Price Caps as Welfare-Enhancing Coopetition*

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Abstract: The paper analyzes the impact of price caps agreed upon by industry participants. Price caps, like mergers, allow firms to solve Cournot's multiple marginalization problem; but unlike mergers, they do not stifle price competition in case of substitutes or facilitate foreclosure in case of complements. The paper first demonstrates this for non-repeated interaction and general demand and cost functions. It then shows that allowing price caps has no impact on investment and entry in case of substitutes. Under more restrictive assumptions, the paper finally generalizes the insights to repeated price interaction, analyzing coordinated effects when goods are not necessarily substitutes.

Keywords: Price caps, information-light regulation, tacit collusion, complements and substitutes, mergers, joint marketing agreements, coopetition.

JEL Codes: D43, L24, L41, O34.

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1 Introduction

Starting with the Sherman Act's Section 1 prohibition of any "contract, combination in the form of trust or otherwise, or conspiracy, in restraint of trade or commerce" (1890), the prevention of lessening of competition through agreements among potential competitors has been one of the two cornerstones of competition policy.[1] Major applications include the antitrust treatment of *mergers* and of *joint marketing*. [Under joint marketing, firms co-sell their goods or licenses either through a joint subsidiary – as in the case of patent pools, through which intellectual property owners sell licenses on bundles of their patents – or through an independent entity]. If improperly structured, such cooperative practices have the potential to lessen competition and harm consumers.

Assessing whether a cooperative agreement is likely to reduce competition and raise price is a notably difficult exercise. First, there is often a shortage of reliable price and demand data, leading to well-known difficulties in assessing the impact of, say, a merger.[2] For new technologies, there may even be no data at all, and yet antitrust authorities have to approve or block acquisitions of startups by incumbents or the formation of patent pools. Accordingly, authorities often have little information as to whether a merger will raise prices substantially; or even whether it will raise prices at all, that is, whether the merger is horizontal (involves substitutes) or not (involves complements)[3].

Second, the pattern of substitutability/complementarity may change over time, and a merger that is desirable at the date of the approval may no longer be so later on. Products evolve, as do usages. For instance, product B may be a complement to product A today, but later become a substitute. Such an allegation was made for instance in the Microsoft case,[4] in which the browser was definitely a complement to the operating system, but was alleged to have the potential to become an operating system itself through the writing of extra code. Similarly, molecules A and B may be jointly needed to cure disease C, but each may in the future suffice to curing disease D. A proper merger assessment therefore may require not only past data, but also unavailable forward-looking ones.

Third, while economists and antitrust practitioners neatly distinguish between "substitutes" and "complements", in many industries products may exhibit dual patterns of complementarity and substitutability: They compete with each other for consumers having selected the technology or the platform to which the products are related; but they also have a joint interest in keeping prices low so as to make the technology or platform attractive against rival options (non-consumption or competing technologies and

[1] Article 101 of the European Treaty provides a similar prohibition in the EU. The other cornestone is the monitoring of abuses of dominant positions (Section 2 of the Sherman Act, Article 102 of the European Treaty).

[2] See, e.g., https://en.wikipedia.org/wiki/Small_but_significant_and_non-transitory_increase_in_price.

[3] Mergers of complements fall in the category of "conglomerate mergers" in antitrust circles.

[4] Technically, this was an abuse of dominant position case, but the same concerns would have emerged in a merger case.

platforms). For instance, a technology built around multiple *patents* held by different owners becomes more attractive when licensing prices decrease, but these patents can also be substitutes in that they enable alternative implementations of given functionalities.[5] Products can be complements at low prices and substitutes at high prices, or the reverse. This means that local measurements of demand elasticities may mislead the observer as to the nature of competition. Existing data, even if available to the antitrust authority or the researcher, again may not tell the entire story.

The purpose of the paper is to add a regulatory instrument to the competition authorities' policy toolbox. The new cooperative arrangement would be an agreement among firms on price caps for their various products. Unlike in a merger or an old-style patent pool, firms would keep control – including over pricing – of their products or licenses and would only be constrained to charge no more than the agreed-upon caps. Also, unlike regulated price caps, the caps would be set by the firms; the validation by competition agencies could take the form of guidelines combined with business review letters, approving the industry-initiated price cap arrangements.

Although economists have neither advocated voluntary price caps nor studied their social desirability, such caps have surreptitiously appeared in the competition policy landscape in at least four guises. Since 2014, European antitrust policy with regards to patent pools requires that patent owners keep ownership of their patent (and therefore can grant licences to them outside the pool) and that the pool unbundle its license offering;[6] thus, aside from a one-stop-shopping transaction-cost benefit, patent pools amount to setting a cap on the price of individual licences. Second, and still in the realm of intellectual property (IP), most standard setting organizations require that IP owners commit to granting licenses either royalty-free or at a non-discriminatory, fair and reasonable (FRAND) price; thus IP owners who consent to such a standard setting process de facto collectively agree on capping their prices. In some cases (e.g., for standard setting organization VITA), firms may even commit to explicit price caps prior to standard approval. Third, a firm or group of firms may release lines of codes under an open source license; they thereby commit to a price cap equal to zero on the basic software and decide

[5] *Contents* offered by a cable or satellite television operator compete among themselves for the attention of the operator's subscribers but are also complements to the extent that increased operator membership benefits all content providers. Likewise, *payment systems* using a common point-of-sale terminal or interface at merchant premises compete for cardholder clientele and usage but share a common interest in merchants' adoption of the terminals. *Health care providers* who are members of a health insurance network vie for patients insured by the network but also depend on rival providers for the attractiveness of the insurance network – see Katz (2011). *Supermarkets* offer competing brands for many product categories, but one-stop shop benefits create complementarities across categories – See Thomassen *et al.* (2017) for a recent empirical analysis accounting for cross-category complementarities. Further illustrations include *music performance rights* (as, say, licensed by Pandora), *alcoholic beverages* (as in the Grand Met-Guinness merger), *retail outlets* (in department stores and commercial malls), *intermodal transportation*, *airline alliances*, or *books, tickets and hotel rooms* (on online platforms).

[6] The European Commission's guidelines on technology transfer agreements has been requiring independent licensing since 2004 and unbundling since 2014. Other juridictions, including the US and Japan, only require independent licensing.

to focus on complementary services. Finally, and relatedly, the supplier (e.g., a printer manufacturer) who commits to dual-or-multiple-sourcing for an add-on (e.g., cartridges) de facto caps the price of the add-on; industry incumbents that opt for an open standard behave similarly.

We argue that, when it is unclear whether products or services are substitutes or complements, and authorities feel hesitant about approving a merger or a joint-marketing alliance, they may well want to consider allowing instead price-caps agreements. To make such a case, and motivated by the lack of data that plagues merger analysis, we analyze the general properties of price-cap coopetition. The intuition for why price caps can be attractive is that they allow producers of complements to cooperate and solve Cournot's double marginalization problem, but do not allow competitors to collude and raise prices of substitutes.

For multiple reasons this intuition requires scrutiny, though. First, under strategic substitutes, imposing a cap on the price of one good may raise the price of another good to such an extent that consumer welfare is reduced. Second, demand may either involve a stable mix of complements and substitutes or exhibit a price-dependent pattern of complementarity/substitutability, and it is not a priori clear how such features affect the desirability of price caps. Third, and from a longer-term perspective, price caps could be used either to monopolize the industry by inducing the exit of some incumbent firms or by stifling their investment, or else to deter entry of new entrants. Fourth, under repeated interactions, price caps may change both the benefit from deviating from a collusive path and the feasible punishments of such deviations. These four extensions will lead us to qualify our analysis and to propose concrete policy recommendations to limit potential harms of price caps.

Section 2 first sets up the model, which allows for asymmetry among firms, for demand substitutability/complementarity, for strategic complementarity/substitutability, as well as for hybrid cases – it indeed provides two examples where these characteristics depend on price levels; in the first example (technology adoption) complementary patents become substitutes as prices rise, whereas in the second example (differentiated goods with network externalities) substitutes become complements at higher prices.

Section 2 then characterizes the set of prices sustainable through price caps in the absence of repeated interaction. In duopoly settings and under an assumption that holds trivially for strategic complements and under reasonable conditions for strategic substitutes, price caps can only improve consumer welfare relative to independent and unconstrained price setting. Furthermore, letting firms negotiate price caps benefits them (and consumers as well, from the previous result) when goods are complements and have no impact when goods are substitutes. So, unlike mergers, price-caps agreements are always socially beneficial. Finally, these insights are extended to symmetric oligopoly settings, to oligopolistic competition with strategic complementarity, and to our examples of hybrid demands.

Section 3 steps back and considers the impact of price caps in the presence of investment, entry and exit decisions. This analysis leads us to issue several caveats for the encouraging results of Section 2 and associated policy recommendations. Section 3.1 first investigates how the *prospect* of price caps affects the incentive to innovate and introduce products. For substitutes, price caps do not affect profits and thus have no impact on entry or investment either. For complements, price caps, when they benefit all firms, enhance product variety by encouraging entry.

Section 3.2 then looks at whether incumbent firms could set price caps so as to reduce competition, either among themselves through a reduced incentive to invest or from rivals who would be discouraged by the prospect of low prices (a collective version of the Modigliani-Sylos Labini limit-pricing paradigm). We show that two conditions are necessary to preserve the benefits of price caps: a) consumers cannot ask courts to enforce the price-caps agreement among firms; b) the agreement becomes void if none of the parties wishes to enforce it.

Section 3.3 then points out that price caps may dominate a merger even when the competition authority knows that products are complements. As is well-known, a merger between producers of complements may raise market power by facilitating foreclosure, that is, by deterring entry (or triggering exit) of competitors: The merged entity may practice technological or commercial (tariff-based) bundling to preserve its dominance at the system level. Price caps, like a merger, solve the double-marginalization problem. But, unlike a merger, they preserve the component producers' autonomy; the latter individually have no incentive to reduce competition among complementary components through closed standards or to cross-subsidize external products to squeeze an entrant. We thus conclude that price caps may have benefits over mergers even in situations where there is no ambiguity about the complementarity pattern.

Price caps might facilitate tacit collusion (called "coordinated effects" in merger analysis) by reducing the set of possible deviations on the equilibrium path or in punishment phases. Section 4 accordingly extends our study to allow firms to coordinate tacitly through repeated interaction. Alas, even in the absence of price caps, the repeated-game literature has focused on the case of substitutes, and often on perfect substitutes. Before trying to assess the desirability of price caps, the paper must therefore start filling the gap and study tacit collusion with arbitrary degrees of substitutability or complementarity. It obtains two sets of results.

Section 4.1 focuses on symmetric stationary paths in symmetric oligopoly settings. In this context, the lessons of the static analysis are confirmed for both substitutes and complements, provided that substitutes are strategic complements (as is usually assumed in economic analysis). The intuition is as follows. What prevents firms from achieving perfect coordination through repeated interaction is their incentives to deviate so as to increase their short-run profits. In the case of substitutes, where collectively firms wish to collude and raise prices, the profitable individual deviations consist in undercutting the collusive prices, and price caps cannot be used to limit or better punish these deviations. By contrast, in the case of complements, firms want to cooperate on lowering their prices so as to eliminate the double marginalization; in this case, price caps actually inhibit deviations from such low prices, both on- and off-equilibrium, and firms' and consumers' interests are aligned in preventing such deviations.

Section 4.2 goes further in the study of tacit collusion for the above-mentioned technology adoption model in which individual users must select a) which licenses to purchase in the technological class and b) whether to adopt the technology at all. The first choice depends on the extent of patent substitutability within the class, while the second captures the complementarity dimension. We measure the "essentiality" of offerings through the reduction in the value of the technology when users forego an offering – for the sake of tractability, users have the same preferences along this dimension, and only differ along another dimension: the cost of adopting the technology, or equivalently their opportunity cost of not adopting another technology. The model allows for a smooth transition between perfect substitutes and perfect complements.

Within this framework, we derive general results about the sustainability of "tacit collusion" (coordinated increase in price) or "tacit cooperation" (coordinated decrease in price), that is, about bad and good coordination through repeated interaction. We then note that price-caps agreements are equivalent to setting a joint-marketing entity combining two features, individual licensing and unbundling, and that both features are needed to ensure that consumer welfare always (weakly) increases under the agreement.

Finally, Section 4.3 discusses the issue of equilibrium selection, and Section 5 concludes.

Related literature. Our paper contributes to three literatures: static oligopoly, tacit collusion in oligopoly, and an emerging literature that looks for information-free (or -light) regulatory tools.

The literature on static oligopoly, well-reviewed by Vives (1999), is large, but has not emphasized the themes of this paper. The study of price-cap-constrained competition in particular is new. By contrast there is a large literature on the impact of mergers under non-repeated interaction. This line of research was initiated by the seminal paper of Farrell and Shapiro (1990), who consider Cournot, homogenous-goods competition, provide a necessary and sufficient condition for a merger to raise price, and warn against the hazards of using concentration indices.

Second, there is an extensive theoretical literature on repeated games, with and without observability of actions,[7] as well as a large theoretical and empirical literature on collusion in oligopoly.[8] Less attention has been devoted to the role of substitutability

[7]See Mailath and Samuelson (2006) for an excellent overview of this literature up to the mid 2000s.

[8]For surveys of this literature, see, for instance, Jacquemin and Slade (1998) and Marshall and Marx (2012).

and complementarity, however, despite the importance of these factors in the antitrust treatment of mergers or marketing alliances. The exception is a literature which, following Deneckere (1983) and Wernerfelt (1989), studies the impact of product differentiation. The conventional view, pioneered by Stigler (1964), is that homogeneous cartels are more stable than non-homogeneous ones (Jéhiel (1992) calls this the principle of minimum differentiation). In the context of symmetric horizontal differentiation, Ross (1992) shows however that stability does not increase monotonically with substitutability, because product differentiation both lowers the payoff from deviation and reduces the severity of punishments (if one restricts attention to Nash reversals; Häckner (1996) shows that Abreu's penal codes can be used to provide more discipline than Nash reversals, and finds that product differentiation facilitates collusion).[9] Building on these insights, Lambertini *et al.* (2002) argue that, by reducing product variety, joint ventures can actually destabilize collusion. In a context of vertical differentiation, where increased product differentiation also implies greater asymmetry among firms, Häckner (1994) finds that collusion is instead easier to sustain when goods are more similar (and thus firms are more symmetric). Building on this insight, Ecchia and Lambertini (1997) note that introducing or raising a quality standard can make collusion less sustainable.

Section 4 departs from the existing literature in several ways. First, it characterizes the scope for tacit coordination in settings with (varying degrees of) complementarity as well as substitutability. Second, it allows for explicit commercial cooperation, such as a price-caps agreement or a patent pool, and studies its impact on the scope for tacit coordination. Finally, it derives the regulatory implications.

Third, the paper contributes to a small but growing literature searching for regulatory rules that require little or no information from regulators; information-free regulatory rules have been studied primarily in the context of intellectual property, including guidelines for joint marketing agreements, with and without market power and vertical integration, and for standard-setting bodies (see Lerner-Tirole (2004, 2015), Boutin (2016) and Reisinger-Tarantino (2017)).

2 Impact of price caps on non-repeated interactions

2.1 Setting

• *Demand and supply.* We consider a classic oligopoly setting with $n \geq 2$ single-product[10] firms, indexed by $i \in \mathcal{N} \equiv \{1, ..., n\}$. Let $C_i(q_i)$ denote the cost of producing a quantity q_i of good i, and $D_i(\mathbf{p})$ the demand for that good, as a function of the vector of prices

[9] Raith (1996) emphasizes another feature of product differentiation, namely, the reduced market transparency that tends to hinder collusion.

[10] See Section 2.5 for a generalization to multi-product firms.

$\mathbf{p} = (p_i)_{i \in \mathcal{N}} \in \mathbb{R}^n_+$. We will assume that, for $i \in \mathcal{N}$, $D_i(\cdot)$ and $C_i(\cdot)$ are both C^2 and:[11]

- $C_i(0) = 0$ and $C_i'(\cdot) \geq 0$;
- $D_i(\cdot) > 0$, $\partial_i D_i(\cdot) < 0$ (individual demands are positive and downward sloping) and $\sum_{j \in \mathcal{N}} \partial_j D_i(\cdot) \leq 0$ (a uniform increase in all prices reduces individual demands)[12];
- the profit function

  $$\pi_i(\mathbf{p}) \equiv p_i D_i(\mathbf{p}) - C_i(D_i(\mathbf{p}))$$

  is strictly quasi-concave in p_i;
- the best-response function[13]

  $$R_i(\mathbf{p}_{-i}) \equiv \arg\max_{p_i} \pi_i(p_i, \mathbf{p}_{-i}),$$

  is well-defined, C^1, and bounded above.

It will be useful to consider the following familiar environments:

(**S**) *Substitutes:* $\partial_j D_i(\cdot) > 0$ for $j \neq i \in \mathcal{N}$;

(**C**) *Complements:* $\partial_j D_i(\cdot) < 0$ for $j \neq i \in \mathcal{N}$.

A given pair of goods are necessarily either substitutes or complements when demands are linear. With more general demands, however, the sign of $\partial_j D_i(\mathbf{p})$ may vary with $\mathbf{p}$ (see Section 2.6 for examples).

(**SC**) *Strategic complementarity:* $\partial_j R_i(\cdot) > 0$ for $j \neq i \in \mathcal{N}$.

(**SS**) *Strategic substitutability:* $\partial_j R_i(\cdot) < 0$ for $j \neq i \in \mathcal{N}$.

In our setting, strategic complementarity (resp., substitutability) amounts to $\partial^2_{ij}\pi_i(\cdot) > 0$ (< 0), and is implied by (**S**) ((**C**)) for linear demand systems and non-increasing returns to scale.[14] More generally, under mild regularity conditions (and indeed, in all

[11] In what follows, $\partial_i F(\mathbf{p})$ denotes the first-order derivative of the function $F(\mathbf{p})$ with respect to the price p_i; likewise, $\partial^2_{ij} F(\mathbf{p})$ denotes the second-order derivative with respect to the prices p_i and p_j.

[12] This condition is automatically satisfied when consumers have unit demands overall: if $\{v_i\}_{i \in \mathcal{N}}$ is a consumer's valuation vector (drawn from a continuous distribution), then $D_i(\mathbf{p}) = \Pr[v_i - p_i \geq \max\{\max_{j \neq i}(v_j - p_j), 0\}]$. It is also satisfied if, for instance, consumers have unit demands for each good and idiosyncratic preferences $v(\#\mathcal{S})$ for any combination $\mathcal{S} \subseteq \mathcal{N}$ (with $v(0) = 0$); we then have:

$$D_i(\mathbf{p}) = \Pr[\max_{\{\mathcal{S} \subseteq \mathcal{N} | i \in \mathcal{S}\}} \{v(\#\mathcal{S}) - \sum_{j \in \mathcal{S}} p_j\} \geq \max_{\{\mathcal{S} \subseteq \mathcal{N} | i \notin \mathcal{S}\}} \{v(\#\mathcal{S}) - \sum_{j \in \mathcal{S}} p_j\}],$$

which decreases when all prices increase uniformly (as this can only induce consumers to switch to smaller baskets). When $n = 2$, the condition is satisfied for any preferences $v(S)$ such that $v(\varnothing) = 0$.

[13] As usual, it is sometimes convenient to express the price vector $\mathbf{p} = (p_1, ..., p_n)$ as $\mathbf{p} = (p_i, \mathbf{p}_{-i})$, where $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$ denotes the vector of all prices but p_i.

[14] We then have $\partial_j \pi_i(\mathbf{p}) = [p_i - C_i'(D_i(\mathbf{p}))]\, \partial_j D_i$ and thus $\partial^2_{ij}\pi_i(\cdot) = [1 - C_i''(\cdot)\, \partial_i D_i]\partial_j D_i > 0$.

standard oligopoly models), prices are strategic complements (substitutes) when goods are substitutes (complements).[15]

Throughout the paper, we assume that there exists a unique Nash equilibrium in the unconstrained pricing game, which we denote by $\mathbf{p}^N = \left(p_i^N\right)_{i\in\mathcal{N}}$. We further suppose that, for $j \in \mathcal{N}$:[16]

$$\sum_{i\in\mathcal{N}\setminus\{j\}} \partial_j R_i\left(\mathbf{p}_{-i}\right) < 1. \tag{1}$$

Finally, we assume that the industry profit

$$\Pi\left(\mathbf{p}\right) = \sum_{i\in\mathcal{N}} \pi_i\left(\mathbf{p}\right)$$

is strictly quasi-concave in $\mathbf{p}$ and achieves its maximum at $\mathbf{p}^M = \left(p_i^M\right)_{i\in\mathcal{N}}$; let $q_i^M \equiv D_i\left(\mathbf{p}^M\right)$ denote the monopoly output of good i.

• *Unconstrained benchmarks.* The following lemmas provide useful properties of the monopoly and Nash outcomes. The first lemma shows that the monopoly outcome lies above firms' best-responses when goods are substitutes. When goods are complements instead, the monopoly outcome lies below at least one firm's best-response, and below all firms' best-responses in the absence of cross-subsidization, that is, if all marginal markups are non-negative. However, with complements, it may be optimal to sell some goods below cost in order to boost the demand for other goods; the prices of the latter goods may then lie above the best-responses.[17]

Lemma 1 (monopoly prices)

(i) $(\mathbf{S}) \Longrightarrow \forall i \in \mathcal{N}, p_i^M > C_i'\left(q_i^M\right)$ *and* $p_i^M > R_i\left(\mathbf{p}_{-i}^M\right)$.

(ii) $(\mathbf{C}) \Longrightarrow \exists\,(i,j) \in \mathcal{N}^2$ *such that* $p_i^M > C_i'\left(q_i^M\right)$ *and* $p_j^M < R_j\left(\mathbf{p}_{-j}^M\right)$*; furthermore, if* $n = 2$*, then* $p_i^M > C_i'\left(q_i^M\right) \Longrightarrow p_j^M < R_j\left(p_i^M\right)$ *for* $j \neq i$.

Proof. See Appendix A. ■

The next lemma shows that firms' best-responses always exceed their marginal costs:[18]

[15] See Vives (1999) for a detailed analysis.

[16] A stronger version, namely, $\sum_{i\in\mathcal{N}\setminus\{j\}} \left|\partial_j R_i\left(\mathbf{p}_{-i}\right)\right| < 1$, suffices to guarantee the existence and uniqueness of the Nash equilibrium, and moreover ensures that it is stable under the standard tâtonnement process; see online Appendix A. However, our analysis does not rely on equilibrium stability in the case of strategic substitutes.

[17] For instance, consider the case of two goods, produced at the same constant unit cost $c > 0$, and with demands respectively given by $D_1\left(p_1,p_2\right)$, with $\partial_2 D_1 < 0$, and $D_2\left(p_1,p_2\right) = \lambda D_1\left(p_2,p_1\right)$, with $\lambda \in (0,1)$. The monopoly prices are then asymmetric, and involve cross-subsidization (namely, $p_2^M < c < p_1^M$) for λ small enough (indeed, p_2^M tends to or is equal to 0 as λ goes to 0). Furthermore, as by construction p_1^M satisfies $\partial_1\pi_1\left(\mathbf{p}^M\right) = -\partial_1\pi_2\left(\mathbf{p}^M\right) = -\left(p_2^M - c\right)\partial_1 D_2\left(\mathbf{p}^M\right) < 0$, it is such that $p_1^M > R_1\left(p_2^M\right)$.

[18] Starting from $\mathbf{p} = \left.\left(p_i,\mathbf{p}_{-i}\right)\right|_{p_i=R_i\left(\mathbf{p}_{-i}\right)}$, the impact of a slight increase in p_i on firm i's profit is given by $\left[p_i - C_i'\left(D_i\left(\mathbf{p}\right)\right)\right]\partial_i D_i\left(\mathbf{p}\right) + D_i\left(\mathbf{p}\right)$. If firm i's margin were non-positive, this impact would be positive (as $\partial_i D_i\left(\cdot\right) < 0 < D_i\left(\cdot\right)$), a contradiction. Hence, $R_i\left(\mathbf{p}_{-i}\right) > C_i'\left(D_i\left(R_i\left(\mathbf{p}_{-i}\right),\mathbf{p}_{-i}\right)\right)$.

Lemma 2 (best responses exceed marginal costs) *For any firm $i \in \mathcal{N}$ and any $\mathbf{p}_{-i} \in \mathbb{R}_+^{n-1}$, $R_i(\mathbf{p}_{-i}) > C_i'(D_i(R_i(\mathbf{p}_{-i}), \mathbf{p}_{-i}))$.*

• *Price-cap constrained game.* Suppose now that, prior to setting their prices, firms can agree on a vector of price caps, $\bar{\mathbf{p}} = (\bar{p}_1, ..., \bar{p}_2)$. They then play a constrained game, which we denote by $G_{\bar{\mathbf{p}}}$, in which they simultaneously set their prices, subject to the agreed-upon price caps. The game G_∞ in which all caps are infinite is the unconstrained (no-price-cap) game.

We say that a vector of prices $\mathbf{p}$ is sustainable through price caps if there is some $\bar{\mathbf{p}}$ such that $\mathbf{p}$ is an equilibrium of $G_{\bar{\mathbf{p}}}$. We do not require the price-cap-constrained equilibrium to be unique; rather, we will provide results that hold regardless of which equilibrium is played. However, we show in the next sub-section that price caps can indeed be chosen so as to induce a unique equilibrium when either $n = 2$ or either (**SC**) or (**SS**) holds.

• *Timing.* The overall game, which we denote by $\mathcal{G}$, unfolds as follows:

1. The competition authority decides whether to allow firms to enter into price-caps agreements.

2. Firms choose price caps if such agreements are allowed.

3. If price-caps agreements are forbidden, or no agreement was signed, then firms play the unconstrained pricing game G_∞; if instead firms agreed on a price cap vector $\bar{\mathbf{p}}$, then they play the constrained pricing game $G_{\bar{\mathbf{p}}}$.

This game form calls for some comments. First, the green light given at stage 1 by the competition authority may introduce specific rules about the mechanisms that can be used to reach such an agreement (see in particular Section 3); however, in line with our information-free objective, we do not allow it to depend on actual levels of the price caps.

Second, we voluntarily refrain from specializing the negotiation process underlying stage 2. The normative focus of our analysis focuses on stage 1 and asks whether price-caps agreements can harm consumers, regardless of how they come about; we thus provide results that hold for any price caps. We therefore do not specify any extensive form for the negotiation; neither do we require that agreements be reached among all firms, or for that matter even within a single coalition of firms.

We also ask whether price-caps agreements can benefit firms themselves, which of course conditions the outcome of the stage-2 negotiation, regardless of the particular bargaining game that is implicit in that stage. The "existence of caps benefitting firms" requires some discussion, for several reasons. a) Reminiscent of the literature on cartel formation or merger negotiations,[19] an efficient agreement may not be reached as some firms try to free ride on the others. For example, with complements, a firm may try to benefit from the other firms' agreement to reduce price through price caps without contributing to the public good themselves. b) Different coalitions may form, that have antagonistic interests. Section 2.7 notes that agreements between platforms and their apps may create a prisoner's dilemma. c) Firms may need to operate lump-sum transfers among themselves in order to reach agreement. For example, a firm may benefit from another firm's lowering its price, but the converse may not hold, in which case some compensation is required. To address this issue, when discussing firms' incentives we will occasionally consider two scenarios, with and without lump-sum transfers at the time of agreement.

Third, we assume that competition authority's objective function is consumer welfare. This implies that when price caps are allowed, they *a fortiori* increase a broader notion of welfare if they raise industry profit.[20]

2.2 Price-cap implementable allocations

The following proposition shows that such price caps can sustain any prices lying below firms' best responses, and only these prices:

Proposition 1 (price-cap implementable allocations)

(i) *The set of prices that are sustainable through price caps is:*

$$\mathcal{P} \equiv \left\{ \mathbf{p} \in \mathbb{R}^n_+ \mid p_i \leq R_i \left(\mathbf{p}_{-i} \right) \text{ for } i \in \mathcal{N} \right\}.$$

(ii) *In particular, the Nash price vector* $\mathbf{p}^N$ *belongs to* $\mathcal{P}$ *and, for any other price vector* p *in* $\mathcal{P}$, $p_i < p_i^N$ *for some* $i \in \mathcal{N}$.

Proof. (i) We first show that price caps can sustain only prices in $\mathcal{P}$. Consider a price vector $\hat{\mathbf{p}}$ that is sustainable through price caps $(\bar{p}_i)_{i \in \mathcal{N}}$. As $\pi_i(\mathbf{p})$ is strictly quasi-concave in p_i, we must have, for $i \in \mathcal{N}$:

$$\hat{p}_i = \arg\max_{p_i \leq \bar{p}_i} \pi_i \left(p_i, \hat{\mathbf{p}}_{-i} \right) = \min \left\{ R_i \left(\hat{\mathbf{p}}_{-i} \right), \bar{p}_i \right\} \leq R_i \left(\hat{\mathbf{p}}_{-i} \right).$$

Hence, $\hat{\mathbf{p}} \in \mathcal{P}$.

Conversely, any price vector $\hat{\mathbf{p}} \in \mathcal{P}$ is sustainable through the vector of price caps $\bar{\mathbf{p}} = \hat{\mathbf{p}}$. To see this, note that, for $i \in \mathcal{N}$: $\bar{p}_i = \hat{p}_i \leq R_i(\hat{\mathbf{p}}_{-i})$; the strict quasi-concavity of $\pi_i(\mathbf{p})$ in p_i then yields the result.

[19] See, e.g., d'Aspremont *et al.* (1982), d'Aspremont and Gabszewicz (1986), Deneckere and Davidson (1985), Donsimoni *et al.* (1986), and Rajan (1989).

[20] As we noted, a coalition of firms may sign an agreement that increases the profit of its members but reduces total industry profit.

(ii) See Appendix B. ■

This proposition already establishes that firms cannot use price caps to raise all prices above their Nash levels. More generally, as price caps prevent firms from charging high prices, they are intuitively unlikely to harm consumers. We explore this further in the next two subsections.

The proof of Proposition 1 shows that any $\hat{\mathbf{p}}$ in $\mathcal{P}$ is sustainable through the vector of price caps $\bar{\mathbf{p}} = \hat{\mathbf{p}}$; however, the constrained game $G_{\hat{\mathbf{p}}}$ may exhibit other equilibria (in which case any other price cap vector $\bar{\mathbf{p}}$ sustaining $\hat{\mathbf{p}}$ would also exhibit other equilibria, as it would impose less stringent constraints on firms' pricing decisions). The following proposition shows that unique implementation can actually be obtained in a wide range of settings:

Proposition 2 (unique implementation) *In any of the following situations, any* $\hat{\mathbf{p}} \in \mathcal{P}$ *is the unique equilibrium of the constrained game* $G_{\hat{\mathbf{p}}}$*:*

(i) *duopoly;*

(ii) (**SC**)*;*

(iii) (**SS**).

Proof. See Appendix C.1. ■

In other settings, however, price caps may not achieve unique implementation. Online Appendix C.2 provides an example with three firms and strictly quasi-concave profit functions for which there is a unique Nash equilibrium, but multiple price-cap-constrained equilibria.

2.3 Duopoly

We focus here on the case of a duopoly. Recall the intuition that price caps do not allow competitors to collude and raise prices of substitutes, but allow producers of complements to cooperate and solve the double marginalization problem. Based on this intuition, one might expect that for any $\bar{\mathbf{p}}$ and any demand system, if $\mathbf{p}$ is an equilibrium of G_∞ and $\mathbf{p}'$ is an equilibrium of $G_{\bar{\mathbf{p}}}$, then $p_i' \leq p_i$ for all i, but this is not true: a reduction in one price may induce other firms to raise their prices; nonetheless, under the following regularity assumption, consumers are necessarily weakly better off under $\mathbf{p}'$ than under $\mathbf{p}$:

Assumption A: For any $i \neq j \in \{1, 2\}$ and any price $p_i \in [0, p_i^N)$, if $R_j(p_i) > p_i^N$, then:

$$R_j'(p_i) > -\frac{D_i(p_i, R_j(p_i))}{D_j(R_j(p_i), p_i)}.$$

Assumption A holds trivially when prices are strategic complements (as then $R_j'(\cdot) > 0$), as is the case with standard theoretical and empirical models of price competition. It also follows from the usual stability condition $\left|R_j'(p_i)\right| < 1$ when demand functions are "quasi-symmetric" in that $p_1^N = p_2^N$ and $p_i < p_j$ implies $D_i(p_i, p_j) \geq D_j(p_j, p_i)$.[21] It also holds for the hybrid demands considered in Section 2.6.[22]

We have:

Proposition 3 (duopoly: price caps benefit consumers) *For any demand system that satisfies Assumption A, any price vector $\mathbf{p} \neq \mathbf{p}^N$ that is sustainable through price caps yields a higher consumer surplus than $\mathbf{p}^N$. Therefore: (i) for any vector of price caps $\bar{\mathbf{p}}$, consumers are weakly better off under $G_{\bar{\mathbf{p}}}$ than under G_∞; and (ii) in $\mathcal{G}$, it is optimal for the competition authority to allow price caps.*

Proof. Consider a price vector $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2)$ in $\mathcal{P} \setminus \{\mathbf{p}^N\}$. From Proposition 1, $\hat{p}_i < p_i^N$ for some $i \in \{1, 2\}$. If the price of the other firm, j, satisfies $\hat{p}_j \leq p_j^N$, then consumers clearly prefer $\hat{\mathbf{p}}$ to $\hat{\mathbf{p}}^N$. Suppose now that $\hat{p}_j > p_j^N$; from Proposition 1 we then have $R_j(\hat{p}_i) \geq \hat{p}_j > p_j^N$, let:[23]

$$\mathring{p}_i \equiv \inf \left\{p_i \geq \hat{p}_i \mid R_j(p_i) \leq p_j^N\right\}.$$

By construction, $\mathring{p}_i \in (\hat{p}_i, p_i^N]$ and $R_j(\mathring{p}_i) = p_j^N$. Letting $S(p_i, p_j)$ denote total consumer surplus, we then have:

$$S(\hat{p}_i, \hat{p}_j) \geq S(\hat{p}_i, R_j(\hat{p}_i)) > S(\mathring{p}_i, R_j(\mathring{p}_i)) \geq S\left(p_i^N, p_j^N\right),$$

where the first inequality follows from Proposition 1, the last follows from $\mathring{p}_i \leq p_i^N$ and $R_j(\mathring{p}_i) = p_j^N$, and the strict one follows from $\mathring{p}_i > \hat{p}_i$ and Assumption A, which together imply:

$$S(\hat{p}_i, R_j(\hat{p}_i)) - S(\mathring{p}_i, R_j(\mathring{p}_i)) = \int_{\hat{p}_i}^{\mathring{p}_i} \left[D_i(p_i, R_j(p_i)) + D_j(R_j(p_i), p_i) R_j'(p_i)\right] dp_i > 0.$$

■

This Proposition shows that, under Assumption A, firms' use of price caps can only benefit consumers. Consider now firms' incentives to introduce price caps.

[21] Fix $p_i < p^N$. As $R_j'(p_i) < 1$, we then have:

$$R_j(p_i) - p_i = \int_{p^N}^{p_i} \left[R_j'(p) - 1\right] dp > 0.$$

Under "quasi-symmetry", we thus have $D_j(R_j(p_i), p_i) \leq D_i(p_i, R_j(p_i))$, which, together with $R_j'(p_i) > -1$, implies that Assumption A is satisfied.

[22] In online Appendix B, we provide a sufficient condition on demand ensuring that Assumption A holds, as well as a counter-example where this assumption does not hold; in the counter-example, price caps increase profits and reduce consumer surplus.

[23] The reasoning that follows relies on the range $[\hat{p}_i, \mathring{p}_i]$, because Assumption A is required to hold only for the prices $p_i < p_i^N$ that satisfy $R_j(p_i) > p_i^N$.

Intuitively, suppliers of substitutes wish to avoid competition and raise prices above the Nash level; in the light of Proposition 1, price caps are unlikely to help them. By contrast, suppliers of complements wish to avoid double marginalization, and price caps can enable them to achieve that. The following Proposition confirms this intuition:

Proposition 4 (duopoly: firms' incentives to adopt price caps)

(i) Under (**S**)*, firms cannot use price caps to increase both of their profits; if in addition* (**SC**) *holds, then firms cannot use price caps to increase any of their profits (and thus, a fortiori, their joint profit).*

(ii) Under (**C**)*, firms can use price caps to increase both profits (and thus, a fortiori, their joint profit); allowing price caps without transfers benefits consumers as well.*

Proof. We start with the observation that $\pi_i(R_i(p_j), p_j)$ increases (resp., decreases) with p_j under (**S**), (resp., under (**C**)); to see this, note that:

$$\begin{aligned}\frac{d}{dp_j}\{\pi_i(R_i(p_j), p_j)\} &= \partial_j \pi_i(R_i(p_j), p_j) \\ &= [R_i(p_j) - C_i'(D_i(R_i(p_j), p_j))]\,\partial_j D_i(R_i(p_j), p_j),\end{aligned}$$

where the first equality follows from the envelope theorem. It follows from Lemma 2 that the last expression has the same sign as $\partial_j D_i(\cdot)$.

(i) Consider a price vector $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2)$ in $\mathcal{P}\backslash\{\mathbf{p}^N\}$. From Proposition 1, $\hat{p}_j < p_j^N$ for some $j \in \{1, 2\}$. Under (**S**), we have, for $i \neq j \in \{1, 2\}$:

$$\pi_i(\hat{p}_i, \hat{p}_j) \leq \pi_i(R_i(\hat{p}_j), \hat{p}_j) < \pi_i\left(R_i\left(p_j^N\right), p_j^N\right) = \pi_i\left(p_i^N, p_j^N\right),$$

where the weak inequality stems from the definition of $R_i(\cdot)$ and the strict inequality follows from $\hat{p}_j < p_j^N$ and $\pi_i(R_i(p_j), p_j)$ being strictly increasing in p_j, as noted above. Therefore, firms cannot use price caps (with or without transfers) to increase both of their profits.

Furthermore, if prices are strategic complements, then any price vector $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2)$ in $\mathcal{P}\backslash\{\mathbf{p}^N\}$ is such that $\hat{p}_i < p_i^N$ for $i = 1, 2$.[24] The above argument then implies that both firms obtain strictly less profit than in the Nash equilibrium. Hence, in that case firms cannot use price caps to increase any of their profits.

(ii) By contrast, under (**C**), there exist prices in $\mathcal{P}$ that increase both firms' profits. To see this, note first that, from Lemma 2, both firms' margins are positive at the Nash equilibrium. It follows that, starting from the Nash equilibrium prices $\left(p_1^N, p_2^N\right)$, a small and uniform reduction in both prices increases both firms' profits, as reducing one firm's

[24] From Proposition 1, this has to be the case for at least one firm i; Proposition 1 and strategic complementarity then together imply that, for the other firm, j: $\hat{p}_j \leq R_j(\hat{p}_i) < R_j\left(p_i^N\right) = p_j^N$.

price has only a second-order effect on the profit of that firm, and a first-order, positive effect on the other firm's profit (as it increases that firm's demand). To conclude the argument, it suffices to note that any $(p_1, p_2) = \left(p_2^N - \varepsilon, p_2^N - \varepsilon\right)$, with $\varepsilon > 0$, belongs to $\mathcal{P}$, as (using condition (1)):

$$R_j(p_i) - p_j = \int_0^{\varepsilon} \left[1 - R_j'\left(p_i^N - x\right)\right] dx > 0.$$

Therefore, there are prices in $\mathcal{P}$ that give both firms more profit than the Nash equilibrium prices.

To conclude the proof, it suffices to note that increasing both firms' profits requires lowering prices below the Nash level. To see this, consider a price vector $\mathbf{p}$ that increases both firms' profits above their Nash levels; we then have, for $i \neq j \in \{1, 2\}$:

$$\pi_i\left(R_i(p_j), p_j\right) \geq \pi_i(\mathbf{p}) \geq \pi_i^N = \pi_i\left(R_i\left(p_j^N\right), p_j^N\right),$$

which, under (**C**), implies $p_j \leq p_j^N$. Hence, $\mathbf{p} \leq \mathbf{p}^N$. ■

Proposition 4 (which does not hinge on Assumption A) shows that: (i) firms will not select price caps when they offer substitutable goods (case (**S**)) – firms would like to raise prices, whereas price caps (which would benefit consumers) can only be used to lower prices; and (ii) price caps enable the firms to cooperate when they offer complements (case (**C**)), in which case firms' interests are aligned with those of consumers – both long for lower prices. Furthermore, price caps benefit consumers whenever they enhance both firms' profits.[25]

Finally, it is interesting to compare the use of price caps with the impact of a merger on firms' pricing policies (in the absence of merger-specific synergies). For the sake of exposition, it is useful to suppose that either:

($\mathbf{M_S}$) $p_i^M \geq p_i^N$ for $i = 1, 2$, with at least one strict inequality, and Assumption A holds;

or:

($\mathbf{M_C}$) $p_i^M \leq p_i^N$ for $i = 1, 2$, with at least one strict inequality, and $p_i^M \leq R_i\left(p_j^M\right)$ for $i \neq j \in \{1, 2\}$.

The first situation (case ($\mathbf{M_S}$)) always arises under (**S**) and (**SC**).[26] However, it can also arise even when (**S**) does not hold. This can be the case, for instance, when goods are substitutes for some prices but complements for other prices. For instance, neither

[25] Which prices firms actually choose to sustain depends on factors such as firms' relative bargaining power or the feasibility of side transfers.

[26] From Lemma 1, under (**S**) the monopoly outcome lies above both firms' best-responses; under (**SC**), this in turn implies that monopoly prices strictly exceed Nash levels.

