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Abstract: This paper explores empirically the link between stocks returns Value-at-Risk (VaR) and the state of financial markets cycle. The econometric analysis is based on a simple vector autoregression setup. Using quarterly data from 1970Q4 to 2008Q4 for France, Germany and the United-Kingdom, it turns out that the k-year VaR of equities is actually dependent on the cycle phase: the expected losses as measured by the VaR are smaller in recession times than expansion periods, whatever the country and the horizon. These results strongly suggest that the European rules regarding the solvency capital requirements for insurance companies should adapt to the state of the financial market's cycle.

Keywords: Expected equities returns, Value at Risk, Financial cycle, Investment horizon, Vector Auto-regression.

JEL classification: G11.

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Introduction

After the renewal of banks regulatory framework with the Basel II agreement in 2003, the European Solvency II committee is currently working on new capital standards for insurance companies.

Recently, the Basel II risk-based capital requirements have been widely criticized because they could exacerbate financial cycles, or more generally business cycle fluctuations (see e.g. Kashyap and Stein [2004], Adrian and Shin [2007, 2008], Plantin, Sapra and Shin [2008], Rochet [2008]). Basically, these authors claim that solvency capital requirements (SCR hereafter) rules which do not depend on the state of the business/financial cycle may lead to large pro-cyclical leverage effects. As a result of such rules, investors demand of securities increases during financial booms which pushes stock prices even upper. Conversely, investors have to sell securities during financial downturns in order to restore their solvency ratios, which precipitates the financial recession.[1] Yet, a cyclical SCR rule allowing for smaller capital requirements during downturns could at least dampen, if not completely eliminate, this procyclical leverage effect.

Providing further support such a cyclical SCR rule, a growing empirical literature points to predictability and mean-reversion in stocks returns (see e.g. Campbell [1991], Campbell [1996], Barberis [2000], or Campbell and Viceira [2002] for U.S. data and Bec and Gollier [2007] for french data). More precisely, excess stock returns risk is found to be mean reverting in the sense that the risk associated with long holding periods is lesser than the one associated with short holding horizons as e.g. the widely scrutinized one-year horizon. Beyond this potential investment horizon effect, returns mean reversion may also imply a cyclical effect. In other words, the financial cycle's position could help predicting future returns and future risk.

Our contribution to this literature is twofold. First, we assess empirically the importance of these cyclical and investment horizon effects for European stock price data. This question is explored by modelling the joint dynamics of excess return of equities

[1] See Adrian and Shin [2007] for a very clear presentation of this procyclical leverage effect.

and an indicator of the financial market cycle from a vector autoregression model. Actually, in the recent empirical literature devoted to asset returns predictability, the vector autoregressive dynamics is often retained. The choice of this representation is basically motivated by the fact that this framework allows for straightforward computation of the conditional first and second-order moments matrices, namely the conditional mean and variance-covariance matrices. Hence, two crucial variables for dynamic portfolio allocation optimization are obtained easily — the time-t conditional expectation (forecast) and conditional variance (risk measure) for asset returns at horizon $t+h$. We also propose a measure of the Value-at-Risk based on the vector autoregression estimates. It is in line with existing measures in that it derives from the empirical distribution of the expected $k-$period returns. Nevertheless, it has the advantage of not imposing any assumption regarding the law of distribution of the sample but relies on bootstrapped quantiles instead. Our second contribution is then to propose a VaR measure which takes the influence of the recent cycle conditions into account, since it is based on the bivariate dynamics of stock returns and financial market cycle. Furthermore, we take advantage of this to propose a cycle-dependent measure of the Solvency Capital Requirement which accounts for the illiquidity risk.

Using quarterly French, German and British data from 1970Q4 on, it turns out that both cyclical and horizon effects do influence the Value-at-Risk: it is higher during booms than during recessions, and lower for long than for short investment horizons. Hence, beyond the fact that constant SCR rules may be destabilizing, they are not even justified by a constant VaR. By contrast, our findings support SCR rules which would be flexible enough so as to take these cyclical and horizon effects into account. This modification of the methodology is countercyclical: it should induce intermediaries to be more conservative in long expansionary phases and to be more risk-taking in downturns.

The paper is organized as follows. Section 1 presents the econometric methodology. Section 2 describes the data used for the vector autoregression presented in Section 3. In Section 4, estimated stocks returns VaR are compared across investment horizons and phases of financial cycle. Section 5 concludes.

1 Vector autoregression modelling of VaR

1.1 The vector autoregressive model

So as to simplify the presentation, and without loss of generality, let us consider the following vector autoregression of order one[2] :

$$\mathsf{z}_t = \Phi_0 + \Phi_1 \mathsf{z}_{t-1} + \mathsf{v}_t, \tag{1}$$

where

$$\mathsf{z}_t = \begin{bmatrix} \mathsf{x}_t \\ \mathsf{s}_t \end{bmatrix}$$

is a $m \times 1$ vector with x_t, the $n \times 1$ vector of log excess returns and s_t the $m-n-1 \times 1$ vector of variables which have been identified as financial markets cycle indicators. In equation (1), Φ_0 is the $m \times 1$ vector of intercepts and Φ_1 is the $m \times m$ matrix of slope coefficients. It is assumed that the roots of the characteristic polynomial $\Phi(z) = I_m - \Phi_1 z$ lie strictly outside the unit circle in absolute value, a condition which rules out nonstationary or explosive behavior in z_t. Finally, v_t is the $m \times 1$ vector of innovations which is assumed to be *i.i.d.* distributed with mean zero and covariance matrix Σ_v.

A very parsimonious version of this autoregressive model will be retained for the evaluation of VaR from French, German and UK data. Let R_{0t} denote the nominal short rate and $r_{0t} = \log(1 + R_{0t})$ the log (or continuously compounded) return on this asset that is used as a benchmark to compute excess returns on equities. Then, with r_{et} the log stock return, let $x_{et} = r_{et} - r_{0t}$ denote the corresponding log excess returns. Finally, let m_{ct} denote the cyclical component of the log price index, to be defined later in the paper. In our empirical work, we will estimate a vector autoregression in which $\mathsf{z}_t = (x_{et}, m_{ct})'$.

[2]The analysis can be easily extended to more than one lag.

