TSE WP working paper series

10-142

Research Group: *Industrial Organization*

February, 2010

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The value of switching costs*

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February 3, 2010

*We would like to thank Andrew Clausen, Philipp Kircher, George Mailath, Curt Taylor, Paul Klemperer, and the participants at seminars where this paper has been presented, especially those of the Economics department at the University of Cambridge.

We study the consequences of heterogeneity of switching costs in a dynamic model with free entry and an incumbent monopolist. We identify the equilibrium strategies of the incumbent and of the entrants and show that the strategic interactions are more complex and more interesting than either in static models or in models where all consumers have the same switching costs. In particular, we prove that even low switching cost customers have value for the incumbent: when there are more of them its profits increase. Indeed, their presence hinders entrants who find it more costly to attract high switching cost customers. This leads to different comparative statics: for instance, an increase in the switching costs of all consumers can lead to a decrease in the profits of the incumbent.

1. Introduction

On February 6, 2007, in the same well-known letter in which he called for an end to DRM (Digital Rights Management) for music distributed in electronic form, Steve Jobs discussed the incumbency benefits that the iPod enjoyed thanks to iTunes' proprietary format (Jobs, 2007). He noticed that

> "[s]ome have argued that once a consumer purchases a body of music from one of the proprietary music stores, they are forever locked into only using music players from that one company. Or, if they buy a specific player, they are locked into buying music only from that company's music store."

He argued that on average there are "22 songs purchased from the iTunes store for each iPod ever sold", and that this implied that "under 3% of the music on the average iPod is purchased from the iTunes store and protected with a DRM." He concluded that there was no lock-in as it is "hard to believe that just 3% of the music on the average iPod is enough to lock users."

In a response to Jobs' statement, Jon Lech Johansen[1] made the following interesting points:

> "Many iPod owners have never bought anything from the iTunes Store. Some have bought hundreds of songs. Some have bought thousands. At the 2004 Macworld Expo, Steve revealed that one customer had bought $29,500 worth of music."

Therefore, the lock-in is non negligible as

> "it's the customers who would be the most valuable to an Apple competitor that get locked in. The kind of customers who would spend $300 on a set-top box."

In essence, Johansen argued that the consumers that matter, those who buy lots of online music, have high switching costs, and therefore that an entrant in the market will face large obstacles attracting them. As we will discuss in Section 4, in the simplest economic model of switching costs, with one incumbent and free entry, Johansen is wrong: heterogeneity of switching costs does not matter. If a proportion $\alpha > 0$ of the agents have switching cost $\sigma > 0$, while the others have no switching cost, then the profits of the incumbent will be equal to $\alpha\sigma$, the average switching cost of all the consumers, multiplied by the number of agents. This would imply

[1] See Johansen (2007). Jon Lech Johansen, also know as "DVD Jon" is a "hacker" made famous by his work on reverse engineering of data formats, and in particular on the DVD licensing enforcement software (see `http://nanocr.eu/`, last visited on 31 January 2010).

that Steve Jobs is not underestimating the value of incumbency by assuming that all consumers have the same switching cost.

We show that this result changes drastically in a dynamic model in which there are new potential competitors in every period; then Johansen is right: the more skewed the distribution of switching costs, the greater the profits of the incumbent. To the best of our knowledge, this fact and the importance of the distribution of switching costs has not been recognized in the literature, despite the existence of a significant body of theory which explores the consequences of consumer switching costs. (We discuss the literature below in Section 2.)

Our results have policy implications. Because, as we show, larger industry wide switching costs can *lower* the profits of the incumbent, competition authorities should not use a *per se* rule that any action which has the effect of raising them is anti-competitive. A more careful evaluation is needed.

We conduct our analysis by constructing a series of models that share the following features: a) the switching costs of consumers are invariant over time; b) at the start of the 'game' there is a single incumbent firm; and c) there is free entry by competing firms in every period. Following much of the literature, we assume that only short term contracts are used and that a consumer's switching cost does not depend on the firm from which it is purchasing.

In Section 3, we introduce our analysis by considering the case where all consumers have the same switching costs σ. In a one period model, the incumbent would charge σ, and, assuming that the mass of consumers is equal to 1, its profit would also be equal to σ. We show that its equilibrium aggregate discounted profit over all periods is also equal to σ when we embed this static model in a dynamic framework, whether the number of periods is finite or, subject to stationarity assumptions, infinite. In the latter case, this implies that the profit of the incumbent is equal to the value of a flow of per period payments equal to $(1-\delta)\sigma$, not to σ! Although this result is very easy to prove, and is implicit in some of the literature, we feel that it is worth stressing as it shows that switching costs are a leaner cash cow than sometimes assumed.

We begin our analysis of the heterogeneity of switching costs in Section 4, where we study the dynamic version of the model which we sketched above when describing the Jobs-Johansen debate: a proportion $\alpha \in (0,1)$ of consumers have a switching cost equal to $\sigma > 0$, while the others have no switching cost. We identify the (stationary) equilibrium of the infinite horizon model. As opposed to the case where all consumers have the same switching cost, the intertemporal profit of the incumbent is greater than the one period profit, although it is smaller than the value of an infinite stream of one period profits. We prove that even zero switching customers have value for the incumbent: when there are more of them its profits increase. Indeed, their presence hinders entrants who find it more costly to attract high switching cost customers.

In order to conduct more complete comparative statics, in Section 5, we generalize the model of Section 4 by assuming that the low switching cost consumers have strictly positive switching costs. For technical reasons, we turn to a two period model. For a large class of parameters decreasing the switching costs of *all* consumers *increases* the profits of the incumbent. By itself, a decrease in the high switching cost decreases the profits of the incumbents. On the other hand, a decrease in the low switching cost increases the eagerness of the less profitable low switching cost consumers to change supplier and makes the entrants less aggressive. This second effect can dominate the first in many non-pathological cases.

The conclusion discusses further research as well as policy implications.

2. Literature

The literature has distinguished switching cost models proper and subscription models: in switching cost models, a firm must charge the same price to both current and new consumers, while in subscription models it can offer different prices to consumers with different purchase histories of its products. Switching cost models were introduced in the economics literature by[2] Klemperer (1987b) (see the surveys of the theoretical literature in Klemperer (1995), Annex A of Office of Fair Trading (2003), and Farrell and Klemperer (2007), and the discussion of policy implications in Office of Fair Trading (2003), especially Annex C). Chen (1997) initiated the investigation of subscription models. We present our model as a switching cost model, but, as we point out later in this section, because of free entry, our results would be the same if the model was a subscription model.

Much of the switching cost literature focuses on two-period duopsony models in which firms choose between charging a high price in order to extract rents from their customers and charging a low price in order to attract customers from their rivals. In this framework, Klemperer (1987a) shows that higher switching costs may make entry more likely, by inducing incumbents to abandon the hope of attracting the customers of other incumbents and therefore choosing higher prices. In our model, where new entrants provide the only effective competition, incumbents never try to attract customers from other incumbents. Our comparative statics are entirely the consequence of the heterogeneity of switching costs. Dubé, Hitsch, and Rossi (2006) present an infinite horizon model where a single consumer has random utility and firms have differentiated products. While their focus is on empirics, they provide numerical examples where prices may fall when switching costs are present.

Farrell and Shapiro (1988), Beggs and Klemperer (1992), Padilla (1995), and Anderson, Kumar, and Rajiv (2004) study infinite horizon switching cost models, in each of these cases with two firms and homogenous switching costs;[3] they focus on the evolution of market shares and on the effect of switching costs on prices.

[2]See also Klemperer (1983) and Klemperer (1986).

[3]Beggs and Klemperer assume that consumers are horizontally differentiated, but that, once they have purchased from a firm, they never buy from another firm.

Klemperer (1986) studies an infinite horizon model with homogeneous switching cost and free entry by firms. By contrast, we focus our analysis on the consequences of the heterogeneity of switching costs in the presence of free entry.

The paper that is closest to ours is Taylor (2003). He analyzes a finite horizon subscription model where consumers have different switching costs and where there is free entry. In his primary model, consumers draw new switching costs from identical, independent distributions in each period. He shows that free entry limits the advantages of incumbency and that a firm makes zero expected profits from the consumers that it attracts from its rivals. In an extension, Taylor examines a two period model with two types of consumers who draw their switching cost (as before, independently in each period) from different distributions. His focus is on the incentives of consumers to build a reputation of having low switching cost in order to get better offers in the future.

In our model, switching costs are constant over time and this implies that it is harder for an entrant to attract the more valuable consumers, those with higher switching costs, than to attract the less valuable customers. As in Taylor, the presence of low switching cost consumers hurts the high switching cost consumers, but in our model, we show that it can also increase the incumbent's profit.

Finally, in our model, because of free entry, incumbent firms find it just as difficult as entrants to attract customers of other firms. Therefore, incumbent firms, just like entrants, make zero profits on customers of other firms, and in equilibrium they ignore them when choosing the price they charge. As a consequence, our model would generate exactly the same results if we transformed it into a subscription model.

3. When all consumers have the same switching cost: You cannot get rich on switching costs alone

In this section, we consider a repeated version of the most standard textbook model of switching cost, with one incumbent and free entry. We show that, in equilibrium, the profit of the incumbent is equal to its profit in the one period version of the game. This is true for all equilibria when there are a finite number of periods, and for stationary equilibria when there are an infinite number of periods. We begin by presenting the one period version of the model and then turn to the repeated game with a finite number of periods.

There is a continuum of consumers with mass normalized to 1, and a good which can be supplied by a number of firms, as we will describe below. Each consumer has a perfectly inelastic demand for one unit of the good, and therefore always buys one unit either from the incumbent or from one of the entrants. In this section only, all consumers have the same switching cost σ. This switching cost is incurred every time a consumer changes from one supplier to another. It reflects industry wide similarities or compatibilities between products, rather than idiosyncrasies of specific sellers. This implies, for instance, that our comparative statics results which describe the consequences in changes of the switching costs bear on circumstances where the cost of changing between any pair or products increase or decrease.