(**S**) nor (**C**) holds in the two examples discussed in Section 2.6, and yet one of ($\mathbf{M_S}$) and ($\mathbf{M_C}$) is satisfied, depending on the specific values of some of the parameters. Note also that, when firms are sufficiently asymmetric, then under (**C**) the monopoly price may lie above the Nash level for one firm, and below it for the other firm.[27] Online Appendix F.3 provides a platform – (large number of) apps example in which, under monopoly pricing, applications are sold at cost (say 0, and thus, below Nash prices), which allows the platform to be priced at a price exceeding the Nash level, as illustrated in Figure 1.

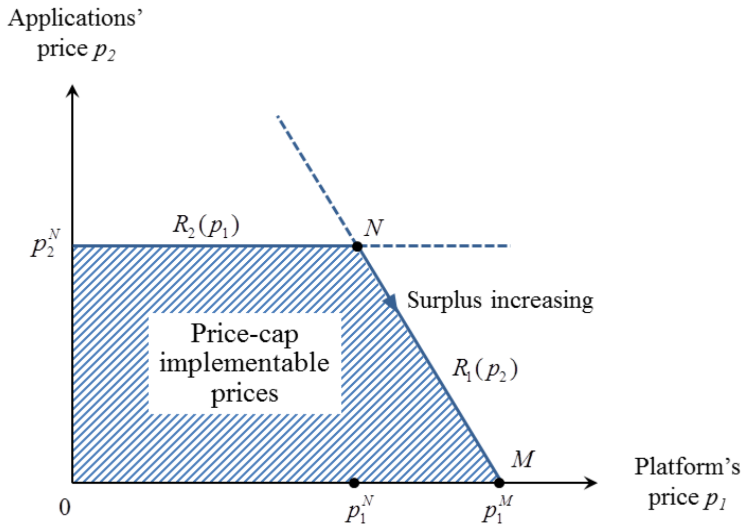

Figure 1: Platform & apps

The analysis above suggests that, in the absence of information about the situation at stake, and in the absence of merger-specific efficiency gains, price caps constitute a socially safer alternative to mergers. Indeed, we have:

Proposition 5 (duopoly: price caps vs. mergers)

(*i*) *Under* ($\mathbf{M_S}$)*, a merger harms consumers, whereas price caps can only benefit them.*

(*ii*) *Under* ($\mathbf{M_C}$)*, allowing a merger or price caps with transfers both yield perfect price cooperation, which benefits firms and consumers.*

Proof. (*i*) Under ($\mathbf{M_S}$), consumers prefer the Nash prices to the monopoly prices, and thus a merger harms them. By contrast, from Proposition 3, price caps can only benefit consumers.

(*ii*) Under ($\mathbf{M_C}$), $R_i\left(p_j^M\right) \geq p_i^M = \bar{p}_i$ for $i \neq j \in \{1,2\}$; from Propositions 1 and 2, charging the monopoly prices thus constitutes the unique equilibrium of $G_{\mathbf{p}^M}$. Thus, when firms can operate transfers at part of the price caps agreement, they will agree on price caps p^M, so as to generate the industry monopoly profit (as would a merger), and use transfers to share it appropriately. ■

[27]Furthermore, as noted at the end of Subsection 2.2, with complements it may be useful to price one good below cost, in which case the price of the other good lies above the best-response.

2.4 Symmetric oligopoly

We now extend the analysis to an arbitrary number of firms, and first focus on symmetric firms and price caps. Specifically, we assume here that the $n \geq 2$ firms:

- face the same cost: $C_i(q_i) = C(q_i)$ for all $i \in \mathcal{N}$;
- face symmetric demands, in the sense that other firms' prices are interchangeable: $D_i(\mathbf{p}) = D(p_i; \mathbf{p}_{-i})$, where $D(p_i; \mathbf{p}_{-i}) = D(p_i; \sigma(\mathbf{p}_{-i}))$ for any permutation $\sigma(\cdot)$ of the prices $\mathbf{p}_{-i}$, for all $i \in \mathcal{N}$ and $(p_i, \mathbf{p}_{-i}) \in \mathbb{R}^n_+$.

It follows that all firms have the same best-response $R(\mathbf{p}_{-i})$, which is moreover invariant under any permutation of the other firms' prices. We further assume that

$$\partial_1 R(\cdot) > -1, \tag{2}$$

where $\partial_1 R(\cdot)$ denotes the partial derivative of $R(\cdot)$ with respect to its first argument (by symmetry, the same condition applies to the other derivatives).[28]

We maintain our general assumptions, which imply that the Nash equilibrium and the monopoly outcome are not only unique but symmetric: $p_i^N = p^N$ and $p_i^M = p^M$; let q^M denote the monopoly quantity. Finally, it will be convenient to denote by $\pi^s(p) \equiv \pi(p, ..., p)$ the individual profit achieved when all firms charge the same price p, and by $R^s(p) \equiv R(p, ..., p)$ the best-response to a uniform price charged by the other firms. Condition (1) implies $R^{s\prime}(\cdot) < 1$.[29]

Intuitively, firms wish to raise prices above the static Nash level when their goods are substitutes, and to lower prices when their goods are complements. Indeed, we have:

Lemma 3 (symmetric oligopoly: profitable prices)

(i) Under (**S**), $p^M > p^N$ *and* $\pi^s(p) \geq \pi^s(p^N) \implies p \geq p^N$;

(ii) Under (**C**), $p^M < p^N$ *and* $\pi^s(p) \geq \pi^s(p^N) \implies p \leq p^N$.

Proof. See Appendix D. ■

From Proposition 1, firms cannot use price caps to raise their prices uniformly above the Nash level. We now show that, conversely, symmetric price caps can only sustain prices below the Nash level, leading to:

[28] This condition is trivially satisfy under (**SC**); under (**SS**), it is implied by the standard stability assumption $\sum_{i \in \mathcal{N} \setminus \{j\}} \partial_j R_i(\mathbf{p}_{-i}) > -1$.

[29] Using symmetry and $R^{s\prime}(p) = \sum_{j \in \mathcal{N} \setminus \{i\}} \partial_j R_i(\mathbf{p}_{-i})$, we have:

$$R^{s\prime}(p) = \frac{1}{n} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} \partial_j R_i(\mathbf{p}_{-i}) = \frac{1}{n} \sum_{j \in \mathcal{N}} \sum_{i \in \mathcal{N} \setminus \{j\}} \partial_j R_i(\mathbf{p}_{-i}) < 1,$$

where the inequality stems from (1).

Proposition 6 (symmetric oligopoly: price caps benefit consumers)

For any symmetric price caps $\bar{\mathbf{p}}$*, consumers are weakly better off under* $G_{\bar{\mathbf{p}}}$ *than under* G_∞*; therefore, it is optimal for the competition authority to allow symmetric price caps.*

Proof. See Appendix E. ■

This Proposition extends Proposition 3 in that symmetric price caps can only result in lower prices and thus benefit consumers. Using Lemma 3, it also implies that firms have no incentives to introduce a price cap under (**S**), and can instead use them to increase their profits under (**C**). Indeed, we have:

Proposition 7 (symmetric oligopoly: firms' incentives)

(*i*) *Under* (**S**)*, firms cannot use symmetric price caps to sustain a more profitable symmetric outcome than that of the Nash equilibrium.*

(*ii*) *Under* (**C**)*, firms can use (symmetric) price caps to sustain the monopoly outcome, which increases their profits and also benefits consumers, compared with the Nash outcome; furthermore, under* (**SS**) *or* (**SC**)*, or when firms face non-decreasing returns to scale, the monopoly outcome is the unique equilibrium of the constrained game* $G_{\mathbf{p}^M}$.[30]

Proof. See Appendix F. ■

Proposition 7 extends Proposition 4: symmetric price caps play no role when firms offer substitutes, and enable firms to achieve perfect cooperation when they offer complements, in which case this cooperation also benefits consumers.

Remark: price caps versus mergers. The above findings also extend the insight that, in the absence of merger-specific efficiency gains, price caps constitute a safer alternative to mergers: (i) they both enable (perfect) socially desirable cooperation in case of complements; and (ii) in case of substitutes, price caps are innocuous whereas mergers harm consumers and social welfare.

2.5 Oligopolistic competition under strategic complementarity

Online Appendix C generalizes the analysis to multiproduct oligopolies when prices satisfy (**SC**) both within and across firms. In the text, and as for the other accounts of online Appendix results, we provide an informal treatment of our analysis and its intuition, and refer the reader to the online Appendix for more detail.

We assume quasi-concave individual profit functions with product-by-product increasing reaction functions (a firm's optimal price on a product is non-decreasing in all prices,

[30]It can be checked that this result also holds as long as costs are not too convex, namely, as long as $C'\left(D\left(p^M,0\right)\right) < p^M$.

its own prices on other products and those of rival firms) and, in the absence of price caps, a unique Nash equilibrium.

Proposition 8 (multi-product firms under (SC)) *With multi-product firms under* (**SC**) *and for any vector of price caps, there exists a unique Nash equilibrium, and the equilibrium prices weakly increase with the price cap vector. Therefore: (i) for any vector of price caps $\bar{\mathbf{p}}$, consumers are weakly better off under $G_{\bar{\mathbf{p}}}$ than under G_∞; and (ii) in $\mathcal{G}$, it is optimal for the competition authority to allow price caps.*

With a single product per firm, the analysis follows from standard supermodularity reasoning. For given prices charged by the other firms, a firm's price reaction under a price cap lies weakly below its unconstrained level; under (**SC**) this induces in turn other firms to charge lower prices and so on.

To extend the analysis to multi-product firms, it is useful to interpret a firm i with m_i products as m_i firms with a single product each and the same objective function. Indeed, any (constrained) best-response of the multi-product firm characterizes an individual best-response for each of these mono-product firms. The only difficulty consists in ensuring that the reverse holds (that is, that any "Nash equilibrium" among these single-product firms constitutes a best-response of the multi-product one), which is indeed the case when the profit of the multi-product firms are quasi-concave in their prices.

Proposition 8 implies that firms have no incentive to adopt price caps when products are substitutes, and benefit from doing so when products are complements:

Corollary 1 (multi-product firms' incentives to set price caps under (SC))

(*i*) *If all goods are substitutes, then price caps cannot increase the profit of any firm.*

(*ii*) *If instead all goods are complements, then price caps can be used to increase all firms' profits.*

Remark: segmented markets. A specific case of a multi-product firm arises when a firm produces a single product and sells it in segmented markets. Under price discrimination, the analysis is the same as in the single product case, as each market is a separate market; so Proposition 8 applies. If the firms set a uniform price across markets, the analysis is again unchanged, as aggregating demands across markets takes the analysis back to a single market, and the (**SC**) property is preserved under aggregation. So Proposition 8 applies and price caps lower prices.

In both cases, suppliers of substitutes would have no incentive to introduce price caps. By contrast, suppliers of complements can benefit from the adoption of price caps under uniform pricing, and under discriminatory pricing as well if price caps, too, can be differentiated across market segments; however, if firms can price discriminate but price caps are restricted to be uniform, then it may be more difficult to find profit-enhancing agreements.[31]

2.6 Hybrid demands

We sketch here two environments exhibiting reversals of the complementarity/substitutability pattern. In the first, offerings are complements at low prices and substitutes at higher prices; the second exhibits the reverse pattern.

2.6.1 Technology adoption

Consider a nested model in which (a unit mass of) users first select among licenses to patents covering a technology and then choose between this technology and selected licenses and an outside option (adopting a competing technology or no technology at all). For tractability we assume a single-dimensional heterogeneity parameter: users differ in their opportunity cost or benefit of adopting the technology, but not in their preferences for the bundles of licenses within the technology. In the context of two symmetrical patents held by two patent owners, the description goes as follows. The users obtain value V from acquiring the two licenses, and $V - e$ from a single one. So $e \in [0, V]$ is an essentiality parameter: $e = 0$ for perfect substitutes and $e = V$ for perfect complements.

A user with adoption cost θ is willing to adopt the complete technology, based on both patents, if and only if $V \geq \theta + P$, where P is the total licensing price; the demand is thus

$$D(P) \equiv F(V - P),$$

where $F(\cdot)$ denotes the cumulative distribution function of the technology adoption cost θ. Similarly, the demand for the incomplete technology at price p is

$$D(p + e) = F(V - e - p).$$

That is, an incomplete technology sold at price p generates the same demand as the complete technology sold at price $p + e$; thus $p + e$ can be interpreted as the "quality-adjusted price."

For low prices ($p_i < e$ for $i = 1, 2$), users secure both licenses and (assuming a zero marginal cost) firm i's profit is given by $p_i F(V - p_1 - p_2)$. So offerings are local

[31] Consider for example the case where two firms (1 and 2) are active in two markets (A and B), in such a way that firm 1's product boosts the sales of firm 2 in market A, and conversely in market B. Specifically, $D_{A1}(\mathbf{p}_A) = \underline{d}(p_{A1})$, $D_{A2}(\mathbf{p}_A) = \lambda(p_{A1})\bar{d}(p_{A2})$, $D_{B2}(\mathbf{p}_B) = \underline{d}(p_{B2})$ and $D_{B1}(\mathbf{p}_B) = \lambda(p_{B2})\bar{d}(p_{B1})$, where $\lambda(\cdot)$ is increasing whereas $\underline{d}(\cdot)$ and $\bar{d}(\cdot)$ are both decreasing in their respective arguments; all costs are zero. In the absence of price caps, the equilibrium involves the "monopoly prices" $p_{A1} = p_{B2} = \underline{p}^m = \arg\max p\underline{d}(p)$ and $p_{A2} = p_{B1} = \bar{p}^m = \arg\max p\bar{d}(p)$. In order to increase both firms' profits, price caps should decrease p_{A1} and p_{B2} below $\underline{p}^m$, which would be costly if the same caps apply to both market segments and $\bar{p}^m > \underline{p}^m$ and $\lambda(p) = \lambda + \varepsilon p$, where $\varepsilon \to 0$ (small complementarity) .

complements: each patent holder wishes that the other owner reduce her price. By contrast, for high prices ($p_i > e$ for $i = 1, 2$), users do not acquire a second license and pick the lower price one if they adopt the technology at all. So offerings are local substitutes.

To ensure the concavity of the relevant profit functions, we will assume that the demand function is well-behaved:

Assumption B: $D(\cdot)$ is twice continuously differentiable and, for any $P \in [0, V]$, $D'(P) < 0$ and $D'(P) + PD''(P) < 0$.

If users buy the two licenses at unit price p, each firm obtains

$$\pi(p) \equiv pD(2p),$$

which is strictly concave under Assumption B; let $p^M \in [0, V]$ and π^M denote the per-patent monopoly price and profit:

$$p^M \equiv \arg\max_p \pi(p), \text{ and } \pi^M \equiv \pi\left(p^M\right) = p^M D(2p^M).$$

If instead users buy a single license at price p, industry profit is

$$\tilde{\pi}(p) \equiv pD(p + e),$$

which is also strictly concave under Assumption B; let $\tilde{p}^M(e)$ denote the monopoly price and $\tilde{\pi}^M(e)$ the total monopoly profit for the incomplete technology:

$$\tilde{p}^M(e) \equiv \arg\max_p pD(p + e), \text{ and } \tilde{\pi}^M(e) \equiv \tilde{\pi}\left(\tilde{p}^M(e)\right) = \tilde{p}^M(e) D(\tilde{p}^M(e) + e).$$

Like $\tilde{\pi}(p)$, $\tilde{\pi}^M(e)$ is decreasing in e: the profit derived from the incomplete technology decreases as each patent becomes more essential.

Consider the static game in which the two firms simultaneously set their prices. When a firm raises its price, either of two things can happen. First, technology adopters may keep including the license in their basket, but because the technology has become more expensive, fewer users adopt it. In reaction to price p_j set by firm j, firm i sets price $r(p_j)$ given by:

$$r(p_j) \equiv \arg\max_{p_i} p_i D(p_i + p_j),$$

which under Assumption B satisfies $-1 < r'(p_j) < 0$ and has a unique fixed point, which we denote by $\hat{p}$:

$$\hat{p} = r(\hat{p}).$$

The two patents are then complements and their prices strategic substitutes. Furthermore,

$\hat{p} > p^M$ due to double marginalization.[32]

Second, technology adopters may stop including the license in their basket; this occurs only when the firm raises its price above e. It follows that the Nash equilibrium is unique and symmetric:[33] both firms charge

$$p^N \equiv \min\{e, \hat{p}\},$$

and face positive demand. We will denote the resulting profit by $\pi^N \equiv \pi\left(p^N\right)$.

Along the lines of our previous results, it is easy to check that price caps can only be beneficial: if $p^N < p^M$, the equilibrium prices under a cap cannot exceed $e = p^N$ (each p_i is equal to the minimum of e and firm i's cap), and so letting firms agree on price caps has no effect on the outcome; if instead $p^N > p^M$, then allowing price caps lead to the lower-price monopoly outcome.

Interestingly, a pool offering the bundle at some pre-agreed price P, together with *independent licensing*, achieves here the same outcome.[34] Independent licensing means that the owners of the patents keep ownership of their patent and therefore can market it outside the pool. A price cap is equivalent to the combination of independent licensing and unbundling, where "unbundling" refers to the requirement that the pool sells individual licenses (at a total price below the bundle price) and not only the bundle; the pool's stand-alone prices then serve as price caps for the independent licensing pricing game. As we will see, independent licensing alone no longer provides a perfect screen under repeated interaction.[35]

2.6.2 Differentiated goods with network externalities

Online Appendix E studies the properties of the Hotelling model augmented with positive network externalities.[36] The one difference with the familiar Hotelling model on a line is that the consumers' valuation is $v + \sigma(q_1 + q_2)$ where $\sigma > 0$ is the club-effects parameter. For low prices, the market is covered and there are no network externalities at the margin, as total demand is fixed and equal to the unit mass of consumers $(q_1 + q_2 = 1)$. So offerings are imperfect substitutes. For high prices the market is not covered and the firms choose their prices as local monopolies. So each locally would like the other firm to lower its price and create more externalities. The reaction curves are represented in Figure

[32] By revealed preference, $p^M D\left(2p^M\right) \geq \hat{p}D\left(2\hat{p}\right) \geq p^M D\left(\hat{p} + p^M\right)$ and thus $D\left(2p^M\right) \geq D\left(\hat{p} + p^M\right)$, implying $\hat{p} \geq p^M$. Assumption B moreover implies that this inequality is strict.

[33] See online Appendix D for a detailed exposition. The symmetric reaction function exhibits a kink, but satisfies Assumption A whenever it is differentiable, as well as for both right- and left-derivatives at the kink.

[34] See Lerner and Tirole (2004). Current antitrust guidelines in Europe, Japan and the US require patent pools to allow independent licensing.

[35] When $n > 2$, a pool with independent licensing still always admits an equilibrium with prices below the Nash prices; but Boutin (2016) shows that it may also admit equilibria that raise prices, and that unbundling destroys these bad equilibria.

[36] For related work, see Stahl (1982) and Grilo *et al.* (2001).

2, which illustrates some of the results obtained in the online Appendix: in particular, for v high enough, the offerings satisfy (**S**) and (**SC**), and the monopoly prices are greater than the Nash prices. By contrast, if v is small enough, then for low prices the offerings locally satisfy (**S**) and (**SC**), but around the Nash prices they satisfy (**C**) instead, and the monopoly prices are lower than the Nash prices.

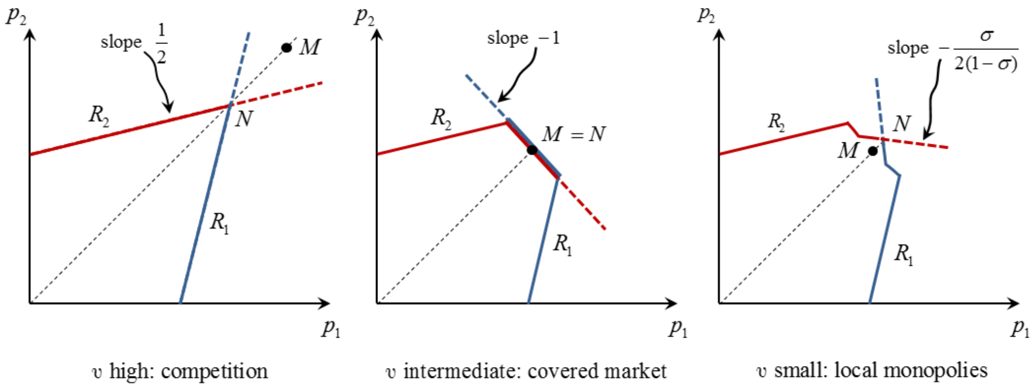

Figure 2: Differentiated goods with network externalities

It is straightforward to check that, in this setting as well, price caps can only benefit consumers: if $p^N < p^M$ (which occurs when v is high), prices are strategic complements, implying that price caps can only lower the equilibrium prices; if instead $p^N > p^M$, then allowing price caps leads to the lower-price monopoly outcome.[37]

Proposition 9 (price-dependent C/S pattern) *The technology adoption and the differentiated goods with network externalities models both exhibit a pattern of price-dependent complementarity/substitutability: goods are complements for low (resp. high) prices in the former (resp. latter) case. In $\mathcal{G}$, it is optimal for the competition authority to allow price caps.*

2.7 Mixture of complements and substitutes

Online Appendix F first provides an *example of a welfare-decreasing price-caps agreement* in a mixed complement-substitute environment. Its jest is simple. Imagine that firms 1 and 2 compete for a clientele on a Hotelling line with imperfect substitutes A_1 and A_2, respectively. There are two other symmetric and captive clienteles, willing to pay some known amount for the combination of products A_i and B_i (viewed as perfect complements). Suppose now that the firms agree on a price cap on goods B_1 and B_2.

[37]In the intermediate case where there exist multiple equilibria, we assume that firms coordinate on the symmetric one, which allows them to share the monopoly profit. Price caps can then only reduce prices and benefit consumers. However, within the set of equilibrium outcomes, moving away from the symmetric one benefits consumers, although this reduces firms' joint profit, and may even lower both of their profits. Thus, if firms were somehow (mis-)coordinating on a highly asymmetric equilibrium, then price caps might be used to induce a less asymmetric outcome that benefits both firms but harms consumers.

The reduced price on good B_i induces firm i to increase its own price on good A_i so as to keep capturing surplus from its captive clientele. This puppy-dog strategy softens price competition in the Hotelling market, and reduces consumer surplus.

To obtain a positive result, we add structure by capturing a familiar environment: a finite number of platforms compete with each other. Each platform has a large number of applications, and each application is supplied by competing app suppliers. So there is competition at the platform and at the app level, and complementarity between platforms and applications. We make the standard regularity conditions (in particular strictly quasi-concave profit functions), and mainly assume that the prices charged by rival suppliers of an app are strategic complements, as are the prices charged by the platforms.

Proposition 10 (platforms and apps) *Consider price caps in the platform-apps model, with* (**SC**) *for platform app supplier prices and for platform prices. There exists a unique price-constrained equilibrium, and equilibrium prices weakly increase with the vector of price caps on platform prices and on apps. Therefore: (i) for any vector of price caps* $\bar{\mathbf{p}}$*, consumers are weakly better off under* $G_{\bar{\mathbf{p}}}$ *than under* G_∞*; and (ii) in* $\mathcal{G}$*, it is optimal for the competition authority to allow price caps.*

With a large number of apps, app suppliers' prices are independent of platform prices. The (**SC**) assumption and Proposition 8 then imply that their prices move monotonically with the caps they face. Platforms compete among each other in quality-adjusted prices (the quality-adjusted price of a platform is its price minus the net consumer surplus derived from its apps). Caps on a platform's app prices lower their prices and improve platform users' experienced quality, leading platforms to charge lower quality-adjusted prices (although higher gross prices). Thus consumers benefit from caps on apps even in the absence of caps on platform prices. Given the (**SC**) assumption on platform prices, this is *a fortiori* the case when platform prices are capped, following the logic of Proposition 8.

The online Appendix also looks at the incentives to adopt caps. Price caps on apps have two effects on app suppliers' profits: they increase the demand for the platform, which unambiguously benefits the app suppliers, at least if they single-home (i.e. produce apps for a single platform; under multi-homing, the gain is weaker or even nil, as a platform's membership gain may be another platform's loss). But enhanced competition among the suppliers of a given app reduces their profit (unless the app supplier is a monopolist in his market niche, in which case a slightly binding cap has only second-order effects). The effect on the platform's profit is unambiguously positive.

Conversely, caps on platform prices benefit app suppliers but do not benefit the platforms themselves, unless the apps agree on sufficiently low caps. More interestingly, platforms face a prisoner's dilemma: each individually wants to agree with its apps suppliers on price caps for the apps and the platform so as to gain market share, reducing the profit of all platforms and all apps in the process.

3 Investment and entry

This section analyses the impact of price caps on investment and entry incentives. We first consider the use of price caps from an *ex post* perspective, once investment or entry decisions have been made, before discussing their possible strategic use, from an *ex ante* perspective, as an instrument to stifle investment or deter entry.

3.1 Post-investment price caps

We consider here a modified version of game $\mathcal{G}$ in which stage 2 comprises two steps:

1. The competition authority decides whether to allow firms to enter into price cap agreements.
2. (a) Firms make investment or entry decisions; these decisions are observed by all firms.

   (b) Firms choose price caps if such agreements are allowed.
3. If price cap agreements are forbidden, or no agreement was signed, then firms play the unconstrained pricing game G_∞; if instead firms agreed on a price cap vector $\bar{\mathbf{p}}$, then they play the constrained pricing game $G_{\bar{\mathbf{p}}}$.

Online Appendix G first introduces investment decisions in the multiproduct oligopoly setting introduced in Section 2.5. These decisions may correspond to entering or staying in the market, developing new products, improving the quality or lowering the production cost of existing ones...; different firms may moreover face different choices. We assume quasi-concavity and strategic complementarity in prices, as well as the existence of a unique continuation price equilibrium (for any investment decisions).

From Proposition 8, producers of substitutes have no incentive to set price caps in stage $2b$, regardless of their investment decisions in stage $2a$. Therefore, allowing price caps has no impact on the set of investment equilibria.

By contrast, in case of complements, price caps help firms solve double marginalization problems; we would thus expect their use to enhance profitability and foster entry. To explore this, we consider a more restrictive *entry/exit scenario* in which each firm must decide whether to be active or not. Active firms can always sign a mutually profitable agreement that benefits all of them.[38] However, if only a subset of firms agree to price caps, we must consider their impact on non-signing firms. We focus here on the case where price caps benefit non-signatories as well:

[38] As in Proposition 4 for a duopoly, a uniform small reduction in prices below the Nash level is both implementable and mutually profitable.

Assumption C: Any active unconstrained firm is at least as well off when other active firms are constrained by price caps than when all active firms are unconstrained.

This condition is likely to hold when firms are in a rather symmetric position. For example, the online Appendix shows that Assumption C holds under (**SS**) when the demand for one firm depends on other firms' prices only through a symmetric aggregator: $D_i(\mathbf{p}) = D(p_i, A(\mathbf{p}_{-i}))$.[39] It also provides an example with very asymmetric impacts (namely, each product constitutes a complement for the next one) where a price-caps agreement can benefit two firms at the expense of a third, non-signing and therefore unconstrained one.

Another difficulty lies in the possible multiplicity of equilibria in stage $2a$. To address this issue, we focus on the case where, as intuition suggests, the development of additional complementary products enhances firms' profits, and show that, for any given market structure that arises in the absence of price caps, allowing price caps yields an equilibrium where at least the same firms are active. This leads to:

Proposition 11 (investment incentives)

(*i*) *Under* (**S**) *and* (**SC**), *the possibility of post-investment price-caps agreements has no impact on investment/entry/exit decisions.*

(*ii*) *Under* (**C**) *and Assumption C, for any entry/exit equilibrium without price caps, there is an equilibrium in which the same firms (and possibly others) are active when price caps are allowed.*

3.2 Pre-investment price caps

We now turn to "ex-ante" price caps. Such price caps could be used as a commitment device to induce exit, deter entry or stifle investment, ultimately hurting consumers. There are two types of concerns, which online Appendix H explores in more detail.

First, incumbents might adopt price caps as a commitment to be tough toward a rival entrant, a collective version of the limit pricing model of Sylos Labini (1957) and Modigliani (1958). However, limit pricing can work only if it is credible (emphasizing this was an achievement of the early game-theoretic IO literature). One way of achieving credibility would be to involve customers, as they would then insist on enforcing the caps.[40] We thus propose a regulation requiring that customers are not part of the price-caps agreement. They cannot sue – or ask for a money-back payment from – a firm that charges a higher price than it promised. This recommendation applies to all customers, at all stages of the production and distribution chain, and not only to final consumers.

[39] See, e.g., Corchón (1994), Acemoglu and Jensen (2013), and Nocke and Schutz (2017).

[40] This arrangement would also be analoguous to the use of long-term contracts as a barrier to entry, in the spirit of Aghion and Bolton (1987).

Second, firms might set very low price caps (say, below minimum average cost) as a commitment to exit/not enter. When lump-sum monetary transfers are allowed, a firm could monopolize the industry by bribing its rival(s) into accepting such a price cap. Even if lump-sum monetary transfers are disallowed (which precludes such self-mutilation by the exiting firms), firms could still allocate territories or market segments among themselves by agreeing to exit-inducing caps on the other's "turf". Intuitively, though, such commitments are not credible, as producers of substitutes have no incentive to enforce the price caps (or, equivalently, want to renegotiate them away). Taking advantage of these incentives leads us to propose a second recommendation, which is that the agreement becomes void if none of the parties wishes to enforce it.

Provided that these two regulations are in place, could price caps hurt consumers by affecting entry, exit or more generally investment? To explore this, we consider the following modified version of game $\mathcal{G}$:

1. The competition authority decides whether to allow firms to enter into price cap agreements.
2. (a) Firms choose price caps if such agreements are allowed.
   (b) Firms make observable investment or entry decisions.
3. (a) If an agreement has been signed, firms choose whether to confirm it; the agreement is enforced if and only if at least one firm confirms it.
   (b) Firms set their prices.

This timing allows firms to sign price-caps agreements in order to influence investment decisions, but rules out non-credible threats by asking them to confirm their willingness to enforce the agreement, once investment decisions have been made. Indeed, Proposition 4 and its variants show that price caps hurt the firms under (**S**) and (**SC**); their incentive is thus to renegotiate away (not enforce) the initial agreement, which undermines any attempt to stifle investment stifling, deter entry or trigger exit.

Proposition 12 (commitment effects of price caps) *Consider the following two prerequisites to setting price caps: a) customers are not part of the price-caps agreements; b) the agreement becomes void if none of the parties wishes to enforce it. Then, when* (**S**) *and* (**SC**) *hold, price caps have no impact on investment/entry/exit and therefore no impact on consumers.*

This Proposition shows that, under (**SC**), suppliers of substitutes cannot strategically enter into price-caps agreements in order to deter entry or expansion, or induce the exit of a competitor. Coalitions involving producers of complements may however have an incentive to adopt price caps for such strategic reasons. For example, an incumbent firm could enter into a price-caps agreement with the supplier of a complementary product, so as to commit itself to maintain lower prices and deter in this way the entry of a potential rival: that supplier would then play a role similar to that of a "customer", by opposing any later attempt to remove the price cap. Competition authorities should therefore remain cautious in such environments.

3.3 Foreclosure

Our analysis so far has been motivated by the observation that mergers between producers of substitutes increase market power. But so do mergers between producers of complements if the merger allows them to foreclose entry (or trigger exit) of competitors. A number of theoretical contributions noted that the Chicago school argument according to which, in a world of complements, favoring an internal division over external suppliers is self-defeating, no longer holds when entry or exit decisions are at stake. Notably, Carlton and Waldman (2002) and Choi and Stefanadis (2001) showed that bundling may allow an integrated incumbent to deter the development of rival systems; even though the development of a single alternative component benefits the incumbent by boosting the value of its other components (the Chicago school argument), the development of a full range of components, creating together an alternative system, destroys the monopoly position.

We will not develop a complete theory of how price caps can advantageously substitute for mergers in such environments, and content ourselves with showing, in the context of the Choi-Stefanadis and the Carlton-Waldman frameworks, that price caps have the potential to promote system expansion without facilitating the foreclosure of rival systems; see in Online Appendix I.

The instrument for foreclosure may be the choice of an incompatible technology over an open standard. Entrants can then make a profit if they develop an entire system alternative, but not through partial entry on a component. So entry is riskier (Choi-Stefanadis) or less profitable if entry in the complementary segments cannot be synchronized (Carlton-Waldman). Independent incumbent producers of complementary goods by contrast are better off picking the open standard, as they create more competition for the complementary product(s). Prohibiting the merger of incumbent producers of complementary products thus reduces the likelihood of foreclosure. To be certain, the merger may also eliminate double marginalization and lower the price of the complementary components. But this property is shared by both a merger and price caps, and price caps are a socially superior way of avoiding double marginalization, as they do not encourage foreclosure.

Alternatively, foreclosure may result from bundling through tariffs: an incumbent can practice cross subsidies to squeeze an entrant in a specific segment. This too is precluded by the absence of merger: While an integrated firm can offset a price reduction on a component by raising price on the other, an independent incumbent is not willing to lose money on its product.

Proposition 13 (price caps versus mergers: foreclosure concerns) *In the Choi-Stefanadis and Carlton-Waldman frameworks, a merger of complements allows foreclosure while price caps do not. Accordingly, price caps are a socially superior way of handling double marginalization.*

4 Impact of price caps on repeated interactions

We now study the scope for tacit coordination through repeated interaction. To do so, in stage 3 of $\mathcal{G}$ we replace the pricing game G_∞ (in the absence of price caps) or $G_{\bar{\mathbf{p}}}$ (if price caps $\bar{\mathbf{p}}$ have been adopted in stage 2 of $\mathcal{G}$) with an infinitely repeated game, G_∞^δ or $G_{\bar{\mathbf{p}}}^\delta$, in which:

- in each period $t = 1, 2, ...$, firms set their prices $\{p_i^t\}_{i \in \mathcal{N}}$ for that period (subject to the price cap $\bar{p}_i$ adopted in stage 2 of $\mathcal{G}$, if any).
- each firm i maximizes the discounted sum of its per-period profits, $\sum_{t \geq 1} \delta^t \pi_i(\mathbf{p}^t)$, where δ is the discount factor, common to all firms.

To avoid technicalities, we focus on pure strategies and, in the case of substitutes, assume that prices are bounded above by an arbitrarily large bound: firms' strategies are of the form $\{p_i^t(\cdot)\}_{t=1,2,...}$, where $p_i^t(\cdot)$ is a mapping from H^t, the set of all possible histories at the beginning of period t, onto $[0, p^{\max}]$, where $p^{\max} > \max\left\{p^N, p^M\right\} (> 0)$.[41] We also focus on the subgame-perfect equilibria of this game.

It is well-known that the repetition of the static Nash equilibrium constitutes a subgame-perfect equilibrium of this repeated game, and that multiple equilibria may exist when the discount factor is not too small. To study the overall impact of price caps on the scope for tacit coordination, we study the impact of price caps on the resulting *equilibrium set*. Price caps can affect this equilibrium set in two ways: price caps limit feasible deviations from the equilibrium path; and they may enlarge the set of feasible punishments following a deviation, by constraining the deviator's possible actions.

4.1 Symmetric oligopoly outcomes

We consider here the symmetric oligopoly setting of Section 2.4 and focus on symmetric price caps and stationary equilibrium paths.

We first note that the static Nash outcome is sustainable, with or without price caps, and that the possibility for the firms to enter into price-caps agreements can only enhance

[41] This upper bound ensures the existence of a worst punishment, which we use for the case of substitutes.

the scope for tacit coordination. Let $\mathcal{P}^+$ (resp., $\mathcal{P}_c^+$) denote the set of symmetric prices p that are weakly more profitable than the static Nash equilibrium (i.e., $\pi^s(p) \geq \pi^s(p^N)$) and can be sustained in the absence of price caps (resp., for some symmetric price caps). Note that $(p^N \in) \mathcal{P}^+ \subseteq \mathcal{P}_c^+$: any price in $\mathcal{P}^+$ remains sustainable when a "high-enough" price cap (e.g., $\bar{p} = p^{\max}$) is introduced. We denote by p^* and by p_c^* the most profitable prices in these sets: $p^* \equiv \arg\max_{p \in \mathcal{P}^+} \pi^s(p)$ and $p_c^* \equiv \arg\max_{p \in \mathcal{P}_c^+} \pi^s(p)$. From Lemma 3, p^* and p_c^* both lie above p^N when the two goods are substitutes, and below p^N when they are complements. Furthermore, if firms can achieve perfect coordination in the absence of price caps, they can do so as well with high enough price caps. The more interesting case is therefore when, in the absence of price caps, firms cannot achieve perfect coordination through repeated interaction (i.e., $p^M \notin \mathcal{P}^+$). We have:

Proposition 14 (screening through price caps under tacit coordination) *For a symmetric oligopoly:*

(i) Under (**S**) *and* (**SC**), $\mathcal{P}_c^+ = \mathcal{P}^+$*: the possibility for firms of setting price caps have no impact on the scope for tacit collusion.*

(ii) Under (**C**)*, if* $p^M \in \mathcal{P}^+$*, then firms can achieve perfect cooperation with and without price caps; if instead* $p^M \notin \mathcal{P}^+$*, then* $(p^N \geq) p^* > p^M = p_c^*$*: price caps enable the firms to achieve perfect cooperation, which also benefits consumers and society. Furthermore, when firms face non-decreasing returns to scale, a price cap* $\bar{p} = p^M$ *yields a unique continuation equilibrium, in which firms repeatedly charge* p^M*.*

Proof. See Appendix G. ■

The intuition underlying Proposition 14 is simple. Under (**S**), firms want to raise prices. However, under (**SC**) the targeted prices lie above firms' best-responses; as a result, price caps cannot be used to limit firms' deviations from these targeted prices and, when prices are strategic complements, they cannot limit deviations from any other feasible price either. Hence, price caps do not facilitate tacit collusion. By contrast, under (**C**), firms produce complements; they thus wish to lower their prices in order to eliminate double marginalization. In the absence of price caps, repeated interaction may not enable firms to achieve perfect coordination, in which case the most profitable sustainable price remains higher than the monopoly level. A symmetric price cap $\bar{p} = p^M$ enables instead the firms to achieve perfect coordination; furthermore, under non-decreasing returns to scale, each firm can again secure its share of the monopoly profit by charging the monopoly price, ensuring that perfect coordination is the unique equilibrium.

4.2 The technology adoption model

To study more fully the impact of price caps on tacit coordination, let us return to the hybrid demand model introduced in Section 2.6.1. Unlike in Section 4.1, we provide a complete characterization of the prices that can be achieved through arbitrary price caps (and so in particular in the absence of any price cap). We also provide extensions to asymmetric demands and to an arbitrary number of firms.[42]

The comparison of p^N, the Nash price, with p^M, the monopoly price, drives the nature of the interaction between the firms, and the coordination that they wish to pursue.[43] *Under rivalry* ($p^N < p^M$, which arises when the essentiality parameter e lies below the monopoly price: $e < p^M$, implying $p^N = e$), the firms wish to *collude* by raising their prices above the static Nash level, which harms consumers and reduces social welfare. Charging a price above $p^N = e$ however induces users to buy at most one license. We will assume that firms can share the resulting profit $\tilde{\pi}(p)$ as they wish.[44] In this "incomplete-technology region", it is optimal for the firms to raise the price up to $\tilde{p}^M(e) \equiv \arg\max_p \{pD(p+e)\}$, if feasible, and share the resulting profit, $\tilde{\pi}^M(e)$. *Under complementors* ($p^N > p^M$, which arises when $e > p^M$), the firms wish to *cooperate* by lowering their prices below the static Nash level, which benefits users as well as firms. Ideally, the firms would reduce the per-patent price down to p^M, and so as to obtain per-firm profit π^M.

4.2.1 Repeated interaction without price caps

We first consider the scope for tacit coordination through repeated interaction, in the absence of price caps. Online Appendix J provides a complete characterization, the key results of which are summarized in Figure 3.

Tacit coordination is easiest, and the gain from coordination highest, when the patents are close to being either perfect substitutes or perfect complements. Tacit coordination is impossible when patents are weak substitutes; raising price then leads users to adopt an incomplete version of the technology, and decreases overall profit. Collusion by contrast is feasible when patents are strong substitutes, and all the more so as they become closer substitutes. Likewise, the scope for cooperation increases as patents become more essential; finally, some cooperation is always feasible when patents are strong complementors.

We now consider the impact of tacit coordination on consumers. To perform a welfare analysis we assume that, whenever equilibria exist that are more profitable than the static Nash outcome, then firms coordinate on one – anyone – of those equilibria.[45]

[42] This section builds on an earlier Discussion Paper entitled "Cooperation vs. Collusion: How Essentiality Shapes Co-opetition".

[43] It is tempting to refer to "substitutes" in case of rivalry and to "complements" in case of complementors. However, in this hybrid demand model, patents are always local complements for low prices, and local substitutes for high prices. For instance, in the case of "weak complementors" (namely, when $p^N = e > p^M$), patents are complements at prices below the Nash level (e.g., at monopoly prices), and local substitutes at higher prices.

[44] In our setting, they can do so by charging the same price $p > e$ and allocating market shares among themselves; more generally, introducing a small amount of heterogeneity in users' preferences would allow the firms to achieve arbitrary market shares by choosing their prices appropriately.

[45] We remain agnostic about equilibrium selection, as the conclusions hold for *any* profitable coordination.