1.2 From vector autoregression to Value-at-Risk

Following Campbell and Viceira [2004], the one-period log returns are added over k successive periods in order to get the cumulative $k-$period log returns. The one corresponding to the log excess return on equities is denoted $x_{et}^k \equiv \mathsf{x}_{e,t+1} + \cdots + \mathsf{x}_{e,t+k}$. The vector autoregression is particularly well suited for forecasting purposes. By forward recursion of equation (1), it is possible to derive the expression of $(\mathsf{z}_{t+1} + \cdots + \mathsf{z}_{t+k})$:

$$\begin{aligned}\mathsf{z}_{t+1} + \cdots + \mathsf{z}_{t+k} &= [k + (k-1)\Phi_1 + (k-2)\Phi_1^2 + \cdots + \Phi_1^{k-1}]\Phi_0 + (\Phi_1^k + \Phi_1^{k-1} + \cdots + \Phi_1)\mathsf{z}_t \\ &\quad + (1 + \Phi_1 + \cdots + \Phi_1^{k-1})\mathsf{v}_{t+1} + (1 + \Phi_1 + \cdots + \Phi_1^{k-2})\mathsf{v}_{t+2} + \cdots \\ &\quad + (1 + \Phi_1)\mathsf{v}_{t+k-1} + \mathsf{v}_{t+k},\end{aligned}$$

or equivalently:

$$\mathsf{z}_{t+1} + \cdots + \mathsf{z}_{t+k} = \left[\sum_{i=0}^{k-1}(k-i)\Phi_1^i\right]\Phi_0 + \left[\sum_{j=1}^{k}\Phi_1^j\right]\mathsf{z}_t + \sum_{q=1}^{k}\left[\sum_{p=0}^{k-q}\Phi_1^p\mathsf{v}_{t+q}\right], \tag{2}$$

where the first two terms on the RHS correspond to the $k-$period conditional mean, $E_t(\mathsf{z}_{t+1} + \cdots + \mathsf{z}_{t+k})$. Finally, the cumulative $k-$period log excess return on equities derives from equation (2) as follows:

$$x_{et}^k = M_r(\mathsf{z}_{t+1} + \cdots + \mathsf{z}_{t+k}), \tag{3}$$

where the selection matrix is defined by $M_r = [\mathbf{I}_{n\times n}\ \mathbf{0}_{n\times(m-n-1)}]$. Dividing both sides of equation (3) by k gives the annualized log excess return.

The value-at-risk obtains straightforwardly from equation (2). The VaR is basically defined as a number such that there is a probability p that a worse excess (log-)return occurs over the next k periods. As such, the VaR is a quantile of this return distribution. The VaR of a long position (left tail of the distribution function) over the time horizon k with probability p may hence be defined from:

$$p = Pr\left[x_{et}^k \leq VaR\right] = F_k(VaR), \tag{4}$$

where $F(\cdot)$ denotes the cumulative distribution function of x_{et}^k. The quantile function is the inverse of the cumulative distribution function from which the VaR obtains:

$$VaR_k(p) = F_k^{-1}(p). \tag{5}$$

Since x_{et}^k is the sum of log excess returns over k periods, it is also the log of the product of the excess returns (not taken in log) over k periods. Hence, the VaR of the corresponding capital requirement simply obtains as:

$$VaR_k^{cr}(p) = \exp(VaR_k(p)) - 1$$

Since we are interested in the value-at-risk for various time horizons, it is desirable to keep an equivalent risk level over all the horizons, which means adjusting p with k. For instance, the $1 - p = 95\%$ level retained in VaR analysis is chosen on a yearly basis. In order to maintain the same yearly probability, the corresponding probability for horizon k must be adjusted accordingly, that is $1 - p = (95\%)^k$. All the computations below will retain this horizon-adjusted probability.

As can be seen from equation (5), such a VaR measure is directly affected by the distribution chosen for $F(\cdot)$. It is now well-known that the normal distribution is not suitable for most speculative assets, even at the quarterly or yearly frequency. Since there is no consensus regarding which alternative distribution to choose, we propose to retain a bootstrap approach relying on the empirical distribution. Basically, this approach consists in resampling S times the residuals estimated from model (1) so as to re-built S simulated sequences of $\frac{1}{k}(\mathsf{z}_{t+1} + \cdots + \mathsf{z}_{t+k})$ using equation (2). The method will be discussed to greater extend below and will be applied to the European data described in the next section.

2 The assets return data

The benchmark asset from which the excess returns on equities will be calculated is a short rate. For France, the 3-month PIBOR rate obtained from Datastream is retained from 1970M11 to 1998M12. It is then continued using the 3-month EURIBOR rate from 1999M1 to 2008M12. For Germany and the United-Kingdom, the money market 3-month rate and the T-bills 3-month rate are respectively retained for the whole sample. The end-of-quarter values from these monthly series are retained to get quarterly observations, and r_{0t} denotes the log return on the 3-month rate.

National data for stock prices and returns come from Morgan Stanley Capital International (MSCI) database and are available since December 1969. More precisely, quarterly stock market data are based on the monthly MSCI National Price and Gross Return Indices in local currency. From these data, a quarterly stock total return series and a quarterly dividend series are obtained following the methodology described in Campbell [1999][3]. Note that we depart from Campbell's approach by not including the tax credits on dividends. Indeed, MSCI calculates returns from the perspective of US investors, so it excludes from its indices these tax credits which are available only to local investors. For e.g. France, Campbell chooses to add back the tax credits quite roughly, by applying the 1992 rate of 33.33% to all the sample. Nevertheless, this rate hasn't remained fixed over the sample considered here (1970Q1—2008Q3). On top of this, the way dividends are taxed has also changed during that period. We couldn't find exact tax rate data for our sample and have chosen to work with data excluding tax credits. For each country, the equities excess return, x_{et}, is then obtained by substracting r_{0t} from the log return on equities.

Finally, we have to find a proxy variable for the financial market cycle. From a practitioner's point of view, a variable such as a moving average of the log of the stock market price index would seem to be a good candidate because of its simplicity. Nevertheless, such kind of proxy variable has the serious drawback that a moving average is backward-looking by nature, and for this reason would always be late compared to the current cycle. Another possibility is to extract the trend component of the log stock market price index using the filter proposed in Hodrick and Prescott [1997]. This filter

[3]See also Campbell's "Data Appendix for *Asset Prices, Consumption and the Business Cycle*", March 1998, downloadable from Campbell's homepage.

is the most used one in the business cycles literature since more than three decades. Since this HP filter uses all the sample to extract the cyclical component, it is well in line with the current cycle contrary to such backward-oriented filtering methods as the moving average class of filters for instance. Nevertheless, this filter is not perfect (see e.g. King and Rebelo [1993], Cogley and Nason [1995], Pederson [2001] and Mise, Kim and Newbold [2005]): its main drawback is the endpoint issue which would make the results regarding, say, the last two years of the sample, unreliable. Finally, we have chosen to follow e.g. Clarida, Gali and Gertler [2000] or Christensen and Nielsen [2009] in estimating the trend component of the log stock price index by $m^*_{ct} = g(t)$ where $g(t)$ is a polynomial in the time index t. Regarding the choice of the stock price index underlying m_{ct}, we have retained the European index provided by MSCI Barra. This Europe Index is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of the developed markets in Europe.[4] More precisely, m_{ct} denotes the cyclical component of the logarithm of the European index, with $g(t)$ a seventh-order polynomial.[5]

Figure 4 in Appendix reports the French, German and UK log returns and the European stock market cycle data under study.