In previous periods, the consumers have bought from the incumbent,[4] firm I. We do not study the process by which firm I became the incumbent, but only the continuation game after entry is possible. In general, at least some of the incumbency rents which we identify would have been dissipated in the competition to become the incumbent.

Let us consider first a one period model with a denumerable number of entrants who can enter the market at zero cost in each period. The main focus of our study is the following "Bertrand" game:

Stage 1: The incumbent and the entrants set prices;

Stage 2: The consumers choose from which firm to buy.

All of our qualitative results also hold true, and are sometimes easier to establish, in the "Stackelberg" version of this game:

Stage 1: The incumbent sets a price;

Stage 2: The entrants set their prices;

Stage 3: The consumers choose from which firm to buy.

Assuming, as we will throughout this paper that all firms have zero marginal cost, it is easy to prove that, in both the Bertrand and Stackelberg versions of the game, there is only one equilibrium, where the incumbent[5] charges σ, the entrants 0, and all consumers buy from the incumbent. We will show that in the repeated version of the game, the discounted intertemporal profit of the incumbent is not increased: it is still equal to σ. One can only pocket the switching cost once.[6]

This is easy to prove when there are two periods. Formally, we expand the game above by assuming that every entrant that has sold to a positive measure of consumers in the first period becomes a second period incumbent, *and* that there are new entrants, again in denumerable number,[7] in the second period. In all the

[4]In the dynamic version of the model, there could be, in some periods $t > 1$, several incumbents, *i.e.,* firms who have sold goods to a positive mass of consumers in previous period.

[5]The results would be the same with several incumbents.

[6]Although the model we use is a trivial extension of the most elementary model of switching costs, we have not found in the literature a clear statement of what happens when this game is repeated, with new entrants in every period; almost all of the literature focuses on the case of duopsony, where the same two firms compete again each other period after period.

[7]For the two period model, two entrants would be enough, but we need more in the infinite horizon models that we will discuss later.

paper, we assume that firms cannot discriminate between consumers, for instance as a function of their past purchasing histories, and also that they cannot commit to prices beyond the current period.

In equilibrium, whether in the Bertrand or Stackelberg model, all second period incumbents charge σ, and make profits equal to σ multiplied by the number of their first period customers. Therefore, competition between first-period entrants pushes the price that they charge down to $-\delta\sigma$, where $\delta \in (0, 1]$ is the discount rate. Consumers know that all incumbents will charge σ in the second period. Hence, firm I will be able to "keep" its customers only by charging a price less than or equal to $-\delta\sigma + \sigma$. It is straightforward to show that that it indeed charges this price, and therefore that it "keeps" all its customers. Hence its discounted profit is

$$(-\delta\sigma + \sigma) + \delta\sigma = \sigma.$$

An easy proof by induction shows that the same result holds with any finite number of periods.

We now show that the same result holds true in the infinite horizon version of this model (we now assume $\delta < 1$). In each period, we assume only a finite number of "active" entrants offer the good. We look for subgame perfect equilibria which satisfy conditions which we describe informally below and define formally in the web appendix of this paper (Biglaiser, Crémer, and Dobos, 2009).

The first condition which we impose eliminates the following type of situations: an entrant makes a better offer than the incumbent, taking into account the fact that the consumers have to pay the switching cost σ. However, every consumer believes that the others will refuse the offer and, therefore, he would be the only one to accept it. Since we assume that firms who do not have a positive measure of consumers at the end of the period are not active in the future,[8] after deviating our consumer will have to pay the switching cost once again in the following period. Therefore, it is an equilibrium for all consumers not to accept the offer. To eliminate this equilibrium we assume that "consumers have mass" by allowing small groups of consumers to coordinate on a strategy: if an (arbitrarily small) group of them is strictly better off purchasing from an entrant, then they do so.[9]

Our second set of conditions define the stationary[10] requirements which we impose on the pricing strategy of the firms: we search for equilibria where the pricing

[8] We make this assumption so that we need not worry about the policy used by a firm that has a set of consumers which is not empty but has measure 0.

[9] In many models of network externalities, it is assumed that the consumers coordinate on the purchasing decision which maximize their utility. We do not make this assumption. In a dynamic model, either we would have to assume that they are able to coordinate on a, potentially infinite, sequence of moves, which requires very strong coordination, or that this coordination has a myopic component, which is not very attractive. Furthermore, as the game progresses even similar consumers can find themselves in situations where they face different payoffs moving forward; their interest might diverge.

[10] In a companion paper, Biglaiser and Crémer (2009) prove that there exist many other equilibria

strategies of active firms only depend on whether consumers with positive switching costs (that is, the profitable consumers) purchased from them in the previous period. In particular, the incumbents who have a positive measure of profitable customers always choose the same price (or the same distribution of prices) whatever the history, and the lowest price charged by an entrant is the same (or has the same distribution) in all periods.

Finally, as is standard in one period Bertrand models with different costs for the different firms, we assume that the firms play undominated strategies.

We now state and sketch the proof of the main result of this section.

Proposition 1. *In both the Stackelberg and the Bertrand models, when all consumers have the same switching costs σ, the intertemporal discounted profit of the incumbent is equal to σ, whatever the number of periods.*

Therefore, as in the two period model, the incumbent can only collect the switching cost once: he only gets one bite at the apple.

The result has been proved with a finite number of periods. When their number is infinite, let Π be the present discounted profit of an incumbent firm which supplied all consumers in the previous period. By the stationarity assumption, this profit is independent of the firm's name and of the date. Entrants are willing to charge $-\delta\Pi$ to attract all the buyers. As in the two period model, consumers know that their welfare in subsequent periods does not depend on the identity of firm they choose to purchase from in the current period, and the incumbent will have to set a price equal to $-\delta\Pi + \sigma$ in order to keep its customers.[11] Hence, the equilibrium profit of the incumbent satisfies

$$\Pi = (-\delta\Pi + \sigma) + \delta\Pi = \sigma.$$

In every period the entrants charge $-\delta\sigma$, while the incumbent charges $\sigma(1-\delta)$, which does yield a discounted profit equal to σ. (See Appendix A for a formal proof.)

4. Heterogeneity of switching costs increases the profits of the incumbent and hurts consumers

We now turn to the main theme of the article: the consequences of heterogenous switching costs. In this section, we study a model with two types of consumers: high switching cost (HSC) consumers, who are a fraction $\alpha \in (0, 1)$ of the population, have a switching cost equal to $\sigma > 0$, while low switching cost (LSC) consumers, who form a fraction $1 - \alpha$ of the population, have a switching cost equal to 0. In the one period

of this game, which satisfy a weaker version of stationarity: although the outcome of the game is stationary (with prices in each period as low as 0 or as high as σ), after a deviation incumbents may charge prices different from the prices along the equilibrium path.

[11] Technically, the incumbent will charge $-\delta\Pi + \sigma$, the entrants charge $-\delta\Pi$ and in the continuation equilibrium, all the consumers buy from the incumbent.

model, competition drives the prices of entrants to 0, while the incumbent charges a price of σ, and obtains a profit equal to $\alpha\sigma$: its profits are the average switching cost of consumers multiplied by their mass. We analyze the infinite horizon version of this game.

4.1. Results

As in the model where consumers have the same switching costs, we restrict attention to equilibria that satisfy the "consumers have mass" condition, the stationarity conditions, and where players do not use weakly dominated strategies.

The following proposition summarizes our results.

Proposition 2. *In the infinite horizon model, where α consumers have switching costs equal to $\sigma > 0$, while the remaining consumers have 0 switching costs, under either Stackelberg or Bertrand competition*

i. the expected profit of the incumbent is

$$\Pi = \frac{\alpha\sigma}{1-\delta+\alpha\delta}. \tag{1}$$

ii. Π is greater than the profit of the incumbent in the one period model, $\alpha\sigma$, but smaller than the value of an infinite stream of one period profits, $\alpha\sigma/(1-\delta)$.

iii. Π is strictly smaller than σ, but $\lim_{\delta\to 1} \Pi = \sigma$ for all α.

Parts (i) and (ii) of the proposition show that, contrary to what happens when all consumers have the same switching costs, the intertemporal profit is not equal to the one period profit, but is *greater*; however the *per period* profit is smaller in the infinite horizon model than in the one period model. Finally, part (iii) shows that when the agents are very patient, the profit of the incumbent is independent of the proportion α of HSC consumers, whereas in the one period model profits are proportional to α. As we will explain below, LSC consumers, who always purchase from the lowest price entrant, make it more costly to attract profitable HSC customers away from the incumbent.

Proposition 2 yields interesting comparative statics, which we summarize in the following corollary.

Corollary 1. *Under the conditions of Proposition 2:*

i. Π is increasing in α, σ and δ;

ii. for a given average level of consumer switching costs, $\alpha\sigma$, the profit of the incumbent, Π, is decreasing in α;

iii. adding LSC consumers without changing the number of HSC consumers increases Π;

iv. under Stackelberg competition, the utility of HSC consumers is an increasing function of α.

Parts (i) and (ii) of the corollary are obvious from equation (1). Part (iii) is easy to prove. Assume that we add a mass $\eta > 0$ of LSC consumers; the total mass of consumers becomes $\eta' = 1 + \eta$ and the proportion of HSC consumers becomes $\alpha' = \alpha/(1 + \eta)$. The new profits are

$$\Pi' = (1+\eta)\frac{\alpha'\sigma}{1-\delta+\alpha'\delta} = \frac{\alpha\sigma}{1-\delta+\frac{\alpha}{1+\eta}\delta},$$

which is increasing in η. LSC consumers are valuable to the incumbent, even though they never buy its product, as they make it more costly for entrants to make aggressive discounts in order to attract HSC customers.