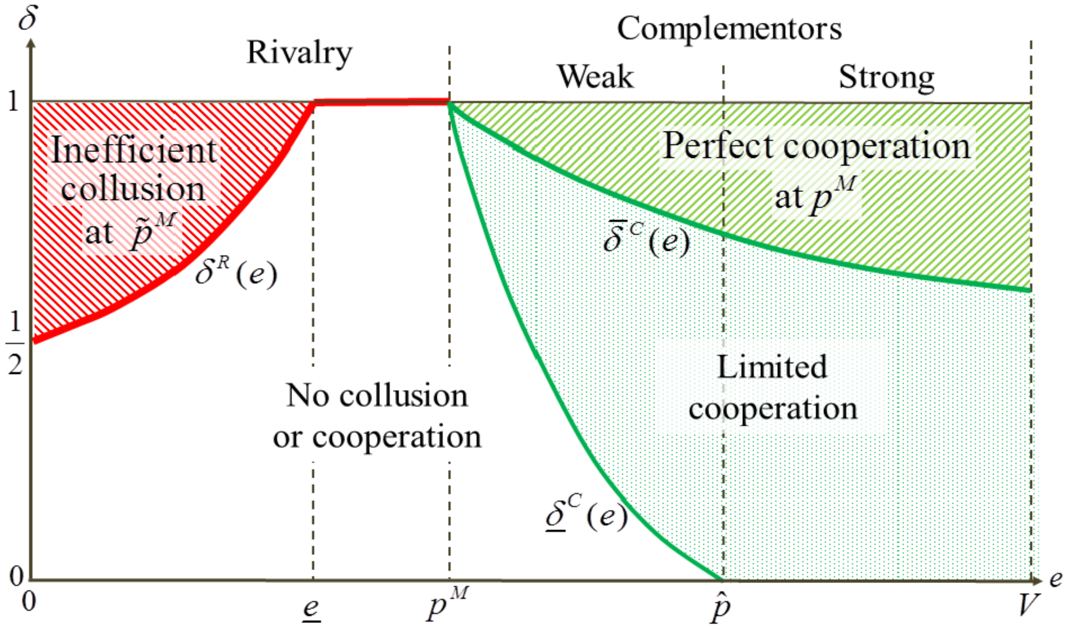

Figure 3: Tacit coordination without price caps in the technology model

Proposition 15 (welfare) *Whenever firms coordinate on an equilibrium that is more profitable than the static Nash benchmark, such tacit coordination:*

(*i*) *harms users and reduces total welfare under rivalry ($e < p^M$).*

(*ii*) *benefits users and increases total welfare for complementors ($e > p^M$).*

Proof. See online Appendix K. ∎

4.2.2 Impact of price caps on repeated interaction

Let us now investigate tacit coordination under price caps. Let $\mathcal{V}_c^+$ denote the set of equilibrium payoffs that are weakly more profitable than Nash payoffs when price caps can be introduced, and v_c^* denote the maximal payoff in this set. We have:

Proposition 16 (benefits of price caps) *Price caps:*

(*i*) *have no impact on profitable collusion in case of rivalry: if $e < p^M$, then $\mathcal{V}_c^+ = \mathcal{V}^+$;*

(*ii*) *enable perfect cooperation, which benefits consumers as well, in case of complementors: if $e \geq p^M$, then $v_c^* = \pi^M$; in particular, introducing a price cap $\bar{p} = p^M$ yields a unique continuation equilibrium, in which firms repeatedly charge p^M.*

It is therefore optimal for the competition authority to allow price caps in $\mathcal{G}$.

Proof. See online Appendix L. ■

Within the context of technology adoption, Proposition 16 extends Proposition 14 in that it considers the entire set of Nash-dominating equilibria (stationary or not, symmetric or not), with and without price caps. The findings can be illustrated by comparing Figure 3 with Figure 4: price caps can only benefit consumers when firms use them to increase their profits; they do not allow for any additional undesired collusion in case of rivalry, and allow instead for perfect, desirable cooperation in case of complementors.

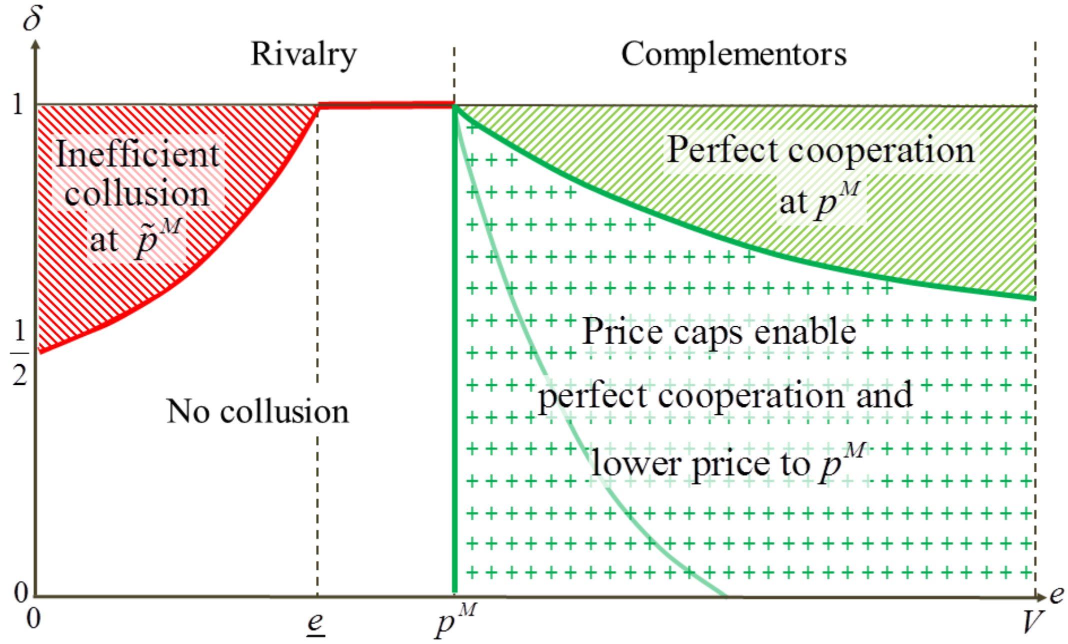

Figure 4: Tacit coordination under price caps in the technology model

Remark: Independent licensing is no longer a perfect screen under repeated interaction. We saw that independent licensing provides a perfect screen under non-repeated interaction: it prevent pools from sustaining any collusion in case of rivalry, and does not prevent pools from achieving perfect cooperation in case of complementors. Alas, as shown in online Appendix M, this is no longer so under repeated interaction. A pool subject to independent licensing still improves cooperation and lowers price for complementors $(e > p^M)$, and it also benefits consumers when (inefficient) collusion would already arise in the absence of a pool, by allowing them to consume both offerings; however, a pool, even subject to independent licensing, may harm consumers by enabling collusion in case of weak rivalry. These insights are illustrated in Figure 5.

Without independent licensing, a pool would enable the firms to sustain the monopoly outcome which also benefits consumers in case of complementors but harms them in case of rivalry. Appending independent licensing does not prevent the pool from achieving the desired cooperation in case of complementors, and in case of rivalry, it enables the firms

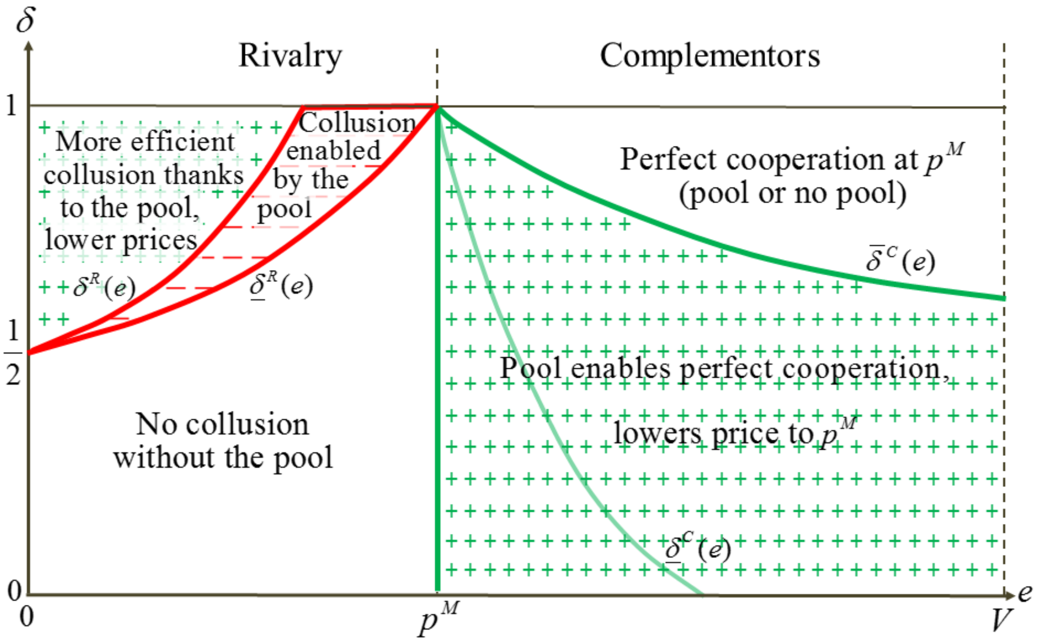

Figure 5: Impact of a pool with Independent Licensing (relative to no pool)

to collude more efficiently (by selling the complete technology), which again benefits consumers when firms could already collude without the pool. However, the pool can also enable the firms to collude when otherwise they could not, in which case it hurts consumers. This is because, by eliminating the inefficiency from selling an incomplete technology (the corollary of an attempt to raise price in the absence of a pool), the pool makes high prices more attractive.

Thus, by relying on independent licensing alone, authorities run the risk of generating some welfare loss by approving a pool of weak substitutes.[46] By contrast, price caps (which, as already noted, amount to appending unbundling to independent licensing) provide a perfect screen.

4.2.3 Asymmetric offerings and oligopoly

Suppose now that essentiality differs across firms: The technology has value $V - e_i$ if the user buys only patent j (for $i \neq j \in \{1, 2\}$); without loss of generality, suppose that $e_1 \geq e_2$. The following proposition shows that price caps still provide a perfect screen. As in Section 4.2.2, let $\mathcal{V}_c^+$ denote the set of pure-strategy equilibrium payoffs that are weakly more profitable than Nash when price caps can be introduced, and v_c^* denote the maximal per firm payoff in this set; we have:

Proposition 17 (asymmetric offerings) *Price caps:*

[46]This occurs only when no collusion is sustainable in the absence of a pool (i.e., $\delta < \delta^R$) *and* the pool enables some collusion (e.g., $\delta \geq \underline{\delta}^R$); the pool is instead beneficial when inefficient collusion was already sustainable (i.e., $\delta \geq \delta^R$) and is neutral when collusion remains unsustainable (i.e., for δ low enough).

(i) *do not affect the scope for profitable collusion in case of rivalry: if* $e_1 + e_2 < P^M \equiv \arg\max_P PD(P)$, *then* $\mathcal{V}_c^+ = \mathcal{V}^+$;

(ii) *enable consumer-welfare-augmenting perfect cooperation in case of complementors: if* $e_1 + e_2 \geq P^M$, *then* $v_c^* = \pi^M$; *in particular, any vector of price caps* $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2)$ *satisfying* $\bar{p}_1 + \bar{p}_2 = P^M$ *and* $\bar{p}_i \leq e_i$ *induces* $\mathbf{p} = \bar{\mathbf{p}}$ *in every period as unique continuation equilibrium.*

Proof. See online Appendix L. ■

Suppose now that there are $n \geq 2$ symmetric firms: The technology has value $V(m)$ if the user buys $m \leq n$ licenses, with $0 = V(0) \leq V(1) \leq ... \leq V(n)$ and $V(n) > 0$. The demand for the bundle of n patents at total price P becomes

$$D(P) \equiv F(V(n) - P),$$

where the c.d.f. $F(\cdot)$ satisfies the same regularity conditions as before (that is, Assumption B holds). Lerner and Tirole (2004) show that, in the unique symmetric static Nash outcome, users buy patents at price $p^N \equiv \min\{\tilde{p}, \hat{p}\}$, where $\tilde{p}$ is the unique price p satisfying $V(n) - np = \max_{m<n}\{V(m) - mp\}$, and where $\hat{p}$ is now defined as $\hat{p} \equiv \arg\max_p \{pD(p + (n-1)\hat{p})\}$.

As in a duopoly, multiple marginalization implies $\hat{p} > p^M \equiv \arg\max_p npD(np)$, leading to three relevant regimes:

- Rivalry when $\tilde{p} < p^M$, implying $p^M > p^N = \tilde{p}$.
- Weak complementors when $p^M < \tilde{p} < \hat{p}$, implying $p^M < p^N = \tilde{p}$.
- Strong complementors when $\tilde{p} \geq \hat{p}$, implying $p^M < p^N = \hat{p}$.

Our previous insights readily extend to any number of patents in the case of complementors. Likewise, in the case of rivalry, we show in Online Appendix N that raising total profit above the static Nash level requires again selling an incomplete bundle. To go further, we focus for simplicity on the stationary symmetric outcomes that can be sustained by reversal to Nash; let $\hat{\mathcal{P}}^+$ (resp., $\hat{\mathcal{P}}_c^+$) denote the set of prices that are weakly more profitable than the static Nash equilibrium and can be sustained in the absence of price caps (resp., with price caps)[47]. We have:

Proposition 18 (oligopoly) *Price caps:*

(i) *do not affect the set of profitable prices that can be sustained by reversal to Nash in case of rivalry: if* $p^N < p^M$, *then* $\hat{\mathcal{P}}_c^+ = \hat{\mathcal{P}}^+$;

(ii) *enable perfect cooperation, which benefits consumers as well, in case of complementors: if* $p^N \geq p^M$, *then introducing a price cap* $\bar{p} = p^M$ *yields a unique continuation equilibrium, in which firms repeatedly charge* p^M.

[47]We allow for asymmetric price caps; however, given the symmetry of the environment, symmetric price caps are as effective as asymmetric ones.

Proof. See online Appendix N. ■

In the rivalry case, in order to increase profits firms must raise prices, which induces users to buy only a subset of patents. But then, a firm cannot profitably deviate by raising further its price, as it would exclude itself from the basket. Price caps thus have no bite on profitable deviations, and so cannot enhance the scope for collusion. By contrast, in the case of complementors, price caps enable the firms to increase their profits by reducing their prices down to the monopoly level, and preventing any profitable deviation towards higher prices.

4.3 Focal points and market transparency

Proposition 16 and its extensions imply that price caps are always socially beneficial even under repeated interaction, provided that firms seize coordination opportunities when these exist. A potential objection to the policy of allowing price caps is that, if instead firms fail to coordinate in the absence of price caps, the latter may provide focal points and facilitate collusion. Caution suggests neither dismissing this possibility nor viewing it as negating the benefits of price caps. First, while progress has lately been made on trying to understand how firms coordinate, our knowledge of the matter is still scant, and the empirical evidence often involves additional features such as information sharing. Second, while communication seems to have an effect on collusion, it is not clear that the type of communication involved in price-cap setting is the relevant channel. Let us elaborate briefly on these two points.

Transparency. It is well-known that "transparency" (making data about prices, outputs, or costs publicly available) has the potential to facilitate tacit collusion: see, e.g., Green and Porter (1984) and Rahman (2014)[48] for formal analyses,[49] and Ivaldi *et al.* (2003), Kühn and Vives (1995), Vives (2007) and Whinston (2006) for policy discussions. Rules restricting information exchange, as they already exist for merger negotiations, could then be useful.

For instance, Albaek *et al.* (1997) study the impact of the Danish antitrust authority's 1993 decision to gather and publish firm-specific transactions prices in the ready-mixed concrete market. Following initial publication, average prices of reported grades increased by 15-20 percent within one year. Similar findings were found in other industries, including for the episode in the US railroad industry when contract disclosure was mandated by the Congress.

[48] Rahman studies a repeated Cournot game with i.i.d. price shocks. He shows that privy messages on the possible existence of a "monitoring phase" that is later made public can enable the firms to better detect deviations and therefore may facilitate tacit collusion. As in the conventional wisdom, ex-post exchange of information may discipline firms.

[49] In a recent paper, Sugaya and Wolitzky (2017) however challenge this common wisdom and show that maintaining privacy may help firms collude by refraining to compete in each other's markets, while better information may make deviations more profitable and thereby hinder collusion.

Testing the impact of (regulatory, rather than negotiated price caps) price caps on collusion is made difficult by the fact that price caps are often accompanied with information disclosure, and so it is difficult to identify the effect of price caps. For example, Genakos et al. (2017) study regulatory caps in the context of the Greek market for fresh fruits and vegetables. Using a diff-in-diff approach (through the comparison with five unregulated fruit prices), they find that prices dropped by 6% when the regulation was lifted and argue convincingly that it facilitated collusion before. This interesting evidence however does not inform us on the impact of price caps *per se*. First, the focal prices had been in place between 1946 and 2011; so it would seem that they would not have disappeared overnight when caps were lifted,[50] and yet most of the price reduction took place within a few weeks. Second, the Greek regulation was a (percentage) markup regulation, and so it involved information sharing about cost and possibly demand (if cost is computed as an average cost).

The field evidence on focal points is therefore difficult to interpret, due to the lack of appropriate counterfactual. Knittel and Stango's classic paper (2003) however find evidence of a focal point effect of regulatory price caps in the credit card industry, in which no information exchange happened.[51] During the sample period, most issuers set rates of interest that matched the ceiling in their states; and interest rates were higher in states with high ceilings than in states with no ceiling.

Laboratory experiments have tried to circumvent this issue, but so far have failed to provide conclusive evidence of collusive, focal-point effects. See, e.g., Engelmann-Müller (2011) for an experiment designed to make collusion easier than in previous attempts, as well as a review of that literature.

Communication of intentions. A potential argument against price caps is that they may[52] necessitate some communication among the parties to fix the caps. There is evidence that communication is the main driver of collusive coordination.[53,54] But again one

[50] Given that cost information was no longer collected after 2011, one would expect that in the longer term, the focal prices would become irrelevant.

[51] As explained carefully by Knittel and Stango, testing the existence of a focal point effect is complex for multiple reasons. First, there were two prices (issuer annual fee and interest rate), one of which is not subject to a cap; the theory of tacit collusion is not well developed for such environments, let alone a theory of focal points. Second, the econometric specification of the dynamic competition model is not straightforward.

[52] In theory, this is not quite the case. On could for example imagine sealed-bid proposals of the type "I will not charge more than x if the other does not charge more than y", with the price caps being set by a computer or blockchain provided that demands are compatible (with a mechanism determining how to use the slack if they are strictly compatible, as in the Nash demand game). A direct negotiation, as it takes place for the formation of patent pools, seems more realistic. It may nonetheless be interesting to design schemes that would limit the amount of direct communication among the firms.

[53] Brandts and Cooper (2007) look at methods of coordination in a one-shot coordination game. They show that the most powerful way of communicating is directly giving instructions (rather than trying things like indirect incentives). In a repeated interaction environment, Fonseca and Normann (2012) show that absent communication collusion seems to be weak except under duopoly, while communication can have important effects even with a reasonably large number of players.

[54] Even cheap-talk communication can facilitate collusion by enhancing market transparency; see Harrington and Skrzypacz (2011) and Awaya and Krishna (2016).

needs to understand the channel in order to know whether the communication that is involved in price cap setting is the actual driver of collusive coordination. The discussion in Kühn and Tadelis (2017), which builds on Cooper and Kühn (2014), suggests that this may not be the case. They argue that "agreements on the collusive price are actually not decisive [...]. What is central to collusion is that subjects have a clear view of the responses of the other parties to the agreement of reactions to possible cheating [...]. First, the most reliable factor to achieve collusive outcomes is communication in which clear punishments are threatened when the explicit agreement on price is violated [...]. The second mechanism that very strongly supports collusion is repeated conversation and feedback about past behavior. In particular, there is frequent and intense verbal punishment by players who complain about the cheating by their counterparts."

Finally, let us note that the advice given in the literature on communication is really about markets in which producers are clearly substitutes; in a sense, our contribution is of interest only when this is not the case. If one is concerned that price caps could facilitate tacit collusion by providing a focal point, screening out such straightforward cases before issuing a business review letter allowing price caps makes good sense.

5 Concluding remarks

Reviews of mergers and joint marketing agreements can be hindered by poor information associated with patchy or non-existent price and demand data as well as time- and price-dependent patterns of complementarity/substitutability. This suggests enriching the antitrust toolbox with new and less information-intensive regulatory instruments. This paper is a first attempt at meeting this challenge. It investigates price caps as a possible alternative to mergers.

We saw that voluntary price caps raise consumer welfare – for quite general demand and cost functions in the case of non-repeated interactions and for the more specific repeated-interactions environments we were able to analyze. We provided a novel analysis, of independent interest, of coordinated effects in an industry in which goods are not necessarily substitutes, let alone perfect substitutes. We issued some caveats, and provided extensions for the cases in which demand may exhibit either a stable mix of complements and substitutes or a price-dependent pattern of complementarity/substitutability. We also analyzed whether price caps could be used either to monopolize the industry by programming some firms' exit, or to stifle the incumbents' investment, or else to deter entry of new entrants, and were led to formulate two policy recommendations to counter potential perverse effects of price caps in this dimension of industry performance. Finally, we showed that price caps substitute advantageously for a merger of complements when the latter enables established firms to foreclose rivals.

This paper is only a first step in an extensive research agenda. We conclude with six important lines for future research. First, the theory of repeated interaction with arbitrary degrees of substitutability and complementarity should be developed for general cost and demand functions. Second, while we have assumed that firms take advantage of existing opportunities to coordinate tacitly, we know little about whether discussions such as those on price caps could enhance market transparency, or create a focal point that would help the firms to indeed achieve tacit collusion. Further experimental work could inform us on this question. Third, we should think beyond mergers and pools. Other joint marketing arrangements, such as alliances for instance, ought to be considered; similarly, "indirect" joint marketing through platforms should receive more attention. Fourth, we could extend the analysis to competitive non-linear pricing and other forms of price discrimination, so as to unveil the proper counterparts of price caps in such settings.[55] Fifth, the analysis should incorporate cost synergies and look at agreements that might increase our confidence in their competitive benefits. Finally, in the paper we consider only the wholesale acceptance of price-caps agreements by the antitrust authority. Section 3.2 qualified this approach by adding two requirements on party enforcement. The desirability of price-caps agreements might also be enhanced by eliciting more information from industry participants; for example, the authority could elicit the effects of price caps set by a sub-coalition of parties on other parties by granting the latter an outright veto (giving non-participants formal authority), by letting the latter express opposition (possibly giving them real authority), or through more sophisticated mechanisms. This research would allow policy makers to run a better-informed horserace among mergers, price caps and more sophisticated collaborative agreements. We leave these and other fascinating aspects of coopetition to future research.

[55] For instance, China Eastern Airlines and Qantas (which are substitutes on the Shanghai-Sydney route and complements on connecting flights) submitted to the Australian competition agency (the ACCC) a coordination agreement covering schedules, frequencies and connection times, but also new fare products and frequent flyer programs – the airline industry is highly prone to yield management and loyalty programs. To accept the agreement, the ACCC imposed minimal quantity requirements (expressed in terms of seat capacity between Shanghai and Sydney, and of aggregate seat capacity between Shanghai and Australia). See ACCC decision N° A91470, available at http://www.accc.gov.au. We thank Graeme Woodbridge for drawing our attention to this case.

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Appendix

A Proof of Lemma 1

We first show that monopoly prices exceed marginal costs for at least one firm. Suppose instead that $p_i^M \leq C_i'\left(q_i^M\right)$ for all $i \in \mathcal{N}$, and consider a small and uniform increase in prices: $dp_i = dp > 0$ for $i \in \mathcal{N}$. We then have $dq_j = \sum_{i \in \mathcal{N}} \partial_i D_j\left(\mathbf{p}^M\right) dp \leq 0$ for all $j \in \mathcal{N}$, and thus:

$$d\Pi = \sum_{j \in \mathcal{N}} \left[p_j^M - C_j'\left(q_j^M\right)\right] dq_j + \sum_{j \in \mathcal{N}} q_j^M dp > 0,$$

a contradiction. Therefore, $p_i^M > C_i'\left(q_i^M\right)$ for some $i \in \mathcal{N}$.

We now show that, under (**S**), $p_i^M > C_i'\left(q_i^M\right)$ for every $i \in \mathcal{N}$. To see this, suppose that there exists a non-empty subset of $\mathcal{N}$, $\mathcal{N}^-$, such that $p_j^M \leq C_j'\left(q_j^M\right)$ for every $j \in \mathcal{N}^-$, and consider a small and uniform increase in these prices: $dp_j = dp > 0$ for $j \in \mathcal{N}^-$. Under (**S**), we then have:

- for $i \in \mathcal{N} \setminus \mathcal{N}^-$, $dq_i = \sum_{j \in \mathcal{N}^-} \partial_j D_i\left(\mathbf{p}^M\right) dp > 0$, as $\partial_j D_i\left(\mathbf{p}^M\right) > 0$ for $j \neq i$.
- for $i \in \mathcal{N}^-$, $dq_i = \sum_{j \in \mathcal{N}^-} \partial_j D_i\left(\mathbf{p}^M\right) dp \leq \sum_{j \in \mathcal{N}} \partial_j D_i\left(\mathbf{p}^M\right) dp \leq 0$.

Therefore:

$$d\Pi = \sum_{j \in \mathcal{N} \setminus \mathcal{N}^-} \underbrace{\left[p_j^M - C_j'\left(q_j^M\right)\right]}_{>0} \underbrace{dq_j}_{>0} + \sum_{j \in \mathcal{N}^-} \underbrace{q_j^M}_{>0} \underbrace{dp}_{>0} + \sum_{j \in \mathcal{N}^-} \underbrace{\left[p_j^M - C_j'\left(q_j^M\right)\right]}_{\leq 0} \underbrace{dq_j}_{\leq 0} > 0,$$

a contradiction. Therefore, under (**S**), $p_i^M > C_i'\left(q_i^M\right)$ for every $i \in \mathcal{N}$.

We now compare monopoly prices to firms' best-responses. The monopoly prices satisfy, for $i \in \mathcal{N}$:

$$0 = \partial_i \Pi\left(\mathbf{p}^M\right) = \partial_i \pi_i\left(\mathbf{p}^M\right) + \sum_{j \in \mathcal{N} \setminus \{i\}} \partial_i \pi_j\left(\mathbf{p}^M\right),$$

and thus:

$$\partial_i \pi_i\left(\mathbf{p}^M\right) = -\sum_{j \in \mathcal{N} \setminus \{i\}} \partial_i \pi_j\left(\mathbf{p}^M\right) = -\sum_{j \in \mathcal{N} \setminus \{i\}} \left\{\left[p_j^M - C_j'\left(q_j^M\right)\right] \partial_i D_j\left(\mathbf{p}^M\right)\right\}. \quad (3)$$

Therefore:

(*i*) Under (**S**), the right-hand side of (3) is negative, as $p_j^M > C_j'\left(q_j^M\right)$, from the first part of the lemma, and $\partial_j D_i(\cdot) > 0$ for $j \neq i \in \mathcal{N}$; hence, for $i \in \mathcal{N}$, we have $\partial_i \pi_i\left(\mathbf{p}^M\right) < 0$, which, together with the quasi-concavity of π_i with respect to p_i, implies $p_i^M > R_i\left(\mathbf{p}_{-i}^M\right)$.

(ii) Suppose that for all $j \in \mathcal{N}$, $p_j^M \geq R_j\left(\mathbf{p}_{-j}^M\right)$, implying $\partial_j \pi_j\left(\mathbf{p}^M\right) \leq 0$. We then have, for $j \in \mathcal{N}$:

$$0 \geq \partial_j \pi_j\left(\mathbf{p}^M\right) = D_j\left(\mathbf{p}^M\right) + \left[p_j^M - C_j'\left(q_j^M\right)\right] \partial_j D_j\left(\mathbf{p}^M\right),$$

and thus, under (**C**), $p_j^M > C_j'\left(q_j^M\right)$ for every $j \in \mathcal{N}$. But then, as $\partial_j D_i\left(\cdot\right) < 0$ for $j \neq i \in \mathcal{N}$ under (**C**), (3) implies $\partial_i \pi_i\left(\mathbf{p}^M\right) > 0$, a contradiction. Hence, the monopoly outcome satisfies $p_j^M < R_j\left(\mathbf{p}_{-j}^M\right)$ for some firm j.

Finally, when $n = 2$, (3) implies, for $j \neq i \in \{1, 2\}$:

$$\partial_j \pi_j\left(\mathbf{p}^M\right) = -\left[p_i^M - C_i'\left(q_i^M\right)\right] \partial_j D_i\left(\mathbf{p}^M\right).$$

Under (**C**), $\partial_j D_i\left(\cdot\right) < 0$ and thus $p_i^M > C_i'\left(q_i^M\right)$ implies $p_j^M < R_j\left(p_j^M\right)$.

B Proof of Proposition 1(ii)

By construction, $\mathbf{p}^N$ lies on firms' best-responses, and thus belongs to $\mathcal{P}$. Consider now a price vector $\hat{\mathbf{p}}$ in $\mathcal{P} \backslash \left\{\mathbf{p}^N\right\}$, and suppose that $\hat{p}_i \geq p_i^N$ for all $i \in \mathcal{N}$. For every $i \in \mathcal{N}$, we then have:

$$\begin{aligned}
\hat{p}_i - p_i^N &\leq R_i\left(\hat{\mathbf{p}}_{-i}\right) - p_i^N, \\
&= R_i\left(\hat{\mathbf{p}}_{-i}\right) - R_i\left(\mathbf{p}_{-i}^N\right) \\
&= \int_0^1 \frac{d}{d\lambda}\left\{R_i\left(\lambda \hat{\mathbf{p}}_{-i} + (1-\lambda)\,\mathbf{p}_{-i}^N\right)\right\} d\lambda \\
&= \int_0^1 \{\sum_{j \in \mathcal{N} \backslash \{i\}} \partial_j R_i\left(\lambda \hat{\mathbf{p}}_{-i} + (1-\lambda)\,\mathbf{p}_{-i}^N\right)\left(\hat{p}_j - p_j^N\right)\} d\lambda.
\end{aligned}$$

Summing up these inequalities for $i \in \mathcal{N}$ yields:

$$\begin{aligned}
\sum_{i \in \mathcal{N}}\left(\hat{p}_i - p_i^N\right) &\leq \sum_{i \in \mathcal{N}} \int_0^1 \{\sum_{j \in \mathcal{N} \backslash \{i\}} \partial_j R_i\left(\lambda \hat{\mathbf{p}}_{-i} + (1-\lambda)\,\mathbf{p}_{-i}^N\right)\left(\hat{p}_j - p_j^N\right)\} d\lambda \\
&= \sum_{j \in \mathcal{N}}\left(\hat{p}_j - p_j^N\right) \int_0^1 \{[\sum_{i \in \mathcal{N} \backslash \{j\}} \partial_j R_i\left(\lambda \hat{\mathbf{p}}_{-i} + (1-\lambda)\,\mathbf{p}_{-i}^N\right)]\} d\lambda \\
&< \sum_{j \in \mathcal{N}}\left(\hat{p}_j - p_j^N\right),
\end{aligned}$$

where the last inequality follows from (1). We thus obtain a contradiction, implying that $\hat{p}_i < p_i^N$ for some $i \in \mathcal{N}$.

C On unique implementation

C.1 Proof of Proposition 2

By assumption, $\mathbf{p}^N$ is the unique equilibrium of game G_∞. Consider a price vector $\hat{\mathbf{p}} \in \mathcal{P} \backslash \left\{\mathbf{p}^N\right\}$ and the associated price-cap-constrained game $G_{\hat{\mathbf{p}}}$. As noted in the proof of Proposition 1, $\hat{\mathbf{p}}$ is sustainable through the vector of price caps $\bar{\mathbf{p}} = \hat{\mathbf{p}}$: Strict quasi-concavity and the fact that each price must be below the firm's reaction curve implies that $\hat{\mathbf{p}}$ is an equilibrium of $G_{\hat{\mathbf{p}}}$. So the question is whether there might exist another equilibrium $\hat{\mathbf{p}}' \neq \hat{\mathbf{p}}$, in which necessarily at least one of the prices is strictly below the cap. We now show that this is never the case in the three settings described in the proposition.

C.1.1 Duopoly

Suppose that $n = 2$. If both prices in $\hat{\mathbf{p}}'$ are strictly lower than their cap, then, because of quasi-concavity, $\hat{\mathbf{p}}'$ is also an unconstrained equilibrium; but then, from Proposition 1, $p_i^N > \hat{p}_i > \hat{p}'_i$ for some $i \in \mathcal{N}$, and thus we have two equilibria of the unconstrained game, a contradiction.

Suppose instead that only one price, say $\hat{p}'_2$, is below the cap while the other, $\hat{p}'_1$, is at the cap; that is: $\hat{p}'_2 < \hat{p}_2$ and $\hat{p}'_1 = \hat{p}_1$. From Proposition 1, $\hat{\mathbf{p}}$ lies below the reaction curves; therefore, we have:

$$\hat{p}'_2 < \hat{p}_2 \leq R_2\left(\hat{p}_1\right) = R_2\left(\hat{p}'_1\right).$$

But then, strict quasi-concavity implies that, in the $\hat{\mathbf{p}}'$ equilibrium, firm 2 could increase its profit by raising its price toward the cap $\hat{p}_2$.

C.1.2 Strategic complementarity

Let $\hat{\imath}$ denote the firm for which the difference $\hat{p}_i - \hat{p}'_i$ is the largest. From the implementability of $\hat{\mathbf{p}}$ and strict quasi-concavity, we have $R_{\hat{\imath}}\left(\hat{\mathbf{p}}_{-\hat{\imath}}\right) \geq \hat{p}_{\hat{\imath}}$ and $R_{\hat{\imath}}\left(\hat{\mathbf{p}}'_{-\hat{\imath}}\right) = \hat{p}'_{\hat{\imath}}$.

Furthermore, letting $\mathbf{p}^\lambda_{-\hat{\imath}} \equiv \lambda \hat{\mathbf{p}}_{-\hat{\imath}} + (1-\lambda)\,\hat{\mathbf{p}}'_{-\hat{\imath}}$, we have:

$$\begin{aligned}
R_{\hat{\imath}}\left(\hat{\mathbf{p}}_{-\hat{\imath}}\right) - R_{\hat{\imath}}\left(\hat{\mathbf{p}}'_{-\hat{\imath}}\right) &= \int_0^1 \frac{d}{d\lambda}\left[R_{\hat{\imath}}\left(\mathbf{p}^\lambda_{-\hat{\imath}}\right)\right] d\lambda \\
&= \int_0^1 \sum_{j \in \mathcal{N}\backslash\{\hat{\imath}\}} \partial_j R_{\hat{\imath}}\left(\mathbf{p}^\lambda_{-\hat{\imath}}\right)\left(\hat{p}_j - \hat{p}'_j\right) d\lambda \\
&\leq \int_0^1 \sum_{j \in \mathcal{N}\backslash\{\hat{\imath}\}} \partial_j R_{\hat{\imath}}\left(\mathbf{p}^\lambda_{-\hat{\imath}}\right)\left(\hat{p}_{\hat{\imath}} - \hat{p}'_{\hat{\imath}}\right) d\lambda \\
&< \hat{p}_{\hat{\imath}} - \hat{p}'_{\hat{\imath}},
\end{aligned}$$

where the weak inequality stems from $\hat{p}_{\hat{\imath}} - \hat{p}'_{\hat{\imath}} \geq \hat{p}_j - \hat{p}'_j$ for every $j \neq \hat{\imath}$, and the strict inequality follows from the assumption $\sum_{j \in \mathcal{N} \setminus \{\hat{\imath}\}} \partial_j R_{\hat{\imath}} (\cdot) < 1$. But then, combining these conditions yields:

$$\hat{p}_{\hat{\imath}} - \hat{p}'_{\hat{\imath}} > R_{\hat{\imath}} (\hat{\mathbf{p}}_{-\hat{\imath}}) - R_{\hat{\imath}} \left(\hat{\mathbf{p}}'_{-\hat{\imath}}\right) \geq \hat{p}_{\hat{\imath}} - \hat{p}'_{\hat{\imath}},$$

a contradiction.

C.1.3 Strategic substitutability

Suppose that the reaction functions satisfy (**SS**). Then, for some $i \in \mathcal{N}$:

$$\hat{p}'_i < \hat{p}_i \leq R_i(\hat{\mathbf{p}}_{-i}) \leq R_i(\hat{\mathbf{p}}'_{-i}),$$

where the last inequality stems from the fact that, by construction, $\hat{\mathbf{p}}'_{-i}$ lies below the vector of price caps $\hat{\mathbf{p}}_{-i}$. But then, strict quasi-concavity implies that firm i could increase its profit by raising its price toward the cap $\hat{p}_i$.

C.2 Example of multiplicity in the price-cap-constrained game

Take three firms $i = 1, 2, 3$ with profits $\pi_1 (\mathbf{p}) = p_1 - (p_1)^2$, $\pi_2 (\mathbf{p}) = p_2 - (p_2)^2 - (1 - p_1)p_2(2p_3 - 1)$ and $\pi_3 = p_3 - (p_3)^2 - (1 - p_1)p_3(2p_2 - 1)$, respectively. The game G_∞, which has strictly concave payoff functions in own price, admits a unique Nash equilibrium: $p_1^N = p_2^N = p_3^N = 1/2$. But if firm 1 faces a price cap $\bar{p}_1 = 0$, then there is a continuum of equilibria, in which firm 1 sets $p_1 = 0$ whereas firms 2 and 3 charge any non-negative prices satisfying $p_2 + p_3 = 1$.

D Proof of Lemma 3

We have:

$$p^M - R^s\left(p^M\right) = \int_{p^N}^{p^M} \left[1 - R^{s\prime}(p)\right] dp,$$

where the integrand of the right-hand side is positive. Hence, the sign of $p^M - p^N$ is the same as that of $p^M - R^s\left(p^M\right)$. Therefore, using Lemma 1, $p^M > p^N$ under (**S**) and $p^M < p^N$ under (**C**). The implication for profitable prices follows from the strict quasi-concavity of the industry profit: $n\pi^s(\hat{p}) \geq n\pi^s\left(p^N\right)$ then implies $\hat{p} \geq p^N$ under (**S**), and $\hat{p} \leq p^N$ under (**C**).

E Proof of Proposition 6

Suppose that all firms face the same price cap $\bar{p}$. If $\bar{p} \leq p^N$, then

$$R^s(\bar{p}) - \bar{p} = \int_{p^N}^{\bar{p}} \left[R^{s\prime}(p) - 1\right] dp \geq 0,$$

where the inequality follows from $R^{s\prime} < 1$ and $\bar{p} \leq p^N$. It follows that $\mathbf{p} = \bar{\mathbf{p}}$ constitutes an equilibrium of $G_{\bar{\mathbf{p}}}$; furthermore, any equilibrium $\hat{\mathbf{p}}$ of $G_{\bar{\mathbf{p}}}$ satisfies $\hat{\mathbf{p}} \leq \bar{\mathbf{p}} \leq \mathbf{p}^N$, and thus consumers are weakly better off under $G_{\bar{\mathbf{p}}}$ than under G_∞.

Consider now the case where $\bar{p} > p^N$. Obviously, $\mathbf{p}^N$ constitutes an equilibrium of $G_{\bar{\mathbf{p}}}$. Suppose now that there exists another equilibrium, $\hat{\mathbf{p}}$, in which some firm charges strictly more than p^N. By strict quasi-concavity, $\hat{p}_i = \min\left\{R_i(\hat{\mathbf{p}}_{-i}), \bar{p}\right\}$ for all $i \in \mathcal{N}$. If $\hat{p}_i = \bar{p}$ for all $i \in \mathcal{N}$, then

$$\bar{p} - R^s(\bar{p}) = \int_{p^N}^{\bar{p}} \left[1 - R^{s\prime}(p)\right] dp > 0,$$

implying that each firm would profitably deviate by cutting its price. Therefore, there exists $\hat{\imath} \in \mathcal{N}$ for which $\hat{p}_{\hat{\imath}} = R(\hat{\mathbf{p}}_{-\hat{\imath}}) < \bar{p}$. Conversely, if $\hat{p}_i = R(\hat{\mathbf{p}}_{-i})$ for all $i \in \mathcal{N}$, then $\hat{\mathbf{p}} = \mathbf{p}^N$, contradicting the assumption that some firm charges more than p^N. Therefore, there exists $\bar{\imath} \in \mathcal{N}$ for which $\hat{p}_{\bar{\imath}} = \bar{p} < R(\hat{\mathbf{p}}_{-\bar{\imath}})$. But then, we have:

$$\begin{aligned}
\bar{p} - \hat{p}_{\hat{\imath}} &\leq R(\hat{\mathbf{p}}_{-\bar{\imath}}) - R(\hat{\mathbf{p}}_{-\hat{\imath}}) \\
&= R\left(\hat{p}_{\hat{\imath}}, \hat{\mathbf{p}}_{-\{\hat{\imath},\bar{\imath}\}}\right) - R\left(\bar{p}, \hat{\mathbf{p}}_{-\{\hat{\imath},\bar{\imath}\}}\right) \\
&= \int_{\bar{p}}^{\hat{p}_{\hat{\imath}}} \partial_1 R\left(p, \hat{\mathbf{p}}_{-\{\hat{\imath},\bar{\imath}\}}\right) dp \\
&> \bar{p} - \hat{p}_{\hat{\imath}},
\end{aligned}$$

where the inequality follows from $\partial_1 R(\cdot) > -1$ and $\bar{p} > \hat{p}_{\hat{\imath}}$. We thus have a contradiction, implying there exists no equilibrium of $G_{\bar{\mathbf{p}}}$ in which a firm charges strictly more than p^N.

