A Theory of Non-Conventional Monetary Policy

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ABSTRACT

The paper provides a conceptual foundation for non conventional monetary policy and analyzes its interaction with bank regulation . In a stylized model with two types of financial intermediaries, regulated banks and non-regulated cash funds, we emphasize the general equilibrium effects of bank capital regulation. We put forward the concept of a "natural" equity to asset ratio for commercial banks. In some situations (which we term "high demand for safe assets") this natural equity to asset ratio is not sufficient to absorb bank losses and guarantee the safety of the savings of depositors and cash investors. In that case non conventional monetary policy interventions are necessary to restore the Pareto optimality of the banking equilibrium.

1 Introduction

After the Global Financial Crisis (GFC) of 2007-2009 and the Great Recession that followed it, the Federal Reserve and other Central Banks have adopted radically new forms of policy intervention. In particular, they have started to purchase massive amounts of risky securities (the so called Quantitative Easing policy) and to collect large volumes of reserves from commercial banks by offering them a relatively attractive remuneration on these reserves. Both policies have resulted in a huge increase in the size of Central Banks' balance sheets (bank reserves on the liability side and risky securities on the asset side), which has been heavily criticized by some commentators.

The objective of this paper is to provide a conceptual foundation for this new type of non conventional monetary policy interventions. We follow the path initiated by Gertler and Karadi (2011), who were the first to propose a model of "unconventional" monetary policy. Our approach is complementary to theirs: whereas they work with a complex (and relatively opaque) DSGE model, because they want to calibrate it for to the US economy and assess the quantitative impact of these policies, we work instead with a stylized theoretical model, because we want to highlight the conceptual foundations for such policies.

Many economists have suggested that the main justification for non conventional monetary policy was that policy rates had reached the zero lower bound for nominal interest rates. We believe the most fundamental justification is elsewhere[1], i.e. what Bernanke (2008) has called the "savings glut". We show that, when there is a large demand for safe assets by short term investors, as compared with the demand for risky assets by long term investors (what we call a "high demand for safe assets"), competition between banks and shadow-banks leads either to a high risk of crisis (as cash investors invest without precaution because they anticipate a bail out by the government in case of low returns on banks' investments) or to a "credit crunch" where bank credit is inefficiently low. We show how non conventional monetary policy interventions may restore Pareto optimality.

The paper proposes a stylized and highly simplified general equilibrium model with one physical good, two dates ($t = 0, 1$) and two types of financial intermediaries: (regulated) banks and (non-regulated) shadow banks. The good can be consumed at $t = 0$ or invested in the real sector. The two types of financial intermediaries collect the savings of two types of infinitely risk averse[2] agents: respectively depositors and cash investors, and a third type of agents, who are risk neutral [3] and buy banks' equity (or risky debt). We call this third type of agents the capital investors.

Banks differ from shadow banks in three respects: first only them can invest in the real sector

[1] This view is shared by Gertler and Karadi (2011) who find that "there are benefits from credit policy even if the nominal interest rate has not reached the zero lower bound", p1.

[2] This symplifying assumption is borrowed from Stein 2012 and Gennaioli, Shleifer and Vishny 2013

[3] This assumption drastically simplifies the analysis.

(i.e., provide loans to small and medium size enterprises), second their deposits are explicitly insured by the government, and third they are regulated. They have to comply with a minimum capital requirement: at least a fraction α of their risky investments has to be financed by equity. Shadow banks (cash funds) collect the savings of cash investors and invest them into wholesale bank deposits (which are not explicity insured by the government) and Treasury bills.

The government sector consists of the Treasury, who finances the government's date $t = 0$ expenditures G(which are taken as exogenous) by issuing Treasury Bills ()which are repaid at $t = 1$ by levying lump sum taxes), and the Central Bank who controls the riskless rate R, the banks' capital ratio α and may also enter into non conventional policies as specified below.

We show that our economies can be characterized by a "natural" equity to asset ratio α^*, interpreted as the optimal share of real investment that should be financed by capital investors. Depending on the risk inherent in the technology, which we measure by the critical equity ratio α_c that allows absorbing losses on real investment with probability one (so to speak, the Value at Risk at level 0), the economy can be in a situation of "high demand for safe assets" (when $\alpha^* < \alpha_c$) or a "low demand for safe assets" otherwise. In the latter case, conventional monetary policy is sufficient to attain a Pareto optimal allocation. In the former case of a "high demand for safe assets", non conventional interventions are needed.

This paper is related to several strands of the literature.

One is coming from international finance: for example Caballero, Fahri and Gourinchas (2008, 2015) are concerned by the excess demand for US government bonds from foreign countries trading in dollars, which keeps the interest rate low. In a similar vein, Krishnamurthy and Vissing Jorgensson (2012) show the existence of an excess demand for safe assets in the US. Pozsar (2012, 2014) provides a global picture of the US financial system, with very useful orders of magnitude for the variables of interest in our analysis.

Also directly related are the papers such as Gorton, Lewellen and Metrick (2013), Gorton and Metrick (2010) (2012) , Gennaioli, Shleifer and Vishny (2012, 2013) that model the relation between global demand for safe assets and the development of the shadow banking system. Gennaioli-Shleifer-Vishny (2012) in particular, is closely related to our approach. They assume that cash investors are irrational and neglect small probability disaster events, while we rationalize their behavior by the expectation of a bail-out of cash funds by the government, but this is essentially equivalent.

Our paper has a lot in common with the recent literature on the macro-prudential regulation of banks (e.g., Hanson, Kashyap and Stein 2011) and its relation with monetary policy (Stein 2012). Like Stein(2012) or DeAngelo and Stulz (2013), we explicitly role of banks as providing liquidity services to depositors. This is the reason why the Modigliani and Miller theorem, which has been so much emphasized by Admati and Hellwig (2013), may not be valid for banks.

We also contribute to the debate about the "right" level of capital for banks (see for example Allen, Carletti and Marquez 2014, Miles, Yang and Marcheggiano 2013) by taking into account general equilibrium effects. It is indeed striking that most of the literature has only considered partial equilibrium models in which interest rates are exogenous. For example, Admati and Hellwig (2013) implicitly consider that bank equity requirements can be increased dramatically without any impact on equilibrium interest rates. By contrast our model puts forward the important notion of a "natural" equity to asset ratio for banks, which is determined by preferences and technology.

Finally, our model provides a simple set-up for exploring the impact of alternative instruments for central banks such as the interest rate on reserves (see Williamson 2015)and a second policy rate that would directly influence bank credit rates rather than through money market rates.

The rest of the paper is organized as follows.

Section 2 shows the fundamental importance of capital regulation in the analysis of bank profit maximization. Section 3 develops the full model and characterizes competitive banking equilibria and Pareto optima. It develops the notion of a "natural" equity to debt ratio for banks. Section 4 is the core of our paper. It shows why non conventional monetary policy may be needed to restore Pareto optimality of the banking equilibrium in a situation of "high demand for safe assets". Section 5 discusses an alternative institutional arrangement, which is an extended version of the 100

2 Bank Profit Maximization

Consider a two-date ($t = 0, 1$) economy with a continuum of small banks that collect funds from depositors, cash investors and capital investors at date 0. They invest these funds in productive projects with risky per-unit payoff $\tilde{a}$ at date 1: we bypass the intermediate step where banks lend their funds to entrepreneurs who undertake risky projects and reimburse the bank when the projects succeed. The random variable $\tilde{a}$ has support $A = [\underline{a}, \infty)$ with $\underline{a} \geq 0$, and continuous density f which can be extended to $\mathbb{R}_+$ when $\underline{a} > 0$ i.e. $f : \mathbb{R}_+ \to \mathbb{R}_+$ and $f(a) = 0$ if $0 \leq a \leq \underline{a}$. We assume that the banks' investment has constant returns, that all banks are identical and have perfectly correlated payoffs. The last property implies that despite the fact that bansk are small and competitive, their decisions generate externalities, as in an economy with "too big to fail" banks.

Banks are created at date 0 by raising equity (E) and debt (D), and investing the funds $K = E + D$ to get the risky payoff $K\tilde{a}$ at date 1. Capital investors, who provide the equity are risk neutral, and are protected by limited liability. To attract these investors, banks have to maximize their expected rate of return on equity, which is denoted R^E and will be the same for all banks at equilibrium. Depositors and cash investors are assumed to be infinitely risk averse. This means that they will only provide funds if the repayment of the funds at date 1 is either backed by safe collateral or, if the debt exceeds the amount of safe collateral, is backed by explicit or implicit government insurance. The consequence of the insurance is that the interest rate R that the bank pays on its debt does not depend on the debt-equity choice of the bank. Let $\hat{a}$ defined by

$$K\hat{a} = RD$$

denote the threshold per unit payoff which suffices to reimburse the banks debt D. If $\hat{a} \leq \underline{a}$ the debt is safe even without insurance and the bank is never bankrupt. If $\hat{a} > \underline{a}$, since shareholders have limited liability, the bank pays its debt only if its realized payoff is at least at the threshold, $a \geq \hat{a}$, and is bankrupt if $a < \hat{a}$. In both cases the expected profit of the bank's shareholders can be written as

$$\int_{\hat{a}}^{\infty} (Ka - RD) f(a) da - R^E E \tag{1}$$

since if $\hat{a} < \underline{a}$, $f(a) = 0$ for $a \in [\hat{a}, \underline{a}]$. The bank chooses its investment and financing $(K, E, D, \hat{a})$ to maximize (1) subject to $K = E + D$, $K\hat{a} = RD$: using these two relations the bank's maximum problem reduces to choosing $(E, D, \hat{a})$ to maximize its expected profit

$$\int_{\hat{a}}^{\infty} (E + D)(a - \hat{a}) f(a) da - R^E E$$

subject to $(E + D)\hat{a} = RD$, $E \geq 0$, $D \geq 0$. The government's insurance of the reimbursement of the bank's debt and the associated constancy of the bank's borrowing rate R has the following important consequence.

Proposition 1. *If $R < \sup\{a | a \in A\}$, then the problem of choosing $E \geq 0$, $D \geq 0$ to maximize the bank's expected profit (1) subject to $K = E + D$ and $K\hat{a} = RD$, has no solution.*

Proof. Suppose $E = 0$, $D > 0$ then $\hat{a} = R$ and the banks profit is $D \int_R^{\infty} (a - R) dF(a)$ which, if there is positive probability that $a > R$, tends to infinity as $D \to \infty$. □

This result stands in sharp contrast to the result that holds under the standard assumption of finance that the price of the risky debt of a corporation with limited liability is the present value of the income stream that it delivers, and thus depends on the probability that the debt is reimbursed. In that setting the bank's profit maximizing problem has a solution—in fact infinitely many solutions, all with zero expected profit and an indeterminate debt-equity ratio[4]: this is the Miller-Modigliani theorem.

[4] Assume for simplicity that $\underline{a} = 0$. In a standard finance framework where agents provide funds to the bank and

The government's insurance of the bank's debt makes it, in the terminology of Gorton (2010), 'information insensitive' in the sense that the interest rate that lenders require is not tied to the riskiness of the debt, so that lenders do not feel the need to get information on the financing and investment strategies of the bank. This lack of dependence of the interest rate on the riskiness of the loan tends to make debt look 'cheap' and leads to strategies where equity tends to zero and debt tends to infinity. This conforms with the commonly held view of bankers that debt is cheaper than equity as a source of funds and suggests (what experience confirms) that bankers, if left to their own choices, choose financing strategies with a lot of debt. As a result regulators typically set minimum equity-to-asset ratios for banks. In the setting of our model this suggests imposing a constraint of the form

$$E \geq \overline{\alpha}\, K, \quad 0 < \overline{\alpha} < 1$$

where $\overline{\alpha}$ is the equity requirement of the bank. The profit maximizing problem of the representative bank thus becomes to choose $(E, D, \hat{a})$ to maximize

$$\Pi = \int_{\hat{a}}^{\infty} (E + D)(a - \hat{a}) f(a) da - R^E E$$

subject to $(E + D)\hat{a} = RD$, $E \geq \overline{\alpha}\,(E + D)$, $E \geq 0$, $D \geq 0$. The equity constraint $E \geq \overline{\alpha}\,(E + D)$ suggests that instead of using equity and debt (E, D) as the bank's choice variables we use (E, α), namely the bank's equity E and its equity/capital ratio $\alpha = \frac{E}{E+D}$: this simple change of variable greatly simplifies the analysis of the bank's decision problem. The variables $(D, \hat{a})$ can then be recovered from (E, α) since $D = (\frac{1-\alpha}{\alpha})E$ and $\hat{a} = (1 - \alpha)R$. Since $E + D = \frac{E}{\alpha}$, if we define the function

$$\Phi(\alpha; R) = \frac{1}{\alpha} \int_{(1-\alpha)R}^{\infty} (a - (1 - \alpha)R) f(a) da, \quad 0 < \alpha \leq 1 \tag{2}$$

then the bank's profit can be decomposed into the product

$$\Pi(E, \alpha) = E \cdot (\Phi(\alpha; R) - R^E), \quad E \geq 0,\ 0 < \alpha \leq 1.$$

The function $\Phi(\alpha; R)$ defines the bank's *expected rate of return on equity* when its equity ratio is α and it faces the interest rate R on its debt. Later in the paper we will find it useful to view Φ

perceive the debt to be risky, the banks profit maximizing problem would be

$$\max_{(E,D',K,\hat{a})} \int_{\hat{a}}^{\infty} K(a - \hat{a})\mu(a) f(a) da - E$$

subject to (i) $K = E + q_D D'$; (ii) $K\hat{a} = D'$ and (iii) $q_D = \int_0^{\hat{a}} \frac{Ka}{D'} \mu(a) f(a) da + \int_{\hat{a}}^{\infty} \mu(a) f(a) da$ where D' is the amount of debt at date 1 and $\mu(a)_{a \in A}$ is the stochastic discount factor, which is constant and equal to $\frac{1}{R^E}$ if investors are risk neutral. It is easy to check that a necessary condition for this problem to have a solution is $1 = \int_0^{\infty} a\, \mu(a) dF(a)$ and that any $E > 0$, $D' > 0$, K, $\hat{a}$ satisfying (i) and (ii) give zero profit and give a solution to the bank's problem, where the price of debt is given by (iii). This is just the Modigliani-Miller theorem in a setting with constant returns.

as a function of the face value $\hat{a} = \frac{DR}{E+D} = (1-\alpha)R$ of the bank's debt (per unit of capital). If we let $r(\hat{a}) = \int_{\hat{a}}^{\infty}(a-\hat{a})f(a)da$ then $\Phi(\alpha,\hat{a}) = (\frac{1}{\alpha})r(\hat{a})$, where $r(\hat{a})$ is the expected return per unit of capital. $r(\hat{a})$ can be viewed as the value of a call option on one unit of bank capital with exercise price equal to the face value of the debt $\hat{a}$: this expresses the property of "limited liability" of the bank—that it defaults on its debt $\hat{a}$ when it cannot pay $(a < \hat{a})$. The bank's return on equity $\Phi(\alpha,\hat{a})$ is a levered multiple of its return to capital, $\Phi(\alpha,\hat{a}) = (\frac{1}{\alpha})r(\hat{a})$ where $(\frac{1}{\alpha})$ is the bank's *equity leverage*.[5] Using the function $\Phi(\alpha,R)$ the bank's maximum problem expressed in terms of (E,α) reduces to

$$\max_{(E,\alpha)} \{E \cdot (\Phi(\alpha;R) - R^E) \,|\, \alpha \geq \overline{\alpha}\} \tag{3}$$

Modulo its choice of E (namely its scale) the bank's problem reduces to the optimal choice of its equity ratio α: this decision depends on the qualitative behavior of the expected rate of return $\Phi(\alpha;R)$. There are three cases, which are distinguished by the magnitude of the interest rate R on the bank's debt relative to the expected return $\mathbb{E}(\tilde{a})$ on its random payoff $\tilde{a}$: (i) $R < \mathbb{E}(\tilde{a})$ (ii) $R = \mathbb{E}(\tilde{a})$, (iii) $R > \mathbb{E}(\tilde{a})$, which we call *low*, *natural* and *high* interest rate cases. The graphs of the expected rate of return on equity $\Phi(\alpha;R)$ for the three cases are shown in Figure 1 , assuming $R < \sup_{a\in A}\tilde{a}$.

