, z = 0) = \theta_1/\theta_2 > 1$ and $\lim_{z\to 1} A(\theta, z) = \infty$. On the other hand, $B(z)$ is decreasing in z with $\lim_{z\to 0} B(z) = \infty$ and $B(z = 1) = 0$. This implies that the solution to this equation is unique. Moreover, z^* is continuous and monotonically decreasing with θ_1/θ_2. Moreover, since $\theta_1 + \theta_2 = 1$, $\theta_2 < \theta_1$ implies that $\theta_2 \in (0, 1/2)$. We know that for $\theta_2 = 1/2$, $z^* = 1 - \sqrt{1/2} < 1/2$. Therefore, for $\theta_2 = 1/2$, it is verified that $\Delta s_2 < 0$. Thus, we only need to check that $\Delta s_2 < 0$ for $\theta_2 = \varepsilon$, where ε is a positive number. In other words, we need to show that $z^* < \varepsilon = \theta_2$. When $\theta_2 = \varepsilon$, the equilibrium threshold z is implicitly defined by $\varepsilon^{\frac{1}{1-z}} = \frac{z-\frac{z^2}{2}}{\frac{1}{2}-\left(z-\frac{z^2}{2}\right)}$. Note that $z = 0$ does not solve this equation. In particular, the left-hand side is larger than the right-hand side. Moreover, it is straightforward to check that $z = \varepsilon$ does not solve this equation either. In addition, the left-hand side is smaller than the right-hand side. Thus, Bolzano's Theorem guarantees that there exists $z^* \in (0, \varepsilon)$ that solves this equation.

Finally, we prove that the relative capital income of country 2 declines with unbundling. That is, $\left(\frac{\rho k_2}{\rho k_1}\right)^{\text{with}} - \left(\frac{\rho k_2}{\rho k_1}\right)^{\text{without}} = \frac{z^{*2}}{1-z^{*2}} - \frac{\theta_2}{\theta_1} < 0$. Notice that $z^* < \theta_2$ directly implies that $\frac{z^{*2}}{1-z^{*2}} - \frac{\theta_2}{\theta_1} < 0$. Rearranging terms and noting that $\theta_1+\theta_2 = 1$, this condition becomes ${z^*}^2 < \theta_2$, which it is true because $z^* < \theta_2 < 1$.

Proof of Proposition 4 and 5 The change in the income share of country j is

$$\Delta s(z) = z\lambda \left(\frac{1}{1-z} - e^{1-z}\right). \tag{E.1}$$

Note that at $\Delta s(1) = \infty$ and that $\Delta s(0) = 0$. Also, note that $1/(1-z)$ is increasing in the relevant domain while e^{1-z} is decreasing. Moreover, at $z = 0$, $e > 1$ and at $z = 1$, $\infty > 1$. Thus, the two curves cross once (actually at a value $1 - W(1) \simeq .43$).

$$\frac{d\Delta s}{dz} = \lambda \left(\frac{1}{(1-z)^2} - e^{1-z}(1-z)\right). \tag{E.2}$$

It is readily verified that the derivative is positive for $z \in (1 - 3W(1/3) \simeq .23, 1]$ and negative otherwise. Moreover $\frac{d\Delta s}{dz}(0) < 0$ and $\frac{d\Delta s}{dz}(1) = \infty$.

The second derivative is

$$\frac{d^2\Delta s}{dz^2} = \lambda \left(\frac{2}{(1-z)^3} + e^{1-z}(2-z)\right), \tag{E.3}$$

which is positive for all $z \in [0, 1]$.

Finally we can analyze the shape of $\Delta s(j)$ given the previous results given that we can write $\Delta s(j(z))$. Recall that the mapping of z to j is continuously decreasing with $s(0) = \infty$

and $s(1) = 0$. This shows that $\Delta s_j(0) = \infty$, $\Delta s_j(j) > 0$ for $j < j(1 - W(1))$ and increasing for $j > j(1 - W(1))$, and $\Delta s_j(\infty) = 0$. Using the implicit function theorem we have that $d\Delta s_j/dj = d\Delta s_j(z(j))/dj$

$$\frac{d\Delta s_j}{dj} = \frac{d\Delta s_j(z(j))}{dz(j)}\frac{dz(j)}{dj} = \frac{d\Delta s_j(z(j))}{dz(j)}\frac{1}{\frac{dj}{dz}}. \tag{E.4}$$

As $\frac{dj}{dz} < 0$, we have that $\frac{d\Delta s_j}{dj}$ is decreasing for $j \in [0, j(1-3W(1/3))$ and increasing thereafter. Moreover, note that as $\frac{d\Delta s_j(z(j))}{dz(j)}\Big|_{z=0}$ is bounded and $\frac{dz(j)}{dj}\Big|_{z=0} = \infty$ we have that $\frac{d\Delta s_j}{dj}\Big|_{j=\infty} = 0$. Thus, as we have a function it cannot be convex throughout its support.

Finally, for the second derivative, using that

$$\frac{d^2 z(j)}{dj^2} = -\frac{d^2 j(z)}{dz^2}\left(\frac{1}{\frac{dj(z)}{dz}}\right)^3 \tag{E.5}$$

we have that

$$\frac{d^2\Delta s_j}{dj^2} = \frac{d}{dj}\left(\frac{d\Delta s_j(z(j))}{dz(j)}\frac{1}{\frac{dj}{dz}}\right) \tag{E.6}$$

$$= \frac{d^2\Delta s_j(z(j))}{dz(j)^2}\left(\frac{1}{\frac{dj}{dz}}\right)^2 - \frac{d\Delta s_j(z(j))}{dz(j)}\frac{d^2 j(z)}{dz^2}\left(\frac{1}{\frac{dj(z)}{dz}}\right)^3. \tag{E.7}$$

The first term is always positive. The second term has the first derivative of the share which is decreasing and then increasing in j, the derivative of $dj(z)/d(z)$ which is always negative and the term

$$\frac{d^2 j(z)}{dz^2} = \frac{1}{\lambda z^2}, \tag{E.8}$$

which is always positive. For $j < j(1-3W(1/3)$, we have that $z > 1-3W(1/3)$, which implies that the second derivative is unambiguously convex. For $j > j(1 - 3W(1/3))$ we have that $z < 1 - 3W(1/3)$ and the sign is ambiguous. We have that

$$\frac{d^2\Delta s_j(z(j))}{dz(j)^2} - \frac{d\Delta s_j(z(j))}{dz(j)}\frac{d^2 j(z)}{dz^2}\frac{1}{\frac{dj(z)}{dz}} = \lambda\frac{1 - e^{1-z}(1-z)^5 + 2z}{z(1-z)^3} \tag{E.9}$$

As $1 + 2z$ is increasing in z and $e^{1-z}(1 - z)^5$ is decreasing, and the numerator evaluated at 0 is negative $1 - e < 0$ and at $1/2$ is positive $2 > e^{1/2}/2$, we have a unique solution such that below a critical threshold the equation is concave. This threshold is given by the solution to $1-e^{1-z}(1-z)^5+2 = 0$ which is approximately $z = .123$ (and thus smaller than $1-3W(1/3)$).

