Downstream Concentration and Endogenous Upstream Supply

Abstract

This paper investigates the effects of buyer power by downstream firms on the entry decisions of upstream suppliers. Under reasonable market conditions, industries with a few downstream buyers induce more entry at the upstream stage than industries with a larger number of firms. In particular, monopsony can be more conducive to upstream entry and lead to higher social welfare than more fragmented industry structures. This seeming paradox arises because a single processor better internalizes the positive effects of entry on later-periods’ supply conditions than a collection of firms. Such a firm rationally restrains exercise of its market power to incentivize more entrants to supply the input. This result is relevant in a number of market settings, including markets for specialized labor and processing markets for agricultural products.

Keywords: industry concentration, monopsony power, hold-up, entry, merger

A widely held belief among economists and policymakers is that the degree of concentration in an industry, e.g., as measured via concentration ratios or the Herfindahl index, is positively related to the extent of market power exercised in that industry and negatively related to the industry’s rate of output and social welfare (Farrell and Shapiro, 1990). This belief has been expressed most famously through reduced-form models of the structure-conduct-performance paradigm that link market outcomes to various structural indicia (Bain, 1968). However, it also emerges in equilibrium under standard structural models of imperfect
competition such as Cournot or Bertrand with differentiated products.\textsuperscript{1} It is also a central tenet in the guidelines to evaluate mergers issued by the antitrust authorities of many countries. For example, the U.S. Department of Justice’s (DOJ) and Federal Trade Commission’s (FTC) Merger Guidelines state:

The Agencies give weight to the merging parties’ market shares in a relevant market, the level of concentration, and the change in concentration caused by the merger [...] Mergers that cause a significant increase in concentration and result in highly concentrated markets are presumed to be likely to enhance market power. (U.S. Department of Justice and Federal Trade Commission, 1997)

Although this belief is most commonly discussed in terms of output markets and seller oligopoly power, it is also routinely assumed to be true for input markets and buyer oligopsony power. In their treatise on monopsony, Blair and Harrison (1993, pp. 82-83) note in considering mergers among buyers that “the critical issue is to determine the level of market power enjoyed by the newly merged firm. The most commonly employed indicator of market power is market share.” Similarly, Noll (2005) argues that the consequences of monopoly and monopsony are the same and that the only basis for differentiating between them is if society places different values on the welfare of upstream suppliers vs. downstream buyers.

The U.S. DOJ and FTC in their Merger Guidelines concur with these views. Following a lengthy discourse on mergers among sellers, they dispatch mergers among competing buyers (section 12) in just 395 words, noting that “the Agencies employ essentially the framework [...] for evaluating whether a merger is likely to enhance market power on the selling side of the market.” The same conclusion applies to evaluation of mergers in the European Union, where the section “Mergers creating or strengthening buyer power in upstream markets” in the European Commission’s competition handbooks consists of three paragraphs, the first of which warns that mergers that strengthen the market power of a buyer may lead to lower input prices through reduced purchases, whereas the second acknowledges that the merged entity may achieve lower input costs without suppressing output, and the third indicates that, in order to evaluate the likely impacts on input procurement, the authorities would

\textsuperscript{1}Farrell and Shapiro (1990) show that the negative correlation between concentration and market performance may not hold if firms are heterogenous or in the presence of economies of scale.
need to conduct an analysis similar to what is described to evaluate the seller-power impacts of a merger (European Commission, 2010).

The goal of this paper is to cast doubt on the applicability of this prevailing paradigm in certain buyer market-power contexts. The economic logic of our argument derives from the fact that buyer power is grounded in the immobility of certain factor inputs that are in the short run largely “captive” to a set of available buyers. For example, consider modern agriculture, which is capital intensive with highly specialized inputs. Farms’ geographic locations are fixed and most are specialized to producing one or a few products, perhaps in rotation. These products are typically bulky and perishable and, hence, difficult and costly to transport (Rogers and Sexton, 1994; Vukina and Leegomonchai, 2006; Crespi et al., 2012). Thus, processing and packing facilities located in geographic proximity to farms, and likely spatially distributed in their own right, will have buyer power over farms located in their vicinity. The same idea applied to labor markets is workers, such as nurses or physicians, with specialized skills but who are geographically immobile in the short run for any of a multitude of possible reasons, who face a limited number of employers demanding those skills within a relevant geographic market.

Buyers in these settings can, and under standard models of imperfect competition will, exploit short-run inelasticities in the supply of these inputs and increase profits by reducing input employment and driving input price below the value of marginal product. Such behavior, however, is likely to have adverse consequences on supply of that input in the geographic market in the long run because input prices that are suppressed relative to the competitive level will incite extant suppliers of those inputs to exit the market and/or fewer new suppliers of the input to enter it, resulting in a form of hold-up problem where upstream agents under-invest in specialized resources relative to the societal optimum (Vukina and Leegomonchai, 2006). A recent report by the Organisation for Economic Co-operation and Development acknowledges the potential effects of buyer power on upstream investment in these terms (O.E.C.D., 2009, pg. 11):

The exercise of buyer power may affect dynamic efficiency by reducing the incentives of upstream firms to invest. The prevailing view is that the exercise of buyer power undermines incentives for suppliers to invest.
This long-run effect reduces the supply of the input to all of the buyers operating in the market (relative to the competitive equilibrium) and increases future input costs.

The problem in the prototypical oligopsony setting is that the long-run effect of suppressing input prices on upstream investment is internalized by any single buyer only to the extent that it impacts the buyer’s own future input costs and the buyer values the future. The impact on all other buyers operating in the same product and geographic market is an externality and is not accounted for in the decision making of any single buyer. The effect is thus very similar to Hardin’s commons (Hardin, 1968). A limited number of buyers “exploit” a given stock of input suppliers in the short run, but the result of their individual optimization decisions is to reduce the input price of the resource relative to the competitive equilibrium and in the long run to diminish the stock of the resource by causing extant suppliers to exit and fewer new suppliers to enter.

The question, thus, is whether fewer buyers and greater buyer concentration in such input markets can improve market performance and mitigate the hold-up problem. The answer is not obvious because greater buyer concentration has offsetting impacts on input employment and upstream entry. With increased concentration, the short-run market power of buyers is magnified, which, absent long-run considerations, would simply reduce input employment. But the greater concentration means that each buyer will internalize to a greater extent the long-run impacts on supply of the input from the exercise of buyer power in the short run.

We seek to address this question in the remainder of this paper. The following section develops a two-period oligopsony model and characterizes the resulting equilibrium. We show that fewer suppliers enter the upstream market than in the competitive benchmark case. Key propositions are then derived establishing necessary and sufficient conditions for upstream entry to be decreasing in the number of downstream buyers. We give just enough structure to our model for these conditions to be expressible as functions of fundamental market parameters, making our results directly relevant for policy. We then develop simulation analyses to further explore the range of market parameters that support this result and the result that economic welfare is decreasing in the number of downstream buyers. Finally, we apply the analysis to U.S. agricultural markets, including some recent merger cases.
1 The Model

Two basic models of oligopsony competition have emerged in the literature—a standard homogeneous-products Cournot or “conjectural variations” model and a spatial model in the tradition of Hotelling (1929) and Salop (1979) where differentiated buyers compete in prices. Examples of the former approach include Bergman and Brännlund (1995), Azzam and Schroeter (1995), Xia and Sexton (2004), and Zhang and Brorsen (2010). Examples of the spatial approach include Alvarez et al. (2000), Zhang and Sexton (2000), and Mérel et al. (2009), all studying buyer power in agricultural markets and Thisse and Zenou (2000), Bhaskar et al. (2002), and Staiger et al. (2010) studying monopsony power in labor markets.

In the interest of simplicity, we develop our formal analysis within the homogenous-products Cournot framework, but the intuition carries over to a differentiated buyer context. Specifically, we extend the standard static model to a two-period framework of oligopsony competition that captures the essential feature of input markets central to this investigation, namely that supply of the input requires committed and immobile resources such as land and sunk capital in agriculture and specialized training for certain labor inputs, resulting in the possibility of short-run “capture” by oligopsonistic downstream buyers. Potential input suppliers anticipate this possibility when making entry decisions, resulting in a form of hold-up problem where too few resources are committed to supply of the input relative to the social optimum wherein the buying sector behaves competitively.

To give context to the model, we develop it within the framework of an agricultural product market where a homogeneous farm product is sold to processors. The number of processing firms, \( M \geq 1 \), is taken as an exogenous and continuous variable that lies at the center of our comparative statics analysis. All processors are identical. Processors convert the farm product input into a finished product according to a fixed-proportions (Leontief) technology. Without any further loss of generality we can set the conversion coefficient to one through choice of units of measurement, so one unit of the farm product input is needed to produce one unit of output. Including a fixed per-unit cost for other inputs adds nothing to the model and is omitted for the sake of simplicity.

In each of two periods, the processing sector faces a downward-sloping demand function
for the finished product.\textsuperscript{2} We parameterize this demand function using the constant-elasticity form: $P(Q) = \alpha Q^\eta$, with $\alpha > 0$ and $\eta > -1$. The parameter $\eta$ represents the price flexibility of demand or inverse price elasticity of demand.

There is a large number of potential suppliers of the farm input to the processing industry, all identical with increasing marginal cost function in any period. Entry into upstream supply of the input requires a fixed cost $F > 0$ due in the first period. This entry cost could for example represent the cost of acquiring land, buildings, and machinery (or specialized training in a labor-market context). The increasing marginal cost of production then reflects decreasing returns to the variable inputs given these fixed assets, as in Perry (1978).

A farmer’s individual variable cost in any given period is parameterized as $c(q) = \frac{1}{1+\epsilon} q^{1+\epsilon}$, where $q$ denotes the quantity of input supplied. Conditional on $N$ farmers having entered the input market, the market inverse supply schedule is thus $p = \left( \frac{Q}{N} \right)^\epsilon$, where $p$ is market price and $Q$ denotes the market quantity of input. The parameter $\epsilon$ represents the price flexibility of supply or inverse price elasticity of supply.