Although they lead to the same profits for the incumbent, the equilibria under Bertrand and Stackelberg competition are very different. Under Stackelberg competition, the incumbent offers the same price in every period, and HSC consumers never change suppliers. On the other hand, in Bertrand competition, the incumbent and the entrants play mixed strategies, and in each period there is a strictly positive probability that all the HSC consumers change suppliers. As switching is socially wasteful and as the profits of the incumbent are the same in these two models, consumer surplus and social welfare is lower under Bertrand than under Stackelberg competition. We summarize this result in the following proposition.

Corollary 2. *In the infinite horizon model, where a proportion α of the consumers have switching costs equal to $\sigma > 0$, while the others have zero switching costs, consumer surplus and welfare is lower under Bertrand competition than under Stackelberg competition.*

Before proceeding, remember that, as we have discussed in Section 3, our comparative statics results assume that the changes in switching costs that apply to all changes from one supplier to another. A consumer who chooses to switch in the first period from the incumbent to an entrant would have to find that his cost of switching once again, to a future entrant, has also increased. On the other hand, the result does not apply if the increase in switching costs applies only to a switch from the incumbent to a period 1 entrant. From a policy point of view, this implies that our theory can illuminate changes which affect the whole industry, for instance changes in standards or new regulations such as number or bank account portability.

We now present an informal proof of Proposition 2, starting with the Stackelberg case, which is easier to analyze. Complete proofs are presented in Appendices B and C.

4.2. Analysis of Stackelberg competition

By stationarity, HSC consumers know that the price that they will face in future periods is independent of the firm they choose in the current period. Hence, they will definitely switch suppliers if the difference of price is strictly greater than σ and definitely not switch if this difference is strictly smaller than σ. In the first period, entrants will be willing to underbid the incumbent by (slightly more than) σ as long as the price it charges is (strictly) greater than $-\delta\Pi + \sigma$. Hence, the incumbent will charge $-\delta\Pi + \sigma$ and sell to the α high cost customers at this price.[12] Therefore,

$$\Pi = \alpha \times (-\delta\Pi + \sigma) + \delta\Pi \Longrightarrow \Pi = \frac{\alpha\sigma}{1 - \delta + \alpha\delta}. \tag{2}$$

This implies that the price charged, in every period, by the incumbent, and paid by the HSC consumers, is equal to

$$p^S \equiv \alpha\sigma \frac{1 - \delta}{1 - \delta + \alpha\delta}. \tag{3}$$

In the next stage of the game one or several entrants charges and attract all the LSC consumers.

4.3. Analysis of Bertrand competition

In the Bertrand game, there is no equilibrium in which the incumbent charges $p^S = -\delta\Pi + \sigma$ and at least one entrant charges $p^S - \sigma = -\delta\Pi$. Indeed, if the incumbent did not retain all the HSC customers, he would have incentives to decrease its price; if it retained them, the entrant would attract only the LSC consumers, who generate no profit in future periods, at a negative price. More generally, it is easy to show that there is no pure strategy equilibrium of the game, but we will still be able to show that the profits of the incumbent are equal to the profits in Stackelberg competition.

We do this by proving that, if Π is the (expected) profit of the incumbent, then $-\delta\Pi + \sigma$ belongs to the support of the distribution of prices that it announces; furthermore when it chooses this price, its HSC customers purchase its product with probability 1. This will imply that equation (2) holds. (More precisely, we will show that $-\delta\Pi + \sigma$ is the lower bound on the support of prices charged by the incumbent, and that when it chooses a price arbitrarily close to this lower bound, it 'keeps' the HSC customers with probability arbitrarily close to 1.)

LSC consumers always purchase from one of the lowest price sellers. By the stationarity hypothesis, HSC consumers who change suppliers can never gain from purchasing from an entrant which does not charge the lowest price: in the next period, any entrant who has attracted customers and become an incumbent will

[12] More precisely, along the equilibrium path the incumbent charges $-\delta\Pi + \sigma$ and the entrants charge 0 (so as not to subsidize LSC consumers who would bring them no future profits). In any continuation equilibrium after one or several entrants charge $-\delta\Pi$, at least some HSC consumers buy from the incumbent. In any continuation equilibrium after the incumbent charges more than $-\delta\Pi + \sigma$, some entrants charge $-\delta\Pi$.

charge the same price. Hence, calling p_E the lowest price charged by an entrant and p_I the price charged by the incumbent, HSC consumers buy from the incumbent if $p_I < p_E + \sigma$ and from one of the lowest price entrants if $p_I > p_E + \sigma$.

In the current period, the aggregate revenues of all the entrants who charges p_E is equal to p_E times the mass of (LSC and HSC) customers that they attract. By stationarity, their total future profits discounted to the next period are smaller than or equal to Π. Any $p_E < -\delta\Pi$ would generate strictly negative discounted profits in the aggregate for the lowest price entrants, and therefore $\underline{b}_E$, the lower bound of the support of the strategies of entrants, is greater than or equal to $-\delta\Pi$.

Clearly, the incumbent never charges less than $\underline{b}_E + \sigma$. Furthermore, $\underline{b}_E$ cannot be strictly greater than $-\delta\Pi$: otherwise, an entrant could attract all the consumers and make strictly positive discounted profits by charging a price in the interval $(-\delta\Pi, \underline{b}_E)$. Thus, $\underline{b}_E = -\delta\Pi$, and it is possible to show that the distribution of the lowest prices charged by the entrants does not have a mass point at this price.[13] Therefore, when the incumbent charges a price close to $-\delta\Pi + \sigma$ (which is the lower bound of the prices it charges), it 'keeps' all the HSC customers, and in all stationary equilibria equation (2) must hold.

5. Lower switching costs for all buyers can lead to higher profits for the incumbent

In Section 4, the switching cost of LSC consumers is equal to zero; with an infinite horizon, we have not been able to extend the analysis[14] to the case where all consumers have strictly positive switching costs. This prevents us from examining questions such as the consequences of an increase in the switching cost of all consumers (which we will show can decrease the profits of the incumbent!). Therefore, in this section we consider a two period model where LSC consumers can have strictly positive switching costs. This leads to new economic insights and to unexpected comparative statics, which are presented in Proposition 3.

[13] If there was such a mass point, for some $\eta > 0$ the incumbent would never choose a price in $(\underline{b}_E + \sigma, \underline{b}_E + \sigma + \eta]$: it could increase its profit by choosing a price slightly smaller than $\underline{b}_E + \sigma$ and selling to its HSC customers with probability 1. Then, entrants who make at best zero profits by charging $-\delta\Pi$ would obtain higher profits by charging any price in the interval $(\underline{b}_E, \underline{b}_E + \eta)$ than by charging $\underline{b}_E$, which establishes the contradiction.

[14] When LSC consumers have zero switching costs, in every period they purchase from one of the firm which charges the lowest price and that price is negative in equilibrium. Therefore, attracting them generates no profit; this fact greatly simplifies the analysis of the infinite horizon model.

5.1. Results and intuition

There are two[15] types of consumers: a mass α of HSC buyers, with a switching cost equal to σ_H, and a mass $(1-\alpha)$ of LSC buyers with a switching cost equal to $\sigma_L \in [0, \sigma_H)$. We assume that σ_L is small, more precisely,

$$\sigma_L < \frac{\alpha\delta}{1+\delta}\sigma_H, \tag{4}$$

which implies $\sigma_L < \alpha\sigma_H$. Thus, in the one period model the incumbent would charge σ_H, sell to all the HSC consumers and to no LSC consumer, and make a profit equal to $\alpha\sigma_H$. (In subsection 5.4, we study environments where inequality (4) does not hold.)

The following proposition states our main result.

Proposition 3. *In the two period model, where a proportion α of consumers have switching costs equal to σ_H while the others have switching costs equal to σ_L with inequality (4) satisfied, the equilibrium profit of the incumbent is*

$$\Pi = \sigma_H \left[\frac{\alpha\sigma_H - \sigma_L}{\sigma_H - \sigma_L}(1 + \delta - \alpha\delta) \right] \tag{5}$$

under either Stackelberg or Bertrand competition. Π *is greater than the one period profit, $\alpha\sigma_H$, and smaller than the discounted value of a flow of one period profit, $\alpha\sigma_H(1+\delta)$.*

We discuss the proof of Proposition 3 in Sections 5.2 and 5.3. Before doing so, we comment on its economic significance.

As in the infinite horizon model of Section 4, the presence of LSC buyers enables the incumbent to generate higher profits than it would receive in the one period model. Furthermore, when specialized to the case $\sigma_L = 0$, equation (5) is consistent with equation (1): it yields $\Pi = \alpha\sigma_H(1 + (1-\alpha)\delta)$, which is equal to the value of a flow of one period profits discounted at the rate of $\delta(1-\alpha)$, as in equation (2). It is also worth noticing that as α converges to 1, Π converges to σ_H, the one period profit, as we would expect from point iii of Proposition 2.