F Proof of Proposition 7

(i) From Proposition 6, price caps can only sustain prices below the Nash level; it follows from Lemma 3 that, under (**S**), firms cannot use price caps to sustain more profitable symmetric outcomes.

(ii) Under (**C**), $R\left(p^M\right) > p^M$ from Lemma 1, and thus $\mathbf{p}^M \in \mathcal{P}$. Furthermore, under either (**SS**) or (**SC**), the monopoly outcome is the unique equilibrium of $G_{\mathbf{p}^M}$. Suppose now that $C''(\cdot) \leq 0$, and firms agreed on price cap $\bar{p} = p^M$. By charging $p_i = p^M$ each firm $i \in \mathcal{N}$ can obtain

$$\pi_i\left(p^M, \mathbf{p}_{-i}\right) = \pi^s\left(p^M\right) + \int_{q^M}^{D_i\left(p^M, \mathbf{p}_{-i}\right)} \left[p^M - C'\left(q_i\right)\right] dq_i \geq \pi^s\left(p^M\right),$$

where the inequality stems from the fact that (a) $p_j \leq \bar{p} = p^M$ for every $j \in \mathcal{N}$ implies $D_i\left(p^M, \mathbf{p}_{-i}\right) \geq D_i\left(p^M, ..., p^M\right)$, as goods are complements, and (b) using Lemma 1 and $C''(\cdot) \leq 0$, $p^M > C'\left(q^M\right) \geq C'\left(q_i\right)$ for $q_i \geq q^M$; hence, $\pi_i\left(p^M, \mathbf{p}_{-i}\right) \geq \pi^s\left(p^M\right)$. As each firm can secure $\pi^s\left(p^M\right)$, it follows that the monopoly outcome constitutes again the unique continuation equilibrium.

G Proof of Proposition 14

(i) We already noted that $\mathcal{P}^+ \subseteq \mathcal{P}_c^+$. Conversely, fix a price $\hat{p} \in \mathcal{P}_c^+$ and an equilibrium sustaining this price $\hat{p}$ thanks to a price cap $\bar{p}$. Letting $\hat{\underline{V}}_i$ denote the lowest sustainable continuation value for firm $i \in \mathcal{N}$, consider the alternative "bang-bang" strategies: (i) along the equilibrium path, firms stick to $\hat{p}$; and (ii) any deviation by firm $i \in \mathcal{N}$ (from the equilibrium path, or for any other history) is punished with the continuation value $\hat{\underline{V}}_i$. These alternative strategies still sustain $\hat{p}$, as any deviation (including from off-equilibrium) is punished at least as severely with the alternative strategies.

Thus, without loss of generality, consider an equilibrium sustaining $\hat{p}$ thanks to the price cap $\bar{p}$, in which any deviation by any firm $i \in \mathcal{N}$ is punished with the continuation value $\hat{\underline{V}}_i$. At any point in time, all deviations from the prescribed continuation path are punished in the same way; hence, the best deviation is the "myopic" deviation that maximizes the current profit. As firms' individual profits are moreover strictly quasi-concave with respect to their own prices, it follows that, for any $t = 1, 2, ...$, and any $h^t \in H^t$, firm i's best deviation from the prescribed price vector $\mathbf{p}^t\left(h^t\right)$ consists in charging:

$$\bar{R}_i\left(\mathbf{p}_{-i}^t\left(h^t\right); \bar{p}\right) \equiv \min\left\{R_i\left(\mathbf{p}_{-i}^t\left(h^t\right)\right), \bar{p}\right\}.$$

By construction, $\bar{p} \geq \hat{p}$; furthermore, as by assumption $\hat{p}$ is at least as profitable as p^N, and the goods are substitutes, $\hat{p} \geq p^N$. We thus have $\bar{p} \geq p^N$, which, using $R^{s\prime}(\cdot) < 1$ and $R^s\left(p^N\right) = p^N$, yields:

$$\bar{p} - R^s\left(\bar{p}\right) = \int_{p^N}^{\bar{p}} \left[1 - R^{s\prime}\left(p\right)\right] dp \geq 0.$$

We thus have $\bar{p} \geq R\left(\bar{p}\right)$ for any firm $i \in \mathcal{N}$. But by construction, any prescribed prices $\mathbf{p}^t\left(h^t\right)$ must satisfy $p_j^t\left(h^t\right) \leq \bar{p}$ for all $j \in \mathcal{N}$. Under (**SC**), we thus have:

$$R_i\left(\mathbf{p}_{-i}^t\left(h^t\right)\right) \leq R^s\left(\bar{p}\right) \leq \bar{p}.$$

It follows that the price cap $\bar{p}$ never limits firms' deviations: in any period $t = 1, 2, ...$, and for any history $h^t \in H^t$, firm i's best deviation from the prescribed price vector $\mathbf{p}^t(h^t)$, for any $i \in N$, is the same as in the absence of any price caps:

$$\bar{R}_i\left(\mathbf{p}_{-i}^t\left(h^t\right); \bar{p}\right) = \min\left\{R_i\left(\mathbf{p}_{-i}^t\left(h^t\right)\right), \bar{p}\right\} = R_i\left(\mathbf{p}_{-i}^t\left(h^t\right)\right).$$

Hence, the same strategies constitute an equilibrium in the absence of price caps, and thus $\hat{p} \in \mathcal{P}^+$.

(ii) We first establish that $p^M \notin \mathcal{P}^+$ implies $p^* > p^M$. Suppose instead that $p^* < p^M$. To prevent a deviation by firm $i \in \mathcal{N}$, there must exist a continuation payoff $\underline{V}_i$ such that:

$$\pi^s\left(p^*\right) \geq \left(1-\delta\right)\pi\left(R^s\left(p^*\right), p^*\right) + \delta\underline{V}_i. \tag{4}$$

However, note that, for $p < p^M$:

$$\frac{d}{dp}\left\{\pi^s\left(p\right) - \left(1-\delta\right)\pi\left(R^s\left(p\right), p\right)\right\} = \left(\pi^s\right)'\left(p\right) - \left(1-\delta\right)\frac{d}{dp}\left\{\pi\left(R^s\left(p\right), p\right)\right\} > 0,$$

where the inequality stems from the fact that $n\pi^s(p)$ increases with p for $p < p^M$, and that, under (**C**), $\pi_i^s\left(R^s\left(p\right), p\right) = \max_{p_i}\pi_i^s\left(p_i, p\right)$ decreases as p increases when the goods are complements:

$$\frac{d}{dp}\left\{\pi\left(R^s\left(p\right), p\right)\right\} = \left[R^s\left(p\right) - C'\left(D\left(R^s\left(p\right), p\right)\right)\right]\sum_{j\in\mathcal{N}\setminus\{i\}}\partial_j D_i\left(R^s\left(p\right), p, ..., p\right) < 0,$$

where the equality follows from the envelope theorem and the inequality from the margins being positive, from Lemma 2, together with $\partial_j D_i\left(\cdot\right) < 0$ under (**C**).

As $p^* < p^M$ by assumption, it follows from (4) that

$$\pi^s\left(p^M\right) \geq \left(1-\delta\right)\pi\left(R^s\left(p^M\right), p^M\right) + \delta\underline{V}_i.$$

Hence, p^M can be sustained as well, a contradiction.

Therefore, $p^M \notin \mathcal{P}^+$ implies $p^* > p^M$; as $R^s\left(p^M\right) > p^M = \bar{p}$ from Lemma 1, it follows from the proof of Proposition 1 that charging p^M in every period constitutes a continuation equilibrium when firms agree on a price cap $\bar{p} = p^M$. This establishes the first half of part (ii).

Turning to the second part of (ii), suppose that the firms agree on a price cap $\bar{p} = p^M$. As no firm can charge more than p^M, if $C'' \leq 0$, then by charging $p_i = p^M$ in every period, each firm $i \in \mathcal{N}$ can secure at least $\pi^s\left(p^M\right)/\left(1-\delta\right)$; hence, the monopoly outcome constitutes the unique continuation equilibrium.

Price Caps as Welfare-Enhancing Coopetition

Online Appendix (Not for publication)

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A Nash equilibrium

We establish here the existence of a unique Nash equilibrium in the setting considered in Section 2.1:

Lemma 4 (Nash equilibrium) *In the setting considered in Section 2.1, if in addition*

$$\forall j \in \mathcal{N}, \sum_{i \in \mathcal{N} \setminus \{j\}} \left| \partial_j R_i \left(\mathbf{p}_{-i} \right) \right| < 1,$$

then there exists a unique static Nash equilibrium, which is moreover "stable" under the standard tâtonnement process.

Proof. As it is never optimal for firm i to charge a negative price, and $R_i(\cdot)$ is bounded above by some finite B_i (which obviously must satisfy $B_i > 0$), we have, for $\mathbf{p}_{-i} \in \mathbb{R}_+^{n-1}$:

$$R_i \left(\mathbf{p}_{-i} \right) \in \mathcal{C}_i \equiv \left[0, B_i \right],$$

where $\mathcal{C}_i$ is a non-empty compact interval of $\mathbb{R}_+$. Note that, by construction, any Nash equilibrium price vector $\mathbf{p}^N = \left(p_i^N \right)_{i \in \mathcal{N}}$ is such that $p_i^N \in \mathcal{C}_i$.

Next, define $\boldsymbol{\phi}(\mathbf{p}) \equiv \left(\phi_i (\mathbf{p}) \right)_{i \in \mathcal{N}}$, where $\phi_i(\mathbf{p}) = R_i \left(\mathbf{p}_{-i} \right)$. $\boldsymbol{\phi}$ is a contraction mapping from $\mathcal{C} \equiv \mathcal{C}_1 \times ... \times \mathcal{C}_n$ to $\mathcal{C}$, endowed with the ℓ_1 norm: for any $\mathbf{p} \in \mathcal{C}$, $\boldsymbol{\phi}(\mathbf{p}) \in \mathcal{C}$ and, in addition, for any $\mathbf{p}' \in \mathcal{C}$:

$$\begin{aligned}
\left\|\boldsymbol{\phi}\left(\mathbf{p}'\right)-\boldsymbol{\phi}\left(\mathbf{p}\right)\right\| &= \sum_{i\in\mathcal{N}}\left|\phi_i\left(\mathbf{p}'\right)-\phi_i\left(\mathbf{p}\right)\right| \\
&= \sum_{i\in\mathcal{N}}\left|R_i\left(\mathbf{p}'_{-i}\right)-R_i\left(\mathbf{p}_{-i}\right)\right| \\
&= \sum_{i\in\mathcal{N}}\left|\int_0^1 \frac{d}{d\lambda}\left\{R_i\left(\lambda\mathbf{p}'_{-i}+(1-\lambda)\,\mathbf{p}_{-i}\right)\right\}d\lambda\right| \\
&\leq \sum_{i\in\mathcal{N}}\int_0^1\{\sum_{j\in\mathcal{N}\backslash\{i\}}\left|\partial_j R_i\left(\lambda\mathbf{p}'_{-i}+(1-\lambda)\,\mathbf{p}_{-i}\right)\right|\left|p'_j-p_j\right|\}d\lambda \\
&= \int_0^1\{\sum_{j\in\mathcal{N}}[\sum_{i\in\mathcal{N}\backslash\{j\}}\left|\partial_j R_i\left(\lambda\mathbf{p}'_{-i}+(1-\lambda)\,\mathbf{p}_{-i}\right)\right|]\left|p'_j-p_j\right|\}d\lambda \\
&\leq \sum_{j\in\mathcal{N}} k\left|p'_j-p_j\right| \\
&= k\left\|p'-p\right\|,
\end{aligned}$$

where:

$$k=\max_{\mathbf{p}\in\mathcal{C},j\in\mathcal{N}}\sum_{i\in\mathcal{N}\backslash\{j\}}\left|\partial_j R_i\left(\mathbf{p}_{-i}\right)\right|<1.$$

It follows from the Banach fixed point theorem that $\boldsymbol{\phi}(\mathbf{p})$ has a unique fixed point in $\mathcal{C}$, $\mathbf{p}^N$, and that any sequence $\{\mathbf{p}_n\}_{n\in\mathbb{N}}$ satisfying $\mathbf{p}_{n+1}=\boldsymbol{\phi}(\mathbf{x}_n)$ converges to this fixed point. Hence, $\mathbf{p}^N$ is the unique Nash equilibrium of the static game, and it is stable under the standard tâtonnement process. ■

B On Assumption A

B.1 A sufficient condition

We show here that Assumption A holds under the following condition:

Assumption A': For any $i \neq j \in \{1,2\}$ and any prices $p_i \in [0, p_i^N)$ and $p_j > p_j^N$:

$$D_j(p_j,p_i)\,\partial^2_{11}D_j(p_j,p_i) < 2\left(\partial_1 D_j(p_j,p_i)\right)^2 - C''_j\left(D_j(p_j,p_i)\right)\left(\partial_1 D_j(p_j,p_i)\right)^3,$$

and:

$$\begin{aligned}
& D_j(p_j,p_i)\left[\partial_1 D_j(p_j,p_i)\,\partial_2 D_j(p_j,p_i)-D_j(p_j,p_i)\,\partial^2_{12}D_j(p_j,p_i)\right] \\
< \; & D_i(p_i,p_j)\left[2\left(\partial_1 D_j(p_j,p_i)\right)^2-D_j(p_j,p_i)\,\partial^2_{11}D_j(p_j,p_i)\right] \\
& +C''_j(D_j)\left(\partial_1 D_j\right)^2\left[D_j(p_j,p_i)\,\partial_2 D_j(p_j,p_i)-D_i(p_i,p_j)\,\partial_1 D_j(p_j,p_i)\right].
\end{aligned}$$

The first part of this assumption amounts to say that, for any given price of the other firm, the profit of a given firm is concave with respect to the price of that firm. It is satisfied, for instance, when the cost function is weakly convex (i.e., $C_i''(\cdot) \geq 0$) and the elasticity of the inverse of the residual demand is lower than 2, as is the case for the demand functions usually considered in oligopoly theory – in particular, it holds whenever the residual demand is log-concave (or equivalently, that the elasticity of its inverse is lower than 1), or it is exponential (and thus log-convex) with an elasticity higher than 1.

The second part of the assumption holds, for instance, when the goods are close to being perfect complements.[1]

Firm j's best-response, $R_j(p_i)$, is characterized by the first-order condition:

$$\left[R_j(p_i) - C_j'(D_j(R_j(p_i), p_i))\right] \partial_1 D_j(R_j(p_i), p_i) + D_j(R_j(p_i), p_i) = 0,$$

which yields (dropping the argument $(R_j(p_i), p_i)$):

$$R_j'(p_i) = -\frac{\partial_1 D_j \partial_2 D_j - D_j \partial_{12}^2 D_j - C_j''(D_j)(\partial_1 D_j)^2 \partial_2 D_j}{2(\partial_1 D_j)^2 - D_j \partial_{11}^2 D_j - C_j''(D_j)(\partial_1 D_j)^3},$$

where the denominator of the right-hand side is positive under Assumption A'.

Therefore, Assumption A amounts to (dropping the argument $(R_j(p_i), p_i)$):

$$\begin{aligned} &D_i\left[2(\partial_1 D_j)^2 - D_j \partial_{11}^2 D_j - C_j''(D_j)(\partial_1 D_j)^3\right] \\ &\quad > D_j\left[\partial_1 D_j \partial_2 D_j - D_j \partial_{12}^2 D_j - C_j''(D_j)(\partial_1 D_j)^2 \partial_2 D_j\right], \end{aligned}$$

which follows from Assumption A'.

B.2 A counter-example

We provide here an example where Assumption A does not hold.

B.2.1 Setting

There are two goods 1 and 2, produced at no cost by two different firms 1 and 2, and a unit mass of consumers, indexed by x, where x is uniformly distributed on $[0, 1]$:

[1] Remember that Assumption A holds trivially when prices are strategic complements, as is usually the case for substitutable goods. Hence, considering Assumption A' is useful only when prices are strategic substitutes, which in turn is mostly relevant when the goods are complements. But for perfect complements, demands are of the form $D_i(p_i, p_j) = D(p_1 + p_2)$, and Assumption A' then boils down to

$$\begin{aligned} D(\cdot)\left[(D'(\cdot))^2 - D(\cdot) D''(\cdot)\right] \quad < \quad & D(\cdot)\left[2(D'(\cdot))^2 - D(\cdot) D''(\cdot)\right] \\ & + C_j''(\cdot)(D'(\cdot))^2\left[D(\cdot) D'(\cdot) - D(\cdot) D'(\cdot)\right], \end{aligned}$$

or $D(\cdot)(D'(\cdot))^2 > 0$, which is trivially satisfied. By continuity, this strict inequality still holds when the above demands are only slightly modified.

- each consumer is willing to buy $1-\lambda$ units of good 1, and the per-unit valuation of consumer x is $v_1(x)=x$; hence, the demand for good 1 is given by:

$$q_1 = D_1(p_1) = \begin{cases} (1-\lambda)(1-p_1) & \text{if } \; p_1 \leq 1, \\ 0 & \text{if } \; p_1 > 1. \end{cases}$$

- each consumer x is willing to buy λ units of good 2, and the per-unit valuation of consumer x is $v_2(x) = 1 - x + xn_1$, where $n_1 = q_1/(1-\lambda)$ denotes the number of consumers buying good 1; hence, in the relevant range $p_1 \in [0,1]$ (so that $n_1 = 1-p_1$ and $v_2(x) = 1 - xp_1$), the demand for good 2 is given by:

$$q_2 = D_2(p_1, p_2) = \begin{cases} \lambda & \text{if } \quad p_2 \leq 1 - p_1, \\ \lambda \dfrac{1-p_2}{p_1} & \text{if } \; 1-p_1 \leq p_2 \leq 1, \\ 0 & \text{if } \quad p_2 \geq 1. \end{cases}$$

B.2.2 Nash equilibrium

The firms' best-responses are as follows:

- firm 1 always charges the monopoly price for its product:

$$R_1(p_2) = p_1^M = \frac{1}{2}.$$

- in the relevant range $p_1 \in [0,1]$, firm 2 never charges a price below $1-p_1$ (as all consumers are buying at that price) and thus:

$$R_2(p_1) = \begin{cases} 1-p_1 & \text{if } \; p_1 \leq \frac{1}{2}, \\ \dfrac{1}{2} & \text{if } \; p_1 \geq \frac{1}{2}. \end{cases}$$

  Therefore, in equilibrium, firm 1 charges the monopoly price:

$$p_1^N = p_1^M = \frac{1}{2},$$

  leading to:

$$n_1^N = n_1^M = \frac{1}{2},$$

  and thus:

$$q_1^N = q_1^M = \frac{1-\lambda}{2}.$$

  In response, firm 2 charges :

$$p_2^N = R_2\left(p_1^M\right) = \frac{1}{2},$$

leading to $n_2^N = 1$ (that is, all consumers buy good 2) and:

$$q_2^N = \lambda.$$

Consumers thus obtain a surplus equal to:

$$S^N = (1-\lambda)\frac{1}{8} + \lambda \int_0^1 \left[\left(1 - \frac{x}{2}\right) - \frac{1}{2}\right] dx = \frac{1+\lambda}{8},$$

whereas firms' profit are given by:

$$\begin{aligned} \pi_1^N &= \pi_1^M = \frac{1-\lambda}{4}, \\ \pi_2^N &= \lambda p_2^N = \frac{\lambda}{2}. \end{aligned}$$

Industry profit and total welfare are thus respectively equal to:

$$\begin{aligned} \Pi^N &= \frac{1+\lambda}{4}, \\ W^N &= \Pi^N + S^N = \frac{3}{8}(1+\lambda). \end{aligned}$$

B.2.3 Price caps

Suppose now that a price cap $\bar{p} \in [0, 1/2]$ is imposed on firm 1: that is, firm 1's price must satisfy $p_1 \leq \bar{p}$. As firm 1's profit is quasi-concave in its price and maximal for $p_1 = p_1^M = 1/2 \geq \bar{p}$, in equilibrium firm 1 finds it optimal to charge a price just satisfying the constraint; that is, it charges:

$$\hat{p}_1(\bar{p}) = \bar{p},$$

leading to

$$\hat{n}_1(\bar{p}) = 1 - \bar{p}$$

and

$$\hat{q}_1(\bar{p}) = (1-\lambda)(1-\bar{p}).$$

In response, firm 2 sells to all consumers (that is, $n_2 = 1$ and $q_2 = \lambda$) by charging a price:

$$\hat{p}_2(\bar{p}) = R_2(\bar{p}) = 1 - \bar{p}.$$

Firms' profits are now given by:

$$\begin{aligned} \hat{\pi}_1(\bar{p}) &= (1-\lambda)\bar{p}(1-\bar{p})\left(\leq \pi_1^N\right), \\ \hat{\pi}_2(\bar{p}) &= \lambda \hat{p}_2(\bar{p}) = \lambda(1-\bar{p}). \end{aligned}$$

If transfers are feasible, the firms will set the price cap so as to maximize the industry profit, equal to:

$$\hat{\Pi}(\bar{p}) = (1-\lambda)\,\bar{p}\,(1-\bar{p}) + \lambda\,(1-\bar{p}) = (1-\bar{p})\,[\lambda + (1-\lambda)\,\bar{p}]\,,$$

leading to:

$$\bar{p}^* = \max\left\{\frac{1}{2}\frac{1-2\lambda}{1-\lambda}, 0\right\}.$$

In particular, $\bar{p}^* = 0$ for $\lambda > 1/2$; in that case, firm 1 sets its price to 0: $\hat{p}_1^* = 0$, whereas firm 2 extracts all the surplus generated by its good: $\hat{p}_2^* = 1$. As a result:

- Price caps enable the firms to increase their joint profits:[2]

$$\hat{\Pi}^* = (1-\lambda)\times 0 + \lambda \times 1 = \lambda > \Pi^N = \frac{1+\lambda}{4}.$$

- This harms consumers when λ is large enough:

$$\hat{S}^* = (1-\lambda)\times\frac{1}{2} + \lambda\times 0 = \frac{1-\lambda}{2},$$

  which is lower than $S^N = (1+\lambda)/8$ whenever $\lambda > 3/5\,(> 1/2)$.[3]

C Multi-product oligopoly under (SC)

We extend here the analysis to multi-product oligopolies where prices are strategic complements (both within and across firms). We show that, under that assumption, price caps cannot generate higher equilibrium prices (regardless of whether goods are complements or substitutes). It follows that price caps can only benefit consumers, and are useful for suppliers of complements, but not for competitors offering substitutes.

C.1 Setting

We consider a multi-product firm oligopoly setting with $n \geq 2$ multi-product firms, indexed by $i \in \mathcal{N} \equiv \{1, ..., n\}$, each producing m_i products, indexed by $j \in \mathcal{M}_i \equiv \{1, ..., m_i\}$; there are thus in total $m \equiv \sum_{i\in\mathcal{N}} m_i$ prices. Let $C_i(\mathbf{q}_i)$, where $\mathbf{q}_i = \left(q_i^j\right)_{j\in\mathcal{M}_i}$, denote firm i's cost of producing each good $j \in \mathcal{M}_i$ in quantity q_i^j, and $\mathbf{D}_i(\mathbf{p}) = (D_i^1(\mathbf{p}), ..., D_i^{m_i}(\mathbf{p}))$ denote the demand for these goods, as a function of the vector of prices $\mathbf{p} = (\mathbf{p}_i)_{i\in\mathcal{N}} \in \mathbb{R}_+^m$, where $\mathbf{p}_i = \left(p_i^j\right)_{j\in\mathcal{M}_i} \in \mathbb{R}_+^{m_i}$ denotes the vector of firm i's

[2] The inequality holds whenever $\lambda > 1/3$, which is implied by $\lambda > 1/2$.

[3] It can however be checked that total welfare is increased:

$$\hat{W}^* = (1-\lambda)\times\frac{1}{2} + \lambda\times 1 = \frac{1+\lambda}{2} > W^N = \frac{3}{8}(1+\lambda)\,.$$

prices. We will assume that, for $i \in \mathcal{N}$, $\mathbf{D}_i(\cdot)$ and $C_i(\cdot)$ are both C^2 and that, for every $i \in \mathcal{N}$:

- the profit function

$$\pi_i(\mathbf{p}) \equiv \sum_{j \in \mathcal{M}_i} p_i^j D_i^j(\mathbf{p}) - C_i(\mathbf{D}_i(\mathbf{p}))$$

  is strictly quasi-concave in $\mathbf{p}_i$;

- for every $j \in \mathcal{M}_j$, the "product-by-product" best-response function[4]

$$r_i^j\left(\mathbf{p}_i^{\mathcal{M}_i \setminus \{j\}}, \mathbf{p}_{-i}\right) \equiv \arg\max_{p_i^j} \pi_i\left(p_i^j, \mathbf{p}_i^{\mathcal{M}_i \setminus \{j\}}, \mathbf{p}_{-i}\right),$$

  where $\left(\mathbf{p}_i^{\mathcal{M}_i \setminus \{j\}}, \mathbf{p}_{-i}\right)$ denotes the vector of all prices but p_i^j, is well-defined and bounded above.

Remark: Note that we consider here firm i's price decision for one of its products, taking as given not only the other firms' prices, but also firm i's own prices for its other products. Furthermore, as the best-response is bounded, it is interior and thus, given the strict quasi-concavity assumption, uniquely characterized by the first-order condition $\partial_{p_i^j} \pi_i\left(r_i^j\left(\mathbf{p}_i^{\mathcal{M}_i \setminus \{j\}}, \mathbf{p}_{-i}\right), \mathbf{p}_i^{\mathcal{M}_i \setminus \{j\}}, \mathbf{p}_{-i}\right) = 0$.

We still focus on strategic complementarity, both "within firms" and "across firms":

(**SC**) *Strategic complementarity:* for every $i \in \mathcal{N}$ and every $j \in \mathcal{M}_i$, $r_i^j(\cdot)$ strictly increases in p_h^k for any $h \in \mathcal{N}$ and any $k \in \mathcal{M}_h$ such that $(h, k) \neq (i, j)$.

Finally, we assume again that there exists a unique Nash equilibrium, and denote by $\mathbf{p}^N = \left(\mathbf{p}_i^N\right)_{i \in \mathcal{N}}$ the equilibrium prices.

C.2 Price caps

Suppose now that each firm $i \in \mathcal{N}$ faces a price cap $\bar{p}_i^j$ for each product j in $\mathcal{M}_i$. Any resulting equilibrium price vector $\hat{\mathbf{p}} = (\mathbf{p}_i)_{i \in \mathcal{N}}$ satisfies $\hat{\mathbf{p}}_i = \bar{\mathbf{R}}_i(\hat{\mathbf{p}}_{-i}; \bar{\mathbf{p}}_i)$ for $i \in \mathcal{N}$, where:

$$\bar{\mathbf{R}}_i(\mathbf{p}_{-i}; \bar{\mathbf{p}}_i) = \left(\bar{R}_i^j(\mathbf{p}_{-i}; \bar{\mathbf{p}}_i)\right)_{j \in \mathcal{M}_i} \equiv \arg\max_{\mathbf{p}_i \leq \bar{\mathbf{p}}_i} \pi_i(\mathbf{p}_i, \mathbf{p}_{-i})$$

denotes firm i's best-response, constrained by the price caps it faces.

We first show that multiple price deviations cannot be more profitable for a firm than isolated ones, so that a firm best responds to its rivals if and only if each of its prices best responds individually to all other prices, including its own. That is, letting

$$\bar{r}_i^j\left(\mathbf{p}_i^{\mathcal{M}_i \setminus \{j\}}, \mathbf{p}_{-i}; \bar{p}_i^j\right) \equiv \arg\max_{p_i^j \leq \bar{p}_i^j} \pi_i\left(p_i^j, \mathbf{p}_i^{\mathcal{M}_i \setminus \{j\}}, \mathbf{p}_{-i}\right)$$

[4] In what follows, $\mathbf{p}^{\mathcal{S}}$ denotes the projection of the vector $\mathbf{p}$ on the subset $\mathcal{S}$; that is: $\mathbf{p}^{\mathcal{S}} = \left(p^j\right)_{j \in \mathcal{S}}$.

denote firm i's constrained product-by-product best-response for good $j \in \mathcal{M}_i$, given all other prices (including its own) and the price cap it faces for that good, we have:

Lemma 5 (constrained best-responses) *For any $i \in \mathcal{N}$, rivals' prices $\mathbf{p}_{-i}$ and price caps $\bar{\mathbf{p}}_i = \left(\bar{p}_i^j\right)_{j\in\mathcal{M}_i}$, firm i's constrained best-response function $\bar{\mathbf{R}}_i\left(\mathbf{p}_{-i};\bar{\mathbf{p}}_i\right)$ can be characterized as the unique fixed point in $\mathcal{M}_i$ of firm i's constrained product-by-product best-responses; that is:*

$$\hat{\mathbf{p}}_i = \bar{\mathbf{R}}_i\left(\mathbf{p}_{-i};\bar{\mathbf{p}}_i\right) \Longleftrightarrow \left\{\hat{p}_i^j = \bar{r}_i^j\left(\hat{\mathbf{p}}_i^{\mathcal{M}_i\setminus\{j\}},\mathbf{p}_{-i};\bar{p}_i^j\right) \text{ for every } j\in\mathcal{M}_i\right\}.$$

Proof. Consider firm i, for given price caps $\bar{\mathbf{p}}_i = \left(\bar{p}_i^j\right)_{j\in\mathcal{M}_i}$ and given rivals' prices $\mathbf{p}_{-i}$. Obviously, each price in firm i's (constrained) best-response is also a (constrained) product-by-product best-response: $\hat{\mathbf{p}}_i = \bar{\mathbf{R}}_i\left(\mathbf{p}_{-i};\bar{\mathbf{p}}_i\right)$ implies $p_i^j = \bar{r}_i^j\left(\mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}},\mathbf{p}_{-i};\bar{p}_i^j\right)$ for every $i\in\mathcal{N}$ and every $j\in\mathcal{M}_i$; uniqueness then follows from the strict quasi-concavity of the profit function. We now show that, conversely, any fixed point of the (constrained) product-by-product best-responses constitutes a best-response for firm i.

Thus, consider a price vector $\hat{\mathbf{p}}_i = \left(\hat{p}_i^j\right)_{j\in\mathcal{M}_i}$ satisfying $\hat{p}_i^j = \bar{r}_i^j\left(\hat{\mathbf{p}}_i^{\mathcal{M}_i\setminus\{j\}},\mathbf{p}_{-i};\bar{p}_i^j\right)$ for every $j\in\mathcal{M}_i$, and suppose that $\hat{\mathbf{p}}_i \neq \check{\mathbf{p}}_i \equiv \bar{\mathbf{R}}_i\left(\mathbf{p}_{-i};\bar{\mathbf{p}}_i\right)$. By construction, both $\hat{\mathbf{p}}_i \leq \bar{\mathbf{p}}_i$ and $\check{\mathbf{p}}_i \leq \bar{\mathbf{p}}_i$, and so $\varepsilon\check{\mathbf{p}}_i + (1-\varepsilon)\hat{\mathbf{p}}_i \leq \bar{\mathbf{p}}_i$ for any $\varepsilon\in(0,1)$. Furthermore, as $\hat{\mathbf{p}}_i$ consists of product-by-product best responses, for every $j\in\mathcal{M}_i$ either $\partial_{p_i^j}\pi_i\left(\hat{\mathbf{p}}_i,\mathbf{p}_{-i}\right)=0$ (if $\hat{p}_i^j = r_i^j\left(\hat{\mathbf{p}}_i^{\mathcal{M}_i\setminus\{j\}},\mathbf{p}_{-i}\right)$, i.e., the price cap $\bar{p}_i^j$ is not binding for $\hat{\mathbf{p}}_i$) or $\partial_{p_i^j}\pi_i\left(\hat{\mathbf{p}}_i,\mathbf{p}_{-i}\right)\left(\check{p}_i^j - \hat{p}_i^j\right) < 0$; furthermore, the latter must hold for some $j\in\mathcal{M}_i$: otherwise, $\hat{\mathbf{p}}_i$ would be firm i's unconstrained best-response (i.e., $\hat{\mathbf{p}}_i = \mathbf{R}_i\left(\mathbf{p}_{-i}\right) \leq \bar{\mathbf{p}}_{-i}$), implying $\check{\mathbf{p}}_i = \mathbf{R}_i\left(\mathbf{p}_{-i}\right) = \hat{\mathbf{p}}_i$, a contradiction. Therefore, for ε positive but small we have:

$$\pi_i\left(\varepsilon\check{\mathbf{p}}_i + (1-\varepsilon)\hat{\mathbf{p}}_i,\mathbf{p}_{-i}\right) - \pi_i\left(\hat{\mathbf{p}}_i,\mathbf{p}_{-i}\right) \simeq \sum_{j\in\mathcal{M}_i}\partial_{p_i^j}\pi_i\left(\hat{\mathbf{p}}_i,\mathbf{p}_{-i}\right)\varepsilon\left(\check{p}_i^j - \hat{p}_i^j\right) < 0.$$

Hence, $\pi_i\left(\hat{\mathbf{p}}_i + \varepsilon\left(\check{\mathbf{p}}_i - \hat{\mathbf{p}}_i\right),\mathbf{p}_{-i}\right) \leq \pi_i\left(\hat{\mathbf{p}}_i,\mathbf{p}_{-i}\right) < \pi_i\left(\check{\mathbf{p}}_i,\mathbf{p}_{-i}\right)$, contradicting the strict quasi-concavity of the profit function π_i.[5] ■

Lemma 5 allows us to treat the present n-player game, where each firm $i\in\mathcal{N}$ has m_i products, as a $m-$player game among single-product firms. Building on this, we now show that price caps cannot be used to raise equilibrium prices:

Proposition 8 (incidence of price caps for multi-product firms under (SC)) *With multi-product firms under* (**SC**) *and for any vector $\bar{\mathbf{p}} = \left(\bar{\mathbf{p}}_i\right)_{i\in\mathcal{N}}$ of price caps, there exists a unique Nash equilibrium, and the equilibrium prices weakly increase with the price cap vector. Therefore: (i) for any vector of price caps $\bar{\mathbf{p}}$, consumers are weakly better off*

[5] We use here the characterizing property that a function $f(x)$ is strictly quasi-concave if and only if, for any $x\neq y$ and $\lambda\in(0,1)$: $f(\lambda x + (1-\lambda)y) > \min\{f(x), f(y)\}$.

under $G_{\bar{\mathbf{p}}}$ than under G_∞; and (ii) in $\mathcal{G}$, it is optimal for the competition authority to allow price caps.

Proof. Fix a vector of price caps $\bar{\mathbf{p}} = (\bar{\mathbf{p}}_i)_{i\in\mathcal{N}}$. From Lemma 5, a price vector $\hat{\mathbf{p}} = (\hat{\mathbf{p}}_i)_{i\in\mathcal{N}} = \left(\left(\hat{p}_i^j\right)_{j\in\mathcal{M}_i}\right)_{i\in\mathcal{N}}$ constitutes a Nash equilibrium for these price caps if and only if it constitutes a Nash equilibrium of the $m-$player game in which each price p_i^j is chosen by a distinct player (subject to the price cap $\bar{p}_i^j$) so as to maximize the profit $\pi_i\left(p_i^j, \mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}, \mathbf{p}_{-i}\right)$, taking as given firm i's other prices, $\mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}$, as well as the other firms' prices, $\mathbf{p}_{-i}$. In the absence of price caps, this player's behavior is given by the best-response function $r_i^j\left(\mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}, \mathbf{p}_{-i}\right)$, which is bounded above by some B_i^j. Without loss of generality, we can thus restrict the price p_i^j to belong to $S_i^j = \left[0, B_i^j\right]$, and can also restrict attention to price caps such that each $\bar{p}_i^j$ belongs to S_i^j (as higher price caps would have no effect). From strict quasi-concavity, when facing the price cap $\bar{p}_i^j$ the constrained best-response can be expressed as:

$$\bar{r}_i^j\left(\mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}, \mathbf{p}_{-i}; \bar{p}_i^j\right) = \arg\max_{p_i^j\leq\bar{p}_i^j} \pi_i\left(p_i^j, \mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}, \mathbf{p}_{-i}\right) = \min\left\{r_i^j\left(\mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}, \mathbf{p}_{-i}\right), \bar{p}_i^j\right\},$$

and is thus increasing in all arguments. Let $\mathbf{S} = \left(\left(S_i^j\right)_{j\in\mathcal{M}_i}\right)_{i\in\mathcal{N}}$ denote the (bounded) relevant set of prices and consider the best-response function $\bar{\mathbf{r}}\left(\mathbf{p};\bar{\mathbf{p}}\right) : \mathbf{S}\times\mathbf{S}\to\mathbf{S}$, where each price p_i^j is given by the constrained best-response $\bar{r}_i^j\left(\mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}, \mathbf{p}_{-i}; \bar{p}_i^j\right)$. Knaster-Tarski's Lemma ensures that, for each $\bar{\mathbf{p}}$, there exists a pure-strategy Nash equilibrium of this $m-$player game. To show that this equilibrium is unique, suppose instead that there are two equilibria, $\hat{\mathbf{p}}$ and $\hat{\mathbf{p}}'$; one of them, say $\hat{\mathbf{p}}'$, must have a lower price for at least one product. Applying the same argument as in the proof of Proposition 2 then leads to a contradiction.[6]

That the unique equilibrium prices increase with every price cap follows from Theorem 2.5.2 of Topkis (1998). The last conclusion follows from the fact that the unconstrained Nash equilibrium prices is sustainable through high enough price caps (e.g., such that $\bar{p}_i^j \geq B_i^j$ for every $i\in\mathcal{N}$ and every $j\in\mathcal{M}_i$). ■

Remark: bundling. The above analysis carries over when firms engage in pure or mixed bundling. For instance, consider a case of *pure bundling* in which firm i offers goods $j\in\mathcal{B}_i\subset\mathcal{M}_i$ as a bundle, and only as a bundle. Let p_i^B denote the price charged for the bundle, and $D_i^B\left(p_i^B, \mathbf{p}_i^{\mathcal{M}_i\setminus\mathcal{B}_i}, \mathbf{p}_{-i}\right)$ the demand for the bundle, $D_i^j\left(p_i^B, \mathbf{p}_i^{\mathcal{M}_i\setminus\mathcal{B}_i}, \mathbf{p}_{-i}\right)$ denote the demand for every other good offered by firm i, for $j\in\mathcal{M}_i\setminus\mathcal{B}_i$, and $D_h^k\left(p_i^B, \mathbf{p}_i^{\mathcal{M}_i\setminus\mathcal{B}_i}, \mathbf{p}_{-i}\right)$ the demand for the other firms' products, for $h\in\mathcal{N}\setminus\{i\}$ and $k\in\mathcal{M}_h$; firm i's cost function remains the same as before, with the

[6]See Section C.1.2 of the Appendix, using firm $\hat{\imath}$ for which the difference between $\hat{p}_i$ and $\hat{p}_i$ is the largest, and noting that strict quasi-concavity and $\hat{p}_{\hat{\imath}}' < \hat{p}_{\hat{\imath}}\,(\leq\bar{p}_{\hat{\imath}})$ together imply $\hat{p}_{\hat{\imath}}' = R_{\hat{\imath}}\left(\hat{p}_{-\hat{\imath}}'\right)$ and $\hat{p}_{\hat{\imath}} \leq R_{\hat{\imath}}\left(\hat{p}_{-\hat{\imath}}\right)$.

caveat that each unit of the bundle requires the production of one unit of each good $j \in \mathcal{B}_i$. The previous analysis then remains valid as long as the reaction functions derived from these adjusted demand and cost functions exhibit strategic complementarity. In the case of mixed bundling, where firm i offers the bundle as well as each product $j \in \mathcal{B}_i$ on a stand-alone basis, a similar reasoning applies, interpreting the bundle as an additional good in firm i's product set.

C.3 Firms' incentives

It follows from the above that, again, firms have an incentive to agree on price caps when they offer complements, but not when they offer substitutes:

Corollary 1 **(multi-product firms' incentives to set price caps under** (SC)**)** *Let $\mathcal{P}$ denote the set of prices that are sustainable through price caps.*

(i) If all goods are substitutes, then price caps cannot increase the profit of any firm; that is, $\hat{\mathbf{p}} \in \mathcal{P}$ implies $\pi_i(\hat{\mathbf{p}}) \leq \pi_i(\mathbf{p}^N)$ for every $i \in \mathcal{N}$.

(ii) If instead all goods are complements, then price caps can be used to increase all firms' profits; that is, there exists $\hat{\mathbf{p}} \in \mathcal{P}$ such that $\pi_i(\hat{\mathbf{p}}) > \pi_i(\mathbf{p}^N)$ for every $i \in \mathcal{N}$.

Proof. If goods are all substitutes, then for any $\hat{\mathbf{p}} \in \mathcal{P}$ and any $i \in \mathcal{N}$ we have:

$$\pi_i(\hat{\mathbf{p}}) \leq \max_{\mathbf{p}_i} \pi_i(\mathbf{p}_i, \hat{\mathbf{p}}_{-i}) \leq \max_{\mathbf{p}_i} \pi_i(\mathbf{p}_i, \mathbf{p}_{-i}^N) = \pi_i(\mathbf{p}^N),$$

where the first inequality reflects the fact that firm i may be constrained by its price caps $\bar{p}_i$, and the second inequality stems from the fact that price caps can only sustain prices that are lower than $\mathbf{p}^N$.