To understand the geometric form of the function $\Phi(\cdot\,;R)$ for all $R > 0$ note first that $\Phi(\alpha;R) > 0$ for $0 < \alpha \leq 1$, $\Phi(1;R) = \mathbb{E}(\tilde{a})$ and

$$\Phi'(\alpha;R) = -\frac{1}{\alpha^2}\psi(\alpha;R) \quad \text{with } \psi(\alpha;R) = \int_{(1-\alpha)R}^{\infty}(a-R)f(a)da$$

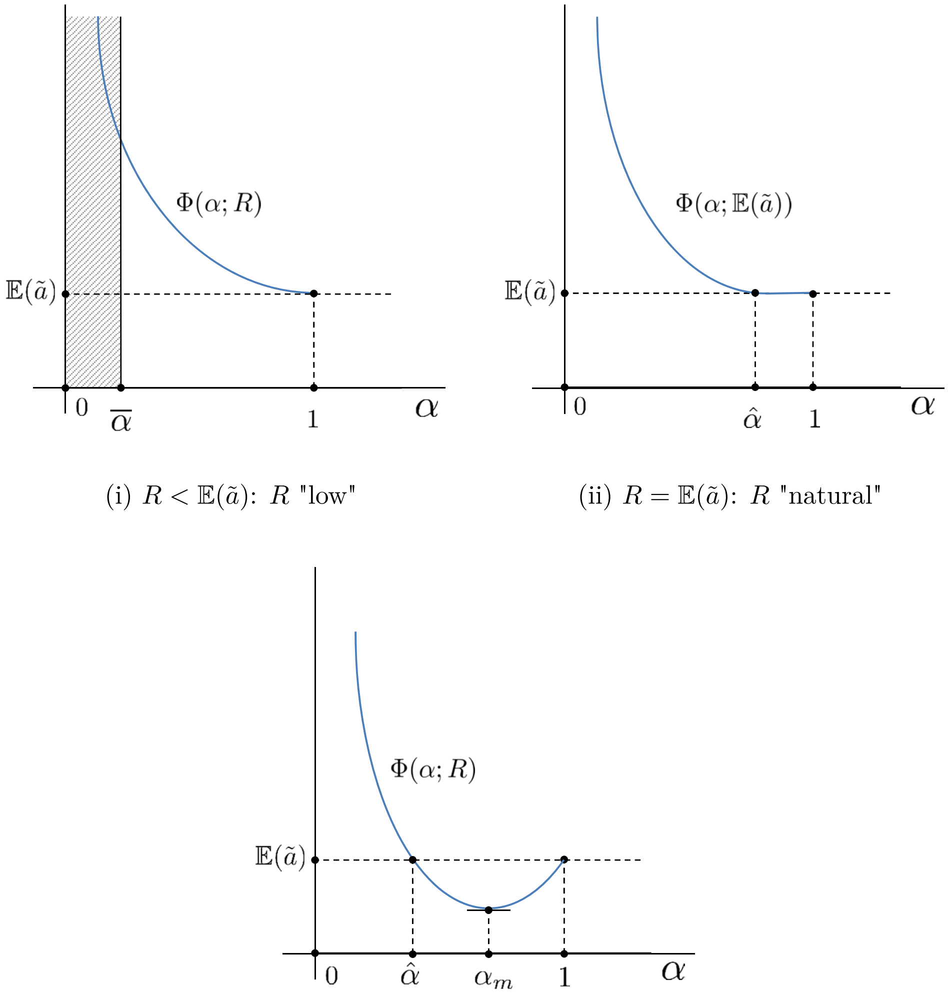
As $\alpha \to 0$, $\psi(\alpha;R) \to \displaystyle\int_R^{\infty}(a-R)f(a)da > 0$, so $\Phi(\alpha;R)$ is decreasing when α is close to zero. $\psi'(\alpha;R) = -\alpha R^2 f((1-\alpha)R)$ so that $\psi(\alpha;R)$ is decreasing if $f((1-\alpha)R) > 0$ or constant if $f((1-\alpha)R) = 0$ and thus attains a minimum for $\alpha = 1$ with value $\psi(1;R) = \mathbb{E}(\tilde{a}) - R$.

(i) If $R < \mathbb{E}(\tilde{a})$, $\psi(\alpha) > 0$ for all $\alpha \in [0,1]$ which implies $\Phi'(\alpha) < 0$ for all $\alpha \in (0,1]$ and the graph of Φ has the form shown in Figure 1 (i).

(ii) If $R = \mathbb{E}(\tilde{a})$, $\psi(1) = 0 = \psi(\alpha)$ for $(1-\alpha)R \leq \underline{a}$. Thus if $\hat{\alpha}$ is defined $(1-\hat{\alpha})R = \underline{a}$, $\Phi(\alpha)$ is decreasing on $(0,\hat{\alpha})$ and constant on $[\hat{\alpha},1]$ and the graph of Φ is as shown in Figure 1 (ii).

(iii) If $R > \mathbb{E}(\tilde{a})$, $\psi(1) < 0$ and there exists $\alpha_m > 0$ such that $\psi(\alpha_m) = 0$ with $\psi(\alpha) > 0$ if $\alpha < \alpha_m$ and $\psi(\alpha) < 0$ if $\alpha > \alpha_m$, since $(1-\alpha)R > \underline{a}$. Thus $\Phi(\alpha)$ is decreasing on $(0,\alpha_m)$ and increasing on $(\alpha_m,1]$. Thus $\Phi(\alpha_m) < \mathbb{E}(\tilde{a})$ and the graph of Φ is as shown in Figure 1 (iii).

[5] Admati-Hellwig (2013,p.177) note that "in a major innovation" in 2010, Basel III proposed fixing a minimum equity requirement of 3% of assets, commenting that "if this number looks outrageously low, it is because it *is* outrageously low". With $\alpha = 0.03$, $(\frac{1}{\alpha}) = 33.3$: the return on equity is more than thirty three times the return on capital.



(i) $R < \mathbb{E}(\tilde{a})$: R "low"

(ii) $R = \mathbb{E}(\tilde{a})$: R "natural"

(iii) $R > \mathbb{E}(\tilde{a})$: R "high"

Figure 1: Bank's expected return on equity $\Phi(\alpha; R)$

We can now readily deduce the bank's choice of equity ratio α which maximizes its expected return on equity (and hence its expected profit) subject to the regulatory requirement $\alpha \geq \overline{\alpha}$. In case (i) since Φ is decreasing the bank chooses $\alpha = \overline{\alpha}$ the lowest permissible ratio (i.e. excluding α in the shaded region). In case (ii) if $\overline{\alpha} < \hat{\alpha}$, the bank chooses $\overline{\alpha}$ and if $\hat{\alpha} \leq \overline{\alpha} < 1$ then the bank is indifferent between all $\alpha \in [\overline{\alpha}, 1]$ which corresponds to the Miller-Modigliani theorem since there is no default. In the high interest rate case (iii), if $\overline{\alpha} < \hat{\alpha}$ (where $\hat{\alpha}$ is defined by $\Phi(\hat{\alpha}; R) = \mathbb{E}(\tilde{a})$) then the bank chooses $\alpha = \overline{\alpha}$. If $\overline{\alpha} > \hat{\alpha}$ then the bank does not borrow, setting $\alpha = 1$, financing all investment by equity.

The bank's maximum problem (3) consists of a joint choice of E and α. For this problem to have a solution the minimum equity requirement $\overline{\alpha}$ imposed by the regulator cannot be chosen independently of (R, R^E). In view of the constant returns to scale assumption when the bank's optimal choice of α is $\overline{\alpha}$ (case (i), (ii) and (iii) with $\overline{\alpha} \leq \hat{\alpha}$) there is a non-trivial solution to the choice of E if and if

$$\Phi(\overline{\alpha}; R) = \frac{1}{\overline{\alpha}} \int_{(1-\overline{\alpha})R}^{\infty} (a - (1-\overline{\alpha})R) f(a) da = R^E \tag{4}$$

i.e. the bank's expected profit is zero. In case (iii) if $\overline{\alpha} > \hat{\alpha}$ the bank's problem has a solution if and only if $R^E = \Phi(1) = \mathbb{E}(\tilde{a})$.

Our analysis of the bank's choice problem can be summarized in the following proposition.

Proposition 2. *Let $(R, R^E, \overline{\alpha})$ denote the loan rate, equity rate and equity-asset ratio faced by a bank, then*

(i) *a necessary condition for the bank's maximum problem to have a solution is that the equity rate R^E satisfy $R^E \geq \mathbb{E}(\tilde{a})$;*

(ii) *if the bank faces a low loan rate $R < \mathbb{E}(\tilde{a})$, or if $R = \mathbb{E}(\tilde{a})$ and $(1-\overline{\alpha})R > \underline{a}$, then there is a non-trivial solution to the bank's maximum problem if and only if the zero profit condition (4) holds. The solution is such that the equity constraint $E \geq \overline{\alpha} K$ binds. E is indeterminate and D is such that $D = (\frac{1-\overline{\alpha}}{\overline{\alpha}})E$;*

(iii) *if $R = \mathbb{E}(\tilde{a})$ and $(1-\overline{\alpha})R \leq \underline{a}$ then the bank's maximum problem has a non-trivial solution if and only if $R^E = \mathbb{E}(\tilde{a})$ and the bank is indifferent between all equity asset ratios $\alpha \in [\overline{\alpha}, 1]$;*

(iv) *if the bank faces a high loan rate $(R > \mathbb{E}(\tilde{a}))$ then there is a critical value $\hat{\alpha}$ of its equity ratio defined by*

$$\Phi(\hat{\alpha}; R) = \frac{1}{\hat{\alpha}} \int_{(1-\hat{\alpha})R}^{\infty} (a - (1-\hat{\alpha})R) f(a) da = \mathbb{E}(\tilde{a}), \quad 0 < \hat{\alpha} < 1 :$$

such that

- *if $\overline{\alpha} \leq \hat{\alpha}$ there is a solution to the bank's maximum problem if $(R, R^E, \overline{\alpha})$ are such that (4) holds: as in(ii) the equity constraint $E \geq \overline{\alpha} K$ binds, E is indeterminate and $D = (\frac{1-\overline{\alpha}}{\overline{\alpha}})E$;*
- *if $\overline{\alpha} > \hat{\alpha}$ there is a solution if $R^E = \mathbb{E}(\tilde{a})$ which consists of equity only, $\alpha = 1$, $D = 0$, E indeterminate.*

The cost of borrowing for the bank is influenced by two components: the interest rate R that it pays on the loan and the proportion of the time (the probability) that it repays its loan. When the interest rate is low ($R < \mathbb{E}(\tilde{a})$) the interest cost is less than the expected return on investment and the greater the proportion financed by debt the higher the profit for the shareholders. Thus the equity requirement $\alpha \geq \overline{\alpha}$ is always binding. When the interest rate is the natural rate ($R = \mathbb{E}(\tilde{a})$), the debt is still "cheap" if the probability of repaying it is less than one, which occurs when the equity requirement $\overline{\alpha}$ is low enough ($\overline{\alpha} < \hat{\alpha}$). The bank still finances as much as possible by debt and the equity requirement is binding. (Proposition 2 (ii)).

When the interest rate is the natural rate and the equity requirement limits the leverage of the bank so that its debt will always be repaid ($(1 - \overline{\alpha})R \leq \underline{a}$) then there is no default and the cost of debt for the bank is "fair". The bank is then indifferent between debt and equity—whatever the equity ratio α the expected revenue is the expected return on the investment. (Proposition 2 (iii)).

When the bank's interest rate exceeds the natural rate ($R > \mathbb{E}(\tilde{a})$) debt can still be "cheap" if the probability of repaying it is sufficiently small. This occurs when the proportion financed by debt is high i.e. when the equity requirement is small ($\overline{\alpha} \leq \hat{\alpha}$), in which case the bank borrows as much as possible and the equity requirement is binding. When the equity requirement is high ($\overline{\alpha} > \hat{\alpha}$) the probability of default is small and the true cost of debt to the bank exceeds its expected return—the debt is too expensive and the banks chooses all equity financing. (Proposition 2 (iv)).

In each of the above cases (ii), (iii), (iv), the bank's expected revenue at the optimal α either exceeds or is at least equal to its expected return $\mathbb{E}(\tilde{a})$ on its investment. Thus to have a solution to profit maximization, the cost of equity R^E must be at least $\mathbb{E}(\tilde{a})$ (Proposition 2 (i)). If the cost of equity were less than $\mathbb{E}(\tilde{a})$, for any value of R, the bank could make a profit by financing investment by equity only, with a cost less than the expected return on the investment and there would be infinite demand for equity.

3 The Model and Banking Equilibrium

Our objective is to study banking equilibria in number of different institutional settings. In this section we present a simple model of banking equilibrium for an institutional setting akin to that in existence in the period preceding the Global Financial Crisis of 2007-2009 . We show that in an economy for which there are real costs of bankruptcy, if it has a high supply of debt then the equilibrium cannot be efficient. In the sections that follow we propose ways of modifying the current system to improve the efficiency of the equilibrium and discuss the advantage and inconvenience involved in adopting each modification.

In each of these institutional settings, the model of the previous section will serve as a canonical model of the bank's choice problem in that, by an appropriate change of variable, the bank's profit maximizing problem can be reduced to the one analyzed in the canonical case. In each of these settings we consider an economy with three types of agents—depositors, institutional cash investors (or rather the cash funds managers which represent them) and investors—banks which channel the funds of these agents into risky productive ventures, and a government which determines interest rates, regulates banks, and perhaps insures the agents who lend to banks. Depositors and cash funds have fundamentally the same objective, they seek a safe haven in which to place their funds; they will thus only lend to banks if they are sure of having their funds returned. Capital investors are more flexible and are prepared to accept risk.

Two important hypotheses lie behind the framework: first, the infinitely risk-averse agents (depositors and cash investors) cannot directly invest in the real sector because firms (which are not explicitly modeled) cannot commit to pay back such a loan in all circumstances; thus it is not feasible for the risk-neutral agents to insure the risk-averse agents. Second, banks are the only institutions with the know-how to invest in productive projects—neither investors nor the government can directly fund productive projects without going through bank intermediaries. We thus abstract from that part of the productive sector which receives market-based financing by issuing traded bonds or equity.