Corollary 3. *Under the hypotheses of Proposition 3*

i. Π *is increasing in α and σ_H and decreasing in σ_L;*

ii. If $\alpha < (\sigma_L + \sigma_H)/2\sigma_H$, which is always satisfied if $\alpha < 1/2$, then an equal increase in σ_H and σ_L leads to a decrease in Π *($\partial\Pi/\partial\sigma_L + \partial\Pi/\partial\sigma_H < 0$).*

[15] We have studied a model with a continuum of switching costs. There exists an equilibrium in pure strategies, which is easier to handle than the mixed strategy equilibrium of this paper, but the study of the consequences of increasing the mass of LSC consumers is more difficult: among other issues, we cannot use the simplifying assumption, which is often made in the literature, that types are uniformly distributed.

iii. If $\sigma_L < \alpha^2 \delta \sigma_H/(1+\delta)$, then a small increase in the number of LSC *consumers increases the profits of the incumbent.*

Without surprise, when α or σ_H increase, the profit of the incumbent increases. To understand why an increase in σ_L decreases profits, we note first that the incumbent will always price in such a way that it sells to no LSC consumer. Let us assume, only for expository purposes, that only one entrant attracted customers in the first period, and let $\gamma' > 0$ be the proportion of the HSC customers that it attracted. Because the LSC consumers are the most eager to switch suppliers, the entrant must also have attracted all of them. Therefore, its second period profit is $\alpha\gamma'\sigma_H$ if it charges σ_H, and $(\alpha\gamma' + (1-\alpha))\sigma_L$ if it charges σ_L. If it has attracted the proportion γ of HSC customers such that

$$\begin{aligned} &\alpha\gamma\sigma_H = (\alpha\gamma + (1-\alpha))\sigma_L \\ &\qquad \iff \gamma \equiv \frac{1-\alpha}{\alpha}\frac{\sigma_L}{\sigma_H - \sigma_L} = \frac{1-\alpha}{\alpha}\left(\frac{\sigma_H}{\sigma_H-\sigma_L} - 1\right), \end{aligned} \tag{6}$$

it will be indifferent between charging σ_L and σ_H. From (6), it is straightforward that an increase in σ_L leads to an increase in γ: the benefits of 'keeping' the LSC customers increases, thus the number of HSC consumers attracted in the first period must increase if the entrant is to be kept indifferent between its two plausible second period strategies. In equilibrium, in the first period a proportion γ of HSC consumers purchase from the entrant: if fewer than this proportion did so, the entrant would charge a low price in the second period, and be very attractive to HSC customers.[16] Therefore when σ_L increases, the first period incumbent loses more customers, which explains the result.

Whether an equal increase in both σ_H and σ_L will increase or decrease the profit of the incumbent will therefore depend on the relative strengths of two opposing effects, which, by (5), can be determined by evaluating the change in $\sigma_H(\alpha\sigma_H - \sigma_L)$. Adding η to both σ_H and σ_L and taking the derivative for $\eta = 0$, we obtain result ii) in Corollary 3: the negative consequences for the incumbent of an increase in σ_L swamps the positive consequences of an equal increase in σ_H when α is small enough.

Part (iii)[17] of the corollary is similar to part (iii) of Corollary 1. Note that it requires a σ_L smaller than the upper bound authorized by equation (4). Indeed, when σ_L is small, the same reasoning as in Section 4 holds: entrants do not want to attract LSC customers, and an increase in their number makes them less aggressive.

[16] As we will see shortly, the entrant mixes between σ_H and σ_L in the second period.

[17] It is easy to prove by computing the value of the derivative of

$$(1+\eta)\sigma_H\left[\frac{\frac{\alpha}{1+\eta}\sigma_H - \sigma_L}{\sigma_H - \sigma_L}(1+\delta - \frac{\alpha}{1+\eta}\delta)\right] = \frac{\sigma_H}{\sigma_H - \sigma_L}(\alpha\sigma_H - (1+\eta)\sigma_L)(1+\delta - \frac{\alpha}{1+\eta}\delta).$$

with respect to η for $\eta = 0$.

On the other hand, when σ_L is larger, LSC customers become valuable enough to entrants that an increase in their number makes them more aggressive.

We now turn to the proof of the Proposition 3.

5.2. Proof of Proposition 3 for the Stackelberg model

In period 2, all the firms which sold strictly positive amounts in period 1 announce their prices first, followed by the entrants, who in equilibrium charge 0. The incumbents charge σ_H if the proportion of HSC buyers among their period 1 consumers is strictly greater than σ_L/σ_H and σ_L if it is strictly less than σ_H/σ_L; if this proportion is exactly equal to σ_H/σ_L, they will be indifferent between σ_L and σ_H, and charge either one of these two prices with probability 1, or mix between the two.

If the firm from which it purchased in period 1 charges σ_H in period 2, a LSC consumer will choose to purchase from a period 2 entrant at a price of 0. Hence, his total period 2 cost will always be exactly σ_L, whatever he does in period 1. As a consequence, if, as above, we denote by p_E the lowest price charged by any entrant in period 1 in response to the period 1 price p_I charged by the incumbent, LSC consumers will purchase from one of the lowest price entrants if $p_E + \sigma_L < p_I$, from the incumbent if $p_E + \sigma_L > p_I$, and from one or the other if $p_E + \sigma_L = p_I$. Effectively, LSC customers minimize their cost in each period. Therefore, in equilibrium, the expected value of the second period price of all the entrants who attract LSC customers in the first period must be equal to each other.

Because second period prices are increasing functions of the proportion of HSC customers in the clientèle of a firm, the HSC customers who purchase from an entrant will also allocate themselves among the lowest cost first period entrants, and it cannot be an equilibrium for these entrants to charge different prices in the second period. Therefore, the pricing strategy of the "successful" entrants will only depend on whether or not in the aggregate they attracted a proportion of the HSC consumers smaller than, equal to, or greater than γ, as defined in (6).

This enables us to prove the following lemma, which describes the continuation payoff of the incumbent as a function of the price it charges in the first period. (The proof is in Appendix D.)

Lemma 1. *For a given price, p_I, charged by the incumbent in the first period:*

i. if $p_I < (1-\delta)\sigma_L$, the incumbent sells to all consumers in period 1 and to all HSC *consumers (at price σ_H) in period 2. Its profit is $p_I + \delta\alpha\sigma_H$.*

ii. if $p_I \in ((1-\delta)\sigma_L, (1-\delta)\sigma_H)$, the incumbent sells to all HSC *consumers in both periods and to no* LSC *consumer in either period. Its profit is $\alpha(p_I + \delta\sigma_H)$.*

iii. if $p_I \in \big((1-\delta)\sigma_H, (1-\alpha\delta)\sigma_H\big)$, the incumbent sells to $\alpha(1-\gamma)$ HSC *consumers at price p_I in period 1 and at price σ_H in period 2, while its sales to* LSC *consumers are equal to 0 in both periods. Its profit is $\alpha(1-\gamma)(p_I + \delta\sigma_H)$.*

iv. if $p_I > (1-\alpha\delta)\sigma_H$, the incumbent has zero sales in both periods.

From Lemma 1, the profits of the incumbent are increasing on the intervals $(-\infty, (1-\delta)\sigma_L)$, $((1-\delta)\sigma_L, (1-\delta)\sigma_H)$ and $\big((1-\delta)\sigma_H, (1-\alpha\delta)\sigma_H\big)$. Given the restrictions that we have imposed on σ_H/σ_L, it is easy to check that it is maximized on the union of these intervals for p_I smaller than and 'very close to' $(1-\alpha\delta)\sigma_H$. Therefore, the only equilibrium of the game has the incumbent charging $(1-\alpha\delta)\sigma_H$ in the first period with the continuation equilibrium described in point (iii), yielding the profits described by equation (5). This proves Proposition 3 for the Stackelberg model.[18]

5.3. Sketch of the proof of Proposition 3 in the Bertrand model

We divide this subsection in two parts. In the first, we provide a short sketch of the proof of equation (5), which is derived formally in Appendix E; in the second part, we describe in detail one of the payoff equivalent equilibria of the game.

5.3.1. Proving equation (5)

There are only mixed strategy equilibria in the Bertrand model, and we use a proof similar to the proof in Section 4.3 to show that the profits of the incumbent are the same as in the Stackelberg model. We sketch the argument in the rest of this subsection and present the full proof in Appendix E.

In period 1, entrants never charge strictly less than $-\alpha\delta\sigma_H$: at this price, they make zero profit even if they attract all the buyers. By exactly the same reasoning as in the infinite horizon case, this price must be in the support of the lowest price charged by the entrants and $-\alpha\delta\sigma_H + \sigma_H$ must be in the support of the period 1 price charged by the incumbent. Because we show that the incumbent never sells to a LSC customer in the period 1, its profit when it charges $-\alpha\delta\sigma_H + \sigma_H$ is

$$\alpha(1-\gamma) \times [\sigma_H(1-\alpha\delta) + \delta\sigma_H)],$$

where $\alpha(1-\gamma)$ is the number of (HSC) customers of the incumbent and $\sigma_H(1-\alpha\delta) + \delta\sigma_H$ is its discounted profit per customer. It is easy to check that this is indeed equal to the Π of equation (5).

5.3.2. What do equilibria look like?

The reasoning above is sufficient to prove equation (5), but does not provide much intuition about the equilibrium strategies of the agents. To help the reader build this intuition we now describe explicitly one equilibrium of the Bertrand game.

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[18] The identification of the equilibrium can be easily extended to the case where the cost of shifting from the incumbent to an entrant is σ'_L, greater but close to σ_L which now is the cost of shifting from a period 1 to a period 2 entrant. The entrants will still charge $-\delta\sigma_L$ in the period 1, and the total cost of the LSC consumers will be σ'_L. Apart from this, the equilibrium, and in particular the profit of the incumbent, will not be affected.

As discussed in section 4.3, in all equilibria the incumbent and the entrants use mixed strategies in period 1. For simplicity, we present an equilibrium where there is only one[19] active entrant, who chooses its price p_E in $[-\alpha\delta\sigma_H, -\delta\sigma_L]$, while the incumbent chooses p_I in $[-\alpha\delta\sigma_H + \sigma_H, -\delta\sigma_L + \sigma_H)$ and at least one other entrant charges $-\delta\sigma_L$ with probability 1. Then, all LSC customers buy from the entrant, and, depending on the difference between p_I and p_E, either all or a fraction γ of HSC customers purchase from the entrant:

- ➢ if $p_I - p_E \geq \sigma_H$, then all HSC consumers buy from the entrant, who therefore charges σ_H in the second period — its second period profit is $\alpha\sigma_H$;
- ➢ if $p_I - p_E < \sigma_H$, a proportion γ purchases from the entrant, who in the second period uses a mixed strategy: he chooses prices σ_L and σ_H with probabilities such that the HSC customers are indifferent between switching and not switching suppliers in period 1 — its second period profit is $(\alpha\gamma + (1-\alpha))\sigma_L = \alpha\gamma\sigma_H$ (in the states of nature where its second period price is σ_H, all the LSC customers switch to a period 2 entrant).