3 Empirical assessment of the influence of the financial market cycle on excess equities log returns

In the sequel, we will consider an autoregressive model for $\mathsf{z}_t = (x^i_{et}, m_{ct})$, $i = FR, GE, UK$ in equation (1). The estimated model writes as follows:

$$\mathsf{z}_t = \Phi_0 + \sum_{i=1}^{n} \Phi_i \mathsf{z}_{t-i} + \mathsf{v}_t. \qquad (6)$$

[4]As of June 2007, the MSCI Europe Index consisted of the following 16 developed market country indices: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

[5]The whole analysis has also been performed using a HP filter to extract the cyclical component, without changing the conclusions.

The lag order n is chosen so as to eliminate residuals serial correlation, which leads to retain one lag for all the models. The estimation results are reported in Table 2, see Appendix. For these VAR(1) systems, the null of no residuals serial correlation up to order 8 is not rejected according to the Portmanteau statistics. It is also worth noticing that both ARCH and White F tests do not reject the homoskedastic null hypothesis in France and Germany, whereas there is an ARCH(2) effect in the residuals from the United Kingdom model. This will be taken into account when bootstrapping the UK residuals in the next section.

So as to check for the dynamic relationship between the market cycle and the excess equities returns, we performed Granger-causality tests. Table 1 reports the corresponding LR statistics and p-values. These statistics are distributed as a Chi-squared with one degree of freedom. As can be seen from this table, the nullity of m_c's coefficients in

Table 1: LR statistitics for Granger (non-)causality tests

	FR	GE	UK
from m_c to x_e	3.05	6.06	9.15
p-value	(0.08)	(0.01)	(0.00)
from x_e to m_c	4.16	1.82	1.49
p-value	(0.04)	(0.18)	(0.22)

the equation of x_e is strongly rejected for Germany and the United Kingdom, whereas it is rejected at the 8%-level in France. On the whole, we may conclude that our proxy variable of the financial market cycle Granger-causes the log excess returns on equities. This confirms the relevance of the joint modelling of these two variables.

This causal link is further confirmed by the impulse response function of the log excess return on equities to an innovation in the market cycle. In order to identify this innovation, we performed a Choleski decomposition of Σ_v — the variance-covariance matrix of the vector autoregression estimated residuals — retaining the following ordering of the variables in the model: (m_{ct}, x^i_{et}). Denoting $\mathsf{v}_t = (v^m_t, v^x_t)'$ the residuals of model (1) for such an ordering of the variables, we define the structural innovations in the market cycle and the returns $\varepsilon_t = (\varepsilon^m_t, \varepsilon^x_t)'$, with $E(\varepsilon\varepsilon') = I$, by:

$$\mathsf{v}_t = G\varepsilon_t,$$

where G is the lower-triangular 2×2 matrix such that $GG' = \Sigma_v$. This choice allows the market's cycle innovations to affect instantaneously the excess return, while the return innovations influence the market cycle after one period only.[6] Figure 1 below reports this impulse response function of the german x_e to a favorable unit shock in the market cycle innovation, together with two-standard deviation confidence interval computed from 10,000 drawings of the estimated residuals. As can be seen from Figure 1, the instantaneous response of the excess return is positive, but then becomes significantly negative for two years before progressively going back to zero. The french and UK return response functions have the same shape, and are respectively a little bit less and more pronounced than the german returns response. Of course, an adverse shock would generate the reverse effect: the log returns would drop the first quarter but then would become positive the next two years before the shock's effect completely vanishes. This figure also reveals that after eight quarters, the impact of the financial cycle innovation on the excess return is not significantly different from zero.

If the dynamics of the log returns is affected by innovations in market cycle, so should be the dynamics of the Value-at-Risk.

4 The dynamics of Value-at-Risk

4.1 The proposed empirical measures of the VaR$_k$

The bootstrap method described below belongs to the multivariate filtered historical simulation (FHS) method presented in Chirstoffersen [2009]. This method consists in simulating future returns from a model using historical return innovations. It is qualified

[6] The results obtained from the alternative identification scheme are qualitatively similar.

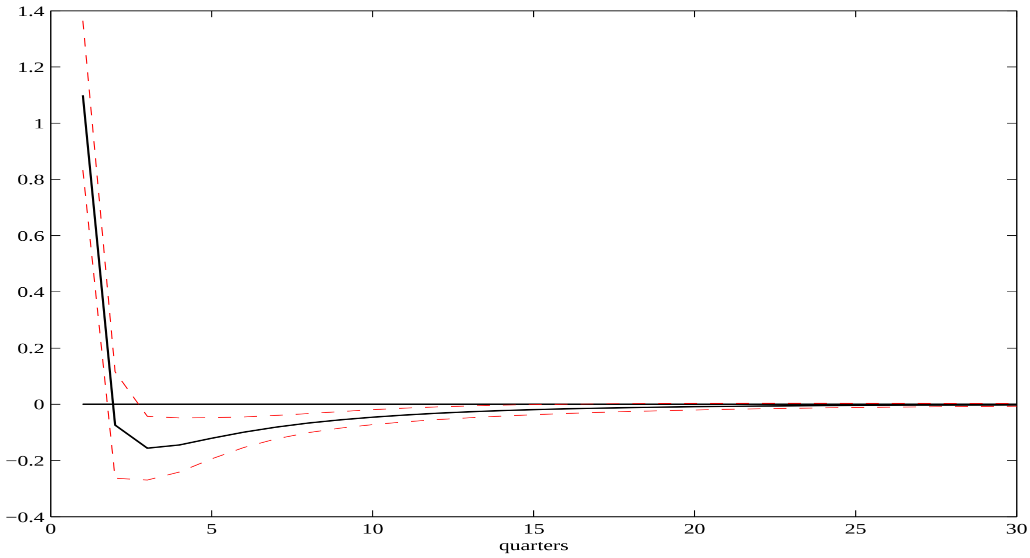

Figure 1: Response of german x_e to a unit shock in ε^m

by “filtered” because it does not use simulations from the set of returns directly, but from the set of shocks, which are basically returns such as filtered by our vector autoregressive model.