F Proofs and detailed derivations in Section 4

F.1 Results in Section 4.1

Note that the general solution to the differential equation is

$$\frac{\lambda(1-z(j))}{C_1} = 1 + W\left(\frac{e^{-1-\frac{\lambda^2(j+C_2)}{C_1}}}{C_1}\right). \tag{F.1}$$

The first boundary condition is that $z(0) = 1$, which yields

$$W\left(\frac{e^{-1-\frac{\lambda^2 C_2}{C_1}}}{C_1}\right) = -1. \tag{F.2}$$

Thus,

$$\frac{e^{-1-\frac{\lambda^2 C_2}{C_1}}}{C_1} = -e^{-1} \tag{F.3}$$

and we can express C_2 as

$$C_2 = \frac{-C_1 \ln(-C_1)}{\lambda^2}. \tag{F.4}$$

Substituting in the general solution and simplifying, we find that

$$j = -\frac{1-z}{\lambda} - \frac{C_1}{\lambda^2}\ln\left(1 - \frac{\lambda(1-z)}{C_1}\right). \tag{F.5}$$

or alternatively,

$$\frac{\lambda(1-z(j))}{C_1} = 1 + W\left(-e^{-1-\frac{\lambda^2 j}{C_1}}\right) \tag{F.6}$$

The second terminal condition is that $z(\underline{j}) = 0$. Thus, substituting in the previous equations we have that

$$\underline{j} = -\frac{1}{\lambda} - \frac{C_1}{\lambda^2}\ln\left(1 - \frac{\lambda}{C_1}\right). \tag{F.7}$$

Note that for $\underline{j} > 0$, it has to be the case that $C_1 > \lambda$ or that $C_1 < 0$. Rearranging, we find that

$$-\lambda(1+\lambda\underline{j}) = C_1 \ln\left(1 - \frac{\lambda}{C_1}\right). \tag{F.8}$$

The left hand side of this expression is negative and decreasing in $\underline{j}$. Note for $\underline{j} \to \infty$ it becomes $-\infty$. This implies that $C_1 = \lambda$ or that $C_1 = -\infty$. The latter case would yield a constant function in the assignment function, which cannot be a solution. Thus, we select $C_1 = \lambda$. Moreover, as the domain of W is $[-1, \infty)$ the branch of C_1 that can solve (D.6) is $C_1 \geq \lambda$. Note that this implies that the solution $C_1(\underline{j})$ is continuous in the parameter $\underline{j}$. For

$0 < \underline{j} < \infty$, as the left hand is negative and decreasing in $\underline{j}$. Moreover, the first derivative of the right hand of (D.8) is

$$\frac{\lambda}{C_1 - \lambda} + \ln\left(\frac{C_1 - \lambda}{C_1}\right). \tag{F.9}$$

Note that this function asymptotes to $+\infty$ when $C_1 \to \lambda$ and to 0 when $C_1 \to \infty$. The second derivative is

$$\frac{-\lambda^2}{C_1(C_1 - \lambda)^2} < 0. \tag{F.10}$$

for all $C_1 \geq \lambda$. Thus, as the function is strictly convex over $[C_1, \infty)$, we have that it is monotonically decreasing. This implies, the first derivative (F.9) is always positive. We have shown that the right hand side of (F.8) is monotonically increasing and convex. Moreover, it is readily verified that the range of the right hand side is $(\infty, 0]$. Hence, the solution to (F.8) exists and is unique. Moreover, as the left hand side is decreasing in $\underline{j}$, we have that $C_1(\underline{j})$ is decreasing in $\underline{j}$.

The solution for $\underline{j} < \infty$ can be characterized implicitly proceeding in an analogous manner as in (F.3) to obtain

$$C_1^* = \frac{\lambda(1 + \lambda\underline{j})}{(1 + \lambda\underline{j}) + W\left(-e^{-(1+\lambda\underline{j})}(1 + \lambda\underline{j})\right)}. \tag{F.11}$$

Note that for $\underline{j} = \infty$, $C_1 = \lambda$ and otherwise $C_1 > \lambda$. If $j \to 0$ we have that $C_1 = \infty$ and we obtain a flat assignment (the only country produces everything). Indeed, C_1 is monotonically declining in $\underline{j}$

Thus, the equilibrium assignment is characterized by

$$j = -\frac{1-z}{\lambda} - \frac{C_1^*(\underline{j})}{\lambda^2} \ln\left(1 - \frac{\lambda(1-z)}{C_1^*(\underline{j})}\right). \tag{F.12}$$

The assignment $j(z, \underline{j})$ is decreasing in j. To see this, take the derivative of (F.12) with respect to $\underline{j}$ to find that

$$\frac{\partial j(z, \underline{j})}{\partial \underline{j}} = \frac{C^{*'}(\underline{j})}{\lambda^2}\left(\frac{(1-z)\lambda}{(1-z)\lambda - C^*(\underline{j})} - \ln\left(1 - \frac{(1-z)\lambda}{C^*(\underline{j})}\right)\right). \tag{F.13}$$

The term inside brackets is always negative (it is minus (F.9)). Thus the partial derivative is negative. As a result, taking the derivative of the inverse function, we have that $z(j; \underline{j})$ is also decreasing in $\underline{j}$. This is illustrated in figure 6a.

Finally, we are interested in computing the cross-partial of $z(j;\underline{j})$. Note that

$$\frac{\partial j(z;\underline{j})}{\partial z} = -\frac{1-z}{C_1(\underline{j}) - \lambda(1-z)}, \tag{F.14}$$

$$\frac{\partial z(j;\underline{j})}{\partial j} = -\frac{C_1(\underline{j}) - \lambda(1 - z(j;\underline{j}))}{1 - z(j;\underline{j})}. \tag{F.15}$$

Taking the derivative of the second equation we find that

$$\frac{\partial^2 z(j;\underline{j})}{\partial z \partial \underline{j}} = \frac{-(1-z)C_1'(\underline{j}) - C_1 \partial z/\partial \underline{j}}{(1-z)^2} > 0$$

as $1 - z \geq 0$, $C_1' < 0$, $C_1 > 0$ and $\partial z/\partial \underline{j} < 0$, the previous equation is unambiguously positive.

