TSE WP working paper series

12-309

Research Group: Behavioral and Experimental Economics

May, 2012

"Cartel Pricing Dynamics, Price Wars and Cartel Breakdown"

Anton-Giulio Manganelli

Toulouse School of Economics (TSE) - Manufacture des Tabacs
Aile Jean-Jacques Laffont - 21, allée de Brienne - 31015 TOULOUSE Cedex 6
Tél : +33 (0) 5 61 12 85 89 - Fax : +33 (0) 5 61 12 86 37 - **www.tse-fr.eu** - contact@tse-fr.eu

Toulouse School of Economics

Cartel Pricing Dynamics, Price Wars and Cartel Breakdown

Anton-Giulio Manganelli[1,2]

Toulouse School of Economics

May 2012

Abstract

This paper gives an unified explanation of some of the most widely known facts of the cartel literature: prices gradually rise, then remain constant, there can be price wars and some cartels break down. In this model consumers are loss averse and efficiency of a competitive fringe is not publicly observable. In the best collusive equilibrium, the price expectation can be so low that loss aversion makes consumers not buy at the maximal collusive price: firms then set a lower price that rises in time with consumers' expectations. This increasing price path is bounded from above by the presence of the fringe. If the fringe sets a low price during a sufficient number of periods, there can be price wars and collusion can eventually break down.

Introduction

The analysis of discovered cartels in the last decades has shown that

1) prices do not directly jump to the maximal level, but have a transition phase during which they gradually rise and eventually remain constant;

2) some cartels succeed in reaching stability, while others break down some time after their formation;

3) some cartels suffer temporary price wars.

---

[1] e-mail: antongiuliomanganelli@hotmail.it.

[2] I am grateful to Roberta Dessì, Natalia Fabra, David Gill, Joseph E. Harrington Jr., Bruno Jullien and Patrick Rey for their useful comments. I thank participants at the Summer School ECORE 2011 in Louvain-la-Neuve, at the EEA Conference 2011 in Oslo and at the EARIE Conference 2011 in Stockholm for useful comments and discussions.

The first fact has been found in many of the largest discovered cartels, both for intermediate products,[3] as well as for final products.[4] The second and the third fact are documented from an empirical (Levenstein (1997)) as well as from a theoretical point of view (Green-Porter (1984), Rotemberg-Saloner (1986), Kandori (1991), Haltiwanger-Harrington (1991))[5].

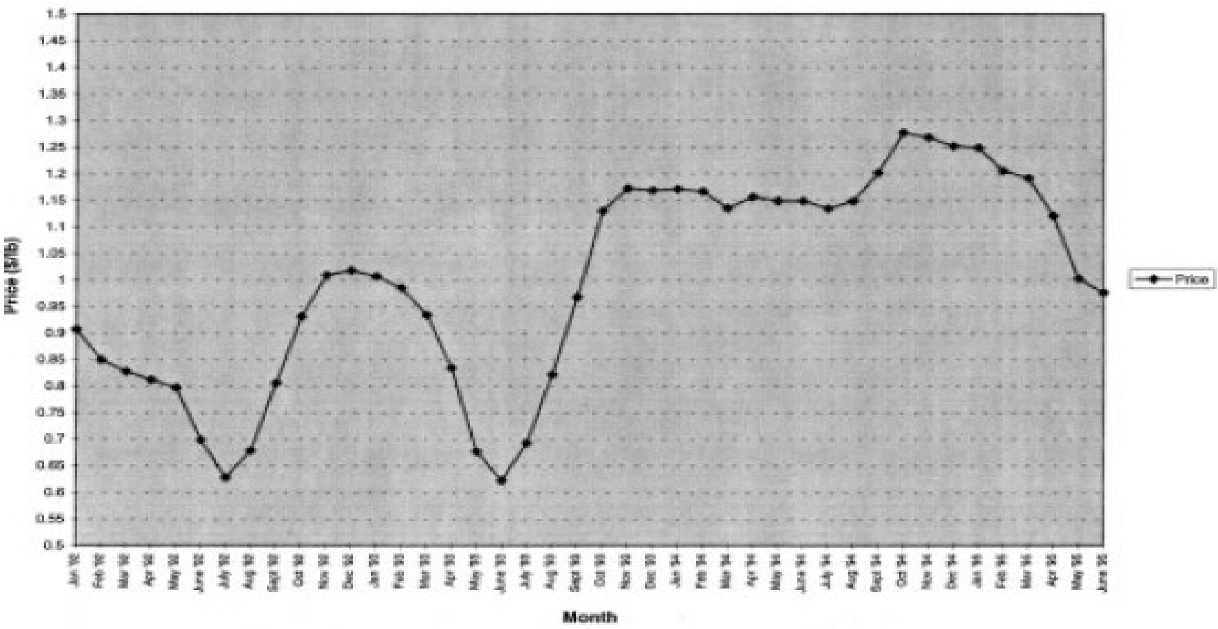


Figure 1. ADM U.S. lysine price, January 1992–June 1995.

(Source: Connor (2001))

Figure 1 shows the dynamics of lysine price between 1992 and 1995. In June 1993 lysine reached the lowest level. In July, the cartel took place and during the following months price continuously rose until November 1993, when it remained stable for ten months. After another rise in price, the cartel collapsed.

The dynamics of prices for the other cartels show similar patterns.[6]

The questions are: why has not the price directly jumped to the maximal level? Why has the cartel

[3] Like the Citric Acid and the Lysine cartel (see Connor (2001) and Levenstein-Suslow (2001))

[4] Like the French mobile cartel (2000-2002, see http://www.autoritedelaconcurrence.fr/pdf/avis/05d65.pdf), the Italian pasta cartel and the German coffee cartel (see http://www.bundeskartellamt.de/wEnglisch/download/pdf/Fallberichte/B11-019-08-ENGLISH.pdf. This cartel served also bulk customers, like hotels and vending machine operators.

[5] A famous case of cartel's breakdown is the Vitamin C cartel, in which the cartel broke down due to the competitive pressure of Chinese manufacturers. The previous cartel firms eventually exited the market, which subsequently resulted in the Chinese manufacturers forming a cartel. For more information, see http://www.nd.edu/~mgrecon/datafiles/articles/vitamins.html for the original cartel and http://www.law.com/jsp/nylj/CaseDecisionNY.jsp?id=1202513757490&In_Re_Vitamin_C_Antitrust_Litigation_MD_BMC_JO&slreturn=1&hbxlogin=1 for the subsequent cartel by the Chinese manufacturers.

[6] The pasta cartel, for example, shows the same features but with a less clear dynamics due to the high variance, over the cartelized period, of durum wheat cost, the principal input in pasta production.

suffered temporary price wars? Why has the cartel eventually broken down? The theories aiming at addressing the first question are basically developed by Harrington (2004, 2005) and Chen-Harrington (2006).

Harrington (2005) analyzes the impact of different Antitrust policies on cartel price dynamics. He shows that a higher damage multiple and probability of detection lower the steady-state price, but also that the level of fines does not alter it. Furthermore, a more competitive benchmark to calculate damages can *increase* the steady-state price. All the results are derived by assuming that the incentive compatibility constraints (henceforth ICCs) are fulfilled and not binding.

Harrington (2004) generalizes the results above by allowing the ICCs to be binding. First, he shows that when ICCs bind, the cartel may rise prices and then *decrease* them towards the steady-state level, in order to maintain the incentive compatibility. Second, Antitrust laws may have a perverse effect, as in some cases they allow the cartel to eventually price *higher*. This result is due to the fact that the risk of detection and penalties can serve to stabilize a cartel and thereby allow it to set higher prices.

Chen and Harrington (2006) motivate the slow rise of prices with the fact that consumers and Antitrust Authorities (henceforth AA) infer the existence of a cartel with a probability depending on the difference between past and present prices. This creates an incentive for raising prices more slowly than they would otherwise, in order to avoid fines.

The second question is addressed by Green and Porter (1984), who explain price wars through imperfect information between the firms in the cartel, and Rotemberg and Saloner (1986), Kandori (1991) and Haltiwanger and Harrington (1991), who explain temporary price wars by assuming a difference between current and future demand: this changes the incentive to deviate from one period to another, and firms may find it optimal to reduce prices in the "boom" periods in order to maintain the sustainability of collusion.

The present model gives an unified explanation for all these facts. Building on insights from the behavioral literature[7], I assume that consumers are *loss averse*. Loss aversion means that sensitivity to losses compared to a reference point is greater than to gains. Consumers are loss averse in the price dimension: price expectations directly enters their utility function, so the higher the difference between the actual and the expected price, the higher the utility loss.[8]

Evidence of loss aversion is widespread both in the experimental and in the empirical literature.

[7]Kahneman and Tversky (1979), Erickson and Johansson (1985), Winer (1986), Kahneman, Knetsch and Thaler (1991), Chi-Kin-Jim and Kalwani (1992), Rotemberg (2004), Koszegi and Rabin (2006), Ellison (2006), Heidhues and Koszegi (2008).

[8] One could argue that this explanation is questionable when the good is intermediate. When buyers are firms, they have a stronger incentive than human buyers to behave as rationally as possible. My theory is indeed more suited to cartels where buyers are final consumers.

Thaler (1980) proposes this concept to explain why consumers often do not behave as consumer's theory predicts. Kahneman et al. (1991) enumerate a number of biases that are not explainable by traditional economic theory, among which loss aversion, providing a series of experiments. Kahneman and Tversky (1991) discuss other experimental evidence and propose a model based on loss aversion that explains these biases by a deformation of the indifference curves about the reference point. Further evidence came in the following years: with empirical data, Bowman et al. (1999) show the existence of loss aversion in saving decisions, Genesove and Mayer (2001) in the housing market and Haigh and List (2005) among professional traders. Novemsky and Kahneman (2006) and Gill and Prowse (2012) provide experimental evidence. Finally, Fox et al. (2007) investigate neural correlates of loss aversion and they show that people typically exhibit greater sensitivity to losses than to equivalent gains.

Koszegi and Rabin (2006) and Koszegi and Heidhues (2008) build formal models on it. Koszegi and Rabin (2006) analyze consumer behavior with loss aversion and endogenous reference point and they show that, when the outcome is uncertain, the willingness to pay increases in the expected price, conditional on purchase. Heidhues and Koszegi (2008) use loss aversion to explain the existence of focal prices[9] in a static game.[10] Koszegi and Rabin (2007), Macera (2009) and Gill and Stone (2010) consider dynamic games with loss aversion and endogenous reference point in consumption plans, labor contracts and tournaments, respectively.