In the second period, the number of input suppliers is given, and the market equilibrium arises from standard Cournot competition among processors, given the aggregate, upward-sloping input supply schedule. The first period is the farm investment period. Processing firms Nash-compete in the quantity of the input to purchase in order to maximize their discounted profit stream, anticipating the effect of their quantity choice on the entry decisions of input suppliers. Suppliers enter until the anticipated discounted stream of individual quasi-rents, given the second-period equilibrium and the first-period quantity, exhausts the entry cost. The discount factor between the two periods, assumed to be common to farmers and processors, is denoted $\delta$.\textsuperscript{3}

There are three essential stages in the game and three related equilibrium conditions: (i) the second-period Cournot equilibrium, conditional on the number of farmers, $N$, (ii) the first-period farmer entry decision, conditional on input demand, $Q_1$, by processors in the

\textsuperscript{2}Although static models of oligopsony routinely assume that demand for the final product is perfectly elastic, a downward-sloping demand is required here for a non-trivial equilibrium to emerge. A downward-sloping demand is also required for a non-trivial equilibrium in the benchmark case where processors behave competitively.

\textsuperscript{3}Although the discount factor could be differentiated across agents, our main point can be made without this complication. A common discount factor also allows us to define inter-temporal welfare in a straightforward fashion.
first period, and (iii) the choice of input quantity by any given processor in the first period, taking as given the quantity purchased by other buyers and anticipating the upstream entry process and the second-period Cournot equilibrium. We solve the game recursively.

1.1 Second-Period Cournot Equilibrium

Each processor chooses the input quantity to maximize second-period profits, given \( N \) and the quantity purchased by other processors. The symmetric equilibrium condition is standard and reads

\[
P(Q_2) \left( 1 + \frac{\eta}{M} \right) - \left( \frac{Q_2}{N} \right)^{\epsilon} \left( 1 + \frac{\epsilon}{M} \right) = 0
\]

where the subscript 2 indicates the second-period equilibrium variables. Using \( P(Q_2) = \alpha Q_2^\eta \), we obtain equilibrium output and farm input employment as follows:

\[
Q_2(N, M) = N^{\epsilon - \eta} \left[ \alpha \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\epsilon}{M}} \right) \right]^{\frac{1}{\epsilon - \eta}}.
\]

The second-period profit of a given processor is

\[
\tilde{\pi}_2 = \frac{Q_2}{M} \left[ P(Q_2) - \left( \frac{Q_2}{N} \right)^{\epsilon} \right] = \frac{Q_2^{1+\epsilon}}{N^{\epsilon}} \left[ \frac{\epsilon - \eta}{M(M+\eta)} \right]
\]

which is positive given \( M \geq 1 \) and \( \eta > -1 \). Substituting for \( Q_2 \) using (2), we can write this processor profit as a function of \( N \) and \( M \):

\[
\tilde{\pi}_2(N, M) = N^{\frac{\epsilon(1+\eta)}{\epsilon - \eta}} \left[ \frac{\epsilon - \eta}{M(M+\eta)} \right] \left[ \alpha \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\epsilon}{M}} \right) \right]^{\frac{1+\epsilon}{\epsilon - \eta}}.
\]

1.2 Entry Process

With \( N \) farmers, the per farmer quasi-rent in a period with aggregate input quantity \( Q \) is \( \frac{\epsilon}{1+\epsilon} \left( \frac{Q}{N} \right)^{1+\epsilon} \). In the first period, farmers enter until the discounted sum of quasi-rents exactly
covers the entry cost:

\[
\frac{\epsilon}{1 + \epsilon} \left( \frac{Q_1}{N} \right)^{1+\epsilon} + \delta \frac{\epsilon}{1 + \epsilon} \left( \frac{\hat{Q}_2(N, M)}{N} \right)^{1+\epsilon} - F = 0
\]  

(4)

where \( Q_1 \) denotes the aggregate input quantity in period 1.

**Lemma 1** Given \( Q_1 \geq 0 \) and \( M \geq 1 \), the number of upstream entrants \( N \) is uniquely determined by equation (4) and is an increasing function of aggregate period 1 quantity.

**Proof.** Using (2) and defining \( f \equiv \frac{F(1+\epsilon)}{\epsilon} \), we can rewrite (4) as

\[
\left( \frac{Q_1}{N} \right)^{1+\epsilon} + \delta \left[ \alpha \left( \frac{1 + \eta M}{1 + \frac{\eta}{M}} \right)^{1+\epsilon/\eta} N^{2(1+\epsilon)/\eta} - f = 0. \right. 
\]

(5)

The left-hand side of (5) is a strictly decreasing function of \( N \), say \( \chi(N) \), with \( \lim_{N \to 0} \chi(N) = +\infty \) and \( \lim_{N \to +\infty} \chi(N) = -f < 0 \). Therefore, equation (5) has a unique solution on \((0, +\infty)\).

Equation (5) implicitly defines \( N \) as a continuously differentiable function of \( Q_1 \) and \( M \), say \( \tilde{N}(Q_1, M) \), and from the implicit function theorem we obtain\(^4\)

\[
\frac{\partial \tilde{N}}{\partial Q_1} = \frac{(\frac{Q_1}{N})^{\epsilon}(\epsilon - \eta)}{-f \eta + \epsilon (\frac{Q_1}{N})^{1+\epsilon}} > 0,
\]

(6)

i.e., the number of farmers entering production of the input is increasing in the magnitude of period 1 purchases.

### 1.3 First-Period Quantity

In the first period, processors compete in input quantities in a Nash fashion. A typical processor \( i \) solves the following program:

\[
\max_{q_i \geq 0} \phi(q_i) \equiv q_i \left[ P(q_i + Q_{-i}) - \left( \frac{q_i + Q_{-i}}{\tilde{N}(q_i + Q_{-i}, M)} \right)^{\epsilon} \right] + \delta \tilde{\pi}_2(\tilde{N}(q_i + Q_{-i}, M), M) \]

(7)

\(^4\)Although equation (5) cannot be solved explicitly for \( N, Q_1 \) can be written as an explicit function of \( N \) and \( M \).
where $Q_{-i}$ denotes the combined input purchases of all processors except the $i^{th}$.

**Lemma 2** The objective function $\phi(q_i)$ in program (7) is twice continuously differentiable and satisfies the following property:

$$\forall q_i \geq 0 \quad \phi'(q_i) = 0 \Rightarrow \phi''(q_i) < 0.$$

**Proof.** See Appendix A.1.

**Corollary 1** If $q_i^* \geq 0$ satisfies $\phi'(q_i^*) = 0$, then $q_i^*$ solves program (7).

**Corollary 2** If $\phi'(0) \leq 0$, then $q_i^* = 0$ solves program (7).

Corollary 1 and Corollary 2 together imply that if the first-order conditions to program (7) are satisfied at any nonnegative quantity, then this quantity indeed solves the first-period program.

Note that the assumption that the processing sector faces a downward-sloping demand for its final product is required for a non-trivial first-period equilibrium to emerge. To see why, suppose that $\eta = 0$ so that $p = \alpha$. Condition (5) implies that the ratio $Q_1/N$ is fixed at the value $\left[ f - \delta \left( \frac{\alpha}{1+\delta} \right)^{\frac{1+\epsilon}{1+\epsilon}} \right]^{\frac{1}{1+\epsilon}}$. This, in turn, implies that the second-period processor profit (3) is proportional to $Q_1$. Hence, a processor’s objective function in (7) is affine in its own first-period quantity $q_i$. Therefore, conditional on other processors’ quantities $Q_{-i}$, a processor will want to purchase either nothing or an infinite amount in the first period. Note that a downward-sloping demand curve is also required for a non-trivial equilibrium to obtain in the benchmark case where processors behave as price-takers in the procurement and final product markets.\(^5\)

1.4 Characterization of the Symmetric Equilibrium

We focus on the symmetric Nash equilibrium where all processors purchase the same positive quantity in the first period. Using symmetry, formulas (3) and (6), and relationship (5), the

\(^5\)Otherwise, the only possible equilibria emerge for $\alpha \leq \left( \frac{f}{1+\delta} \right)^{\frac{1+\epsilon}{1+\epsilon}}$, in which case there are either zero (if the inequality is strict) or an undetermined number of entrants (if the condition is satisfied with equality).
first-order condition for inter-temporal profit maximization can be written as:

\[
P(Q_1) \left(1 + \frac{\eta}{M}\right) - \left(\frac{Q_1}{N}\right) = 1 - \frac{\epsilon(\eta(M - 1 + \epsilon) + \epsilon)}{M(M + \eta)} \left(1 + \frac{\alpha N^\eta \left(\frac{1 + \frac{\eta}{1 + \epsilon}}{1 + \frac{\eta}{M}}\right)^{1+\epsilon}}{-f N + \delta \alpha N^\eta \left(\frac{1 + \frac{\eta}{1 + \epsilon}}{1 + \frac{\eta}{M}}\right)^{1+\epsilon}}\right) = 0 \quad (8)
\]

where $\tilde{N}$ is the function of $Q_1$ and $M$ implicitly defined by (5). However, since $Q_1$ can be written explicitly as a function of $N$ through relationship (5), it is convenient to express condition (8) in terms of $N$ rather than $Q_1$:

\[
1 - \frac{\epsilon(\eta(M - 1 + \epsilon) + \epsilon)}{M(M + \eta)} \left[\frac{\alpha N^\eta \left(\frac{1 + \frac{\eta}{1 + \epsilon}}{1 + \frac{\eta}{M}}\right)^{1+\epsilon}}{-f N + \delta \alpha N^\eta \left(\frac{1 + \frac{\eta}{1 + \epsilon}}{1 + \frac{\eta}{M}}\right)^{1+\epsilon}}\right] = 0. \quad (9)
\]

**Proposition 1** Whenever $\eta(M - 1 + \epsilon) + \epsilon > 0$, there exists a unique symmetric Nash equilibrium.

**Proof.** Denote by $\tilde{N}$ the value of $N$ that solves $f - \delta \left[\alpha N^\eta \left(\frac{1 + \frac{\eta}{1 + \epsilon}}{1 + \frac{\eta}{M}}\right)^{1+\epsilon}\right] = 0$. (From relationship (5), this value corresponds to the case where $Q_1 = 0$.) When $\eta(M - 1 + \epsilon) + \epsilon > 0$, the expression on the left-hand-side of equation (9) is a strictly increasing function of $N$ on $(\tilde{N}, +\infty)$. Its limit as $N \to \tilde{N}$ is equal to $-\infty$ and its limit as $N \to +\infty$ equals one. Therefore, equation (9) has a unique solution in the interval $(\tilde{N}, +\infty)$, and at this solution processors purchase a positive quantity in the first period. From Corollary 1, this unique solution indeed represents a Nash equilibrium.