For the southern countries, those countries with $j > \underline{j}$ where $\theta(\underline{j}) = \underline{\theta}$, the world income share is just the relative number of varieties

$$s_j^B = \frac{\mu_j}{\int \mu_j dj} = \lambda \exp(-\lambda j). \tag{F.16}$$

For the northern countries $j < \underline{j}$, the income share calculation differs from (17) because the demand of intermediates comes only for countries that are integrated in the global supply chain. Following the same steps as above, it is straightfoward to derive that the income share is

$$s_j^B = -\left(z_j^B\right)' \left(1 - \frac{\int_{\underline{j}}^{\infty} \mu_j dj}{\int_0^{\infty} \mu_j dj}\right), \tag{F.17}$$

where z_j^B is the equilibrium assignment of intermediates when only northern countries can trade intermediates.

Characterization of the changes in the world income distribution The change in the world income distribution is thus given by

$$\Delta s_j(z) = -z'(j) - \left((1 - \mathbb{1}_{\underline{j}}) z'(j;\underline{j}) \left(1 - e^{-\lambda \underline{j}}\right) + \mathbb{1}_{\underline{j}} \lambda e^{-\lambda j}\right) \tag{F.18}$$

where $\mathbb{1}_{\underline{j}}$ is an indicator function that takes value of 1 if $j > \underline{j}$ and zero otherwise. The first term $z'(j)$ refers to the derivative of (26), which is the particular case $z'(j;\underline{j} = \infty)$. From

(D.17) we have that $z'(j) < z'(j;\underline{j})$. However, the presence of the extra term, implies that

$$\frac{d^2 z(j;\underline{j})\left(1-e^{-\lambda\underline{j}}\right)}{dj d\bar{j}} = \frac{d}{d\underline{j}}\left(\lambda - \frac{C_1\underline{j}}{1-z(j;\underline{j})}\right)(1-e^{-\lambda\underline{j}}) \tag{F.19}$$

$$= \frac{-C_1'(1-z) - C_1\partial z/\partial\underline{j}}{(1-z)^2} > 0 \tag{F.20}$$

as $C_1 > \lambda$, $0 < z < 1$, $C_1' < 0$ and $\partial z\partial/\underline{j} < 0$.[2] This implies, that for $j < \underline{j}$ the equilibrium with the integrated world generates a higher share than the equilibrium in which the south does not participate in unbundling. Thus, this shows that the "North" always increases its share of world output with the south joining the global supply chain.

For the south there are two possible cases, either all countries lose or some lose and the southern countries with highest TFP win. To see this, we show that the income shares with complete unbundling is decreasing in j faster than without the south joining the global supply chain. And that depending on $\underline{j}$, the income share of the most productive southern country without unbundling can be either higher or lower than in the final equilibrium.

We analyze when the two curves cross. Note that the income share before we have the South joining

$$s(j) = \lambda e^{-\lambda j}$$

can be expressed in terms of the ex-post assignment $j(z)$, to obtain

$$s(z) = \lambda e^{1-z+\ln z}.$$

The income share when all countries join is

$$s^{unbundling}(z) = \frac{\lambda z}{1-z}. \tag{F.26}$$

[2] One can further characterize the function

$$\frac{d^2 z(j;\underline{j})\left(1-e^{-\lambda\underline{j}}\right)}{dj^2} = -\frac{j''(z)}{(j'^3} \tag{F.21}$$

$$= C_1\frac{C_1-(1-z)\lambda}{(1-z)^3} > 0 \tag{F.22}$$

$$\frac{d^3 z(j;\underline{j})\left(1-e^{-\lambda\underline{j}}\right)}{dj^3} = -j'''(z'^4 - 3j''z''(z'^2 \tag{F.23}$$

$$= \frac{3C_1}{(1-z)(C_1-\lambda(1-z))^5} - 2\lambda C_1\frac{(C_1-\lambda(1-z))}{(1-z)^4} \tag{F.24}$$

$$\frac{d^2 z(j;\underline{j})\left(1-e^{-\lambda\underline{j}}\right)}{dj^2 d\underline{j}} = \frac{-(2C_1-\lambda(1-z))(1-z)C_1' + C_1\partial z/\partial\underline{j}(-3C_1+2\lambda(1-z)}{(1-z)^4} > 0. \tag{F.25}$$

The result that these derivatives are unambiguously signed follows from the fact that $C_1 > \lambda$, $0 < z < 1$, $C_1' < 0$ and $\partial z\partial\underline{j} < 0$.

Equating the previous two equations, we find that the solution is

$$\tilde{z} = 1 - W(1) \tag{F.27}$$

Thus, this implies that in order to have a crossing $\underline{j}$ has to be less than

$$j(z = 1 - W(1)) = \frac{1 - W(1) - \ln(1 - W(1)) - 1}{\lambda} = -\frac{W(1) + \ln(1 - W(1))}{\lambda}. \tag{F.28}$$

Otherwise there is not solution because the two lines do not cross. This completes the proof of the proposition

F.2 Proof of Propositions in Section 4.2

Proof of Proposition 7 Consider an increase in γ. From equation (30), the overall effect on (30) is ambiguous,

$$\frac{ds(j)}{d\gamma} = -\lambda \frac{\mathcal{I}_\gamma(1-z) - \mathcal{I}z_\gamma}{(\mathcal{I}(z)(1-z))^2}. \tag{F.29}$$

Note however that at the very top $z(j = 0) = 1$, thus the top country increases its share unambiguously. Moreover, as at $j = \infty$, $z = 0$, we have that $z_\gamma(j = \infty) = 0$, thus the worst country unambiguously loses income share. As the $s(j)$ is continuous the first result follows.

For the particular case described in (29) we have that

$$j(z) = \int_z^1 \frac{(1-x)}{\lambda(x-\gamma)} dx = \frac{z - 1 - (1-\gamma)\ln\left(\frac{z-\gamma}{1-\gamma}\right)}{\lambda}. \tag{F.30}$$

This equation defines implicitly $z(j, \gamma)$. We find that

$$\frac{dz}{d\gamma} = 1 + \ln\left(\frac{z-\gamma}{1-\underline{z}}\right)\frac{z-\gamma}{1-z} > 0 \tag{F.31}$$

which is positive for all $z \in [\gamma, 1)$ and zero at $z = 1$. Moreover, this derivative is monotonically decreasing in z. In this case the index of the hazard rate is γ, and

$$\frac{d\mathcal{I}}{d\gamma} = \frac{1 - z_\gamma}{(z - \underline{z})^2}. \tag{F.32}$$

Finally note that the income share has to be normalized by the support of the distribution

$$s(z;\gamma) = \frac{\lambda}{\mathcal{I}(z,\gamma)(1 - z(\gamma))(1-\gamma)}. \tag{F.33}$$

Thus, as

$$\frac{ds(j)}{d\gamma} > 0 \Longleftrightarrow \mathcal{I}z_\gamma(1-\gamma) + \mathcal{I}(1-z) - \mathcal{I}_\gamma(1-z)(1-\gamma) > 0. \tag{F.34}$$

Substituting, and arranging the terms, we find that the derivative is

$$\frac{(z-1)(z+\gamma-2)+(1-\gamma)^2\ln\left(\frac{z-\gamma}{1-\gamma}\right)}{(1-z)(z-\gamma)}$$

It is readily verified that this is a continuous function on its domain. Also, for $z=\gamma$, this function is $-\infty$, also for $z=1$, the function is positive and equal to $(1+\gamma)/(1-\gamma)$. Taking the first order derivative of this expression and equating it to zero, it can be verified that it only has an interior extremum at

$$z^* = \frac{1+\gamma}{2} \tag{F.35}$$

and this is a maximum. This implies that this function only crosses once the zero in the relevant domain at a value $z < z^*$.