I will instead develop an infinite-time horizon dynamic model with endogenous reference point that focuses on collusion. The reference point, i.e. the expected price for the present period, is updated in a Bayesian way in every period, using the information available up to that moment. It can thus change in every period and consumers are fully rational.

The reference point is based on an exogenous probability that a competitive fringe sells the good at a low price. This probability is unknown to both firms and consumers. They have a common prior about two possible probabilities. If the fringe sets the high (low) price, consumers and firms update their beliefs by giving more weight to the lower (higher) probability of the low cost.

Thus, in the best collusive equilibrium, the larger the number of periods when the fringe sets the high price, the more consumers expect a high price in the future: that is, depending on fringe's behavior, the reference point can rise, which reduces the effect of loss aversion, or fall, making the sustainability of collusion harder. Two key insights of the model are that: i) the effect of loss aversion can be so large that firms just gradually rise prices, so as to keep selling to consumers, and

[9] Prices that are equal across differentiated goods, even if their costs of production differ.

[10] Heidhues and Koszegi interpret the reference point as consumers' "lagged rational expectation", but since the setting is static, the reference point is exogenous.

ii) firms can sustain collusion if and only if the probability of a low fringe's price is sufficiently low.

If the fringe continues setting the high price, consumers update their price beliefs upwards, thus allowing firms to further increase prices, up to a point where consumers become willing to pay the maximal collusive price. The transition path is then over. On the other hand, if the fringe sets the low price during a sufficient number of periods, firms rationally anticipate that the fringe is too efficient to make continuation profits high enough to deter a deviation, thus collusion breaks down,. This model can also explain why there are temporary price wars.

The paper is organized as follows: Section 1 presents the mechanism underlying the cartel pricing dynamics of collusive prices. Section 2 gives an intuition for one of the possible reasons of a cartel breakdown. Section 3 shows some simulations. Section 4 analyses the robustness of the model and discusses possible modifications. Section 5 concludes.

1. Setup

Consider an infinitely repeated Bertrand game. In each period, a mass one of consumers has unit demand for an homogeneous[11] good, produced by 2 firms[12] and by a competitive fringe. All consumers have the same willingness to pay, which depends on to the intrinsic utility and loss aversion. If they buy, they obtain:

$$u_t = \nu - p_t - \lambda(p_t - E[p_t])^{13}. \tag{1}$$

The parameter ν represents the maximal willingness to pay absent loss aversion; p_t is the price paid in period t; $\lambda \geq 0$ is the loss aversion coefficient: the higher λ, the higher the utility loss in paying a price higher than the expected one; $E_{t-1}[p_t]$ is the expected price in t, given the information available up to period $t-1$. If they do not buy, they get $u_t=0$.

Firms have a common and uniform marginal cost $\underline{c}$. The cost of the competitive fringe can change in every period: with a probability $\underline{\mu}$ or $\overline{\mu}>\underline{\mu}$, it is $\underline{c}$, and with 1-$\underline{\mu}$ (or 1-$\overline{\mu}$) it is $\overline{c} > \underline{c}$. The high

[11]The results would qualitatively hold also if goods were differentiated.

[12]All the results would hold qualitatively unchanged also with n firms.

[13]A more precise formulation would be $U_t = \nu - p_t - \lambda(p_t - E[p_t])I(p_t > E[p_t]) + \kappa(E[p_t] - p_t)I(p_t < E[p_t])$, where I is the indicator function and κ the "gain" parameter (traditionally in the literature we have κ<λ). However, as prices are higher than expectations when the fringe sets the high price and as consumers' behavior is not affected by this change in the utility functions, I will use its short version (1).

probability $\overline{\mu}$ is assumed to be strictly smaller than 1.

Neither consumers nor firms know this probability: they believe that the true probability is $\underline{\mu}$ with probability ρ and $\overline{\mu}$ with probability (1-ρ). The fringe sets a price equal to its cost.

Clearly, the maximal price that collusive firms can set in equilibrium is $\overline{c}$, as this is the maximal price set by the competitive fringe. If a firm deviates, they revert to Nash equilibrium forever (which constitutes the maximal punishment).

The parameters (ρ, $\underline{\mu}$, $\overline{\mu}$, $\underline{c}, \overline{c}$) are common knowledge.

Firms' only strategic variable is price[14]. Their strategy is then, for any i=1,2 and t, p^*_{it}: $[\hat{\rho}_t] \rightarrow R_+$, where $\hat{\rho}_t$ is firms' (and consumers') belief in t, updated with the information up to t-1, that the true probability of a low price by the fringe is $\underline{\mu}$.

Consumers' decision is whether to buy from firm 1, firm 2, the fringe or not buy at all. So their strategy is described by σ_t : $[p_{1t}, p_{2t}, p_{Ft}] \rightarrow [1, 2, F, 0]$ ∀ t.

In each period, consumers buy the good with the lowest price, if their utility is greater or equal than 0. So firm i's demand is D_{it}=1 if $p_{it}<p_{jt}$ for $i \neq j$, $u_t \geq 0$ *and* the fringe sets $p_{Ft}=\overline{c}$.

D_{it}=1/2 if $p_{it}=p_{jt}$ for $i \neq j$, $u_t \geq 0$ *and* the fringe sets $p_{Ft}=\overline{c}$.

D_{it}=0 if $p_{it}>p_{jt}$ for $i \neq j$ or $u_t<0$ *or* the fringe sets $p_{Ft}=\underline{c}$.

So the stage game profits are $\pi_{it}=(p_{it}-\underline{c})$ if $p_{it}<p_{jt}$ for i≠j, $u_t \geq 0$ *and* the fringe sets $p_{Ft}=\overline{c}$;

$\pi_{it}=\frac{(p_{it}-\underline{c})}{2}$ if $p_{it}=p_{jt}$ for i≠j, $u_t \geq 0$ *and* the fringe sets $p_{Ft}=\overline{c}$;

π_{it}=0 if $p_{it}>p_{jt}$ for any i≠j or $u_t<0$ *or* the fringe sets $p_{Ft}=\underline{c}$.

I will chacterize the best Perfect Bayesian equilibrium from the point of view of firms.

1.1 Timing of the game

In t=1:

1) The probability that the fringe draws the low cost is $\mu \in \{\underline{\mu}, \overline{\mu}\}$, where $\mu=\underline{\mu}$ has probability ρ. Consumers form price expectation $E_0[p_1]$. The cost for the competitive fringe is $c_t \in \{\underline{c}, \overline{c}\}$ where $\underline{c}$ has probability μ.
2) The fringe draws c_t and sets its price $p_{Ft}=c_t$.
3) Firms observe c_t and set their price.
4) Consumers observe prices and do their purchase decision.
5) Stage game payoff are realized.

[14] To keep things simple, I assume that firms cannot invest in advertising to influence consumers' beliefs.

In $t \geq 2$ all the steps are the same, except step 1 that becomes "1*) Past history is common knowledge. c_t is redrawn according to the μ drawn in t=1. Consumers update their $E_{t-1}[p_t]$ by updating their beliefs about μ given the price of the fringe".[15]

1.2 Firms' Problem, Price Expectations and Prices

The cartel chooses an infinite price path to maximize the expected sum of discounted profits. The problem each firm *i* wants to solve is:[16]

$$Max_{p^*_{it}} \sum_{t=0}^{\infty} \delta^t \pi_t \left(\hat{p}_{t,\tau,\hat{t}}, p_{Ft}\right) \quad (2)$$

$$s.t. \quad \sum_{t=0}^{\infty} \delta^t \pi_t \left(\hat{p}_{t,\tau,\hat{t}}, p_{Ft} = \overline{c}\right) > 2\pi_1\left(\hat{p}_{t,\tau,\hat{t}}, p_{Ft} = \overline{c}\right) \quad [ICC]$$

$$p_t^* \leq \overline{c} \ \ \forall\, t \quad (2b)$$

$$u_t \geq 0 \ \forall\, t \quad (2c)$$

In each period, when the fringe draws the high cost a firm may want to deviate[17]. This ICC has a role in the breakdown of the cartel and in the possible price wars. If the ICC is not fulfilled, firms understand that there is an incentive to undercut the collusive price, so they just set the competitive price: $p_t=\underline{c}$. Note that this ICC varies over time, as it depends on the updated belief over the fringe's efficiency $\hat{p}_{t,\tau,\hat{t}}$. I assume in the following that the ICC is fulfilled; I analyze in Section 2 what happens when this is not the case.
The constraint $\hat{p}_t \leq \overline{c} \ \ \forall\, t$ is due to the fact that if firms set a higher price than fringe's high cost, they sell nothing. The constraint $u_t \geq 0 \ \forall\, t$ means that firms want consumers to buy the good.
To make the problem interesting, I do the following assumption.

ASSUMPTION 1. $\overline{c}<v$;

ASSUMPTION 2. i) *Perfect collusion is sustainable if $\mu=\underline{\mu}$ is common knowledge, but not if $\mu=\overline{\mu}$.* ii) *Perfect collusion is sustainable for the initial belief ρ.*

[15] I assume that firms know the current cost draw of the fringe because they have a good knowledge of the industry. All the results would still hold also if also firms had varying costs, provided that their range is sufficiently smaller than the one of the fringe. Assuming this consists in assuming that firms are less vulnerable to industry-wide shocks, say because of economies of scale, a better knowledge of the industry etc. For simplicity of exposition I assume that firms' cost variation is zero.
[16] When firms compete, firms just set the stage-game profit-maximizing price.
[17] There is no point in deviating when the fringe sets the low price, as deviation profits would be zero.

In the best collusive equilibrium, in *t*=1 consumers know that market price will be $p_t = \underline{c}$ if and only if the fringe sets $p_{Ft} = \underline{c}$. From the consumers' point of view, this event happens with a probability $\rho\underline{\mu} + (1-\rho)\overline{\mu}$. If, on the other hand, $p_{Ft} = \overline{c}$, then firms set $p_1 = p_1^*$, where p_1^* is the price that solves (2); this simply consists in maximizing current profits, as current firms' price has no impact on consumers' beliefs, and so on future prices.