Therefore, in equilibrium, a proportion at least equal to γ of the HSC consumers purchase from the entrant in period 1.

The entrant chooses p_E according to the following distribution G_E, which has a mass point at $-\delta\sigma_L$:

$$G_E(p_E) = \begin{cases} \dfrac{p_E + \alpha\delta\sigma_H}{p_E + (1+\delta)\sigma_H} & \text{if } p_E \in [-\alpha\delta\sigma_H, -\delta\sigma_L), \\ 1 & \text{if } p_E = -\delta\sigma_L. \end{cases} \tag{7}$$

Then, if the incumbent chooses any $p_I \in [\sigma_H(1-\alpha\delta), \sigma_H - \delta\sigma_L)$, its expected discounted profit is

$$G_E(p_I - \sigma_H) \times 0 + (1 - G_E(p_I - \sigma_H)) \times (1-\gamma)(p_I + \delta\sigma_H) = (1-\gamma)\sigma_H(1+\delta-\alpha\delta). \tag{8}$$

To see why the incumbent must choose a price in the interval $[\sigma_H(1-\alpha\delta), \sigma_H - \delta\sigma_L)$, we check for possible deviations. A) It is not profitable for the incumbent to choose a price greater than or equal to $\sigma_H - \delta\sigma_L$, as this implies $p_I - p_E \geq \sigma_H$ with probability 1, and no sales! B) To show that it is not profitable to decrease prices below $\sigma_H(1-\alpha\delta)$, we proceed in two steps. a) First, note that by charging $\sigma_H(1-\alpha\delta)$, the incumbent sells to a proportion $1-\gamma$ of HSC customers. Claim E 2 in Appendix E shows that to increase its sales above this number the incumbent needs to choose $p_I \leq \sigma_H(1-\delta)$, which implies that as long as it does not sell to LSC customers, its profit, $\alpha(p_I + \delta\sigma_H)$, is at most $\alpha\sigma_H$ which is smaller than $(1-\gamma)\sigma_H(1+\delta-\alpha\delta)$, by (4). b) Second, in order to sell to LSC customers, the incumbent needs to make their total costs, over both periods, less than σ_L, which is the upper bound of their

[19] Our equilibrium is also an equilibrium if there are several active entrants and they each choose a mixed strategy such that the distribution of the minimum of the prices they charge is the function G_E defined below.

cost if they switch to the entrant in the first period. Given that they will switch in period 2 when it charges σ_H, this necessitates $p_I \leq (1-\delta)\sigma_L$, which leads to profits $p_I + \delta\alpha\sigma_H$ smaller than the profits when using the equilibrium strategy.

Similarly, in our equilibrium the incumbent chooses p_I according to the distribution

$$G_I(p_I) = \frac{p_I - \sigma_H(1-\alpha\delta)}{p_I - \sigma_H(1-\alpha\delta) + ((1-\alpha)+\alpha\gamma)(\sigma_H - p_I - \delta\sigma_L)}.$$

Then, the profit of the active entrant is

$$G_I(p_E + \sigma_H) \times (p_E + \delta\sigma_L)(1-\alpha+\alpha\gamma) + (1 - G_I(p_E+\sigma_H))(p_E + \delta\alpha\sigma_H) = 0$$

when it chooses a price in $[-\alpha\delta\sigma_H, -\delta\sigma_L]$, and smaller than or equal to 0 when it chooses a price outside of this interval (the presence of another entrant who charges $-\delta\sigma_L$ is crucial for this last point).

In all equilibria p_I will be distributed according to G_I and p_E, interpreted as the lower bound of the prices of the active entrants, will be distributed according to G_E. We will let the interested reader convince himself of this fact.

Figure 1 shows the equilibrium strategies with $\sigma_H = 1$, $\sigma_L = .2$, $\alpha = .4$ and $\delta = 1$.

5.4. Equilibrium with large σ_L

For completeness, we now turn to a discussion of the equilibrium when equation (4) does not hold. Proofs and more details can be found in Appendix E.

If σ_L is very large, *i.e.,* greater than $\alpha\sigma_H$, then everything happens as if all the consumers were LSC consumers: the incumbent charges $\sigma_L(1-\delta)$ in period 1, σ_L in period 2, and sells to all consumers. Its profit is σ_L.

If σ_L/σ_H is smaller than but close enough to α, then there is a pure strategy equilibrium where in period 1 the incumbent sells to all the HSC consumers at a price $\sigma_H(1-\delta)$ and the entrants sell to all the LSC consumers at a price $-\delta\sigma_L$. In period 2, the incumbent charges σ_H and 'keeps' all the HSC customers — its profit over both periods is therefore $\alpha\sigma_H$. The best alternative strategy for the incumbent would be to charge $\sigma_H - \delta\sigma_L$ in period 1, and sell to a proportion $1-\gamma$ of the HSC consumers. This strategy becomes more attractive as σ_L decreases, and dominates when $\sigma_L/\sigma_H < x_C$, where $x_C \in (\delta\alpha/(1+\alpha), \alpha)$ solves

$$x_C\left[1+\delta+\alpha\delta-\alpha\right] = \delta\left[\alpha + {x_C}^2\right]. \tag{9}$$

When $\sigma_L/\sigma_H \in (\delta\alpha/(1+\delta), x_C)$, we show in Appendix E that there exists the same mixed strategy equilibrium as when $\sigma_L < \frac{\alpha\delta}{1+\delta}\sigma_H$. The difference is that now the one period game profit is larger than the equilibrium profit and thus we need to check for deviations where the incumbent could retain all the HSC customers.[20]

[20] In Appendix E we only prove that there exists an equilibrium which satisfy these properties, *not* that *all* equilibria do although we believe that this may well be the case.

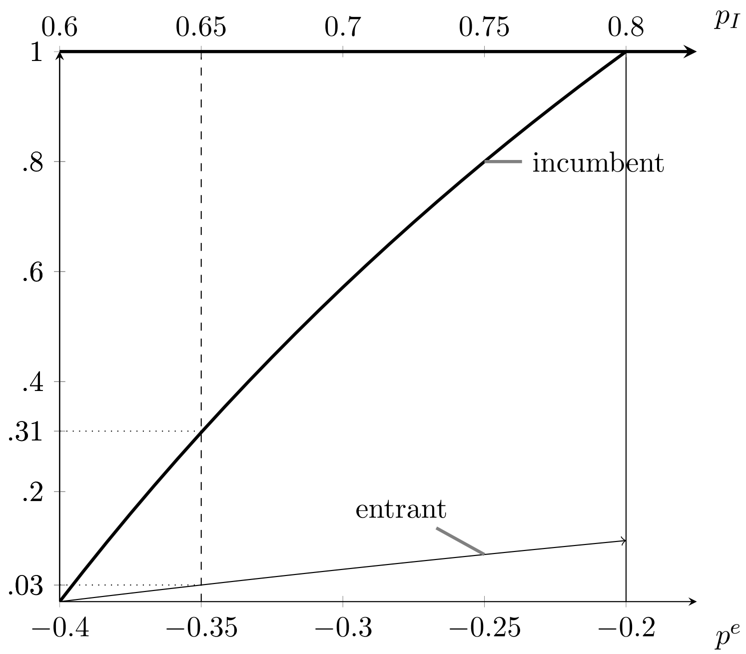

Figure 1: This figure represents the probability distributions in the mixed strategies of the incumbent and the entrant with $\sigma_H = 1$, $\sigma_L = .2$, $\alpha = .4$ and $\delta = 1$, which implies $\gamma = 37.5\%$. For instance, reading along the vertical dashed line, if $p_E = -0.35$, we obtain $G(p_E) \approx 0.03$, which implies that if the incumbent chooses $p_I = 0.65 = -0.35 + \sigma_H$, then it looses all its HSC customers with probability 3% and sells to a proportion $1 - \gamma$ of them with probability 97%. Similarly, if the entrant chooses $p_E = -0.35$, it sells to a proportion γ of HSC customers with a probability 31% and to all of them with probability 69%.

6. Conclusion

A significant body of theory explores the consequences of consumer switching costs: it highlights the role of "bargain then rip-off" pricing patterns, where a firm makes very profitable introductory offers and raises its price in subsequent periods. To the best of our knowledge, the fact that the distribution of switching costs changes considerably the way in which these strategies play out has not been pointed out. We hope the present paper will contribute to close this gap. Taking into account the heterogeneity of switching costs has enabled us to identify very rich strategic interactions between the incumbent and the entrants and led to surprising comparative statics.

As we have seen, our analysis supports Johansen's insight that the distribution of switching costs might be important in the music player industry. However, our interpretation is different than his: the heterogeneity of switching costs could be beneficial to Apple not so much because it implies that there exist a subset of consumers with very high switching costs, but rather because the presence of customers with low switching costs makes an aggressive pricing strategy potentially very costly for an entrant.

The liberalized UK domestic gas and electricity markets analyzed by NERA in Office of Fair Trading (2003) appears to broadly fit the context we consider: the product is homogenous, discrimination between old and new customers was not an option, and entrants had to attract customers away from the historical incumbent (British Gas and the public electricity suppliers) as there were practically no unattached customers. Entrants offered prices below cost, and a fortiori below those of the incumbent(s), which saw their market share decrease. Our analysis shows that information on the distribution of switching costs, for which no data is given, should have been gathered and that its consequences for the strategy of the entrants should have been considered.

