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Subadditivity Tests for Network Separation Using a Generalized McFadden Cost Function

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Abstract

The paper describes a pair of subadditivity tests for network costs that can be used to evaluate the technological feasability of network separation. The tests can be implemented with a Generalized McFadden cost function. It illustrates the tests for *orthogonal separation* and *operational complementarity* with an analysis of U.S. freight railroads. The methodological conclusion is that Baumol's 1977 test is directly relevant to the policy alternatives for network natural monopoly. The empirical results support and extend our earlier findings [Ivaldi and McCullough (2001)] regarding the feasibility of vertical and horizontal separation of U.S. rail operations.

1. Introduction

There are two policy alternatives to state ownership for dealing with monopoly networks. One is traditional economic regulation. The other is competition policy, a set of government activities ranging from antitrust enforcement to mandated open access to establish competition on previously monopolized networks. Though some view competition policy as a *substitute* for regulation, others argue that the two forms of network oversight are best seen as *complements*.[1]

This paper focuses on certain technological aspects of network oversight, particularly networks such as electricity and railroads where service-related operations and infrastructure maintenance both play an important role in determining costs. These networks pose a special challenge for competition policy because they may resist separation of network monopoly into a common infrastructure entity and competing operating entities. If, for example, there are vertical economies of scope *between* operations and infrastructure, such as those found by Kaserman and Mayo (1991) between the generation stage and the transmission/distribution stages of electricity, there may be a loss of technical efficiency if the two are separated. Or, if there are economies of joint production *among* operational activities, such as those found by Ivaldi and McCullough (2001) among differentiated services on U.S. freight railroads, then firms operating on network infrastructure may still be natural monopolies. In the former case, administrative regulation of an integrated network may *substitute* for competition policy, while in the latter case regulation of operating entities may *complement* it.

This paper demonstrates that the subadditivity test introduced by Baumol (1977) can be extended using the Generalized McFadden (GM) cost function to evaluate the technological feasibility of network separation. Section 2 describes a pair of subadditivity tests for network

[1] See EC(2000) for a justification of this view.

costs that are directly linked to the policy alternatives for natural monopoly. Section 3 shows how the tests can be implemented with the multiproduct GM cost function introduced by Kumbhakar (1994). Section 4 applies the tests to evaluate recent competition proposals for U.S. freight railroads. The methodological and empirical conclusions are summarized in Section 5. The empirical results support and extend our earlier finding [Ivaldi and McCullough(2001)] that Class I freight railroads have an operating structure similar to that of large commercial airlines, and that competition policy, though technologically feasible, would need to be complemented by regulation of operations as well as infrastructure.

2. Two Subadditivity Tests for Network Separation

Scale and scope are the standard measures to describe production economies, but these are only *descriptive* measures. The *proscriptive* technological measure is subadditivity of the cost function. It is subadditivity (and *not* scale or scope) which finally determines whether an output vector *y* can be produced more cheaply by a single firm than by any group of firms.[2] This in turn determines whether an industry is a natural monopoly and potentially subject to some form of industry oversight.[3]

Baumol's test of cost subadditivity for multiproduct production is as follows.

Test of subadditivity:

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[2] Panzar (1989) analyzes the relationship of subadditivity to economies of scale and scope.
[3] This is only a partial indication of natural monopoly since it is focused on technology and not demand. For a test that takes the resulting market structures into account see Gasmi, Laffont, and Sharkey (1997)

Let $C(y)$ represent the total costs associated with production of output vector y. The overall network cost function $C(y)$ is subadditive if for any and all $y^i \neq y$ s.t. $\Sigma y^i = y$, $C(y) < \Sigma C(y^i)$.

If $C(y)$ is subadditive and y is significant relative to market demand, there may be a need for public ownership or industry oversight. Oversight can take the form of administrative regulation and/or competition policy, but since the shortcomings of traditional regulation are well known, the presumption has been in favor of competition policy. This typically requires the separation of a network monopoly into a regulated or publicly supported infrastructure component and competing operating components with access to infrastructure.