The FHS method described in Chirstoffersen [2009] would amount in our case to the following: First, using random draws from a uniform distribution, the estimated residuals of model (6) are resampled S times. Using these S series of v^s together with the estimated parameters of model (6) and the observed value of z_{t-1}, in equation (3), S hypothetical sequences of x_{et}^k are obtained. The $VaR_k(p)$ then obtains by retaining — amongst these S simulated sequences — the value of return such that there is a probability p that a worse value occurs at horizon k. This method clearly accounts for the uncertainty of the shocks realization. However, by setting $\mathsf{z}_{t-1}^s = \mathsf{z}_{t-1}$, it makes the VaR measure strongly dependent on the last available observations. In order to illustrate this, Figure 2 reports this date-dependent VaR measure calculated from 300,000 simulations for the one-year investment horizon and for all t from 1980Q1 to 2008Q4. For each date t, we have estimated model (6) from 1973Q1 until t and obtained the $k-$year

VaRs by the bootstrap method described above. Note that for the UK model, the bootstrap procedure is adapted to account for the residuals heteroskedasticity following the lines described in e.g. Cavaliere, Rahbek and R. [2008]: instead of being resampled, the estimated VAR residuals are multiplied by a Gaussian i.i.d. $\mathcal{N}(0,1)$ sequence so that the resulting simulated residuals keep the same heteroskedastic features as the estimated ones. These figures also plot the ex-post observed values of $\exp(x_{et}^k) - 1$. In all these countries, the one-year VaR(95%) under-estimates the one-year stock return during the 2001-2002 recession episode. For France and Germany, the same occurs with the 1986-1987 downturn. Table 3 in appendix reports the percentage of violations, i.e. the percentage of VaRs above the corresponding ex-post observed return, for the $VaR(95\%)$'s up to five years. It turns out that the model for french returns performs remarkably well for the one and two-year horizons, while it becomes too conservative for longer horizons. The UK model is also quite good in reproducing the expected percentages of violations even though it is slightly too conservative at the one year horizon. Finally, the German model is slightly too liberal for the one and two-year horizons.

Nevertheless, since we aim at evaluating the impact of the financial cycle on the VaR for various investment horizons, we would rather control for the position in the cycle. We will do this by setting the excess return to its sample average, i.e. $\mathsf{x_e}_t^s = \bar{\mathsf{x}}_\mathsf{e}$, while fixing the market cycle indicator respectively to its mean (mid-cycle measure), to its mean plus one or two standard deviation (one or two standard expansion case) and to its mean minus one or two standard deviation (one or two standard recession case).

4.2 Empirical measures of VaR$_k$ across investment horizon and financial cycle

The results reported below were obtained for $S = 300,000$ simulations for each $k = 1, \cdots, 20$ years, from which we picked up the corresponding $(1 - 95^k\%)$ quantile for each VaR_k^{cr}. Figure 3 plots the five measures of VaR_k^{cr} described above, namely the mid-

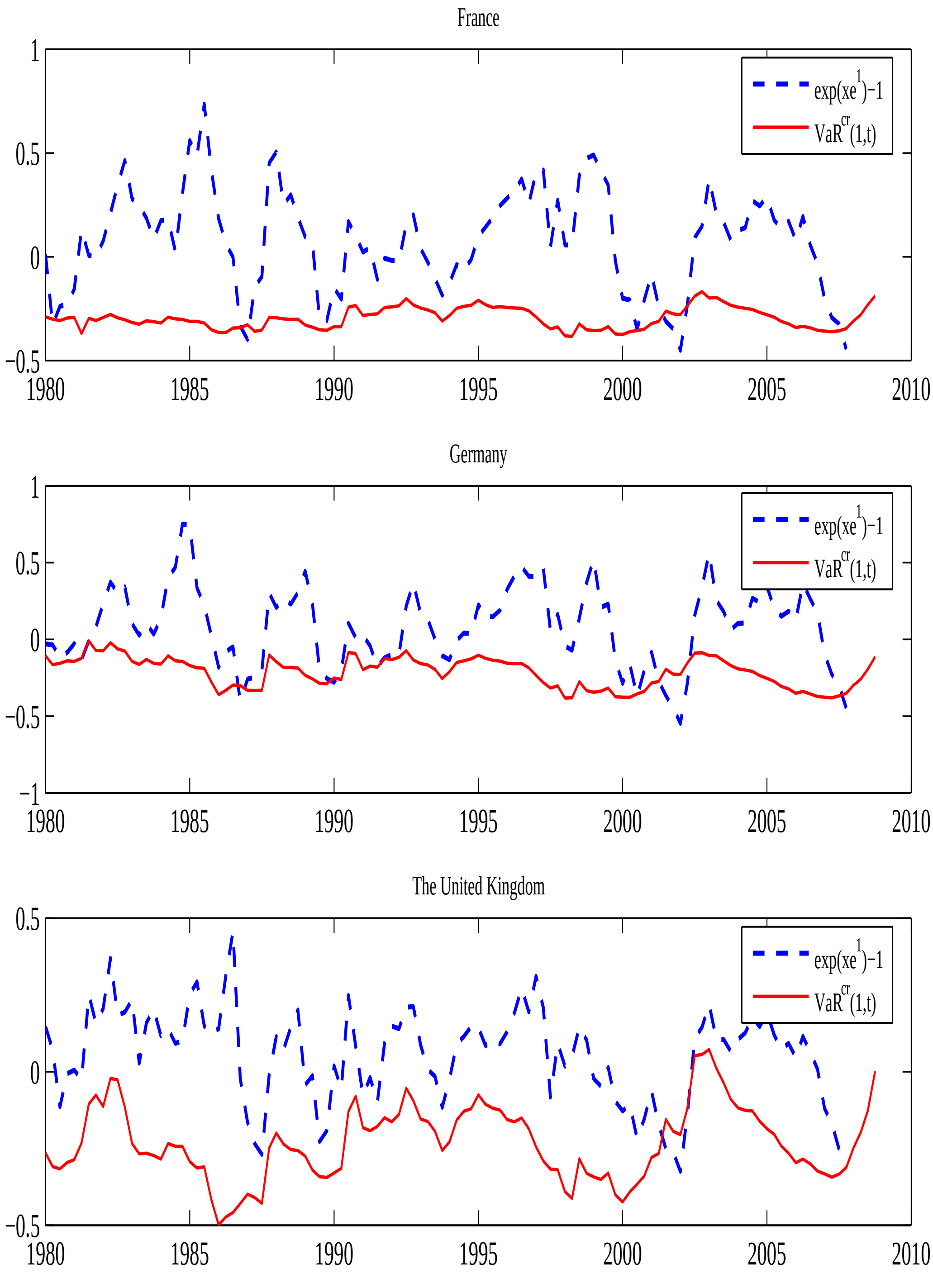

Figure 2: One-year VaR($95^k\%$) and observed one-year returns

cycle, the one and two standard expansions and the one and two standard recessions against holding horizons up to twenty years. The corresponding figures are reported in Tables 4 to 6 in Appendix, up to the twenty-year horizon. The first important result