Equation (9) thus implicitly defines $N$ as a continuously differentiable function of $M$ and $\eta$, say $\tilde{N}(M, \eta)$, on the set $[1, +\infty) \times (-1, 0)$. Note that given $\epsilon$ and $M$, the condition $\eta(M - 1 + \epsilon) + \epsilon > 0$ is satisfied as long as $\eta$ is small enough in absolute value, i.e., final product demand is sufficiently elastic, and in what follows we assume that this condition holds.

**Assumption 1** $\eta(M - 1 + \epsilon) + \epsilon > 0$.

At the symmetric Nash equilibrium, a typical processor earns a positive discounted profit stream. To see why, note that conditional on $Q_{-i} > 0$, processor $i$ could always choose $q_i = 0$,
which would yield a discounted profit stream $\phi(0) = \delta \tilde{\pi}_2(\tilde{N}(Q_{-i}, M), M) > 0$.

**Proposition 2**  The Nash equilibrium characterized by (9) is such that the number of upstream entrants is less than in the competitive equilibrium where processors behave as price-takers in the procurement and final product markets. Processors employ more input in the first period than in the second period.

**Proof.** See Appendix A.2.

Proposition 2 establishes that the symmetric Nash equilibrium always involves fewer upstream firms than if processors were behaving competitively. This result, while consistent with the view that processors may hold up farmers’ investments, stands in sharp contrast to the conventional view that monopsonistic behavior does not lead to long-run market distortions in the absence of Ricardian rents (Noll, 2005). This view, however, is based on the notion that the long-run supply curve of a competitive upstream sector is flat at the minimum of the long-run average cost. This long-run, “one shot” representation of upstream supply obliterates the very fact that capital investments must be made prior to production, allowing processors to then hold up farmers’ investments. In our dynamically explicit model, upstream suppliers earn zero profit in the long run, and thus any profits earned by processors are indeed being extracted from consumers. But this is not to say that monopsony power is absent or rendered ineffective by upstream entry, as our sequential game explicitly allows for the short-run capture of initial investments.

Yet, it is fair to ask how our dynamic equilibrium would change if we assumed away any seller power by processors. This is easily done by treating processors as price-takers in the sale of the processed product while requiring that the product price clears the market in each period. Ensuring the existence and uniqueness of a symmetric Nash equilibrium requires a slightly stronger assumption on model parameters, but the under-investment result stands.

**Assumption 2**  $\eta (M + \epsilon) + \epsilon > 0$.

**Proposition 3**  Under Assumption 2, if processors behave as price-takers in the final product market, there exists a unique symmetric Nash equilibrium. The number of upstream entrants is less than in the competitive equilibrium, and processors employ more input the first period than in the second period.
Proof. See Appendix A.3.

To illustrate how buyer power may affect upstream entry in the presence of short-run hold up, Figure 1 depicts the equilibrium short-run supply curves resulting from the dynamic game under three behavioral assumptions: (i) competitive behavior, (ii) buyer power only, and (iii) buyer and seller power. The model parameters are $\alpha = F = 1$, $\delta = 0.95$, $\eta = -0.10$, $\epsilon = 8$, and $M = 1$. The figure illustrates the fact that even in the absence of seller power, short-run monopsony results in reduced upstream entry.

Having established that buyer power affects upstream entry in the presence of hold up on farm investments, in the following section we investigate the conditions under which greater buyer concentration, i.e., smaller $M$, may increase first-period entry, that is, $\frac{\partial N}{\partial M} < 0$. We do so in the case where processors exercise buyer and seller power, but the intuition carries over to the case where they behave as price-takers in the final product market. Given relationship (5), it is clear that whenever $\frac{\partial N}{\partial M} < 0$, greater buyer concentration will cause both larger $N$ and larger $Q_1$, while the net effect on $Q_2$ will be ambiguous. Such ambiguity arises because greater buyer concentration in this case will cause both larger second-period supply and greater buyer power. As a way to resolve this ambiguity, in Section 4 we investigate the effect of buyer concentration on intertemporal welfare, and show numerically that welfare
itself depends on $M$ in a non-monotone way.

2 Comparative Statics

The assumption that demand slopes down ($\eta < 0$), while necessary to obtain a non-trivial equilibrium, gives processors selling power for the final product and creates incentives for them to reduce output and thus input employment, i.e., to exercise seller oligopoly power, in both periods. Such incentives are stronger the more concentration there is in the processing sector (that is, the smaller $M$ is). In order to curtail this demand-driven incentive to reduce input employment (and thus upstream entry), we concentrate on the case where the demand flexibility $\eta$ is small, i.e., demand is highly elastic. We derive an explicit sufficient condition for the critical comparative static $\frac{\partial N}{\partial M} < 0$ to hold for small enough values of $\eta$. We prove the existence of a sizable region of the parameter space in which an increase in processor concentration results in more upstream entry and thus more resources committed to production of the agricultural input.

The case where $\eta$ is small is of considerable empirical relevance because it pertains to cases where the relevant market for the input is limited geographically due to immobility of the input, but the geographic market for the output is much broader in its scope and the $M$ processors who comprise the buyers in the input market face competition from many other firms located elsewhere for sale of the finished product. This is the typical case for agricultural products, which often are bulky and highly perishable and, thus, transportable for only short distances (Rogers and Sexton, 1994; Noll, 2005; Vukina and Leegomonchais, 2006; Crespi et al., 2012), whereas the finished product in canned, chilled, or frozen form is more compact, less perishable and much more readily transportable, causing those markets to be national or international in geographic scope.\footnote{The parallel condition in a labor-market context would be employers who faced an upward-sloping supply for labor inputs due to workers’ geographic immobility, differentiation among firms (Hamilton et al., 2000), or search costs (Burdett and Mortensen, 1998), but who sold finished products in markets of much broader geographic scope.}
2.1 Analysis when $\eta \to 0$

If $\eta = 0$, the terms in $N$ vanish in equation (9), and therefore the function $\hat{N}(M, \eta)$ is not defined at $\eta = 0$. This observation leads us to make the following change of variable: $\nu \equiv \frac{\alpha N}{f^{\eta}}$. For all $-1 < \eta < 0$, $N$ solves (9) if and only if $\nu$ solves the following equation:

\[
1 - \frac{\left[ \frac{\epsilon(M-1+\eta+\epsilon)}{M(M+\eta)} \right]}{-\eta + \epsilon} - \frac{\left[ 1 - \delta \left[ \nu \left( \frac{1+\eta}{1+\epsilon} \right) \right]^{\frac{1+\epsilon}{1-\eta}} \right]}{1 - \delta \left[ \nu \left( \frac{1+\eta}{1+\epsilon} \right) \right]^{\frac{1+\epsilon}{1-\eta}}} = 0.
\]

Equation (10) implicitly defines $\nu$ as a continuously differentiable function $\hat{\nu}(M, \eta)$ on the set $[1, +\infty) \times (-1, 0)$, and $\hat{\nu}(M, 0)$ solves

\[
1 - \frac{\epsilon \delta M^2 \left( \frac{\nu}{1+\epsilon} \right)^{\frac{1+\epsilon}{1-\eta}}}{1 - \delta \left( \frac{\nu}{1+\epsilon} \right)^{\frac{1+\epsilon}{1-\eta}}} - \frac{\nu}{1 - \delta \left( \frac{\nu}{1+\epsilon} \right)^{\frac{1+\epsilon}{1-\eta}}} = 0.
\]

For all $(M, \eta) \in [1, +\infty) \times (-1, 0)$, we have that $\hat{\nu}(M, \eta) = \frac{\alpha \hat{N}(M, \eta)}{f^{\eta}}$, and therefore $\frac{\partial \hat{\nu}}{\partial M}(M, \eta) = \frac{\alpha N}{f^{\eta}} \hat{N}(M, \eta)^{-1} \frac{\partial \hat{N}}{\partial M}(M, \eta)$. Since $\eta < 0$, the sign of $\frac{\partial \hat{N}}{\partial M}$ is opposite of that of $\frac{\partial \hat{\nu}}{\partial M}$.

**Definition 1** We say that the critical comparative static holds in the vicinity of $\eta = 0$ if

\[
\exists \tilde{\eta} < 0 \text{ such that } \tilde{\eta} < \eta < 0 \Rightarrow \frac{\partial \hat{N}}{\partial M}(M, \eta) < 0.
\]

We now investigate the sign of $\frac{\partial \hat{\nu}}{\partial M}$ in the vicinity of $\eta = 0$. Because $\hat{\nu}$, as opposed to $\hat{N}$, is defined at $\eta = 0$, and $\frac{\partial \hat{\nu}}{\partial M}$ is continuous, we just have to analyze the sign of $\frac{\partial \hat{\nu}}{\partial M}(M, 0)$. By applying the implicit function theorem to equality (11), we show in Appendix A.4 that $\frac{\partial \hat{\nu}}{\partial M}(M, 0)$ has the same sign as the expression $-M^2 - 1 + 2M + \epsilon + \delta \left( \frac{\nu}{1+\epsilon} \right)^{-\frac{1+\epsilon}{1-\eta}} (M^2 - 2M - \epsilon)$. The critical comparative static $\frac{\partial \hat{N}}{\partial M} < 0$ holds in the vicinity of $\eta = 0$ if and only if $\frac{\partial \hat{\nu}}{\partial M}(M, 0) > 0$, that is,

\[
-M^2 - 1 + 2M + \epsilon > \delta \left( \frac{\nu}{1+\epsilon} \right)^{\frac{1+\epsilon}{1-\eta}} (-M^2 + 2M + \epsilon).
\]
Proposition 4 A necessary condition for the critical comparative static to hold in the vicinity of $\eta = 0$ is that $-M^2 - 1 + 2M + \epsilon > 0$, that is, $\epsilon > (M - 1)^2$.