Underlying this policy, however, are two critical technological assumptions which can also be evaluated using the subadditivity criterion:

*Test I (Orthogonal Separation): Let y^S and $y^{\mathbf{T}}$ represent an orthogonal partition of the output vector **y** into operational activities y^j ($\Sigma y^j = y^S$) and infrastructure-related activities y^k ($\Sigma y^k = y^T$). Network operational costs and infrastructure costs are subadditive if $C(y) < C(y^S, 0) + C(0, y^T)$.*

This is actually a subset of Baumol's test since $y^{\mathbf{j}}$ and $y^{\mathbf{k}}$ are among the vectors in y^i . If this test indicates there are vertical economies of scope *between* operations and infrastructure, there will be a loss of technical efficiency if infrastructure and operations are separated. This may be offset by competition effects, but if the loss of technical efficiency is large, administrative regulation of an integrated monopoly may be the better policy.

Test II (Operational Complementarity): Let y^S and $y^{\mathbf{T}}$ represent an orthogonal partition of the output vector y as in Test I. The operating cost function $C(y^S)$ is subadditive if $C(y^S) < \Sigma C(y^j)$.

This is a test of economies of joint production *among* operational activities. If this test establishes that the operational cost function $C(y^S)$ is subadditive, competition policy will have limited impact because firms operating on network infrastructure will still be natural monopolies through some level of output y^{S*}. There may be a need for administrative regulation of operating entities as a complement to competition policy.

A similar test could be used to establish whether the separated infrastructure cost function $C(y^k)$ is subadditive. This is not strictly necessary, however, since under competition policy there are anti-exclusionary grounds for requiring administrative regulation or public provision of infrastructure.

3. Econometric Implementation with the Generalized McFadden Cost Function

The best known application of Baumol's subadditivity test in a network context is Evans and Heckman (1984) which uses a translog cost function to simulate one-firm versus two-firm cost outcomes for the Bell telecommunications system. The paper's finding that the Bell system was not a natural monopoly is disputed by Roller (1990) which reports the results of the same simulations with the same data using a quadratic cost function. The Evans and Heckman finding is reaffirmed though by Shin and Ying (1992) which uses a translog cost function with different data to conduct one-firm versus two-firm simulations. All three studies are limited, however, by choice of functional form. Proper econometric testing for subadditivity requires a cost function which is globally concave in input prices (to allow for cost extrapolations beyond the region of estimation) and which permits the assignment of zero output values (to be consistent with the requirements of Baumol's definition). Neither the translog nor the quadratic flexible form meets both of these requirements.

To implement our tests we adopt the multiproduct GM cost function introduced by Kumbhakar (1994) which is an extension of the single product GM derived from McFadden (1978) and introduced by Diewert and Wales (1987). Let w be an n-dimensional vector of input prices, t a q-dimensional vector of quasi-fixed technological factors, y an r-dimensional vector of outputs. Define z as the m-dimensional vector ($m = q + r$) that includes y and t. The cost function is

$$C = \alpha' w + 0.5 \frac{w' \Delta w}{\theta' w} \left(\beta' y\right)^2 + w' \Lambda z + 0.5(\theta' w) z' \Gamma z \tag{1}$$

where α is an unconstrained n-dimensional parameter vector, Δ an n x n symmetric parameter matrix, Λ an n x m parameter matrix of nonnegative elements, Γ an m x m symmetric parameter matrix, and θ is an n x 1 vector of fixed parameters.