If goods are all complements, the reasoning used in the proof of Proposition 4 extends to the case of an oligopoly: starting from the Nash equilibrium prices $\mathbf{p}^N$, reducing all prices by a small amount ε increases all firms' profits, as firms' margins are positive from Lemma 2, and reducing one firm's price has only a second-order effect on the profit of that firm, and a first-order positive effect on the other firms' profits. To conclude the argument, it suffices to show that the new price vector, $\hat{\mathbf{p}}(\varepsilon) = \left((\hat{p}_i^j(\varepsilon))_{j \in \mathcal{M}_i} \right)_{i \in \mathcal{N}}$, where $\hat{p}_i^j(\varepsilon) = p_i^{jN} - \varepsilon$, belongs to $\mathcal{P}$; indeed, we have, for $i \in \mathcal{N}$ and $j \in \mathcal{M}_i$:

$$R_i^j(\hat{\mathbf{p}}_{-i}(\varepsilon)) - \hat{p}_i^j(\varepsilon) = \int_0^\varepsilon [1 - \sum_{h \in \mathcal{N} \setminus \{i\}} \sum_{k \in \mathcal{M}_h} \partial_{p_h^k} R_i^j(\hat{\mathbf{p}}_{-i}(x))] dx > 0,$$

where the inequality stems from (1). From Propositions 1 and 2, $\hat{\mathbf{p}}(\varepsilon)$ is the unique equilibrium of $G_{\hat{\mathbf{p}}(\varepsilon)}$. ■

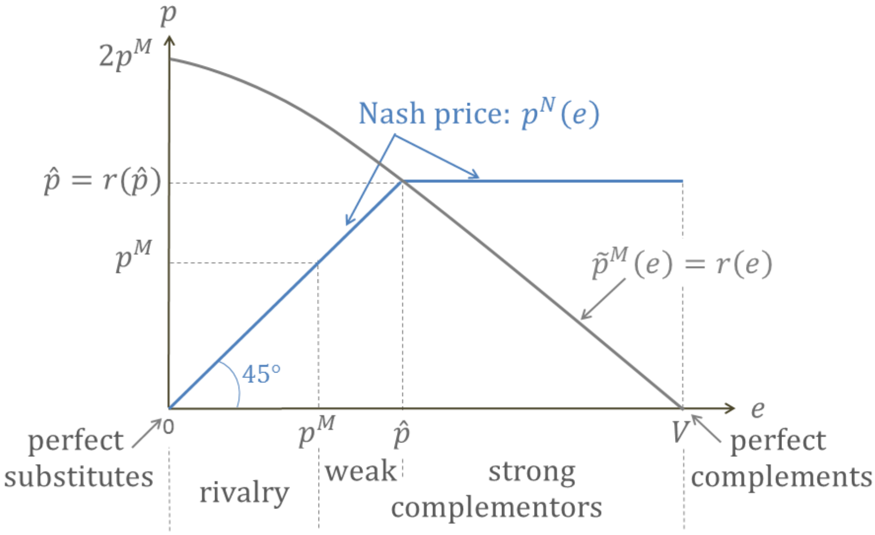

Figure 6: Nash equilibrium in the technology model

D Nash equilibrium in the technology adoption model

Consider, in the technology adoption environment described in Section 4.2, the static game in which the two firms simultaneously set their prices. Without loss of generality we can require prices to belong to the interval $[0, V]$.

From the discussion presented in the text, firm i's best response to firm j setting price $p_j \leq e$ is to set

$$p_i = \min\{e, r(p_j)\},$$

where

$$r(p) = \arg\max_p \{p_i D(p_i + p)\}$$

satisfies

$$-1 < r'(p_j) < 0$$

and has a unique fixed point $\hat{p} > p^M$.

When instead $p_j > e$, then firm i faces no demand if $p_i > p_j$ (as users buy only the lower-priced license), and faces demand $D(p_i + e)$ if $p_i < p_j$. Competition then drives prices down to $p_1 = p_2 = e$. Hence, the Nash equilibrium is unique and such that both firms charge $p^N \equiv \min\{e, \hat{p}\}$.

Figure 6 summarizes this analysis.

E Hotelling with club effects

E.1 Model

Consider the following symmetric duopoly setting, in which for notational simplicity costs are zero (i.e., $C_i(q_i) = 0$ for $i = 1, 2$) and:

- As in the standard Hotelling model, a unit mass of consumers is uniformly distributed along a unit-length segment; the two firms are located at the two ends of the segment, and consumers face a constant transportation cost per unit of distance, which is here normalized to 1.
- Unlike in the Hotelling model, however, consumers enjoy club effects: their gross surplus is $v + \sigma Q$, where $v > 0$, $\sigma \in (0, 1/2)$ reflects the magnitude of these positive externalities and $Q = q_1 + q_2$ denotes the total number of consumers.

For low enough prices, the entire market is covered (i.e., $Q = 1$), and as long as prices are not too asymmetric, each firm faces the classic Hotelling demand given by:

$$D_i^H(p_1, p_2) \equiv \frac{1 - p_i + p_j}{2}.$$

This case arises as long as $|p_1 - p_2| < 1$ (to ensure that the market is shared) and $v + \sigma \geq (1 + p_1 + p_2)/2$ (to ensure that the market is covered). The two goods are then substitutes: $\partial_2 D_i = 1/2 > 0$.

By contrast, for high enough prices, firms are local monopolies and each firm i faces a demand satisfying:

$$q_i = v + \sigma Q - p_i,$$

where now $Q = q_1 + q_2 < 1$. As long as both firms remain active, their demands are then given by:

$$D_i^m(p_1, p_2) \equiv \frac{v - (1 - \sigma) p_i - \sigma p_j}{1 - 2\sigma}.$$

This case arises as long as $Q < 1$ and $q_i \geq 0$, which amounts to $p_1 + p_2 > 2v - (1 - 2\sigma)$ and $(1 - \sigma) p_i + \sigma p_j \leq v$ for $i \neq j \in \{1, 2\}$. The goods are then complements: $\partial_j D_i = -\sigma/(1 - 2\sigma) < 0$.

E.2 Best-responses

We now study firm i's best-response to the price p charged by firm j. For the sake of exposition, we will focus on the range $p \in [0, v]$.[7]

[7] It can be checked that the Nash prices and the monopoly prices lie indeed in this range.

Consider first the case where firm j charges a price $p_j \in [0, v-(1-\sigma)]$, so that it would serve the entire market if firm i were to charge a prohibitive price.[8] In this case, the market remains fully covered whatever price firm i chooses to charge, and it is optimal for firm i to obtain a positive share of that market. Firm i will thus seek to maximize $p_i D_i^H(p_1, p_2)$ and choose to charge:

$$p_i = R^H(p_j) \equiv \arg\max_{p_i} \left\{ p_i D_i^H(p_i, p_j) \right\} = \frac{1+p_j}{2}.$$

Consider now the case where firm j charges a price $p_j \in (v-(1-\sigma), v]$; the market is then covered only if firm i charges a sufficiently low price, namely, if

$$p_i \leq \tilde{p}(p_j) \equiv 2v - (1-2\sigma) - p_j.$$

In this range, firm i seeks to maximize $p_i D_i^H(p_1, p_2)$, which is maximal for $p_i = R^H(p_j)$. If instead firm i chooses to charge a higher price, the firms are local monopolies; furthermore, as $p_j \leq v$, firm j's market share remains positive, whatever price firm i chooses to charge.[9] As it is optimal for firm i to maintain a positive market share as well, the demand will be given by $D_i^m(p_i, p_j)$. In this range, firm i will thus seek to maximize $p_i D_i^m(p_i, p_j)$, which is maximal for:

$$p_i = R^m(p_j) \equiv \arg\max_{p_i} \{ p_i D_i^m(p_i, p_j) \} = \frac{v - \sigma p_j}{2(1-\sigma)}.$$

Note that:

- The profit functions $p_i D_i^H(p_1, p_2)$ and $p_i D_i^m(p_1, p_2)$ are concave with respect to p_i in their respective relevant ranges.
- $\tilde{p}(v-(1-\sigma)) = v + \sigma > R^m(v-(1-\sigma)) = R^H(v-(1-\sigma)) = (v+\sigma)/2$.
- $\tilde{p}' = -1 < (R^m)' = -\sigma/2(1-\sigma) < 0 < (R^H)' = 1/2$.

It follows that $R^H(p_j) > R^m(p_j)$ in the range $p_j \in (v-(1-\sigma), v]$, and $\tilde{p}(p_j) > R^H(p_j)$ in the beginning of that range. This, in turn, implies that firm i's best response is given by:

$$p_i = R(p_j) \equiv \min \left\{ R^H(p_j), \max\{\tilde{p}(p_j), R^m(p_j)\} \right\}.$$

More precisely:

- If $\tilde{p}(v) \geq R^H(v)$, which amounts to $v \geq 3 - 4\sigma$, then $(R^m(p_j) <) \, R^H(p_j) < \tilde{p}(p_j)$, and thus $R(p_j) = R^H(p_j)$;

[8] To see this, note that under full participation (i.e., $Q = 1$), the consumer who the farthest away from firm j is willing to pay $v + \sigma - 1 \geq p_j$ for firm j's product.

[9] To see this, note that even if $q_i = 0$, the consumer who's the nearest to firm j is willing to pay at least $v \geq p$ for firm j's product.

- If $R^H(v) > \tilde{p}(v) \geq R^m(v)$, which amounts to $3 - 4\sigma > v \geq 2 - 4\sigma$, then there exists $\check{p} = 4(v+\sigma)/3 - 1$ such that $R^H(\check{p}) = \tilde{p}(\check{p})$, and thus:
  - For $p_j \leq \check{p}$, we have again $(R^m(p_j) <) R^H(p_j) \leq \tilde{p}(p_j)$; hence, $R(p_j) = R^H(p_j)$;
  - For $p_j > \check{p}$, $R^m(p_j) < \tilde{p}(p_j) < R^H(p_j)$; hence, $R(p_j) = \tilde{p}(p_j)$;
- Finally, if $R^m(v) > \tilde{p}(v)$, which amounts to $2 - 4\sigma > v$, then there also exists $\mathring{p} = [(3-4\sigma)v - 2(1-\sigma)(1-2\sigma)] / (2-3\sigma)$ such that $R^m(\mathring{p}) = \tilde{p}(\mathring{p})$, and thus:
  - For $p_j \leq \check{p}$, we still have $(R^m(p_j) <) R^H(p_j) \leq \tilde{p}(p_j)$; hence, $R(p_j) = R^H(p_j)$;
  - For $\check{p} < p_j < \mathring{p}$, we still have $R^m(p_j) < \tilde{p}(p_j) < R^H(p_j)$; hence, $R(p_j) = \tilde{p}(p_j)$;
  - For $p_j \geq \mathring{p}$, we now have $\tilde{p}(p_j) \leq R^m(p_j) < R^H(p_j)$; hence, $R(p_j) = R^m(p_j)$.

It follows that for low prices (namely, for $p_j < \min\{\check{p}, v\}$), goods are substitutes and prices are strategic complements: $\partial_j D_i = \partial_j D_i^H > 0$ and $R' = \left(R^H\right)' > 0$. By contrast, whenever $v < 2 - 4\sigma$ (implying $\mathring{p} < v$), then for high enough prices (namely, for $p_j > \mathring{p}$), goods are complements and prices are strategic substitutes: $\partial_j D_i = \partial_j D_i^m < 0$ and $R' = (R^m)' < 0$.

E.3 Monopoly prices

Conditional on covering the entire market, it is optimal to raise prices until the marginal consumer is indifferent between buying or not: indeed, starting from a situation where $q_1 + q_2 = 1$, and the marginal consumer strictly prefers buying, increasing both prices by the same amount does not affect the firms quantities, q_1 and q_2, but increase both of their margins. Hence, without loss of generality, we can focus on situations such that, for some $q_1 \in [0,1]$: $q_2 = 1 - q_1$ and $p_i = v + \sigma - q_i$ for $i = 1, 2$. Total profit, as a function of q_1, is then given by:

$$\Pi^H(q_1) = (v + \sigma - q_1) q_1 + [v + \sigma - (1 - q_1)](1 - q_1) = v + \sigma - q_1^2 - (1 - q_1)^2.$$

This profit is concave in q_1 and reaches its maximum for $q_1 = 1/2$, where it is equal to

$$\Pi_H^M \equiv v + \sigma - \frac{1}{2}.$$

Alternatively, the firms may choose to cover only part of the market, in which case they are both local monopolies. For any $Q = q_1 + q_2 \in [0, 1)$ and any $q_1 \in [0, Q]$, the associated quantity for firm 2 is then $q_2 = Q - q_1$ and the associated prices are $p_i = v + \sigma Q - q_i$; the resulting industry profit is therefore given by:

$$\Pi = (v + \sigma Q - q_1) q_1 + [v + \sigma Q - (Q - q_1)](Q - q_1) = (v + \sigma Q) Q - q_1^2 - (Q - q_1)^2.$$

For any $Q \in [0,1)$, it is thus optimal to choose $q_1 = q_2 = Q/2$, which yields an industry profit equal to:

$$\Pi^m(Q) = (v + \sigma Q)\, Q - \frac{Q^2}{2}.$$

This profit is concave in Q and coincides with Π_H^M for $Q = 1$; furthermore, ignoring the constraint $Q \leq 1$, it is maximal for $Q = v/(1-2\sigma)$. Therefore, the monopoly outcome is $q_i^M = q^M = Q^M/2$, where:

$$Q^M = \min\left\{\frac{v}{1-2\sigma}, 1\right\},$$

and the associated prices are $p_1^M = p_2^M = p^M$, where:

$$\begin{aligned} p^M &= v + \sigma Q^M - q^M = v - (1-2\sigma)\, q^M \\ &= \begin{cases} \frac{v}{2} & \text{if } \ v < 1-2\sigma \\ v - \left(\frac{1}{2} - \sigma\right) & \text{if } \ v \geq 1-2\sigma \end{cases} = \max\left\{\frac{v}{2}, v - \left(\frac{1}{2} - \sigma\right)\right\}. \end{aligned}$$

E.4 Nash equilibrium

We first note that, in equilibrium, both firms must obtain a positive market share. Starting from a situation where all consumers are inactive, each firm could profitably attract some consumers by charging a price slightly below v. Furthermore, in a candidate equilibrium in which only one firm attracts consumers, this firm must charge a non-negative price, otherwise it would have an incentive to raise its price in order to avoid making a loss; but then, the other firm could profitably deviate, as charging a price only slightly higher would attract some consumers.

It can also be checked that, when one firm charges $p \leq v$, then the profit of the other firm is globally quasi-concave in the relevant price range $[0, v+\sigma]$. To see this, note that the profit functions $p_i D_i^H(p_i, p_j)$ and $p_i D_i^m(p_i, p_j)$ are both strictly concave in their relevant ranges; the conclusion then follows from the fact that, at the boundary between these two ranges, $D_i^H(p_1, p_2) = D_i^m(p_1, p_2)$ and $\partial_i D_i^H(p_1, p_2) > \partial_i D_i^m(p_1, p_2)$.

Suppose first that, at the Nash equilibrium prices, the market is fully covered *and* the marginal consumer strictly prefers buying (from either firm) to not buying. As both firms must be active, their demands are given by $D_i^H(\cdot)$, and remain so around the Nash prices. Therefore, their best-responses are given by $R^H(\cdot)$. It follows that the Nash equilibrium price is then symmetric, with both firms charging the standard Hotelling price

$$p^H = 1.$$

Conversely, both firms charging p^H is indeed an equilibrium if and only if the consumer that is at equal distance from the two firms (thus facing a transportation cost equal to $1/2$) then strictly prefers to be active, that is, if and only if $p^H + 1/2 < v + \sigma Q|_{Q=1}$, which amounts to:

$$v > \frac{3}{2} - \sigma \left(> 1 = p^H\right).$$

When instead the market is not fully covered at the Nash equilibrium prices, firms' best-responses are given by $R^m(\cdot)$. It follows that the Nash equilibrium price is again symmetric, with both firms charging

$$p^m = \frac{v}{2-\sigma} \left(< v\right).$$

Conversely, both firms charging p^m is indeed an equilibrium if and only if the market is not fully covered at these prices, that is, if and only if $p^m + 1/2 > v + \sigma Q|_{Q=1}$, which amounts to:

$$v < \frac{2-\sigma}{1-\sigma}\left(\frac{1}{2} - \sigma\right).$$

Finally, the Nash equilibrium can also be such that the entire market is "barely" covered, in that the marginal consumer is just indifferent between buying or not. The prices are then such that $p_i^N = v + \sigma - q_i^N$ and satisfy (as $q_1^N + q_2^N = 1$):

$$p_1^N + p_2^N = 2(v+\sigma) - 1. \tag{5}$$

Furthermore, no firm $i = 1, 2$ should benefit from a small deviation; as the market would remain covered if firm i lowers its price, but not so if it increases its price, we must have:

$$\begin{aligned}
\partial_i \pi_i (p_1, p_2)|_{p_j = p_j^N, p_i = p_i^{N-}} &= \frac{\partial}{\partial p_i}\left\{p_i D_i^H (p_1, p_2)\right\}\Big|_{p_j = p_j^N, p_i = p_i^N} = q_i^N - \frac{p_i^N}{2} \geq 0, \\
\partial_i \pi_i (p_1, p_2)|_{p_j = p_j^N, p_i = p_i^{N+}} &= \frac{\partial}{\partial p_i}\left\{p_i D_i^m (p_1, p_2)\right\}\Big|_{p_j = p_j^N, p_i = p_i^N} = q_i^N - \frac{1-\sigma}{1-2\sigma} p_i^N \leq 0.
\end{aligned} \tag{6}$$

Summing-up these conditions for $i = 1, 2$, and using $q_1^N + q_2^N = 1$ and (5), yields:

$$\frac{2-\sigma}{1-\sigma}\left(\frac{1}{2} - \sigma\right) \leq v \leq \frac{3}{2} - \sigma.$$

Figure 1 from Section 2.6 illustrates three possible configurations.

• In the first situation, v is sufficiently high (namely, $v > 3 - 4\sigma\,(> 3/2 - \sigma)$) that firms always compete for consumers in the relevant price range $[0, v]$. The goods are thus substitutes $\left(\partial_j D_i = \partial_j D_i^H = 1/2 > 0\right)$, and their prices are strategic complements $\left(R_i' = \left(R^H\right)' = 1/2 > 0\right)$. Furthermore, the monopoly prices lie above the Nash level: $p^M = v + \sigma - 1/2 > p^N = p^H = 1$.

• In the second, intermediate situation, firms compete again for consumers when prices are low, as in the previous situation. However, for higher price levels, firms best-respond to each other so as to maintain full participation; as a result the goods are at the boundary between substitutes and complements[10] and their prices become strategic substitutes ($R_i' = \tilde{p}' = -1 < 0$). While there are multiple Nash equilibria, they all involve the same total price, and the symmetric Nash equilibrium coincides with the monopoly outcome. As firms are symmetric, it is natural to focus on the symmetric Nash equilibrium, which moreover maximizes industry profit: $p^M = p^N = v + \sigma - 1/2$.

• Finally, in the last situation v is sufficiently low (namely, $v < 2 - 4\sigma$) that firms become local monopolies for high enough prices. The goods then become complements ($\partial_j D_i = \partial_j D_i^m = -\sigma / (1 - 2\sigma) < 0$) and their prices are again strategic substitutes ($R_i' = (R^m)' = -\sigma/2(1-\sigma) < 0$); the monopoly prices then lie below the Nash level: $p^M = v/2 < p^N = v/(2-\sigma)$.

E.5 Price caps

We now study the impact of price caps on the equilibrium prices and profits. As already noted, in the relevant price range each firm's profit function is quasi-concave with respect to the price of that firm. It follows that firms' constrained best responses are of the form $\bar{R}_i(p_j; \bar{p}_i) = \min\{R(p_j), \bar{p}_i\}$. Building on this insight, we now consider the three configurations identified above.

• When v is high enough (namely, $v > 3/2 - \sigma$), the monopoly price lies above the Nash level and, for prices below the Nash level, the goods are substitutes and their prices are strategic complements. It follows that firms have no incentives to adopt price caps, as they can only result into (weakly) lower prices and profits for both firms.

• For intermediate levels of v, firms best-respond to each other so as to maintain full participation. Compared with symmetric Nash equilibrium, which coincides with the monopoly outcome, price caps can only result into lower and more asymmetric prices. Indeed, for any prices $(\hat{p}_1, \hat{p}_2)$ lying below firms' best responses:

- the average is lower than the Nash level: $\hat{p} \equiv (\hat{p}_1 + \hat{p}_2)/2 < p^N$;
- there is asymmetry: $\hat{p}_1 \neq \hat{p}_2$.

It follows that, compared with the symmetric Nash equilibrium without price caps, these price caps can only benefit consumers; to see this, it suffices to decompose the move from (p^N, p^N) to $(\hat{p}_1, \hat{p}_2)$ as:

- a first move from (p^N, p^N) to $(\hat{p}, \hat{p})$, which obviously benefits consumers, as $\hat{p} \leq p^N$;
- an additional move from $(\hat{p}, \hat{p})$ to $(\hat{p}_1, \hat{p}_2)$, which also benefits consumers – keeping the total price constant maintains participation, and among those outcomes

[10] Namely:
$\partial_j D_i(p_i^N, p_j^{N-}) = \partial_j D_i^H(p_i^N, p_J^N) = 1/2 > 0 > \partial_j D_i(p_i^N, p_j^{N+}) = \partial_j D_i^m(p_i^N, p_J^N) = -\sigma/(1-2\sigma)$.

consumers favor asymmetry.[11]

• Finally, when v is low enough (namely, $v < 2 - 4\sigma$), the monopoly price lies below the Nash level and, for prices below the Nash level, the goods are complements and their prices are strategic substitutes. Introducing price caps then lowers the higher of the two equilibrium prices and, while this may be partially compensated by a limited increase in the other price, consumers are always (weakly) better off than in the absence of price caps.[12] Furthermore, firms can use price caps to maintain the monopoly outcome, which, compared with the outcome in the absence of price caps, strictly increases both firms' profits and strictly enhances consumer surplus.

F Complements and substitutes

F.1 Welfare-reducing price caps: an example

We provide here an example with both complements and substitutes, in which the prices of complements exhibit strategic substitutability, in such a way that capping the prices of some of the goods may induce undesirable price increases for other goods – thus violating the spirit of Assumption A.

There are two firms, each producing (costlessly) two goods:

- firm 1 produces goods A_1 and B_1;
- firm 2 produces goods A_2 and B_2.

Let p_{A_1}, p_{A_2}, p_{B_1}, p_{B_2} denote the prices of the four goods.

Consumers are atomless and divided into three groups:

- A mass ε of consumers are only interested in goods A_1 and B_1, which are perfect complements and worth v to them: that is, consumers are willing to buy one unit of both goods as long as $p_{A_1} + p_{B_1} \leq v$.
- A mass ε of consumers are only interested in goods A_2 and B_2, which are perfect complements and worth v to them: that is, consumers are willing to buy one unit of both goods as long as $p_{A_2} + p_{B_2} \leq v$.

[11] Among the prices that satisfy $p_1 + p_2 = 2\hat{p}$, the symmetric outcome ($p_1 = p_2 = \hat{p}$) is the one that maximizes consumer surplus – to see this, note that consumer surplus can be expressed as $\int_0^x tydy + \int_x^1 t(1-y)\,dy = tx^2/2 + t(1-x)^2/2$, where x denotes the location of the marginal consumer that is indifferent between buying or not, and this expression is maximal for $x = 1/2$.

[12] To see this, note that price caps can only reduce the total price (which increases total participation and enhances consumer surplus among symmetric price configurations) and moreover result into asymmetric prices, which, keeping total price constant, generates higher consumer surplus than the symmetric configuration.

- A unit mass of consumers are only interested in goods A_1 and A_2, which are imperfect substitutes for them: the two goods are at the end of a Hotelling segment, along which consumers are uniformly distributed; that is, the demand from these consumers for good A_i is given by, for $i \neq j \in \{1, 2\}$:

$$D_{A_i}\left(p_{A_i}, p_{A_j}\right) = \frac{1}{2} - \frac{p_{A_i} - p_{A_j}}{2t},$$

where t denote the transportation parameter reflecting the degree of differentiation between the two goods, and satisfies $t < v$.

The two firms are therefore competing with goods A_1 and A_2 for consumers of the third group; we will refer to these goods as "competitive". In addition, each firm i offers good B_i as a perfect complement to its competitive good to a distinct group of consumers, over which it has monopoly power; we will refer to goods B_1 and B_2 as "non-competitive".

F.1.1 Nash equilibrium

In the absence of price caps, and as long as the prices of the competitive goods do not exceed v, each firm can charge a total price of v to the consumers interested in its non-competitive good (by charging them $p_{B_i} = v - p_{A_i}$); hence, firm i's profit is given by:

$$\pi_i = p_{A_i} D_{A_i}\left(p_{A_i}, p_{A_j}\right) + \varepsilon v.$$

It follows that the standard Hotelling result prevails: each firm i offers its competitive product at a price equal to t; hence,

$$\begin{aligned} p_{A_1}^N &= p_{A_2}^N = p_A^N = t, \\ p_{B_1}^N &= p_{B_2}^N = p_B^N = v - t. \end{aligned}$$

Each firm earns a profit equal to

$$\pi^N = \frac{t}{2} + \varepsilon v,$$

whereas consumers obtain an aggregate surplus equal to:

$$S^N = 2\int_0^{1/2} \left(V - p_A^N - tx\right) dx + 0 + 0 = V - \frac{5t}{4},$$

where V denotes consumers' value for the competitive good, and is supposed to be large enough to ensure that all the market is always served.

F.1.2 Price caps

Suppose now that the firms face a price cap set to zero on their non-competitive goods: $\bar{p}_{B_1} = \bar{p}_{B2} = 0$. As long as their still compete for consumers of the third group, firm i's profit is now given by:

$$\bar{\pi}_i = p_{A_i} D_{A_i}\left(p_{A_i}, p_{A_j}\right) + \varepsilon p_{A_i} = p_{A_i}\left(\frac{1}{2} - \frac{p_{A_i} - p_{A_j}}{2t}\right) + \varepsilon p_{A_i},$$

leading to:

$$\hat{p}_{A_1} = \hat{p}_{A_2} = \hat{p}_A = (1 + 2\varepsilon)\, t,$$

and:

$$\pi_1 = \pi_2 = \hat{\pi} = (1 + 2\varepsilon)^2 \frac{t}{2} = \pi^N + \varepsilon\left[2t\,(1 + \varepsilon) - v\right].$$

This constitutes indeed an equilibrium as long as:

- Consumers are still buying the non-competitive goods, which requires:

$$v \geq (1 + 2\varepsilon)\, t.$$

- Firms do not prefer to focus on the demand for the non-competitive goods, which requires:

$$\varepsilon v \leq (1 + 2\varepsilon)^2 \frac{t}{2}.$$

As total welfare remains unchanged, in this equilibrium consumers obtain a surplus equal to:

$$\hat{S} = S^N - 2\varepsilon\left[2t\,(1 + \varepsilon) - v\right]. \tag{7}$$

Therefore, if

$$(1 + 2\varepsilon)\, t \leq v < \min\left\{\frac{2 + 2\varepsilon}{1 + 2\varepsilon}, \frac{1 + 2\varepsilon}{2\varepsilon}\right\}(1 + 2\varepsilon)\, t,$$

price caps enable the firms to increase their profits at the expense of consumers. As ε goes to zero, these conditions boil down to to $t \leq v < 2t$ and thus characterize a non-empty set of parameters.

Remark: welfare. Total welfare is here unaffected because total demand is inelastic; making the aggregate demand of the last group of consumers (for whom the goods A_1 and A_2 are substitutes) slightly elastic[13] would yield a reduction in total welfare as well.

Remark: bundling. Allowing the firms to engage in (mixed) bundling would not affect the analysis. In the absence of price caps, each firm i can (and does) extract all the surplus from consumers interested in buying both of its goods by charging them an adequate price on good B_i; hence, offering goods A_i and B_i as a bundle, in addition to offering them on a stand-alone basis, could not increase firm i's profits. When instead a price cap prevents

[13] Following Bénabou and Tirole (2016), a simple way is to introduce outside options, $\tilde{A}_1$ and $\tilde{A}_2$, also located at the two ends of the segment and giving consumers a random value.

firm i from charging a positive price on good B_i, the firm derives all of its profit from selling good A_i (both to consumers interested in buying A_i only, and to those interested in the bundle $A_i - B_i$); offering A_i and B_i as a bundle could not increase this profit, as consumers' arbitrage would prevent firm i from charging more for the bundle than it does for good A_i alone.

F.2 Platforms and apps

We show here that the main insights carry over to a class of situations involving both complements and substitutes. Specifically, we consider a setting in which platforms seek to attract developers for a variety of applications.

F.2.1 Setting

There are n platforms, indexed by $i \in \mathcal{N} \equiv \{1, ..., n\}$. Each platform i charges a price P_i and hosts a continuum of applications, indexed by $x \in [0, 1]$; for each application x, there are $m_{i,x}$ developers, indexed by $j \in \mathcal{M}_{i,x} \equiv \{1, ..., m_{i,x}\}$. The per-user demand for application j is given by $d_{i,x}^j (\mathbf{p}_{i,x})$, where $\mathbf{p}_{i,x} = \left(p_{i,x}^j\right)_{j \in \mathcal{M}_{i,x}} \in \mathbb{R}_+^{m_{i,x}}$ denotes the vector of prices for the application, and $\partial_{p_{i,x}^k} d_{i,x}^j (\mathbf{p}_{i,x}) > 0$ for any $k \in \mathcal{M}_{i,x} \setminus \{j\}$ – that is, developers offer (imperfect) substitutes. Let $s_{i,x} (\mathbf{p}_{i,x})$ denote the consumer net surplus generated by application x on platform i, as a function of the prices $\mathbf{p}_{i,x}$, and

$$S_i (\mathbf{p}_i) = \int_0^1 s_{i,x} (\mathbf{p}_{i,x}) \, dx$$

denote the aggregate net surplus that consumers can derive from the applications running on platform i. Letting $\tilde{P}_i = P_i - S_i (\mathbf{p}_i)$ denote platform i's quality-adjusted price, the demand for that platform is then given by $D_i \left(\tilde{\mathbf{P}}\right)$, where $\tilde{\mathbf{P}} = \left(\tilde{P}_i\right)_{i \in \mathcal{N}}$ and $\partial_{\tilde{P}_h} D_i \left(\tilde{\mathbf{P}}\right) > 0$ for any $h \in \mathcal{N} \setminus \{i\}$.[14] All costs are normalized to zero.

Remark: application multi-homing. For the sake of exposition, we will suppose that applications single-home – that is, each particular app is present on a single platform. However, given our assumption of atomistic apps, the pricing analysis (with or without price caps) does not depend on whether they multi-home or single-home (as long as they can charge platform-specific prices). By contrast, as discussed in Section F.2.4 of this online Appendix, applications' multi-homing decisions may affect their incentives to set price caps.

Remark: complements and substitutes. This setting exhibits substitution among platforms, as well as among the developers of any given application; by contrast, it features complementarity between a platform and its applications, as well as among these applications (and thus, among their developers). Furthermore, the analysis that follows

[14] The demand for application x on platform i is therefore given by $D_i \left(\tilde{\mathbf{P}}\right) d_{i,x}^j (\mathbf{p}_{i,x})$.

applies unchanged if some of the "applications" are actually (atomistic) "components" of a given (non-atomistic) application. For example, for any given (finite or infinite) partition $\mathcal{X} = \{\mathcal{X}_l\}_{l\in\mathcal{L}}$ of $[0,1]$, we could interpret $\mathcal{L}$ as the set of applications running on a given platform (with possibly different sets across platforms); in this interpretation, for any $l \in \mathcal{L}$ and any $x \in \mathcal{X}_l$, the developers in $\mathcal{M}_{i,x}$ are working on component x of application l. Developers then offer substitutes if they work on the same component of an application, and complements if they work on different components or different applications.

We maintain the following assumptions:

- *Applications.* For every $x \in [0,1]$, every $i \in \mathcal{N}$ and every $j \in \mathcal{M}_{i,x}$:

  – The profit function
  $$\pi_{i,x}^j(\mathbf{p}_{i,x}) \equiv p_{i,x}^j d_{i,x}(\mathbf{p}_{i,x})$$
  is strictly quasi-concave in $p_{i,x}^j$;

  – The reaction function
  $$r_{i,x}^j\left(\mathbf{p}_{i,x}^{-j}\right) \equiv \arg\max_{p_{i,x}^j} \pi_{i,x}^j\left(p_{i,x}^j, \mathbf{p}_{i,x}^{-j}\right)$$
  is uniquely defined for any prices $\mathbf{p}_{i,x}^{-j}$ of the rival application developers; it is moreover differentiable and bounded above, and satisfies:

  (**sc**) *Strategic complementarity across developers*:
  $$\partial_{p_{i,x}^h} r_{i,x}^j(\cdot) > 0 \text{ for any } h \in \mathcal{M}_{i,x} \setminus \{j\}.$$

  – Equilibrium: strategic complementarity yields the existence of a fixed point of the function $\mathbf{p}_{i,x} \longrightarrow \mathbf{r}_{i,x}(\mathbf{p}_{i,x}) \equiv \left(r_{i,x}^j\left(\mathbf{p}_{i,x}^{-j}\right)\right)_{j\in\mathcal{M}_{i,x}}$, for every platform i and every application x. For the sake of exposition we assume that this fixed point is unique, and denote it by $\mathbf{p}_{i,x}^N$.

- *Platforms.* For every $i \in \mathcal{N}$ and any net surplus $S_i \in \mathbb{R}_+$:

  – The profit function[15]
  $$\Pi_i\left(\tilde{\mathbf{P}}; S_i\right) \equiv \left(\tilde{P}_i + S_i\right) D_i\left(\tilde{\mathbf{P}}\right)$$
  is strictly quasi-concave in $\tilde{P}_i$;

[15] As will become clear, platform i's pricing decision amounts to choosing the quality-adjusted price $\tilde{P}_i = P_i - S_i$; its profit can thus be expressed as $P_i D_i\left(\tilde{\mathbf{P}}\right) = (P_i + S_i) D_i\left(\tilde{\mathbf{P}}\right)$.

– The reaction function

$$R_i\left(\tilde{\mathbf{P}}_{-i}; S_i\right) = \arg\max_{\tilde{P}_i} \Pi_i\left(\tilde{\mathbf{P}}; S_i\right)$$

is uniquely defined, differentiable and bounded above, and satisfies:

(**SC**) *Strategic complementarity across platforms:*

$$\partial_{\tilde{P}_h} R_i\left(\tilde{\mathbf{P}}_{-i}; S_i\right) > 0 \text{ for any } h \in \mathcal{N} \setminus \{i\}.$$

– Equilibrium: strategic complementarity yields the existence of a fixed point of the function $\tilde{\mathbf{P}} \longrightarrow \mathbf{R}\left(\tilde{\mathbf{P}}, \mathbf{S}\right) \equiv \left(R_i\left(\tilde{\mathbf{P}}_{-i}, S_i\right)\right)_{i\in\mathcal{N}}$, for any $\mathbf{S} = (S_i)_{i\in\mathcal{N}}$; for the sake of exposition, we assume that for $\mathbf{S}^N \equiv \left(S_i\left(\mathbf{p}_i^N\right)\right)_{i\in\mathcal{N}}$ this fixed point is unique, and denote it by $\tilde{\mathbf{P}}^N$.

The timing is as follows:

- Stage 1: platforms and application developers all set their prices simultaneously; all prices are public.
- Stage 2: consumers learn their private benefits for the various platforms and choose which platform to join, if any; they also choose whether to buy the applications developed for the chosen platform.

Remark: Decomposing stage 1 into two distinct stages, where platforms set their prices before application developers do, would not affect the analysis. Likewise, decomposing stage 2 into two distinct stages, where consumers first choose among platforms, before buying the apps, would not affect the analysis either.

F.2.2 Nash equilibrium

As applications are atomistic, a single developer's price has no impact on platform adoption; therefore, for every platform i and every application x, in stage 2 each developer $j \in \mathcal{M}_{i,x}$ seeks to maximize $\pi_{i,x}^j\left(p_{i,x}^j, \mathbf{p}_{i,x}^{-j}\right) D_i$, taking $D_i = D_i\left(\tilde{\mathbf{P}}\right)$ as fixed, and thus chooses $p_{i,x}^j = r_{i,x}^j\left(\mathbf{p}_{i,x}^{-j}\right)$. The above assumptions then imply that the equilibrium prices are uniquely given by $\mathbf{p}_{i,x}^N$. It follows that joining platform $i \in \mathcal{N}$ gives a consumer a net surplus given by $S_i^N \equiv S_i\left(\mathbf{p}_i^N\right)$, where $\mathbf{p}_i^N = \left(\mathbf{p}_{i,x}^N\right)_{x\in[0,1]}$ denotes the vector of equilibrium prices for the applications running on the platform.

Given its rivals' prices, the profit of platform i can be expressed as:

$$P_i\tilde{D}_i(\mathbf{P}) = \left(\tilde{P}_i + S_i^N\right) D_i\left(\tilde{\mathbf{P}}\right) = \Pi_i\left(\tilde{\mathbf{P}}; S_i^N\right).$$

As the platform's price has no incidence on the surplus generated by the applications, we can take the quality-adjusted price as the relevant decision variable. It follows from the above that platform i will choose P_i so as to induce a quality-adjusted price equal to[16] $\tilde{P}_i = R_i\left(\tilde{\mathbf{P}}_{-i}; S_i^N\right)$. The above assumptions then imply that the equilibrium prices are uniquely given by $\mathbf{P}^N = \tilde{\mathbf{P}}^N + \mathbf{S}^N$.

F.2.3 Price caps

Suppose now that each firm faces a price cap. Let $\bar{\mathbf{P}} = \left(\bar{P}\right)_{i\in\mathcal{N}}$ denote the vector of price caps for the platforms; likewise, for every platform i and every application x on that platform, let $\bar{\mathbf{p}}_{i,x} = \left(\bar{p}_{i,x}^j\right)_{j\in\mathcal{M}_{i,x}}$ denote the vector of price caps for that application, and $\bar{\mathbf{p}}_i = \left(\bar{\mathbf{p}}_{i,x}\right)_{x\in[0,1]}$ denote the vector of price caps for all applications running on that platform. The next proposition shows that these price caps can only benefit consumers.

Proposition 10 (platforms and apps) *For any price caps $\bar{\mathbf{P}} = \left(\bar{\mathbf{P}}_i\right)_{i\in\mathcal{N}}$ and $\bar{\mathbf{p}} = \left(\bar{\mathbf{p}}_i\right)_{i\in\mathcal{N}}$, there exists a unique price-constrained equilibrium, and equilibrium prices weakly increase with the vector of price caps on platform prices and on apps. Therefore: (i) for any vector of price caps $\left(\bar{\mathbf{P}}, \bar{\mathbf{p}}\right)$, consumers are weakly better off under $G_{\left(\bar{\mathbf{P}}, \bar{\mathbf{p}}\right)}$ than under G_∞; and (ii) in $\mathcal{G}$, it is optimal for the competition authority to allow price caps.*

Furthermore, these elements weakly increase with $\bar{P}$ and $\bar{p}$; hence, the introduction of price caps can only benefit consumers.

Proof. It is straightforward to check that the equilibrium prices of any given application x on any given platform i depend only on $\bar{\mathbf{p}}_{i,x}$, and not on the other price caps. From Proposition 8, there exists a unique Nash equilibrium, and the equilibrium prices weakly increase with $\bar{\mathbf{p}}_{i,x}$. It follows that the equilibrium net surpluses that consumers derive from the applications decrease with $\bar{\mathbf{p}}_{i,x}$ – in particular, they are all (weakly) larger than the Nash levels.

Let $\hat{\mathbf{p}} = \left(\hat{\mathbf{p}}_{i,x}\right)_{i\in\mathcal{N}, x\in[0,1]}$ denote a Nash equilibrium sustainable through the applications' price caps $\bar{\mathbf{p}}$, and $\hat{\mathbf{S}} = \left(\hat{S}_i\right)_{i\in\mathcal{N}}$, where

$$\hat{S}_i = \int_0^1 s_{i,x}\left(\hat{\mathbf{p}}_{i,x}\right) dx,$$

denote the associated equilibrium net surpluses. The quality-adjusted prices $\tilde{\mathbf{P}} = \left(\tilde{P}_i\right)_{i\in\mathcal{N}}$ are now equal to:

$$\tilde{P}_i = P_i - \hat{S}_i.$$

Using again these quality-adjusted prices as strategic decision variables, platform i there-

[16] That is, it will choose $P_i = R_i\left(\tilde{\mathbf{P}}_{-i}; S_i^N\right) + S_i^N$.

fore seeks to solve:

$$\max_{P_i \le \bar{P}_i} P_i D_i \left(\tilde{\mathbf{P}}\right) = \max_{\tilde{P}_i \le \bar{P}_i - \hat{S}_i} \Pi_i \left(\tilde{P}_i, \tilde{\mathbf{P}}_{-i}; \hat{S}_i\right).$$

From quasi-concavity, platform i will thus choose:

$$\tilde{P}_i = \bar{R}_i \left(\tilde{\mathbf{P}}; \hat{S}_i, \bar{P}_i\right) \equiv \min \left\{\bar{P}_i - \hat{S}_i, R_i \left(\tilde{\mathbf{P}}; \hat{S}_i\right)\right\}. \quad (8)$$

This reaction function still exhibits strategic complementarity across platforms: rivals' prices only affect the second term, which satisfies $\partial_{\tilde{P}_k} R_i \left(\tilde{\mathbf{P}}; \hat{S}_i\right) > 0$ for $h \in \mathcal{N} \setminus \{i\}$. Furthermore, the first term in the right-hand side of (8) increases with $\bar{P}_i$, and both terms decrease with $\hat{S}_i$: this is obvious for $\bar{P}_i - \hat{S}_i$, and for $R_i \left(\tilde{\mathbf{P}}; \hat{S}_i\right)$ this follows from

$$\partial^2_{\tilde{P}_i S_i} \Pi_i \left(\tilde{\mathbf{P}}; S_i\right) = \partial_{\tilde{P}_i} D_i \left(\tilde{\mathbf{P}}\right) < 0.$$

The best-response function $\bar{R}_i(\cdot)$ of each platform i is therefore increasing in $\bar{P}_i$ and decreasing in $\hat{S}_i$. Strategic complementarity then yields the result:

- For any equilibrium net surpluses $\hat{\mathbf{S}} = \left(\hat{S}_i\right)_{i \in \mathcal{N}}$ sustainable through the application price caps $\bar{\mathbf{p}}$, and any platform price caps $\bar{\mathbf{P}}$, there exist a unique Nash equilibrium for the platforms' quality-adjusted prices, and the equilibrium quality-adjusted prices weakly increase with $\bar{\mathbf{P}}$.