Infinitely risk averse depositors deposit their funds with banks and cash funds lend to banks, despite the fact that they know banks will invest these funds in risky ventures, because the deposits are explicitly insured by the government (FDIC insurance for the US) and the cash funds are either protected by the presence of safe collateral, or if they lend more than the safe collateral, because they believe they are *implicitly* insured i.e. the government will rescue the banks if their assets prove insufficient for pay back their loans. Such a belief was essentially confirmed in 2008, since in order to avoid a collapse of the financial system, the government either directly bailed out the failing institutions or, via the Central Bank, purchased the assets serving as collateral for their loans to increase their resale value. Because lenders are explicitly or implicitly "insured" there is no possibility of runs on the banks. The cost of runs can however be incorporated into the model as a loss in output when the government has to step in to pay the banks' debt, i.e. it can be incorporated into the bankruptcy cost, which is parametrized by the coefficient of loss γ: the cost γ will be greater when the default involves both cash funds and deposits, than when only deposits are involved. Thus embedded in this first version of the model is the assumption that runs are so costly for the economy that the government insures depositors and cash funds to avoid the possibility of a run.

We now describe the characteristics and decisions made by the three groups of agents, depositors, cash fund managers and investors.

Depositors The representative depositor has an endowment of funds $w_{d\,0}$ (the single good) at date 0 and no endowment at date 1. The depositor places funds in a bank so as to be able to transfer them to date 1 for consumption and to make use of the payment services provided by the bank at that date. The utility the agent derives from the consumption stream $x_d = (x_{d\,0}, \widetilde{x}_{d1})$ consisting of the consumption $x_{d\,0}$ at date 0 and the random consumption $\widetilde{x}_{d1}$ at date 1 is given by

$$u_d(x_{d\,0}) + \min\{\widetilde{x}_{d1}\} + \rho \min\{\widetilde{x}_{d1}\} \tag{5}$$

where u_d is a concave increasing function, $\min\{\widetilde{x}_{d1}\}$ expresses the agents' infinite risk aversion and $\rho(\min\{\widetilde{x}_{d1}\})$ denotes the convenience yield obtained from the transaction services offered by the banks at date 1: for simplicity we assume ρ is linear. Payment services exist only for deposits; for example, if the depositor gets the funds $\min\{\widetilde{x}_{d1}\}$ from investing in government bonds then the third term in (5) is zero—no convenience yield is obtained from holding government bonds. If R^d denotes the interest rate paid by banks on deposits, depositing the amount d in a bank generates the consumption stream $x_d = (w_{d\,0} - d, R^d d)$ from which a depositor derives the utility

$$u_d(w_{d\,0} - d) + (1 + \rho)R^d d$$

The date 0 utility function u_d models the opportunity cost of depositors and replaces the frequently made assumption that depositors have access to a safe storage technology.

Cash funds In addition to the (insured) deposits of households, banks have access to a large supply of funds from a variety of institutional investors (corporations, wealth managers, money market funds,...) through what is generally referred to as the wholesale money market. Like depositors these institutional investors insist on the strict safety and liquidity of their funds, the mandate of their managers being: "do not lose" (Pozsar (2015)). This insistence on safety and liquidity made these funds vulnerable to runs which in the recent financial crisis were halted by actions of Central Banks and Treasuries, confirming the perception that these funds are "implicitly" insured by the macro prudential policy of the government. To capture the role of these investors as purveyors of funds to the banking sector we introduce a group of agents that we call "cash investors", who will exclusively invest in cash funds. For the sake of simplicity we will use the term cash funds as a synonym for cash investors reserve the term "investor" exclusively for capital investors, i.e. the risk neutral agents who accept to invest in risky equity. The representative cash fund has a date 0 endowment $w_{c\,0}$ and like a depositor infinite risk aversion with utility function

$$u_c(x_{c\,0}) + \min\{\widetilde{x}_{c1}\}$$

where u_c is a concave increasing function which models the opportunity cost of their funds. It follows that cash funds will only lend under the form of sure debt. If R^c denotes the interest rate that they receive (from banks or government bonds) the representative cash fund will choose c to maximize

$$u_c(w_{c\,0} - c) + R^c c$$

If the date 0 utility functions u_d and u_c of depositors and cash funds satisfy Inada conditions

$$u_d'(x_{d\,0}) \to \infty \text{ as } x_{d\,0} \to 0, \quad u_c'(x_{c\,0}) \to \infty \text{ as } x_{c\,0} \to 0$$

then the solutions of their maximization problems are characterized by the first-order conditions

$$u_d'(w_{d\,0} - d) = (1+\rho)R^d,$$
$$u_c'(w_{c\,0} - c) = R^c.$$

Investors To keep the number of different types of agents to a minimum we assume that investors play two roles: they represent both the agents who are long-term investors accepting to take risks and the taxpayers. Investors have an endowment stream $w_i = (w_{i\,0}, w_{i1})$ where w_{i1} is non risky and a risk neutral (date 1) utility function

$$u_i(x_{i\,0}) + \mathbb{E}(\tilde{x}_{i1})$$

where u_i is a concave increasing function satisfying the Inada condition, and similarly to the other agents, represents the opportunity cost of their date 0 funds. Investors can place their funds either in the equity of banks or in government bonds or can lend to the banks on the same terms as cash funds. If they buy the equity of a bank they receive the payoff $V(a)$ per unit of equity, where a denotes a realization of the random payoff $\widetilde{a}$, and if they lend without risk they receive R^c per unit of loan. If c_i denotes the funds placed in riskless lending by the representative investors and if e denotes the amount invested in bank equity, then the problem of an investor is to choose (c_i, e) to maximize

$$u_i(w_{i\,0} - c_i - e) + \mathbb{E}(w_{i1} - t(a) + V(a)e + R^c c_i)$$

where $t(a)$ is the tax (or subsidy) from the government at date 1. Define the expected return on equity $R^E = \mathbb{E}(V(\widetilde{a}))$ then the first-order conditions characterizing the solution of the investor's maximum problem are

$$u_i'(w_{i\,0} - c_i - e) = R^c, \text{ if } R^c = R^E \text{ or } c_i = 0 \text{ if } R^c < R^E$$
$$u_i'(w_{i\,0} - c_i - e) = R^E.$$

We do not consider the case $R^c > R^E$ for which $c_i > 0$ and $e = 0$, since banks must have positive equity in equilibrium. If $R^E = R^c$ then we assume investors only invest in equity (i.e. $c_i = 0$) and this is without loss of generality under assumptions that we will introduce shortly.

There is a unit mass of each of the three types of agents and a unit mass of banks, to which we now turn our attention.

Banks Banks collect the deposits (d), the equity (e) and a part (c_b) of the lending of cash funds (the rest finances the government) and invest the proceeds $K = d + e + c_b$ in risky projects with payoff $\widetilde{a}$ per unit of capital at date 1. The random variable $\widetilde{a}$ is as described in Section 2 with support on the interval $[\underline{a}, \infty)$ and with continuous density $f(a)$ extended to the interval $[0, \infty)$. The safe part $K\underline{a}$ of their date 1 payoff can be interpreted as the safe component which can be pledged as collateral for borrowing from cash funds (akin to the senior tranche of the banks securitized assets). Cash funds will lend an amount in excess of this sure component i.e. $R^c c_b > K\underline{a}$ only if they are sure to recover their funds. In this section we consider two possibilities, with or without *implicit* insurance for the cash funds. If there is implicit insurance, cash funds believe that the government will reimburse their loans if the banks default and this belief is realized. In this case they may accept to lend more than the value of the safe collateral $K\underline{a}$, i.e. they accept risky collateral. We call the insurance "implicit" because there is no explicit contract or insurance premium attached to it. An example of implicit insurance is the belief that the government will bailout too-big-to-fail banks if they are in difficulty. However if the government makes it credible that it will not intervene in case of banks' default, then there is no implicit insurance, and the cash funds will not lend to the banks more than the sure collateral $K\underline{a}$. As for the depositors, we assume that they are explicitly insured (FDIC)[6]. In addition banks provide payment services to depositors which cost them μ per unit of spending by a depositor at date 1. To recoup some of the cost incurred by the taxpayers to pay the banks' debts in case of bankruptcy, the government charges an insurance premium π per unit of debt at date 1.[7]

Introducing cash funds into the model serves to capture the change in banking from traditional banking based on deposits to modern banking based on securitization of assets and collateralized borrowing on the wholesale money market. Such loans are safe as long as the collateral retains its value: when collateral is at risk of losing value (low return on bank assets) the Central Bank may intervene on the security markets to enhance their value (and liquidity) and avoid runs on the wholesale money market. Such interventions, and sometimes more direct interventions by the Treasury in times of crisis, are what justify the assumption of implicit insurance of the loans by

[6]Large uninsured deposits enter as "implicitly" insured cash funds loans since these loans are unsecured and uninsured. The implicit insurance of such deposits was made explicit after the financial crisis when temporarily (up to December 2012) all the non-interest-bearing accounts of banks were insured for an unlimited amount.

[7]If FDIC were a standard insurance company the premium would only be charged on the value of the insured deposits: in the model we follow the practice in the US, by which the FDIC premium is charged on *all* the bank's debt.

cash funds in our model.

Banks are required to hold at least a minimal level of equity $E \geq \overline{\alpha} K$ where $0 < \overline{\alpha} \leq 1$: this is in line with current regulation (Basel accords), and as we saw in Section 2, is needed to have a solution to the bank's maximum problem when the interest rate charged on its loans does not adjust to the riskiness of its portfolio (E, α).

Let R^d denote the return promised on deposits, R^c the rate on cash funds and R^E the required rate of return on equity. The bank acts in the best interests of its shareholders, the investors, and chooses (d, c_b, E, K) to maximize

$$\int_{\hat{a}}^{\infty} (Ka - \mu R^d d - (1+\pi)(R^d d + R^c c_b)) f(a) da - R^E E$$

under the constraints $d \geq 0,\ c_b \geq 0,\ E \geq 0,$

$$K = d + c_b + E, \quad K\hat{a} = \mu R^d d + (1+\pi)(R^d d + R^c c_b],\ E \geq \overline{\alpha}\, K.$$

Since deposits and cash funds are perfect substitutes for investment, both source of funds will be used only if they have the same cost

$$(1+\mu+\pi) R^d = (1+\pi) R^c.$$

If we let $D = d + c_b$ and $R = (1+\pi)R^c = (1+\mu+\pi)R^d$ then the bank's problem is the problem studied in Section 2. From Proposition 2 the profit of the bank is zero, the scale of its investment is indeterminate and the bank chooses $\alpha = \overline{\alpha}$, unless both the interest rate R and the equity requirement $\overline{\alpha}$ are too high ($R > \mathbb{E}(\tilde{a}), \overline{\alpha} > \hat{\alpha}(R)$), in which case it chooses $\alpha = 1$ (see Figure 1(iii)). Since this latter case is incompatible with equilibrium, we only consider the case where $\alpha = \overline{\alpha}$.

The return to an equity holder is the random variable $V(a)$ defined by

$$V(a) = \begin{cases} \frac{K}{E}(a - \hat{a}), & \text{if } a \geq \hat{a}, \\ 0, & \text{if } a \leq \hat{a}. \end{cases} \quad \text{where } \hat{a} = (1-\overline{\alpha})R \tag{6}$$

Government In broad terms the government in our model combines the role a fiscal authority which finances government expenditures and the role of a Central Bank (backed by the fiscal authority) which conducts monetary and macro-prudential policies. The government is assumed to have exogenously given expenditure (G) that the fiscal authority finances by issuing bonds B at date 0 for the value $B = G$, imposing taxes on investors at date 1 to pay for the government debt. It also insures banks' deposits, imposing an insurance premium π per dollar of debt due due at date 1, and reimburses depositors when banks go bankrupt. The insurance premium decreases the taxes that taxpayers have to pay when banks are not bankrupt, while reimbursing the depositors in low realizations of $\tilde{a}$ requires additional taxes. The Central Bank fixes the interest rate R^B on government bonds and the minimum equity requirement $\overline{\alpha}$ for banks. If the cash funds are reimbursed at date 1 when banks are bankrupt, taxes are increased to cover the cost. The taxes imposed at date 1 in outcome a are thus given by

$$t(a) = \begin{cases} R^B B - \pi(R^d d + R^c c_b), & \text{if } a \geq \hat{a}, \\ R^B B + (1+\mu)R^d d + R^c c_b - (1-\gamma)Ka, & \text{if } a \leq \hat{a}. \end{cases} \tag{7}$$

where $1-\gamma$ is the recovery rate on output when there is bankruptcy. We could introduce a separate group of agents who pay taxes (when $t(a) > 0$) or receive payments (when $t(a) < 0$). However it is simpler to assume directly that these taxes are paid by the investors who have sufficient resources $w_{i1} > 0$ to pay for them at date 1.

Assumptions We introduce assumptions on agents' endowments which ensure that there exist equilibria with positive debt and equity for banks. In this model with constant returns in technology and linear date 1 preferences, there is a *natural rate of interest* $R^* = \mathbb{E}(\tilde{a})$ determined by the technology which is the expected return at date 1 from a one unit investment of the good at date 0. This is the benchmark interest rate that we use to express the willingness of agents to supply debt and equity in the economy.

Assumption 1. (a) $u_i'(w_{i0}) < \mathbb{E}(\tilde{a})$; (b) $w_{i1} > (w_{d0}(1+\mu) + w_{c0})\mathbb{E}(\tilde{a})$

Assumption (1)(a) guarantees that investors want to invest in the technology even if the profit of banks is not increased by leverage, while (b) guarantees that investors have sufficient resources at date 1 to reimburse the maximum that can be due to depositors and cash investors.

In keeping with the recent literature on shadow banking which emphasizes the magnitude of the funds on the wholesale money market seeking a safe haven, we assume that (short-term) government bonds do not absorb all funds that cash investors are willing to lend.

Assumption 2. $u_c'(w_{c0} - B) < \mathbb{E}(\tilde{a})$

Under this assumption, for all interest rates R^B such that $u_c'(w_{c0} - B) < R^B \leq \mathbb{E}(\tilde{a})$, the cash investors want to lend an amount which exceeds the supply of (short-term) government bonds B.

Deposits differ from government bonds by the payment services they offer, modeled by the convenience yield $\rho R^d d$. To ensure that in equilibrium deposits are positive and preferred by depositors to government bonds we assume

Assumption 3. (a) $u_d'(w_{d0}) < u_c'(w_{c0} - B)$; (b) $\rho > \mu$.

Assumption 3(a) ensures that for interest rates R^B such that cash funds absorb the government bonds B, depositors would want to buy government bonds if they did not have any other choice. Assumption 3(b) ensures that for such interest rates depositors prefer to place their funds as deposits with banks.

Finally we restrict our attention to equilibria such that

$$u_c'(w_{c0} - B) \leq R^B \leq \mathbb{E}(\tilde{a}) \tag{8}$$

that is, we are interested in "low interest rate" equilibria where government bonds do not offer a rate of return in excess of the expected rate of return in production.

Banking Equilibrium For this economy a *banking equilibrium* consists of interest rates (R^B, R^c, R^d), equity requirement $\overline{\alpha}$, rate of return on equity R^E, deposit insurance rate π, choices $(d, c, e, E, D, K, \hat{a})$, and taxes t such that

(i) $R^B = R^c$ (cash investors are indifferent between government bonds and lending to banks);

(ii) $R^d = \frac{R^c(1+\pi)}{1+\pi+\mu}$ (banks are indifferent between deposits and borrowing from cash funds);

(iii) d is optimal for depositors given R^d

(iv) c is optimal for cash investors given R^c

(v) $D = d + c - B$, E, and $K = D + E$ are optimal for the representative bank faced with interest rates (R^d, R^c), deposit insurance premium π, required rate of return on equity R^E and equity constraint $E \geq \overline{\alpha}\, K$;

(vi) $E = e$ and e is optimal for capital investors given the rate of return R^E on equity;

(vii) $x_{i1}(a) = w_{i1} - t(a) + V(a)e$ where $V(a)$ is given by (6) and $t(a)$ given by (7).