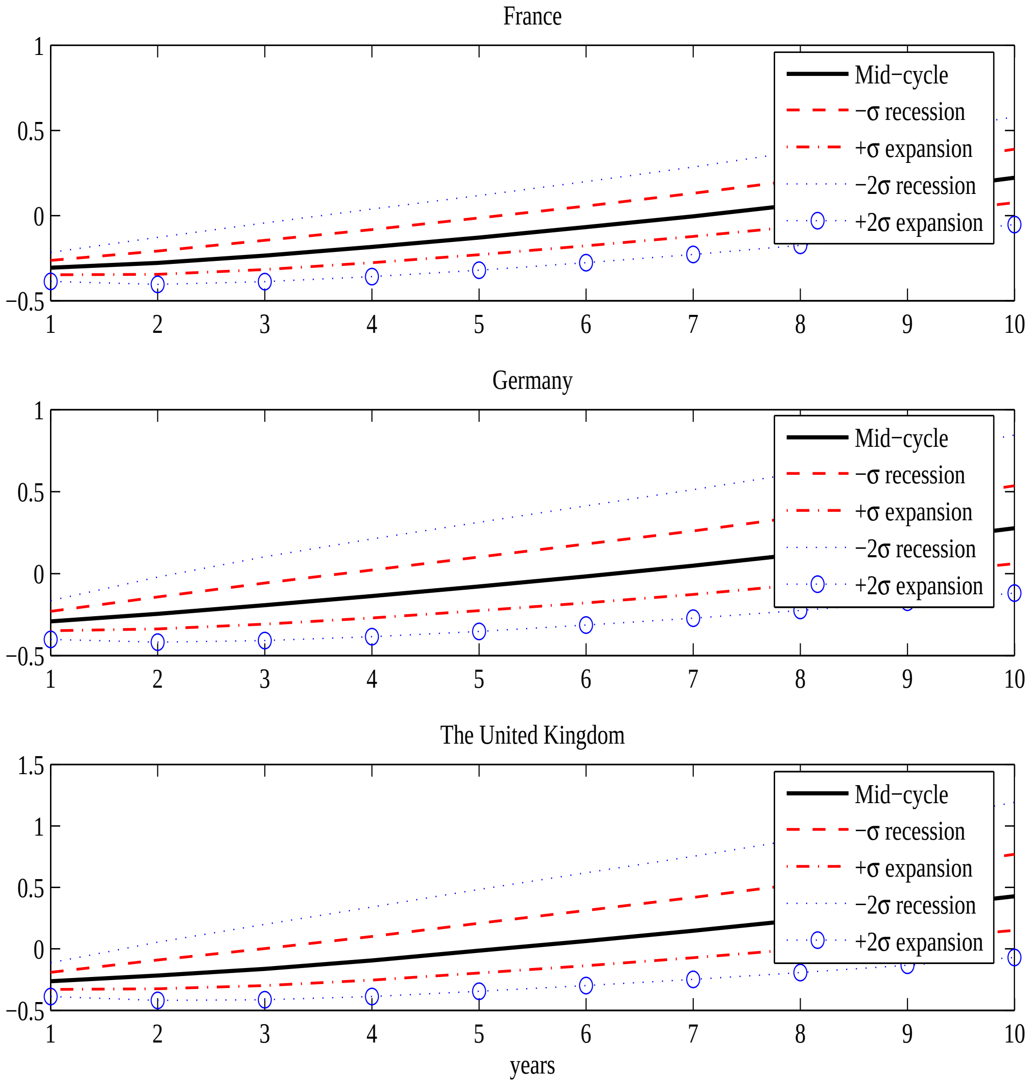

Figure 3: Value-at-Risk(95^k%) across cycle and horizons

emerging from this figure is that whatever the investment horizon, the VaR depends on the position in the financial cycle. For all countries and horizons, the VaR is stronger in expansion than in recession. The VaR's gap between recession and expansion times at the one-year horizon ranges from around 8.4% in France to 13.7% in the United Kingdom, while it is 11.8% in Germany, for the one-standard deviation case. It ranges from 17.1% in France to 27.6% in the United Kingdom for the two-standard deviation case. In all the countries considered here, this gap widens with the holding horizon. The lower cyclical impact found on French returns may stem from the fact that this is the country in which the European financial cycle indicator has the lowest explanatory power for the excess returns. Overall, these results suggest that a rule imposing the same solvency capital requirement whatever the state of the financial market cycle could actually be pro-cyclical.

The second important result concerns the dynamics of the VaR across investment horizons. In a previous study (see Bec and Gollier [2007]), mean-reversion was found in log returns on French equities relatively to other assets returns: their relative risk was found decreasing with the holding period. This is confirmed by the results in Figure 3. Indeed, the worst expected loss in terms of capital requirement, at the $(1 - 0.95^k)-$percent level, decreases with the investment horizon. In all countries, starting from a one-standard recession, it becomes a gain after two to five years according to our estimates. These results are quite robust to the estimation period.

As a further check, the simulations were also performed with re-estimation of the vector autoregression for each $s \in S$ so as to take the parameters estimates uncertainty into account — which is not done in the common FHS approach. Indeed, the impact of parameter uncertainty on the conclusions regarding the horizon effect has been stressed in a recent empirical work by Pastor and Stambaugh [2009]. Nevertheless, as shown in Figure 5 and Tables 7 to 9 reported in appendix, both the cyclical and horizon effects are robust when the parameters uncertainty is taken into account.

5 Concluding remarks

The vector autoregressive joint modelling of stocks excess returns and financial market cycle indicator reveals that the latter helps predicting the former. Put in other words, the financial market cycle variable Granger-causes the excess returns on equities. Since the Value-at-Risk is evaluated from the expected excess returns, it is also influenced by the state of the financial cycle. The gap found between the VaR evaluated at a one-standard recession and the one measured at a one-standard expansion might be as high as 13.7% at the one-year horizon. Our results provide support to the claim that fixed solvency capital requirements may have important procyclical consequences on the dynamic investment strategies of the financial intermediaries. They also suggest some predictability in French, German and British equities returns since they point to a decrease in the VaR as the holding period increases. One limit of the approach retained here is that it assumes the existence of financial markets cycles without explaining it. A better understanding of this phenomenon is a challenging question on our research agenda.