For C to provide a second order approximation to an arbitrary cost function C^* it must contain $(n + m)(n + m + 1) / 2$ free parameters. The cost function in (1) contains $(n + m)(n + m + 1)/2 + m$ parameters so it is flexible. It is also homogeneous and monotonic in w, and it is concave in w if the estimated matrix Δ is negative semidefinite. If not, concavity can be imposed by setting $\Delta = -BDB'$ where B is a lower triangular matrix with the sum of its diagonal elements equal to 1 and D is a nonnegative diagonal matrix.[4] C also permits assignment of zero output values.

To estimate C we use the vector of n factor demands which contains all of the cost function parameters. The demand vector derived by Shepherd's lemma is

[4] This parameterization is from Wiley, Schmidt and Bramble (1973).

$$X = \alpha + \left[\frac{\Delta w}{\theta ' w} - 0.5 \frac{(w' \Delta w)\theta}{(\theta ' w)^2} \right] (\beta ' y)^2 + \Lambda z + 0.5 \theta (z' \Gamma z) \tag{2}$$

To evaluate subadditivities, we begin by using the estimated parameters in (1) to identify the (quasi-)fixed costs that would be incurred at zero output levels. We then "aggregate up" from this base, comparing the projected cost levels that result as the level and composition of output change. Projected fixed costs are

$$\hat{C}^{FIXED} = w'(\hat{\Lambda}_t ' t + 0.5 \; t' \hat{\Gamma}_t t) \tag{3}$$

where the term in parentheses is the vector of projected factor demands in (2) but using only the technological components (and not the output-related components) of *z*. Notice that these are costs associated with production which are fixed but not sunk.[5] If there are fixed sunk costs in addition to these, the full cost specification would be $C(y) = S + C^M$ where *S* is the opportunity cost of a fixed-and-sunk component and C^M captures fixed-but-not-sunk and variable cost components.

Projected stand-alone variable costs for the output vector *y** are

$$\hat{C}^{SA} = w' \left\{ \left[\frac{\hat{\Delta} w}{\hat{\theta} ' w} - 0.5 \frac{(w' \hat{\Delta} w)\hat{\theta}}{(\hat{\theta} ' w)^2} \right] (\hat{\beta} ' y^*)^2 + \hat{\Lambda}_{z^*} ' z^* + z^{*\prime} \hat{\Gamma}_{z^*} z^* \right\} \tag{4}$$

[5] See Mas-Collel, Whinston, and Green (1990), p. 145, for a discussion of fixed costs which can be sunk or not sunk.

where the term in brackets is the projected factor demand vector but with those elements of *y* that do not belong to *y** set to zero and with the elements of *t* entering only to the extent that they interact with *y** elements.

The GM's ability to project fixed-but-not-sunk costs which are not sunk separately from stand-alone variable costs is an important feature. It means that as we simulate changes in costs that result from combining or separating network activities, we are able to distinguish effects that are purely output-related from those that might involve the duplication of fixed-but-not-sunk costs. For example, we might expect the technology $C^M(y_1,0) + C^M(0,y_2)$ to have higher unsunk fixed costs than $C^M(y_1,y_2)$, but we might not expect these costs to double. The GM allows us to use (3) simulate a range of unsunk fixed costs.

4. U.S. Railroad Operations Above-the-Rail and Below-the-Wheel

The U.S. railroad industry consists primarily of private sector freight railroads operating under a relaxed regulatory scheme imposed by the Staggers Rail Act of 1980.[6] Output levels have remained fairly constant since Staggers, and overall rate levels have declined, but the number of Class I railroads has also dropped from 38 to eight.[7] The consolidation trend has led to legislative calls for an open access rail system in the U.S.