(i) reflects the fact that because of the explicit insurance given by collateral and/or the implicit insurance of the government for less secure forms of debt, buying government bonds and lending to banks are perfect substitutes for cash investors. (ii) reflects the fact that cash funds and deposits are perfect substitutes for investment by banks and thus must have the same cost. The other conditions incorporate the optimization of the depositors, cash investors, capital investors and banks given the prices that they face and the market clearing conditions. Assumptions 1-3 imply that $d > 0$, $c \geq B$, $e > 0$, and $x_{i1}(a) > 0$ for all $a \geq \underline{a}$. If a banking equilibrium is such that $(c - B)R^B > K\underline{a}$, the equilibrium exists only under the assumption that the government implicitly insures the cash funds. On the other hand if the banking equilibrium satisfies the reverse inequality, it is an equilibrium with or without implicit insurance of the cash funds.

Replacing the optimality requirements by equivalent first-order conditions and incorporating the market clearing conditions, the equations that characterize an equilibrium are

$$u'_d(w_{d\,0} - d) \quad = \quad R^B \frac{(1+\rho)(1+\pi)}{1+\pi+\mu} \tag{9}$$

$$u'_c(w_{c\,0} - c) \quad = \quad R^B \tag{10}$$

$$\frac{1}{\overline{\alpha}} \int_{\hat{a}}^{\infty} (a - \hat{a}) f(a) da \quad = \quad R^E, \quad R^E \geq \mathbb{E}(\tilde{a}), \quad \hat{a} = (1-\overline{\alpha})R, \quad R = (1+\pi)R^B \tag{11}$$

$$u'_i\Big(w_{i\,0} - \frac{\overline{\alpha}}{1-\overline{\alpha}} D\Big) \quad = \quad R^E, \quad \text{with } D = d + c - B \tag{12}$$

Under the assumptions on u_d, ρ and u_c, the equations (9) and (10) have a unique solution. Let $d(R^B, \pi)$ denote the supply of deposits when depositors are given the interest rate $\frac{R^B(1+\pi)}{1+\pi+\mu}$ (i.e. the solution to equation (9)) and let $c(R^B)$ denote the supply function of funds by the cash investors (the solution to equation(10)), then

$$D(R^B, \pi) = d(R^B, \pi) + c(R^B) - B \tag{13}$$

which denotes the total supply of debt to the banks when the interest rate is R^B and the insurance premium is π, incorporates the solutions to equations (9) and (10). As a result the equilibrium equations reduce to the zero profit condition for the bank (11) and the first-order condition for the investors (12) with $D = D(R^B, \pi)$. We can now establish two properties of banking equilibria.

(i) Banking equilibria can be parametrized by (R^B, π) i.e. the government has only two degrees of freedom. The equity requirements $\overline{\alpha}$ for banks cannot be freely chosen since there is a unique value of $\overline{\alpha}$ such that there exists a banking equilibrium compatible with the government's policy $(R^B, \pi, \overline{\alpha})$.

(ii) Banking equilibria are of three types:

  (1) equilibria in which there is never bankruptcy;

  (2) equilibria in which bankruptcy can occur and when it occurs the collateral is sufficient to pay the cash funds and only depositors need to be rescued by government insurance;

  (3) equilibria in which bankruptcy can occur and when it occurs both cash funds and depositors need to be rescued by the government.

All equilibria are inefficient except equilibria of the first type with $R^B = \mathbb{E}(\widetilde{a})$, $\pi = 0$.

Property (i) is somewhat surprising since in most discussions of banking regulation it is implicitly assumed that the regulator can choose equity requirements as high as desired. As for Property (ii) we argue below that in the current environment it is vey unlikely that $R^B = \mathbb{E}(\widetilde{a})$, $\pi = 0$ leads to an equilibrium without bankruptcy. Thus with the standard institutional setting that we have just discussed, it is not possible to obtain an efficient banking equilibrium.

Given the exogenous choice of policy (R^B, π, α), equations (9)-(12) express the conditions of compatibility that must be satisfied to obtain a banking equilibrium. Equations (9) and (10) give the net supply of debt $D(R^B, \pi) = d(R^B, \pi) + c(R^B) - B$ made available by depositors and cash investors, given that the latter have already invested a portion of their funds in government debt B. To have an equilibrium it must be optimal for the banks to use this supply of debt, while respecting their equity requirement α (or the equity-debt ratio $\frac{\alpha}{1-\alpha}$). Thus investors must want to supply the equity $e = \frac{\alpha}{1-\alpha} D(R^B, \pi)$ to the banks. Let

$$s(\alpha, D) = u_i'\Big(w_{i\,0} - \frac{\alpha}{1-\alpha} D\Big)$$

denote the return on equity (supply price) required by investors to supply the equity $\frac{\alpha}{1-\alpha}D$: thus $s(\alpha, D(R^B, \pi))$ is the return they require to supply the equity $\frac{\alpha}{1-\alpha}D(R^B, \pi)$ which complements the funds provided by depositors and cash investors. In equilibrium this return must be equal to the return on equity $\Phi(\alpha; R)$ of banks when faced with the equity requirement α and cost of debt $R = R^B(1+\pi)$. Equations (11) and (12) require that these two rates of return be the same

$$s(\alpha, D(R^B, \pi)) = \Phi(\alpha; R^B(1+\pi)) \geq \mathbb{E}(\widetilde{a}) \tag{14}$$

where $\Phi \geq \mathbb{E}(\widetilde{a})$ must hold for the banks' decision to be optimal. We study the existence of equilibrium taking (R^B, π) as fixed and seek an equilibrium requirement α satisfying (14). For a solution to (14) to exist the insurance premium π changed by the Central Bank must not be "too high". For a high cost $R = R^B(1+\pi)$ for the banks causes their return $\Phi(\alpha; R)$ to fall so that (14) only holds for a value $\overline{\alpha}$ for which $\Phi(\alpha; R) < \mathbb{E}(\widetilde{a})$. To define more precisely what "not too high" means, let $\alpha^{\min} = \alpha^{\min}(D)$ be the value of α satisfying

$$u_i'\Big(w_{i\,0} - \frac{\alpha^{\min}}{1-\alpha^{\min}} D\Big) = \mathbb{E}(\widetilde{a}) \tag{15}$$

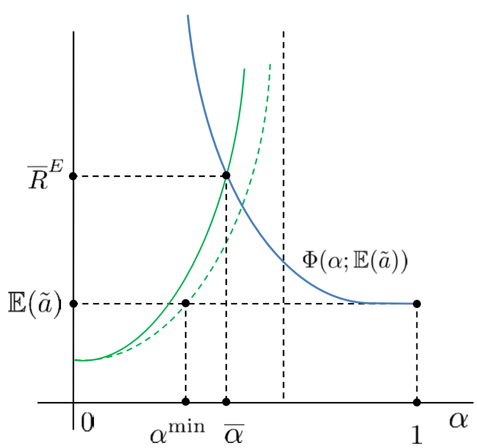
Definition. *Given $R^B \leq \mathbb{E}(\widetilde{a})$, we say that the insurance premium π is "not-too-high" for R^B if* $\alpha = \alpha^{min}(D(R^B, 0)) \quad \Rightarrow \quad \Phi(\alpha; R^B(1+\pi)) \geq \mathbb{E}(\widetilde{a})$.

It is easy to check that given R^B the set of not-too-high π is an interval which is never empty since it contains $\pi = 0$ and $R^B < \mathbb{E}(\widetilde{a})$ is non degenerate. For some economies (which we later call "high-equity economies") the interval may reduce to 0 for $R^B = \mathbb{E}(\tilde{a})$ but in this case bankruptcy never occurs and $\pi = 0$ is the natural choice of insurance premium. We can now give the following result which gives conditions under which banking equilibria exist.

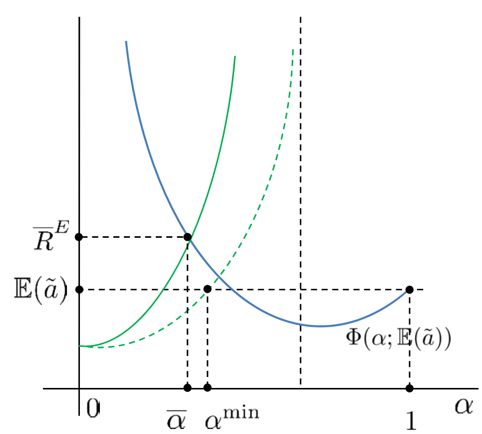
Proposition 3. *Let Assumptions 1–3 hold and let R^B be in the interval (8). If the insurance premium π is not-too-high given R^B then there is a unique equity requirement $\overline{\alpha}$ such that there is a banking equilibrium associated with the policy $(R^B, \pi; \overline{\alpha})$.*

Proof: (see Appendix)

The intuition underlying the proof of Proposition 3 can be understood geometrically using Figures 2(a) and 2(b). Recall the investors supply price (return) curve $s(\alpha, D(R^B, \pi)) = u_i'\Big(w_{i\,0} - \dfrac{\alpha}{1-\alpha} D(R^B, \pi)\Big)$. Since the interest rate $R^d = \frac{1+\pi}{1+\pi+\mu}$ offered to depositors is increasing in π, $D(R^B, \pi) > D(R^B, 0)$: thus $s(\alpha, D(R^B, \pi)) > s(\alpha, D(R^B, 0))$. For a given interest rate R^B, the same two supply curves corresponding to $\pi > 0$ and $\pi = 0$ are shown in both Figures. Figure 2(a) shows the bank's return $\Phi(\alpha; R)$ for $R = R^B(1+\pi) \leq \mathbb{E}(\tilde{a})$, while 2(b) shows $\Phi(\alpha; R)$ for $R = R^B(1+\pi) > \mathbb{E}(\tilde{a})$ but with π being not-too-high so that $\Phi(\alpha^{\min}(D(R^B, 0)); R) > \mathbb{E}(\tilde{a})$: as a result at the intersection $s(\overline{\alpha}, D(R^B, \pi)) = \Phi(\overline{\alpha}; R) \geq \mathbb{E}(\tilde{a})$.



(a) $R \leq \mathbb{E}(\tilde{a})$



(b) $R > \mathbb{E}(\tilde{a})$

Figure 2: Banking equilibrium

In practice the insurance premium charged by the FDIC is small so the condition that π is not-too-high is likely to be a reasonable assumption as a model of the current US banking system.

Types of banking equilibria The three types of banking equilibria identified above can now be characterized as follows, and serve to explain why cash funds are willing to lend to the banks at the same interest rate R^B that they lend to the government, despite the fact that the banks can end up in bankruptcy:

Equilibria of type 1: $(1-\overline{\alpha})R^B(1+\pi) \leq \underline{a} \iff \hat{a} \leq \underline{a}$.
For this type of equilibrium there is never bankruptcy: the supply of savings by cash investors and depositors is sufficiently low at the interest rate R^B for the sure part $K\underline{a}$ of the payoff of the bank to cover the requisite reimbursement to cash investors and depositors at date 1. Banks can finance their debt at the rate R^B since their debt is sure. In this case it is natural that $\pi = 0$.

Equilibria of type 2: $(1-\overline{\alpha})R^B(1+\pi) > \underline{a}$ ($\iff \hat{a} > \underline{a}$) and $R^B c_b(R^B) \leq K\underline{a}$.
For this type of equilibrium there is bankruptcy for low realizations of $\widetilde{a}$, but there is enough sure collateral to insure the cash investors, provided their debt has priority over deposits in case of bankruptcy: this will be the case if cash investors lend to banks through repo markets. Then the cash investors will be willing to lend at the rate R^B. Only depositors need to be reimbursed in case of a bad realization of $\tilde{a}$, and the insurance premium serves to reduce the expected cost of the rescue to the taxpayer.

Equilibria of type 3: $R^B c_b(R^B) > K\underline{a}$.
For this type of equilibrium there is bankruptcy (when $a < \hat{a}$) and the government pays back both depositors and the unsecured component of the investments of cash investors. The cash investors are willing to lend to the banks at the interest rate R^B provided they feel confident that their funds will be reimbursed either directly via the collateral or indirectly via the implicit insurance of the government. Such equilibria are however "fragile" in that they depend on the lenders trust in the implicit insurance and for this reason in practice they are equilibria which can be subject to runs. Instead of modeling runs explicitly we assume that the bankruptcy cost (γ) associated with type 3 equilibria is higher than the cost associated with equilibria of type 2, for which depositors have insurance and thus have no reason to run.

For an economy with fixed characteristics, the type of equilibrium which prevails depends on the interest rate R^B. For the same economy, lowering the interest rate may (depending on the magnitude of the change) shift the equilibrium to an equilibrium of lower type. In particular for an economy satisfying Assumptions 1-3, an interest rate sufficiently close to the low end of the interval (8) generates an equilibrium of type 1 or 2.

Since during the financial crisis, the US government (CB and Treasury) had to intervene to prevent banks (at least the largest ones) from defaulting on their debts to the wholesale money market, we may interpret the situation prevailing at that time as a type-three equilibrium where part of the wholesale money-market lending was unsecured. This type of equilibrium is justified in the model by the assumption that there is implicit insurance by the government: this assumption is an abstract way of modeling a variety of explanations which have been proposed to explain why banks were able to borrow so much on the wholesale money market—either that they were believed to be "too-big-to fail", or that lenders had become lulled into a false sense of security so that debt was "information insensitive" (Gorton-Metrick (2010, 2012), or simply that many lenders did not understand the magnitude of the risks to which banks were exposed (Gennaioli-Shleifer-Vishny (2012)). What is important for our analysis is that at the interest rate R^B cash investors want to invest more than what can be absorbed by government bonds (B) and the safe debt of banks ($K\underline{a}$).

Many of the recent proposals for improving the safety of the banking system involve regulating the terms on which banks can incur debt and can be interpreted in this model as ways of moving from a type-three equilibrium where the costs of bankruptcy are high to a type-two equilibrium where they are lower: regulation of the repo markets to ensure that collateral is safe and that the haircuts are sufficient; preventing banks from using short-term unsecured debt, forcing them instead to draw on equity or corporate bonds, or bonds convertible to equity when banks have difficulty paying. Such a regulatory framework can be captured in our model by assuming that the government makes no implicit insurance so that cash investors will limit their investments in banks to an amount which is secured by fully safe collateral, $R^B c_b(R^B) \leq K\underline{a}$. If the characteristics of the economy are such that with implicit insurance the equilibrium is of type three then the interest rate R^B (chosen by the CB) will need to decrease to support a type-two equilibrium in which cash investors reduce their lending to banks. We show below that regulations which restrict the possible equilibria to type-two equilibria, although they reduce the probability of bankruptcy, making the banking system safer and reducing the cost of bankruptcy to the taxpayer, do not lead to efficient equilibria. All three types of equilibria are inefficient, except the special case where the equilibrium associated with the policy $(R^B, \pi) = (\mathbb{E}(\widetilde{a}), 0)$ is of type 1, but this requires that the supply of safe debt and deposits be very low in a way that we make precise below.