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Appendix

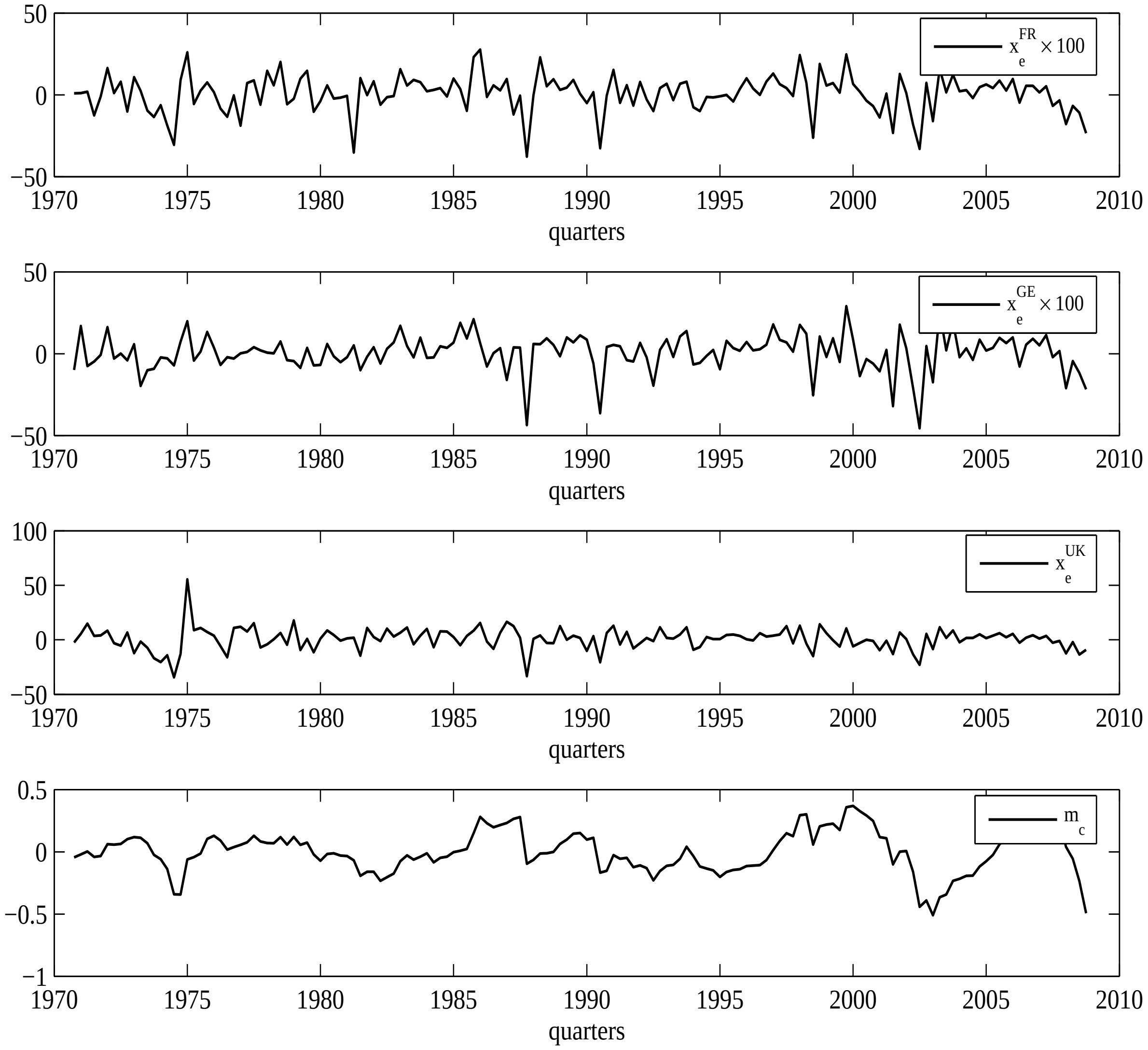

Figure 4: The data (1970Q4—2008Q4)