In this section we apply the subadditivity tests using the data from our earlier paper with the GM functional form. The data that we use are from regulatory reports filed by major U.S. freight railroads for the period 1978-1997. The model we propose is

[6] Intercity passenger trains operated by the National Railroad Passenger Corp. (AMTRAK) account for less than one percent of rail revenues.

$$C^M = C^M\left(y_B, y_V, y_E, y_I, w_L, w_E, w_F, w_M; H, R, T, \theta\right) \tag{5}$$

where:

C^M	=	variable (and sunk) opportunity costs of doing business,
y_B	=	car-miles of bulk traffic (i.e. open hopper, closed hopper, tank),
y_V	=	car-miles of intermodal and auto-carrier traffic,
y_E	=	car-miles of general traffic (gondolas and box cars),
y_I	=	replacement ties installed in a given year,
w_L	=	index of labor prices,
w_E	=	index of equipment prices,
w_F	=	index of fuel prices,
w_M	=	index of material prices and other input prices,
H	=	average length of haul,
R	=	miles of road operated,
T	=	counter for years.
θ	=	vector of fixed effect parameters.

More detailed explanation of the choice of variables and a description of their sources and construction are in the earlier paper.[8]

The full system includes four factor demand equations defined by (2) above, and four additional demand equations for the endogenous output variables y_B, y_V, y_E and y_I.[9] The system

[7] These are railroads with annual revenues of $250 million or more. Surface Transportation Board (2000) analyzes rates and output levels. See Wilson (1997) for a discussion of rail performance since deregulation.

is estimated using full information maximum likelihood with a correction for first-order serial correlation within the factor demand equations. The parameters θ are fixed at the average values of input quantities.[10] The estimated cost model is globally concave in *w*, and the full parameter set (Table A1) is generally consistent with earlier rail cost models including the translog estimates in Ivaldi and McCullough (2001).[11] Of particular interest, the highlighted second-order parameter estimates show cost *complementarities* between bulk operations (y_B) and intermodal operations (y_V), and between bulk and general freight (y_E), and *anticomplementarities* between infrastructure outputs (y_I) and each of the operational outputs.[12] There are *anticomplementarities* between intermodal and general freight but these are not statistically significant.

We now use (3) and (4) to conduct simulations of the type used by Evans and Heckman and others to evaluate single firm versus two firm subadditivity. For these simulations we restrict the feasible output levels of *y* to the 217 (of 301) observations in our data for which estimated marginal costs are simultaneously positive.[13] We use sample mean values of the elements of *w* and *t* for each projection. To test whether the rail costs are overall subadditive we evaluate.

$$C^M(y_B, y_V, y_E, y_I) \le C^M[\alpha y_B, \beta y_V, \gamma y_E, \delta y_I] + C^M[(1-\alpha)y_B, (1-\beta)y_V, (1-\gamma)y_E, (1-\delta)y_I]$$

[8] We assume a vertical production process in which quasi-fixed land and other inputs (fuel, materials, labor, equipment) are first converted into infrastructure outputs and then converted into car-miles.
[9] See Berndt et. al. (1993) for a discussion of the output endogeneity problem. The additional variables in the demand equations are annual system-wide population and system-wide coal consumption for each railroad.
[10] See Diewert and Wales, op. cit., p.49.
[11] The characteristic roots for Δ are -0.0448, -0.0005, -0.0778, and -0.0109.
[12] In Ivaldi and McCullough (2001) we speculate on the possible sources of complementarities among operational outputs and anticomplementarities between infrastructure and operations.
[13] Individually, the estimated marginal costs of y_B are positive for 266 observations, y_V for 269 observations, y_E 271, and y_I 291. Most of the negative MC projections are for observations distant from the means of the data. Though the GM is more than Diewert flexible it is not flexible enough for Sobolev estimations. See Flroens, Ivaldi and Larribeau (1996).

where the parameters $\alpha, \beta, \gamma, \delta$ takes values 0, 0.33, 0.66 and 1, and where the unsunk fixed cost factors for the two-firm scenarios are 1.0, 1.33, 1.66, and 2.0 times single firm costs. For *orthogonal separation* (Test I) we test

$$C^M(y_B, y_V, y_E, y_I) \leq C^M[y_B, y_V, y_E, 0] + C^M[0,0,0,y_I]$$

using the same unsunk fixed cost factors. For *operational complementarity* (Test II), we evaluate

$$C^M(y_B, y_V, y_E, 0) \leq C^M[\alpha \ \ y_B, \beta \ \ y_V, \gamma \ \ y_E, 0] + C^M[(1-\alpha) \ \ y_B, (1-\beta) \ \ y_V, (1-\gamma) \ \ y_E, \ 0]$$

where the parameters $\alpha, \beta, \gamma, \delta$ and the unsunk fixed cost factors have the same values as in the test of general subadditivity**.** The results are presented in table 1.