Proof. Suppose that $-M^2 - 1 + 2M + \epsilon \leq 0$. Then, if $-M^2 + 2M + \epsilon \geq 0$, inequality (12) cannot be satisfied. Therefore, we must have $-M^2 + 2M + \epsilon < 0$, and thus $-M^2 - 1 + 2M + \epsilon < 0$ as well. But $0 < \delta \left( \frac{\nu}{1 + \frac{\epsilon}{M^2}} \right)^{\frac{1+\epsilon}{\epsilon}} < 1$, so inequality (12) cannot be satisfied either.

Corollary 3 For $M \geq 2$, individual farmer supplies must be inelastic for the critical comparative static to occur in the vicinity of $\eta = 0$.

Proposition 5 The condition $\epsilon \geq \frac{(M-1)^2 + \sqrt{(M-1)^4 + 4M^2}}{2}$ is sufficient for the critical comparative static to hold in the vicinity of $\eta = 0$.

Proof. Suppose that $\epsilon > (M - 1)^2$. Then, the critical result holds if and only if

$$\delta \left( \frac{\nu}{1 + \frac{\epsilon}{M^2}} \right)^{\frac{1+\epsilon}{\epsilon}} < \frac{-M^2 - 1 + 2M + \epsilon}{-M^2 + 2M + \epsilon}.$$ 

For the equilibrium condition (11) to hold, the sum of the first two terms on its RHS must be positive, which implies that $\delta \left( \frac{\nu}{1 + \frac{\epsilon}{M^2}} \right)^{\frac{1+\epsilon}{\epsilon}} < \frac{1}{1 + \frac{\epsilon}{M^2}}$. It is easy to check that $\frac{1}{1 + \frac{\epsilon}{M^2}} \leq \frac{-M^2 - 1 + 2M + \epsilon}{-M^2 + 2M + \epsilon}$ whenever $\epsilon \geq \frac{(M-1)^2 + \sqrt{(M-1)^4 + 4M^2}}{2}$. The result follows.

![Figure 2: Occurrence of the critical comparative static when $\eta \to 0$](image-url)
Proposition 6  If $(M, \epsilon)$ is such that $M > 1$ and the critical comparative static holds in the vicinity of $\eta = 0$, then the critical comparative static holds in the vicinity of $\eta = 0$ for all $(M', \epsilon)$ such that $1 \leq M' \leq M$.

Proof. Suppose that $\frac{\partial \hat{\nu}}{\partial M}(M, 0) > 0$ for some $(M, \epsilon)$ with $M > 1$. Then, rearranging condition (12) we must have that $\epsilon - (M - 1)^2 > \frac{\delta^{1+\frac{\nu}{1+\frac{\nu}{1-\delta}}}}{1-\delta^{1+\frac{\nu}{1+\frac{\nu}{1-\delta}}}}$. Given that $\frac{\partial \hat{\nu}}{\partial M}(M, 0) > 0$ it is clear that reducing $M$ will reduce the right-hand side of the previous inequality. It will also increase the left-hand side, thus the condition will continue to be satisfied.

Figure 2 delineates the set of values of $M$ and $\epsilon$ for which $\frac{\partial \hat{\nu}}{\partial M}(M, 0) > 0$, for a discount factor $\delta = 0.95$. All values located above the solid line in $(M, \epsilon)$ space support occurrence of the critical comparative static, indicating that for small enough values of $\eta$ the result $\frac{\partial \hat{\nu}}{\partial M} < 0$ will obtain. The dashed (resp. dotted) line represents the minimum levels of $\epsilon$ for which the necessary (resp. sufficient) condition identified in Proposition 4 (resp. Proposition 5) is fulfilled. For our choice of $\delta$, the actual necessary and sufficient condition (represented by the solid line) is close to the necessary condition $\epsilon > (M - 1)^2$, that is, the necessary condition is “tight” or “almost sufficient.” Notably, the threshold value of $\epsilon$ is increasing with $M$, a direct consequence of Proposition 6.

3  Simulations

Propositions 4 and 5 in the previous section establish necessary and sufficient conditions for higher concentration among buyers to cause greater entry among sellers as $\eta$ approaches zero. In what follows we conduct simulation analyses for parameterizations of markets that are consistent with typical agricultural production settings in order to further investigate the conditions under which this result arises.

3.1  Effect of the Demand Flexibility

The preceding analysis was done in the vicinity of $\eta = 0$, that is, when product demand is almost perfectly elastic. Because less than perfectly elastic demand increases processors’
incentives to reduce output (and thus input employment) due to market power in the output market, we expect increasing values of $|\eta|$ to shrink the parameter space that supports the critical comparative static $\frac{\partial \hat{N}}{\partial M} < 0$. This is because in the output market there is no entry effect (demand is identical in both periods), and thus processors do not have conflicting incentives regarding their output decisions. As such, absent the input market considerations studied here, greater processor concentration would always lead to less output, and therefore less input employment.

We explore the impact of $\eta$ in Figure 3 which depicts the threshold values of $\epsilon$ beyond which $\frac{\partial \hat{N}}{\partial M} < 0$, for $\delta = 0.95$ and nonzero values of the demand flexibility $\eta$. Indeed, less elastic output demand reduces the parameter space supporting the critical comparative static, as the frontier moves upwards. Yet, for the range of $M$ and $\eta$ considered, the critical comparative static holds for plausible market parameters. For instance, for $\eta = -0.10$ and $M = 2$, the threshold supply flexibility is 3.14. In addition, the threshold supply flexibility is an increasing function of $M$, which implies that the comparative static $\frac{\partial \hat{N}}{\partial M} < 0$ is relevant for incremental (as opposed to merely marginal) increases in industry concentration. Note that Assumption 1, which ensures a unique symmetric equilibrium, is satisfied here for all combinations of $(M, \epsilon)$ that support the critical comparative static. Specifically, all values of $(M, \epsilon)$ lying above the dark solid line in Figure 3 satisfy this assumption for $\eta = -0.20$, and thus also for lower values of $\eta$.

Figure 3: Occurrence of the critical comparative static for $\eta < 0$
Figure 4: Effect of processor concentration on upstream entry ($\delta = 0.95$, $\epsilon = 8.0$)

Figure 4 depicts the equilibrium number of farmers $\hat{N}$ as a function of $M$, for $\epsilon = 8$ and selected values of the demand flexibility. (We have set $\alpha = 1$ and $F = 1$ as normalizations.\textsuperscript{7}) Here we see that highly concentrated markets with $M = 2$ or even $M = 1$ induce more farmers to enter than markets with a moderate number of processors. For instance, when $h = -0.15$ a monopsony market has more farmers than a market with 8 processors. The curves in Figure 4 can be related to the corresponding thresholds in Figure 3. For instance, for $h = -0.10$, Figure 3 shows that when $\epsilon = 8$ the critical comparative static holds until $M \approx 3.1$, which is the point at which Figure 4 shows a reversal in the effect of processor concentration on $\hat{N}$.

### 3.2 Effect of the Upstream Supply Flexibility

Figure 5 depicts the equilibrium number of upstream entrants as a function of processor concentration, for selected values of $\epsilon$, $\delta = 0.95$, and $\eta = -0.08$. The figure shows, for instance, that for $\epsilon = 4$, a monopsonistic market will induce more upstream entry than a market with up to 7 processing firms.

\textsuperscript{7}The set of values of $(M, \epsilon, \eta)$ that support the critical comparative static is invariant to the choice of parameters $\alpha$ and $F$. As should be clear from equation (9), parameters $\alpha$ and $f$ (and thus $F$) only affect the scale of $N$. In Figures 4 and 5, given our model parameterization differences in $N$ across values of $\eta$ are meaningless; only variations in $N$ across values of $M$, for given $\eta$, matter.
Figure 5: Effect of processor concentration on upstream entry (δ = 0.95, η = −0.08)

3.3 Effect of the Discount Factor

If δ = 0, then the equilibrium condition (9) becomes \( \alpha N^\eta (1 + \frac{\eta}{M}) = (f)^{\frac{\epsilon-\eta}{1+\epsilon}} \), and in that case \( \frac{\partial \hat{N}}{\partial M} > 0 \). Therefore, the critical comparative static can occur only if δ > 0, i.e., if processors value the future. More generally we expect that for η < 0, an increase in δ increases the size of the parameter space which supports the critical comparative static. That is, for given \( M \) and \( \eta \), we expect the threshold value of the supply flexibility \( \epsilon \) to decrease monotonically with δ.

Figure 6: Occurrence of the critical comparative static for selected discount factors (η = −0.10)
Figure 6 offers support for this intuition by depicting the effect of the discount factor \( \delta \) on the set of parameter values \((M, \epsilon)\) that support the critical comparative static, for \( \eta = -0.10 \). As \( \delta \to 0 \), the set of \((M, \epsilon)\) values that support the critical result is reduced, as the frontier shifts upwards.\(^8\)

4 **Welfare Effects**

The result that the number of upstream input suppliers \( N \) may increase when the number of processors \( M \) decreases does not necessarily imply that social welfare also increases. With larger \( N \), the short-run upstream supply curve is flatter in each period, which implies that any given quantity of input can be acquired at a cheaper price, leading to a higher welfare potential. (The quasi-rents to upstream suppliers are exhausted by the fixed entry cost.) However, the quantities purchased in each period will also change with \( M \). As argued above, if \( N \) increases then \( Q_1 \) must also increase, but the effect on \( Q_2 \) is ambiguous. Therefore, \( N \) may not be a perfect indicator of welfare. Yet social welfare is likely the key criterion for antitrust authorities to consider, e.g., when evaluating a proposed merger between firms. As such, it is useful to investigate whether social welfare itself may be a non-monotone function of processor concentration.