- As the set of equilibrium net surpluses $\hat{\mathbf{S}} = \left(\hat{S}_i\right)_{i \in \mathcal{N}}$ weakly decrease with $\bar{\mathbf{p}}$, the overall equilibrium quality-adjusted prices weakly increase with both $\bar{\mathbf{P}}$ and $\bar{\mathbf{p}}$.

■

F.2.4 Firms' incentives

Other things equal, introducing caps on platforms' prices increases applications' profits, by increasing the number of platforms' users, but reduces platforms' profits, both by constraining their pricing decisions and by making their rivals more aggressive: for any $\tilde{\mathbf{P}} \le \tilde{\mathbf{P}}^N$,

$$\max_{\tilde{P}_i \le \bar{P}_i - S_i^N} \Pi_i \left(\tilde{P}_i, \tilde{\mathbf{P}}_{-i}; S_i^N\right) \le \max_{\tilde{P}_i} \Pi_i \left(\tilde{P}_i, \tilde{\mathbf{P}}_{-i}; S_i^N\right) \le \max_{\tilde{P}_i} \Pi_i \left(\tilde{P}_i, \tilde{\mathbf{P}}_{-i}^N; S_i^N\right) = \Pi_i^N,$$

where the second inequality is strict whenever $\tilde{\mathbf{P}} \le \tilde{\mathbf{P}}^N$.

By contrast, introducing caps on applications' prices can increase not only platforms' profits, by boosting their demands thanks to the greater net surplus that consumers derive from the apps, but it can also benefit the apps, by increasing the number of users.

More precisely, consider the introduction of price caps $\bar{\mathbf{p}}_i$ on the applications running on platform i:

- This increases the net surplus S_i generated by these apps, which increases platform i's profit by expanding its demand.
- As noted in the proof of Proposition **10**, $\partial_{S_i} R_i \left(\tilde{\mathbf{P}}_{-i}; S_i \right) \leq 0$; therefore, the increase in the net surplus S_i expands the equilibrium number of users on platform i.

The extent to which this increase in user participation can offset the direct negative impact of the price caps $\bar{\mathbf{p}}_i$ on applications' per-user profitability depends on several factors:

- Consider first the polar case where:[17]
  - there is a single developer for each application, so that
$$p_{i,x}^N = p_{i,x}^M \equiv \arg\max_{p_{i,x}} p_{i,x} d_{i,x} \left(p_{i,x} \right);$$
  - applications single-home – that is, the applications running on platform i run only on that platform.

  In this case, the increase in platform i's user participation can indeed benefit the applications running on that platform. In particular, introducing price caps $\bar{\mathbf{p}}_i$ that are slightly below $\mathbf{p}_i^M$ is likely to have only a second-order effect on applications' per user profit (as these profits are maximal under monopoly), but a first-order effect on platform i's quality-adjusted price, and thus on the number of its users; hence, the applications are likely to benefit from the introduction of such caps.
- The potential benefit of users' greater participation is however diluted for multi-homing applications. For example, if the applications are present on all platforms, then this demand expansion effect arises only if users' aggregate participation is elastic. Otherwise, the number of users would remain unchanged, and the negative effect of price caps on per user profits would prevail.
- This potential benefit of users' greater participation is also lower in case of competition among developers; in particular, introducing a price cap slightly lower than the Nash level would then have a first-order effect on applications' per-user profitability as well as on user participation.

The elasticity of users' aggregate demand for platforms is also a key factor for the profitability of price caps at the industry level. For example, if total platform participation

[17] See for instance the monopoly example studied in the next section.

is inelastic, then a platform may still benefit unilaterally from capping the prices of its applications, but this would be at the expense of the other platforms: the industry as a whole would not benefit from introducing price caps.

Consider for instance the following example, where a unit mass of consumers consider joining one of the platforms – and only one: consumers do not derive any benefit from joining additional platforms, and thus single-home;[18] each user obtains a benefit θ_i from joining platform i, and these private benefits are randomly drawn from a common distribution, with cumulative distribution $F(\cdot)$ and density $f(\cdot)$ over some Θ, with independent drawn across both platforms and consumers. The demand for platform i is then given by:

$$\begin{aligned} D_i\left(\tilde{\mathbf{P}}\right) &= \Pr\left[\theta_i - \tilde{P}_i > \theta_j - \tilde{P}_j \text{ for every } j \in \mathcal{N}\setminus\{i\}\right] \\ &= \int_\Theta \prod_{j\in\mathcal{N}\setminus\{i\}} F\left[\theta_i - \tilde{P}_i + \tilde{P}_j\right] f(\theta_i)\, d\theta_i, \end{aligned}$$

and satisfies, for any symmetric prices $\tilde{\mathbf{P}}^s = \left(\tilde{P}^s, ..., \tilde{P}^s\right)$:

$$\begin{aligned} D_i\left(\tilde{\mathbf{P}}^s\right) &= \int_\Theta F^{n-1}(\theta) f(\theta)\, d\theta = \frac{1}{n}, \\ \partial_{\tilde{P}_i} D_i\left(\tilde{\mathbf{P}}^s\right) &= -\int_\Theta \prod_{j\in\mathcal{N}\setminus\{i\}} F^{n-2}(\theta) f^2(\theta)\, d\theta = -\lambda, \end{aligned}$$

where the constant λ is positive and does not depend on the actual level of the quality-adjusted price $\tilde{P}^s$.

Suppose now symmetric caps on the applications result in the same equilibrium net surplus S for each platform. Any resulting symmetric price equilibrium $\tilde{\mathbf{P}}^s = \left(\tilde{P}^s, ..., \tilde{P}^s\right)$ satisfies the first-order condition:

$$0 = \partial_{\tilde{P}_i} \Pi_i\left(\tilde{\mathbf{P}}^s; S\right) = D_i\left(\tilde{\mathbf{P}}^s\right) + \tilde{P}^s \partial_{\tilde{P}_i} D_i\left(\tilde{\mathbf{P}}^s\right) = \frac{1}{n} - \lambda \tilde{P}^s,$$

and thus:

$$\tilde{P}^s = \frac{1}{\lambda n}.$$

It follows that any increase in the net surplus generated by the applications is entirely passed on to consumers; introducing such price caps would thus benefit consumers at the expense of the applications' profits, without any impact on the profitability of the platforms.

[18] For example, joining the platform may involve substantial fixed costs (learning how to use it, set-up costs, and so forth), accounted for in the definition of the private benefit θ from single-homing, but making multi-homing undesirable. In some cases, multi-homing may be infeasible (e.g., for broadband Internet access, consumers may choose among alternative suppliers, but only one at a time can operate the local connection to the home).

Such a situation generates a prisoners' dilemma: each platform would have an incentive to introduce price caps on its own applications (and would be willing to compensate the applications, in case this negatively affect their profits), but the benefit to that platform would come at the expense of the other platforms and their own applications, so that the industry profit would be reduced as a result.

F.3 Platform & apps: the monopoly case

We consider here a particular case of the setting considered in the previous section, which is used in Section 2.3 of the main text to illustrate the possibility that, even with complements, a monopoly price may lie above the Nash level for one of the products (cf. Figure 1). To see this, we suppose here that there is a single platform as well as a single developer per application. We further assume for simplicity that all applications face the same demand $d(p)$, which is downward-sloping (i.e., $d(p) < 0$). It follows that, in the absence of any price caps, all applications will charge the same price. To be consistent with the notation used in Section 2.3, we will denote the price of the platform by p_1 and that of the applications by p_2.

F.3.1 Complementarity between the platform and the applications

We first check that the platform and the applications are indeed *complements*. Letting

$$s(p_2) \equiv \int_{p_2}^{+\infty} d(p)\, dp$$

denote the additional surplus that platform users derive from applications, the demand for the platform is then given by:[19]

$$D_1(p_1, p_2) \equiv D(p_1 - s(p_2)),$$

where $D(\tilde{p})$ denotes the demand for the platform, as a function of the quality-adjusted price $\tilde{p} = p_1 - s(p_2)$. The demand for each application is thus given by:

$$D_2(p_2, p_1) \equiv D(p_1 - s(p_2))\, d(p_2),$$

and it satisfies:

$$\partial_2 D_1(p_1, p_2) = \partial_1 D_2(p_2, p_1) = D'(p_1 - s(p_2))\, d(p_2) < 0.$$

[19] As all applications are charging the same price p_2, $s(p_2)$ represents both the per-application net surplus, and the total net surplus that consumers derive from the applications.

F.3.2 Best-responses

We now turn to firms' reaction functions, and first note that the applications' best-response is flat (i.e., $R_2(p_1) = 0$); indeed, each application wishes to maximize its per-user profit, which amounts to choosing a price equal to:

$$p_2^N \equiv \arg\max_{p_2} p_2 d(p_2).$$

Note that $p_2^N > 0$: the applications could not obtain any profit by charging a non-positive price, whereas they can secure a positive profit by charging any positive price.[20]

Consider now the platform's best-response to the application price p_2. The profit of the platform can be expressed as:

$$\pi_1(p_1, p_2) \equiv p_1 D_1(p_1, p_2) = p_1 D(p_1 - s(p_2)).$$

Maximizing this profit amounts to

$$\log(\pi_1(p_1, p_2)) = \log(p_1) + L(p_1 - s(p_2)),$$

where $L(\tilde{p}) \equiv \log(D(\tilde{p}))$ denotes the logarithm of the demand for the platform, and:

$$\partial^2_{p_1 p_2} \log(\pi_1(p_1, p_2)) = L''(p_1 - s(p_2)) d(p_2).$$

It follows that, from the platform's standpoint, prices are strategic substitutes (i.e., $R_1'(\cdot) < 0$) whenever the demand for the platform is log-concave (i.e., $L''(\cdot) < 0$).

It follows from the above that, in equilibrium, the applications charge $p_2^N > 0$ whereas the platform charges $p_1^N \equiv R_1(p_2^N)$.

Finally, we check that Assumption (A) is satisfied. It is obvious for the applications, as $R_2'(p_1) = 0$, and for the platform it follows from the fact that, as noted in Section F.2, the platform's optimal quality-adjusted price is increasing in the net surplus generated by the applications; hence, in response to a decrease in the application price p_2, and thus to an increase in the surplus s, the platform increases its own price – as $R_1'(p_2) < 0$ – but not so as to offset entirely the consumers' benefit from the reduction in the price p_2.

F.3.3 Monopoly outcome

We now characterize the monopoly outcome. We start with the observation that, in order to maximize the industry profit, it is optimal to sell the applications at cost. To see this, let us again index the applications by $x \in [0, 1]$; for a given platform price p_1 and given

[20]Note that $d(\cdot) \geq 0$ and $d'(\cdot) < 0$ together imply $d(\cdot) > 0$.

application prices $(p_{2x})_{x\in[0,1]}$, the industry profit can then be expressed as:

$$\Pi\left(p_1,(p_{2x})_x\right)=\left[p_1+\int_0^1 p_{2x}d\left(p_{2x}\right)dx\right]D\left(p_1-s\right),$$

where

$$s=\int_0^1 s\left(p_{2x}\right)dx$$

denotes consumers' expected surplus from the applications. Replacing these prices with $\tilde{p}_2=0$ and $\tilde{p}_1=p_1+s(0)-s$ does not affect the number of platform users (the quality-adjusted price remains equal to p_1-s), and thus the impact on industry profit is equal to:

$$\begin{aligned}\Delta\Pi &= \left\{s(0)-s-\int_0^1 p_{2x}d\left(p_{2x}\right)dx\right\}D\left(p_1-s\right)\\ &= \left\{\int_0^1\left[s(0)-s\left(p_{2x}\right)-p_{2x}d\left(p_{2x}\right)\right]dx\right\}D\left(p_1-s\right)\\ &> 0,\end{aligned}$$

where the inequality stems from the fact that the total surplus from the applications, $s(p)+pd(p)$, is maximal under marginal cost pricing, i.e., for $p=0$.

The monopoly prices are thus $p_2^M=0$ and

$$p_1^M\equiv\arg\max_{p_1}\left\{\Pi\left(p_1,p_2=0\right)\right\}=\arg\max_{p_1}\left\{p_1D\left(p_1-s(0)\right)\right\}=R_1(0).$$

The monopoly outcome therefore lies (weakly) below firms' best-responses (more precisely, $p_2^M<R_2\left(p_1^M\right)$ and $p_1^M=R_1\left(p_2^M\right)$). However, as $p_2^M=0<p_2^N$, the application price is (strictly) lower than its Nash level $\left(p_2^M<p_2^N\right)$, but the opposite holds for the platform: as $R_1'(\cdot)<0$, we have:

$$p_1^M=R_1(0)>R_1\left(p_2^N\right)=p_1^N.$$

Figure 1 illustrates these insights.

F.4 Discussion

The two situations considered in Section F.1 and in Sections F.2-F.3 both exhibit a combination of complements and substitutes. In both instances, capping the prices of substitutes would benefit consumers, but is not appealing to the firms.[21] By contrast, capping the prices of (some of the) complements has a more ambiguous effect on firms

[21] In the example considered in Section F.1, capping the prices of the competitive goods, A_1 and A_2, would reduce the profits derived from the consumers of the third group (for which these goods are substitutes), without any off-setting increase in the profits derived from the other consumers (for which these goods are complements to the non-competitive goods).

and consumers: the profit may be reduced on the goods for which a cap is introduced, but it increases for their complements, thanks to an expansion in their demands. Consumers also benefit from the lower prices on the goods for which a cap is introduced, but may face higher prices for their complements.

In the example studied in Section F.1, consumers and firms have indeed perfectly conflicting interests, and introducing price caps on the non-competitive goods can benefit either the consumers or the firms. In essence, price caps benefit the firms (at the expense of consumers) when the spirit of Assumption A is not satisfied, namely, when[22]

$$\frac{\Delta p_A}{\Delta p_B} < -\frac{D_B}{D_A}, \tag{9}$$

where, for $L = A, B$, D_L denotes the aggregate demand for good L (i.e., the sum of the demands for L_1 and L_2), whereas Δp_L denotes the variation in the prices of these goods, following the introduction of a uniform price cap on B_1 and B_2.[23]

By contrast, the spirit of Assumption A is automatically satisfied in the platforms and apps settings: as the applications are atomistic, each individual application price has no influence on platform participation, and so each application seeks to maximize its per-user profit, which does not depend on platforms' prices; hence, platforms' prices have no impact on applications' prices, that is, applications' best-responses are "flat", and Assumption A is trivially satisfied. More generally, we would expect the spirit of Assumption A to hold as long as there are multiple applications, so that their pricing decisions are primarily driven by competition among developers, rather than by the impact of their prices on platform participation (and thus, indirectly, by platforms' prices).

G Post-investment price caps

G.1 Substitutes

Consider the multi-product firm oligopoly setting developed in online Appendix C, in which each firm $i \in \mathcal{N}$ can offer a set $\mathcal{M}_i \equiv \{1, ..., m_i\}$ of products, and now suppose that in addition firms must make investment decisions. These decisions may correspond to entering or staying in the market, developing new products, improving the quality or lowering the production cost of existing ones; different firms may moreover be facing different choices.

Let $\mathcal{I}_i$ denote the set of feasible investment decisions for firm i, and $\mathbf{I} = (I_1, ..., I_n) \in \mathcal{I} = \mathcal{I}_1 \times ... \times \mathcal{I}_n$ denote the vector of these decisions. Firm i's production cost is now given by $C_i(\mathbf{q}_i; I_i)$, and the demand for firm i's goods is $\mathbf{D}_i(\mathbf{p}; \mathbf{I}) = (D_i^1(\mathbf{p}; \mathbf{I}), ..., D_i^{m_i}(\mathbf{p}; \mathbf{I}))$.

[22]Because of the inelasticity of the demand, the condition involves discrete rather than marginal price changes.

[23]In the example, $D_A = 1 + 2\varepsilon$ and $D_B = 2\varepsilon$, whereas $\Delta p_A = 2\varepsilon t$ and $\Delta p_B = t - v$. Condition (9) thus amounts to $v < 2(1 + \varepsilon)t$, the condition under which price caps reduce consumer surplus (see (7)).

As before, we will assume that, for all $\mathbf{I} \in \mathcal{I}$ and $i \in \mathcal{N}$, $\mathbf{D}_i(\cdot)$ and $C_i(\cdot)$ are both C^2 and that, for every $i \in \mathcal{N}$:

- the profit function $\pi_i(\mathbf{p};\mathbf{I}) \equiv \sum_{j\in\mathcal{M}_i} p_i^j D_i^j(\mathbf{p};\mathbf{I}) - C_i(\mathbf{D}_i(\mathbf{p};\mathbf{I});I_i)$ is strictly quasi-concave in $\mathbf{p}_i$;
- for every $j \in \mathcal{M}_j$, the "product-by-product" best-response function $r_i^j\left(\mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}, \mathbf{p}_{-i};\mathbf{I}\right) \equiv \arg\max_{p_i^j} \pi_i\left(p_i^j, \mathbf{p}_i^{\mathcal{M}_i\setminus\{j\}}, \mathbf{p}_{-i};\mathbf{I}\right)$ is well-defined and bounded above.

We further focus on substitutes and strategic complementarity, and assume price equilibrium uniqueness:

- For every $i \in \mathcal{N}$ and any $\mathbf{I} \in \mathcal{I}$:
  - (**S**) products are *substitutes*: $\partial_{p_j} D_i(\cdot) < 0$ for $j \neq i \in \mathcal{N}$.
  - (**SC**) prices are *strategic complements:* $\partial_{p_j} R_i(\cdot) > 0$ for $j \neq i \in \mathcal{N}$.
- For any investment decisions $\mathbf{I} \in \mathcal{I}$, in the absence of price caps there exists a unique Nash equilibrium in prices, which we denote by $\mathbf{p}^{\mathbf{I}} = \left(p_i^{\mathbf{I}}\right)_{i\in\mathcal{N}}$.

Suppose that investment decisions are publicly made in stage $2a$, and firms can then agree on price caps in stage $2b$, before setting prices in stage 3. From Proposition 8, for any vector of investment decisions $\mathbf{I} \in \mathcal{I}$ made in stage $2a$, firms have no incentive to adopt price-caps agreements in stage $2b$; therefore, in stage 3 the continuation price equilibrium is $\mathbf{p}^{\mathbf{I}}$, as in when price caps are not allowed. It follows that thus allowing price caps in stage 1 has no impact on the set of investment and price equilibria in stages 2 and 3.

G.2 Complements

G.2.1 On Assumption C

Suppliers of complements can always sign a mutually profitable agreement that benefit all of them: as shown in the proof of Corollary 1, starting from the Nash equilibrium prices $\mathbf{p}^N$, reducing all prices by a small amount ε is sustainable through price caps, and it increases all firms' profits, as firms' margins are positive from Lemma 2, and reducing one firm's price has only a second-order effect on the profit of that firm, and a first-order positive effect on the other firms' profits.

Other agreements may not share this feature: The price-caps agreement signed by a coalition of firms may benefit them, but hurt others. For example, suppose that there are three firms $i = 1, 2, 3$ producing at no cost products $1, 2, 3$ respectively, and facing demand $D_i(\mathbf{p}) = d_i(p_{i-1}) - p_i$ (with the convention that $0 = 3$), where $d_i(\cdot) > 0 > d_i'(\cdot)$

(that is, product i is a complement for product $i+1$). It is easy to show that this leads to best-responses $R_i(\mathbf{p}_{-i}) = d_i(p_{i-1})/2$ and to a unique Nash equilibrium $\mathbf{p}^N$.[24] Introducing a price cap $\bar{p}_1$ slightly below p_1^N then induces firm 1 to set $p_1 = \bar{p}_1$, which in turn leads firm 2 to raise p_2 slightly above p_2^N and firm 3 to reduce p_3 slightly below p_3^N. As a result, firms 1 and 2 benefit from the introduction of the cap (as their demands are boosted by the reductions in p_1 and p_3, respectively, and because they either best-respond or are close to their best-response), whereas firm 3 is hurt (as its demand is harmed by the increase in p_2).

The fact that price caps may benefit a coalition at the expense of outsiders need not imply that they will be adopted by the coalition, however; other agreements may be more profitable and benefit others as well. For example, in the above example, introducing the price cap $\bar{p}_1$ led to small reductions $\Delta_1 \left(= p_1^N - \bar{p}_1\right)$ and Δ_3 in the prices of firms 1 and 3, and to a small increase in the price of firm 2; a uniform slight reduction in all three prices, by $\Delta \equiv \max\{\Delta_1, \Delta_3\}$, would instead benefit firm 3 as well, while giving at least the same benefits to firms 1 and 2. We will not develop here a full-fledged model of negotiations over price caps, and simply assume that any price-caps agreement benefits non-signatories:

Assumption C: Any active unconstrained firm is at least as well off when other active firms are constrained by price caps than when all active firms are unconstrained.

Intuitively, this Assumption is likely to hold when firms are in a rather symmetric position. For example, under (**SS**) it holds for symmetric demands with an "aggregative" nature, that is, when there exist an aggregator $A(p_1, ..., p_{n-1})$, which is symmetric and increasing in all prices, and a function $D(p, A)$, which decreases with both p and A, such that

$$D_i(\mathbf{p}) = D(p_i, A(\mathbf{p}_{-i})).$$

A classic example is the linear demand $D_i(\mathbf{p}) = d - ap_i - b\sum_{j\in\mathcal{N}\setminus\{i\}} p_j$ (with $d > 0$ and $a > b > 0$).[25]

Suppose for simplicity that all firms are active.[26] In the absence of price caps, the resulting equilibrium is symmetric ($p_i = p^N$ for all $i \in \mathcal{N}$) and satisfies $p^N = R\left(\mathbf{p}_{-i}^N\right)$; we will assume that the symmetric best-response satisfies $\partial_1 R(\cdot) \in (-1, 0)$;[27] that

[24] To see this, it suffices to note that $\rho(p_1) \equiv R_1(R_3(R_2(p_1))) - p_1 = d_1(d_3(d_2(p_1)/2)/2)/2 - p_1$ is such that $\rho(0) > 0$ and $\rho'(p_1) = d_1'(\cdot)\, d_3'(\cdot)\, d_2'(p_1)/8 - 1 < -1$. Therefore, there exists a unique p_1^N satisfying $\rho(p_1) = 0$; the price vector $\left(p_1^N, p_2^N \equiv d_2\left(p_1^N\right)/2, p_3^N \equiv d_3\left(p_2^N\right)/2\right)$ then constitutes the unique Nash equilibrium.

[25] For substitutes, such demand systems include multinomial logit ($D_i(\mathbf{p}) = d\exp(a - bp_i)/\sum_{j\in\mathcal{N}}\exp(a - bp_j)$, with $a, b, d > 0$) and CES ($D_i(\mathbf{p}) = dp_i^{-\sigma}/\sum_{j\in\mathcal{N}} ap_j^{1-\sigma}$, with $a, d > 0$ and $\sigma > 1$). See Nocke and Schutz (2017) for a recent analysis of such aggregative demand systems.

[26] The reasoning applies to any smaller set of active firms, with the convention that $p_i = +\infty$ for any inactive firms.

[27] $\partial_1 R$ denotes here the partial derivative of $R(\cdot)$ with respect to its first argument; by symmetry, it applies to the other arguments as well.

is, prices are strategic substitutes (**SS**), but they do not respond excessively to each other – the latter condition is implied by the usual stability condition which requires $\sum_{j\in\mathcal{N}\setminus\{i\}} \partial_j R(\mathbf{p}_{-i}) > -1$.

Suppose now that firms are constrained by price caps $\{\bar{p}_i\}_{i\in\mathcal{N}}$, where $\bar{p}_i = +\infty$ for at least one firm, and let $\hat{\mathbf{p}}$ and $\hat{\boldsymbol{\pi}}$ denote the resulting equilibrium prices and profits. We first note that all unconstrained firms charge the same price $\hat{p}$. To see this, suppose that two unconstrained firms i and j charge different prices, e.g., $\hat{p}_i > \hat{p}_j$. Using symmetry, we then have:

$$
\begin{aligned}
\hat{p}_i - \hat{p}_j &= R(\hat{\mathbf{p}}_{-i}) - R(\hat{\mathbf{p}}_{-j}) \\
&= R\left(\hat{p}_j, \hat{\mathbf{p}}_{-\{i,j\}}\right) - R\left(\hat{p}_i, \hat{\mathbf{p}}_{-\{i,j\}}\right) \\
&= \int_{\hat{p}_i}^{\hat{p}_j} \partial_1 R\left(p, \hat{\mathbf{p}}_{-\{i,j\}}\right) dp \\
&< \hat{p}_i - \hat{p}_j,
\end{aligned}
$$

a contradiction.

Next, we show that the symmetric unconstrained price $\hat{p}$ lies above the highest binding price cap. To see this, let $\mathcal{C}$ denote the set of firms for which the price cap is binding, $\bar{\imath}$ denote the firm with the highest binding price cap (that is, $\bar{p}_{\bar{\imath}} = \max_{i\in\mathcal{C}}\{\bar{p}_i\}$), and suppose that $\hat{p} < \bar{p}_{\bar{\imath}}$. We then have $\bar{p}_{\bar{\imath}} \leq R(\hat{\mathbf{p}}_{-\bar{\imath}})$ and $\hat{p} = R(\hat{\mathbf{p}}_{-i})$ for any $i \in \mathcal{N}\setminus\mathcal{C}$; therefore:

$$
\begin{aligned}
\bar{p}_{\bar{\imath}} - \hat{p} &\leq R(\hat{\mathbf{p}}_{-\bar{\imath}}) - R(\hat{\mathbf{p}}_{-i}) \\
&= R\left(\hat{p}, \hat{\mathbf{p}}_{-\{i,\bar{\imath}\}}\right) - R\left(\bar{p}_{\bar{\imath}}, \hat{\mathbf{p}}_{-\{i,\bar{\imath}\}}\right) \\
&= \int_{\bar{p}_{\bar{\imath}}}^{\hat{p}} \partial_1 R\left(p, \hat{\mathbf{p}}_{-\{i,\bar{\imath}\}}\right) dp \\
&< \bar{p}_{\bar{\imath}} - \hat{p},
\end{aligned}
$$

a contradiction.

We thus have $p_i = \bar{p}_i \leq \hat{p}$ for all $i \in \mathcal{C}$, and $p_i = \hat{p}$ for all $i \in \mathcal{N}\setminus\mathcal{C}$. From (**SS**), $\hat{p} < p^N$ would then imply $\hat{p}_i < p_i^N$ for all $i \in \mathcal{N}$ and thus, for any $j \in \mathcal{N}\setminus\mathcal{C}$: $\hat{p} = R(\hat{\mathbf{p}}_{-i}) > R\left(\mathbf{p}_{-i}^N\right) = p^N$, a contradiction. Therefore, $\hat{p} \geq p^N$. Finally, for aggregative games the best-response $R(\cdot)$ is of the form $R(\mathbf{p}_{-i}) = \hat{R}(A(\mathbf{p}_{-i}))$, where $\hat{R}' < 0$ from (**SS**). Therefore, for any unconstrained firm $i \in \mathcal{N}\setminus\mathcal{C}$: $A(\mathbf{p}_{-i}) = \hat{R}^{-1}(\hat{p}) \leq \hat{R}^{-1}\left(p^N\right) = A\left(\mathbf{p}_{-i}^N\right)$, implying that firm i obtains at least as much profit as in the unconstrained Nash equilibrium.[28]

[28] To see this, note that the price at which firm i can sell any quantity q_i is decreasing with $A(\mathbf{p}_{-i})$.

G.2.2 Entry/exit game

Consider a setting in which each firm $i \in \mathcal{N}$ must decide whether to enter (or stay in) the market and suppose further that Assumption C holds. We first note that this assumption implies that, as intuition suggests, the development of complementary products boosts demand and enhances profits. To see this, consider two situations which only differ in that one firm (firm i, say), is either active or not, and let $\hat{p}_i$ denote firm i's (unconstrained) equilibrium price when it is active. From the standpoint of the other firms, the entry of firm i has the same impact as imposing a price cap $\bar{p}_i = \hat{p}_i$ on a firm producing the same good as firm i but with a very large marginal cost: that firm would thus charge $+\infty$ in the absence of a cap, and $\hat{p}_i$ when facing the cap.

Let $\mathcal{A}$ denote the set of active firms in an equilibrium that arises in the absence of price caps; each firm in $\mathcal{A}$ is thus better off being active (given the presence of the others), and it would benefit from the presence of any additional firms. Hence, if price caps are now allowed, each firm in $\mathcal{A}$ finds it profitable to be active if the others do, regardless of the decisions of firms outside $\mathcal{A}$, and regardless of any price caps that the other firms may agree to. The possibility of price caps can moreover be used to increase all active firms' profits (as noted at the beginning of Section G.2.1), and from Assumption C this may induce some of the outsiders to enter.

Summarizing this discussion yields Proposition 11.

H Pre-investment price caps

A potential concern is the use of (artificially low) price caps as a way of softening competition, by inducing exit, deterring entry or stifling investment.

A first issue is the possible use of price caps as a commitment to maintain low prices, so as to deter entry or discourage investment. This is indeed a serious concern if firms can sign long-term contracts with their customers: firms could then credibly commit themselves to maintain low prices, for example, by adopting most favored nation clauses promising a compensation for any price increase: this would *de facto* allow customers to buy at the initially agreed price caps, even if these caps are then renegotiated away. In such a case, incumbent firms could adopt low price caps so as to deter entry, as in the limit pricing model of Sylos Labini (1957) and Modigliani (1958). Ruling out this possibility leads to:

Policy recommendation 1: Customers are not part of the price-caps agreements.

Second, a low price cap (possibly against compensation) may act as a commitment to exit the market. Indeed, if firm i accepts a price cap $\bar{p}_i$ that is too low for operating profitably (e.g., lower than its minimum average cost), it will then choose to leave the market. Firms could therefore use such price-caps agreements so as to bribe some rivals out of the market; likewise, incumbent firms could induce potential entrants to stay out of the market.[29] These commitments are not credible, however, as the firms would have no incentive to enforce the price caps. Taking advantage of these incentives leads to:

Policy recommendation 2: The agreement becomes void if none of the parties wishes to enforce it.

This requirement implies that, in order to remain in place, a price-caps agreement must be "confirmed" by at least one party to the agreement; this contributes to undermine the credibility of the "threats" discussed above.

We now show that these policy recommendations indeed alleviate the above concerns about the use of price caps as a way to deter entry or stifle investment.

Consider the same setting as in Section G.1 (with quasi-concavity, strategic complementarity and unique continuation price equilibria), but modify the overall game $\mathcal{G}$ as follows:

2. (a) Firms choose price caps if such agreements are allowed.
   (b) Firms make observable investment or entry decisions.
3. (a) If an agreement has been signed, firms choose whether to confirm it; the agreement is enforced if and only if at least one firm confirms it.
   (b) Firms set their prices.

This timing allows the firms to sign price-caps agreements in order to influence investment decisions, but rules out non-credible threats by asking firms to confirm their willingness to enforce the agreement, once investment decisions have been made. To avoid coordination issues, we rule out weakly dominated strategies.

From Proposition 8, for any vector of investment decisions $\mathbf{I} \in \mathcal{I}$ made in stage $2b$, in stage $3a$ firms have no incentive to enforce any price-caps agreement, regardless of what they may have agreed to in stage $2a$; therefore, in stage $3b$ the continuation price equilibrium is $\mathbf{p}^{\mathbf{I}}$, as when price caps are not allowed. It follows that allowing price caps in stage 1 has no impact on the set of investment and price equilibria in stages 2 and 3.

Summarizing the above analysis yields Proposition 12.

[29] Consider for example a symmetric duopoly in which each firm faces a constant marginal cost c and a fixed cost $f > 0$, and obtains a gross profit $\pi^D > f$. If either firm were alone, it would instead obtain the monopoly profit π^M, where, due to competition $\pi^M > 2\pi^D$. Firm 1, say, would then have an incentive to "bribe" firm 2 into a price-caps agreement of the form $(\bar{p}_1 \geq p^M, \bar{p}_2 \leq c)$: this would induce firm 2 to exit, and enable firm 1 to increase its profit by $\Delta\pi_1 = \pi^M - \pi^D$; as $\Delta\pi_1 > \pi_2 = \pi^D - f$, these price caps, together with any transfer $T \in (\pi_2, \Delta\pi_1)$, would make both firms better off. Furthermore, transfers may no longer be needed when several markets are involved, as price caps could then be used to divide these markets in a mutually profitable way.

I Foreclosure

Let us first recall the jest of the Choi-Stefanadis foreclosure model.

• *Integration and foreclosure.* There are two firms: an incumbent and a potential entrant. An integrated incumbent costlessly produces two perfect complements, A and B. We first assume an inelastic demand for the system: A and B together bring value v to consumers. An entrant can invest I to develop with probability $\rho \in (0,1)$ product A', which is an alternative to A and brings extra surplus $\Delta \in (I/\rho, \min\{I/\rho^2, v\})$; and similarly with product B' (A' and B' combined thus deliver consumer value $v + 2\Delta$). The two R&D processes are independent.

Prior to the entrant deciding whether to undertake R&D on A', B' or both, the incumbent makes a technological choice: it can choose an open standard, in which case consumers can mix and match developed products as they like (e.g., combine A and B', for value $v + \Delta$); alternatively, it can choose a closed standard, in which case A and B can only be consumed together (combining A and B', say, thus brings no value). The technological choice is costless. It is easy to check that the entrant always invests in both markets or none.

The entrant obtains no profit when both R&D projects fail, and 2Δ when both succeed. When a single R&D project succeeds, under a closed standard the entrant obtains again no profit. Under an open standard, the incumbent and the entrant are in a Nash demand game. As Nash observed, introducing a small noise on consumer valuation would deliver equal sharing $(v + \Delta)/2$, except that the incumbent can secure v (regardless of the entrant's price) by charging v for the monopolized component and offering the other one at cost. As $\Delta < v$, the resulting outcome is that the incumbent obtains the base value v and the entrant obtains its added value Δ.

It follows that, under an open standard, the entrant invests and obtains a profit equal to

$$\rho^2 (2\Delta) + 2\rho(1-\rho)(\Delta) + (1-\rho)^2 (0) - 2I = 2(\rho\Delta - I) > 0,$$

and the incumbent thus obtains:

$$\rho^2 (0) + 2\rho(1-\rho) v + (1-\rho)^2 (v) = \left(1 - \rho^2\right) v.$$

Under a closed standard, entry becomes riskier and the entrant does not invest, as

$$\rho^2 (2\Delta) - 2I = 2\left(\rho^2\Delta - I\right) < 0.$$

The incumbent's profit is then $v > (1-\rho^2) v$. So the incumbent is better off preventing investment by choosing the closed standard, as entry may lead to full system competition.

• *Absence of merger.* Suppose now that A and B are produced by two distinct incumbent firms, a and b, and each can choose an open or closed standard. It is then a dominant strategy for the incumbent firms to choose the open standard. For example, if A' is developed but not B', then with an open standard b can appropriate the entire surplus v: the entrant charges Δ, a charges 0, and b can charge v as consumers are willing to pay $v+\Delta$ for the pair $\{A', B\}$; with a closed standard, b can only appropriate $v/2$.[30] If A' is not developed, or both A' and B' are developed, then b's choice of technology is irrelevant. We thus conclude that no foreclosure occurs under separate ownership. Furthermore, allowing price-caps agreements does not enable the incumbents to deter entry, as the adoption of price caps can only boost the demand for the alternative components.

• *Downward sloping demand.* An elastic demand creates a private and social benefit from either a merger or price caps: the elimination of the double marginalization. However, price caps are a socially superior way of avoiding double marginalization, as they do not enable foreclosure.

To illustrate this, suppose that:

- There are two types of consumers: a fraction f have value v_H, whereas the others have value v_L, where $0 < v_L < v_H$ and there is double marginalization by independent producers:
$$v_L > f v_H > \frac{1+f}{2} v_L. \tag{10}$$
- The R&D cost can take two values, 0 (with probability γ) and I (with probability $1-\gamma$).

We consider three scenarios: (i) in the *benchmark* case, the two components are initially produced by independent firms; (ii) in case of a *merger*, these incumbents are integrated; (iii) in the *price caps* scenario, the independent incumbents can enter into price-caps agreements. The timing is as follows.

- Stage 0: incumbent firms choose between open and closed standards.
- Stage 1: the entrant decides whether to invest.
- Stage 2: R&D outcomes are observed by all firms; in the last scenario, they can moreover agree on price caps
- Stage 3: firms set their prices.

From the above analysis, when both alternative components are developed, price competition drives the incumbent firms' prices down to 0 (note that incumbents would never agree on negative price caps). When a single alternative component is developed and the monopolized component opted for an open standard (which is the case in the absence of a merger), price competition drives again the price of the incumbent component

[30] The two incumbents are again in a Nash demand game, and introducing a small noise on consumer valuation then delivers equal sharing.

down to 0. The producer of the monopolized component (or the integrated firm, in case of a merger) charges v_L as, from first inequality in (10), the corresponding profit, v_L, exceeds the profit derived from targeting the high-end segment, fv_H. The entrant obtains Δ under an open standard, and 0 otherwise.

In the absence of any alternative component, the equilibrium prices vary across scenarios. In the benchmark case, each incumbent charges $v_H/2$ and thus obtains a profit equal to $fv_H/2$. This indeed constitutes an equilibrium, as serving the low-end segment would require charging $v_L - v_H/2$ and thus generate a profit $v_L - v_H/2$, which, from the second inequality in (10), is lower than $fv_H/2$. To see that each incumbent charging $v_L/2$ and obtaining a profit $v_L/2$ is not an equilibrium,[31] it suffices to note that deviating and targeting the high-end segment would generate a profit equal to $f(v_H - v_L/2)$, which under (10) is higher than $v_L/2$.

In case of a merger, the integrated incumbent charges v_L as, from first inequality in (10), the corresponding profit, v_L, exceeds the profit derived from targeting the high-end segment, fv_H. For the same reason, in the price caps scenario, the incumbents agree on price caps equal to $v_L/2$.

Building on these insights:

- In the benchmark scenario, the entrant invests and consumers obtain an expected surplus equal to:

$$S^* = 2\rho(1-\rho) f(v_H - v_L) + \rho^2 [fv_H + (1-f) v_L].$$

- In the merger scenario, the integrated incumbent opts for a closed standard and the entrant does not invest; the merger however eliminates double marginalization and consumers thus obtain an expected surplus equal to:

$$\begin{aligned} S^m &= f(v_H - v_L) \\ &= S^* + (1-\rho)^2 f(v_H - v_L) - \rho^2 v_L. \end{aligned}$$

- In the price caps scenario, the entrant again invests *and* price caps eliminate double marginalization when R&D projects fail; as a result, consumers obtain an expected surplus equal:

$$\begin{aligned} S^p &= (1-\rho)^2 f(v_H - v_L) + 2\rho(1-\rho) f(v_H - v_L) + \rho^2 [fv_H + (1-f) v_L] \\ &= S^* + (1-\rho)^2 f(v_H - v_L) \\ &= S^m + \rho^2 v_L. \end{aligned}$$

Whether the merger benefits consumers depend on the balance between the ex-

[31] The same argument as before allows us to focus on symmetric equilibria.

pected gain from eliminating double marginalization in case of failed R&D projects, $(1-\rho)^2 f(v_H - v_L)$, and the harm resulting from the loss of competition in case of successful projects, $\rho^2 v_L$. By contrast, allowing price caps enables the firms to eliminate double marginalization without giving them incentives to opt for closed standards and deter entry in the other component. Hence, price caps do benefit consumers, and constitute a better alternative to mergers.

• *Bundling and price squeezes.*

Finally, let us ignore technological choices (that is, the only standard is an open one) but assume that the incumbent can commit to specific pricing policies.[32] A first option it to engage in *pure bundling* (i.e., to sell the two products only as a bundle). This is irrelevant when the entrant develops 0 or 2 products, but forces consumers to buy the bundle $\{A, B\}$ when the entrant develops only one product; contrary to the case of a closed standard, the entrant can still sell its component (which consumers can use as a replacement of the bundled component), but the profitability of doing so however depends on the level of production costs. To see this, suppose now that all goods are produced at the same constant unit cost c, and interpret the above values as "net" of this cost. Absent entry, the incumbent sells the bundle at price $2c + v$; when instead the entrant develops both products, it sells them at total price $2c + \Delta$. Consider now the case when A', say, is developed but not B'. Absent bundling, the incumbent sells B at price $c + v$ and the entrant sells A' at price $c + \Delta$. In case of bundling, the incumbent sells A and B at bundled price $2c + v$; the entrant can then sell A' at price Δ, but earns a profit of $\Delta - c$ (instead of Δ, absent bundling). Thus, when c is high (e.g., $c > \Delta$), bundling plays the same role of a closed standard: it deprives the entrant of a profit when it develops a single product, and therefore deters entry.