F.3 Derivation of Results in Section 4.3

The productivity distribution moves from being distributed exponential with parameter λ_1 to $\lambda_2 < \lambda_1$. The change in the assignment function is readily computed as

$$\Delta j = \left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)(z-\ln z-1). \tag{F.36}$$

Thus, except for $z=1$, for which $\Delta j = 0$ we have that the change is positive. Not only that, as $\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)$ and

$$\frac{d}{dz}(z-\ln z-1) = 1-\frac{1}{z} < 0 \tag{F.37}$$

Thus, the poorest countries are climbing up the ladder of global supply chains and producing higher z intermediates. The change in the income share is given by

$$\frac{ds(j)}{d\lambda} = -\frac{d}{d\lambda}\left(\lambda - \frac{\lambda}{1+W(-e^{-1-\lambda j})}\right) = -1 + \frac{\lambda j W(-e^{-1-\lambda j})}{(1+W(-e^{-1-\lambda j}))^3} + \frac{1}{1+W(-e^{-1-\lambda j})}. \tag{F.38}$$

Note that $\frac{ds(0)}{d\lambda} = \infty$ and $\frac{ds(\infty)}{d\lambda} = 0$. The second term is negative and increasing. It asymptotes towards $-\infty$ for $j=0$ and towards zero as $j \to \infty$. The last term is positive and decreasing, asymptotically towards 1 as $j \to \infty$. The ratio of these last two terms is

$$\frac{\lambda j W(-e^{-1-\lambda j})}{(1+W(-e^{-1-\lambda j})^2} \tag{F.39}$$

It is negative, increasing and bounded between $[-.5, 0]$. Perhaps the simplest way is to analyze it is to realize that

$$\frac{ds(j)}{d\lambda} = \frac{s_j}{\lambda} + \frac{\lambda j W(-e^{-1-\lambda j})}{(1+W(-e^{-1-\lambda j}))^3}. \tag{F.40}$$

We have that the first term is positive and decreasing and dominates for $j \to 0^+$, meaning that the function is decreasing in 0^+. As s_j is decreasing and the second term is increasing but negative the overall behavior is ambiguous. However, it must exist a region in which $\frac{ds(j)}{d\lambda}$ as the overall integral of s_j is one, so if some countries increase their share some others have to lose it. Expressing s_j we find that the sign of the derivative coincides with the sign of

$$\frac{\lambda j}{(1 + W(-e^{-1-\lambda j}))^2} - 1, \tag{F.41}$$

or alternatively, whether

$$\lambda j - (1 + W(-e^{-1-\lambda j}))^2 \lesseqgtr 0. \tag{F.42}$$

Note that both terms are equal to zero for $j = 0$. The slope of the first term is λ, while

$$\frac{d}{dj}(1 + W(-e^{-1-\lambda j})^2 = -2\lambda W(-e^{1-j\lambda}). \tag{F.43}$$

It is readily verified that

$$-2\lambda W(-e^{1-j\lambda}) > \lambda \tag{F.44}$$

if and only if $j \in [0, \frac{-1+2\ln(2)}{2\lambda})$. Thus, we have that

$$\lambda j - (1 + W(-e^{-1-\lambda j}))^2 < 0 \tag{F.45}$$

for $j \in [0, \frac{-1+2\ln(2)}{2\lambda})$. Moreover, as λj grows at a slower speed than $(1 + W(-e^{-1-\lambda j})^2$ for all $\frac{-1+2\ln(2)}{2\lambda}$, it means that will exist a unique $j^\dagger > \frac{-1+2\ln(2)}{2\lambda}$ such that for all $j < j^\dagger$ equation (F.42) is negative, and positive for $j > j^\dagger$. Moreover, this implies that

$$\frac{d^2 s(j^\dagger)}{d\lambda dj} < 0 \tag{F.46}$$

as otherwise it would not be possible to reach zero as $j \to \infty$. This observation, joint with the fact that

$$\frac{d^2}{dj^2}\left(\frac{ds(j)}{dj}\right)\bigg|_{j=0} > 0 \tag{F.47}$$

implies that the function ds/dj is convex for $j \in [0, j^{\dagger\dagger})$ with $j^{\dagger\dagger} > j^\dagger$ and concave thereafter.

We finally compare this trickle-down process of technology with what would happen in a world without unbundling. In this case, income distribution is given by $s(j) = \lambda e^{-\lambda j}$. So

$$\frac{d}{d\lambda}(\lambda e^{-\lambda j}) = (1 - \lambda j)e^{-\lambda j}. \tag{F.48}$$

Thus, we see that countries with $j < 1/\lambda$ increase their and the rest decrease their share. For the shape of the change, we have that

$$\frac{d}{dj}\left(\frac{d}{d\lambda}(\lambda e^{-\lambda j})\right) = (-2+\lambda j)e^{-\lambda j}\lambda < 0 \iff j < 2/\lambda, \tag{F.49}$$

$$\frac{d^2}{dj^2}\left(\frac{d}{d\lambda}(\lambda e^{-\lambda j})\right) = (3-\lambda j)e^{-\lambda j}\lambda^2 < 0 \iff j > 3/\lambda. \tag{F.50}$$

Thus the function is decreasing for $j < 2/\lambda$ and then increasing, convex for $j < 3/\lambda$ and then concave. We compare the threshold for which countries increase their share in the equilibrium without unbundling with the one for the equilibrium with unbundling.