Pareto optimal allocations. To study the normative properties of a banking equilibrium, we examine the first-order conditions for Pareto optimality and compare them with the FOCs satisfied at an equilibrium. An interior Pareto optimal allocation consists of consumption streams and investment

$$(x_{d\,0}, x_{d1}, x_{c\,0}, x_{c1}, x_{i\,0}, (x_{i1}(a))_{a\in A}, K) \gg 0$$

which maximize social welfare

$$\beta_d[u_d(x_{d\,0}) + x_{d1} + \rho\, x_{d1}] + \beta_c[u_c(x_{c\,0}) + x_{c1}] + \beta_i[u_i(x_{i\,0}) + \int_0^\infty x_{i1}(a) f(a) da]$$

subject to the date 0 and date 1 resource constraints

$$\begin{aligned} &x_{d\,0} + x_{c\,0} + x_{i\,0} + K + G = w_0 \equiv w_{d\,0} + w_{c\,0} + w_{i\,0} \\ &(1+\mu)x_{d1} + x_{c1} + x_{i1}(a) = w_{i1} + Ka, \quad a \in A \end{aligned} \tag{16}$$

where $(\beta_d, \beta_c, \beta_i) \gg 0$ are the relative weights of the agents. We have incorporated into the description of the allocation the property that the date 1 consumption streams of depositors and cash investors must be non-random because of their infinite risk aversion. The necessary and sufficient conditions for an interior Pareto optimum are given by

$$\frac{1+\mu}{1+\rho}\, u'_d(x_{d\,0}) = u'_c(x_{c\,0}) = u'_i(x_{i\,0}) = \mathbb{E}(\widetilde{a}). \tag{17}$$

and the resource constraints (16).

Natural Rate of Return, Natural and Critical Equity Ratios Suppose we attempt to decentralize the Pareto optimal allocation defined by (17), as a banking equilibrium. Then we must choose $(R^B, \pi) = (\mathbb{E}(\widetilde{a}), 0)$ and the optimal deposit of the depositors is characterized by

$$u'_d(w_{d\,0} - d^*) = \mathbb{E}(\widetilde{a}) \frac{1+\rho}{1+\mu} \quad \Leftrightarrow \quad d^* = d(\mathbb{E}(\widetilde{a}), 0)$$

where $d(\mathbb{E}(\widetilde{a}), 0)$ denotes the supply function of depositors. The optimal lending c^* of cash investors is given by $u'_c(w_{c\,0} - c^*) = \mathbb{E}(\widetilde{a}) \;\Leftrightarrow\; c^* = c(\mathbb{E}(\widetilde{a}))$, so that the optimal supply of debt is $D^* = d(\mathbb{E}(\widetilde{a}), 0) + c(\mathbb{E}(\widetilde{a})) - B$. In the same way the optimal supply of equity e^* is given by $u'_i(w_{i\,0} - e^*) = \mathbb{E}(\widetilde{a}) \;\Leftrightarrow\; e^* = e(\mathbb{E}(\widetilde{a}))$: note that since investors are risk neutral there is no risk premium in the Pareto optimal allocation.

We call $\mathbb{E}(\widetilde{a})$ the *natural rate of return* of the economy since it is the expected return on investment and by (17) is the rate that must be earned by all agents contributing to the financing of investment at the Pareto optimal allocation. We call the proportion of the funds contributed by the investors

$$\alpha^* = \frac{e^*}{e^* + D^*} = \frac{e(\mathbb{E}(\widetilde{a}))}{e(\mathbb{E}(\widetilde{a})) + D(\mathbb{E}(\widetilde{a}), 0)}$$

the *natural equity ratio* of the economy; in the same way we call $1 - \alpha^* = \frac{D^*}{e^* + D^*}$ the *natural debt ratio*. The proportions $(\alpha^*, 1 - \alpha^*)$ of equity and debt supplied at the natural rate $\mathbb{E}(\widetilde{a})$ depend on the endowment and preference characteristics of the agents.

On the other hand the technology defines the minimum equity ratio α_c (or the maximum debt ratio $1-\alpha_c$) above which (below which) there is never bankruptcy: in order that banks can pay their debt in all circumstances when the interest rate is $R^B = \mathbb{E}(\widetilde{a})$ we must have $K\underline{a} \geq (1-\alpha)K\mathbb{E}(\widetilde{a}) \Leftrightarrow \underline{a} \geq (1-\alpha)\mathbb{E}(\widetilde{a})$. Thus there is a critical equity ratio

$$\widehat{\alpha}_c = 1 - \frac{\underline{a}}{\mathbb{E}(\widetilde{a})} \tag{18}$$

with the property that if the equity ratio α exceeds $\widehat{\alpha}_c$ ($\alpha \geq \widehat{\alpha}_c$) there is never bankruptcy.[8] $\widehat{\alpha}_c$ depends only on the characteristics of the banks' random return $\widetilde{a}$ and can be considered as a normalized measure of the downside risk of $\widetilde{a}$: it satisfies $0 \leq \widehat{\alpha}_c \leq 1$; $\widehat{\alpha}_c = 0 \Leftrightarrow \underline{a} = \mathbb{E}(\widetilde{a})$ corresponds to zero risk and $\widehat{\alpha}_c = 1 \Leftrightarrow \underline{a} = 0$ corresponds to maximum risk. Let us show that whether or not a banking equilibrium is Pareto optimal depends on the relation between α^* and $\widehat{\alpha}_c$.

Banking Equilibria and Pareto Optimality If a banking equilibrium is to be Pareto optimal then, by (17), the interest rate and insurance premium must satisfy $(R^B, \pi) = (\mathbb{E}(\widetilde{a}), 0)$. The properties of the return on equity $\Phi(\alpha; \mathbb{E}(\widetilde{a}))$ were studied in Section 2, where it was shown that if $0 < \alpha < \widehat{\alpha}_c$ then $\Phi(\alpha; \mathbb{E}(\widetilde{a})) > \mathbb{E}(\widetilde{a})$, and if $\widehat{\alpha}_c \leq \alpha \leq 1$ then $\Phi(\alpha; \mathbb{E}(\widetilde{a})) = \mathbb{E}(\widetilde{a})$: the notation $\widehat{\alpha}$ was used instead of $\widehat{\alpha}_c$, but the definition is the same, namely $(1-\widehat{\alpha}_c)\mathbb{E}(\widetilde{a}) = \underline{a}$.

The set of all economies $\mathcal{E}$ fall into two categories: those for which the preference-endowment-risk characteristics are such that $\alpha^* < \widehat{\alpha}_c$ and those for which $\alpha^* \geq \widehat{\alpha}_c$: we call them *high debt* and *high equity economies* respectively.

- *High debt economies:* $\alpha^* < \widehat{\alpha}_c$. For these economies the characteristics are such that the supply of equity is (relatively) small, the supply of debt is (relatively) large and the risk in the technology is (relatively) high. We argue below that the current high demand for safe debt makes this the relevant case for modeling the current situation (in the US): thus for brevity we attribute the inequality $\alpha^* < \widehat{\alpha}_c$ to the large supply of debt $(1-\alpha^* > 1-\widehat{\alpha}_c)$.

- *High equity economies:* $\alpha^* \geq \widehat{\alpha}_c$. By contrast for these economies it is useful to attribute the inequality to a relatively high supply of equity.

Proposition 4. (i) *In a high debt economy, no banking equilibrium is Pareto optimal.* (ii) *In a high equity economy the policy* $(R^B, \pi, \alpha) = (\mathbb{E}(\widetilde{a}), 0, \alpha^*)$ *leads to a Pareto optimal banking equilibrium.*

[8] If banks make a large number of loans, there is a Law of Large Numbers at work for idiosyncratic risks, so that the risks in $\tilde{a}$ can thought of as aggregate risks. If, as a rough back-of-the-envelope calculation, we assume that one dollar invested cannot lose more than 20% ($\underline{a} = .8$) and the expected return on investment is 3% ($\mathbb{E}(\widetilde{a}) = 1.03$), then the critical equity requirement $\hat{\alpha}_c$ above which there is no bankruptcy is 22%. This is in the ball park of the Total Loss Absorbing Capacity recently proposed by the Federal Reserve for large (GSIB) banks.

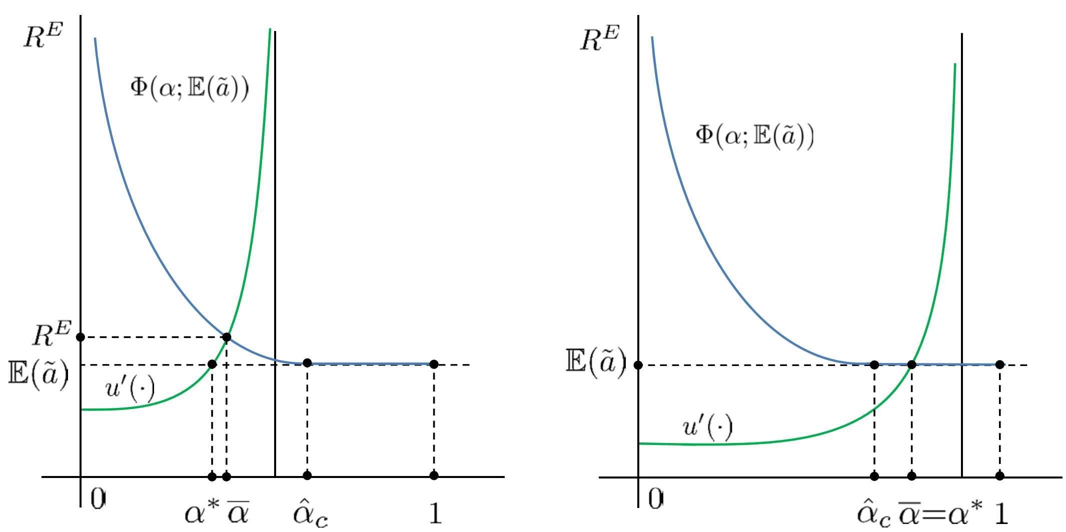
Proof. As we noted above a banking equilibrium requires $(R^B, \pi) = (\mathbb{E}(\widetilde{a}), 0)$. In this case the supply of debt is $D^* = D(\mathbb{E}(\widetilde{a}), 0)$ and it follows from the definition of α^* that $s(\alpha^*, D^*) = \mathbb{E}(\widetilde{a})$ i.e. $\alpha = \alpha^*$ is the value of α for which the curve $s(\alpha, D^*) = u_i'(w_{i\,0} - \frac{\alpha}{1-\alpha} D^*)$ intersects the line $R^E = \mathbb{E}(\widetilde{a})$. The equilibrium equity requirement $\overline{\alpha}$ associated with the policy $(R^B, \pi) = (\mathbb{E}(\widetilde{a}), 0)$ is given by the intersection of the curve $s(\alpha, D^*)$ and the curve $\Phi(\alpha; \mathbb{E}(\widetilde{a}))$. The equilibrium is shown for the case $\alpha^* < \widehat{\alpha}_c$ in Figure 3(a) and for the case $\alpha^* \geq \widehat{\alpha}_c$ in Figure 3(b).

(i) When $\alpha^* < \widehat{\alpha}_c$ the two curves meet for $\overline{\alpha}$ such that $\alpha^* < \overline{\alpha} < \widehat{\alpha}_c$ since $\mathbb{E}(\widetilde{a}) = s(\alpha^*, D^*) < \Phi(\alpha^*; \mathbb{E}(\widetilde{a}))$ and $s(\widehat{\alpha}_c, D^*) > \Phi(\widehat{\alpha}_c; \mathbb{E}(\widetilde{a})) = \mathbb{E}(\widetilde{a})$ (where $s(\widehat{\alpha}_c, D^*)$ may be infinite). Thus the equilibrium associated with the policy $(R^B, \pi, \alpha) = (\mathbb{E}(\widetilde{a}), 0, \overline{\alpha})$ is not Pareto optimal since

- $u_i'(x_{i\,0}) = u_i'(w_{i\,0} - \frac{\overline{\alpha}}{1-\overline{\alpha}} D^*) > \mathbb{E}(\widetilde{a})$ so (17) does not hold
- $\overline{\alpha} < \widehat{\alpha}_c \;\Rightarrow\; \widehat{a} > \underline{a}$ so bankruptcy can occur i.e. there is a loss $\gamma K a$ whenever $a < \widehat{a}$.

(ii) When $\alpha^* \geq \widehat{\alpha}_c$, $s(\alpha^*, D^*) = \mathbb{E}(\widetilde{a}) = \Phi(\alpha^*; \mathbb{E}(\widetilde{a}))$: thus the policy $(\mathbb{E}(\widetilde{a}), 0, \alpha^*)$ leads to a Pareto optimal banking equilibrium

□



(a) High debt economy $\alpha^* < \widehat{\alpha}_c$.

(b) High equity economy $\alpha^* \geq \widehat{\alpha}_c$.

Figure 3: Banking equilibrium with policy $(R^B, \pi) = (\mathbb{E}(\widetilde{a}), 0)$.

If we view our model of banking equilibrium as an abstract and stylized representation of the current banking system in the US, then it should be clear that the relevant case is where $\alpha^* < \widehat{\alpha}_c$: if $\alpha^* \geq \widehat{\alpha}_c$ then only equilibria without bankruptcy (i.e. of type 1) can occur. Furthermore if the supply of debt and deposits had been small, and most of the banking system had been financed by equity, then there would not have been a banking crisis in 2008. A number of recent papers have highlighted the importance of the “safe asset phenomenon": Pozsar (2012), (2014)) stresses the importance of the fact that money market funds or more generally cash pools have very substantial amounts of money that they seek to lend safely and in liquid form.[9] In practice this means that in addition to buying short-term government bonds, the cash pools lend to large institutions for short periods, often with collateral—and that the supply of these funds inevitably encourages high leverage by banks, shadow banks and investment funds.

Since the financial crisis much of the focus of bank regulation has been on increasing the safety of the financial system because of the high perceived costs of the crisis in terms of lost output.The current trend in regulation is to require that a large part of the financing of banks to come from equity and long-term risky corporate bonds. Such regulation, and in addition the regulation of the repo markets, does not however take into account that the buyers of equity and long-term bonds are distinct from the suppliers of funds on the wholesale money market. In our model the first are the risk-neutral investors, while the second are the infinitely risk-averse cash funds. Mandating that the financing of the banking sector comes from deposits, safely collateralized short-term debt, long-term risky bonds and equity will take the economy from the current type-three equilibrium, where the taxpayer has to rescue both depositors and cash funds in the case of bad outcomes, to a type-two equilibrium where only depositors need to be rescued. The economy will indeed be safer, but the interest rate will have to be very low, to induce cash funds to cut back on their supply of debt. Moreover the investment will have to be low, since a low leverage implies a low return on equity, which in turn implies that investors will only supply a small amount of equity (and/or buy a small amount of risky corporate bonds).

Another approach mentioned among others by (Pozsar (2014)) consists in increasing the supply of short-term government bills by tilting the maturity structure of government debt towards the short end. In our model this would mean considering economies with larger values of the (short-term) government debt B. However given the magnitudes involved[10], it does not seem realistic that

[9]Gorton-Metrick (2010) and Gennaioli-Schleifer-Vishny (2012) argue that the high demand for a safe asset in large part serves to explain the emergence of the shadow banking system. In an international context Caballero-Fahri-Gourinchas (2015) argue that funds seeking a safe haven serve to explain the movement of international global imbalances over the last 30 years and the low interest rates of the last ten years.

[10]Pozsar (2014) estimates the amount that institutional cash investors placed in safe short-term liquid instruments in 2013 as approximately 6 trillion, while the amount of Treasury Bills outstanding was 1.6 trillion.

short-term government debt could absorb all the funds in the wholesale money market. Under the current institutional framework there is thus an unavoidable trade-off between safety and efficiency.