In the beginning of every period t≥2, consumers update their beliefs about the true probability of the low cost μ by looking at the fringe's price. Call $\hat{\rho}_{t,\tau,t}$ the updated consumers' subjective probability that the true μ is $\underline{\mu}$ after having seen τ periods of $p_{Ft} = \underline{c}$ and $\hat{t}$ periods of $p_{Ft} = \overline{c}$. This $\hat{\rho}_{t,\tau,t}$ has a crucial role: it enters the price expectation expression.

LEMMA 1*: If consumers observe the fringe setting p_{Ft}=$\underline{c}$ during τ periods and p_{Ft}=$\overline{c}$ during $\hat{t}$ periods, independently from their order, then the updated $\hat{\mathrm{p}}_{t,\tau,t}$ becomes*

$$\hat{p}_{t,\tau,\hat{t}} = \frac{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}}{\rho\underline{\mu}^{\tau}\left(1-\underline{\mu}\right)^{\hat{t}}+(1-\rho)\overline{\mu}^{\tau}(1-\overline{\mu})^{\hat{t}}} \tag{3}$$

Proof. See Appendix 1

One can easily check that $\frac{d\,\hat{p}_{t,\tau,\hat{t}}}{d\hat{t}}$>0 and $\frac{d\,\hat{p}_{t,\tau,\hat{t}}}{d\tau}$<0. This Lemma explains how the belief over $\underline{\mu}$ evolves over time, after any sequence of prices by the fringe.

I search for the best Perfect Bayesian equilibrium for the firms. In this equilibrium 1) each firm maximizes its own profits, given the fringe and the other firm's behavior, 2) consumers maximize their utility, given fringe's and firms' prices, 3) consumers beliefs are consistent with the equilibrium being played.

In particular, point 3 means that, when firms collude, consumers expect collusion to take place, while when they compete, consumers expect competition. This is a basic difference between this theory and Chen and Harrington (2006). They assume that consumers do not have correct beliefs about the equilibrium played; they try to infer whether firms collude or not by observing price differences over time. The bigger the difference between today's and yesterday's price, the higher the consumers' belief over collusion. In our paper, consumers know what equilibrium is being played.

In the competitive equilibrium, loss aversion and the uncertainty over the fringe's efficiency have no impact over the equilibrium outcomes, that remain the standard Bertrand ones: 1) $p^*_{it}=p^*_{jt}=\underline{c}$ ∀ t, i=1,2, i≠j, 2) $D_i=D_j=1/2$, 3) $E_{t-1}[p_t]=\underline{c}$.

In the best collusive equilibrium, when collusion is sustainable: 1) $p_t^*=p^*_{it}=p^*_{jt}$ ∀ t, i=1,2, i≠j, where $p_t^*=\bar{\bar{p}}_t\equiv\max\{p|u_t=0, \overline{c}\}$, 2) $D_i=D_j=1/2$. When collusion is not sustainable, firms just maximize current profits: $p^*_{it}=p^*_{jt}=\underline{c}$. The difference between *competition* and *collusion not sustainable* is that, under the second, if the belief $\hat{p}_{t,\tau,\hat{t}}$ over μ becomes sufficiently high, firms can set price above cost again. This is the idea of temporary price wars: firms set $p_t^*=\underline{c}$ during some periods (the ones when the belief $\hat{p}_{t,\tau,\hat{t}}$ over the fringe's inefficiency μ is too low). If $\hat{p}_{t,\tau,\hat{t}}$ never become sufficiently high anymore, the outcome is the cartel breakdown (i.e. price equal to marginal cost forever).

Any deviation from this makes firms revert to the competitive equilibrium forever (grim trigger strategy). $E_{t-1}[p_t]$ is given by the following Lemma; this will determine also the optimal price p_t^*.

LEMMA 2: *In the best collusive equilibrium, price expectation has the following form:*

$$E_{t-1}[p_t] = \left[\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu} + \left(1-\hat{\rho}_{t,\tau,\hat{t}}\right)\overline{\mu}\right]\underline{c} + [1-\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu} - \left(1-\hat{\rho}_{t,\tau,\hat{t}}\right)\overline{\mu}]p_t^* \tag{4}$$

where $t=\tau+\hat{t}+1$.

COROLLARY: combining (3) and (4), price expectation increases (decreases) after a period with high (low) fringe's price, that is $\frac{d\mathrm{E}_{t-1}[\mathrm{p}_t]}{d\hat{t}}>0$ and $\frac{d\mathrm{E}_{t-1}[\mathrm{p}_t]}{d\tau}<0$.

Every time consumers observe $p_{Ft}=\overline{c}$, they expect to pay a higher price than in the previous period, because they correctly give more weight to the lower probability that the fringe will set the low price. Being it common knowledge, firms can raise prices while keeping consumers' participation constraint still binding. The higher the number of periods during which consumers see high fringe's prices, the higher the expected future price, and so the actual firms' price (and vice versa).

LEMMA 3: *In the best collusive equilibrium, firms set the first period price equal to:*

$$p_1^* = \bar{\bar{p}}_t \equiv \min\{\frac{\nu+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]}, \overline{c}\} \quad (5)$$

where $\hat{p}_t := \frac{\nu+\lambda[\rho\underline{\mu}+(1-\rho)\overline{\mu}]\underline{c}}{1+\lambda[\rho\underline{\mu}+(1-\rho)\overline{\mu}]}$ is the *transitory phase price* and $\tilde{\overline{p}}_t = \overline{c}$ is the *constant phase price.*

Proof. See Appendix 2.

PROPOSITION 1: *Collusive prices have a transitory phase if and only if customers' willingness to pay is not too large:*

$$\overline{c} < \nu < \overline{c}(1+\lambda) - \lambda\left[\rho\underline{\mu} + (1-\rho)\overline{\mu}\right]\frac{m+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]} + [1-\rho\underline{\mu}-(1-\rho)\overline{\mu}]\underline{c}.$$

Proof. See Appendix 3.

LEMMA 4: *The transitory phase price $\hat{p}_1$ depends negatively on the probabilities $\underline{\mu}$ and $\overline{\mu}$ that the fringe is efficient, the size of loss aversion λ and positively on the belief ρ that the fringe is inefficient:*

$$\frac{d\hat{p}_1}{d\underline{\mu}} < 0; \qquad \frac{d\hat{p}_1}{d\overline{\mu}} < 0; \qquad \frac{d\hat{p}_1}{d\rho} > 0; \qquad \frac{d\hat{p}_1}{d\lambda} < 0.$$

The reason is that if consumers believe that the probabilities that the fringe sets the low price are high (via μ or ρ), they expect a low price, which makes them suffer high losses for a high price: this, in turn, forces firms to lower the price in order to keep consumers willing to buy.

A high loss aversion makes an intuitive result: $\hat{p}_1$ will be smaller, for any price expectation, as a higher λ means that consumers suffer a greater utility loss. This forces firms to lower prices to keep consumers willing to buy.

1.2.2 t≥2

Iterating the Bayesian updating procedure, we obtain the following proposition.

PROPOSITION 2: *In the best collusive equilibrium, after τ low and $\hat{t}$ high fringe's prices, the collusive price firms set in period t is*

$$p_t^* = \min\left\{\frac{\nu+\lambda\left[\frac{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}}{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}+(1-\rho)\overline{\mu}^{\tau}(1-\overline{\mu})^{\hat{t}}}\underline{\mu}+\left(1-\frac{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}}{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}+(1-\rho)\overline{\mu}^{\tau}(1-\overline{\mu})^{\hat{t}}}\right)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\frac{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}}{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}+(1-\rho)\overline{\mu}^{\tau}(1-\overline{\mu})^{\hat{t}}}\underline{\mu}+\left(1-\frac{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}}{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}+(1-\rho)\overline{\mu}^{\tau}(1-\overline{\mu})^{\hat{t}}}\right)\overline{\mu}\right]},\overline{c}\right\} \quad (7)$$

The left term in the parenthesis is the transitory phase price $\hat{p}_t$. When $p_t^* = \hat{p}_t$, (2b) is binding and (2c) is not. The price $\hat{p}_t$ rises with $\hat{t}$ because, after every $p_{Ft}=\overline{c}$, consumers update their beliefs about ρ upwards, making their utility loss due to loss aversion smaller. This lets firms increase price while keeping $u_t=0$. When $p_t^*=\overline{c}$, instead, price reached its maximum level: in that moment the binding constraint is (2b) and, normally, (2c) does not.[18]

The gradual rise of prices is a phenomenon that occurs when the fringe sets the high price and firms collude. If the fringe sets the low price, market price suddenly falls; afterwards, if the fringe sets again the high price, market price gradually rises again, because consumers buy from the incumbent firms for any price smaller than $\overline{c}$ and such that their $u_t \geq 0$. So there is an asymmetry in market price's behavior: price slowly rises when a cartel is present, but it falls quickly when the fringe sets the low price. I will come back on this in Section 2 when discussing price wars and cartel's breakdown.

Given equation (7), we get a result for the variance in the long run:

PROPOSITION 5*: Assume that there is some positive, albeit small, randomness on incumbents' cost. If the cartel is stable, then there exists a number of periods $T^*(t,\tau,\hat{t})$ after which, when the cartel is operative, we have $Var(P^{Comp})>Var(p_t^*)$, i.e. $\lim_{t\to\infty} Var(p_t^*)\to 0$.*

Proof. See Appendix 6.

[18] The fact that p_t^* reaches $\overline{c}$ depends on the parameters. If $\underline{\mu}$ is sufficiently high, even an arbitrarily high $\hat{t}$ will *not* make consumers so pessimistic that they accept to pay $p_t = \overline{c}$.-

2.1 Sustainability: sufficient condition

Up to now this theory could have perfectly applied also to a monopoly. Indeed, up to now the ICC is assumed to be fulfilled. By the first part of Assumption 2, we know that the collusive price is sustainable if and only if $\rho \geq \rho^*$, i.e. if and only if the belief about the low probability is sufficiently high. Here I analyze the conditions that assure that the ICC is fulfilled and I describe what happens when they are violated.

PROPOSITION 3: *In the best collusive equilibrium, the ex ante expected price is non-increasing on time.*

Proof. See Appendix 5.