We evaluate equation (F.42) at $j = 1/\lambda$,

$$\lambda/\lambda - (1 - W(-e^{-1-\lambda/\lambda})^2 \simeq 1 - (1 + .1586)^2 = -.34 < 0 \tag{F.51}$$

this shows that the range of countries that increase their income share is larger in the equilibrium without unbundling than with unbundling. Moreover, as both changes in the share are convex in this regime and we have that the slope in the positive region of $ds/d\lambda$ is higher in the equilibrium with unbundling. Next we evaluate the minimum value of ds/dj without unbundling $j = 2/\lambda$ at $d^2s/djds$ with unbundling

$$\left.\frac{d}{dj}\frac{ds^{unbundling}(j)}{d\lambda}\right|_{j=2/\lambda} = 2\lambda W(-1/e^3)^2 \frac{4 + W(-1/e^3)}{(1 + W(-1/e^3))^5} \simeq .029\lambda > 0. \tag{F.52}$$

G Derivation of World Output

G.1 Ex-ante Identical Countries

We start analyzing the equilibrium in which there is no unbundling of production. The price index for the final good is

$$\ln p_j = \frac{1}{N}\sum_{j=1}^{J}\int_0^{\mu_j} \ln p_j(v)dv = \frac{1}{N}\sum_{j=1}^{J}\mu_j \ln p_j(v).$$

The price of variety v in steady-state is

$$p_j(v) = \exp\left[\int_0^1 \ln p_j(z)dz\right] = \frac{1}{\theta_j}(r_j w_j)^{1/2} = \frac{1}{\theta_j}\left(\frac{1}{2}\mu_j\frac{Y^{\text{World}}}{N}\rho\right)^{1/2}$$

Using that $\mu_j = \kappa\theta_j$, $N = \sum_j \mu_j$, and introducing the notation $\theta_j = \theta_i \equiv \theta$, the price index simplifies to

$$\ln p_j = \frac{1}{J\kappa\theta}J\kappa\theta \ln p(v) = \ln p(v) = \frac{1}{2}\ln\left(\frac{Y^{\text{World}}\rho}{2J}\right) - \ln\theta$$

Normalizing the price index of the final good to 1, we find that

$$Y^{\text{World}} = \frac{2J\theta^2}{\rho}.$$

Next, we compute the world output in the steady-state equilibrium with unbundling. Note that the price index for the final good is as in the equilibrium without unbundling. Normalizing the price of the final good to 1 implies that

$$0 = \sum_{j=1}^{J} \mu_j \ln p_j(x) = \kappa\theta \sum_{j=1}^{J} \ln p_j(v)$$

The price of variety v in steady-state is

$$\begin{aligned} \ln p(v) &= \sum_{j=1}^{J} \int_{z_j}^{z_{j-1}} dz \left(z \ln r_j + (1-z)\ln w_j - \ln\theta\right) \\ &= \frac{1}{2}\ln\rho - \ln\theta + \sum_{j=1}^{J} \int_{z_j}^{z_{j-1}} dz \left((1-z)\ln w_j\right) \end{aligned}$$

The wage (bill) paid in country j is

$$w_j = \int_{z_j}^{z_{j-1}} dz(1-z)Y^{\text{World}} = \Delta Y^{\text{World}} = \frac{Y^{\text{World}}}{2J},$$

where we have used that $\int_{z_j}^{z_{j-1}} dz(1-z)$ is equalized across countries in the equilibrium with symmetric countries. In the proof of Proposition 2 we show that the value of this integral is $1/2J$. As a result,

$$\ln p(v) = \frac{1}{2}\ln\rho - \ln\theta + J\frac{1}{2J}\ln\left(\frac{Y^{\text{World}}}{2J}\right).$$

Thus, all varieties have the same price. Substituting in the definition of the price index of the final good, we obtain that

$$Y^{\text{World}} = \frac{2J\theta^2}{\rho}.$$

G.2 Heterogeneous Countries

G.2.1 Two-countries

We first derive the world output under the two trade regimes for the case of two countries. To derive world output without unbundling, we proceed in an analogous manner as above.

Remember that the price of final good in country j is

$$p_j = \exp\left[\frac{1}{N}\sum_{j=1}^{J}\int_0^{\mu_j}\ln p_j(v)dx\right] = \exp\left[\frac{1}{N}\sum_{j=1}^{J}\mu_j \ln p_j(v)\right].$$

where $p_j(v) = \exp\left[\int_0^1 \ln p_j(z)dz\right] = \frac{1}{\theta_j}(r_j w_j)^{1/2}$. In the steady-state, $r_j = \rho$. Thus,

$$\ln p_j = \frac{1}{2}\frac{1}{N}\sum_{j=1}^{J}\mu_j\left[\ln\rho + \ln w_j - 2\ln\theta_j\right].$$

We assume that the final good is the numéraire, thus,

$$0 = \sum_{j=1}^{J}\mu_j\left[\ln\rho + \ln w_j - 2\ln\theta_j\right]$$

Using that $\mu_j = \kappa\theta_j$, $w_j = \frac{1}{2}\mu_j\frac{Y^{world}}{N}$ and $N = \sum_j \mu_j$ we find that

$$0 = \sum_{j=1}^{J}\theta_j\left[\ln\rho + \ln\frac{1}{2}\frac{\theta_j}{\sum_j\theta_j} + \ln Y^{world} - 2\ln\theta_j\right].$$

Solving for world income,

$$Y^{\text{World, without}} = \frac{2}{\rho}\sum_{j=1}^{J}\theta_j\prod_{j=1}^{J}\theta_j^{\frac{\theta_j}{\sum_{j=1}^{J}\theta_j}}.$$

We see that the distribution of θ_j affects the world income level in two different ways. First, the aggregate level of productivity matters, or alternatively, the average productivity of a country in the world, as $\sum_{j=1}^{J}\theta_j = J\frac{\sum_{j=1}^{J}\theta_j}{J} = J\bar{\theta}$. The second element captures the distribution of θ_j across countries. This second term rewards inequality in the world, as it achieves a minimum for the case in which the distribution is of θ is uniform.

The case reported in the main text is for only two countries with the normalization $\theta_1+\theta_2 = 1$,

$$Y^{\text{World, without}} = \frac{2}{\rho}\theta_1^{\theta_1}\theta_2^{\theta 2}.$$

We next turn to the derivation of world output with unbundling. Remember that $\theta_1 > \theta_2$, this implies that country 1 produces intermediates from z^* to 1 and country 2 the rest. We can now write $\ln p(v)$ as

$$\ln p(v) = \sum_{j=1}^{J} \int_{z_j}^{z_{j-1}} dz \left(z \ln r_j + (1-z) \ln w_j - \ln \theta_j \right).$$

Using that

$$\begin{aligned} w_1 &= \int_{z^*}^{1} (1-z) dz Y^{\text{World, with}} = \frac{(1-z)^2}{2} Y^{\text{World, with}}, \\ w_2 &= \int_{0}^{z^*} (1-z) dz Y^{\text{World, with}} = \frac{z(2-z)}{2} Y^{\text{World, with}}. \end{aligned}$$

the expression for the price index becomes

$$\frac{1}{2}\ln\rho - z^* \ln\theta_2 - (1-z^*)\ln\theta_1 + \frac{z^*(2-z^*)}{2}\ln\left(\frac{z^*(2-z^*)}{2}\right) + \frac{(1-z^*)^2}{2}\ln\left(\frac{(1-z^*)^2}{2}\right) + \frac{1}{2}\ln Y^{\text{World, with}}.$$