Social surplus in a period \( i = 1, 2 \), is defined as the benefit from input employment minus variable production costs:

\[
S_i = \int_0^{Q_i} \alpha Q_i^{\eta} dQ - \frac{N}{1 + \epsilon} \left( \frac{Q_i}{N} \right)^{1+\epsilon}.
\]

Total welfare is the discounted sum of period surpluses minus the sum of the fixed costs:

\[
W = \int_0^{Q_1} \alpha Q_1^{\eta} dQ - \frac{N}{1 + \epsilon} \left( \frac{Q_1}{N} \right)^{1+\epsilon} + \delta \left[ \int_0^{Q_2} \alpha Q_2^{\eta} dQ - \frac{N}{1 + \epsilon} \left( \frac{Q_2}{N} \right)^{1+\epsilon} \right] - NF
\]

\(^8\)It is clear from Figure 6 that as \( \delta \) gets smaller, the frontier moves upwards at an increasing rate. This result is expected, since for \( \delta = 0 \) conditional on \( M \) the critical comparative static does not occur for any value of \( \epsilon \), that is, the frontier moves towards infinity. Importantly, however, the figure also shows that the frontier is relatively invariant to moderate changes in the discount value from the base case of \( \delta = 0.95 \).
which after rearrangement can be written as

\[ \hat{W}(M) = \frac{\alpha \hat{N}^{1+\eta}}{1+\eta} \left\{ \left[ f - \delta \left[ \alpha \hat{N}^{\eta} \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\eta}{M}} \right) \right]^{\frac{1+\eta}{1-\eta}} \right]^{\frac{1+\eta}{1-\eta}} + \delta \left[ \alpha \hat{N}^{\eta} \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\eta}{M}} \right) \right]^{\frac{1+\eta}{1-\eta}} \right\} - \hat{N} f \]

where \( \hat{N} \) is the implicit function of \( M \) and \( \eta \) defined by (9).

\[ \hat{W}(M) = \frac{\alpha \hat{N}^{1+\eta}}{1+\eta} \left\{ \left[ f - \delta \left[ \alpha \hat{N}^{\eta} \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\eta}{M}} \right) \right]^{\frac{1+\eta}{1-\eta}} \right]^{\frac{1+\eta}{1-\eta}} + \delta \left[ \alpha \hat{N}^{\eta} \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\eta}{M}} \right) \right]^{\frac{1+\eta}{1-\eta}} \right\} - \hat{N} f \]

In what follows we set \( \alpha = 1 \) and \( F = 1 \) as normalizations. These normalizations only affect the scale of \( W \).

\[ \hat{W}(M) = \frac{\alpha \hat{N}^{1+\eta}}{1+\eta} \left\{ \left[ f - \delta \left[ \alpha \hat{N}^{\eta} \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\eta}{M}} \right) \right]^{\frac{1+\eta}{1-\eta}} \right]^{\frac{1+\eta}{1-\eta}} + \delta \left[ \alpha \hat{N}^{\eta} \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\eta}{M}} \right) \right]^{\frac{1+\eta}{1-\eta}} \right\} - \hat{N} f \]

Figure 7 depicts the function \( \hat{W}(M) \) for selected values of \( \eta \), where we have set \( \delta = 0.95 \) and \( \epsilon = 8 \). The figure shows that welfare itself may be a non-monotone function of processor concentration, and that in some instances very concentrated structures lead to higher social welfare than moderately concentrated ones. For instance, when \( \eta = -0.10 \) monopsony yields higher welfare than an industry with 8 processing firms.

In order to illustrate how upstream supply elasticities affect the industry’s performance, Figure 8 depicts \( \hat{W}(M) \) for selected values of \( \epsilon, \delta = 0.95, \) and \( \eta = -0.08 \).

Figure 4 provides an alternative perspective on welfare by showing, for given \( M \) and selected demand flexibilities, the threshold values of \( \epsilon \) above which \( \frac{\partial W}{\partial M} < 0 \).

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9See Appendix A.5 for the derivation.
10Given our parameterization, differences in \( W \) across values of \( \eta \) or \( \epsilon \) are meaningless; only differences across values of \( M \) matter.
11Although not represented here, the effects of \( M \) on \( Q_1 \) and \( Q_2 \) are also non-monotone.
5 Applications

5.1 Agricultural Markets

Results from the analytical and simulations sections demonstrate that entry into production of the input and economic welfare are more likely to be increasing in the extent of concentration in the downstream buying industry the more inelastic the supply of the input and the greater the \textit{ex ante} degree of concentration in the industry, measured for purposes of the model by the number of buyers. Agricultural product markets often represent such settings. Inelastic supply (flexible prices) is a widely accepted stylized fact of agricultural product markets. Indeed, inelastic supply, along with inelastic demand, slow demand growth, and rapid supply growth, constitute the four market characteristics that define what is known as the “farm problem,” e.g., in Gardner (1992). Table 1 provides recent estimates of short-run acreage supply elasticities for a representative sample of annual and perennial crops and animal products. For nearly all of the commodities considered the elasticities are 0.5 or less (flexibilities 2.0 or more).

Farm product procurement markets are also likely to be highly concentrated due, as noted, to their generally limited geographic dimensions based upon high costs of transporting such products in their raw form. Further, such markets are narrow in product dimensions as well due to the highly specialized nature of processing facilities. Thus, finished products such
Figure 9: Market parameters where welfare is increasing in buyer concentration

Note: For values of $(M, \epsilon)$ above the depicted frontiers welfare increases in buyer concentration.

as different meats or different vegetables that might compete in the same output markets due to close substitutability in consumption will not compete as raw products because a facility built to process one particular product, e.g., cattle, cannot normally handle another farm product such as hogs. Because reporting on industry concentration levels is done at the national level and focuses on output markets, there is little formal evidence on rates of concentration that are applicable to farm input markets in general.\footnote{Crespi et al. (2012) report and discuss concentration rates in U.S. food product markets and their relevance to farm product procurement markets.}

5.2 Merger Analysis

We apply the previous analysis to three recent U.S. merger cases wherein the U.S. DOJ challenged the proposed mergers at least in part on the grounds that the merger would have negative impacts on input prices. In each case we argue that it is plausible that the conventional analysis applied to the case by the DOJ was erroneous because it ignored the impact the merger would have had on firms’ long-run incentives.

In U.S. v. Cargill, Incorporated and Continental Grain Company the DOJ asserted that “the grain trading business at certain levels is highly concentrated. Cargill and Continental compete to purchase corn, soybeans, and wheat in numerous rail terminal, river elevator,
Table 1: Price elasticity of supply estimates for selected agricultural commodities and port elevator markets throughout the country where they are two of a small number of competitors.” The DOJ argued that geographic markets were limited due to “costly and time consuming” transportation and identified several regional procurement markets for corn, wheat, and soybeans which it considered to be highly concentrated. For example, the DOJ claimed that a post-merger Cargill would have a 94% share of soybean purchases in the Pacific Northwest and 53% of Pacific Northwest corn purchases. It further stated that, within the Central California port range market, a merged Cargill-Continental firm would be a virtual monopsonist. Applying traditional market-power analysis, the DOJ claimed the merger would “substantially lessen competition for purchases of corn, soybeans, and wheat in each of the relevant geographic markets, enabling it unilaterally to depress the prices paid to farmers.”

Table 1 does not include estimates of supply elasticities for U.S. corn, wheat, or soybeans, but the values reported in the European Union for corn (0.08) and wheat (0.12) indicate that the supply is likely highly inelastic. The combination of the high rates of concentration reported by the DOJ and the inelastic commodity supplies suggested by Table 1 put these procurement markets well within the range of values wherein the market performance mea-
sured in terms of number of farmers and overall welfare might have been increased by the merger, contrary to the analysis put forth by the DOJ.\textsuperscript{13}

In \textit{U.S. et al. v. JBS S.A. and National Beef Packing Company, LLC} the DOJ challenged a merger between JBS and National, the third and fourth largest U.S. beef packers, respectively, alleging that if the merger were approved, over 80\% of U.S. beef production would be controlled by a three-firm oligopoly. The DOJ argued that the purchase of fed cattle constituted a relevant antitrust product market, and the central “High Plains” region and Southwest region of the U.S. comprised relevant geographic markets for fed cattle. Based upon the DOJ’s analysis, a three-firm oligopsony would prevail in the High Plains, and the Southwest would be a near monopsony if the merger were approved. In either market the DOJ forecast “less aggressive competition and lower prices for feedlots and producers of fed cattle.”

Cattle supply response is complicated by dynamic considerations discussed by Rosen et al. (1994) among others. Nonetheless, short-run supply is highly inelastic as exemplified by Marsh’s estimate of 0.26 reported in Table 1 (Marsh, 2003). This proposed merger was ultimately abandoned due to the opposition of the U.S. DOJ and several states’ attorneys general, but the combination of the highly inelastic short-run supply and the high prevailing concentration ratios in the regional cattle procurement markets suggest that this proposed merger might have increased entry, production, and overall welfare, as the merged company internalized more of the long-run impacts of its procurement practices.

In \textit{U.S. and State of Texas v. Aetna, Inc. and Prudential Insurance Company of America}, the U.S. DOJ challenged a proposed merger between HMO providers in part on the claim that the proposed merger would depress competition and price in the markets for physician services in the areas of Houston and Dallas, Texas. According to the DOJ the merger would have increased Aetna’s market share in the HMO sector from 44\% to 63\% (26\% to 42\%) in Houston (Dallas). The DOJ thus argued that “the proposed acquisition would give Aetna the ability to unduly depress physician reimbursement rates in Houston and Dallas, likely leading to a reduction in quantity or degradation in the quality of physicians’ services.” Although the DOJ presented no estimates of the elasticity of supply of physician services in these areas,\textsuperscript{13}

\textsuperscript{13}The DOJ ultimately approved the merger after the merging parties agreed to divest themselves of various grain and soybean processing facilities in several states.
its analysis presumed that the supply was quite inelastic: “[nor] will such a price decrease cause physicians to stop providing their services or shift towards other activities in numbers sufficient to make such a price reduction unprofitable.” Although the buyer concentration rates at issue here were somewhat lower than in the aforementioned agricultural market cases, these concentration levels, coupled with a sufficiently inelastic physician supply in these areas, could plausibly have caused the merger to increase physician entry and supply in the affected areas.\footnote{Although the merger was eventually allowed to proceed, Aetna was required to divest itself of its interests in the Dallas and Houston areas.}

Worth emphasizing is that in the proposed JBS and Aetna mergers the DOJ was also alleging that seller market power would be enhanced through the merger. Our arguments apply only to the DOJ’s analysis of buyer power implications of the proposed mergers.