We now turn towards a discussion of questions which are open for research. First, we have used a very stark model, with free entry and "many" entrants in every period. Much of the literature on switching costs has emphasized models where a limited number of incumbents compete over time, trying to vie for each other's consumers. It is important to study the robustness of the conclusions of that part of the literature to heterogeneity in switching.

On the theoretical side, we have not been able to identify the equilibria in a infinite horizon model, except in the case where the switching cost of the LSC consumers is equal to 0. Solving this problem raises interesting, but difficult, questions; in particular, we are not sure that a stationary equilibrium exists, or we do not even know what would be the appropriate definition of stationarity for that case.

Finally, network externalities often play a role similar to switching costs — they have sometimes been called 'collective' switching costs. In future work, we plan to study models where agents have different trade-offs between size of network and prices; we believe that phenomena similar to those analyzed in the current paper can be identified.

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Appendices

A. Equilibrium in the infinite horizon model when all consumers have the same switching cost

In this appendix, we prove that the equilibrium price p^* in the Bertrand competition infinite horizon game where all consumers have the same switching cost σ is equal to $\sigma(1-\delta)$.

This implies Proposition 2. The Stackelberg case is very similar and somewhat easier to prove; we leave it to the reader.

Claim A 1. $p^* \leq \sigma(1-\delta)$.

Proof. Consumers who purchase from an incumbent incur a discounted cost $p^*/(1-\delta)$. In the current period, consumers who purchase from an entrant who charges p' face a disutility of $p' + \sigma$. By stationarity, in each subsequent period they pay p^*; hence, their total discounted disutility is

$$p' + \sigma + \frac{\delta p^*}{1-\delta}$$

Consumers necessarily switch if[21]

$$p' + \sigma + \frac{\delta p^*}{1-\delta} < \frac{p^*}{1-\delta}.$$

Hence, for any $\varepsilon > 0$, an entrant who would charge $p^* - \sigma - \varepsilon$ would attract customers, and obtain profits equal to the mass of consumers it attracts multiplied by

$$p^* - \sigma - \varepsilon + \frac{\delta p^*}{1-\delta}.$$

Writing that this expression is negative for all $\varepsilon > 0$ yields the result. □

Claim A 2. $p^* \geq \sigma(1-\delta)$.

Proof. In any period, the lowest priced entrant must charge $p^* - \sigma$: otherwise the incumbent could increase its price without loosing customers.[22] If the entrant attracted customers at this price it would make profits equal to the mass of these customers multiplied by $p^* - \sigma + \delta p^*/(1-\delta)$, which must be non negative for $p^* - \sigma$ to be undominated. This proves the claim. □

Together, Claims A 1 and A 2 prove that p^* is equal to $\sigma(1-\delta)$.

[21] This is true because of the "consumers have mass" assumption informally introduced on page 6 in Section 3 and more formally in Biglaiser et al. (2009): because an arbitrarily small group of consumers would find it optimal to purchase from the entrant, they will do it.

[22] Out of equilibrium, where there can be several incumbents, the reasoning would have to hold for any incumbent.

B. Stackelberg equilibrium in the infinite horizon model with two levels of switching costs

In this appendix, we present a formal version of the proof sketched in 4.2. We will do so by proving that in all equilibria the equilibrium price charged by an incumbent, p^*, is equal to p^S as defined in (3).

We will call p_E the minimum of the prices charged by any entrant (this minimum exists as we are identifying equilibria with a finite number of active entrants in each period).

Claim B 1. $p^* \leq p^S$.

Proof. By stationarity, if $p_E < p^* - \sigma$ all consumers purchase from one of the lowest price entrants. The sum of the profits of these entrants is $p_E + \alpha\delta p^*/(1-\delta)$ multiplied by the mass of consumers which they have attracted. This expression must be negative for all $p < p^* - \sigma$, otherwise there would be a feasible and profitable path to entry. Therefore,

$$p^* - \sigma + \frac{\delta\alpha p^*}{1-\delta} \leq 0 \Longleftrightarrow p^* \leq \sigma\frac{1-\delta}{1-\delta+\alpha\delta} = p^S$$

□

Claim B 2. $p^* \geq p^S$.

Proof. We will show that if $p^* < p^S$, a deviation by the incumbent to any $p' \in (p^*, p^S)$ would be profitable. Indeed, entrants (there could be only one of them) who would respond by charging $p' - \sigma$ or less would generate aggregate discounted profits of at most

$$p' - \sigma + \delta\frac{\alpha p^*}{1-\delta}.$$

(This is their profit if they attract all the HSC consumers.) If both p' and p^* are strictly smaller than p^S, this expression is strictly negative. Therefore, at least one of the entrants would be making strictly negative profits; the deviation by the incumbent is profitable, as entrants would not be able to respond and attract consumers profitably. □

Claims B 1 and B 2 imply $p^* = p^S$.

C. Bertrand equilibrium in the infinite horizon model with two levels of switching costs

In this appendix, we begin by proving that equation (1) must hold for any Bertrand equilibrium. We then show that there indeed exists an equilibrium by exhibiting one.

C.1. Proof of equation (1)

Most of the work consists in computing bounds on the distribution of prices charged by the firms: $\underline{b}_I$ and $\overline{b}_I$, respectively the lower and upper bounds of the support of the prices charged by incumbent(s) as well as $\underline{b}_E$ and $\overline{b}_E$ the lower and upper bounds of the distribution of the lowest price charged by the entrants.

Claim C 1. Incumbents have strictly positive profits, which implies $\underline{b}_I > 0$.

Proof. We proceed by contradiction, and show that incumbents cannot have profits equal to 0. Remember that we are looking for stationary equilibria: any entrant who would attract consumers would become an incumbent in the next period, and therefore make 0 profits. It is therefore a dominated strategy for entrants to offer a negative price. Now assume that an incumbent deviates in period t and charges $p' \in (0, (1-\delta)\sigma)$. An upper bound for the total cost that an HSC consumer can incur by purchasing from the incumbent in period t and switching in period $t+1$ is $p' + \delta\sigma < \sigma$, which is a lower bound of the cost that he would incur by purchasing from an entrant in period t. Therefore there exists a profitable deviation for the incumbent. □

Claim C 1 implies that in equilibrium LSC consumers never buy from the incumbent.

Claim C 2. In any period, the expected discounted profit of entrants is equal to 0.

Proof. The expected discounted profits of entrants are the same for all prices in ($\underline{b}_E$, $\overline{b}_E$). If the distribution of p_E does not have a mass point at $\overline{b}_E$, then an entrant's expected profit who chooses this price is 0. And there cannot be a mass point at $\overline{b}_E$; otherwise, an entrant who would charge a price slightly below $\overline{b}_E$ would make strictly higher payoffs than charging $\overline{b}_E$, which cannot be true in equilibrium. □

Claim C 3. $\overline{b}_E \leq 0$.

Proof. By Claim C 1, with $\overline{b}_E > 0$, an entrant could make a positive profit by charging a price in $(0, \min\{\underline{b}_I, \overline{b}_E\})$, attract the LSC consumers (and, maybe, some HSC consumers) with probability 1 and make strictly positive profits, which establishes the contradiction due to Claim C 2. □

Claim C 4. The expected discounted profits of an incumbent from any period t are independent of the number of its LSC customers in period $t-1$.

Proof. By Claims C 1 and C 3, LSC customers never purchase from an incumbent. □

This enables us to define, without ambiguity, Π as the profit of an incumbent from whom all the HSC consumers purchased in the previous period. We will now proceed to demonstrate that the incumbent equilibrium profit is the same in the Bertrand and Stackelberg models.

Claim C 5. $\underline{b}_E \geq -\delta\Pi$.

Proof. In every state of nature where $p_E \in [\underline{b}_E, -\delta\Pi)$, the aggregate expected discounted profits of the lowest price entrants would be strictly negative as their current period losses exceeds their future discounted profits. □

Claim C 6. $\underline{b}_I \geq \underline{b}_E + \sigma$.

Proof. By announcing any price strictly smaller than $\underline{b}_E + \sigma$, the incumbent "leaves money on the table". □

Claim C 7. $\underline{b}_E \leq -\delta\Pi$.

Proof. Otherwise, a price in the interval $(-\delta\Pi, \underline{b}_E)$ would allow an entrant to become the lowest price entrant with probability 1 while underpricing incumbents by more than σ (by Claim C 6). It would make strictly positive profits, which establishes the contradiction by Claim C 2. □

Claims C 5 and C 7 and the fact that entrants use a mixed strategy prove

$$\underline{b}_E = -\delta\Pi \leq \bar{b}_E. \tag{C 1}$$

Now, we pin down the lower bound on the prices charged by incumbents. Claim C 6 and equation (C 1) imply the following claim.

Claim C 8. $\underline{b}_I \geq -\delta\Pi + \sigma$.

Claim C 9. $\underline{b}_I = -\delta\Pi + \sigma = -\underline{b}_E + \sigma$.

Proof. By equation (C 1) and Claim C 8, we only need to prove $\underline{b}_I \leq -\delta\Pi + \sigma$. If this were not true, by (C 1), an entrant could make positive profits by choosing a price in $(-\delta\Pi, \min\{\underline{b}_I - \sigma, \underline{b}_E\})$, which contradicts Claim C 2. □

Claim C 10. When p_I converges to $\underline{b}_I$ from above, the proportion of HSC customers who choose to purchase from an incumbent converges to 1.