Notice that when this simulation procedure is applied to firm-level observations, as it is here, the results comprise a local test of whether firm costs are subadditive and not a test of natural monopoly at the industry level. The tests reported here also hold the magnitudes in *t* fixed, and *t* includes a measure of network size (*R*). In fact, one would expect network size to vary with industry structure and this would affect projected total costs. [These are $C = \rho R + C^M(y,w,t)$, where ρR represents the fixed sunk cost of land and C^M represents unsunk fixed costs and variable costs.] There is no effort here to model the longer run effects of changes in *R* (or ρR). Nevertheless, the GM results do provide an good representation of the local cost effects of network size (*R*) as production is "disintegrated". The effect of network size on unsunk fixed costs is explicitly modeled by varying λ. Since the elements of *t* (including *R*) only enter equation 4 interactively (multiplicatively) with the elements of y^*, the role of the vectors α, β, χ, and δ is to apportion the effect of *both* output and network size on stand alone costs.

The separation results are restated in table 2 where we use (3) and (4) to estimate fixed and stand-alone variable costs for the "average firm" in our restricted sample of 217 observations. We project these costs for scenarios ranging from the current integrated system to one which is fully diversified. For analytical simplicity we assume that the diversifications are orthogonal and that there is no duplication of unsunk fixed cost (λ = 1) or adjustment in network size (*R*). The projections suggest that a fully integrated system would have a slight cost advantage over a vertically separated system where the operating companies provided bulk, intermodal and general freight services. However, the margin indicates there would not be a significant loss of *technical* efficiency if operations were separated from infrastructure. With our sample of integrated firms we cannot assess whether *transactions costs* associated with separated operations would be higher (or lower) than they are in integrated firms. Teece (1980) argues that these are likely to be more important than scale or scope in evaluating vertical integration.

Table 1. Subadditivity Tests: One-firm versus Two-firm Scenarios

	Total Cases	**Subadditive Cases** (λ = Factor for Fixed Costs)			
		λ = 1.0	λ = 1.33	λ = 1.66	λ = 2.00
General Subadditivity	27,776	19,252 (69.31%)	27,434 (98.77 %)	27,662 (99.59 %)	27,753 (99.92 %)
Test I: Orthogonal Separation	217	11 (5.07 %)	142 (65.44 %)	162 (74.65 %)	198 (91.24 %)
Test II: Operational Complementarity	6,944	6,296 (90.67 %)	6,944 (100 %)	6,944 (100 %)	6,944 (100 %)

Table 2. Cost Subadditivity on U.S. Freight Railroads

Projected Stand-alone Costs (millions of $97)

	CLOSED ACCESS	OPEN ACCESS OPERATING ALTERNATIVES				
Stand-alone Costs		**Integrated Freight**	**Diversified TOFC**	**Diversified General**	**Diversified Bulk**	**Fully Diversified**
Fixed Costs (not sunk)		191,598	191,598	191,598	191,598	191,598
Infrastructure		146,128	146,128	146,128	146,128	146,128
Bulk					433,749	433,749
Intermodal			130,979			130,979
General				451,333		451,333
Bulk & General			633,038			
Bulk & TOFC				505,019		
General & TOFC					554,701	
Bulk & General &TOFC		678,519				
TOTAL	**992,440**	**1,016,245**	**1,101,743**	**1,294,078**	**1,326,176**	**1,353,787**