6 Conclusion

In this paper, we have investigated in a dynamic framework how downstream concentration in an industry may affect entry incentives at the upstream stage. Our parameterized model is a natural two-period extension of the prototypical Cournot oligopsony model that allows for the short-run hold up of upstream investments by downstream buyers. The impact of such hold up on suppliers’ willingness to participate in the market is endogenized through suppliers’ inter-temporal zero-profit condition. As such, the model parsimoniously captures two essential incentives facing oligopsonistic buyers: short-run benefits from reducing input employment and long-run incentives to secure input supplies.

When such incentives are at play in a relatively unconcentrated processing industry, long-run incentives are not sufficient to reverse the short-run incentive of buyers to exercise the buyer power at their disposal, thereby causing reduced input employment as concentration increases, and the traditional view that rising concentration diminishes upstream entry, production of the input, and social welfare applies. However, our model reveals the existence of market settings where long-run incentives do dominate, resulting in a positive effect of increased buyer concentration on upstream entry. More specifically, for already-concentrated industries and sufficiently inelastic upstream supplies, further concentration might induce
more upstream entry and increase social welfare.

The range of model parameters supporting such scenarios includes values that are relevant in many empirical settings, including markets for agricultural inputs where individual (farm-level) supplies are typically inelastic in the short run, buyers are often few in a given geographical area, and demand for the finished product is highly elastic (for instance, because it is sold in a much broader geographic market than the input procurement market).

Our result is particularly relevant for antitrust, because merger policy focuses on industries that are already highly concentrated. As we have shown, an increase in industry concentration in such settings may not be incompatible with increased social welfare, independent of the existence of scale economies or other efficiency gains enabled by the merger (Williamson, 1968; Farrell and Shapiro, 1990). In addition, our parameterized model provides guidance for identifying such market settings.

References


**Mathematical Appendix**

**A.1 Proof of Lemma 2**

From (6), it is clear that the function \( \bar{N} \) is twice continuously differentiable with respect to \( Q_1 \). In addition, the function \( \bar{\pi}_2 \) defined in (3) is clearly twice continuously differentiable with respect to \( N \). Therefore, the function \( \phi \), as a composite of twice continuously differentiable functions, is itself twice continuously differentiable. Denoting \( Q_1 = q_i + Q_{-i} \), we have:

\[
\phi'(q_i) = \alpha Q_1^{\eta-1} (Q_1 + \eta q_i) - \left( \frac{Q_1}{N} \right)^\epsilon \left( 1 + \frac{\epsilon q_i}{Q_1} \right) + \frac{\partial \bar{N}}{\partial Q_1} \left\{ \frac{\epsilon q_i}{N} \left( \frac{Q_1}{N} \right)^\epsilon + \frac{\delta \epsilon (1 + \eta)}{M(M + \eta)} \left[ \alpha N^{\eta} \left( \frac{1 + \eta}{1 + \frac{\eta}{M}} \right) \right]^{\frac{1 + \eta}{M - \eta}} \right\}. \tag{A-1}
\]
Taking the derivative with respect to \(q_i\), we obtain:

\[
\phi''(q_i) = \alpha \eta Q_1^{\eta-2} (2Q_1 + (\eta - 1)q_i) - \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon-1} \left[ 2 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} - \frac{q_i}{Q_1} \right] + \frac{\epsilon q_i}{Q_1} \left[ 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right] + \frac{\partial \tilde{N}}{\partial Q_1} \left\{ \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon} \left[ 1 + \frac{\epsilon q_i}{Q_1} (1 + \epsilon q_i) \frac{\partial \tilde{N}}{\partial Q_1} \right] \right\} \left[ \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon} \left[ 1 + \frac{\epsilon q_i}{Q_1} (1 + \epsilon q_i) \frac{\partial \tilde{N}}{\partial Q_1} \right] \right\} \right]
\]

where \(\frac{\partial \tilde{N}}{\partial Q_1}\) is given by (6) and

\[
\frac{\partial^2 \tilde{N}}{\partial Q_1^2} = \frac{\partial \tilde{N}}{\partial Q_1} \left( \frac{\epsilon}{Q_1} \right) \left[ 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right] \left[ 1 - \frac{(1 + \epsilon) \left( \frac{Q_1}{N} \right)^{1+\epsilon}}{-f \eta + \epsilon \left( \frac{Q_1}{N} \right)^{1+\epsilon}} \right].
\]

Suppose now that \(\phi'(q_i) = 0\) for some \(q_i \geq 0\). We need to show that \(\phi''(q_i) < 0\). The first term in (A-2) is clearly negative since \(|\eta| < 1\) and \(q_i \leq Q_1\). We concentrate on the remaining terms. First note that \(\frac{\partial \tilde{N}}{\partial Q_1} > 0\) and \(1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} = -\frac{\eta \left[ f - (\frac{Q_1}{N})^{1+\epsilon} \right]}{-f \eta + \epsilon \left( \frac{Q_1}{N} \right)^{1+\epsilon}} > 0\), while the sign of \(\frac{\partial^2 \tilde{N}}{\partial Q_1^2}\) is ambiguous. The term on the third line of (A-2) clearly contributes negatively to \(\phi''(q_i)\), therefore we can ignore it. Denoting by \(A\) the term on the fourth line of (A-2), we thus have that

\[
\phi''(q_i) < A - \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon-1} \left[ 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} + \frac{q_i}{Q_1} \frac{\partial \tilde{N}}{\partial Q_1} \right] - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} + \frac{q_i}{Q_1} \left( \frac{Q_1}{N} \right)^2 \left( \frac{\partial \tilde{N}}{\partial Q_1} \right)^2 - \frac{\epsilon q_i}{Q_1} \frac{\partial \tilde{N}}{\partial Q_1} \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right)
\]
that is,

$$\phi''(q_i) < A - \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon^{-1}} \left[ 1 - 2 \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} + \frac{\epsilon q_i}{Q_1} \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right)^2 \right]$$

$$+ 1 - \frac{q_i}{Q_1} + \frac{q_i}{Q_1} \left( \frac{Q_1}{N} \right)^2 \left( \frac{\partial \tilde{N}}{\partial Q_1} \right)^2$$

and, adding and subtracting \((\frac{Q_1}{N})^2 \left( \frac{\partial \tilde{N}}{\partial Q_1} \right)^2\) inside the square bracket,

$$\phi''(q_i) < A - \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon^{-1}} \left[ 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right] \left( 1 + \frac{\epsilon q_i}{Q_1} \right) \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right)$$

$$+ \left( 1 - \frac{q_i}{Q_1} \right) \left( 1 + \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right) \right]$$

\(\text{(A-4)}\)

The second term on the right-hand side of inequality (A-11) is clearly negative, so we only need to show that this term dominates term \(A\) in absolute value, whenever \(A > 0\). Using (A-3), we obtain

$$A = \frac{\partial \tilde{N}}{\partial Q_1} \left\{ \frac{\epsilon q_i}{N} \left( \frac{Q_1}{N} \right)^{\epsilon} + \frac{\delta \epsilon (1 + \eta)}{M(M + \eta)} \left[ \alpha N^n \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\epsilon}{M}} \right) \right]^{\frac{1+\epsilon}{\epsilon^{-1}}} \right\} \left( \frac{\epsilon}{Q_1} \right) \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right)$$

$$\times \left[ 1 - \frac{(1 + \epsilon) \left( \frac{Q_1}{N} \right)^{1+\epsilon}}{-f \eta + \epsilon \left( \frac{Q_1}{N} \right)^{1+\epsilon}} \right] .$$

\(\text{(A-5)}\)

The positive factor \(\frac{\partial \tilde{N}}{\partial Q_1} \left\{ \frac{\epsilon q_i}{N} \left( \frac{Q_1}{N} \right)^{\epsilon} + \frac{\delta \epsilon (1 + \eta)}{M(M + \eta)} \left[ \alpha N^n \left( \frac{1 + \frac{\eta}{M}}{1 + \frac{\epsilon}{M}} \right) \right]^{\frac{1+\epsilon}{\epsilon^{-1}}} \right\} \) in (A-5) appears in (A-1), and since \(\phi'(q_i) = 0\) it must be less than \(\left( \frac{Q_1}{N} \right)^{\epsilon} \left( 1 + \frac{\epsilon q_i}{Q_1} \right) \). Therefore, whenever \(A > 0\) it must be that

$$A < \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon^{-1}} \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right) \left( 1 + \frac{\epsilon q_i}{Q_1} \right) \left[ 1 - \frac{(1 + \epsilon) \left( \frac{Q_1}{N} \right)^{1+\epsilon}}{-f \eta + \epsilon \left( \frac{Q_1}{N} \right)^{1+\epsilon}} \right] .$$

\(\text{(A-6)}\)
Given inequality (A-11), term $A$ will be dominated when $1 - \frac{(1+\epsilon)(\frac{Q_1}{N})^{1+\epsilon}}{f\eta+\epsilon(\frac{Q_1}{N})^{1+\epsilon}} < 1 - \frac{Q_1}{N} \frac{\partial N}{\partial Q_1}$, that is, $1 + \epsilon > \epsilon - \eta$, which is true since $|\eta| < 1$.

### A.2 Proof of Proposition 2

Given the structure of the model, in a competitive equilibrium the market quantity is the same in both periods and is equal to $\alpha \frac{1+\eta}{\eta} N^{\frac{1+\eta}{\eta}}$. The zero-profit condition then implies an equilibrium number of firms $N^c = \alpha^{\frac{1}{\eta}} \left( \frac{1+\delta}{\delta} \right)^{\frac{1-\eta}{\eta(1+\delta)}} > 0$.