Rearranging, we obtain that

$$Y^{\text{World, with}} = \frac{2(1-z^*)^{-2(1-z^*)^2}\left((2-z^*)z^*\right)^{-z^*(2-z^*)}}{\rho} \theta_1^{2-2z^*} \theta_2^{2z^*}.$$

Using the equilibrium definition of z^* that

$$\left(\frac{\theta_1}{\theta_2}\right)^{\frac{1}{1-z}} = \frac{(1-z)^2}{z(2-z)},$$

we can simplify this expression to

$$Y^{\text{World, with}} = \frac{2}{\rho z^*(2-z^*)} \theta_1^{1-z^*} \theta_2^{1+z^*}.$$

Comparison of the world output with and without unbundling Using again the equilibrium definition, we find that world output with unbundling can also be written as

$$Y^{\text{World, with}} = \frac{1}{\rho C} \theta_1^{2A} \theta_2^{-2B}$$

,whereh $A = \frac{1-z^*+\frac{z^{*2}}{2}}{1-z^*}$, $B = \frac{\frac{z^{*2}}{2}}{1-z^*}$ and $C = \int_{z^*}^{1}(1-z)dz$. Therefore, world output increases if $\frac{\theta_1^{2A-\theta_1}}{\theta_2^{2B+\theta_2}} > 2C$. Note that $0 < z^* < \theta_2 < 1/2$ implies that $C < 1/2$ and $A - B > 1/2$. Thus, $2A - \theta_1 > 2B - \theta_2$. Given that $\theta_1 > \theta_2$ it follows that $\frac{\theta_1^{2A-\theta_1}}{\theta_2^{2B+\theta_2}} > 1 > 2C$.

Comparison of the income of country 2 in the two steady-state equilibria The income of country 2 can be computed by multiplying the world income by the world income share,

$$\begin{aligned} Y_2^{\text{without}} &= \frac{2\theta_2}{\rho}\theta_1^{\theta_1}\theta_2^{\theta 2}, \\ Y_2^{\text{with}} &= \frac{2}{\rho(2-z^*)}\theta_1^{1-z^*}\theta_2^{1+z^*}. \end{aligned}$$

where we have kept using the normalization that $\theta_1 + \theta_2 = 1$. The reader can check that the result extends to the unnormalized case. Taking the ratio of these two incomes, using the normalization and simplifying, we have that

$$\frac{Y_2^{\text{with}}}{Y_2^{\text{without}}} = \left(\frac{1-\theta_2}{\theta_2}\right)^{\theta_2 - z^*} \frac{1}{2-z^*} = \left(\frac{1-\theta_2}{\theta_2}\right)^{\theta_2} \left(\frac{1-\theta_2}{\theta_2}\right)^{-z^*} \frac{1}{2-z^*}.$$

By assumption $\theta_1 > \theta_2$. Thus, the first term is greater than one. As $0 < z^* < 1$, the second and third terms are less than one. The maximum value of the first term can be computed by taking its derivative,

$$\frac{d}{d\theta_2}\left(\frac{1-\theta_2}{\theta_2}\right)^{\theta_2} = \frac{\left(\frac{1}{\theta_2}-1\right)^{\theta_2}\left(\theta_2 \log\left(\frac{1}{\theta_2}-1\right) - \log\left(\frac{1}{\theta_2}-1\right)+1\right)}{\theta_2 - 1},$$

which is maximized at a value of $\theta_2 \simeq 0.2178$, with corresponding value of 1.3211. On the other hand, the biggest number that can be attained by the term $\frac{1}{2-z^*}$ is given by the value of z^* that corresponds to the symmetry breaking case, as we have shown that $z^* \leq 1 - \sqrt{1/2} \simeq .2929$. Thus, we can establish an upper bound on the ratio of incomes as follows

$$\frac{Y_2^{\text{with}}}{Y_2^{\text{without}}} = \underbrace{\left(\frac{1-\theta_2}{\theta_2}\right)^{\theta_2}}_{\leq 1.32} \underbrace{\left(\frac{1-\theta_2}{\theta_2}\right)^{-z^*}}_{\leq 1} \underbrace{\frac{1}{2-z^*}}_{\leq \frac{1}{1+\sqrt{1/2}}} \leq .773 < 1.$$

Since this upper bound is strictly less than one, it proves that real income declines in country 2 with unbundling.

G.2.2 A large number of countries

We next compute the world output under the two trade regimes for the case of a large number of heterogenous countries.

Normalizing the price index of the final good to 1, we have that

$$\ln p_j = 0 = \frac{1}{N}\int_0^\infty dj \mu_j \ln p_j(v).$$

In the steady without unbundling, the price of intermediate z in country j is $\theta^{-1} w_j^{1-z} \rho^z$. Thus, the price of variety v is

$$p_j(v) = \exp\left[\int_0^1 \ln p_j(z) dz\right] = \frac{1}{\theta_j} (\rho w_j)^{1/2}.$$

Substituting in the price index we have that

$$0 = \int_0^\infty \lambda e^{-\lambda j} \left(\frac{\ln \rho}{2} + \frac{\ln w_j}{2} - \ln \theta_j \right) dj = \frac{\ln \rho}{2} - \int_0^\infty \lambda e^{-\lambda j} \ln \left(\lambda e^{-\lambda j} \right) + \int_0^\infty \lambda e^{-\lambda j} \left(\frac{\ln w_j}{2} \right) dj.$$

Substituting the equilibrium wage rate, $w_j = \frac{1}{2} \mu_j \frac{Y^{world}}{N} = \frac{1}{2} \theta_j Y^{world}$ and using that $\int_0^\infty \lambda e^{-\lambda j} (\ln \lambda - \lambda j) dj = -1 + \ln \lambda$, we find that

$$\begin{aligned} 0 &= \frac{\ln \rho}{2} + 1 - \ln \lambda + \frac{1}{2} \int_0^\infty \lambda e^{-\lambda j} \ln \left(\frac{1}{2} \theta_j Y^{world} \right) \\ &= \frac{\ln \rho}{2} + 1 - \ln \lambda + \frac{1}{2} \ln \left(\frac{\lambda}{2} Y^{world} \right) - \frac{\lambda}{2} \int_0^\infty \lambda e^{-\lambda j} j dj \\ &= \frac{\ln \rho}{2} + 1 - \ln \lambda + \frac{1}{2} \ln \left(\frac{\lambda}{2} Y^{world} \right) - \frac{1}{2}. \end{aligned}$$

Isolating Y^{World} we find that

$$Y^{\text{World,without}} = \frac{2\lambda}{e\rho}$$

where e is the base of the natural logarithm.