In the next two sections we propose two alternative ways of broadening the array of policy instruments available to the Central Bank, in ways that make it possible to improve on the banking equilibria of this section for economies with high demand for a safe asset.

4 Asset Purchases and Interest on Reserves

In an economy with high demand for a safe and liquid form of debt there are two difficulties that need to be resolved to improve on the standard banking equilibrium.

- Lower the amount of short-term debt used by the banks to a safe level without excessively reducing the short-term interest rate.
- Increase the amount of equity that investors provide (to replace the reduced debt) despite the fact that lower leverage inevitably implies a lower return on equity.

The first system that we propose solves these two problems by having the Central Bank use the surplus of funds provided by depositors and cash funds to increase the supply of equity to the banks: this is achieved by using these funds to purchases the risky securities that would otherwise be bought by the investors. We call this system Asset Purchases–Interest on Reserves, or more briefly the APIR system. We show that in our simple model such a system can lead to an efficient equilibrium. We also discuss why first best efficiency may not be attainable in an economy with a richer financial structure than in our stylized model. What is of special interest in the APIR system is its essential simplicity and the fact that it calls for the combined use of two policy instruments both of which are currently available to the Central Bank—asset purchases and the payment of interest on reserves.

In this section we introduce three changes in the banking model of the previous section. First, the Central Bank accepts whatever funds (deposits) the banks wish to place as reserves with the Central Bank on which they are paid the interest rate R^r. Second, the Central Bank makes use of these reserves to purchase risky securities from the private sector (the investors) using the dividends from the securities to pay interest on the reserves. Third, we assume that no insurance premium is charged on the banks debt ($\pi = 0$): this is because we are interested in showing that there is a policy which can lead to an equilibrium in which there is never bankruptcy, in which case $\pi = 0$ is natural. We also know that $\pi = 0$ is necessary if the FOC for Pareto optimality is to be satisfied by depositors. Finally, we want other equilibria in which banks only want to use a portion of the debt supplied by cash funds and want to pass the rest through placing it as reserves at the Central Bank: if $\pi > 0$ the bank will be forced to pay the insurance premium on any such funds and will not want to act as the pass-through vehicle for the cash founds.[11] Thus it will be simplest for the equilibrium analysis if we directly assume $\pi = 0$.

Let us see how the above modifications alter the decisions made by the different actors in the model.

Depositors and cash funds Here there is essentially no change. Depositors continue to place deposits in the banks benefitting from the convenience yield of the payment-system they offer. Cash funds buy government bonds and lend the rest of their funds to banks. If their supply of funds at the interest rate R^B exceeds B then they must be indifferent between lending to banks or buying bonds so that in this case equilibrium requires $R^c = R^B$ and for banks to accept deposits we must have $R^d = \frac{R^B}{1+\mu}$.

Banks Banks choose debt D, equity E and the amount of reserves M to place at the Central Bank, investing $K = D + E - M$ in risky projects. They take the cost of debt R^B, the interest rate R^r on reserves and the cost of equity R^E as given. The payoff per unit of bank equity is

$$V(a) = \begin{cases} \frac{Ka - R^B D + R^r M}{E}, & \text{if } a \geq \hat{a}, \\ 0, & \text{if } a \leq \hat{a}, \end{cases}$$

where $\hat{a}$ is the bankruptcy threshold defined by

$$K\hat{a} + R^r M = R^B D.$$

Banks maximize the expected payoff to shareholders net of the cost of equity, under the equity requirement $E \geq \overline{\alpha} K$. If $R^r < R^B$ they choose $M = 0$; if $R^r > R^B$ they choose $K = 0$, $D = M = \infty$ and there cannot be an equilibrium. If $R^r = R^B$, banks are indifferent between all combinations (D, M) given the same value to $D = \tilde{D} - M$. $R^r = R^B$ is the only case compatible with an equilibrium with positive reserves. When $R^r = R^B$, the problem of choosing $(\tilde{D}, E, K)$ for a bank is exactly the same as that studied in Section 2 with $R = R^B$.

Government As before the Treasury finances government expenditure by borrowing with the interest R^B chosen by the Central Bank. In addition the Central Bank accepts deposits of banks

[11] The Federal Reserve was permitted by Congress in 2008 to pay interest on reserves to enable the Fed to put a floor under the short-term interest rate. The excess supply of short-term funds at that time had made it difficult to raise and/or control the short-term interest rate. Only banks however are permitted to earn interest on reserves and banks are unwilling to act as pass-through for wholesale funds (since $\pi > 0$) (see Williamson (2015)). As a result the Central Bank resorted to the device of reverse repo to enable (registered) money market funds to lend (up to 3 tr\$) to the Central Bank to more effectively put a floor on the short rate—i.e. to prevent the demand by MMFs for short term government bonds from driving the short rate to become negative.

as reserves on which it pays the interest rate $R^r = R^B$. The reserves M are then used to buy risky securities from the private sector (the investors). In our stylized model the only risky securities are those issued by banks—either equity or risky bonds with the same expected return as equity and no payoff in case of bankruptcy, which are thus equivalent to equity. The Central Bank uses M to buy some of the holdings of these securities by investors and receives the payoff $V(a)M$ at date 1. This is used to pay back the reserves with interest, any surplus going to the Treasury to reduce taxes. However when $V(a)M = 0$ and the Central bank needs to pay back reserves with interest, taxes are used the finance the Central Bank. The taxes needed to balance the government budget are

$$t(a) = \begin{cases} R^B B - V(a)M + MR^B, & \text{if } a \geq \hat{a}, \\ R^B B - (Ka - \tilde{D}R^B) + MR^B, & \text{if } a < \hat{a}, \end{cases}$$

that is, we maintain the assumption that depositors and cash funds are insured by the government either explicitly (depositors) or by some collateral or implicitly (cash funds). The government continues (as in the previous section) to impose a minimum equity-capital ratio $\overline{\alpha}$.

Investor Investors choose to invest an amount e in the risky securities issued by the banks and then to sell an amount M to the Central Bank. Thus investors choose (e, M) to maximize

$$u_i(w_{i\,0} - e + M) + (e - M)\mathbb{E}(V(\tilde{a}))$$

which only depends on the difference $\tilde{e} = e - M$; thus investors are indifferent among all combinations (e, M) which give the same value to $e - M$.

APIR Equilibrium In a APIR equilibrium depositors, cash funds and investors maximize their utilities and bands maximize expected profit: all choices are compatible. The equations of APIR equilibrium are

$$\begin{aligned} u_d'(w_{d\,0} - d) &= \frac{R^B}{1+\mu} \\ u_c'(w_{c\,0} - c) &= R^B \\ \frac{1}{\overline{\alpha}}\int_{\hat{a}}^{\infty}(a-\hat{a})f(a)da &= R^E, \quad R^E \geq \mathbb{E}(\tilde{a}), \quad \text{with } \hat{a} = (1-\overline{\alpha})R^B \\ u_i'(w_{i\,0} - \tilde{e}) &= R^E, \\ \tilde{D} &= d + c - B - M, \quad \tilde{e} = e - M, \quad K = E + \tilde{D}, \quad \tilde{e} = \frac{\overline{\alpha}}{1-\overline{\alpha}}\tilde{D} \end{aligned}$$

As in the previous section the Fed has two policy instruments $(R^B, \overline{\alpha})$. However, for the standard banking equilibrium there is a unique choice $\overline{\alpha}$ compatible with R^B, while under the APIR system the two instruments are essentially independent. More precisely, if $\alpha^m(R^B)$ denotes the unique equity requirement such that the policy $(R^B, \pi, \alpha) = (R^B, 0, \alpha^m(R^B))$ is compatible with a (standard) banking equilibrium, then any equity requirement which exceeds $\alpha^m(R^B)$ is compatible with a APIR equilibrium.

Proposition 5. *Let $\mathcal{E}$ be an economy satisfying Assumptions 1–3 and let R^B be in the interval* (8). *For any $\overline{\alpha}$ in the interval*

$$\alpha^m(R^B) \leq \overline{\alpha} \leq 1$$

there exists a APIR equilibrium for the policy $(R^B, \overline{\alpha})$.

Proof. Since $\pi = 0$ let $D(R^B) = D(R^B, 0)$ denote supply of debt by depositors and cash funds (as in previous section with $\pi = 0$). Let $(R^B, \overline{\alpha})$ denote a Central Bank policy and let $R^E = \Phi(R^B, \overline{\alpha})$. Since $R^B \leq \mathbb{E}(\tilde{a})$, $R^E \geq \mathbb{E}(\tilde{a})$ (see Proposition 2). Let $\tilde{e}$ be such that $u_i'(w_{i\,0} - \tilde{e}) = R^E$, then $\tilde{e} \geq 0$ by Assumption 1. Let M be such that $\tilde{e} = \frac{\overline{\alpha}}{1-\overline{\alpha}} D(R^B - M) \;\Leftrightarrow\; \overline{\alpha}\, M = \overline{\alpha}\, D(R^B) - (1-\overline{\alpha})\tilde{e}$: $M \geq 0 \;\Leftrightarrow\; \overline{\alpha}\, D(R^B) \geq (1-\overline{\alpha})\tilde{e}$. If $\overline{\alpha} = \alpha^m = \alpha^m(R^B)$ then $\tilde{e} = \frac{\alpha^m}{1-\alpha^m} D(R^B)$ and $M = 0$; this is the standard banking equilibrium. If $\overline{\alpha} > \alpha^m$ then $\tilde{e} < \frac{\alpha^m}{1-\alpha^m} D(R^B) \;\Leftrightarrow\; (1-\overline{\alpha})\,\tilde{e} < \frac{1-\overline{\alpha}}{1-\alpha^m}\,\alpha^m D(R^B) < \overline{\alpha} D(R^B)$ i.e. $M > 0$. Thus, if $\overline{\alpha} \geq \alpha^m$, $(R^B, \overline{\alpha})$ is a policy compatible with a APIR equilibrium $(\tilde{D} = D(R^B) - M,\, K = \tilde{e} + \tilde{D})$. □

The payment of interest on the reserves permits the debt used by banks to be lowered and the remainder to be absorbed as reserves at the Central Bank. Equity can be increased despite the lower return on equity due to the reduced leverage, because parts of the equity or risky securities issued by banks are sold to the Central Bank. Since the equity requirement can be made as high as needed, bankruptcy for the banks can be avoided and a first best equilibrium can be achieved.

Corollary 1. *Every economy $\mathcal{E}$ satisfying Assumption 1–3 has a Pareto optimal APIR equilibrium.*

Proof. Let $R^B = \mathbb{E}(\tilde{a})$ and let $\overline{\alpha} = \hat{\alpha}_c$ i.e. $(1-\overline{\alpha})\mathbb{E}(\tilde{a}) \leq \underline{a}$. The FOC for Pareto optimality are satisfied and there is no loss due to bankruptcy. □

The APIR equilibrium of Corollary 1 is fair for the taxpayer. By buying private securities the Central Bank makes a profit $M(V(a) - \mathbb{E}(\tilde{a}))$ when the realization of $\tilde{a}$ is favorable and has to use taxes to pay $M(V(a) - \mathbb{E}(\tilde{a}))$ when $V(a) < \mathbb{E}(\tilde{a})$. However, since $\mathbb{E}(V(a)) = \mathbb{E}(\tilde{a})$ the expected contribution of the taxpayers is equal to zero.

The result of Corollary 1, that the APIR system can lead to a first best equilibrium without bankruptcy, seems too good to be true, so let us discuss its applicability. The realistic aspect of the model is the IR part. The Federal Reserve began paying interest on reserves in 2008 and has since come to adopt IR as a standard instrument of monetary policy. This policy is supplemented by a policy of accepting funds from qualified Money Market Funds in the form of reverse repo transactions; the Fed uses the securities that it has purchased in prior QE episodes as collateral to borrow funds from Money Market Funds in as large amounts as the Money Market Funds want to lend up to the total value of the securities serving as collateral (currently 3 tr. \$). These policies are clearly directed to absorb the excess supply of funds which tend to depress the short-term interest rates when they cannot find an alternative safe haven in the banking system.

The least realistic part of the model is the purchase of risky assets by the Central Bank—the AP part. To simplify the model we have considered only the risky securities issued by the banking sector, while in practice there is a wide array of securities available to investors. As a result, in our model the Central Bank policy of purchasing risky securities directly increases the demand for equity and subordinated debt issued by the banks. Typically however, the Federal Reserve has restricted its purchase of risky securities to long term government bonds and mortgage backed securities issued by the Government Sponsored Agencies rather than buying risky securities issued by the private sector. It is therefore much less clear that the investors' funds freed by the purchase of the Federal Reserve will end up being channeled into the securities issued by the banks, since there are many other uses for these funds. But then if the supply of equity is lower than assumed, a high equity requirement, like the one considered in Corollary 1 will result in lower investment by banks, since a small amount of available equity and a high equity requirement imply that the amount of debt that the banking sector can use is small. Thus, without the assurance that the AP policy actually increases the supply of equity to banks, the APIR policy may indeed increase the safety of the banking system, but at the cost of decreasing investment and activity in the economy.

The merit of this section lies less in providing a practical way of obtaining first best efficiency in the banking sector, than in clarifying the conditions needed for regulation requiring a high equity-to-debt ratio to lead to an efficient outcome. Not only must excess debt be absorbed—which is currently achieved by the payment of interest on reserves and reverse repo borrowing from the Money Market Funds—but also a way must be found of increasing the demand for equity and equity-like securities issued by banks.

5 Flexible 100% Reserve System

If the ability of a QE policy to deliver the additional equity financing to banks is in doubt, there is an alternative solution for improving on the standard banking equilibrium which does not seek to substantially change the proportion of short-term debt and equity in the financing of the banks. The idea is to draw on the proposal of Irving Fisher and the Chicago School in the 1930s to make the Central Bank the only permissible issuer of safe, short-term, liquid debt to the private sector.

Thus all funds that depositors and cash funds place with the banks, must be placed as reserves at the Central Bank, which pays interest on the reserves at the rate R^B. In contrast to the original 100% Reserve System proposed in the 1930s, under our system the Central Bank lends these funds back to the banks at an interest rate R^D which may be higher than R^B. For this reason we call this system the *Flexible 100% Reserve System* or the *Flex System* for short.

In contrast to the APIR system, the Flex system does not prevent banks from defaulting on their debt. However when they do default, they default on the Central Bank and not on the private sector, in particular they do not default on the potentially runnable debt supplied by the cash funds. This absence of runs should substantially reduce the costs of bankruptcy. Thus in this system we assume that the cost γ^f of bankruptcy under the Flex system is smaller than under the banking equilibria of Section 3. In fact, we study two cases: the ideal limit case where $\gamma^f = 0$ to understand what the best interest rate policy (R^B, R^D) should be in this case, and then study how this policy changes when there are positive resource costs of bankruptcy $\gamma^f > 0$.

The Flex system modifies the current banking system as follows.

(1) Banks are only permitted to have two sources of funds
   - (short-term) borrowing from the Central Bank.
   - equity and risky corporate bonds convertible to equity in case of bankruptcy (i.e. junior to borrowing from the Central Bank).

(2) All funds placed in banks by depositors are transferred to a reserve account with the Central Bank: depositors' funds cannot be used by the banks (100% reserve system). Private banks remain the front end for depositors and continue to provide payment services for which they are paid by depositors. The Central Bank pays the interest rate R^r on reserves.