Significant losses of technical efficiency *would* occur if (as a result of competition policy) there were entry into the operating market and a diversification of operating entities. The least costly diversified scenario would involve intermodal-only firms operating on the same infrastructure with larger firms that combined bulk and general freight services.[14] Separate bulk operations, depicted in the last two columns, would be most costly overall.[15]

5. Conclusion

Our analysis has focused on the technological determinants of network oversight, especially on networks with strong operational and infrastructure maintenance components. We show that Baumol's subadditivity test for natural monopoly can be straightforwardly extended to analyze whether a particular network technology will resist vertical separation into infrastructure and operating components. We use the multiproduct GM cost function, which is globally concave in input prices and which allows us to project fixed-but-not-sunk costs separately from stand-alone variable costs, to implement tests for *orthogonal separation* and *operational complementarity*.

When we apply these tests to U.S. freight railroads, we do not find there would be a significant loss of technical efficiency if operations were separated from infrastructure (Test I), but we also find that the railroad operational cost function is itself subadditive (Test II). Separating out of intermodal-only operations would be least costly from an efficiency standpoint,

[14] This scenario would have some policy appeal given the unique marketing and operational requirements of intermodal freight and its potential for diverting truck traffic from the highways.
[15] This finding could have a bearing on the current regulatory procedures of the Surface Transportation Board (STB) in the U.S. which uses an accounting-based stand-alone cost test to assess rail coal rates. It is unlikely that the accounting procedure measures the significant cross-cost effects.

while separating out of bulk operations would be most costly. Our results suggest that competition policy is technically feasible for U.S. freight railroads but that it would need to be complemented by regulation of operations and (in the North American context) regulation of infrastructure as well. It is beyond the scope of this paper to ask whether such a dual system would be better than the current system which regulates integrated firms.