LEMMA 6: *In a generic period t, a* sufficient *condition to fulfil the ICC of the firms' maximization problem is:*

$$\hat{\rho}_{t,\tau,\hat{t}} \geq \rho^*{}_S = \frac{1-2\delta+2\overline{\mu}\delta}{\delta(\overline{\mu}-\underline{\mu})} \tag{8}$$

Proof. See Appendix 7a.

2.2 Sustainability: necessary condition

In the previous subsection we have found a sufficient condition for the sustainability of the collusive price: finding a necessary *and* sufficient condition is very complicated[19] and we shall be content of having, further than the sufficient one, a necessary condition.

By the second part of Assumption 2, the initial ρ is above ρ^*. Then collusion price is sustainable and firms set the price as explained up to now. Every time the fringe sets the low price, however, $\hat{\rho}_{t,\tau,\hat{t}}$ is updated downwards (see (3)). Intuitively, it can happen that, after a certain number of low prices by the fringe, $\hat{\rho}_{t,\tau,\hat{t}}$ becomes so low that the collusive price is not sustainable anymore. In that

[19] A necessary and sufficient condition is very cumbersome to obtain for a number of reasons. First, the number of periods of low prices by the fringe needed to violate the ICCs depends not only on the parameter set, but also on *t*: the more information we have, the less informative is every new period. Second, the updated $\hat{\rho}_{t,\tau,\hat{t}}$ depends on its past value, making a calculation of a general formula very complicated. Third, continuation payoffs depend on all the possible combinations of prices by the fringe from the current *t* to infinity.

case, as long as $\hat{\rho}_{t,\tau,\hat{t}}$ becomes smaller than ρ*, firms revert to the static Nash equilibrium price, because they recognize that, otherwise, each firm has an incentive to deviate.

Analytically, we have the following. We know that the ex ante expected price is not increasing over time, so expected profits do not increase neither.

A *necessary* condition for sustainability is that deviation yields lower profits than continuing to set the collusive price *assuming that continuation profits are equal to the present stage-game profit*. This is just necessary for two reasons. First, Proposition 3 shows that ex ante expected prices are non-increasing over time. Here we assume them, and so also ex ante expected profits, to be constant over time, so we are possibly overestimating the continuation profits from collusion. Indeed, we are not considering the cap on prices (and profits) that the fringe exerts on firms. Second, we assume the collusive price to be sustainable forever, if firms stick to the best collusive equilibrium. Both assumptions make continuation profits after sticking to the collusive pricing higher than they really are, so the condition is just necessary, closer to be sufficient the farther the price is from $\overline{c}$ and the farther ρ is from ρ*.

LEMMA 7: *In a generic period t, a* necessary *condition to fulfil the ICC of the firms' maximization problem is:*

$$\hat{p}_{t,\tau,\hat{t}} \geq \rho^*{}_N = \frac{1-2\delta+\overline{\mu}\delta}{\delta(\overline{\mu}-\underline{\mu})} \tag{9}$$

Proof. See Appendix 7b.

Being a necessary condition, if $\hat{p}_{t,\tau,\hat{t}}$ is smaller than $\rho^*{}_N$, collusion is not sustainable, but not vice versa.

One can check that $\frac{d\rho^*{}_N}{d\delta} = \frac{d\rho^*{}_S}{d\delta} = -\frac{1}{\delta^2(\overline{\mu}-\underline{\mu})} < 0$, so a higher δ makes $\rho^*{}_N$ and $\rho^*{}_S$ smaller and by the same amount. The more firms are patient, the more the fringe can be efficient. The difference between $\rho^*{}_N$ and $\rho^*{}_S$ is $\frac{\overline{\mu}}{(\overline{\mu}-\underline{\mu})}$, a constant that does not depend on δ.

Firms' prices could nevertheless be higher than costs again if the fringe sets the high price during a number of periods that makes $\hat{p}_{t,\tau,\hat{t}}$ bigger than ρ*. However, differently from Rotemberg and

Saloner (1986) and Haltiwanger and Harrington (1991)[20], future prices are expected to be the same as present ones, because in expectations $\hat{p}_{t,\tau,\hat{t}}$ does not change over time. So, reducing prices does not make the ICC easier to fulfill, because future expected profits are the same as current ones for any $\hat{p}_{t,\tau,\hat{t}}$. So firms, in the best collusive equilibrium, will just switch from the collusive price to the competitive price and vice versa[21]. This consists in temporary price wars, that are not, then, due to imperfect monitoring or to a strategic reduction of price in order to keep collusion sustainable, like in the previous literature, but to the fact that, given the fringe's believed efficiency, firms recognize that each one would have an incentive to deviate.

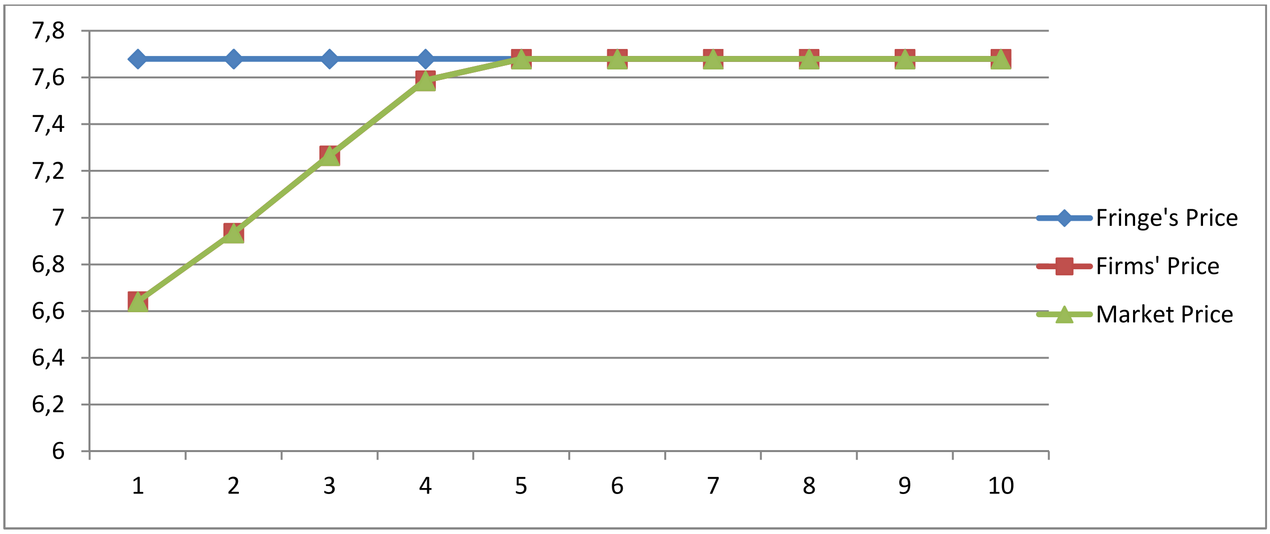
3. Numerical examples

Define δ^*_N as the minimum discount factor s. t. the necessary condition (9) is fulfilled for $\hat{p}_{t,\tau,\hat{t}}=1$. Assume that $\nu=10$, $\lambda=2$, $\bar{c}=7{,}68$, $\underline{c}=2$, $\rho=\frac{1}{4}$, $\bar{\mu}=\frac{1}{2}$, $\underline{\mu}=\frac{1}{8}$ and that δ satisfies Assumption 2.

One can easily verify that the conditions of Proposition 1 are fulfilled, so collusive prices cannot jump directly to $\bar{c}$. In the following numerical simulations I show how the price pattern looks like for different realizations of the fringe's cost.

[20] In Rotemberg and Saloner (1986) demand is determined in a i.i.d. fashion in every period, so future demand is independent from the present one. They show that the ICC is more difficult to fulfill when the present demand is high, because the incentive to deviate is higher. In Haltiwanger and Harrington (1991) the evolution of demand is deterministic and they show that collusion is more difficult to sustain when future demand is low, because the foregone profits are lower. In both these models future demand is expected to be different from the present one, which creates the different incentives to deviate depending on the present state. In my model, the present price is expected to be the best prediction of future prices too, so there is no room for temporary price cuts to reduce the temptation to deviate and "wait for better days".

[21] When, for any fringe's cost, collusive price would not be sustainable, consumers just expect $E_{t-1}[p_t]=\underline{c}$; when a high fringe's cost makes collusive price sustainable, consumers expectations are still (4).



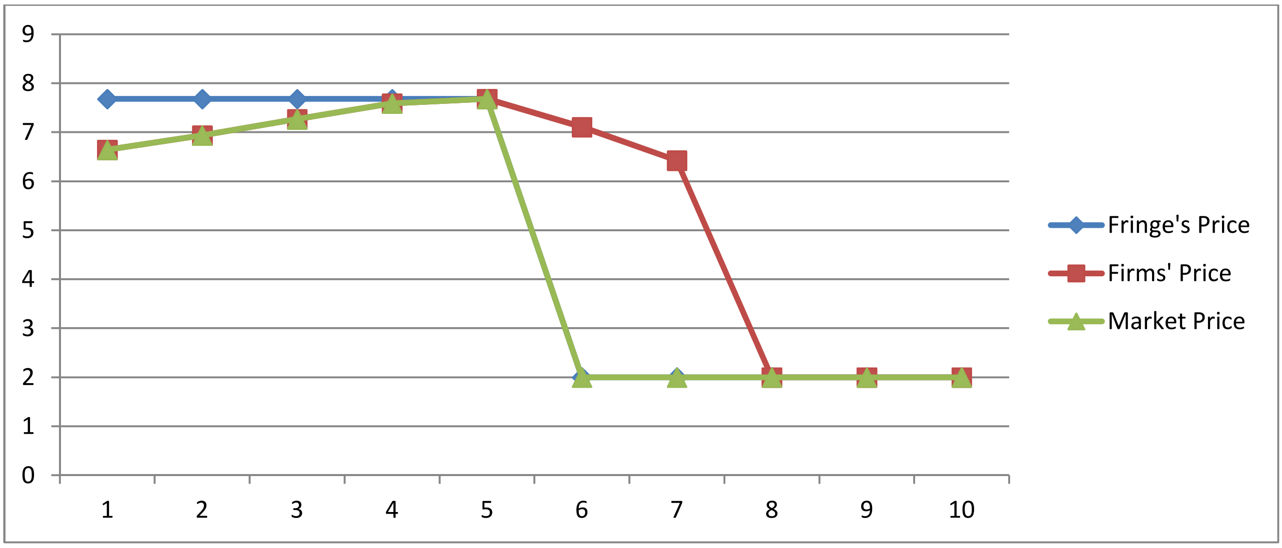
Simulation 1 (successful cartel): the fringe sets always the high cost

The sequence of firms' prices when the fringe always sets p_{Ft}=$\overline{c}$. Price rises until period t=5 and then remains constant and equal to $\overline{c}$=7,68.

