“Honor thy father and thy mother” or Not: Uncertain family aid and the design of social long term care insurance*

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Abstract

We study the role and the design of long-term care insurance programs when informal care is uncertain; with and without active actuarially-fair private insurance markets against dependency. Three types of public insurance policies are considered: (i) a topping-up scheme, (ii) an opting-out scheme, and (iii) an opting-out-cum-transfer scheme which combines elements of the first two. A topping-up scheme can never do better than private insurance; opting out and opting-out-cum-transfer schemes can because they provide some insurance against the default of informal care. Long-term care policies have different implications for crowding out. A topping-up policy entails crowding out at both intensive and extensive margins and an opting-out policy leads to crowding out solely at the extensive margin. The opting-out feature of an opting-out-cum-transfer policy too leads to crowding out at the extensive margin, but its transfer element leads to crowding out at the intensive margin and crowding in at the extensive margin.

JEL classification: H2, H5.

Keywords: Long term care, uncertain altruism, private insurance, public insurance, topping up, opting out, opting-out-cum-transfers.

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1 Introduction

The interaction between market, state and family in providing protection against lifetime risks is a topic of immense interest in economics. In earlier times, family was the exclusive provider of social protection; then came the state and the market. The emergence of these latter institutions has been both a cause and a contributing factor to the decline of family involvement. This has given rise to many questions about the role that each of these three institutions can or should play as the society's risk insurer. Prescribing an exact mission for each institution is of course a somewhat impossible task. Instead, economists have mostly concerned themselves with the issue of crowding out—the withdrawal of the market or the family from providing services when the state provides them (seemingly for free).[1]

The risk of old-age dependency, and provision of long-term care (LTC), is a particular lifetime risk that has garnered a lot of attention in recent years. LTC is different from—albeit often complementary to—health care, particularly terminal care or hospice care. It concerns the dependent elderly who need help to carry out their daily activities (and may or may not require medical care). Providing this type of assistance is labor intensive and often quite costly, specially in severe cases of dependency that call for institutional care.

Currently, dependency presents the elderly with a significant financial risk of which social insurance covers only a small part.[2] As to the private insurance markets, health insurers typically reimburse services deemed to be of medical nature; they do not cover LTC costs. At the same time, private insurance markets dedicated solely to the provision of LTC are thin and very expensive. As a consequence, individuals often have to rely on their own private savings or on the informal care their family members provide which continues to represent a significant part of total LTC provision; see Norton (2000, 2016). This is often insufficient and leaves the elderly who cannot count on family solidarity without proper care.

Various societal trends point to an accelerating decline in family involvement. Family solidarity closely depends on the survival of a spouse and on the geographical proximity of children. Over the past few decades, we have seen an increasing number of elderly living alone because of divorce and widowhood. As to children, childless families are not infrequent and the mobility of children can make nursing assistance somewhat impossible. Increased female labor force participation, population aging, and drastic changes in family values are other contributing factors. Moreover, long-run trends aside, informal care is subject to many random shocks. There are pure demographic factors such as widowhood, absence, or loss of children; divorce and migration too can be put in this category. Children's financial problems, and conflicts within the family, might also prevent children from helping their parents.

The decline in informal care, whatever the reason, makes the need for formal LTC insurance—private or social—a very pressing issue in the coming decades. To be sure, there are two sources of uncertainty giving rise to the problem of formal LTC provision: One is the state of health

[1]On the LTC programs' crowding out of family provision or the purchase of private insurance, see, e.g., Cremer *et al.* (2012b) and Grabowski *et al.* (2012).

[2]In the US, Medicare does not cover LTC; Medicaid, which is offered to families with minimal private sources, does.

in old age and the other the availability of informal care from the family. As far as private insurance markets are concerned, while they could potentially provide coverage against the risk of dependency per se, the uncertainty associated with the level of informal care appears to be a mostly uninsurable risk. This particular form of market failure creates a potential role for public intervention. However, it is unlikely that public administrators have better information than parents themselves about their prospects of receiving informal care in case of dependency. That is, the government cannot condition its assistance to the elderly directly on the default of altruism; only on old-age dependency. Consequently, public intervention would not lead to a first-best outcome either. Nevertheless an interesting policy question arises. Can, under these circumstances, the government design second-best policies that might do better than the partial insurance private insurance markets against dependency provide (in terms of coverage and/or costs)? This is the question that lies at the heart of our study.