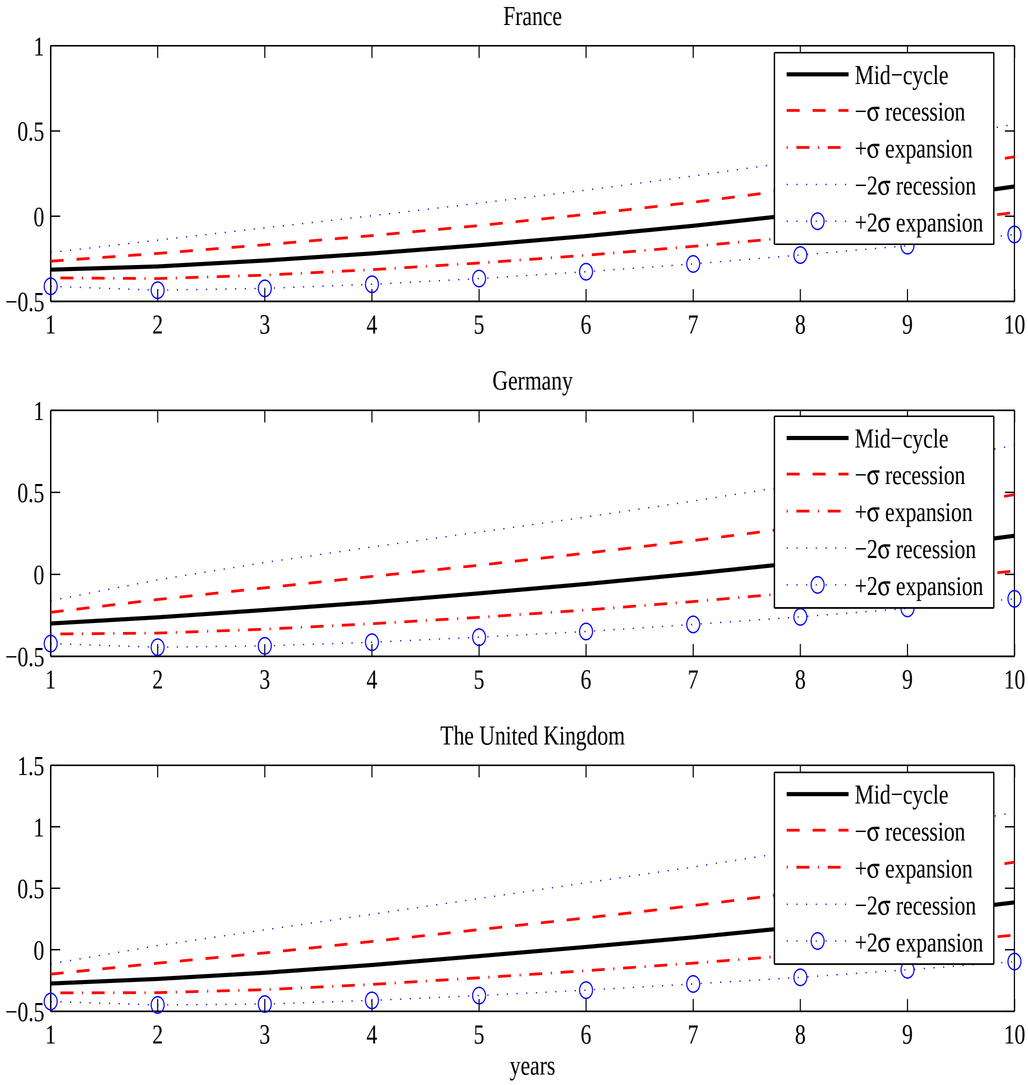

Figure 5: Value-at-Risk($95^k\%$) when taking parameters uncertainty into account

Table 2: VAR estimation results

	France		Germany		UK	
	$m_{c,t}$	$x_{e,t}^{FR}$	$m_{c,t}$	$x_{e,t}^{GE}$	$m_{c,t}$	$x_{e,t}^{UK}$
$m_{c,t-1}$	0.86	-0.10	0.87	-0.14	0.88	-0.15
	[19.77]	[-1.75]	[20.14]	[-2.46]	[20.32]	[-3.02]
$x_{e,t-1}^{FR}$	0.13	0.10				
	[2.04]	[1.20]				
$x_{e,t-1}^{GE}$			0.09	0.08		
			[1.35]	[0.94]		
$x_{e,t-1}^{UK}$					0.09	0.15
					[1.22]	[1.81]
c	-0.00	0.57	-0.00	0.57	-0.00	0.73
	[-0.50]	[0.60]	[-0.44]	[0.63]	[-0.47]	[0.91]
R-squared	0.75	0.02	0.75	0.04	0.75	0.07
Log-likelihood	158.67	-586.04	157.50	-581.61	157.34	-560.86
ARCH(1) p-val.	0.42	0.97	0.53	0.62	0.71	0.67
ARCH(4) p-val.	0.71	0.82	0.68	0.35	0.82	0.00
Q(8) p-val.	0.54	0.76	0.49	0.49	0.43	0.98
White F p-val.	0.12	0.51	0.26	0.30	0.23	0.05

Student's t-statistics in [ ].

Table 3: Percentage of violations for $VaR(95\%)$

	1 year	2 years	3 years	4 years	5 years
$p = 0.95$					
Expected % of violations	5.00	9.75	14.26	18.55	22.62
France	5.36	8.33	9.65	12.00	14.58
Germany	8.93	12.04	15.38	17.00	19.79
United Kingdom	3.57	10.18	12.50	17.00	20.83

The expected percentage of violations is given by $(1 - p^k)$.

Table 4: French Value-at-Risk

	mid-cycle	$-\sigma$ recession	$+\sigma$ expansion	-2σ recession	$+2\sigma$ expansion
Years					
1	-0.30611	-0.26366	-0.34802	-0.21627	-0.38715
2	-0.27821	-0.20842	-0.34465	-0.12825	-0.40361
3	-0.23525	-0.14588	-0.31676	-0.04316	-0.38757
4	-0.18424	-0.08179	-0.27707	0.03769	-0.35798
5	-0.12915	-0.01447	-0.23044	0.11707	-0.32040
6	-0.06826	0.05556	-0.17827	0.19847	-0.27678
7	-0.00419	0.13037	-0.12286	0.28538	-0.22799
8	0.06613	0.21033	-0.06252	0.37837	-0.17445
9	0.14165	0.29718	0.00383	0.47426	-0.11604
10	0.22168	0.38941	0.07573	0.57948	-0.05351
11	0.31089	0.48696	0.15308	0.69348	0.01285
12	0.40374	0.59695	0.23891	0.81446	0.08595
13	0.50667	0.71083	0.32555	0.94959	0.16448
14	0.61710	0.83931	0.42417	1.08982	0.24888
15	0.73611	0.97432	0.52691	1.24574	0.33967
16	0.86251	1.11812	0.64009	1.40576	0.43899
17	0.99823	1.27672	0.76231	1.58438	0.54554
18	1.14808	1.44362	0.89059	1.77637	0.65845
19	1.30955	1.62669	1.03072	1.98423	0.78501
20	1.48202	1.82309	1.18410	2.20767	0.91819

The figures correspond to $VaR_k^{cr}(1 - 0.95^k)$.

Table 5: German Value-at-Risk

	mid-cycle	$-\sigma$ recession	$+\sigma$ expansion	-2σ recession	$+2\sigma$ expansion
Years					
1	-0.29207	-0.23066	-0.34868	-0.16462	-0.40115
2	-0.24611	-0.14222	-0.33808	-0.02167	-0.41825
3	-0.19314	-0.05748	-0.30851	0.10229	-0.40919
4	-0.13720	0.02131	-0.27094	0.21111	-0.38483
5	-0.07801	0.10075	-0.22691	0.31276	-0.35308
6	-0.01689	0.18079	-0.17867	0.41239	-0.31450
7	0.04837	0.25979	-0.12703	0.51158	-0.27168
8	0.11951	0.34782	-0.06752	0.61486	-0.22532
9	0.19530	0.43865	-0.00619	0.72564	-0.17449
10	0.27711	0.53573	0.06066	0.84531	-0.11905
11	0.36181	0.64016	0.13052	0.97140	-0.05981
12	0.45246	0.75131	0.20870	1.10635	0.00361
13	0.55259	0.87134	0.29100	1.25071	0.07057
14	0.65708	1.00070	0.37960	1.40681	0.14242
15	0.76896	1.13575	0.47368	1.56679	0.22181
16	0.89146	1.28529	0.57293	1.74680	0.30508
17	1.02215	1.43914	0.68268	1.93804	0.39449
18	1.16318	1.61088	0.79921	2.13922	0.49166
19	1.31077	1.79060	0.92458	2.35119	0.59636
20	1.47416	1.98748	1.05881	2.58889	0.70935

See Table 4.