Simulation 1 represents the case of a cartel that is always operative from period 1 to 10. It can represent the lysine cartel in Figure 1. When the fringe always sets p_{Ft}=$\overline{c}$, price rises from period 1 to period 5, after which it remains constant. But we see that between period 1 to period 4 it grows in a faster way than between period 4 to 5: the reason is that between period 1 and 4 the constraint that binds is (2c), while from period 5 on the constraint that binds is (2b). From period 1 to 4 firms make consumers pay the price that makes them indifferent between buying or not (u_t=0 for t=1,2,3,4). Since period 5 on consumers are so pessimistic about μ that they lose so little utility in paying a high price that they are willing to pay p_t=$\overline{c}$ too. From that period on, the force that constrains firms' price is the competitive fringe.

One could see that Figure 2 resembles quite closely the actual rise of lysine[22] price of Figure 1: there, we have some periods (from July to October 1993) in which price rose at an almost constant rate, then a much smaller increase in November and then an almost constant price during other ten months; in Simulation 1 we see the same behavior, i.e. some periods of the transitory phase at an almost constant pace, then a slighter increase and thereafter a constant price.

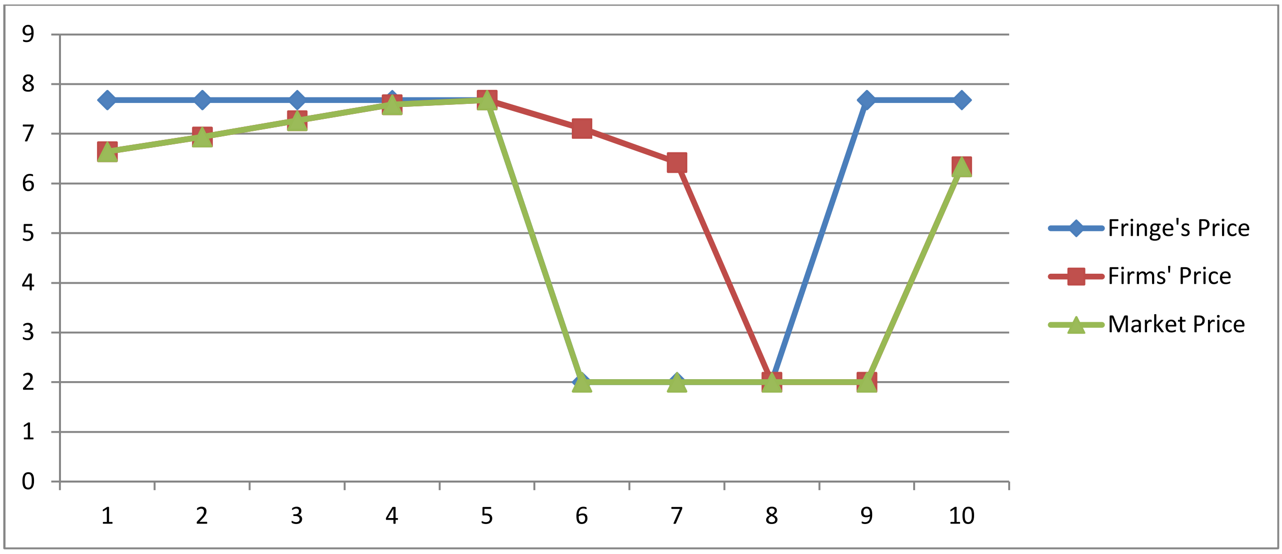
[22] As mentioned before, also the cartelized goods for final consumers show the same qualitative patterns.



Simulation 2 (cartel breakdown): after some periods of p_{FT}=$\underline{c}$, the collusive price is not sustainable anymore

The sequence of firms' prices when the fringe initially sets the high price and then the low one during several periods. Firms initially just lower the price; afterwards, the cartel is not operative.

In this simulation the fringe sets the high price during five periods and then, during other five periods, it sets the low price. Firms' price rises in the first five periods in the way explained in simulation 1; then it gets lower, after every period of low fringe's price. At some point (depending on δ), $\hat{p}_{t,\tau,\hat{t}}$ will be so low that the ICC will not be fulfilled, so collusive price is not sustainable and firms set the competitive price (the cartel is not operative).



Simulation 3 (price war): after some periods of low fringe's price, collusion is not sustainable anymore, but then the cartel becomes operative again.

The sequence of firms' prices when the fringe initially sets the high price, then the low one during several periods and then the high one again. Firms initially just lower the price; afterwards, collusive price is not sustainable, but after some periods of the high fringe's price, it can be sustained again.

Simulation 3 shows the price path of a temporary price war, followed by a reversal to collusive price. After some periods of low fringe's price, collusive price is not sustainable anymore, like in simulation 2; here, in period 9 the fringe sets again the high price, which makes $\hat{p}_{t,\tau,\hat{t}}$ higher, but still not sufficient to make the collusive price sustainable. A further period of high price makes $\hat{p}_{t,\tau,\hat{t}}$ sufficient to make it sustainable again, so firms rise the price again according to the collusive price equation (7). As we can see from the graph, there is a lag between the fringe's price and firms' price. Firms' price in some sense follows the fringe's price: when the fringe's one falls, firms' price still remains to relatively high levels (collusion remains sustainable, price just falls slightly to keep consumers on their u_t=0 level) and only after some periods it falls to the competitive level[23]. When the fringe raises the price, firms' price can remain during some periods to the competitive level, until $\hat{p}_{t,\tau,\hat{t}}$ raises to a level sufficient to make collusion sustainable again.
So we have:

PROPOSITION 4: *There exists parameter constellations for which the best collusive equilibrium exists. In the this equilibrium:*
1) prices have a transitory and then a constant phase,
2) no firm deviates for $\hat{p}_{t,\tau,\hat{t}}$ greater than a threshold, but the collusive price is not sustainable for $\hat{p}_{t,\tau,\hat{t}}$ smaller than a threshold ρ^*,
3) firms can have temporary or permanent cartel breakdowns,
4) price variance is smaller under an operative cartel than under competition.

4. Robustness and discussion

In this section I will discuss how these results are qualitatively robust to a number of variations of the model. The driving forces for the transitory phase are loss aversion and the uncertainty over the fringe's efficiency, and the one for the cartel's breakdown is the uncertainty over μ.
My model assumes, for simplicity, that the mass of consumers willing to buy the product is fixed. In order to relax this, we could introduce heterogeneity in consumers' maximal willingness to pay v_i or assume that there is an heterogeneous exit option if the good is not bought. These two modifications create similar results. Firms would sell the good to a possibly smaller number of consumers, but the transitory phase of prices, their smaller variance, the temporary price wars and the cartel's breakdown would remain qualitatively unchanged.

[23] If we allow firms to reduce their price in order to keep the ICCs binding, this effect would be lighter but still present: prices would still be smaller than the level needed to keep u_t=0.

One could introduce heterogeneity in the loss aversion λ_i or allow for differentiated products. We would get the same result: firms would compute the new optimal equilibrium price (and also the static Nash equilibrium prices, for the differentiated products case, in order to check that the ICC is fulfilled) but the qualitative results would remain the same.
A generalization on firms' number of course do not qualitatively change the results: a higher number of firms just makes the discount factor necessary to sustain collusion higher.
One could assume adaptive expectations on the consumers' side[24]: prices would still gradually rise and then be constant at $\overline{c}$.[25]
The fringe can also be interpreted as a less efficient firms that is unwilling to enter the cartel, or as a firm that uses the "hit and run" strategy[26]: the fringe is taken as a "black box" that embeds all the external competitive pressure that firms might face.
The fact that firms do not know μ has no importance for the transitory phase, but it does have for the breakdown of the cartel. If firms know μ, then the cartel is sustainable or not since the beginning of the game: the fact that the fringe sets the low price has no effect on the sustainability of collusion, because firms do no Bayesian updating at all.
The fact that fringe's cost (and so the price) is seen by the firms before they set their own prices has an impact on the necessary condition of the sustainability of perfect collusion, but not on the price dynamics. The transitory price pattern just depends on loss aversion combined with Bayesian updating and the variance depends on the fact that prices of an operative cartel tends towards a finite value, so they are both independent from the timing of the pricing decisions.
This model accounts for the fact that some cartels are successful in achieving higher prices and reaching stability, while others do not. These two possibilities are represented by the long run behavior when the probability of the low cost μ is, respectively, $\underline{\mu}$ and $\overline{\mu}$. Although price wars are always possible, in the first (second) case, the fringe does not (does) exert sufficient competitive pressure to make the cartel unstable in the long run.
The model can also account for price wars followed by reversals to collusive pricing. If the fringe sets the low price during a sufficiently high number of periods, firms may revert to the static Nash equilibrium pricing, while if the fringe sets the high price during a sufficient number of periods collusive price can become sustainable again.
This model addresses the issue of dynamics. The reference point is endogenous, not assumed. Koszegi and Heidhues (2008) elegantly showed the effects of loss aversion in a static variation of

[24] $E_{t-1}[p_t]=p_{t-1}$.
[25] This case is discussed in Appendix 8.
[26] i.e. selling at low cost on the market, then exit. This strategy is common, for example, among low cost airlines.

the Salop model (1979) to explain the rationale for focal prices, but did not explicitly consider a dynamic environment: their reference point is the "lagged rational expectation" and they do not investigate how this is formed. Koszegi and Rabin (2006) analyze how loss aversion impacts purchase and working decisions, taking the rational expectation over outcomes as the reference point, but still in a static environment. Koszegi and Rabin (2007) and Macera (2009) do consider a dynamic game with loss aversion, but in a different framework.

This paper is the first, at my knowledge, that deals with collusion and loss aversion. It is also the first one that explains the gradual rise of prices with consumers holding correct beliefs over the equilibrium being played by firms.

This model also gives some testable predictions, that can be compared to the ones in Harrington (2004, 2005) and Chen and Harrington (2006). My model predicts that the gradual rise of prices is independent from the existence of an AA, while in the models above an AA is needed and the stronger it is (in terms of probability of detection and damage multiples), the slower is the price rise. One could test data from different countries and test whether different antitrust policies have a different impact on the price path.