To study the role and design of LTC policies when altruism is uncertain, we consider a single generation consisting of parents and children. Initially, we concentrate on the welfare of the parents over their life cycle. Subsequently, in a final section, we investigate how the results might be affected when social welfare also includes the children's utility. Parents work, consume, and save for their retirement when young. In old age, they face a risk of becoming dependent. The probability of dependency is exogenously given and known. On the other hand, when making their savings and insurance decisions, parents do not know if their children would take care of them should they become dependent. Nor do they know the extent of the assistance if it is forthcoming. We represent this uncertainty by a single parameter called the children's "degree of altruism" and assume that it is continuously distributed over some interval.[3] Our conception of altruism is broad. It includes willingness to help as well as the required financial ability to provide care (which may entail, beyond some level, reducing one's labor supply).

As our starting point, we show that even if private insurance markets for dependency exist and are actuarially fair, they leave dependent parents who end up without informal care under-insured. Then, having established that uncertainty in altruism creates a potential role for public provision of LTC, we study the design of public LTC policies. Specifically, we consider three schemes—two of which are often used in connection with provision of private goods by the public sector. In one, referred to as a topping-up scheme (TU), the transfer to dependent parents is conditional on dependency alone. This kind of transfer can be supplemented by informal and market care. In the second, referred to as an opting-out scheme (OO), LTC benefits are exclusive and cannot be topped up. The OO scheme can, for instance, provide free or subsidized institutional care. Third, we consider a more refined OO policy that allows parents to choose between two options, say, a monetary help for care provided at home and a nursing home care provided on an opting out basis. This policy, which we call an "opting-out-cum-transfers" scheme (OC), combines the offer of an exclusive LTC to whoever wants to opt in with a transfer to those who opt out. Interestingly, the transfer can be positive or negative (in the form of a tax). The rationale for, and implications of, all three schemes are studied with and without actuarially fair private insurance markets for dependency.

[3]One can also think of this parameter as indicating the inverse of a child's cost of providing care.

The question of LTC provision under uncertain altruism has previously been studied by Cremer *et al.* (2014, 2017). Our paper is different from theirs in two major respects. First, they assume that altruism is a binary variable: Children are either altruistic with some known degree or not altruistic at all. In our setup, the altruism parameter is a continuum. Modeling a continuous distribution for the degree of altruism is not simply an academic exercise. It brings to light the various tradeoffs involved within and across different LTC schemes and plays a fundamental role in policy design. The very distinction between crowding out at the extensive and intensive margins, that has important and different implications for each of the schemes we study, is not even meaningful in the binary model. This is particularly true for the opting-out-cum-transfer policies; the tradeoffs we identify there are completely obscure in the binary model. Second, within the framework of the binary model, Cremer *et al.* (2017) compares TU and OO policies and Cremer *et al.* (2014) concentrates only on OO.[4] Neither paper studies the opting-out-cum-transfer policies.[5]

Different LTC schemes may coexist within a given country; although, in practice, most are of the TU type.[6] These include (possibly means-tested) cash transfers like APA (Allocation Personalisée d'Autonomie) in France, the "Pflegegeld" in Germany, and "Assegno d'Accompagnamento" in Italy. They also include in-kind transfers like "meals on wheels" or formal home care services provided for free, or at subsidized rates, in most European countries. Scandinavian countries offer a choice between formal care provided at home or at institutions. Institutionalized elderly may have to pay a rent and may be granted a personal-need allowance to pay for residual consumption. It is nevertheless the case that, even in Scandinavian countries where LTC insurance is primarily based on formal care provision, dependent individuals continue to rely heavily on informal care; see Karlsson *et al.* (2010).

The closest example of a OO scheme is the formal care that nursing homes provide (even though in practice relatives provide some additional informal care like visits and assistance during meals etc.) The pure OO policy considered in this paper is a theoretical limiting case. But real world policies are clearly not optimal (and keep being reformed). In most countries, public nursing facilities are of poor quality and chronically under-staffed because of insufficient funding as well as lack of sufficient supply of caregivers in the labor market. Offering nursing home care as a last resort, combined with incentives to stay home if dependency is not too severe, serves as an example of an opting-out-cum-transfer policy (with a positive TU component). This type of policy, aimed at promoting informal care, is increasingly being put in place. Some examples include LTC leaves for working children that enables them to combine care with a professional activity (the Netherlands), cash transfers to the elderly being cared for by family

[4]This latter paper also allows for the parents to affect the children's caregiving decisions thus making the probability that children provide care is endogenous.