Table 6: UK Value-at-Risk

	mid-cycle	$-\sigma$ recession	$+\sigma$ expansion	-2σ recession	$+2\sigma$ expansion
Years					
1	-0.26393	-0.19159	-0.32966	-0.11199	-0.38808
2	-0.21611	-0.09083	-0.32484	0.05368	-0.41935
3	-0.16227	0.00289	-0.29919	0.19996	-0.41445
4	-0.09387	0.10151	-0.25394	0.34063	-0.38706
5	-0.01391	0.20847	-0.19661	0.48367	-0.34478
6	0.06399	0.31175	-0.13645	0.61970	-0.29872
7	0.14686	0.41768	-0.07222	0.75203	-0.24791
8	0.23588	0.52924	-0.00168	0.89329	-0.19284
9	0.32836	0.64478	0.07245	1.03721	-0.13387
10	0.42730	0.76961	0.15138	1.19161	-0.06975
11	0.53274	0.90138	0.23720	1.35587	-0.00141
12	0.64921	1.04536	0.33157	1.53449	0.07404
13	0.77534	1.20068	0.43194	1.72551	0.15514
14	0.90863	1.36979	0.53955	1.93144	0.24405
15	1.05608	1.55054	0.65562	2.15448	0.33848
16	1.21141	1.74176	0.78195	2.39252	0.43832
17	1.37745	1.95057	0.91838	2.65259	0.55024
18	1.55852	2.17239	1.06362	2.92867	0.66654
19	1.75267	2.41701	1.22060	3.23085	0.79454
20	1.96445	2.67799	1.39158	3.55673	0.93367

See Table 4.

Table 7: French Value-at-Risk (parameters uncertainty)

	mid-cycle	$-\sigma$ recession	$+\sigma$ expansion	-2σ recession	$+2\sigma$ expansion
Years					
1	-0.31401	-0.26420	-0.36311	-0.21421	-0.41226
2	-0.29496	-0.21970	-0.36713	-0.14174	-0.43559
3	-0.26139	-0.16849	-0.34664	-0.06956	-0.42374
4	-0.21966	-0.11405	-0.31457	0.00176	-0.39965
5	-0.17135	-0.05572	-0.27606	0.07556	-0.36659
6	-0.11723	0.00943	-0.23023	0.15235	-0.32606
7	-0.05654	0.08171	-0.17788	0.23523	-0.28092
8	0.01156	0.16121	-0.12046	0.32479	-0.22857
9	0.08697	0.24931	-0.05396	0.42623	-0.17303
10	0.17399	0.34770	0.02045	0.53963	-0.10821
11	0.26710	0.45973	0.10466	0.66519	-0.03676
12	0.37436	0.58228	0.19702	0.80822	0.04542
13	0.49423	0.72032	0.30095	0.96520	0.13596
14	0.62857	0.87314	0.41824	1.14565	0.23775
15	0.77963	1.04771	0.54965	1.34281	0.35251
16	0.94941	1.24139	0.69417	1.56539	0.48175
17	1.13870	1.45816	0.86185	1.81736	0.62844
18	1.35317	1.70487	1.05418	2.10629	0.79257
19	1.59564	1.98619	1.26430	2.42112	0.97765
20	1.86485	2.29806	1.50109	2.77758	1.18346

See Table 4.

Table 8: German Value-at-Risk (parameters uncertainty)

	mid-cycle	$-\sigma$ recession	$+\sigma$ expansion	-2σ recession	$+2\sigma$ expansion
Years					
1	-0.29977	-0.23167	-0.36362	-0.16053	-0.42315
2	-0.26274	-0.15440	-0.35892	-0.03393	-0.44552
3	-0.21800	-0.08247	-0.33544	0.07218	-0.43661
4	-0.17046	-0.01332	-0.30198	0.16655	-0.41422
5	-0.11635	0.05504	-0.26241	0.25711	-0.38419
6	-0.05958	0.12931	-0.21740	0.34929	-0.34876
7	0.00361	0.20524	-0.16592	0.44571	-0.30590
8	0.07202	0.28898	-0.10989	0.54987	-0.25970
9	0.14941	0.38288	-0.04776	0.66441	-0.20819
10	0.23445	0.48566	0.02089	0.78687	-0.15020
11	0.32968	0.59792	0.09903	0.92424	-0.08483
12	0.43353	0.72439	0.18377	1.07507	-0.01474
13	0.54787	0.85986	0.27803	1.24225	0.06558
14	0.67420	1.01394	0.38083	1.42522	0.15182
15	0.81632	1.18285	0.50018	1.62921	0.25100
16	0.97539	1.37342	0.62949	1.84953	0.35690
17	1.14767	1.57865	0.77359	2.10612	0.47750
18	1.33907	1.81371	0.93346	2.38463	0.60929
19	1.55823	2.07320	1.10871	2.69625	0.75734
20	1.79473	2.36428	1.30868	3.04484	0.92634

See Table 4.

Table 9: UK Value-at-Risk (parameters uncertainty)

	mid-cycle	$-\sigma$ recession	$+\sigma$ expansion	-2σ recession	$+2\sigma$ expansion
Years					
1	-0.27507	-0.19795	-0.35045	-0.11471	-0.42073
2	-0.23643	-0.10857	-0.34891	0.03384	-0.44917
3	-0.18707	-0.02462	-0.32442	0.16134	-0.44088
4	-0.12445	0.06680	-0.28256	0.28893	-0.41241
5	-0.05186	0.16305	-0.22805	0.41727	-0.37242
6	0.02201	0.25813	-0.17118	0.54460	-0.32894
7	0.10060	0.35919	-0.10853	0.67282	-0.27895
8	0.18895	0.46839	-0.03999	0.81102	-0.22351
9	0.28158	0.58512	0.03536	0.95645	-0.16344
10	0.38536	0.71154	0.11873	1.11696	-0.09661
11	0.49804	0.85473	0.21000	1.29387	-0.02171
12	0.62356	1.00892	0.31188	1.48601	0.06053
13	0.76511	1.18400	0.42699	1.70135	0.15276
14	0.92093	1.37591	0.55388	1.94341	0.25612
15	1.09376	1.58759	0.69140	2.20537	0.36945
16	1.28533	1.82707	0.84786	2.50067	0.49686
17	1.50217	2.09404	1.02386	2.83593	0.63898
18	1.74117	2.38393	1.21926	3.20029	0.79817
19	2.01132	2.71948	1.43810	3.60353	0.97532
20	2.31281	3.09239	1.68384	4.06573	1.17359

See Table 4.