Assuming the existence of an AA does not necessarily change the results from this model: even if an AA is sure of the existence of a cartel by looking at the price path, in order to convict the firms it needs hard evidence. Firms might tacitly collude and avoid it, or the best collusive equilibrium can still be sustainable if the expected fine is sufficiently low.

My model also predicts that external competitive pressure can lead the cartel to temporary price wars and eventually to break down. This is the case of the Vitamin C cartel, where the Chinese manufacturers remained outside the cartel and increased constantly their market share. Eventually the cartel broke down.[27]

6. Conclusion

The model analyzed here explains, through loss aversion and uncertainty over a fringe's efficiency, the dynamic pattern of cartel prices, temporary price wars and cartel breakdowns. The gradual rise of prices is well known in the cartel literature, but up to now only explanations based on the fear of Antitrust fines have been given. My explanation is, on the contrary, based on consumers' tastes. Consumers dislike to pay a price higher than the expected one and this can force firms, in order to sell the good, not to jump to the maximal collusive level, but to raise prices smoothly. When the

[27] For more details, see footnote 5.

external competitive pressure is weak, consumers become more pessimistic towards the price they will pay, as they rationally update downwards the probability that the fringe will set the low price. This makes them willing to pay more, as the utility loss due to loss aversion is smaller. When the fringe, instead, sets the low price repeatedly, firms become more pessimistic about the value of colluding, possibly leading to price wars and the collapse of the cartel.

This model yields some testable predictions, for example that an increased competitive pressure may bring the cartel to breakdown. Empirical evidence from the Vitamin C cartel seems to confirm this claim. Also, the Antitrust enforcement should have a limited effect on the speed of the price rise. It would be interesting to test this claim empirically.

REFERENCES:

Abrantes R., Froeb L., Geweke J. and Taylor C. (2006), "A Variance Screen for Collusion", *International Journal of Industrial Organization*, Vol. 24, Issue 3, May, 467-486.

Besanko, D., Spulber, D. F., (1989), "Antitrust Enforcement under Asymmetric Information", *Economic Journal*, Royal Economic Society, vol. 99(396), pages 408-25, June.

Blinder A.S., Canetti E.R.D., Lebow D.E. and Rudd J.B. (1998), "Asking about Prices: a New Approach to Understanding Price Stickiness", New York, Russell.

Bowman D., Minehart D., Rabin M., (1999), "Loss Aversion in a Consumption Savings Model", *Journal of Economic Behavior & Organization*, Vol. 38 (1999) 155±178

Chen J. and Harrington J. Jr. (2006), "Cartel Pricing Dunamics with Cost Variability and Endogenous Buyes Detection", *International Journal of industrial Organization*, 24, 1185-1212.

Connor J.M. (1998), "What Can We Learn from the ADM Global Price Conspiracies?", Staff Paper 98-14.

Connor J.M. (2001), "Our Customers Are Our Enemies: the Lysine cartel of 1992-1995", *Review of Industrial Organization* 18, 5-21

Ellison G. (2006), "Bounded Rationality in Industrial Organization", in Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress, ed. Richard Blundell, Whitney Newey and Torsten Persson. Cambridge: Cambridge University Press.

Erickson G.M. and Johansson J. K. (1985), "The Role of Price in Multi-Attribute Product Evaluations", *Journal of Consumer Research*, 12(2): 195-199.

Fox C.R., Poldrack R.A., Tom S.M. and Trepel C., (2007), "The Neural Basis of Loss Aversion in Decision-Making Under Risk", *Science*, Vol. 315 26 January.

Genesove D. and Mayer C. (2001), "Loss Aversion and Seller Behavior: Evidence from the Housing Market", *The Quarterly Journal of Economics,* 116 (4): 1233-1260.

Gill D. and Prowse V. (2012), "A Structural Analysis of Disappointment Aversion in a Real Effort Competition", *American Economic Review*, 102(1): 469-503.

Gill D. and Stone R. (2010), "Fairness and Desert in Tournaments", *Games and Economic Behavior*, 69, 346-364.

Green E.J. and Porter R.H. (1984), "Noncooperative Collusion under Imperfect Information", *Econometrica*, Vol. 52, No. 1. Jan., 87-100.

Haigh M.S., List J.A. (2006), "Do Professional Traders Exhibit Myopic Loss Aversion? An Experimental Analysis", *The Journal of Finance*, Vol. 60, No. 1, February.

Haltiwanger J. and Harrington J. Jr. (1991), "The Impact of Cyclical Demand Movement on Collusive Behavior", *RAND Journal of Economics*, 22, 89-106.

Harrington J. Jr. (2004), "Cartel Pricing Dynamics in the Presence of an Antitrust Authority", *RAND Journal of Economics*, 35, 651-673.

Harrington J. Jr. (2005), "Optimal Cartel Pricing in the Presence of an Antitrust Authority", *International Journal of Industrial Organization*, 46, 145-169.

Heidhues P. and Koszegi (2008), "Competition and Price Variation When Consumers Are Loss Averse". *American Economic Review*, 98(4): 1245-1268.

Kahneman, Daniel; Knetsch, Jack L. and Thaler, Richard H. **(1991)**, "The Endowment Effect, Loss Aversion, and Status Quo Bias: Anomalies." *Journal of Economic Perspectives*, Winter 5(1): 193-206.

Kahneman, Daniel and Tversky, Amos (1979). "Prospect Theory: An Analysis of Decision under Risk." *Econometrica*, March, 47(2): 263-91.

Kalwani M. U. and Chi Kin Yim (1992), "Consumer Price and Promotion Expectations: An Experimental Study", *Journal of Marketing Research*, 29(1): 90-100.

Kandori M. (1991), "Correlated Demand Shocks and Price Wars During Booms", *Review of Economic Studies*, 58, 171-180.

Koszegi B. and Rabin M. (2006), "A Model of Reference-Dependent Preferences". *Quarterly Journal of Economics*, 121(4): 1133-1165.

Koszegi B. and Rabin M. (2007), "Reference-Dependent Risk Attitudes". *American Economic Review*, 97(4): 1047-1073.

Levenstein M. and Suslow V. Y. (2001), "What Determines Cartel Success?" *Journal of Economic Literature*, Vol. 44, No. 1, March, 43-95(53).

Levenstein M. (1997), "Price Wars and the Stability of Collusion: A Study of the Pre-World War I Bromine Industry?" The *Journal of Industrial Economics*, Vol. 45, No. 2, Jun., 117-137.

Macera R., (2011), "Intertemporal Incentives with Expectation-Based Reference-Dependent Preferences", mimeo.

Novemsky N. and Kahneman D. (2005), "The Boundaries of Loss Aversion", *Journal of Marketing Research*, Vol. 17, May, 119-128.

Rotemberg J. (2004), "Fair Pricing", National Bureau of Economic Research Working Paper 10915.

Rotemberg J. and Saloner G. (1986), "A Supergame Theoretic Model of Price Wars during Booms", American Economic Review, 76, 390-407.

Salop S. (1979), "Monopolistic Competition with Outside Goods", *The Bell Journal of Economics*, Vol. 10, No. 1, Spring, 141-156.

Thaler R., (1980), "Toward a Positive Theory of Consumer Choice", Journal of Economic Behavior and Organization, Vol. 1, Issue 1, 39-60.

Tversky A. and Kahneman D., "Loss Aversion in Riskless Choice: A Reference-Dependent Model", *The Quarterly Journal of Economics*, Vol. 106, No. 4. (Nov., 1991), pp. 1039-1061.

Winer R. S. (1986), "A Reference Price Model of Brand Choice for Frequently Purchased Products", *Journal of Consumer Research*, 13(2):250-256.

APPENDIX 1: EVOLUTION OF BELIEFS

LEMMA 1*: If consumers observe the fringe setting $p_{Ft}=\underline{c}$ during τ periods and $p_{Ft}=\overline{c}$ during $\hat{t}$ periods, independently from their order, then the updated $\hat{p}_{t,\tau,t}$ becomes*

$$\hat{p}_{t,\tau,\hat{t}} = \frac{\rho\underline{\mu}^{\tau}(1-\underline{\mu})^{\hat{t}}}{\rho\underline{\mu}^{\tau}\left(1-\underline{\mu}\right)^{\hat{t}}+(1-\rho)\overline{\mu}^{\tau}(1-\overline{\mu})^{\hat{t}}}$$

Proof. The possibilities that make the fringe set $p_{Ft}=\overline{c}$ are: 1) $(\mu = \underline{\mu},\ c_t = \overline{c})$; 2) $(\mu = \overline{\mu},\ c_t = \overline{c})$.

The possibilities that make the fringe set $p_{Ft}=\underline{c}$ are: 1) $(\mu = \underline{\mu},\ c_t = \underline{c})$; 2) $(\mu = \overline{\mu},\ c_t = \underline{c})$.

By observing that in *t=1* the fringe sets, say, $p_{F1}=\overline{c}$, consumers update their belief about μ: knowing that $\Pr(\mu=\underline{\mu})=\rho$, $\Pr(a=\overline{a})=1-\rho$, $\Pr(c_t=\underline{c})=\mu$, $\Pr(c_t=\overline{c})=1-\mu$, by applying the Bayes' rule the updated belief about $\underline{\mu}$ is $\hat{\rho}_{1,0,1}=\dfrac{\rho(1-\underline{\mu})}{\rho\left(1-\underline{\mu}\right)+(1-\rho)(1-\overline{\mu})}$. If, on the contrary, consumers see $p_{F1}=\underline{c}$, their updated probability will be $\hat{\rho}_{1,1,0}=\frac{\rho\underline{\mu}}{\rho\underline{\mu}+(1-\rho)\overline{\mu}}$.

Iterating this procedure, we get the result.