As argued in the context of Proposition 1, in the symmetric Nash equilibrium processors earn a positive discounted profit stream. We will show that the competitive equilibrium maximizes the number of entrants subject to the discounted profit stream of processors being nonnegative, which suffices to prove the result. Consider the following optimization program:

$$\max N \quad \text{subj. to} \quad \left\{ \begin{array}{l}
(\frac{Q_1}{N})^{1+\epsilon} + \delta \left( \frac{Q_2}{N} \right)^{1+\epsilon} \geq f \\
\alpha Q_1^\eta - \left( \frac{Q_1}{N} \right)^\epsilon \frac{Q_1}{M} + \delta \left[ \alpha Q_2^\eta - \left( \frac{Q_2}{N} \right)^\epsilon \frac{Q_2}{M} \right] \geq 0
\end{array} \right\} \quad \text{[\lambda]} \quad \text{[\mu]} \quad \text{(A-7)}$$

where the first constraint indicates that a typical upstream supplier must earn a nonnegative profit and the second constraint indicates that a processor must earn a nonnegative profit. Both constraints are satisfied in the symmetric Nash equilibrium characterized by equation (9). Given that $\hat{N} > \tilde{N} > 0$, we can restrict the domain of $N$ in program (A-7) to $N \geq N_0$, where $\tilde{N} < N_0 < \hat{N}$. Program (A-7) can then be reformulated as

$$\max N \quad \text{subj. to} \quad \left\{ \begin{array}{l}
(\frac{Q_1}{N})^{1+\epsilon} + \delta \left( \frac{Q_2}{N} \right)^{1+\epsilon} \geq f \\
\alpha Q_1^{1+\eta} - \frac{Q_1^{1+\epsilon}}{N^{\epsilon}} + \delta \left[ \alpha Q_2^{1+\eta} - \frac{Q_2^{1+\epsilon}}{N^{\epsilon}} \right] \geq 0
\end{array} \right\} \quad \text{[\lambda]} \quad \text{[\mu]} \quad \text{(A-8)}$$

It is clear that program (A-8) admits a solution. Since the constraint set is nonempty, the only way that a solution may not exist is that $N$ takes arbitrarily large values with $(Q_1, Q_2, N)$ remaining in the constraint set. This cannot happen for the following reason. The first constraint indicates that if $N$ grows arbitrarily large, either $Q_1$ or $Q_2$ or both must
also grow arbitrarily large. Suppose that either of them does, say \( Q_1 \), while \( Q_2 \) does not, so that \( \frac{Q_1}{N} \to 0 \). Then, for the second constraint to hold, it must be that \( \frac{Q_2}{N} \to 0 \) because \( Q_1^\eta \to 0 \). But then the first constraint cannot be satisfied. Letting both \( Q_1 \) and \( Q_2 \) grow arbitrarily large does not resolve the contradiction. Therefore, program (A-8) admits an interior solution. This solution must satisfy the necessary first-order conditions:

\[
\begin{align*}
\lambda \left[ \frac{(1+\varepsilon)Q_1^{\eta}}{N^{1+\varepsilon}} \right] + \mu \left[ \alpha(1+\eta)Q_1^\eta - \frac{(1+\varepsilon)Q_1}{N^{\varepsilon}} \right] &= 0 \\
\lambda \left[ \frac{(1+\varepsilon)Q_2^{\eta}}{N^{1+\varepsilon}} \right] + \mu \left[ \alpha(1+\eta)Q_2^\eta - \frac{(1+\varepsilon)Q_2}{N^{\varepsilon}} \right] &= 0 \\
\lambda \left[ \frac{(1+\varepsilon)Q_1^{\eta} + \delta Q_2^{\eta}}{N^{1+\varepsilon}} \right] + \mu \left[ \frac{(1+\varepsilon)Q_1^{\eta} + \delta Q_2^{\eta}}{N^{1+\varepsilon}} \right] &= -1
\end{align*}
\]

where the Lagrange multipliers are both nonnegative. The third relationship implies that \( \lambda > 0 \) and either \( Q_1 > 0 \) or \( Q_2 > 0 \) or both. If \( \mu = 0 \), then since one of the \( Q_i \) must be positive, the first two relationships lead to \( \lambda = 0 \), which is a contradiction. Therefore \( \lambda \) and \( \mu \) are both nonzero and we must have \( \frac{\alpha(1+\eta)Q_1^\eta}{Q_1^{\eta}} = \frac{\alpha(1+\eta)Q_2^\eta}{Q_2^{\eta}} \), which implies \( Q_1 = Q_2 = Q \). Using the two program constraints we then have \( N = \left( \frac{1+\delta}{f} \right)^{1+\varepsilon} Q \) and \( \alpha Q^\eta = \left( \frac{Q}{N} \right)^{\varepsilon} \), from which it is easy to deduce that \( N = N_c \).

To prove the second part of the proposition, we directly compare the two quantities. The second-period quantity is given by (2), while the first-period quantity can be obtained from (5) as \( Q_1 = N \left[ f - \delta \left[ \alpha N^\eta \left( \frac{1+\eta}{1+\varepsilon} \right)^{1+\varepsilon} \right] \right]^{1+\varepsilon} \). It is easy to check that \( Q_1 > Q_2 \Leftrightarrow \left[ \alpha N^\eta \left( \frac{1+\eta}{1+\varepsilon} \right)^{1+\varepsilon} \right]^{1+\varepsilon} < \left( \frac{f}{1+\delta} \right)^{1+\varepsilon} \). Suppose that the converse were true, that is, \( \left[ \alpha N^\eta \left( \frac{1+\eta}{1+\varepsilon} \right)^{1+\varepsilon} \right]^{1+\varepsilon} \geq \left( \frac{f}{1+\delta} \right)^{1+\varepsilon} \). The left-hand side of equation (9) being a decreasing function of \( \alpha N^\eta \), it would be less than

\[
1 - \frac{\epsilon(\eta(M-1+\varepsilon)+\varepsilon)}{M(M+\eta)} \left[ f - \delta \frac{f}{1+\delta} \right]^{1+\varepsilon} - \left[ f - \delta \frac{f}{1+\delta} \right]^{1+\varepsilon} - \frac{\epsilon f}{1+\delta} - \frac{\epsilon f}{1+\delta}
\]

that is, less than

\[
1 - \frac{\epsilon(\eta(M-1+\varepsilon)+\varepsilon)}{M(M+\eta)} \delta - \left( 1 + \frac{\epsilon}{M} \right)
\]

clearly a negative number under Assumption 1. But then \( N \) could not satisfy (9), a contra-
A.3 Proof of Proposition 3

The proof follows the same steps as for the case where processors have buyer and seller power. In the absence of seller power, the second-period quantity is 
\[
\tilde{Q}_2(N, M) = \left( \frac{\alpha}{1 + \frac{\epsilon}{M}} \right)^{\frac{1}{\epsilon - \eta}} N^{\frac{\epsilon}{\epsilon - \eta}}
\]
and the second-period processor profit is 
\[
\tilde{\pi}_2 = \frac{\epsilon}{M^2} \left( \frac{\alpha}{1 + \frac{\epsilon}{M}} \right)^{\frac{1}{\epsilon - \eta}} N^{\frac{(1 + \eta)}{\epsilon - \eta}}.
\]
The first-period number of farmers \(\tilde{N}(Q_1, M)\) is uniquely determined by the relationship
\[
\left( \frac{Q_1}{N} \right)^{1+\epsilon} + \delta \left( \frac{\alpha}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon - \eta}} N^{\frac{2(1+\eta)}{\epsilon - \eta}} = f.
\]
The first-period objective function of a typical processor \(i\) is
\[
\psi(q_i) = p_1 - \left( \frac{q_i + Q - i}{N(q_i + Q - i, M)} \right)^{\epsilon} + \delta \tilde{\pi}_2(\tilde{N}(q_i + Q - i, M), M)\]
where \(p_1\) is the output price in period 1.

**Lemma 3** The function \(\psi(q_i)\) is twice continuously differentiable and satisfies the following property:
\[
\forall q_i \geq 0 \quad \psi'(q_i) = 0 \Rightarrow \psi''(q_i) < 0.
\]

**Proof.** Denoting \(Q_1 = q_i + Q - i\), we have that
\[
\psi'(q_i) = p_1 - \left( \frac{Q_1}{N} \right)^{\epsilon} \left( 1 + \frac{\epsilon q_i}{Q_1} \right) + \frac{\partial \tilde{N}}{\partial Q_1} \left\{ \frac{\epsilon q_i}{N} \left( \frac{Q_1}{N} \right)^{\epsilon} + \frac{\delta \epsilon^2 (1 + \eta)}{M^2 (\epsilon - \eta)} \left( \frac{\alpha N^\eta}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon - \eta}} \right\}.
\]

\(A-9\)
Taking the derivative with respect to $q_i$, we obtain:

$$
\psi''(q_i) = -\frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon-1} \left[ 2 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} - \frac{q_i}{Q_1} + \frac{\epsilon q_i}{Q_1} \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right) \right] + \frac{\partial \tilde{N}}{\partial Q_1} \left\{ \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon} \left[ 1 + \frac{\epsilon q_i}{Q_1} - \frac{(1 + \epsilon) q_i}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right] + \frac{\delta \epsilon^2 (1 + \eta) \eta (1 + \epsilon)}{M^2 (\epsilon - \eta)^2 N} \left( \frac{\alpha N^\eta}{1 + \frac{\epsilon}{M}} \right)^{\frac{2}{1 + \eta}} \frac{\partial \tilde{N}}{\partial Q_1} \right\} + \frac{\partial^2 \tilde{N}}{\partial Q_1^2} \left\{ \frac{\epsilon q_i}{N} \left( \frac{Q_1}{N} \right)^{\epsilon} \frac{\delta \epsilon^2 (1 + \eta)}{M^2 (\epsilon - \eta)} \left( \frac{\alpha N^\eta}{1 + \frac{\epsilon}{M}} \right)^{\frac{2}{1 + \eta}} \right\} \right)
$$