Another option for the incumbent is to commit to (entry-contingent) prices. Even if it is constrained by a system-wide no-loss condition (e.g., if regulators can demonstrate the existence of a financial loss, although not on a given product line, cross-subsidies being hard to monitor), it can use this instrument to extract (all of part of) the added value brought by the entrant when a single R&D project succeeds. For example, when A' is developed but not B', offering A at below-cost price $c - s$, where $s \in [0, \Delta]$, forces the entrant to sell A' at price $\Delta - s$. Opting for s close to Δ thus acts like bundling or the choice an incompatible technology: the price squeeze deprives the entrant of any profit when it develops a single product and not the entire system itself, which deters entry. However, as entry is welfare-enhancing, a better option consists in setting s so as to induce the entrant to invest, and appropriate all or most of the expected profit.[33]

• *Sequential entry.* Carlton and Waldman (2002) consider a related setting, and show that an integrated incumbent may again deter entry when it is sequential rather than

[32] One may have in mind an incumbent facing repeated entry in new segments and developing a reputation for these practices.

[33] That is, s should be set to that $\rho(1-\rho)s = \rho\Delta - I$.

uncertain. To see this, consider a two-period ($t = 1, 2$) variant of the above setting in which: (i) R&D is always successful (i.e., $\rho = 1$); and (ii) developing B' is possible in both periods, at cost I_B, whereas developing A' can only take place in period 2, at cost I_A. In this case, if the development costs satisfy $I_A/\Delta < 1$ and $I_B/\Delta < 1 + \delta$, then with an open standard E develops B' in period 1 and A' in period 2, but if in addition $(I_A + I_B)/\Delta > 2$, then, with a closed standard, E does not develop any product. By contrast, in case of independent incumbents, b's choice of standard is irrelevant, and a opts for an open standard, inducing entry, in order to appropriate the full value v in the first period: in this way, a obtains a profit equal to v, which exceeds its total discounted profit under foreclosure, which is equal to $(1 + \delta) v/2$.

J Repeated interaction in the technology adoption model

Suppose that the firms play the technology adoption game repeatedly, with discount factor $\delta \in (0, 1)$. Let

$$v \equiv (1 - \delta) \Sigma_{t \geq 0} \delta^t \frac{\pi_1^t + \pi_2^t}{2}$$

denote the average of firms' discounted profits over a pure-strategy equilibrium path, $\mathcal{V}^+$ denote the set of these equilibrium payoffs that are weakly more profitable than Nash (i.e., such that $v \geq \pi^N$), and v^* denote the maximal equilibrium payoff.[34] Tacit coordination raises profits only if $v^* > \pi^N$.

The location of e affects not only the nature of tacit coordination, but also the minmax profit:

Lemma 6 (minmax) *Let $\underline{\pi}$ denote the minmax profit.*

(i) If $e \leq \hat{p}$, the static Nash equilibrium (e, e) gives each firm the minmax profit: $\underline{\pi} = \pi^N = \pi(e)$.

(ii) If $e > \hat{p}$, the minmax profit is the incomplete-technology per-period monopoly profit: $\underline{\pi} = \tilde{\pi}^M(e) < \pi^N = \pi(\hat{p})$.

Proof. To establish part (i), note that firm i can secure its presence in the users' basket by charging e, thus obtaining $eD(e + p_j)$ if $p_j \leq e$ and $eD(2e)$ if $p_j > e$. Either way it can secure at least $\pi(e) = eD(2e)$. Because for $e \leq \hat{p}$ this lower bound is equal to the static Nash profit, we have $\underline{\pi} = \pi^N = \pi(e)$.

[34] This maximum is well defined, as the set $\mathcal{V}^+$ of Nash-dominating subgame perfect equilibrium payoffs is non-empty (it includes π^N) and compact (see Mailath and Samuelson (2006), chapter 2). Also, although we restrict attention to pure-strategy subgame perfect equilibria here, the analysis could be extended to public mixed strategies (where players condition their strategies on public signals) or, in the case of private mixed strategies, to perfect public equilibria (relying on strategies that do not condition future actions on private past history); see Mailath and Samuelson (2006), chapter 7.

We now turn to part (ii). If firm j sets a price $p_j \geq e$, firm i can obtain at most $\max_{p \leq p_j} pD(e+p) = \tilde{\pi}^M(e)$ (as $\tilde{p}^M(e) = r(e) < \hat{p} < e \leq p_j$). Setting instead a price $p_j < e$ allows firm i to obtain at least $\max_{p \leq e} pD(p_j + p) > \max_{p \leq e} pD(e+p) = \tilde{\pi}^M(e)$. Therefore, setting any price above e minmaxes firm i, which then obtains $\tilde{\pi}^M(e)$. ■

Hence, when $e \leq \hat{p}$, the static Nash equilibrium (e, e) yields the minmax profit; it thus constitutes the toughest punishment for both firms. When instead $e > \hat{p}$, each firm can guarantee itself the incomplete-technology monopoly profit $\tilde{\pi}^M(e)$, which is then lower than the profit of the static Nash equilibrium $(\hat{p}, \hat{p})$; Abreu (1988)'s optimal penal codes can then be used to sustain the toughest punishment.

We now characterize the scope for tacit coordination in the case of rivalry and of complementors.

a) Rivalry: $p^N < p^M$ This case arises when $e < p^M$, implying $p^N = e$ and $\underline{\pi} = \pi^N = \pi(e)$; collusion then implies selling the incomplete technology, and the loss in demand due to partial consumption grows with essentiality. In particular, if e is close to p^M, the Nash equilibrium payoff $\pi(e)$ approaches the highest possible profit π^M, whereas pricing above e substantially reduces the demand for the patents; as each firm can guarantee itself $\pi(e)$, there is no collusion. Specifically, this occurs when patents are weak substitutes, namely, when $e \geq \underline{e}$, where $\underline{e}$ is the unique solution to

$$\tilde{\pi}^M(\underline{e}) = 2\pi(\underline{e}).$$

By contrast, for e close to 0, this loss in demand is small and the Nash profit is negligible; and so collusion, if feasible, is attractive for the firms. Because users then buy only one license, each firm can attract all users by slightly undercutting the collusive price. Like in standard Bertrand oligopolies, *maximal* collusion (on $\tilde{p}^M(e)$) is sustainable whenever *some* collusion is sustainable. As symmetric collusion is easier to sustain, and deviations are optimally punished by reverting to static Nash behavior, such collusion is indeed sustainable if:

$$\frac{\tilde{\pi}^M(e)}{2} \geq (1-\delta)\,\tilde{\pi}^M(e) + \delta\pi(e) \iff \delta \geq \delta^R(e) \equiv \frac{1}{2}\frac{1}{1 - \frac{\pi(e)}{\tilde{\pi}^M(e)}}, \tag{11}$$

where $\delta^R(e)$ is increasing in e and exceeds 1 for $e \geq \underline{e}$. Building on these insights, we have:

Proposition 19 (rivalry) *When $e < \underline{e}$ and $\delta \geq \delta^R(e)$, $\mathcal{V}^+ = \left[\pi^N, v^*\right]$, and $v^* = \tilde{\pi}^M(e)/2$: tacit collusion is feasible and the most profitable collusion occurs at price $\tilde{p}^M(e)$; otherwise, the unique equilibrium is the repetition of the static Nash one.*

Proof. Let $\pi_i(p_i, p_j)$ denote firm i's profit. Prices such that $\min\{p_1, p_2\} \leq e$ cannot yield greater profits than the static Nash:

- If $p_1, p_2 \leq e$, total price P is below $2e$; as the aggregate profit $PD(P)$ is concave in P and maximal for $P^M = 2p^M > 2e$, total profit is smaller than the Nash level.

- If instead $p_i \leq e < p_j$, then

$$\pi_1(p_1, p_2) + \pi_2(p_2, p_1) = p_i D(e + p_i) \leq eD(2e) \leq 2eD(2e) = 2\pi^N,$$

  where the first inequality stems from the fact that the profit $\tilde{\pi}(p) = pD(e+p)$ is concave in p and maximal for $\tilde{p}^M(e) = r(e)$, which exceeds e in the rivalry case (as then $e < p^M < \hat{p} = r(\hat{p})$).

Therefore, to generate more profits than the static Nash profit in a given period, both firms must charge more than e; this, in turn, implies that users buy at most one license, and thus aggregate profits cannot exceed $\tilde{\pi}^M(e)$. It follows that collusion cannot enhance profits if $\tilde{\pi}^M(e) \leq 2\pi^N = 2\pi(e)$. Keeping V and thus p^M constant, increasing e from 0 to p^M decreases $\tilde{\pi}^M(e) = \max_p pD(p+e)$ but increases $\pi(e)$; as $\tilde{\pi}^M(0) = 2\pi\left(p^M\right) = 2\pi^M$, there exists a unique $\underline{e} < p^M$ such that, in the range $e \in \left[0, p^M\right]$, $\tilde{\pi}^M(e) < 2\pi^N$ if and only if $e > \underline{e}$.

Thus, when $e > \underline{e}$, the static Nash payoff π^N constitutes an upper bound on average discounted equilibrium payoffs. But the static Nash equilibrium here yields minmax profits, and thus also constitutes a lower bound on equilibrium payoffs. Hence, π^N is the unique average discounted equilibrium payoff, which in turn implies that the static Nash outcome must be played along any equilibrium path.

Consider now the case $e < \underline{e}$, and suppose that collusion raises profits: $v^* > \pi^N$, where, recall, v^* is the maximal average discounted equilibrium payoff. As v^* is a weighted average of per-period profits, along the associated equilibrium path there must exist some period $\tau \geq 0$ in which the aggregate profit, $\pi_1^\tau + \pi_2^\tau$, is at least equal to $2v^*$. This, in turn, implies that users must buy an incomplete version of the technology; thus, there exists p^* such that:

$$\tilde{\pi}(p^*) = \pi_1^\tau + \pi_2^\tau \geq 2v^*.$$

By undercutting its rival, each firm i can obtain the whole profit $\tilde{\pi}(p^*)$ in that period; as this deviation could at most be punished by reverting forever to the static Nash behavior, a necessary equilibrium condition is, for $i = 1, 2$:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\tilde{\pi}(p^*) + \delta\underline{\pi},$$

where $v_i^{\tau+1}$ denotes firm i's continuation equilibrium payoff from period $\tau + 1$ onwards. Combining these conditions for the two firms yields:

$$(1-\delta)\tilde{\pi}(p^*) + \delta\underline{\pi} \leq (1-\delta)\frac{\pi_1^\tau + \pi_2^\tau}{2} + \delta\frac{v_1^{\tau+1} + v_2^{\tau+1}}{2} \leq (1-\delta)\frac{\tilde{\pi}(p^*)}{2} + \delta\frac{\tilde{\pi}(p^*)}{2},$$

where the second inequality stems from $v_1^{\tau+1} + v_2^{\tau+1} \leq 2v^* \leq \pi_1^\tau + \pi_2^\tau = \tilde{\pi}(p^*)$. This condition amounts to

$$\left(\delta - \frac{1}{2}\right) \tilde{\pi}(p^*) \geq \delta \underline{\pi} = \delta \pi(e), \tag{12}$$

which requires $\delta \geq 1/2$ (with a strict inequality if $e > 0$). This, in turn, implies that (12) must hold for $\tilde{\pi}^M(e) = \max_p \tilde{\pi}(p)$:

$$\left(\delta - \frac{1}{2}\right) \tilde{\pi}^M(e) \geq \delta \pi(e). \tag{13}$$

Conversely, if (13) is satisfied, then the stationary path $\left(\tilde{p}^M(e), \tilde{p}^M(e)\right)$ (with equal market shares) is an equilibrium path, as the threat of reverting to the static Nash behavior ensures that no firm has an incentive to deviate:

$$\frac{\tilde{\pi}^M(e)}{2} \geq (1-\delta)\tilde{\pi}^M(e) + \delta\pi(e),$$

or

$$\delta \geq \delta^R(e) \equiv \frac{1}{2}\frac{1}{1 - \frac{\pi(e)}{\tilde{\pi}^M(e)}}.$$

Finally, $\delta^R(e)$ increases with e, as $\pi(e)$ increases with e in that range, whereas $\tilde{\pi}^M(e) = \max_p\{pD(p+e)\}$ decreases as e increases. ■

Hence, greater essentiality hinders collusion, which is not feasible if $e \geq \underline{e}$; furthermore, as the threshold $\delta^R(e)$ increases with e, for any given $\delta \in (1/2, 1)$, in the entire rivalry range $e \in \left[0, p^M\right]$ there exits a unique $\hat{e}(\delta) \in (0, \underline{e})$ such that collusion is feasible if and only if $e < \hat{e}(\delta)$. This is because the toughest punishment, given by the static Nash profit, becomes less effective as essentiality increases; although the gains from deviation also decrease, which facilitates collusion, this effect is always dominated.

b) Complementors: $p^M < p^N$ This case arises when $e > p^M$. Like when $e \in [\underline{e}, p^M]$, selling the incomplete technology cannot be more profitable than the static Nash outcome.[35] Firms can however increase their profit by *lowering* their price below the Nash level. Furthermore, when demand is convex, it can be checked that cooperation on some total price $P < 2e$ is easiest when it is symmetric (i.e., when $p_i = P/2$). As $p^N = \min\{e, \hat{p}\}$, we can distinguish two cases:

- *Weak complementors:* $e < \hat{p}$, in which case $p^M < p^N \equiv e$. The static Nash equilibrium $p^N = e$ still yields minmax profits and thus remains the toughest punishment in case of deviation. As $p_j \leq e < \hat{p} = r(\hat{p}) < r(p_j)$, firm i's best deviation then consists

[35] This follows from Lemma 6 for $e > \hat{p}$. For $p^M < e \leq \hat{p}$, we have $\tilde{p}^M(e) = r(e) \geq r(\hat{p}) = \hat{p} \geq e$ and thus $e + \tilde{p}^M(e) \geq 2e > 2p^M = P^M$; hence, $\tilde{\pi}^M(e) = \tilde{p}^M(e) D\left(e + \tilde{p}^M(e)\right) < \left(e + \tilde{p}^M(e)\right) D\left(e + \tilde{p}^M(e)\right) \leq 2eD(2e)$, where the first inequality stems from $e > 0$ and the second one from the fact that the aggregate profit $PD(P)$ is concave in P and maximal for $P^M < e + \tilde{p}^M(e)$.

in charging e. In particular, *perfect* cooperation on p^M is sustainable if and only if:

$$\pi^M \geq (1-\delta)\, eD\left(p^M + e\right) + \delta\pi(e), \tag{14}$$

which is satisfied for δ close enough to 1.

The following proposition characterizes the scope for tacit coordination in this case:

Proposition 20 (weak complementors) *When $p^M < e \leq \hat{p}$:*

(*i*) *Perfect cooperation on price p^M is feasible (i.e., $v^* = \pi^M$) if and only if*

$$\delta \geq \overline{\delta}^C(e) \equiv \frac{eD\left(p^M + e\right) - \pi^M}{eD\left(p^M + e\right) - \pi(e)},$$

where $\overline{\delta}^C(e)$ lies strictly below 1 for $e > p^M$, and is decreasing for e close to p^M.

(*ii*) *Furthermore, if $D'' \geq 0$, then profitable cooperation is sustainable (i.e., $v^* > \pi^N$) if and only if*

$$\delta \geq \underline{\delta}^C(e),$$

where $\underline{\delta}^C(e)$ lies below $\overline{\delta}^C(e)$, is decreasing in e, and is equal to 0 for $e = \hat{p}$. The set of sustainable Nash-dominating per-firm payoffs is then $\mathcal{V}^+ = [\pi(e), v^*(e,\delta)]$, where $v^*(e,\delta) \in (\pi(e), \pi^M]$ is (weakly) increasing in δ.

Proof. (*i*) That perfect cooperation (on $p_i^t = p^M$ for $i = 1,2$ and $t = 0,1,\ldots$) is sustainable if and only if

$$\delta \geq \overline{\delta}^C(e) = \frac{eD\left(p^M + e\right) - \pi^M}{eD\left(p^M + e\right) - \pi(e)} = \frac{1}{1 + \frac{\pi^M - \pi(e)}{eD(p^M+e) - \pi^M}}$$

derives directly from (14).

For $e \in (p^M, \hat{p}]$, $\pi^M > \pi(e)$ and $eD\left(p^M + e\right) > \pi^M$ (as $r\left(p^M\right) > r(e) \geq \hat{p} \geq e$); therefore, $\overline{\delta}^C(e) < 1$. Also, for ε positive but small, we have:

$$\overline{\delta}^C\left(p^M + \varepsilon\right) \simeq \frac{1}{1 - \frac{\pi''(p^M)}{D(2p^M) + p^M D'(2p^M)} \frac{\varepsilon}{2}},$$

which decreases with ε, as $\pi''\left(p^M\right) < 0$ and

$$D\left(2p^M\right) + p^M D'\left(2p^M\right) = -p^M D'\left(2p^M\right) > 0.$$

(*ii*) Suppose that collusion enhances profits: $v^* > \pi^N = \pi(e)$. In the most profitable collusive equilibrium, there exists again some period τ in which the average profit is at least v^*. And as $v^* > \pi(e) > \tilde{\pi}^M(e)/2$,[36] users must buy the complete technology in that period; thus, each firm i must charge a price p_i^τ not exceeding e, and the average price $p^\tau = \frac{p_1^\tau + p_2^\tau}{2}$ must moreover satisfy

$$\pi(p^\tau) = \frac{\pi_1^\tau + \pi_2^\tau}{2} \geq v^*.$$

As $p_j^\tau \leq e \leq \hat{p} = r(\hat{p}) \leq r(p_j^\tau)$, firm i's best deviation consists in charging e. Hence, to ensure that firm i has no incentive to deviate, we must have:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta) eD\left(p_j^\tau + e\right) + \delta \underline{\pi}.$$

Combining these conditions for the two firms yields, using $\pi(p^\tau) = \frac{\pi_1^\tau + \pi_2^\tau}{2}$ and $\underline{\pi} = \pi(e)$:

$$(1-\delta) e \frac{D(p_1^\tau + e) + D(p_2^\tau + e)}{2} + \delta\pi(e) \leq (1-\delta)\pi(p^\tau) + \delta \frac{v_2^{\tau+1} + v_2^{\tau+1}}{2} \leq \pi(p^\tau),$$

where the second inequality stems from $\frac{v_1^{\tau+1} + v_2^{\tau+1}}{2} \leq v^* \leq \pi(p^\tau)$. If the demand function is (weakly) convex (i.e., $D'' \geq 0$ whenever $D > 0$), then this condition implies $H(p^\tau; e, \delta) \geq 0$, where

$$H(p; e, \delta) \equiv \pi(p) - (1-\delta) eD(p+e) - \delta\pi(e). \tag{15}$$

Conversely, if $H(p^*; e, \delta) \geq 0$, then the stationary path (p^*, p^*) is an equilibrium path.

Summing-up, when $D'' \geq 0$, $v^* > \pi^N$ if and only if there exists $p^* < e$ satisfying $\pi(p^*) > \pi^N$ and $H(p^*; e, \delta) \geq 0$. By construction, $H(e; e, \delta) = 0$. In addition,

$$\frac{\partial H}{\partial p}(p; e, \delta) = D(2p) + 2pD'(2p) - (1-\delta) eD'(p+e).$$

Hence, $D'' \geq 0$ and Assumption A (which implies that $PD'(P)$ decreases with P) ensure that

$$\frac{\partial^2 H}{\partial p^2}(p; e, \delta) < 0.$$

Therefore, if $J(e, \delta) \geq 0$, where:

$$J(e, \delta) \equiv \frac{\partial H}{\partial p}(e; e, \delta) = D(2e) + (1+\delta) eD'(2e),$$

then no cooperation is feasible, as then $H(p; e, \delta) < 0$ for $p < e$. Conversely, if $J(e, \delta) < 0$, then tacit cooperation on p^* is feasible for $p^* \in \left[\underline{p}(e, \delta), e\right]$, where $p = \underline{p}(e, \delta)$ is the unique solution (other than $p = e$) to $H(p; e, \delta) = 0$. Note that

$$\frac{\partial J}{\partial \delta}(e, \delta) = eD'(2e) < 0,$$

[36] See footnote 35.

and

$$J(e,0) = D(2e) + eD'(2e) \geq 0,$$

as $e \leq \hat{p} \leq r(e)$, whereas

$$J(e,1) = D(2e) + 2eD'(2e) < 0,$$

as $e > p^M$. Therefore, there exists a unique $\underline{\delta}^C(e)$ such that tacit cooperation can be profitable for $\delta > \underline{\delta}^C(e)$. Furthermore, Assumption A implies that $eD'(2e)$ is decreasing and so

$$\frac{\partial J}{\partial e}(e,\delta) = 2D'(2e) + (1+\delta)\frac{d}{de}(eD'(2e)) < 0.$$

Hence the threshold $\underline{\delta}^C(e)$ decreases with e; furthermore, $\underline{\delta}^C(\hat{p}) = 0$, as $J(\hat{p},0) = D(2\hat{p}) + \hat{p}D'(2\hat{p}) = 0$ (as $\hat{p} = r(\hat{p})$).

Finally, when $\delta > \underline{\delta}^C(e)$, the set of sustainable Nash-dominating per-firm payoffs is $[\pi(e), v^*(e,\delta)]$, where $v^*(e,\delta) \equiv \pi\left(\max\left\{p^M, \underline{p}(e,\delta)\right\}\right)$, and $\underline{p}(e,\delta)$ is the lower solution to $H(p;e,\delta) = 0$; as H increases in δ,[37] $\underline{p}(e,\delta)$ decreases with δ and thus $v^*(e,\delta)$ weakly increases with δ. ■

- *Strong complementors:* $e > \hat{p}$, in which case $p^M < p^N = \hat{p}$. Starting from a symmetric price $p \in \left[p^M, p^N\right]$, the best deviation profit is then given by $\max_{\tilde{p} \leq e} \tilde{p}D(\tilde{p}+p)$. The static Nash equilibrium $(\hat{p},\hat{p})$ however no longer yields the minmax payoff, equal here to the incomplete-technology monopoly profit: $\underline{\pi} = \tilde{\pi}^M(e)$; Abreu (1988)'s optimal penal codes then provide more severe punishments than the static Nash outcome. If firms are sufficiently patient, these punishments can be as severe as the minmax profits,[38] in which case perfect cooperation on p^M is sustainable if in addition:

$$\pi^M \geq (1-\delta)\max_{\tilde{p} \leq e} \tilde{p}D(\tilde{p}+p) + \delta\tilde{\pi}^M(e).$$

In order to characterize the scope for tacit coordination in this case, we first show that Abreu's penal codes (even when restricting attention to symmetric on- and off-equilibrium paths) can sustain minmax profits when firms are sufficiently patient:

Lemma 7 (minmax with strong complementors) *The minmax payoff is sustainable whenever*

$$\delta \geq \underline{\delta}(e) \equiv \frac{\tilde{\pi}^M(e) - \pi(e)}{\pi(\hat{p}) - \pi(e)},$$

where $\underline{\delta}(e) \in (0,1)$ *for* $e \in (\hat{p}, V)$, *and* $\underline{\delta}(V) = \lim_{e \longrightarrow \hat{p}} \underline{\delta}(e) = 0$.

[37] For any $p < e$:

$$\frac{\partial H}{\partial \delta}(p;e,\delta) = e\left[D(p+e) - D(2e)\right] > 0.$$

[38] See Lemma 7 below.

Proof. In order to sustain the minmax profit $\underline{\pi} = \tilde{\pi}^M(e)$, consider the following two-phase, symmetric penal code. In the first phase (periods $t = 1, ..., T$ for some $T \geq 1$), both firms charge e, so that the profit is equal to $\pi(e)$. In the first period of the second phase (i.e., period $T+1$), with probability $1-x$ both firms charge e, and with probability x they switch to the best collusive price that can be sustained with minmax punishments, which is defined as:

$$p^C(e, \delta) \equiv \arg\max_p pD(2p),$$

subject to the constraint

$$(1-\delta) \max_{\tilde{p} \leq e} \tilde{p}D(p+\tilde{p}) + \delta \underline{\pi} \leq pD(2p). \quad (16)$$

Then, in all following periods, both firms charge p^C. Letting $\Delta = (1-\delta)x\delta^T + \delta^{T+1} \in (0, \delta)$ denote the fraction of (discounted) time in the second phase, the average discounted per-period punishment profit is equal to

$$\pi^p = (1-\Delta)\pi(e) + \Delta\pi\left(p^C\right),$$

which ranges from $\pi(e) < \underline{\pi} = \tilde{\pi}^M(e)$ (for $T = +\infty$) to $(1-\delta)\pi(e) + \delta\pi\left(p^C\right)$ (for $T = 1$ and $x = 1$). Thus, as long as this upper bound exceeds $\tilde{\pi}^M(e)$, there exists $T \geq 1$ and $x \in [0, 1]$ such that the penal code yields the minmax: $\pi^p = \tilde{\pi}^M(e) = \underline{\pi}$.

As p^C satisfies (16), the final phase of this penal code (for $t > T+1$, and for $t = T+1$ with probability x) is sustainable. Furthermore, in the first $T+1$ periods the expected payoff increases over time (as the switch to p^C comes closer), whereas the maximal profit from a deviation remains constant and equal to $\max_{p \leq e} pD(e+p) = \tilde{\pi}^M(e)$ (as $\tilde{p}^M(e) = r(e) < e$ for $e > \hat{p}$). Hence, to show that the penal code is sustainable it suffices to check that firms have no incentive to deviate in the first period, which is indeed the case if deviations are punished with the penal code:

$$\tilde{\pi}^M(e) = (1-\Delta)\pi(e) + \Delta\pi\left(p^C\right) \geq (1-\delta)\tilde{\pi}^M(e) + \delta\tilde{\pi}^M(e) = \tilde{\pi}^M(e).$$

There thus exists a penal code sustaining the minmax whenever the upper bound $(1-\delta)\pi(e) + \delta\pi\left(p^C\right)$ exceeds $\tilde{\pi}^M(e)$; as by construction $\pi\left(p^C\right) \geq \pi^N = \pi(\hat{p})$, this is in particular the case whenever

$$(1-\delta)\pi(e) + \delta\pi(\hat{p}) \geq \tilde{\pi}^M(e),$$

which amounts to $\delta \geq \underline{\delta}(e)$. Finally:

- $\underline{\delta}(e) \in (0, 1)$ for any $e \in (\hat{p}, V)$, as then:

$$\pi(\hat{p}) = \max_p pD(\hat{p}+p) > \tilde{\pi}^M(e) = \max_p pD(e+p) > \pi(e) = eD(2e);$$

- $\underline{\delta}(V) = 0$, as $\tilde{\pi}^M(V) = \pi(V) = 0$, and

$$\lim_{e \longrightarrow \hat{p}} \frac{\tilde{\pi}^M(e) - \pi(e)}{\pi(\hat{p}) - \pi(e)} = \left. \frac{\frac{d\tilde{\pi}^M(e)}{de} - \frac{d\pi(e)}{de}}{-\frac{d\pi(e)}{de}} \right|_{e=\hat{p}} = \frac{D(2\hat{p}) + \hat{p}D'(2\hat{p})}{D(2\hat{p}) + 2\hat{p}D'(2\hat{p})} = 0,$$

  where the last equality stems from $\hat{p} = r(\hat{p}) = \arg\max_p pD(\hat{p} + p)$.

■

The following proposition now characterizes the scope for tacit coordination in case of strong complementors:

Proposition 21 (strong complementors) *When $e > \hat{p}$:*

(i) $v^* > \pi^N$: some profitable cooperation is always sustainable. Perfect cooperation on price p^M is feasible (i.e., $v^* = \pi^M$) if $\delta \geq \overline{\delta}^C(e)$, where $\overline{\delta}^C(e)$ continuously prolongs the function defined in Proposition 20, lies strictly below 1, and is decreasing for e close to V.

(ii) Furthermore, if $D'' \geq 0$, then there exists $v^*(e,\delta) \in (\pi^N, \pi^M]$, which continuously prolongs the function defined in Proposition 20 and is (weakly) increasing in δ, such that the set of Nash-dominating sustainable payoffs is $\mathcal{V}^+ = [\pi(\hat{p}), v^*(e,\delta)]$.

Proof. (i) We first show that, using reversal to Nash as punishment, firms can always sustain a stationary, symmetric equilibrium path in which they both charge constant price $p < \hat{p}$, for p close enough to $\hat{p}$. This amounts to $\hat{K}(p; e, \delta) \geq 0$, where

$$\hat{K}(p; e, \delta) \equiv \pi(p) - (1 - \delta)\pi^D(p; e) - \delta\pi(\hat{p}),$$

where

$$\pi^D(p; e) \equiv \max_{\tilde{p} \leq e} \tilde{p}D(p + \tilde{p}) = \begin{cases} r(p)D(p + r(p)) & \text{if} \quad r(p) \leq e, \\ eD(p + e) & \text{if} \quad r(p) > e. \end{cases}$$

Because $\pi^D(\hat{p}; e) = \pi(\hat{p})$, $\hat{K}(\hat{p}; e, \delta) = 0$ for any e, δ. Furthermore:

$$\frac{\partial \hat{K}}{\partial p}(\hat{p}; e, \delta) = \pi'(\hat{p}) - (1 - \delta)\hat{p}D'(2\hat{p}),$$

which using $\pi'(\hat{p}) = \hat{p}D'(2\hat{p})$, reduces to:

$$\frac{\partial \hat{K}}{\partial p}(\hat{p}; e, \delta) = \delta\hat{p}D'(2\hat{p}) < 0.$$

Hence, for p close to $\hat{p}$, $\hat{K}(p; e, \delta) > 0$ for any $\delta \in (0, 1]$. If follows that cooperation on such price p is always sustainable, and thus $v^* > \pi^N$.

We now turn to perfect cooperation. Note first that it can be sustained by the minmax punishment $\underline{\pi} = \tilde{\pi}^M(e)$ whenever

$$\pi^M \geq (1-\delta)\,\pi^D\left(p^M; e\right) + \delta\tilde{\pi}^M(e),$$

or:

$$\delta \geq \overline{\delta}_1^C(e) \equiv \frac{\pi^D\left(p^M; e\right) - \pi^M}{\pi^D\left(p^M; e\right) - \tilde{\pi}^M(e)}.$$

Conversely, minmax punishments can be sustained using Abreu's optimal symmetric penal code whenever

$$(1-\delta)\,\pi(e) + \delta\pi^M \geq \tilde{\pi}^M(e), \tag{17}$$

or:

$$\delta \geq \overline{\delta}_2^C(e) \equiv \frac{\tilde{\pi}^M(e) - \pi(e)}{\pi^M - \pi(e)}.$$

Therefore, we can take $\overline{\delta}^C(e) \equiv \max\left\{\overline{\delta}_1^C(e), \overline{\delta}_2^C(e)\right\}$.

As $\overline{\delta}_1^C(\hat{p}) > \overline{\delta}_2^C(\hat{p}) = 0$ and $\overline{\delta}_1^C(V) > \overline{\delta}_2^C(V) = 0$, $\overline{\delta}^C(e) = \overline{\delta}_1^C(e) \geq \overline{\delta}_2^C(e)$ for e close to $\hat{p}$ and for e close to V. Furthermore, as $\tilde{\pi}^M(e)$ is continuous and coincides with $\pi(e)$ for $e = \hat{p}$, and $\pi^D\left(p^M; e\right) = eD\left(p^M + e\right)$ as long as $e < r\left(p^M\right)$ (where $r\left(p^M\right) > \hat{p}$), $\overline{\delta}_1^C(e)$ continuously prolongs the function $\overline{\delta}^C(e)$ defined in Proposition 20. Finally, both $\overline{\delta}_1^C(e)$ and $\overline{\delta}_2^C(e)$ lie below 1 (as $\tilde{\pi}^M(e) \leq \tilde{\pi}^M(\hat{p}) = \pi(\hat{p}) < \pi^M = \pi\left(p^M\right)$).

Finally, we note that

$$\overline{\delta}_1^C(e) = \frac{1}{1 + \dfrac{\pi^M - \tilde{\pi}^M(e)}{\pi^D\left(p^M; e\right) - \pi^M}}$$

decreases with e for $e \geq r\left(p^M\right)$: $\pi^D\left(p^M; e\right) = r\left(p^M\right) D\left(p^M + r\left(p^M\right)\right)$ does not vary with e whereas $\tilde{\pi}^M(e) = \max_p pD(e+p)$ decreases with e; and so $\overline{\delta}_1^C(e)$ decreases with e.

(ii) As in the case of weak complementors, selling the incomplete technology cannot be more profitable than the static Nash:

$$\tilde{\pi}^M(e) = \max_p pD(e+p) < 2\pi^N = 2\pi(\hat{p}) = 2\max_p pD(\hat{p}+p).$$

Therefore, if collusion enhances profits ($v^* > \pi^N$), there must exist some period $\tau \geq 0$ in which each firm i charges a price p_i^* not exceeding e, and the average price $p^* = \frac{p_1^*+p_2^*}{2}$ moreover satisfies

$$\pi(p^*) = \frac{\pi_1^\tau + \pi_2^\tau}{2} \geq v^*.$$

To ensure that firm i has no incentive to deviate, and for a given punishment payoff $\underline{v}$, we must have:

$$(1-\delta)\,\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\,\pi^D\left(p_j^*; e\right) + \delta\underline{v}.$$

Combining these conditions for the two firms yields:

$$(1-\delta)\frac{\pi^D\left(p_i^*;e\right)+\pi^D\left(p_j^*;e\right)}{2}+\delta\underline{v}\leq(1-\delta)\,\pi\left(p^*\right)+\delta\frac{v_2^{\tau+1}+v_2^{\tau+1}}{2}\leq\pi\left(p^*\right), \tag{18}$$

where the last inequality stems from $\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2}\leq v^*\leq\pi\left(p^*\right)$. But the deviation profit $\pi^D\left(p;e\right)$ is convex in p when $D''\geq 0$,[39] and thus condition (18) implies $\underline{K}\left(p^*;e,\delta,\underline{v}\right)\geq 0$, where

$$\underline{K}\left(p;e,\delta,\underline{v}\right)\equiv\pi\left(p\right)-(1-\delta)\max_{\tilde{p}\leq e}\tilde{p}D\left(p+\tilde{p}\right)-\delta\underline{v}. \tag{19}$$

Conversely, if $\underline{K}\left(p^*;e,\delta,\underline{v}\right)\geq 0$, then the stationary path $\left(p^*,p^*\right)$ is an equilibrium path.

For any δ, from Lemma 7 the minmax $\tilde{\pi}^M\left(e\right)$ can be used as punishment payoff for e close to $\hat{p}$; the sustainability condition then amounts to $K\left(p;e,\delta\right)\geq 0$, where

$$K\left(p;e,\delta\right)\equiv\pi\left(p\right)-(1-\delta)\max_{\tilde{p}\leq e}\tilde{p}D\left(p+\tilde{p}\right)-\delta\tilde{\pi}^M\left(e\right).$$

Using $\tilde{\pi}^M\left(e\right)=\max_p pD\left(e+p\right)$ and noting that $\hat{p}=r\left(\hat{p}\right)<e$ implies $\pi\left(\hat{p}\right)=\max_p pD\left(\hat{p}+p\right)=\max_{p\leq e}pD\left(\hat{p}+p\right)$ for $\delta>0$, we have:

$$K\left(\hat{p};e,\delta\right)=\delta\left[\max_p pD\left(\hat{p}+p\right)-\max_p pD\left(e+p\right)\right]>0.$$

Furthermore, K is concave in p if $\pi^D\left(p;e\right)$ is convex in p, which is the case when $D''\geq 0$. Thus, there exists $\underline{p}(e,\delta)\in[p^M,\hat{p})$ such that cooperation at price p is feasible if and only if $\underline{p}(e,\delta)\leq p<\hat{p}$, and the set of sustainable Nash-dominating per-firm payoffs is then $\left[\pi\left(e\right),v^*\left(e,\delta\right)\right]$, where $v^*\left(e,\delta\right)\equiv\pi\left(\max\left\{p^M,\underline{p}\left(e,\delta\right)\right\}\right)$. Furthermore, using $\tilde{p}^M\left(e\right)=r\left(e\right)<\hat{p}<e$; we have, for $p<\hat{p}<e$:

$$\frac{\partial K}{\partial\delta}\left(p;e,\delta\right)=\pi^D\left(p;e\right)-\tilde{\pi}^M\left(e\right)=\max_{\tilde{p}\leq e}\tilde{p}D\left(p+\tilde{p}\right)-\max_{\tilde{p}}\tilde{p}D\left(e+\tilde{p}\right)>0.$$

Therefore, $\underline{p}\left(e,\delta\right)$ decreases with δ, and thus $v^*\left(e,\delta\right)$ weakly increases with δ. Finally, note that $K\left(p;\hat{p},\delta\right)=H\left(p;\hat{p},\delta\right)$, where H is defined by (15); hence the function $v^*\left(e,\delta\right)$ defined here prolongs that of Proposition 20.

The function $v^*\left(e,\delta\right)=\pi\left(\max\left\{p^M,\underline{p}\left(e,\delta\right)\right\}\right)$ remains relevant as long as the minmax

[39] In the range where $r\left(p\right)<e$, $\frac{\partial\pi^D}{\partial p}\left(p;e\right)=r\left(p\right)D'\left(p+r\left(p\right)\right)$ and thus (using $-1<r'<0$):

$$\frac{\partial^2\pi^D}{\partial p^2}\left(p;e\right)=r'D'+rD''(1+r')>0.$$

In the range where $r\left(p\right)>e$, $\frac{\partial\pi^D}{\partial p}\left(p;e\right)=eD'\left(p+e\right)$ and thus π^D is convex if $D''\geq 0$. Furthermore, the derivative of π^D is continuous at $p=p_e\equiv r^{-1}\left(e\right)$:

$$\lim_{\substack{p\to p_e\\ p<p_e}}\frac{\partial\pi^D}{\partial p}\left(p;e\right)=\lim_{p\to p_e}eD'\left(p+e\right)=eD'\left(p_e+e\right)=\lim_{p\to p_e}r\left(p\right)D'\left(p+r\left(p\right)\right)=\lim_{\substack{p\to p_e\\ p>p_e}}\frac{\partial\pi^D}{\partial p}\left(p;e\right).$$

$\tilde{\pi}^M(e)$ is sustainable. When this is not the case, then $\underline{v}$ can be replaced with the lowest symmetric equilibrium payoff, which, using Abreu's optimal symmetric penal code, is of the form $(1-\delta)\pi(p^p)+\delta\pi(p^*)$, where p^p is the highest price in $[\hat{p}, e]$ satisfying $\pi^D(p^p; e) - \pi(p^p) \leq \delta[\pi(p^*) - \pi(p^p)]$, and p^* is the lowest price in $\left[p^M, \hat{p}\right]$ satisfying $\pi^D(p^*; e) - \pi(p^*) \leq \delta[\pi(p^*) - \pi(p^p)]$; we then have $v^*(e, \delta) = \pi(p^*)$ and the monotonicity stems from p^* and p^p being respectively (weakly) decreasing and increasing with δ. ■

Together, Propositions 20 and 21 lead to:

Proposition 22 (complementors) *When $p^M < p^N$:*

(i) There exists $\overline{\delta}^C(e) < 1$ and $\underline{\delta}^C(e) < \overline{\delta}^C(e)$, where $\overline{\delta}^C(e)$ is decreasing for e close to p^M, close to $\hat{p}$ and close to V, and $\underline{\delta}^C(e)$ is decreasing in e, and equal to 0 for $e = \hat{p}$, such that

- *perfect cooperation on price p^M is feasible (i.e., $v^* = \pi^M$) whenever $\delta \geq \overline{\delta}^C(e)$;*
- *profitable cooperation is sustainable (i.e., $v^* > \pi^N$) whenever $\delta \geq \underline{\delta}^C(e)$.*

(ii) Furthermore, if $D'' \geq 0$, then there exists $v^*(e,\delta) \in (\pi^N, \pi^M]$, which is (weakly) increasing in δ, such that the set of Nash-dominating sustainable payoffs is $\mathcal{V}^+ = \left[\pi^N, v^*(e,\delta)\right]$.

By contrast with the case of rivalry, where collusion inefficiently induces users to adopt the incomplete technology, avoiding double marginalization unambiguously raises profits here. It follows that some cooperation (and even perfect cooperation) is always sustainable, for any degree of essentiality, when firms are sufficiently patient; furthermore, in the case of strong complementors (i.e., $e > \hat{p}$), firms can always sustain some cooperation on a price $p < p^N = \hat{p}$, regardless of their discount factor: this is because starting from the static Nash price $\hat{p}$, a small reduction in the price then generates a first-order increase in profits, but only a second-order incentive to deviate.

K Proof of Proposition 15

When users acquire both licenses at total price P, welfare has the familiar expression:

$$W(P) = S(P) + PD(P),$$

where $S(P) \equiv \int_P^V D(\tilde{P})d\tilde{P}$. When instead users acquire a single license at price p, welfare is

$$\tilde{W}(p) = S(p+e) + pD(p+e).$$

Thus under rivalry ($e < p^M$), welfare is $W(2e)$ in the absence of collusion and $\tilde{W}(p)$ in the collusive outcome, for some $p > e$. Note that

$$\tilde{W}(p) = W(p+e) - eD(p+e).$$

This expression identifies the two facets of the collusive cost. First, the total price, $p+e$, exceeds the competitive price $2e$ as $p > e$. Second, there is a foregone surplus e on actual consumption $D(p+e)$ due to incomplete consumption. Collusion harms consumers and reduces total welfare under rivalry.

In the case of complementors, tacit coordination is profitable when firms cooperate in offering the complete technology at a price lower than the static Nash price; it then benefits users and increases total welfare.