For the equilibrium with unbundling, we start by noting that the price of intermediate z is

$$\ln p(z) = (1 - z) \ln w_j + z \ln \rho - \ln \theta_j.$$

The equilibrium wage is given by $w_j = -z'(1 - z) Y^{\text{World}}$. As we did in the main text, note that the derivative $z'(j)$ can be written in terms of z as

$$\frac{dz(j)}{dj} = -\frac{\lambda z}{1 - z}.$$

Normalizing the price index to one, implies that the price of each variety is also equal to one (as we saw in the previous section). The marginal cost of producing a variety is given by

$$\ln p(v) = 0 = \int_0^1 \left((1 - z) \ln \left(\frac{\lambda z}{1 - z} (1 - z) Y^{\text{World, with}} \right) + z \ln \rho - \ln \theta_{j(z)}) \right) dz.$$

Using the equilibrium assignment, the last term can be expressed as $\ln \theta_j = \ln \lambda - z + \ln z + 1$.

Integrating the previous equation we find that

$$0 = \frac{1}{4}\left(-4\ln\lambda + 2\ln\rho + 2\ln\left(\lambda Y^{\text{World, with}}\right) - 1\right)$$

and isolating for Y^{World} we obtain that

$$Y^{\text{World, with}} = \sqrt{e}\frac{\lambda}{\rho},$$

where e is the base of the natural logarithm.

H South joins the global supply chain: an alternative approach

In Section 4.1, we compared two extreme cases. We assume, for simplicity, that southern countries either fully participated or did not participated in intermediates trade. In practice, southern countries increased their participation in intermediates trade more gradually. One could think of a country as composed by a mass of firms and a fraction of these firms participating in unbundling. This fraction is larger in more productive countries. In this section we assume that the South joining the global supply is an increase in this fraction. We find the same qualitative results as in the all-or-nothing case.

Suppose that for each country j we have a mass one of identical firms and that the fraction of those that are participating in intermediates trade is given by

$$\exp(-\mu j),$$

where $\mu > 0$. This implies that in country $j = 0$, all firms participate and at $j = \infty$, none participates. This changes the assignment function to

$$\left(\frac{\theta_j}{\theta_i}\right)^{\frac{1}{1-z}} = \frac{\Delta_j(z)e^{-\mu i}}{\Delta_i(z)e^{-\mu j}}$$

This generates the following assignment necessary condition

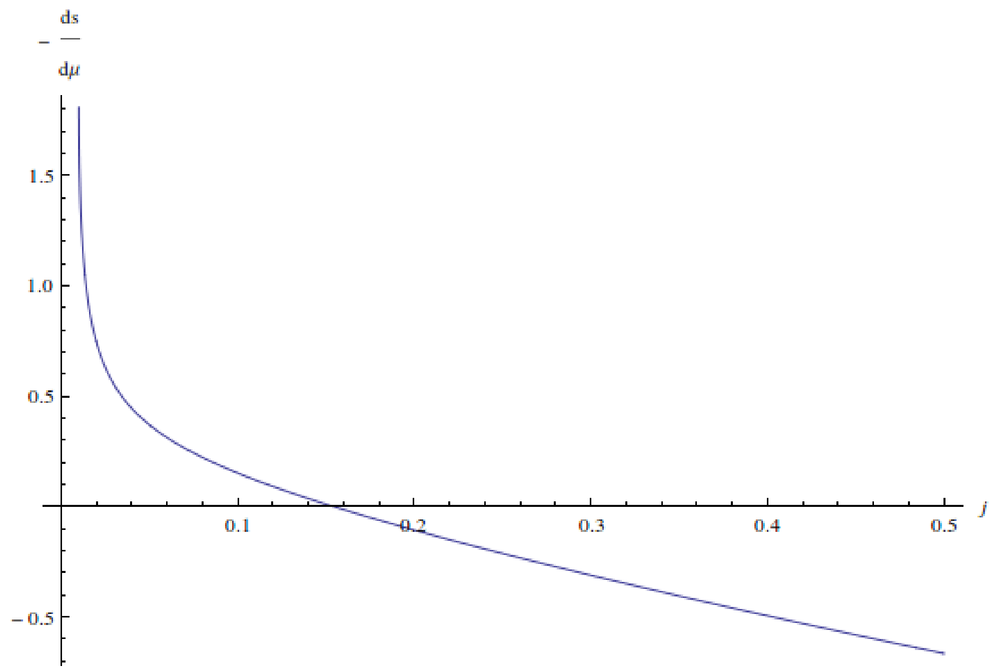
$$(1 - z(j))\left(\frac{z''(j)}{z'(j)} + \mu\right) - z'(j) = -\lambda$$

The income share of country j given μ is

$$s(z) = \frac{z}{1-z}(\lambda + \mu) + \frac{2C_1 - \mu z^2}{1-z}$$

where C_1 is a constant determined by the boundary conditions with the property that $C_1 = 0$ for $\mu = 0$ (which delivers the baseline case analyzed in Section 3.2.2). We can solve numerically

Figure H.1: Change in the WID with a decrease in μ



the differential equation and compute the change in the world income shares as a result of a decrease in μ. We find the same qualitative results. In this case there is no discontinuity between northern and southern countries. Figure H.1 shows $-ds(j)/d\mu$.

References

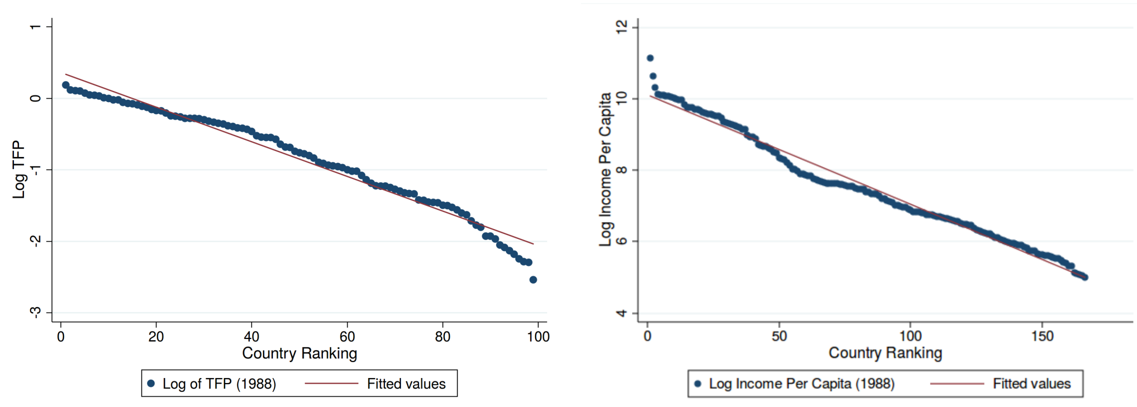
Hall, R. E. and C. I. Jones (1999): "Why Do Some Countries Produce So Much More Output Per Worker Than Others?" *The Quarterly Journal of Economics*, 114, 83–116.