(A-10)

where $\frac{\partial \tilde{N}}{\partial Q_1}$ is given by (6) and $\frac{\partial^2 \tilde{N}}{\partial Q_1^2}$ is given by (A-3). Now suppose that $\psi'(q_i) = 0$. We need to show that $\psi''(q_i) < 0$. The term on the third line of (A-10) clearly contributes negatively to the sum, therefore we can ignore it. Denoting by $B$ the term on the fourth line of (A-10), we thus have that

$$
\psi''(q_i) < B - \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon-1} \left[ 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} + 1 - \frac{q_i}{Q_1} + \frac{\epsilon q_i}{Q_1} \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right) \right] - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} + \frac{q_i}{Q_1} \left( \frac{Q_1}{N} \right)^2 \left( \frac{\partial \tilde{N}}{\partial Q_1} \right)^2 - \frac{\epsilon q_i}{Q_1} \frac{\partial \tilde{N}}{\partial Q_1} \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right) \right]
$$

that is,

$$
\psi''(q_i) < B - \frac{\epsilon}{N} \left( \frac{Q_1}{N} \right)^{\epsilon-1} \left[ 1 - 2 \frac{Q_1 \frac{\partial \tilde{N}}{\partial Q_1}}{N} + \frac{\epsilon q_i}{Q_1} \left( 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1} \right)^2 \right] + 1 - \frac{q_i}{Q_1} + \frac{q_i}{Q_1} \left( \frac{Q_1}{N} \right)^2 \left( \frac{\partial \tilde{N}}{\partial Q_1} \right)^2 \right] \right]
$$
and, adding and subtracting $\left(\frac{Q_1}{N}\right)^2 \left(\frac{\partial \tilde{N}}{\partial Q_1}\right)^2$ inside the square bracket,

$$\psi''(q_i) < B - \frac{\epsilon}{N} \left(\frac{Q_1}{N}\right)^{\epsilon-1} \left(1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1}\right) \left[1 + \frac{\epsilon q_i}{Q_1}\right] \left(1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1}\right)$$

$$+ \left(1 - \frac{Q_i}{Q_1}\right) \left(1 + \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1}\right)$$

(A-11)

The second term on the right-hand side of inequality (A-11) is clearly negative, so we only need to show that this term dominates term $B$ in absolute value, whenever $B > 0$. Using (A-3), we obtain

$$B = \frac{\partial \tilde{N}}{\partial Q_1} \left\{ \frac{\epsilon q_i}{N} \left(\frac{Q_1}{N}\right)^{\epsilon} + \frac{\delta \epsilon^2 (1 + \eta)}{M^2 (\epsilon - \eta)} \left[\alpha N^\eta \left(1 + \frac{\eta}{M}\right)^{1+\epsilon}\right] \right\} \left(\frac{\epsilon}{Q_1}\right) \left(1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1}\right)$$

$$\times \left[1 - \frac{(1 + \epsilon) \left(\frac{Q_1}{N}\right)^{1+\epsilon}}{-f \eta + \epsilon \left(\frac{Q_1}{N}\right)^{1+\epsilon}}\right].$$

(A-12)

The positive factor $\frac{\partial \tilde{N}}{\partial Q_1} \left\{ \frac{\epsilon q_i}{N} \left(\frac{Q_1}{N}\right)^{\epsilon} + \frac{\delta \epsilon^2 (1 + \eta)}{M^2 (\epsilon - \eta)} \left[\alpha N^\eta \left(1 + \frac{\eta}{M}\right)^{1+\epsilon}\right] \right\}$ in (A-12) appears in (A-9), and since $\psi'(q_i) = 0$ it must be less than $\left(\frac{Q_1}{N}\right)^{\epsilon} \left(1 + \frac{\eta}{Q_1}\right)$. Therefore, whenever $B > 0$ it must be that

$$B < \frac{\epsilon}{N} \left(\frac{Q_1}{N}\right)^{\epsilon-1} \left(1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1}\right) \left(1 + \frac{\epsilon q_i}{Q_1}\right) \left[1 - \frac{(1 + \epsilon) \left(\frac{Q_1}{N}\right)^{1+\epsilon}}{-f \eta + \epsilon \left(\frac{Q_1}{N}\right)^{1+\epsilon}}\right].$$

(A-13)

Given inequality (A-11), term $B$ will be dominated when $1 - \frac{(1 + \epsilon) \left(\frac{Q_1}{N}\right)^{1+\epsilon}}{-f \eta + \epsilon \left(\frac{Q_1}{N}\right)^{1+\epsilon}} < 1 - \frac{Q_1}{N} \frac{\partial \tilde{N}}{\partial Q_1}$, that is, $1 + \epsilon > \epsilon - \eta$, which is true since $|\eta| < 1$.

From Lemma 3, we deduce that whenever $\psi'(q_i) = 0$ for $q_i \geq 0$, then $q_i$ solves the first-stage program of processor $i$. In a symmetric Nash equilibrium, an interior solution (with
\( q_i > 0 \) must satisfy the equilibrium condition

\[
P(Q_1) - \left( \frac{Q_1}{N} \right)^{\epsilon} \left[ 1 - \frac{\epsilon(\eta(M + \epsilon) + \epsilon)}{M^2} \left( f - \frac{(Q_1)^{1+\epsilon}}{N} \right) \right] = 0
\]

a condition that is easily expressed in terms of \( N \) as

\[
1 - \frac{\epsilon(\eta(M + \epsilon) + \epsilon)}{M^2} \delta \left( \frac{\alpha N \eta}{1+\epsilon} \right) \delta \left( \frac{\alpha N \eta}{1+\epsilon} \right) \frac{1+\epsilon}{\epsilon - \eta} = 0.
\]

Under Assumption 2, the left-hand side of (A-14) is an increasing function of \( N \) on the interval \((\bar{N}, +\infty)\), where \( \bar{N} \) solves \( f - \delta \left( \frac{\alpha N \eta}{1+\epsilon} \right) \frac{1+\epsilon}{\epsilon - \eta} = 0 \). Its limit as \( N \to +\infty \) is \(-\infty\) and its limit as \( N \to \bar{N} \) is one, therefore there exists a unique \( N \in (\bar{N}, +\infty) \) that solves equation (A-14).

From Section A.2, it is clear that this equilibrium \( N \) is lower than \( N^c \). It remains to be shown that in equilibrium \( Q_1 > Q_2 \). It is easy to show that \( Q_1 > Q_2 \Leftrightarrow \left( \frac{\alpha N \eta}{1+\epsilon} \right) \frac{1+\epsilon}{\epsilon - \eta} < \frac{f}{1+\delta} \).

Suppose the converse were true, that is, \( \left( \frac{\alpha N \eta}{1+\epsilon} \right) \frac{1+\epsilon}{\epsilon - \eta} \geq \frac{f}{1+\delta} \). The left-hand side of (A-14) being a decreasing function of \( N \), it would be less than

\[
1 - \frac{\epsilon(\eta(M + \epsilon) + \epsilon)}{M^2} \frac{\delta f}{1+\delta} \left[ f - \delta f \left( \frac{f}{1+\delta} \right) \frac{1+\epsilon}{\epsilon - \eta} \right] = 0.
\]

that is, less than

\[
1 - \frac{\epsilon(\eta(M + \epsilon) + \epsilon)}{M^2} \frac{\delta f}{1+\delta} \left[ f - \delta f \left( \frac{f}{1+\delta} \right) \frac{1+\epsilon}{\epsilon - \eta} \right] = 0.
\]

\[
1 - \frac{\epsilon(\eta(M + \epsilon) + \epsilon)}{M^2} \frac{\delta f}{1+\delta} \left[ f - \delta f \left( \frac{f}{1+\delta} \right) \frac{1+\epsilon}{\epsilon - \eta} \right] = 0.
\]

clearly a negative number under Assumption 2. But then \( N \) could not solve (A-14).
A.4 Sign of $\frac{\partial \hat{\nu}}{\partial M} (M, 0)$

Start by rewriting condition (11) as $F(\nu, M) = 0$, with

$$F(\nu, M) = \left[ 1 - \delta \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right) \right]^{\frac{1+\epsilon}{\epsilon}} - \frac{\epsilon \delta}{M^2} \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right) - \nu.$$

It is straightforward to see that $\frac{\partial F}{\partial \nu} < 0$, so we just need to analyze the sign of $\frac{\partial F}{\partial M}$. Denoting $D = 1 - \delta \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon}} > 0$, we have

$$\frac{\partial F}{\partial M} = -\frac{\delta \epsilon}{M^2} \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon}} D^{\frac{1}{1+\epsilon}} - \frac{1}{D^{\frac{1}{1+\epsilon}}} \left\{ D^{\frac{1}{1+\epsilon}} \left[ \frac{\delta \epsilon (1 + \epsilon)}{M^4} \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right) \right]^{\frac{1+\epsilon}{\epsilon}} - 2 \frac{\delta \epsilon}{M^3} \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon}} D^{\frac{1}{1+\epsilon}} - \frac{\delta \epsilon}{M^2} \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon}} \right\}$$

an expression that has the same sign as $-1 - \frac{1+\epsilon}{M^2} + \frac{\epsilon}{M} (1 + \frac{\epsilon}{M}) - D^{-\frac{1}{M^2}} \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon}}$, that is, $D (-M^2 - 1 + 2M + \epsilon) - \delta \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon}}$ and, using the definition of $D$, the same sign as $-M^2 - 1 + 2M + \epsilon + \delta \left( \frac{\nu}{1 + \frac{\epsilon}{M}} \right)^{\frac{1+\epsilon}{\epsilon}} (M^2 - 2M - \epsilon)$. 
A.5 Derivation of $\hat{W}(M)$

Using equations (2) and (4), we have:

$$W = \frac{\alpha Q_1^{1+\eta}}{1+\eta} - \frac{N}{\epsilon} \left[ F - \frac{\delta \epsilon}{1+\epsilon} \left( \frac{Q_2}{N} \right)^{1+\epsilon} \right] + \frac{\delta \alpha Q_2^{1+\eta}}{1+\eta} - \frac{\delta N}{1+\epsilon} \left( \frac{Q_2}{N} \right)^{1+\epsilon} - NF$$

$$= \frac{\alpha N^{1+\eta}}{1+\eta} \left[ F \left( \frac{1+\epsilon}{\epsilon} \right) - \delta \left( \frac{Q_2}{N} \right)^{1+\epsilon} \right]^{\frac{1+\eta}{1+\epsilon}} + \frac{\delta \alpha Q_2^{1+\eta}}{1+\eta} - NF \left( \frac{1+\epsilon}{\epsilon} \right)$$

$$= \frac{\alpha N^{1+\eta}}{1+\eta} \left\{ f - \delta \left[ \alpha N^\eta \left( \frac{1+\frac{\eta}{M}}{1+\frac{\epsilon}{M}} \right) \right]^{\frac{1+\eta}{1+\epsilon}} + \delta \left[ \alpha N^\eta \left( \frac{1+\frac{\eta}{M}}{1+\frac{\epsilon}{M}} \right) \right]^{\frac{1+\eta}{1+\epsilon}} \right\} - Nf.$$