Proof. The expected number of HSC customers of the incumbent is decreasing in p_I. Therefore, if the claim is not true, the proportion of HSC consumers who choose to purchase from the incumbent is bounded above by some $\eta < 1$. This implies that when the price it charges converges to $\underline{b}_I$, the total discounted profits of the incumbent, if it sold to a proportion ζ of HSC customers in the previous period, converges to[23]

$$\zeta(\eta\alpha\underline{b}_I + \eta\delta\Pi) < \zeta(\alpha\underline{b}_I + \delta\Pi);$$

by Claim C 9 it can guarantee itself a profit arbitrarily close to the right hand side of this inequality by charging a price below, but very close to $\underline{b}_I$, which establishes the contradiction. □

[23] If some of the HSC consumers do not purchase from the incumbent, none of the LSC consumers will.

Claim C 10 implies that, in any equilibrium, the profits of the incumbent take the form described by equation (2). We now must prove that there exists such an equilibrium. To do this we begin by deriving the distribution of prices that must prevail in any equilibrium.

C.2. Distribution of prices in equilibrium

Assuming that an equilibrium does exist, we compute the distribution of p_E and p_I. In C.3, we will show that these distributions do indeed constitute an equilibrium.

We begin by computing the distribution of prices announced by the incumbent. Standard arguments show that this distribution G_I of p_I has no mass point on $(\underline{b}_I, \overline{b}_I)$. The zero profit condition for the entrants implies

$$G_I(p_E + \sigma) \times [(1-\alpha)p_E] + (1 - G_I(p_E + \sigma)) \times [p_E + \alpha\delta\Pi] = 0 \quad \forall p_E \in (\underline{b}_E, \overline{b}_E)$$

$$\Longrightarrow G_I(p_I) = \frac{p_I - \sigma + \delta\Pi}{\alpha(p_I - \sigma) + \delta\Pi} \quad \forall p_I \in (\underline{b}_I, \overline{b}_I). \tag{C 2}$$

Because $\lim_{p_I \to \underline{b}_I^+} G_I(p_I) = 0$ and $\lim_{p_I \to \overline{b}_I^-} G_I(p_I) = 1$, the function G_I has no mass point.

Similarly, the distribution G_E of p_E is determined by the fact that the profits of the incumbent are equal to Π, for all prices in $[\underline{b}_I, \overline{b}_I]$, and therefore

$$G_E(p_E) = 1 - \frac{\Pi}{\alpha(p_E + \sigma) + \delta\Pi} \text{ for } p_E \in (\underline{b}_E, \overline{b}_E). \tag{C 3}$$

Because

$$\lim_{p_E \to 0^-} G_E(0) = \frac{\alpha\sigma - (1-\delta)\Pi}{\alpha\sigma + \delta\Pi} < 1;$$

the distribution G_E has a mass point at $p_E = 0$.

C.3. Existence of an equilibrium

We have proved that if there exists an equilibrium that satisfies our assumptions, the distribution of prices must satisfy equations (C 2) and (C 3). We now prove that there does indeed exist such an equilibrium; this is a proof by construction: we exhibit the strategies followed by the agents.

In this equilibrium the consumers who buy from an entrant always buy from the same (lowest price) entrant. The analysis which we have conducted to derive (C 2) and (C 3), shows that these strategies are best responses for all the agents when there is only one incumbent.

We need to examine the continuation equilibrium when there are several incumbents, for two reasons: a) for the consumers to find it optimal to coordinate on buying from one incumbent, it must be the case that it is not a profitable deviation for a small mass of HSC consumers to purchase from another firm than the other HSC consumers;

b) we have imposed the requirement that all incumbents, *i.e.*, all firms that have sold to HSC consumers in the previous period, use the same pricing strategy. As we will see, the fact that the strategies of the firms satisfy b) provides an easy proof of point a).

Let us therefore assume that in one period there are $n \geq 2$ incumbents. It is straightforward to see that if the distribution of p_E is G_E, then all the incumbents are indifferent between all prices in $[\underline{b}_I, \overline{b}_I]$. We now show that the profits of the lowest price entrants are equal to 0 if all the incumbents choose the strategy described by (C 2).

Let α_i be the mass of HSC consumers of incumbent $i = 1 \ldots, n$ in the previous period. The lowest price entrant sells to all the LSC consumers and to the mass of HSC consumers who were in the previous period clients of firms who choose in the current period a price $p_i \geq p_E + \sigma$. Because it will follow the same strategy as a unique incumbent, and because the distribution of prices of the entrant is independent of the number of incumbents, its profits discounted to the future of next period will be $\beta\Pi/\alpha$.

Therefore, for given prices by the incumbents, the profit of the entrant is

$$(1-\alpha)p_E + \sum_{\{i|p_i > p_E + \sigma\}} (\alpha_i p_E + \frac{\alpha_i}{\alpha}\delta\Pi) = (1-\alpha)p_E + \sum_{\{i|p_i > p_E + \sigma\}} \alpha_i \times \left(p_E + \frac{\delta\Pi}{\alpha}\right)$$

$$= (1-\alpha)p_E + \sum_i s_i(p_i) \left(p_E + \frac{\delta\Pi}{\alpha}\right),$$

where s_i is the random variable, of expected value $\alpha_i(1 - G_I(p_E + \sigma))$, that takes the value α_i for $p_i \geq p_E + \sigma$ and 0 otherwise. The p_i's are independently distributed, and therefore the expected value of $\sum s_i(p_i)$ is $\alpha(1 - G_I(p_E + \sigma))$, and the expected profit of the lowest price entrant, conditional of the fact that it has chosen a price of p_E, is

$$(1-\alpha)p_E + (\alpha p_E + \delta\Pi)(1 - G_I(p_E + \sigma)),$$

which, by equation (C 2), is equal to 0.

The fact that all incumbents use the same pricing strategy implies that HSC consumers have no incentive to deviate from the focal strategy that we described above: in subsequent periods, they would face the same distribution of prices both from the firm they purchased from in previous periods and from the entrants.

D. Equilibrium in the two period Stackelberg model

In this section, we prove Lemma 1. We begin by establishing three claims; the first one is part *iv* of the lemma.

Claim D 1. If $p_I > (1 - \alpha\delta)\sigma_H$, the incumbent has zero sales in both periods.

Proof. If $p_I > (1-\alpha\delta)\sigma_H$, a unique lowest price entrant who would charge $p_E \in (-\alpha\delta\sigma_H, p_I - \sigma_H)$ would make strictly positive profits equal to $p_E + \delta\alpha\sigma_H$, as it would attract all consumers in period 1. Free entry prevents this, and therefore in the continuation game, one or several entrants must charge $-\alpha\delta\sigma_H$, and attract all the consumers while making zero profits. □

Claim D 2. If $p_I < (1-\alpha\delta)\sigma_H$, then no entrant attracts enough HSC consumers in period 1 that it finds it optimal to charge σ_H with probability 1 in period 2.

Proof. Assume that entrant $\tilde{e}$ attracted a large enough proportion of HSC customers that it found it optimal to charge σ_H in period 2. Because LSC consumers always find it strictly more profitable to switch suppliers than do HSC consumers, the incumbent would have no LSC customers and, therefore, HSC customers can guarantee themselves a second price of σ_H by "staying with" the incumbent. Therefore, entrant $\tilde{e}$ must have chosen a period 1 price $p_{\tilde{e}} \leq p_I - \sigma_H < -\delta\alpha\sigma_H < -\delta\sigma_L$. This implies that no entrant finds it profitable to attract period 1 customers and charge σ_L in period 2: all entrants that attract customers must choose the same strategy as $\tilde{e}$, and in the aggregate their profits are smaller than $p_{\tilde{e}} + \delta\alpha\sigma_H < 0$, which establishes the contradiction. □

Claim D 3. If $p_I < (1-\delta)\sigma_H$, all HSC consumers purchase from the incumbent in period 1.

Proof. By Claim D 2, any period 1 entrant who has attracted consumers in period 1 charges σ_L with positive probability in period 2. Therefore, its second period profit will be σ_L times the mass of consumers it attracted in the first period and, by free entry, its period 1 price must be $-\delta\sigma_L$. The total discounted cost for a HSC consumer who would purchase from a period 1 entrant would therefore be at least $(-\delta\sigma_L + \sigma_H) + \delta\sigma_L = \sigma_H$ (it would be greater if in period 2 the entrant charged σ_H with a probability in $(0,1)$). If he purchases from the incumbent, his total cost is $p_I + \delta\sigma_H$, and therefore strictly lower, which establishes the claim. □

Parts *i* and *ii* of the lemma follow immediately from Claim D 3.

If $p_I \in \big((1-\delta)\sigma_H, (1-\alpha\delta)\sigma_H\big)$, HSC consumers prefer to purchase from an entrant if its period 2 price is σ_L and from the incumbent if the entrant's period 2 price is σ_H. Therefore, there can be an equilibrium only if the entrants play a mixed strategy in period 2, which is feasible only if in period 1 they attract a proportion γ of the HSC consumers. This establishes part *iii* of the lemma and completes the proof.

E. Equilibrium in the two period Bertrand model

In subsection E.1, we begin by deriving some properties of the equilibrium of the two period Bertrand model which hold for any values of the parameters. Then, in E.2, we specialize the model to the case where equation (4) holds and prove Proposition 3. Finally, in E.3, we conduct the analysis which leads to the "large σ_L" results discussed in 5.4.

E.1. Some properties of the equilibrium in the two period Bertrand model

Because of free entry, period 2 entrants choose a price equal to 0. As in the Stackelberg case, a period 2 incumbent charges σ_L or σ_H depending on whether the proportion of its HSC customers in period 1 was less or greater than σ_L/σ_H, and, clearly, the period 1 incumbent will charge σ_H in period 2.