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Table A1. Parameter Results

Param	Estimate	Std Error	Param	Estimate	Std Error	Param	Estimate	Std Error
Wl	155134.7	67962	Wf*T	-2764.73	800.9	Wl*H*Yb	-0.00723	0.0403
Wf	16037.42	6102.6	Wf*T*T	194.4848	64.9913	Wl*H*Ye	-0.03139	0.0398
We	-93710.5	43456.6	We*T	9033.738	4700.8	Wl*H*Yv	-0.07491	0.0396
Wm	-99749.9	52712.8	We*T*T	-577.276	296.2	Wl*H*Yi	-7.93795	16.9403
Wl*Wl	-0.04436	0.0111	Wm*T	13884.27	6240	Wf*T*Yb	-0.00361	0.000701
Wl*Wf	-0.00406	0.00162	Wm*T*T	-1046.55	472.8	Wf*T*Ye	-0.00279	0.000696
Wl*We	-0.00428	0.00341	Wl*R	19039.29	11521.1	Wf*T*Yv	-0.00088	0.000967
Wf*Wf	-0.00417	0.000667	Wl*R*R	-1908.96	980	Wf*T*Yi	-0.30608	0.2474
Wf*We	0.004346	0.000935	Wf*R	-3866.33	984.2	Wf*R*Yb	0.003217	0.000864
We*We	-0.00821	0.00284	Wf*R*R	266.3787	88.3966	Wf*R*Ye	-0.00119	0.000806
Wl*Wm	0.003463	0.0102	We*R	9232.81	2408.7	Wf*R*Yv	-0.00733	0.00131
Wf*Wm	-0.00152	0.00163	We*R*R	-429.605	199.9	Wf*R*Yi	-0.16282	0.2621
We*Wm	-0.00122	0.00334	Wm*R	7690.292	10206.9	Wf*H*Yb	0.000126	0.00388
Wm*Wm	-0.07744	0.0153	Wm*R*R	498.0913	857.6	Wf*H*Ye	0.001196	0.00386
Wl*Yb	0.104605	0.1329	Wl*H	-1427.3	25060.6	Wf*H*Yv	-0.00735	0.00385
Wl*Ye	0.25824	0.1253	Wl*H*H	4322.702	7392.9	Wf*H*Yi	-1.06098	1.4926
Wl*Yv	0.247698	0.174	Wf*H	-961.708	2062.1	We*T*Yb	0.001083	0.00168
Wl*Yi	20.41114	48.4499	Wf*H*H	-300.17	598.4	We*T*Ye	0.005595	0.00168
Wf*Yb	0.02887	0.0135	We*H	2140.612	4976.9	We*T*Yv	-0.00472	0.00243
Wf*Ye	0.064498	0.0135	We*H*H	-248.087	1308.1	We*T*Yi	0.156296	0.4934
Wf*Yv	0.125118	0.0187	Wm*H	13885.22	21779.2	We*R*Yb	0.002419	0.00122
Wf*Yi	4.43449	4.4561	Wm*H*H	-524.672	6756	We*R*Ye	-0.00209	0.00118
We*Yb	-0.02892	0.0232	Wl*T*R	-1049.75	401.1	We*R*Yv	0.001081	0.00213
We*Ye	0.024056	0.0237	Wl*T*H	-879.59	871.6	We*R*Yi	0.093933	0.3098
We*Yv	-0.01217	0.0393	Wl*R*H	2448.885	1972.6	We*H*Yb	0.001184	0.00773
We*Yi	1.766886	7.8797	Wf*T*R	164.3964	35.2903	We*H*Ye	-0.0155	0.00721
Wm*Yb	0.107562	0.1199	Wf*T*H	71.32097	72.9208	We*H*Yv	0.006814	0.00809
Wm*Ye	0.19935	0.1127	Wf*R*H	647.1652	185.1	We*H*Yi	-3.86229	2.8731
Wm*Yv	0.21305	0.1382	We*T*R	-232.081	87.3522	Wm*T*Yb	0.009543	0.00657
Wm*Yi	27.07178	49.3366	We*T*H	51.3543	174.8	Wm*T*Ye	-0.0081	0.00635
Yb*Yb	**2.05E-13**	**7.17E-14**	We*R*H	850.3551	396.7	Wm*T*Yv	0.00874	0.00936
Ye*Ye	**1.08E-13**	**6.19E-14**	Wm*T*R	-166.873	339.2	Wm*T*Yi	-6.56137	2.7305
Yv*Yv	**2.38E-13**	**1.20E-13**	Wm*T*H	-200.137	740.4	Wm*R*Yb	0.006485	0.00636
Yi*Yi	**-2.98E-08**	**8.05E-09**	Wm*R*H	-621.991	1825.2	Wm*R*Ye	-0.00099	0.00522
Yb*Ye	**-2.69E-13**	**5.70E-14**	Wl*T*Yb	-0.00151	0.0079	Wm*R*Yv	-0.00593	0.00724
Yb*Yv	**-6.27E-14**	**5.39E-14**	Wl*T*Ye	-0.00681	0.00754	Wm*R*Yi	-2.60343	1.7087
Yb*Yi	**2.28E-11**	**1.72E-11**	Wl*T*Yv	0.016771	0.0111	Wm*H*Yb	-0.00085	0.0364
Ye*Yv	**3.45E-14**	**6.12E-14**	Wl*T*Yi	4.916959	2.9799	Wm*H*Ye	0.059095	0.0352
Ye*Yi	**3.13E-11**	**1.41E-11**	Wl*R*Yb	0.018957	0.00696	Wm*H*Yv	-0.06024	0.0343
Yv*Yi	**6.83E-11**	**2.30E-11**	Wl*R*Ye	0.009752	0.00604	Wm*H*Yi	23.93022	15.941
Wl*T	-22627.2	8447.2	Wl*R*Yv	-0.00263	0.00869	Yi	74.6653	86.0439