(3) Cash funds place funds directly in a reserve account with the Central Bank and/or buy government bonds. Since the two are perfect substitutes for cash funds they earn the same rate of return $R^r = R^B$: the (short-term) interest rate on reserves chosen by the Central Bank determines the (short-term) interest rate.

(4) Investors buy banks' equity and/or their corporate bonds for which they earn the same rate of return R^E.

(5) The Central Bank lends back to the banks, at the rate R^D, the funds placed on reserve accounts by the depositors and cash funds. The loan is subject to the stipulated equity requirement $\overline{\alpha}$, i.e. the Central Bank lends at most $(1 - \overline{\alpha})$ the value of the bank's assets. When the banks are subject to a low realization a of their payoff and cannot reimburse

their debt to the Central Bank they are placed in "conservatorship", investors (equity and bondholders) receive nothing and the government uses the recoverable assets Ka and taxes to pay depositors and cash funds.

(6) The Central Bank has three policy instruments $(R^B, R^D; \overline{\alpha})$ of which only two, the interest rates (R^B, R^D), are independent since the equity requirement $\overline{\alpha}$ has to adjust to R^D to make it possible to have equilibrium on the equity/risky-bond market.

We call a system satisfying (1)-(6) a *Flexible (100%) Reserve System (Flex)* since all safe debt must be placed on Reserve with the Central Bank, and the system is "flexible" since it permits the Central Bank to lend back these funds to the private sector banks. To describe an equilibrium of the Flex system we need to describe more precisely the behavior of the different economic entities.

Banks When the banks receive deposits from depositors, the funds are transferred to an account at the Central Bank, but the banks continue to manage the accounts and provide payment services. Since a constant marginal cost μ is incurred per unit of expenditure of depositors at date 1, the banks must be compensated for the cost. Thus when a depositor places deposits d, the amount $\frac{d}{1+\mu}$ goes on his/her account at the Central Bank and $\frac{\mu d}{1+\mu}$ goes to the account of the servicing bank. At date 1 the depositor can spend $\frac{dR^B}{1+\mu}$ and the bank receives $\frac{\mu d R^d}{1+\mu}$ which is exactly the cost of providing the payment services. Banks make zero profit on deposits and are indifferent on the amount of deposits they receive.

In addition to providing payment services, the banks (as before) invest in risky production projects. They have two sources of funds: let D denote the amount they borrow from the Central Bank at the rate R^D and let E denote the amount of equity they obtain from risk neutral investors who anticipate the expected return R^E. To simplify the exposition, and without loss of generality, we assume that all funds obtained from investors are received in the form of equity since in this model investors are indifferent between equity and risky bonds provided they have the same expected return. The analysis of the bank's behavior when faced with the loan rate R^D, the equity rate R^E and the equity requirement $\overline{\alpha}$ is given by the analysis of Section 2 which is summarized in Proposition 2 with $R = R^D$.

Government budget balance Let $D = d + c - B$ denote the total funds placed on reserve accounts with the Central Bank, d the amount by depositors and $c - B$ the net amount by cash funds, where the amount B of their funds has already been used to purchase the government bonds. The Central Bank pays the depositors and cash funds the rate R^B and lends the funds D at the rate R^D to the banks. Three cases can arise: (i) $R^D < R^B$ (ii) $R^D = R^B$ (iii) $R^D > R^B$. (i) implies that the Central Bank makes a loss for sure which has to be paid by taxpayers: we eliminate this case. In case (ii) where $R^D = R^B$ the Flex equilibrium (defined below) reduces to the banking equilibrium studied in Section 3 with $\pi = 0$. Since we are interested in high-safe-asset demand (HS) economies in which the standard banking equilibria are inefficient, we also eliminate this case. We thus focus on case (iii) where $R^D > R^B$. When the realized outcome a of the bank's payoff $\widetilde{a}$ is favorable i.e. when $Ka \geq R^D D$ (where K denotes the capital invested by the banks), then the Central Bank makes a profit $(R^D - R^B)D$. This profit is transferred to the Treasury which uses it to reduce taxes. However when the realized outcome a is low i.e. when $Ka < R^D D$, banks default and the Central Bank recovers the value $(1-\gamma^f)Ka$ of the bank's assets and the Treasury raises taxes to reimburse depositors and cash funds. Thus the taxes paid by investors at date 1 are given by

$$t(a) = \begin{cases} R^B B - (R^D - R^B)D, & \text{if } Ka \geq R^D D \\ R^B B - ((1-\gamma^f)Ka - R^D D), & \text{if } Ka < R^D D \end{cases} \tag{19}$$

The first term is the cost of reimbursing the Treasury's debt B, and the second is the profit or loss made by the Central Bank acting as the banks' banker.

Agents Depositors behave as in Section 3 facing the "discounted" interest rate $R^d = \frac{R^B}{1+\mu}$ i.e. the riskless rate R^B discounted by the cost μ of managing their funds at date 1. Cash funds behave as in Section 3 facing the interest rate $R^c = R^B$. Investors behave as in Section 3 anticipating the expected return on equity $R^E = \Phi(\overline{\alpha}; R^D)$.

Equations of Flex equilibrium An *equilibrium of the Flexible (100%) Reserve System* consists of interest rates (R^B, R^D), equity requirements $\overline{\alpha}$, rate on return on equity R^E, and actions (d, c, e, K, D, E) such that

(i) $u'_d(w_{d\,0} - d) = \dfrac{1+\rho}{1+\mu} R^B \quad \Leftrightarrow \quad d = d(R^B)$

(ii) $u'_c(w_{c\,0} - c) = R^B \quad \Leftrightarrow \quad c = c(R^B)$

(iii) $D = d + c - B$

(iv) $E = \dfrac{\overline{\alpha}}{1-\overline{\alpha}} D, \quad \dfrac{1}{\overline{\alpha}} \displaystyle\int_{(1-\overline{\alpha})R^D}^{\infty} (a - (1-\overline{\alpha})R^D) f(a) da = R^E, \quad R^E \geq \mathbb{E}(\tilde{a}), \quad K = D + E$

(v) $e = E, \qquad u'_i(w_{i\,0} - e) = R^E \quad \Leftrightarrow \quad e = e(R^E)$

(vi) $x_{i1}(a) = w_{i1} - t(a) + V(a)e \quad$ with $V(a)$ given by (6) and $t(a)$ by (19).

As we have seen in Section 3, there are two sources of inefficiency in a standard banking equilibrium of a high-debt economy. The first is that the interest rate and the return on equity cannot simultaneously be at the optimal level; the second is that there is an output loss due to bankruptcy. To show that the Flex system can solve the first inefficiency, we assume that the bankruptcy cost in the Flex equilibrium is small (actually at the limit, zero) so that if the rates of return for debt and equity are correct, the equilibrium is Pareto optimal.

5.1 Flex Equilibrium with $\gamma^f = 0$

In the Flex system the Central Bank can choose two interest rates: the rate R^B paid to savers (depositors and cash funds) and the rate R^D charged to banks: this makes it possible to charge banks the "true cost" of their funds i.e. to take into account that they only pay R^D when they are not bankrupt. Providing the Central Bank with this additional flexibility permits the Flex system, with an appropriate choice of policy $(R^B, R^D; \overline{\alpha})$, to achieve a Pareto optimal equilibrium.

Proposition 6. *In a high debt economy there exists a Central Bank policy* $(R^{B*}, R^{D*}; \alpha^*)$ *for which the associated Flex equilibrium is Pareto optimal.*

Proof. We need to show that we can find a solution of the equations (i)-(vi) of a Flex equilibrium which also satisfies the FOC (17) for Pareto optimality. If the short rate R^B is chosen so that $R^B = \mathbb{E}(\tilde{a})$, then (i) and (ii) imply that the FOC for Pareto optimality are satisfied for depositors and cash funds. We will show that the return on equity R^E can also be chosen so that $R^E = \mathbb{E}(\tilde{a})$, in which case the FOC for Pareto optimality for investors is also satisfied. Thus we need to show that there exists (R^D, α) such that (iii), (iv), (v) of a Flex equilibrium are satisfied when $R^E = \mathbb{E}(\tilde{a})$. This is equivalent to showing that there exists a solution (R^*, α^*) of the pair of equations

$$\Phi(\alpha; R) = \mathbb{E}(\tilde{a}) \tag{20}$$

$$u_i'\Big(w_{i\,0} - \frac{\alpha}{1-\alpha}\, D(\mathbb{E}(\tilde{a}))\Big) = \mathbb{E}(\tilde{a}) \tag{21}$$

or, in the notation of Section 3, $s(\alpha, D^*) = \Phi(\alpha, R) = \mathbb{E}(\tilde{a})$, with $D^* = D(\mathbb{E}(\tilde{a}))$. A similar system of equations (with $R^E \geq \mathbb{E}(\tilde{a})$) has been studied in Section 3. α^* is solution of (21) if and only if $\alpha^* = \alpha^{min}(D^*)$, where α^{min} is defined in (15). Then R^* must be solution of $\Phi(\alpha^*, R) = \mathbb{E}(\tilde{a})$, which implies that $R^* \geq \mathbb{E}(\tilde{a})$ (see Figure 1). If the economy is a hight debt economy, α^* is small and $R > \mathbb{E}(\tilde{a})$ (see Figure 3).

To show that we can find a solution of (20) when $\alpha = \alpha^*$, consider the bank's expected return $\Phi(\alpha^*; R)$ viewed as a function of the rate R charged by the Central Bank i.e. the function

$$R \to \Phi(\alpha^*; R) = \frac{1}{\alpha^*} \int_{(1-\alpha^*)R}^{\infty} (a - (1-\alpha^*)R) f(a) da.$$

This is a continuous decreasing function since $\frac{\partial \Phi(\alpha^*;R)}{\partial R} = -\left(\frac{1-\alpha^*}{\alpha^*}\right) \int_{(1-\alpha^*)R}^{\infty} f(a) da < 0$. If $R \to \infty$ then $\Phi(\alpha^*; R) \to 0$, and for $R = \mathbb{E}(\tilde{a})$, $\Phi(\alpha^*; R) > \mathbb{E}(\tilde{a})$ since $\alpha^* < \hat{\alpha}_c$, where $\hat{\alpha}_c$ is defined by

(18). Thus there exists $R^* > \mathbb{E}(\tilde{a})$ such that $\Phi(\alpha^*; R^*) = \mathbb{E}(\tilde{a})$ and the Central Bank policy $(R^{B*}, R^{D*}; \alpha^*) = (\mathbb{E}(\tilde{a}), R^*; \alpha^*)$ leads to a Pareto optimal Flex equilibrium. □

Figure 4 gives a geometric interpretation of Proposition 6. It shows the same economy as in Figure 3(a) for which the banking equilibrium where banks pay the interest rate $R^B = \mathbb{E}(\tilde{a})$ on their debt is such that $R^E > \mathbb{E}(\tilde{a})$, and is thus not Pareto optimal. In the Flex equilibrium the Central Bank increases the interest rate charged to banks from $\mathbb{E}(\tilde{a})$ to R^{D*}, shifting the banks' expected return curve downward so that it passes through the point $(\alpha^*, \mathbb{E}(\tilde{a}))$, thereby achieving Pareto optimality.

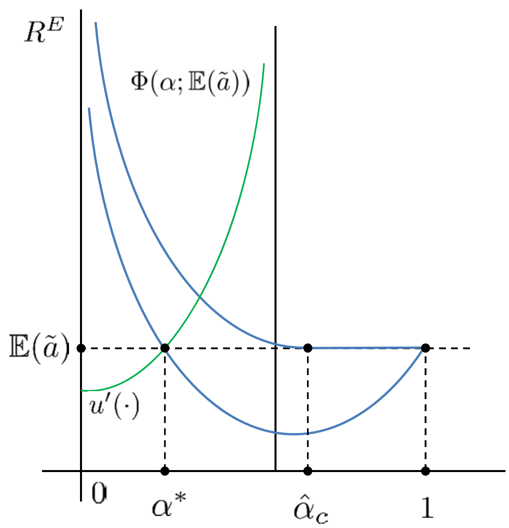


Figure 4: Pareto optimal Flex equilibrium.

The equation which defines the rate R^* charged to banks

$$\mathbb{E}(\tilde{a}) = \frac{1}{\alpha^*} \int_{(1-\alpha^*)R^*}^{\infty} (a - (1-\alpha^*)R^*) f(a) da$$

can be written as

$$\begin{aligned}(1-\alpha^*)R^* \int_{(1-\alpha^*)R^*}^{\infty} f(a)da &= \int_{(1-\alpha^*)R^*}^{\infty} af(a)da - \alpha^* \mathbb{E}(\tilde{a}) \\ &= (1-\alpha^*)\mathbb{E}(\tilde{a}) - \int_{\underline{a}}^{(1-\alpha^*)R^*} af(a)da\end{aligned}$$

which is equivalent to

$$R^* \int_{(1-\alpha^*)R^*}^{\infty} f(a)da + \frac{1}{1-\alpha^*} \int_{\underline{a}}^{(1-\alpha^*)R^*} af(a)da = \mathbb{E}(\tilde{a}) \tag{22}$$

The first term on the left is the expected payment per unit of debt made by the banks when they reimburse their debt, and the second is what banks pay indirectly per unit of debt when they are bankrupt (i.e. the recovery rate of the Central Bank on their assets). The sum of these two terms is the *effective cost of (a unit of) debt* for the banks. Pareto optimality requires that the Central Bank charge the banks the rate $R^{D*} = R^*$ such that *their effective cost of debt equals the natural rate of interest* $\mathbb{E}(\tilde{a})$. When the banks are charged the rate $R^{D*} = R^*$, and $R^B = R^{B*} = \mathbb{E}(\tilde{a})$, (19) and (22) imply that $\mathbb{E}(t(a)) = BR^{B*}$: thus the additional expected cost to the taxpayers of enabling the banking system to play its role of risk transformation is zero. In the Flex equilibrium banks are charged a loan rate R^{D*} such that the expected surplus for the taxpayers in good times covers the expected shortfall in bad times.

5.2 Flex Equilibrium with $\gamma^f > 0$

In the previous section we showed how the Flex System gives the Central Bank sufficient additional control over banks to permit the banking system to induce a Pareto optimal outcome, when no resource costs are incurred in the event of bankruptcy. We now study how the Central Bank's policy choice $(R^B, R^D; \alpha)$ in a Flex equilibrium changes when the occurrence of bankruptcy entails resource costs.

In the banking system studied in Section 3, the creditors of the banks are agents and institutions (depositors and cash funds) in the private sector. In such a setting the costs of bankruptcy proceedings can be extensive as shown by the willingness of the Federal Reserve and the Treasury to rescue the financial system when there is a possibility of widespread bankruptcy. Since under the Flex System the only creditor of the banks is the Central Bank and since in the event of bankruptcy the Central Bank can immediately place a bank in conservatorship (or in the current terminology in "resolution"), the bankruptcy costs incurred should be substantially reduced. However the change in ownership structure following bankruptcy always involves some costs, which we model here by assuming that a proportion $0 \leq \gamma^f < 1$ of output is lost and the remainder is appropriated by the Central Bank when bankruptcy occurs.