L Proof of Propositions 16 and 17

We prove Proposition 17 in the extended setting described in Section 4.2.3, in which firms may have asymmetric offerings; this, in turn, establishes Proposition 16 for the case of symmetric offerings.

The case of complementors (part (ii), where $e_1 + e_2 \geq P^M = 2p^M$) is straightforward, as any vector of price caps $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2)$ satisfying $\bar{p}_1 + \bar{p}_2 = P^M$ and $\bar{p}_i \leq e_i$ induces $\mathbf{p} = \bar{\mathbf{p}}$ as unique continuation equilibrium: starting from any price vector $\mathbf{p} \leq \bar{\mathbf{p}}$, any firm offering $p_i < \bar{p}_i$ would have an incentive to increase its price towards $\bar{p}_i$, as (using $-1 < r'(p_j) < 0$) $\bar{p}_i + \bar{p}_j = P^M = 0 + r(0) < \bar{p}_j + r(\bar{p}_j)$ implies $\bar{p}_i < r(\bar{p}_j) \leq r(p_j)$, for $i \neq j \in \{1, 2\}$.

We now turn to the case of rivalry (part (i), where $e_1 + e_2 < P^M$). We first show that, as noted in the text, this implies that both firms are constrained in the static Nash equilibrium. Indeed, if both firms were unconstrained, then we would have $p_1^N = p_2^N = \hat{p} = r(\hat{p}) \leq e_2 \leq e_1$ and thus $e_1 + e_2 \geq 2\hat{p} > P^M$, a contradiction. If instead firm i is unconstrained whereas firm j is constrained, for some $i \neq j \in \{1, 2\}$, then $p_j^N = e_j$ and $p_i^N = r(e_j) \leq e_i$; hence, $e_i + e_j \geq r(e_j) + e_j > 0 + r(0) = P^M$, again a contradiction. Therefore, it must be the case that both firms are constrained: $p_i^N = e_i \leq r(e_j)$ for $i \neq j \in \{1, 2\}$.

This, in turn, implies that reducing prices below their Nash levels would reduce both firms' profits: for any $\mathbf{p} \leq \mathbf{p}^N = \mathbf{e} = (e_1, e_2)$, we have $p_i \leq e_i \leq r(e_j)$ for $i \neq j \in \{1, 2\}$, and thus: $\pi_i(\mathbf{p}) \leq \pi_i(p_i, e_j) \leq \pi_i(e_i, e_j)$, where the first inequality stems from $p_j \leq e_j$, and the second one from $p_i \leq e_i \leq r(e_j)$ and quasi-concavity. Furthermore, offering a price $\bar{p}_i > V$ would be irrelevant. Thus, without loss of generality, suppose now that a price cap $\bar{p}_i \in [e_i, V]$ is introduced for each patent $i = 1, 2$.

Next, we show that the minmax profits: (a) are the same as without price caps, and (b) can be sustained by the repetition of the (unconstrained) static Nash outcome,

$\mathbf{p}^N = \mathbf{e}$. To establish (a), it suffices to note that the minmaxing strategy $p_j = e_j \ (\leq \bar{p}_j)$ remains available to firm i's rival, and firm i's best response, $p_i = e_i \ (\leq \bar{p}_i)$, also remains available. To establish (b), it suffices to note that the static Nash outcome $\mathbf{p}^N = \mathbf{e}$ remains feasible, and that deviations are only more limited than in the absence of price caps.

We now show that any profitable collusion that is sustainable through price caps is also sustainable without them. Recall that the set of pure-strategy equilibrium payoffs can be characterized as the largest self-generating set of payoffs, where, as minmax profits are sustainable, a self-generating set of payoffs $\tilde{W}$ (where $\tilde{W} = W$ in the absence of price caps, and $\tilde{W} = W^c$ with price caps) is such that, for any payoff (π_1, π_2) in $\tilde{W}$, there exists a continuation payoff (π_1^*, π_2^*) in $\tilde{W}$ and a price profile $(p_1^*, p_2^*) \in \tilde{\mathcal{R}}_1 \times \tilde{\mathcal{R}}_2$, where $\tilde{\mathcal{R}}_i$ is the set of *relevant* prices for firm i (more on this below), that satisfy, for $i \neq j \in \{1, 2\}$:

$$\pi_i = (1-\delta)\, \pi_i \left(p_i^*, p_j^*\right) + \delta \pi_i^* \geq \max_{p_i \in \tilde{\mathcal{R}}_i} \pi_i \left(p_i, p_j^*\right) + \delta \underline{\pi}_i. \tag{20}$$

To establish that the equilibrium payoffs that are weakly more profitable than Nash under price caps are also equilibrium payoffs without price caps, it suffices to show that any self-generating set with price caps $(\bar{p}_1, \bar{p}_2)$ satisfying $\bar{p}_i \in [e_i, V]$ for $i = 1, 2$, is also a self-generating set in the absence of price caps.

In the absence of price caps, without loss of generality the set of relevant prices for firm i is $\mathcal{R}_i \equiv [0, V]$; when a price cap $\bar{p}_i$ is introduced, then the set of relevant prices becomes $\mathcal{R}_i^c \equiv [0, \bar{p}_i]$. Consider now a self-generating set W^c for given price caps $(\bar{p}_1, \bar{p}_2)$ satisfying $\bar{p}_i \in [e_i, V]$ for $i = 1, 2$, and given payoffs $(\pi_1, \pi_2) \in W^c$, with associated payoffs $(\pi_1^*, \pi_2^*) \in W^c$ and prices $\left(\underline{p}_1^*, \underline{p}_2^*\right) \in \mathcal{R}_1^c \times \mathcal{R}_2^c$ satisfying, for $i \neq j \in \{1, 2\}$, $\underline{p}_i^* \leq \bar{p}_i$ and

$$\pi_i = (1-\delta)\, \pi_i \left(\underline{p}_i^*, \underline{p}_j^*\right) + \delta \pi_i^* \geq \max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i \left(\underline{p}_i, \underline{p}_j^*\right) + \delta \underline{\pi}_i. \tag{21}$$

By construction, the associated price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$ also belongs to $\mathcal{R}_1 \times \mathcal{R}_2$. However, the gain from a deviation may be lower than in the absence of price caps, as the set of relevant deviating prices is smaller. To conclude the proof, we now show that, for any $\left(\underline{p}_1^*, \underline{p}_2^*\right) \in \mathcal{R}_1^c \times \mathcal{R}_2^c$ satisfying (21), there exists $(p_1^*, p_2^*) \in \mathcal{R}_1 \times \mathcal{R}_2$ satisfying

$$\pi_i = (1-\delta)\, \pi_i \left(p_i^*, p_j^*\right) + \delta \pi_i^* \geq \max_{p_i \in \mathcal{R}_i} \pi_i \left(p_i, p_j^*\right) + \delta \underline{\pi}_i. \tag{22}$$

For this, it suffices to exhibit a profile $(p_1^*, p_2^*) \in \mathcal{R}_1 \times \mathcal{R}_2$ yielding the same profits (i.e., $\pi_i \left(p_i^*, p_j^*\right) = \pi_i \left(\underline{p}_i^*, \underline{p}_j^*\right)$ for $i = 1, 2$) without increasing the scope for deviations (i.e., $\max_{p_i \in \mathcal{R}_i} \pi_i \left(p_i, p_j^*\right) \leq \max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i \left(\underline{p}_i, \underline{p}_j^*\right)$ for $i = 1, 2$). We can distinguish four cases for the associated price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$:

Case a: $\underline{p}_1^* \leq e_1, \underline{p}_2^* \leq e_2$. In that case, we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$; as firm i's profit from deviating to p_i is then given by

$$\pi_i\left(p_i, \underline{p}_j^*\right) = \begin{cases} p_i D\left(\underline{p}_j^* + p_i\right) & \text{if } p_i \le e_i \\ 0 \text{ otherwise} \end{cases},$$

the best deviation is

$$\arg\max_{p_i \le e_i} p_i D\left(\underline{p}_j^* + p_i\right) = e_i,$$

which belongs to both $\mathcal{R}_i$ and $\mathcal{R}_i^c$. Hence, $\max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \in \mathcal{R}_i} \pi_i\left(p_i, \underline{p}_j^*\right)$.

Case b: $\underline{p}_i^* - e_i \le 0 < \underline{p}_j^* - e_j$, for $i \ne j \in \{1,2\}$. In that case, the profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$ yields profits $\pi_j\left(\underline{p}_j^*, \underline{p}_i^*\right) = 0$ and $\pi_i\left(\underline{p}_i^*, \underline{p}_j^*\right) = \underline{p}_i^* D\left(e_j + \underline{p}_i^*\right)$, and best deviations are respectively given by:

$$\begin{aligned} \arg\max_{p_j} \pi_j\left(p_j, \underline{p}_i^*\right) &= \arg\max_{p_j \le e_j} p_j D\left(\underline{p}_i^* + p_j\right) = e_j, \\ \arg\max_{p_i} \pi_i\left(p_i, \underline{p}_j^*\right) &= \arg\max_{p_i \le \underline{p}_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{\underline{p}_j^* + e_i - e_j, p_i^M\right\}. \end{aligned}$$

As $e_j \in \mathcal{R}_j \cap \mathcal{R}_j^c$, $\max_{\underline{p}_j \in \mathcal{R}_j^c} \pi_j\left(\underline{p}_j, \underline{p}_i^*\right) = \max_{p_j \in \mathcal{R}_j} \pi_j\left(p_j, \underline{p}_i^*\right)$. Therefore, if $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^M\right\} \le \bar{p}_i$ (and thus $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^M\right\} \in \mathcal{R}_i \cap \mathcal{R}_i^c$), we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$, as then we also have $\max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \in \mathcal{R}_i} \pi_i\left(p_i, \underline{p}_j^*\right)$. If instead $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^M\right\} > \bar{p}_i$, then we can pick $p_i^* = \underline{p}_i^*$ and $p_j^* \in (e_j, e_j + \bar{p}_i - e_i)$:[40] the profile (p_1^*, p_2^*) yields the same profits as $\left(\underline{p}_1^*, \underline{p}_2^*\right)$, and, as the best deviations are the same, with or without price caps:

$$\begin{aligned} \arg\max_{p_j} \pi_j\left(p_j, p_i^*\right) &= \arg\max_{p_j} \pi_j\left(p_j, \underline{p}_i^*\right) = e_j \in \mathcal{R}_j \cap \mathcal{R}_j^c, \\ \arg\max_{p_i} \pi_i\left(p_i, p_j^*\right) &= \arg\max_{p_i \le p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{p_j^* + e_i - e_j, p_i^M\right\} \in \mathcal{R}_i \cap \mathcal{R}_i^c, \end{aligned}$$

as $\min\left\{p_j^* + e_i - e_j, p_i^M\right\} \le p_j^* + e_i - e_j < \bar{p}_i$.

Case c: $0 < \underline{p}_i^* - e_i = \underline{p}_j^* - e_j$. In that case, we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$, as best deviations consist in undercutting the other firm, and this is feasible with or without price caps.

Case d: $0 < \underline{p}_i^* - e_i < \underline{p}_j^* - e_j$, for $i \ne j \in \{1,2\}$. In that case, the same payoff could be sustained through $p_i^* = \underline{p}_i^*$ and $p_j^* = \underline{p}_i^* + e_j - e_i \left(< \underline{p}_j^*\right)$, with the convention that technology adopters, being indifferent between buying a single license from i or from j, all favor i: the profile (p_1^*, p_2^*) yields the same profits as $\left(\underline{p}_1^*, \underline{p}_2^*\right)$, $\pi_j = 0$ and $\pi_i = \underline{p}_i^* D\left(e_j + \underline{p}_i^*\right)$, but reduces the scope for deviations, which now boil down to undercutting

[40] This interval is not empty, as $\bar{p}_i \ge e_i$ by assumption.

the rival:

$$\begin{aligned}
\max_{p_j \in \mathcal{R}_j} \pi_j\left(p_j, p_i^*\right) &= \max_{\underline{p}_j \in \mathcal{R}_j^c} \pi_j\left(\underline{p}_j, \underline{p}_i^*\right) = \max_{p_j \leq \underline{p}_i^* + e_j - e_i} p_j D\left(e_i + p_j\right), \\
\max_{p_i \in \mathcal{R}_i} \pi_i\left(p_i, p_j^*\right) &= \max_{p_i \leq p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) \leq \max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \leq \underline{p}_j^* + e_i - e_j} p_i D\left(e_j + p_i\right).
\end{aligned}$$

This moreover implies that, as in case c above, these best deviations were already feasible with price caps. Indeed, as $p_k^* = p_h^* + e_k - e_h$, for $h \neq k \in \{1, 2\}$, we have:

$$\begin{aligned}
\arg\max_{p_j} \pi_j\left(p_j, p_i^*\right) &= \arg\max_{\underline{p}_j} \pi_j\left(\underline{p}_j, \underline{p}_i^*\right) = \arg \max_{p_j \leq \underline{p}_i^* + e_j - e_i} p_j D\left(e_i + p_j\right) = \min\left\{p_j^*, p_j^M\right\}, \\
\arg\max_{p_i} \pi_i\left(p_i, p_j^*\right) &= \arg \max_{p_i \leq p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{p_i^*, p_i^M\right\},
\end{aligned}$$

where $\min\left\{p_j^*, p_j^M\right\} \in \mathcal{R}_j \cap \mathcal{R}_j^c$, as $\min\left\{p_j^*, p_j^M\right\} \leq p_j^* < \underline{p}_j^* \in \mathcal{R}_j^c \left(\subset \mathcal{R}_j\right)$, and likewise $\min\left\{p_i^*, p_i^M\right\} \in \mathcal{R}_i \cap \mathcal{R}_i^c$, as $\min\left\{p_i^*, p_i^M\right\} \leq p_i^* = \underline{p}_i^* \in \mathcal{R}_i^c \left(\subset \mathcal{R}_i\right)$.

M Screening through independent licensing

Let us introduce a pool subject to independent licensing in the repeated game considered in Section 4.2.1. The pool sets the price of the bundle[41] and specifies a sharing rule for its dividends: some fraction $\alpha_i \geq 0$ (with $\alpha_1 + \alpha_2 = 1$) goes to firm i. In addition, each pool member can offer licenses on a stand-alone basis if it chooses to. The game thus operates as follows:

1. At date 0, the firms form a pool and fix a pool price P for the bundle, as well as the dividend sharing rule.

2. Then at dates $t = 1, 2, \ldots$, the firms non-cooperatively set prices p_i^t for their individual licenses; the profits of the pool are then shared according to the agreed rule.

We characterize below the set of equilibria that are sustainable through a pool subject to independent licensing; comparing it to the equilibria without a pool, or sustainable through a pool not subject to independent licensing, leads to the following proposition:

Proposition 23 (screening through independent licensing) *Independent licensing provides a useful but imperfect screen:*

(i) Appending independent licensing to a pool is always welfare-enhancing.

Relative to the absence of a pool:

[41] It can be checked that the firms cannot gain from asking the pool to offer unbundled prices as well.

(*ii*) *In case of complementors, a pool with independent licensing enables the firms to achieve perfect cooperation, which is welfare-enhancing.*

(*iii*) *In case of rivalry, if some collusion is already sustainable without a pool, then a pool with independent licensing enables the firms to collude more efficiently, which results in lower prices and is thus welfare-enhancing; however, there exists* $\underline{\delta}^R(e)$, *which increases from* $\underline{\delta}^R(0) = 1/2$ *to* 1 *as* e *increases from* 0 *to* p^M, *and lies strictly below* $\delta^R(e)$ *for* $e \in (0, p^M)$, *such that, for* $\delta \in [\underline{\delta}^R(e), \delta^R(e))$, *the pool raises prices by enabling the firms to collude.*

To establish this Proposition, we first characterize the scope for tacit coordination for rival and complementary patents, before drawing the implications for the impact of a pool subject to independent licensing.

M.1 Rivalry: $e < p^M$

The firms can of course collude as before, by not forming a pool or, equivalently, by setting the pool price P at a prohibitive level ($P \geq V$, say); firms can then collude on selling the incomplete technology if $\delta \geq \delta^R(e)$. Alternatively, they can use the pool to sell the bundle at a higher price:

Lemma 8 *In order to raise firms' profits, the pool must charge a price* $P^P > 2p^N = 2e$.

Proof. Suppose that the pool charges a price $P^P \leq 2e$, and consider a period t, with individual licenses offered at prices p_1^t and p_2^t. Let $p^t = \min\{p_1^t, p_2^t\}$ denote the lower one.

• Users buy the complete technology from the pool only if $P^P \leq p^t + e$; the industry profit is then $P^P D(P^P) \leq 2\pi^N = 2\pi(e)$, as the aggregate profit function $PD(P)$ is concave and maximal for $2p^M > 2e \geq P^P$.

• Users buy the complete technology by combining individual licenses only if $p_i \leq e$ for $i = 1, 2$, in which case $p_1 + p_2 \leq 2e$ and the industry profit is $(p_1 + p_2) D(p_1 + p_2) \leq 2\pi^N$.

• Finally, users buy an incomplete version of the technology only if $p^t + e \leq P^P$, which in turn implies $p^t \leq e$ (as then $p^t \leq P^P - e$, and by assumption $P^P \leq 2e$); the industry profit is then $p^t D(p^t + e) \leq (p^t + e) D(p^t + e) \leq 2\pi^N$, as $p^t + e \leq 2e$.

Therefore, the industry profit can never exceed the static Nash level. ■

Thus, to be profitable, the pool must adopt a price $P^P > 2e$. This, in turn, implies that the repetition of static Nash outcome through independent licensing remains an equilibrium: If the other firm offers $p_j^t = e$ for all $t \geq 0$, buying an individual license from firm j (corresponding to quality-adjusted total price $2e$) strictly dominates buying from the pool, and so the pool is irrelevant (firm i will never receive any dividend from the pool); it is thus optimal for firm i to set $p_i^t = e$ for all $t \geq 0$. Furthermore, this individual licensing equilibrium, which yields $\pi(e)$, still minmaxes all firms, as in every period each firm can secure $eD\left(e+\min\left\{e,p_j^t\right\}\right) \geq \pi(e)$ by undercutting the pool and offering an individual license at price $p_i^t = e$.

Suppose that tacit coordination enhances profits: $v^* > \pi^N = \pi(e)$, where v^* denotes the maximal average discounted equilibrium per firm payoff. In the associated equilibrium, there exists some period $\tau \geq 0$ in which the aggregate profit, $\pi_1^\tau + \pi_2^\tau$, is at least equal to $2v^*$. If users buy an incomplete version of the technology in that period, then each firm can attract all users by undercutting the equilibrium price; the same reasoning as before then implies that collusion on $p_i^t = \tilde{p}^M(e)$ is sustainable, and requires $\delta \geq \delta^R(e)$.

If instead users buy the complete technology in period τ, then they must buy it from the pool,[42] and the per-patent price $p^P \equiv P^P/2$ must satisfy:

$$2\pi\left(p^P\right) = \pi_1^\tau + \pi_2^\tau \geq 2v^* > 2\pi(e),$$

implying $p^P > e$. In order to undercut the pool, a deviating firm cannot charge more for its individual license than p^D, the price that leaves users indifferent between buying the incomplete technology from the firm and buying the complete technology from the pool; that is, the price p^D is such that:

$$(V-e) - p^D = V - 2p^P,$$

or $p^D = 2p^P - e\,(> e)$; by offering its individual license at this price, the deviating firm obtains a profit equal to:

$$\pi^D = \left(2p^P - e\right) D\left(2p^P\right) = \pi\left(p^P\right) + \left(p^P - e\right) D\left(2p^P\right) > \pi\left(p^P\right). \tag{23}$$

Thus, for the price p^P to be sustainable, there must exist continuation payoffs $\left(v_1^{\tau+1}, v_2^{\tau+1}\right)$ such that, for $i = 1, 2$:

$$(1-\delta)\,\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\left[\pi\left(p^P\right) + \left(p^P - e\right) D\left(2p^P\right)\right] + \delta\pi(e).$$

Combining these two conditions and using $\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2} \leq v^* \leq \frac{\pi_1^\tau+\pi_2^\tau}{2} = \pi\left(p^P\right)$ yields:

$$\pi\left(p^P\right) \geq (1-\delta)\left[\pi\left(p^P\right) + \left(p^P - e\right) D\left(2p^P\right)\right] + \delta\pi(e). \tag{24}$$

Conversely, a pool price $p^P \in (e, p^M]$ satisfying this condition is *stable*: a bundle price $P^P = 2p^P$, together with an equal profit-sharing rule and firms charging high enough individual prices (e.g., $p_i^t \geq V$ for all $t \geq 0$), ensures that no firm has an incentive to undercut the pool, and each firm obtains $\pi\left(p^P\right)$. To see this, it suffices to note

[42] Users would combine individual licenses only if the latter were offered at prices not exceeding e; hence, the total price P would not exceed $2e$. But $PD(P) = \pi_1^\tau + \pi_2^\tau \geq 2v^* > 2\pi(e)$ implies $P > 2e$.

that the expression of π^D given by (23) represents the highest deviation profit when $p^P \leq p^M$, as the deviating profit $pD(p+e)$ is concave and maximal for $\tilde{p}^M(e) = r(e)$, and $e + \tilde{p}^M(e) = e + r(e) \geq 0 + r(0) = 2p^M$ implies $\tilde{p}^M(e) > 2p^M - e \geq 2p^P - e$. Building on this insight yields:

Proposition 24 (pool in the rivalry region) *Suppose $e \leq p^M$. As before, if $\delta \geq \delta^R(e)$ the firms can sell the incomplete technology at the monopoly price $\tilde{p}^M$ and share the associated profit, $\tilde{\pi}^M$. In addition, a per-license pool price p^P, yielding profit $\pi(p^P)$, is stable if* (24) *holds. As a result:*

(i) Perfect collusion (i.e., on a pool price $p^P = p^M$) is feasible if

$$\delta \geq \bar{\delta}^P(e) \equiv \frac{1}{2 - \frac{e}{p^M - e}\frac{D(2e) - D(2p^M)}{D(2p^M)}},$$

where the threshold $\bar{\delta}^P(e)$ is increasing in e.

(ii) If the firms can already collude without a pool (i.e., if $\delta \geq \delta^R(e)$), then the pool enables them to sustain a more profitable collusion, which benefits consumers as well.

(iii) There exists $\underline{\delta}^R(e)$, which coincides with $\delta^R(e)$ for $e = 0$, and lies strictly below $\delta^R(e)$ for $e > 0$, such that some collusion (i.e., on a stable pool price $p^P \in (e, p^M]$) is feasible when $\delta \geq \underline{\delta}^R(e)$.

Proof. (i) We have established that a pool price p^P is stable if and only if $L(p^P; e, \delta) \geq 0$, where

$$\begin{aligned} L(p; e, \delta) &\equiv \pi(p) - (1-\delta)\left[\pi(p) + (p-e)D(2p)\right] - \delta\pi(e) \\ &= \delta p D(2p) - (1-\delta)(p-e)D(2p) - \delta e D(2e). \end{aligned}$$

In the particular case of perfect substitutes (i.e., $e = 0$), this expression reduces to $(2\delta - 1)\pi(p) \geq 0$. Therefore, any pool price $p^P \geq 0$ is stable – including the monopoly price p^M – if and only if $\delta \geq 1/2$. For $e > 0$, sustaining a price $p^P \in (e, p^M]$ requires $\delta > 1/2$:

$$L(p; e, \delta) = (2\delta - 1)\left[pD(2p) - eD(2e)\right] + (1-\delta)e\left[D(2p) - D(2e)\right],$$

where the second term is negative and, in the first term, $\pi(p) > \pi(e)$.

In particular, collusion on p^M is feasible if $L(p^M; e, \delta) \geq 0$, or:

$$\delta \geq \bar{\delta}^P(e) = \frac{(p^M - e)D(2p^M)}{(p^M - e)D(2p^M) + \pi^M - \pi(e)} = \frac{1}{2 - \frac{e}{p^M - e}\frac{D(2e) - D(2p^M)}{D(2p^M)}},$$

where

$$\frac{d\bar{\delta}^P}{de}\left(e,\bar{\delta}^P(e)\right)=-\frac{\dfrac{\partial L}{\partial e}\left(p^M;e,\bar{\delta}^P(e)\right)}{\dfrac{\partial L}{\partial \delta}\left(p^M;e,\bar{\delta}^P(e)\right)}.$$

Clearly $\partial L/\partial\delta>0$. Furthermore

$$\frac{\partial L}{\partial e}\left(p^M;e,\bar{\delta}^P(e)\right)=[1-\bar{\delta}^P(e)]D(2p^M)-\bar{\delta}^P(e)\pi'(e).$$

Using the fact that $L\left(p^M;e,\bar{\delta}^P(e)\right)=0$,

$$\frac{\partial L}{\partial e}\left(p^M;e,\bar{\delta}^P(e)\right)\propto[\pi^M-\pi(e)-(p^M-e)\pi'(e)]<0,$$

from the concavity of π. And so

$$\frac{d\bar{\delta}^P}{de}>0.$$

(ii) In the absence of a pool, collusion is inefficient (users buy only one license) and is therefore unprofitable (and thus unsustainable) when $\tilde{\pi}^M(e)\le 2\pi^N=2\pi(e)$ (i.e., $e\ge\underline{e}$). When instead

$$\tilde{\pi}^M(e)>2\pi^N=2\pi(e),\tag{25}$$

then (i) inefficient collusion on $p\in(e,\tilde{p}^M(e)]$ is profitable for p close enough to $\tilde{p}^M(e)$; in this case, maximal collusion (on $\tilde{p}^M(e)$) is sustainable whenever some collusion is sustainable, and it is indeed sustainable if $\delta\ge\delta^R(e)$. We now show that the pool then enables the firms to sustain a more efficient and more profitable collusion, which benefits consumers as well as the firms. To be as profitable, the pool must charge a price P^P satisfying:

$$P^P D\left(P^P\right)\ge\tilde{\pi}^M(e).$$

Let $\tilde{P}(e)$ denote the lowest of these prices, which satisfies $\tilde{P}D\left(\tilde{P}\right)=\tilde{\pi}^M(e)$.[43] The pool price $\tilde{p}(e)=\tilde{P}(e)/2$ is stable if and only if $L(\tilde{p}(e),e,\delta)\ge 0$, which amounts to:

$$\begin{aligned}0\le G(e,\delta)\;&\equiv\;\delta\tilde{p}(e)D\left(\tilde{P}(e)\right)-(1-\delta)(\tilde{p}(e)-e)D\left(\tilde{P}(e)\right)-\delta eD(2e)\\&=\;(2\delta-1)\left[\frac{\tilde{\pi}^M(e)}{2}-\pi(e)\right]+(1-\delta)e\left[D\left(\tilde{P}(e)\right)-D(2e)\right].\end{aligned}$$

[43] In the rivalry region, we have that $e<p^M<\hat{p}<r(e)=\tilde{p}^M(e)$; hence, the left-hand side increases from

$$2\pi^N=2eD(2e)<2\tilde{\pi}^M(e)=2r(e)D(e+r(e))$$

to $\Pi^M=2\pi^M>\tilde{\pi}^M(e)$ as P increases from $2e$ to $2p^M$; there thus exists a unique $P\in\left(2e,2p^M\right)$ satisfying $PD(P)=\tilde{p}^M(e)D\left(\tilde{p}^M(e)+e\right)$.

We have:

$$\begin{aligned}\frac{\partial G}{\partial \delta}(e,\delta) &= \tilde{p}(e) D\left(\tilde{P}(e)\right) + (\tilde{p}(e) - e) D\left(\tilde{P}(e)\right) - eD(2e) \\ &= \left[\frac{\tilde{\pi}^M(e)}{2} - \pi(e)\right] + \left(1 - \frac{e}{\tilde{p}(e)}\right)\frac{\tilde{\pi}^M(e)}{2} \\ &> 0,\end{aligned}$$

where the inequality follows from $\tilde{\pi}^M > 2\pi(e)$ (using (25)), which in turn implies $e < \tilde{p}(e)$ (as $2\tilde{p}D(2\tilde{p}) = \tilde{\pi}^M > 2\pi(e) = 2eD(2e)$, and the profit function $PD(P)$ is concave); as

$$G(e, 1/2) = \frac{e}{2}\left[D\left(\tilde{P}(e)\right) - D(2e)\right] < 0 < G(e,1) = \frac{\tilde{\pi}^M(e)}{2} - \pi(e),$$

where the inequalities follow again from $\tilde{\pi}^M(e) > 2\pi(e)$ and $e < \tilde{p}(e)$, then some collusion is feasible if δ is large enough, namely, if $\delta \geq \underline{\delta}_1^R(e)$, where:

$$\underline{\delta}_1^R(e) \equiv \frac{[\tilde{p}(e) - e] D\left(\tilde{P}(e)\right)}{\tilde{\pi}^M(e) - \pi(e) - eD\left(\tilde{P}(e)\right)}.$$

From the proof of Proposition 19, the inefficient collusion on $\tilde{p}^M(e)$ is instead sustainable (i.e., $\delta \geq \delta^R(e)$) when:

$$0 \leq \tilde{G}(e,\delta) \equiv (2\delta - 1)\frac{\tilde{\pi}^M(e)}{2} - \delta\pi(e).$$

In the case of perfect substitutes, this condition boils down again to $\delta \geq 1/2$. Therefore, when collusion is sustainable without the pool, the pool enables the firms to sustain perfect efficient collusion. Furthermore, for $e > 0$, $G(e,\delta) - \tilde{G}(e,\delta) = (1-\delta) eD\left(\tilde{P}(e)\right) > 0$ and thus, if some collusion is sustainable without a pool, then the pool enables again the firms to sustain a more efficient and more profitable collusion: as $G(e,\delta) > 0$ in this case, it follows that a pool price p^P slightly higher (and thus more profitable) than $\tilde{p}$ is also stable. Finally, note that the (quality-adjusted) price is lower when collusion is efficient: the most profitable sustainable price lies below P^M,[44] and

$$P^M = 0 + r(0) < e + r(e) = \tilde{p}^M(e) + e.$$

[44] A price $P^P > P^M$ cannot be the most profitable stable price:

$$L\left(p^M; e,\delta\right) - L\left(p^P; e,\delta\right) = (2\delta - 1)\left[\pi^M - \pi\left(p^P\right)\right] + (1-\delta) e\left[D\left(P^M\right) - D\left(P^P\right)\right],$$

which is positive for $P^P > P^M$, as $\pi^M \geq \pi\left(P^P\right)$ and $D\left(P^M\right) > D\left(P^P\right)$. Hence, whenever a pool price $P^P > P^M$ is stable, then $P = P^M$ is also stable.

(iii) Note that $L(e; e, \delta) = 0$ for all e. Therefore, some collusion is sustainable (i.e., there exists a stable pool price $p^P \in (e, p^M))$ whenever $I(e) > 0$, where

$$I(e, \delta) \equiv \frac{\partial L}{\partial p}(e; e, \delta) = (2\delta - 1)D(2e) + 2\delta e D'(2e).$$

We have:

$$\frac{\partial I}{\partial \delta}(e, \delta) = 2\left[D(2e) + eD'(2e)\right] > 0,$$

where the inequality follows from $e < r(e)$ (as here $e < p^M\ (< \hat{p})$); as

$$I(e, 1/2) = eD'(2e) < 0 < I(e, 1) = D(2e) + 2eD'(2e),$$

where the last inequality stems from $e < p^M$, then some collusion is feasible if δ is large enough, namely, if $\delta \geq \underline{\delta}_2^R(e)$, where:

$$\underline{\delta}_2^R(e) \equiv \frac{1}{2} \frac{1}{1 + \frac{eD'(2e)}{D(2e)}}. \tag{26}$$

Furthermore:

$$\frac{\partial I}{\partial e}(e, \delta) = 2(3\delta - 1)\left[D'(2e) + \frac{\delta}{3\delta - 1} 2eD''(2e)\right].$$

But $D'(2e) + 2eD''(2e) < 0$ from Assumption B and $\delta/(3\delta - 1) < 1$ from $\delta > 1/2$; and so

$$\frac{\partial I}{\partial e}(e, \delta) < 0,$$

implying that the threshold $\underline{\delta}_2^R(e)$ increases with e; it moreover coincides with $\delta^R(0) = 1/2$ for $e = 0$, and is equal to 1 for $e = p^M$ (in which case $D(2e) + 2eD'(2e) = 0$, and thus $I(p^M, \delta) = -(1 - \delta) D(2p^M)$).

To conclude the argument, it suffices to note that the statement of part (iii) holds for $\underline{\delta}^R(e) = \min\left\{\underline{\delta}_1^R(e), \underline{\delta}_2^R(e)\right\}$:

- For $e = 0$, perfect collusion is sustainable for $\delta \geq 1/2$, which coincides with the range where inefficient collusion at $\tilde{p}^M(e)$ would be sustainable without a pool.
- For $e \in (e, \underline{e})$ (in which case, without a pool, inefficient collusion at $\tilde{p}^M(e)$ is sustainable if and only if $\delta \geq \delta^R(e)$), $\underline{\delta}^R(e) \leq \underline{\delta}_1^R(e) < \delta^R(e)$.
- Finally, for $e \in [\underline{e}, p^M]$, no collusion is sustainable in the absence of a pool, whereas a pool enables the firms to collude on some price $p^P \in (e, p^M]$ whenever $\delta \geq \underline{\delta}^R(e)$, where $\underline{\delta}^R(e) \leq \underline{\delta}_2^R(e) < 1$.

■

Remark: If $D'' \leq 0$, then L is concave in p.[45] Hence, in that case, some collusion is feasible *if and only if* $\delta \geq \underline{\delta}_2^R(e)$, where $\underline{\delta}_2^R(e)$ lies strictly below $\delta^R(e)$ for $e \in \left(0, p^M\right)$ and increases from $\delta^R(0) = 1/2$ to 1 as e increases from 0 to p^M.

M.2 Weak or strong complementors: $p^M \leq e$

In case of complementary patents, a pool enables the firms to cooperate perfectly:

Proposition 25 (pool with complements) *With weak or strong complementors, a pool allows for perfect cooperation (even if independent licensing remains allowed) and gives each firm a profit equal to* π^M.

Proof. Suppose that the pool charges $P^M = 2p^M$ for the whole technology and shares the profit equally. No deviation is then profitable: as noted above, the best price for an individual license is then $\tilde{p} = 2p^M - e$ (that is, the pool price minus a discount reflecting the essentiality of the foregone license), which is here lower than p^M (since $p^M \leq e$) and thus yields a profit satisfying:

$$\left(2p^M - e\right) D\left(2p^M\right) < p^M D\left(2p^M\right) = \pi^M.$$

■

M.3 Impact of a pool subject to independent licensing

Comparing the most profitable equilibrium outcomes with and without a pool (subject to independent licensing) yields the following observations:

• In the rivalry region, a pool can only benefit users whenever some collusion would already be sustained in the absence of a pool (i.e., when $\delta \geq \delta^R(e)$). In this case, a pool enables the firms to sustain a more efficient collusion, which is more profitable but also benefits users: they can then buy a license for the complete technology at a price $P \leq P^M = 2p^M$, which is preferable to buying a license for the incomplete technology at price $\tilde{p}^M(e)$: as $r'(\cdot) > -1$,

$$e + \tilde{p}^M(e) = e + r(e) > 0 + r(0) = P^M = 2p^M.$$

• By contrast, when collusion could not be sustained in the absence of a pool (i.e., when $\delta < \delta^R(e)$), then a pool harms users whenever it enables the firms to sustain some collusion, as users then face an increase in the price from $p^N(e) = e$ to some $p > e$. This

[45] As $pD(2p)$ is concave from Assumption B and $\delta > 1/2$, we have:

$$\frac{\partial^2 L}{\partial p^2}(p, e, \delta) = (2\delta - 1)(pD(2p))'' + 4(1-\delta)eD''(2p) < 0.$$

happens in particular when $\delta \in \left[\underline{\delta}^R(e), \delta^R(e)\right)$ (if $D''(\cdot) \leq 0$, it happens only in this case), where $\underline{\delta}^R(e)$ increases from $\delta^R(0) = 1/2$ to 1 as e increases from 0 to p^M, and lies strictly below $\delta^R(e)$ for $e \in \left(0, p^M\right)$.

• With weak or strong complementors, a pool enables perfect cooperation and benefits users as well as the firms: in the absence of the pool, the firms would either not cooperate and thus set $p = p^N(e) = \min\{\hat{p}, e\} > p^M$, or cooperate and charge per-license price $p \in [p^M, p^N)$, as opposed to the (weakly) lower price, p^M, under a pool.

Finally, note that, in the absence of the independent licensing requirement, a pool would always enable the firms to achieve the monopoly outcome. Appending independent licensing is therefore always welfare-enhancing, as it can only lead to lower prices in the case of rivalry, and does not prevent the firms from achieving perfect cooperation in the case of complementors.

N Proof of Proposition 18

We start by noting that, if all patents are priced below $\tilde{p}$, then technology adopters acquire all licenses:

Lemma 9 *Offering each license i at a price $p_i \leq \tilde{p}$ induces users to acquire all of them.*

Proof. Without loss of generality, suppose that the patents are ranked in such a way that $p_1 \leq ... \leq p_n$. If users strictly prefer acquiring only $m < n$ licenses, we must have:

$$V(m) - \sum_{k=1}^{m} p_k > V(n) - \sum_{k=1}^{n} p_k \iff \sum_{k=m+1}^{n} p_k > V(n) - V(m).$$

From from the definition of $\tilde{p}$, we also have:

$$V(n) - n\tilde{p} \geq V(m) - m\tilde{p} \iff V(n) - V(m) \geq (n-m)\tilde{p}.$$

Combining these conditions yields:

$$\sum_{k=m+1}^{n} p_k > (n-m)\tilde{p},$$

implying that some licenses are priced strictly above p. Conversely, if all licenses are priced below $\tilde{p}$, users are willing to acquire all of them. ■

To establish part (i) of Proposition 18, suppose that $p^N < p^M$ (which implies $p^N = \tilde{p}$ and $\pi^N = \tilde{p}D(n\tilde{p})$), that each firm faces a given price cap $\bar{p}_i$, and consider a stationary symmetric path in which all firms repeatedly charge the same price p^* (which thus must satisfies $p^* \leq \bar{p}_i$ for $i \in \mathcal{N}$), and obtain the same profit $\pi^* > \pi^N = \tilde{p}D(n\tilde{p})$. We first note that this last condition requires selling an incomplete bundle:

Lemma 10 *When $p^N < p^M$, generating more profit than the static Nash level requires selling less than n licenses.*

Proof. Suppose that a price profile $(p_1, ..., p_n)$ induces users to acquire all n licenses. The aggregate profit is then $\Pi(P) = PD(P)$, where $P = \sum_{k=1}^{n} p_k$ denotes the total price. But this profit function is concave in P under Assumption B, and thus increases with P in the range $P \leq P^M = np^M$. From Lemma 9, selling all n licenses require $P \leq n\tilde{p}$, where by assumption $n\tilde{p} < P^M$; therefore, the aggregate profit $PD(P)$ cannot exceed that of the (unconstrained) static Nash, $n\tilde{p}D(n\tilde{p})$. ■

From Lemma 10, $\pi^* > \pi^N$ implies that users must buy $m^* < n$ patents; Lemma 9 then implies $p^* > \tilde{p}$; the per-firm equilibrium profit is then:

$$\pi^* = \frac{m^*}{n} p^* D(m^* p^* + V(n) - V(m^*)).$$

Furthermore, as $\bar{p}_i \geq p^* > \tilde{p}$ for all $i \in \mathcal{N}$, the price caps do not affect the static Nash equilibrium, in which all firms still charge $p^N = \tilde{p}$. The price p^* can therefore be sustained by reversal to Nash if and only if:

$$\pi^* \geq (1 - \delta) \pi^D(p^*) + \delta \pi^N,$$

where $\pi^N = \tilde{p}D(n\tilde{p})$ and $\pi^D(p^*)$ denotes the most profitable deviation from p^*, subject to charging a price $p^D \leq \bar{p}_i$. But as the deviating price must lie below p^* (otherwise, the member's patent would be excluded from users' basket), it is not constrained by the price cap $\bar{p}_i \geq p^*$; therefore, the deviation cannot be less profitable than in an alternative candidate equilibrium in which, in the absence of price caps, all members would charge p^*. Hence, price caps cannot sustain higher symmetric prices than what the firms could already sustain in a symmetric equilibrium in the absence of price caps.

To establish part (*ii*) of the Proposition, suppose that all firms face the same price cap $\bar{p} = p^M < p^N = \min\{\tilde{p}, \hat{p}\}$. As no firm can charge more than $\bar{p} < \tilde{p}$, Lemma 9 implies that, by charging $p_i = p^M$, each firm i can ensure that technology adopters acquire its license, and thus secure a profit at least equal to:

$$\pi_i = p^M D(p^M + \sum_{j \in \mathcal{N} \setminus \{i\}} p_j) \geq p^M D(p^M + (n-1)\bar{p}) = \pi^M = p^M D(P^M).$$

As each firm can secure π^M, and the industry profit is maximal for P^M, it follows that the unique candidate equilibrium is such that each firm charges $\bar{p} = p^M$. Conversely, all firms charging p^M indeed constitutes an equilibrium: a deviating firm can only charge a price $p < \bar{p} = p^M$, and the deviating profit is thus given by:

$$pD\left(p + (n-1)\, p^M\right).$$

The conclusion then follows from the fact that this profit is concave in p, and maximal for (using $r'(\cdot) < 0$ and $p^M < \hat{p}$):

$$r\left((n-1)\, p^M\right) > r\left((n-1)\, \hat{p}\right) = \hat{p} > p^M = \bar{p}.$$