APPENDIX 2: FIRST PERIOD PRICE

LEMMA 3: *In the best collusive equilibrium, firms set the first period price equal to:*

$$p_1^* = \bar{\bar{p}}_t \equiv \min\{\frac{\nu+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]},\overline{c}\} \tag{5}$$

where $\hat{p}_t := \frac{\nu+\lambda[\rho\underline{\mu}+(1-\rho)\overline{\mu}]\underline{c}}{1+\lambda[\rho\underline{\mu}+(1-\rho)\overline{\mu}]}$ is the *transitory phase price* and $\hat{\bar{p}}_t=\overline{c}$ is the *constant phase price.*

We have two different cases:

i) (2c) slacks: $u_1 \geq 0$ when $p_1=\overline{c}$, in which case $p_1^* = \overline{c}$ (due to the constraint of the fringe);

ii) (2c) binds: $u_1<0$ when $p_1=\overline{c}$, in which case $p_1^* = \hat{p}_1$ is the price that makes $u_1=0$[28].

In this second case, we just have to solve for $u_1=0$. Using (1) we get $\hat{p}_1 = \frac{\nu+\lambda E_0[p_1]}{1+\lambda}$.

Given the expression (4) and substituting it here above, we get

$$\hat{p}_1 = \frac{\nu+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]}. \tag{6}$$

In general, the actual price p_1^* will be the minimum between $\overline{c}$ and $\hat{p}_1$ for three reasons. First, a price higher than $\overline{c}$ would yield zero profits due to the fringe; second, a price higher than $\frac{\nu+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]}$ would make firms sell no good, as $u_1<0$; third, a price lower than $\frac{\nu+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]}$ does not maximize profits.

[28] If the fringe sets the low price, firms have no incentive to deviate to a lower price: they make $\pi=0$ by sticking to this strategy; they make $\pi=0$ by matching fringe's price and they make negative profits by undercutting it.

APPENDIX 3: TRANSITORY PHASE

PROPOSITION 1: *Collusive prices have a transitory phase if and only if customers' willingness to pay, absent loss aversion, is not too large:*

$$\overline{c} < v < \overline{c}(1+\lambda) - \lambda\left[\rho\underline{\mu} + (1-\rho)\overline{\mu}\right]\frac{m+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\rho\underline{\mu}+(1-\rho)\overline{\mu}\right]} + [1-\rho\underline{\mu}-(1-\rho)\overline{\mu}]\underline{c}.$$

Proof. The first inequality is Assumption 1; the second one impedes firms to directly jump to the maximal collusive price $\overline{c}$. This condition basically says that $u_1<0$ when $p_1 = \overline{c}$, so p_1^* will be equal to $\hat{p}_1$. We are in this case if and only if $\overline{c} < m < \overline{c}(1+\lambda) - \lambda E_0[\hat{p}_1]$. Substituting $E_{1,bF}[\hat{p}_1]$ with its expression (4), we get

$\overline{c} < m < \overline{c}(1+\lambda) - \lambda\left[\rho_{1,\tau,\hat{t}}\underline{\mu} + \left(1-\rho_{1,\tau,\hat{t}}\right)\overline{\mu}\right]\hat{p}_1 + [1-\rho_{1,\tau,\hat{t}}\underline{\mu} - \left(1-\rho_{1,\tau,\hat{t}}\right)\overline{\mu}]\underline{c}.$

Substituting $\hat{p}_1$ with its expression (6), we get the result.

APPENDIX 4: DURATION OF THE TRANSITION

Here I briefly discuss the minimal duration of the transition period. In order to explain how it depends on the parameters, define T as

$$T = \{t \in N \mid p_t^* = \overline{c}, \qquad p_t^* = \hat{p}_{t-1} < \overline{c},\ \ p_{Ft} = \overline{c}\ \ \forall t\}$$

So T represents the minimal[29] number of periods after which the high price reaches $\overline{c}$. Using (7), T is the solution of the following system of equations:

1) $$\frac{m+\lambda\left[\frac{\rho\left(1-\underline{\mu}\right)^T}{\rho\left(1-\underline{\mu}\right)^T+(1-\rho)(1-\overline{\mu})^T}\underline{\mu}+\left(1-\frac{\rho\left(1-\underline{\mu}\right)^T}{\rho\left(1-\underline{\mu}\right)^T+(1-\rho)(1-\overline{\mu})^T}\right)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\frac{\rho\left(1-\underline{\mu}\right)^T}{\rho\left(1-\underline{\mu}\right)^T+(1-\rho)(1-\overline{\mu})^T}\underline{\mu}+\left(1-\frac{\rho\left(1-\underline{\mu}\right)^T}{\rho\left(1-\underline{\mu}\right)^T+(1-\rho)(1-\overline{\mu})^T}\right)\overline{\mu}\right]} > \overline{c} \qquad (10a)$$

[29] Minimal because we assume here that the fringe always sets the high price.

2) $$\frac{m+\lambda\left[\frac{\rho\left(1-\underline{\mu}\right)^{T-1}}{\rho\left(1-\underline{\mu}\right)^{T-1}+(1-\rho)(1-\overline{\mu})^{T-1}}\underline{\mu}+\left(1-\frac{\rho\left(1-\underline{\mu}\right)^{T-1}}{\rho\left(1-\underline{\mu}\right)^{T-1}+(1-\rho)(1-\overline{\mu})^{T-1}}\right)\overline{\mu}\right]\underline{c}}{1+\lambda\left[\frac{\rho\left(1-\underline{\mu}\right)^{T-1}}{\rho\left(1-\underline{\mu}\right)^{T-1}+(1-\rho)(1-\overline{\mu})^{T-1}}\underline{\mu}+\left(1-\frac{\rho\left(1-\underline{\mu}\right)^{T-1}}{\rho\left(1-\underline{\mu}\right)^{T-1}+(1-\rho)(1-\overline{\mu})^{T-1}}\right)\overline{\mu}\right]}<\overline{c} \qquad (10b)$$

An explicit solution is quite complicated. Nevertheless, we can state some properties of T.

LEMMA 5: *Let T be defined as above. If T exists, it increases with the efficiency of the fringe, loss aversion and the fringe's high cost; it decreases with the belief that the fringe is inefficient, the low fringe's (and firms') cost and the willingness to pay absent loss aversion. Formally:*

$$\frac{dT}{d\underline{\mu}}>0, \qquad \frac{dT}{d\overline{\mu}}>0, \qquad \frac{dT}{d\rho}<0, \qquad \frac{dT}{d\lambda}>0, \qquad \frac{dT}{d\underline{c}}<0, \qquad \frac{dT}{d\overline{c}}>0, \qquad \frac{dT}{d\nu}<0.$$

A higher ρ and a lower $\underline{\mu}$ and $\overline{\mu}$ makes T smaller because, *ceteris paribus*, consumers expect a higher price because of a lower probability of low fringe's price, so their utility loss due to loss aversion is smaller. This makes the convergence to $\overline{c}$ faster. A higher ν makes T smaller too because the higher intrinsic utility makes $u_t>0$ in a shorter number of periods.
A higher λ makes T larger, because it makes the utility loss due to loss aversion higher for any price expectation. A higher $\overline{c}$ also makes T larger. The reason why $\underline{c}$ and $\overline{c}$ have different effects on T is that $\underline{c}$ directly enters the expectation expression (4), while $\overline{c}$ enters the system of disequations (10a-10b) on the right side. So firstly it does not impact the price expectation in the transitory phase and secondly it makes necessary a higher T to make the left hand expression in (10a) greater than $\overline{c}$. The effect of a larger $\overline{c}$ is towards a larger T because the expression in the left side of (10a) grows as T grows and $\overline{c}$ is present only on its right.

APPENDIX 5: EXPECTED PRICE PATH

PROPOSITION 3: *In the best collusive equilibrium, the ex ante expected price is non-increasing on time.*

Proof .The proof consists of two subcases: the transitory phase and the constant phase[30].

Define $\hat{\hat{\rho}}_{t,\tau,\hat{t}} := E_{t-1}[\hat{\rho}_{t,\tau,\hat{t}}|\underline{\mu}]$. The ex ante expected price in t+1 is:

$$E_{t-1}[p_{t+1}] \;=\; \left[(\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}\underline{\mu} + \left(1-\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}\right)\overline{\mu}\right]\underline{c} + \left[1-\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}\underline{\mu} - \left(1-\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}\right)\overline{\mu}\right]p^*_{t+1}\}. \qquad (11)$$

We have gradual rise of prices in expectations if $E_{t-1}[p_{t+1+k}] > E_{t-1}[p_{t+k}]$ $\forall$ *t, k≥0*. But note that:

$$\hat{\hat{\rho}}_{t+1,\tau,\hat{t}} = \rho\{\underline{\mu}[\frac{\hat{\rho}_{t,\tau,\hat{t}}\,\underline{\mu}}{\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu}+(1-\hat{\rho}_{t,\tau,\hat{t}})\overline{\mu}}]+(1-\underline{\mu})[\frac{\hat{\rho}_{t,\tau,\hat{t}}(1-\underline{\mu})}{\hat{\rho}_{t,\tau,\hat{t}}(1-\underline{\mu})+(1-\hat{\rho}_{t,\tau,\hat{t}})(1-\overline{\mu})}]\}+$$

$$+(1-\rho)\{\overline{\mu}\,[\frac{\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu}}{\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu}+(1-\hat{\rho}_{t,\tau,\hat{t}})\overline{\mu}}]+(1-\overline{\mu})[\frac{\hat{\rho}_{t,\tau,\hat{t}}(1-\underline{\mu})}{\hat{\rho}_{t,\tau,\hat{t}}(1-\underline{\mu})+(1-\hat{\rho}_{t,\tau,\hat{t}})(1-\overline{\mu})}]\}$$

After some algebra, we see that $\hat{\hat{\rho}}_{t+1,\tau,\hat{t}} = \hat{\rho}_{t,\tau,\hat{t}}$. Trivially, as beliefs are ex ante expected to be constant over time, so are prices, as they just depend on beliefs. The fact that a cap $\overline{c}$ exists implies that the transitory phase price is, in expectations, not increasing over time.

Second, in the constant price case, price remains unchanged or falls if the fringe sets during a sufficient number of periods the low price. Also in this case price is non-increasing, which proves the claim.

COROLLARY: *In the best collusive equilibrium, when $\mu=\underline{\mu}$, the ex ante expected price rises over time.*

Proof. The ex ante expected price in t+1 conditioned on $\mu=\underline{\mu}$ is:

$$E_{t-1}[p_{t+1}] \;=\; \left[(\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}\underline{\mu} + \left(1-\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}\right)\overline{\mu}\right]\underline{c} + \left[1-\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}\underline{\mu} - \left(1-\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}\right)\overline{\mu}\right]p^*_{t+1}\}. \qquad (12)$$

We have gradual rise of prices in expectations if $E_{t-1}[p_{t+1+k}] > E_{t-1}[p_{t+k}]$ $\forall$ *t, k≥0.*

In order to show this, we proceed in three steps.