This implies that, as in the Stackelberg case, in period 1 LSC consumers will optimally behave as if they were myopic, switching to one of the lowest price entrants if the difference between its price and the incumbent's price is greater than σ_L and not switching if this difference is smaller than σ_L. It also implies that any HSC consumer who does not buy from the incumbent in period 1 also buys from one of the lowest priced entrants. Indeed, any other entrant would attract only HSC customers, and hence charge σ_H in period 2.

We are now ready to study the pricing behavior of the firms in period 1. We begin by Claim E 1 which describes the behavior of entrants. Next, in Claims E 2 and E 3 we describe properties of the incumbent's first period demand function. They help us characterize the strategy of the incumbent, which enables us to compute the lower bound on its prices of Claim E 6.

Claim E 1. In period 1, any active entrant charges a price in $[-\delta\alpha\sigma_H, -\delta\sigma_L]$.

Proof. Any entrant who has attracted consumers in period 1 will charge at least σ_L in period 2. Therefore, competition and free entry will ensure that in period 1 no entrant which charges more than $-\delta\sigma_L$ attracts a positive measure of customers with positive probability. If the lower priced entrants charge prices strictly smaller than $-\delta\alpha\sigma_H$, their aggregate profit is negative by the same line of reasoning as in the proof of Claim D 2. □

Claim E 2. If the incumbent charges a price strictly greater than $\sigma_H(1-\delta)$ in period 1, then it sells to at most $(1-\gamma)\alpha$ HSC customers.

Proof. Assume $p_I > (1-\delta)\sigma_H$. Because $-\delta\sigma_L + \sigma_L < \sigma_H(1-\delta)$, by Claim E 1 all the LSC consumers, who act myopically in the first period, purchase from entrants. If on the aggregate the entrants attract a proportion of HSC customers smaller than γ, at least one of them will have a proportion of period 1 HSC customers strictly smaller than σ_L/σ_H and therefore charge σ_L with probability 1 in period 2. HSC customers would find this entrant more attractive than the incumbent as $(-\delta\sigma_L + \sigma_H) + \delta\sigma_L < \sigma_H(1-\delta) + \delta\sigma_H$, which establishes the contradiction. □

Claim E 3. If in period 1 the incumbent charges a price strictly smaller than $\sigma_H(1-\alpha\delta)$, then it sells to at least $(1-\gamma)\alpha$ HSC customers.

Proof. If this were not the case, at least one of the entrants would attract enough HSC customers in the first period to charge σ_H in the second period; by Claim E 1, these HSC customers would incur total discounted costs equal to at least $-\delta\alpha\sigma_H+\sigma_H+\delta\sigma_H$, which is strictly larger than the total discounted costs that they would incur from buying from the incumbent in both periods. □

Claims E 2 and E 3 show that for $p_I \in (\sigma_H(1-\delta), \sigma_H(1-\alpha\delta))$, the incumbent sells to $(1-\gamma)\alpha$ customers. This implies the following claim.

Claim E 4. The incumbent will never choose a first period price in $(\sigma_H(1-\delta), \sigma_H(1-\alpha\delta))$.

Claim E 5. By choosing p_I below but 'close to' $\sigma_H(1-\alpha\delta)$, the incumbent can guarantee itself discounted profits arbitrarily close to $(1-\gamma)\alpha\sigma_H(1-\alpha\delta+\delta)$.

Proof. It sells to at least $(1-\gamma)\alpha$ HSC consumers at price $(1-\alpha\delta)\sigma_H$ in period 1 and at price σ_H in period 2. □

Claim E 6. $\underline{b}_I \leq \sigma_H(1-\alpha\delta)$.

Proof. The incumbent makes strictly positive profits. This implies that p_E is not strictly smaller than $p_I - \sigma_H$ with probability 1. However, if $\underline{b}_I > \sigma_H(1-\alpha\delta)$ an entrant could charge a price in $(-\alpha\delta\sigma_H, \underline{b}_I+\sigma_H)$ and obtain strictly positive expected profits. In the states of nature where it is not the lowest priced entrant, it would attract no consumers and make a profit equal to 0. When it is the lowest price entrant, which would happen with strictly positive probability by Claim E 1, it would undercut the other entrants and also undercut the incumbent by more than σ_H; its discounted profit would be strictly positive, which establishes the contradiction. □

E.2. Equilibrium in the two period Bertrand model for small σ_L

In subsection E.1, we have not used any restrictions on ratio of switching costs, σ_L/σ_H. We now restrict the analysis of the cases where equation (4) ($\sigma_L < \alpha\delta\sigma_H/(1+\delta)$) holds, which will enable us to prove Proposition 3.

Claim E 7. If equation (4) holds, then at equilibrium $\underline{b}_I > \sigma_H(1-\delta)$.

Proof. From (4) and Claim E 6, we have $\underline{b}_I < \sigma_L(1-\delta)$. By Claim E 1, this implies that the incumbent never sells to any LSC consumers. By Claim E 2 if the incumbent chooses $p_I > \sigma_H(1-\delta)$, at least $\alpha\gamma$ HSC consumers buy from a period 1 entrant. Thus, the highest profit the incumbent could make while selling to all HSC consumers in period 1 is $\sigma_H(1-\delta)+\delta\sigma_H = \alpha\sigma_H$. Using Claim E 5, the incumbent can improve its profit by charging a price larger than $\sigma_H(1-\delta)$, since equation (4) is equivalent to $\alpha\sigma_H < (1-\gamma)\alpha\sigma_H(1-\alpha\delta+\delta)$. □

Claims E 4, E 6 and E 7 imply $\underline{b}_I = \sigma_H(1 - \alpha\delta)$ if $\sigma_L/\sigma_H < \delta\alpha/(1+\delta)$. By Claim E 2 this implies that the discounted profit of the incumbent is bounded above by

$$(1-\gamma)\alpha\sigma_H(1 - \alpha\delta + \delta\sigma_H) = (1-\gamma)\alpha(\underline{b}_I + \delta\sigma_H).$$

By Claim E 5, this quantity is also an lower bound on the profit, and this proves Proposition 3.

E.3. Equilibrium in the two period Bertrand model when σ_L/σ_H is greater than or equal to $\delta\alpha/(1+\delta)$

In this subsection, we prove the results discussed in subsection 5.4. Note that we are less ambitious than in E.2: we are only trying to identify one equilibrium for each value of σ_H/σ_L, not to characterize all the equilibria. We present the results under the form of three claims, starting with the largest value of σ_L/σ_H.

Claim E 8. If $\sigma_L/\sigma_H > \alpha$, then the two period Bertrand game has a unique equilibrium in which the incumbent charges $\sigma_L(1-\delta)$ in period 1 and σ_L in period 2. It sells to all consumers and its profits are σ_L.

As in the one period model, when $\sigma_L > \alpha\sigma_H$ the incumbent and the entrant act as if there were only LSC customers in the economy. We leave the proof of the claim to the reader.

For $\sigma_L/\sigma_H \in (x_C, \alpha)$, with x_C defined by (9), we establish the following claim:

Claim E 9. If $\sigma_L/\sigma_H \in [x_C, \alpha]$, there exists a pure strategy equilibrium in which the incumbent, whose profits are $\alpha\sigma_H$, sells to the HSC consumers in both periods, at prices respectively equal to $\sigma_H(1-\delta)$ and σ_H. All LSC customers purchase from entrants at price $-\delta\sigma_L$ in period 1 and at price σ_L in period 2.

Proof. We show that the strategies described in the claim form an equilibrium. The LSC customers are clearly better off switching in period 1. The strategy of the HSC customers is a best response to the strategy of the other agents as they are indifferent between purchasing from the incumbent in both periods and switching to an entrant in the first period — in both cases their total discounted costs are equal to σ_H.

This indifference of HSC consumers implies that the incumbent would loose at least a proportion γ of its customers if it increased its period 1 price. It is straightforward to see that, under these circumstances, its most profitable increase in price is to $\sigma_H - \delta\sigma_L$. This deviation is unprofitable as long as

$$\alpha\sigma_H \geq (1-\gamma)\alpha(\sigma_H - \delta\sigma_L + \delta\sigma_H) = [\sigma_H - \delta\sigma_L + \delta\sigma_H]\,\alpha \left[\frac{\alpha\sigma_H - \sigma_L}{\alpha(\sigma_H - \sigma_L)}\right]$$
$$\iff \sigma_L/\sigma_H\,[1 + \delta + \alpha\delta - \alpha] \geq \delta\left[\alpha + (\sigma_L/\sigma_H)^2\right],$$

and therefore, by (9), holds on $[x_C, \alpha]$. A small decrease in period 1 price obviously decreases the profits of the incumbent. A decrease to $\sigma_L(1-\delta)$ allows it to sell to all consumers in period 1, but decreases its profits.

Finally, it is easy to show that the entrants strategy is indeed a best response to the strategies of the other agents. □

Claim E 10. The equilibrium described in 5.3.2 is also an equilibrium when $\sigma_L/\sigma_H \in [\alpha\delta/(1+\delta), x_C]$.

Proof. The proof is exactly the same as in 5.3.2, except that we need to be a bit more careful when showing that the incumbent does not gain by deviating to p_I in $(\sigma_L(1-\delta), \sigma_H(1-\delta)]$. The incumbent sells to all the HSC customers if

$$p_I + \delta\sigma_H < p_E + \sigma_H + \delta\sigma_L \iff p_I < p_E + (1-\delta)\sigma_H + \delta\sigma_L,$$

This implies that if $p_I \in [(1-\alpha\delta-\delta)\sigma_H + \delta\sigma_L, (1-\delta)\sigma_H)$, the incumbent sells to all the HSC consumers with probability strictly between 0 and 1 — in the other states of nature, it sells to a proportion $1-\gamma$ of them. From (7), one can show that this implies that the profits of the incumbent are increasing in this range, and that, because $\alpha < \sigma_L/\sigma_H$, they are always smaller than the putative equilibrium profit, which establishes the claim. □