Jones, C. I. (1995): "R&D-Based Models of Economic Growth," *Journal of Political Economy*, 103, 759–84.

I Additional figures

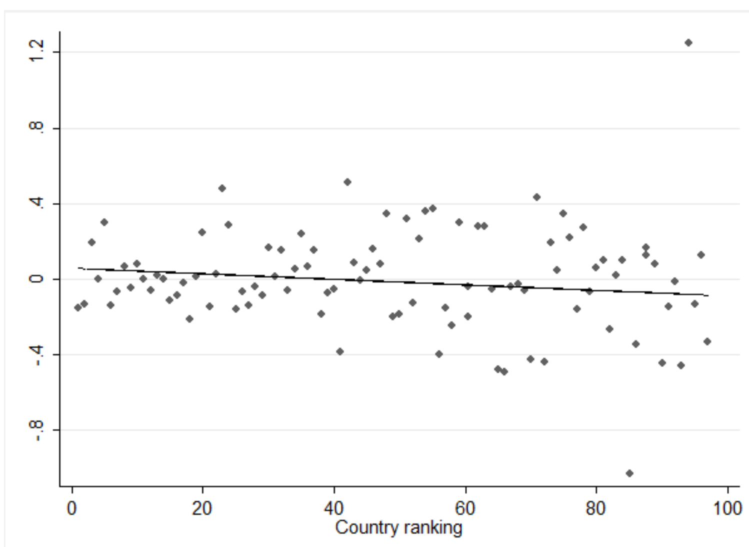
Figure I.1: Distribution of TFP and World Income Share

(a) TFP (b) World Income Shares



Notes: TFP data are obtained from Hall and Jones (1999). Income per capita data are obtained from the World Bank WDI.

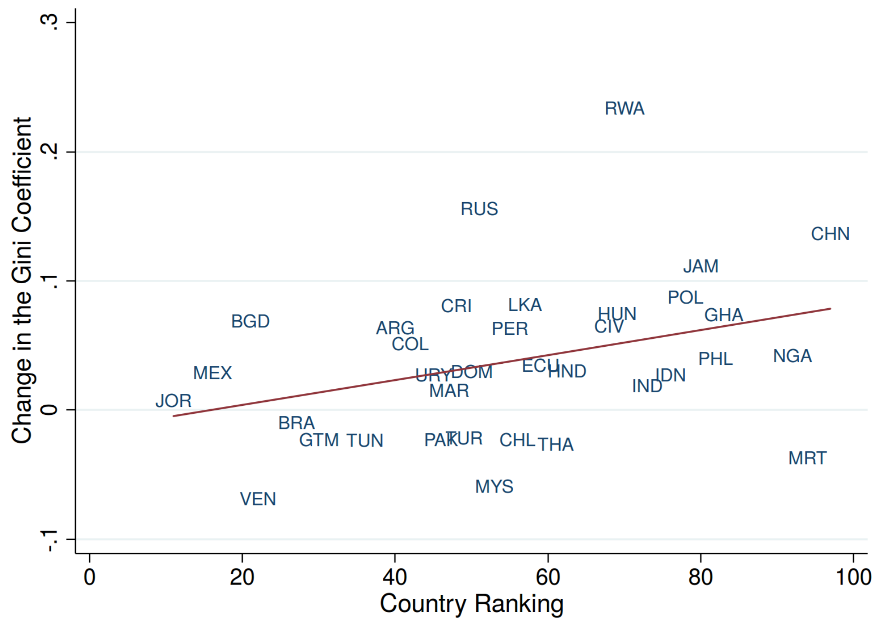
Figure I.2: GDP per capita Growth (1990-2008): Difference from the mean



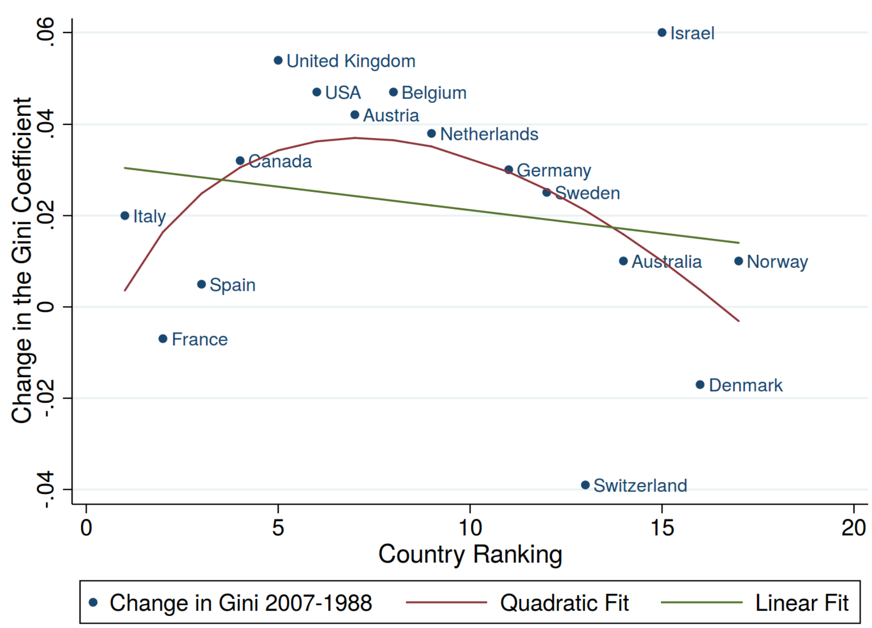
Source: GDP per capita (PPP) is obtained from World Development Indicators (World Bank). Country ranking is the TFP ranking from Hall and Jones (1999). The line represents the predicted values of a linear of regression of both variables, excluding China (the point in the upper-left-side). The negative coefficient is significant at 90%. Without excluding China, the coeficient remains negative but it is not significantly different from zero.

Figure I.3: Changes in Within-Country Inequality

(a) Countries with less than half U.S. income per capita (WDI data)



(b) Countries with more than half U.S. income per capita (LIS data)



Notes: The Gini coefficient is obtained from World Development Indicators (WDI) for countries with less than 50% of U.S. income per capita in 1990. The WDI does not provide a time series for richer countries. We use the Luxembourg Income Studies (LIS) data instead, waves 1988 and 2007 (LIS data provides information on very few poor countries). The country ranking is computed from the TFP ranking in Hall and Jones (1999). As the data from the WDI is quite sparse, we use data for the period 1981-1988 as initial points. If we have more than one observation, we use the average. Likewise, for the final Gini coefficient we use a window from 2001 to 2008.