[30] i.e. when $p_t{}^*=\overline{c}$

1) Consider $\hat{\hat{\rho}}_{t,\tau,\hat{t}}$. Being it conditioned on $\mu=\underline{\mu}$, it can be written as

$$\hat{\hat{\rho}}_{t+1,\tau,\hat{t}} = \underline{\mu}[\frac{\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu}}{\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu}+(1-\hat{\rho}_{t,\tau,\hat{t}})\overline{\mu}}]+(1-\underline{\mu})[\frac{\hat{\rho}_{t,\tau,\hat{t}}(1-\underline{\mu})}{\hat{\rho}_{t,\tau,\hat{t}}(1-\underline{\mu})+(1-\hat{\rho}_{t,\tau,\hat{t}})(1-\overline{\mu})}]. \quad (13)$$

The first (second) part of the right hand side is the probability that the low (high) cost is drawn, multiplied by the updated $\hat{\rho}_{t,\tau,\hat{t}}$.

Note that $\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}>\hat{\rho}_{t,\tau,\hat{t}}$. Indeed, after some algebra, we get

$$\{\underline{\mu}[\frac{\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu}}{\hat{\rho}_{t,\tau,\hat{t}}\underline{\mu}+(1-\hat{\rho}_{t,\tau,\hat{t}})\overline{\mu}}]+(1-\underline{\mu})[\frac{\hat{\rho}_{t,\tau,\hat{t}}(1-\underline{\mu})}{\hat{\rho}_{t,\tau,\hat{t}}(1-\underline{\mu})+(1-\hat{\rho}_{t,\tau,\hat{t}})(1-\overline{\mu})}]\}-\hat{\rho}_{t,\tau,\hat{t}} = (1-\rho)^2\rho(\overline{\mu}-\underline{\mu})^2>0.$$

2) Given (7), $p_t^*>\underline{c}$ $\forall$ *t*. Now consider expression (13). By step 1, as *t* grows, $\hat{\hat{\rho}}_{t+1,\tau,\hat{t}}$ also grows $\forall$ *t*. This means that more and more weight is given to $\underline{\mu}$. Given that $\overline{\mu}>\underline{\mu}$, more and more weight is given to p_{t+1}^*, with respect to $\underline{c}$. By step 2, this means that $E_{t-1}[p_{t+1}]>E_{t-1}[p_t]$.

3) Iterating the process by plugging $\hat{\hat{\rho}}_{t+2,\tau,\hat{t}}$ in $\hat{\rho}_{t+1,\tau,\hat{t}}$ in (13) and so on, we obtain the result.

This shows us that, when $\mu=\underline{\mu}$, the gradual rise of prices is an event that in the long run will occur. With an analogous reasoning, one can easily show that prices eventually drop when $\mu=\overline{\mu}$. These two phenomena describe cartel price behavior when a cartel is successful and stable ($\mu=\underline{\mu}$) as well as when a cartel eventually collapses due to the competition of other firms ($\mu=\overline{\mu}$).

APPENDIX 6: PRICE VARIANCE

Another fact is that, after the transition phase, prices in a cartelized industry have a smaller variance than in a competitive one. This has been analyzed in Abrantes-Metz et al. (2006) and Blinder et al. (1998). A little extension of the model can take this into account.

DEFINITION: a cartel is stable in the long run *if the probability of the low cost is* $\mu=\underline{\mu}$.

The fact that the cartel is stable in the long run means that in the long run collusion is sustainable (see Assumption 2).

Now we extend the model to allow for an analysis of the price variance while the cartel is stable and operative, which is the empirical fact of the literature. We relax the assumption that firms have a constant and uniform cost c=$\underline{c}$ and we allow for a small randomness in their cost draw: they still have the same cost, but it may change over time. Assume now that they have $c_t=\underline{c}$ with a probability

β and $c_t=\tilde{c}$ with a probability 1-β, where $\tilde{c}<p_1^*<\overline{c}$[31]. I am then assuming a small cost randomness for incumbent firms, such that their high cost is smaller than the fringe's one and the first collusive price. We can justify this by the fact that incumbent firms have a better knowledge of the technology needed to produce the good than the fringe, so that their high marginal cost is smaller than fringe's one.

We can see that the price variance of a stable cartel that sells positive quantities, after possibly a sufficient number of periods, is smaller than the variance of the competitive price: we know that, if T exists, after a certain number of periods of $p_{F,t}=\overline{c}$[32], the price of a cartel reaches $\overline{c}$ and then remains constant; or, if T does not exist, it will increase at a slower and slower rate (converging to 0). So, after a sufficient number of periods, the price variance is 0 or tends towards 0. When firms compete, on the contrary, price will simply match cost, as we have Bertrand competition in the stage game: so with a probability β we have $p_t^{Comp}=\underline{c}$ and with probability 1- β we have $p_t^{Comp}=\tilde{c}$. Price variance then remains always positive.

PROPOSITION 5*: Assume that there is some positive, albeit small, randomness on incumbents' cost. Then there exists a number of periods $T^*(t,\tau,\hat{t})$ after which, when the cartel is operative, we have $Var(P^{Comp})>Var(P^{Coll})$.*

APPENDIX 7: CONDITIONS FOR SUSTAINABILITY

a) LEMMA 6: *In a generic period t, a* sufficient *condition to fulfil the ICC of the firms' maximization problem is:*

$$\hat{p}_{t,\tau,\hat{t}} \geq \rho^*_S = \frac{1-2\delta+2\overline{\mu}\delta}{\delta(\overline{\mu}-\underline{\mu})} \tag{8}$$

Proof. Assume that, if $p_{Ft}=\underline{c}$ in any period, continuation profits are zero.[33] Given this assumption,

[31] This assumption consists in a "very small" randomness on incumbents' cost. I do this assumption to avoid technical problems and to add robustness to my claim that cartel price variance is smaller. The technical problem is that if $\tilde{c}$ were greater than $p_{1,0,1}^*$, then competitive firms would not produce when they have the high cost draw in the first period. This would complicate the analysis without adding anything interesting. The robustness is that, *if* I show that cartel price variance is higher, although firms' costs variance is so small, *a fortiori* it will be smaller if firms' costs vary more, because the pass-through is maximal under competition.

[32] Equal to T if we consider the beginning of the game.

[33] In general profits are higher: firms gain positive profits every time the fringe sets the high price, if collusion is still

we can assume that profits are constant and positive for any t with $p_{Ft}=\overline{c}$. Define

$\alpha_t := \text{Prob}(p_{Ft}=\overline{c}) = 1-\hat{p}_{t,\tau,\hat{t}}\underline{\mu} - (1-\hat{p}_{t,\tau,\hat{t}})\overline{\mu}$. So the sufficient ICC becomes $2\pi \leq \pi + \frac{\delta\alpha}{1-\delta\alpha}\pi$.

Using the definition of α and after some algebra, we get the result.

b) LEMMA 7: *In a generic period t, a* necessary *condition to fulfil the ICC of the firms' maximization problem is:*

$$\hat{p}_{t,\tau,\hat{t}} \geq \rho^*_N = \frac{1-2\delta+\overline{\mu}\delta}{\delta(\overline{\mu}-\underline{\mu})} \tag{9}$$

Proof. Assuming that perfect collusion is always sustainable and that expected future profits are constant, the ICC is

$$2\pi \leq \pi+\delta\alpha\pi+\delta^2[\alpha(\alpha\pi)+(1-\alpha)(\alpha\pi)]+\delta^3\{\alpha[\alpha(\alpha\pi)+(1-\alpha)(\alpha\pi)]+(1-\alpha)[\alpha(\alpha\pi)+(1-\alpha)(\alpha\pi)]\}+\ldots$$

This yields $2\pi \leq \pi+\delta\alpha\pi(1+\delta+\delta^2+\delta^3+\ldots)$, that, after substituting for α_t, gives the result.

APPENDIX 8: ADAPTIVE EXPECTATIONS

All these results are robust also to assuming that consumers have adaptive expectations. Relaxing rational expectations (and Bayesian updating on the consumers' side), assume that consumers expect $E_{t-1}[p_t]=\min\{p_t^*, p_{Ft}^*\}$.

From $u_t=0$, we have $\hat{p}_1 = \frac{\nu+\lambda E_0[p_1]}{1+\lambda}$. Given adaptive expectations and that we do not have a price prior to p_1, we must assume an expected price for p_1. In order to keep it general, just assume that $E_0[p_1]=k$, where $\underline{c}\leq k\leq\overline{c}$. We want to show that, for any k, results are qualitatively the same as under Bayesian updating and rational expectations.

We get $\hat{p}_1 = \frac{\nu+\lambda k}{1+\lambda}$. In the best collusive equilibrium, when the fringe sets the high price, consumers buy from firms at a price that sets their $u_t=0$. So, using (1), we get $\hat{p}_2 = \frac{\nu(1+\lambda)+\lambda\nu+\lambda^2 k}{(1+\lambda)^2}$. Iterating this procedure, we get $\hat{p}_{\hat{t}} = \frac{\nu\sum_{l=0}^{\hat{t}-1}(1+\lambda)^l\lambda^{\hat{t}-1-l}+\lambda^{\hat{t}}k}{(1+\lambda)^{\hat{t}}}$. This is the

sustainable.

price firms set after $\hat{t}$-*1* periods of high price by the fringe. When there has been at least one low price by the fringe, k is replaced by $\underline{c}$[34].

One can easily check that $\frac{d\hat{p}_{\hat{t}}}{d\hat{t}}>0$, so after every period of high fringe's price, firms' prices rise. Still there is the cap $\overline{c}$, due to the fringe, so still prices can rise up to $\overline{c}$, after which they remain constant.

For the breakdown, nothing changes, as in this issue what matters is *firms*' expectations. Given that here we only changed consumers' expectations, firms will still be able to collude if $\hat{p}_{t,\tau,\hat{t}} \geq \rho^*$ and revert to competition (temporarily or permanently, depending on the fringe) otherwise.

[34] Because in the period after the low price, consumers' expected price is $E_{t-1}[p_t]=\underline{c}$.