Given the presence of these bankruptcy costs, a Central Bank choice of policy $(R^B, R^D; \alpha)$ is called a *second-best policy* if it maximizes social welfare under the constraint that the allocation is obtained as an Flex equilibrium of the banking system. Such a policy is a solution of the problem

$$\begin{aligned}\max_{(R^B,R^D;\alpha)} \; &\beta d \left[u_d(w_{d\,0} - d(R^B)) + \frac{R^B}{1+\mu}\, d(R^B) + \rho\Big(\frac{R^B}{1+\mu}\, d(R^B)\Big)\right] \\ &+ \left[u_c(w_{c\,0} - c(R^B)) + R^B c(R^B)\right] \\ &+ \left[u_i\Big(w_{i\,0} - \frac{\alpha}{1-\alpha}\, D(R^B)\Big) + \int_{\underline{a}}^{\infty} x_{i1}(a) f(a) da\right]\end{aligned} \tag{23}$$

subject to

- $K = \dfrac{1}{1-\alpha}\, D(R^B)$
- $x_{i1}(a) = \begin{cases} Ka - (D(R^B) + B)R^B, & \text{if } a \geq (1-\alpha)R^D \\ Ka(1-\gamma^f) - (D(R^B) + B)R^B, & \text{if } a < (1-\alpha)R^D \end{cases}$
- $u_i'\Big(w_{i\,0} - \dfrac{\alpha}{1-\alpha}\, D(R^B)\Big) = \dfrac{1}{\alpha}\displaystyle\int_{(1-\alpha)R^D}^{\infty} (a - (1-\alpha)R^D) f(a) da$
- $\dfrac{1}{\alpha}\displaystyle\int_{(1-\alpha)R^D}^{\infty} (a - (1-\alpha)R^D) f(a) da \geq \mathbb{E}(\tilde{a})$

The expression for the date 1 consumption of investors $x_{i1}(a)$ takes into account our earlier assumption that investors also play the role of taxpayers. Thus their date 1 consumption is the output produced minus the consumption of the lenders (depositors and cash funds).

The analysis of the first-order conditions for a second best policy are more straightforward if we use the variables $(R^B, \hat{a}, \alpha)$ rather than $(R^B, R^D; \alpha)$: in view of the bankruptcy threshold relation $\hat{a} = (1-\overline{\alpha})R^D$ there is a one-to-one map between the two. In terms of these new variables the maximum problem can be written as

$$\begin{aligned}\max_{(R^B,\hat{a},\alpha)} \; &\beta d \left[u_d(w_{d\,0} - d(R^B)) + \frac{R^B}{1+\mu} d(R^B) + \rho\Big(\frac{R^B}{1+\mu} d(R^B)\Big)\right] \\ &+ \left[u_c(w_{c\,0} - c(R^B)) + R^B c(R^B)\right] \\ &+ \left[u_i(w_{i\,0} - \frac{\alpha}{1-\alpha}\, D(R^B)) + \frac{D(R^B)}{1-\alpha}\Big(\mathbb{E}(\tilde{a}) - \gamma^f \int_{\underline{a}}^{\hat{a}} a f(a) da\Big) - R^B\Big(D(R^B) + B\Big)\right]\end{aligned} \tag{24}$$

subject to

- $u_i'\Big(w_{i\,0} - \dfrac{\alpha}{1-\alpha}\, D(R^B)\Big) = \dfrac{1}{\alpha}\displaystyle\int_{\hat{a}}^{\infty} (a - \hat{a}) f(a) da$
- $\dfrac{1}{\alpha}\displaystyle\int_{\hat{a}}^{\infty} (a - \hat{a}) f(a) da \geq \mathbb{E}(\tilde{a})$

An analysis of the first-order conditions for this maximum problem leads to the following proposition.

Proposition 7. *Let $(R^B_{sb}, R^D_{sb}; \alpha_{sb})$ denote a second-best policy for a high debt economy. Then*

(i) *for any $\gamma^f \in [0,1]$ the choice $(R^D_{sb}; \alpha_{sb})$ is such that there is a positive probability of bankruptcy at equilibrium: $\hat{a}_{sb} > \underline{a}$;*

(ii) *the interest rate R^B_{sb} on reserves and government bonds is lower than the first-best interest rate $R^{B*} = \mathbb{E}(\tilde{a})$ and is given by*

$$R^B_{sb} = \mathbb{E}(\tilde{a}) - L(\hat{a}_{sb}) - L'(\hat{a}_{sb}) \frac{\int_{\hat{a}_{sb}}^{\infty} (a - \hat{a}_{sb}) f(a) da}{\int_{\hat{a}_{sb}}^{\infty} f(a) da} \tag{25}$$

where $L(\hat{a}) = \gamma^f \int_{\underline{a}}^{\hat{a}} a f(a) da$ denotes the expected loss when there is bankruptcy, per unit of capital invested.

(iii) *the second best equity ratio α_{sb} is greater than the first-best equity ratio α^*, the bankruptcy threshold $\hat{a}_{sb}$ is lower than the first best bankruptcy threshold $\hat{a}^*$, and the interest rate R^D_{sb} charged to banks is lower than the first-best rate R^{D*}.*

Proof. (see Appendix).

Property (i) is somewhat surprising. Intuition might suggest that if the proportion of output lost γ^f is sufficiently high then the best solution would be to reduce the interest rates (R^B, R^D) and to increase the equity requirement to the point where bankruptcy and hence its induced losses would no longer occur. However the loss in utility of the lenders and the decrease in investment makes such a solution suboptimal. The formula (25) in (ii) expresses the second best optimality of debt—namely that the marginal cost of an additional unit of debt equals its marginal benefit in terms of additional output, given the bankruptcy costs. If debt is increased by dD, the marginal cost for lenders is $R^B dD$. The induced increase in capital $dK = dD$ has a direct marginal benefit $(\mathbb{E}(\tilde{a}) - L(\hat{a}) dK$, from the increase in output at date 1. However if the debt is increased without changing equity there is a change in the equity ratio α and in the bankruptcy level $\hat{a}$. Since $D = (1 - \alpha)K$ and $dD = dK$, $d\alpha = -\frac{\alpha dD}{K}$. Since E does not change, the return on equity $\Phi(\alpha, \hat{a})$ does not change which implies that

$$-\frac{d\alpha}{\alpha^2} \int_{\hat{a}}^{\infty} (a - \hat{a}) f(a) da - \frac{d\hat{a}}{\alpha} \int_{\hat{a}}^{\infty} f(a) da = 0$$

Since $d\alpha = -\frac{\alpha dD}{K}$, $d\hat{a}$ is given by

$$d\hat{a} = \frac{-\dfrac{dD}{K} \displaystyle\int_{\hat{a}}^{\infty} (a - \hat{a}) f(a) da}{\displaystyle\int_{\hat{a}}^{\infty} f(a) da}. \tag{26}$$

The increase in the bankruptcy level induces an additional marginal loss in output $KL'(\hat{a})d\hat{a}$. Replacing $d\hat{a}$ by its value in (26) shows that the last term in (25) is the additional loss of output due to the increase in the bankruptcy level induced by the increase in debt.

Although there is still bankruptcy in the second best equilibrium, Property (iii) shows that the probability of bankruptcy is lower and the equity-debt ratio is higher that in the first-best equilibrium. Since $R^B_{sb} < R^{D*} = \mathbb{E}(\tilde{a})$, $D_{sb} = D(R^B_{sb}) < D(R^{B*}) = D^*$: the Central Bank lends less to banks than in the first best. The constraint that the return on equity is at least $\mathbb{E}(\tilde{a})$ implies that the supply of equity is at least that of the first best: $e(R^B_{sb}) \geq e(\mathbb{E}(\tilde{a})$. It follows that the debt-equity ratio $\frac{1-\alpha}{\alpha}$ is lower that in the first best, or equivalently $\alpha_{sb} > \alpha^*$: the share of equity in the financing of investment is increased relative to the first best. We noted in Section 3 that the return on equity can be decomposed into the product of the leverage multiplier $\frac{1}{\alpha}$ and the return on capital $r(\hat{a}) = \int_{\hat{a}}^{\infty}(a - \hat{a})f(a)da$, where the return on capital is a strictly decreasing function of $\hat{a}$. Since the leverage multiplier is lower in the second best than in the first-best while the return on equity is as large, it must be that the return on capital is higher, which is possible only if the bankruptcy level is lower than in the first-best: $\hat{a}_{sb} < \hat{a}^*$. This in turn implies that the probability of bankruptcy is lower in the second-best than in the first-best equilibrium. Finally, the return on equity $\Phi(\alpha, R^D)$ viewed as a function of the equity ratio and the rate of interest charged to banks is decreasing in each variable. Since $\alpha_{sb} > \alpha^*$, and $\Phi(\alpha, R^D) \geq \mathbb{E}(\tilde{a}) = \Phi(\alpha^*, R^{D*})$ does not decrease in the second best equilibrium, it must be that R^D decreases to reestablish a sufficient return on equity. Thus $R^D_{sb} < R^{D*}$: banks pay a lower rate for these loans in the second-best than in the first-best allocation.

6 Conclusion

This paper makes two main contributions: first we emphasize the general equilibrium effects of bank capital regulation and put forward the notion of "natural" equity to asset ratio for banks. This natural equity ratio depends on agents'preferences and endowments, and on the technology. Second, we show that there are situations, which we term "high demand for safe assets" where this natural equity to asset ratio is insufficient to cover losses on productive investments so as to respond to the needs of depositors and cash investors. In such a situation, some form of policy intervention by the central bank is needed. One possibility is to restore Pareto optimality is what we call APIR (Asset Purchase and Interest on Reserves). This policy looks very similar to the type of non conventional monetary policy interventions that the Fed and other central banks have implemented since the GFC. Another possibility, which we call the Flex system, is more revolutionary as it would correspond to an extended version of the Chicago plan (100To some extent, our model provides a rationalization of what central banks have started doing (QE) and what they are trying to do (forward guidance). However, we need to refine this model so as to make the assessments of the two systems more realistic.

Appendix

Proof of Proposition 3. Let R^B be fixed in the interval (8) and fix $\pi \geq 0$ not-too-high. Consider the two functions

$$\begin{aligned} h(\alpha) &= s(\alpha, D(R^B,\pi)) - \Phi(\alpha; R^B(1+\pi)) \\ \tilde{h}(\alpha) &= s(\alpha, D(R^B,0)) - \Phi(\alpha; R^B(1+\pi)) \end{aligned}$$

where $s(\alpha, D) = u_i'\Big(w_{i\,0} - \dfrac{\alpha}{1-\alpha}D\Big)$ defined on $\Big[0, \frac{w_{i\,0}}{w_{i\,0}+D}\Big)$. Since the return to depositors $R^d = \frac{1+\pi}{1+\pi+\mu}$ is increasing in π, $D(R^B,\pi)$ is increasing in π. Since $s(\alpha, D)$ is increasing in D, $h(\alpha) \geq \tilde{h}(\alpha)$ for all $\alpha > 0$ for which both functions are defined. Let $\alpha^{\min} = \alpha^{\min}(D(R^B,\pi))$. By assumption (i.e. π is not-too-high) $\tilde{h}(\alpha^{\min}) \leq 0$. If $\widetilde{\alpha}$ satisfies $\tilde{h}(\widetilde{\alpha}) = 0$ then $\widetilde{\alpha} \geq \alpha^{\min}$. $\widetilde{\alpha}$ exists since $\tilde{h}(\alpha) \to -\infty$ as $\alpha \to 0$ and $\tilde{h}(\alpha) \to \infty$ as $\alpha \to \frac{w_{i\,0}}{w_{i\,0}+D}$. Moreover, $\widetilde{\alpha}$ is unique. Since $\widetilde{\alpha} \geq \alpha^{\min}$, $\Phi(\widetilde{\alpha}; R) = s(\widetilde{\alpha}, D(R^B,0)) \geq s(\alpha^{\min}, D(R^B,0)) = \mathbb{E}(\tilde{a})$, where $R = R^B(1+\pi)$. Thus Φ is decreasing in α (see Proposition 2) and hence h is increasing on $(0,\widetilde{\alpha})$. For $\alpha > \widetilde{\alpha}$, $s(\alpha, D(R^B,0)) > s(\widetilde{\alpha}, D(R^B,0))$ and $\Phi(\alpha;R) \leq \Phi(\widetilde{\alpha};R)$, since $\Phi(\alpha;R)$ can be increasing in α only when $\Phi(\alpha;R) < \mathbb{E}(\tilde{a})$.

Since $h(\alpha) \geq \tilde{h}(\alpha)$, $h(\widetilde{\alpha}) \geq 0$. As $\alpha \to 0$, $h(\alpha) \to -\infty$ so that there exists $\overline{\alpha} \in (0, \widetilde{\alpha}]$ such that $h(\overline{\alpha}) = 0$. Since $\overline{\alpha} \leq \widetilde{\alpha}$, $\Phi(\overline{\alpha}; R) \geq \Phi(\widetilde{\alpha}; R) \geq \mathbb{E}(\tilde{a})$, so that $(R^B, \pi, \overline{\alpha})$ satisfies (14) and an equilibrium exists. The same reasoning that shows $\widetilde{\alpha}$ is unique shows that $\overline{\alpha}$ is unique.

Proof of Proposition 7. Let μ and ν denote the multipliers associated with the constraints of the maximum problem (24). Taking derivatives of the Lagrangian with respect to the variables $(R^B, \hat{a}, \alpha)$ leads to the following FOCs

$$(1-\alpha)R^B + \alpha(u_i' + \mu u_i'') - (\mathbb{E}(\tilde{a}) - L(\hat{a})) = 0 \tag{27}$$

$$(\mu - \nu)\int_{\hat{a}}^{\infty} f(a)da - \frac{\alpha}{1-\alpha}D(R^B)L'(\hat{a}) = 0 \tag{28}$$

$$\frac{D(R^B)}{(1-\alpha)^2}\Big(\mathbb{E}(\tilde{a}) - L(\hat{a}) - (u_i' + \mu u_i'')\Big) + \frac{\mu-\nu}{\alpha^2}\int_{\hat{a}}^{\infty}(a-\hat{a})f(a)da = 0 \tag{29}$$

where the subscript 'sb' for second best has been omitted and the arguments of the function u_i' and u_i'' at their second best values have been omitted. Note that the complete definition of $L(\hat{a})$ is

$$L(\hat{a}) = \begin{cases} \gamma^f \displaystyle\int_{\underline{a}}^{\hat{a}} af(a)da & \text{if} \quad \hat{a} \geq \underline{a} \\ 0 & \text{if} \quad \hat{a} \leq \underline{a} \end{cases}$$

Since we have assumed $f(\underline{a}) = 0$, L is differentiable on $[0, \infty)$.

Replacing $(\mu - \nu)$ in (29) by its value in (28) and then $(u_i' + \mu u_i'')$ in (27) by its value in (29) leads to the formula (25) in (ii) of Proposition 7. To show that $\hat{a} > \underline{a}$, note that if $\hat{a} \leq \underline{a}$, $L'(\hat{a}) = 0$ which implies $\mu - \nu = 0$ (by (28)), which in turn implies $u_i' + \mu u_i'' = \mathbb{E}(\tilde{a})$ (by (29) and $L(\hat{a}) = 0$). Then (27) implies $R^B = \mathbb{E}(\tilde{a})$. Either $\nu > 0$ or $\nu = 0$. If $\nu > 0$ then $R^E = \mathbb{E}(\tilde{a})$, which, in a high debt economy is impossible with $\alpha < 1$, a necessary condition to accommodate the demand for safe debt. If $\nu = 0$ then $\mu = 0$, and from (27) $u_i' = \mathbb{E}(\tilde{a})$, which again is impossible for a high debt economy. This proves (i) of Proposition 7. Properties (iii) are proven in the text after the Proposition.

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