[5]Our paper also contributes to the general literature on in-kind versus cash transfers which has extensively studied the properties of TU and OO schemes both from a positive and a normative perspective. On the normative side, for instance, Blomquist and Christiansen (1998) show that both regimes can be optimal (to supplement an optimal income tax) depending on whether the demand for the publicly provided good increases or decreases with labor. From a positive perspective, TU regimes may emerge from majority voting rules, as shown by Epple and Romano (1996). For a review of the literature, see Currie and Gahvari (2008).

[6]For an overview of different policies and financing models in the EU, see Lipszyc *et al.* (2012) and European Commission (2013).

members (Germany), training and support services for caregivers, and respite care for families taking care of dependent individuals (Sweden).[7]

A major concern often raised in the LTC literature, regarding the efficacy of public programs, is that of the crowding out of informal care; see Cremer *et al.* (2012). It is important to distinguish between two types of crowding out: at the "intensive margin" and at the "extensive margin". The intensive margin refers to the reduction in the informal care children provide when social LTC becomes available. Crowding out at the extensive margin occurs when some children are dissuaded from providing any informal care. We show that TU and OO policies have different effects on informal care. Whereas OO crowds out informal care only at the extensive margin, TU entails crowding out both at the intensive and the extensive margins. Given this property, one might be tempted to think that the OO always dominates the TU policy. However, as the general theory of the second best has taught us, this type of reasoning is faulty. In our model, if the share of non-altruistic children in the population is large, the crowding out at the intensive margin is rather small in the aggregate and that might make TU the preferable policy.

The most interesting tradeoffs arise under an opting-out-cum-transfer scheme. As with the pure OO policy, the offer of an exclusive LTC entails crowding out at the extensive margin. On the other hand, cash transfers to dependent parents who opt out have the opposite effect.[8] This is because such a transfer lowers the children's cost of providing informal care thus encouraging more children to opt out and assist their parents. Nevertheless cash transfers continue to entail crowding out at the intensive margin. The opting-out-cum-transfers scheme enables the children who already assist their parents under a pure OO system to cut their transfers by the same amount that the government gives to parents (keeping the parents' consumption levels unchanged).

That the crowding outs associated with transfer component of an opting-out-cum-transfer scheme go in opposite directions tells us that there is a likelihood that the transfer should be negative. Put differently, the optimal policy may require the parents who opt out should be taxed rather than subsidized (paid in the second period). The upshot is that the dependent parents who opt out should be given a positive transfer if the extensive margin effect dominates, and be taxed if the intensive margin effect dominates. Effectively, this enables the government to treat ex-ante identical parents differently (as far as their tax and transfers are concerned).

Finally, we consider two extensions to our base model and examine the implication of each for our results. In one, we incorporate actuarially fair insurance markets for dependency in our model. We find that the laissez-faire allocation in this case is identical to the outcome of the TU policy under our base model. This tells us that with actuarially fair insurance markets there is no role for a TU policy. By contrast, an OO policy or an opting-out-cum-transfer policy preserve their potential welfare-enhancing role. That is, even in the presence of actuarially fair insurance markets for dependency, the may lead to an outcome preferable to the laissez-faire equilibrium.

[7] For a survey of these policies in OECD countries, see Gori *et al.* (2016).

[8] Transfers under this scheme differ from cash transfers under TU in that the latter *gives them to everyone.*

The second extension concerns the treatment of children's utility in the social welfare function. Crowding out, while bad for parents, may be beneficial to children because it can mitigate the cost that dependency imposes on informal caregivers. To examine this, we extend our base model and consider a broader social welfare function which includes the utility of children as well. This calls for a higher level of government assistance under TU as compared to our base model. On the other hand, the expansion of public LTC is not necessarily warranted for the other two policies even though the basic tradeoffs outlined for the base model remains the same. These latter two policies have mixed effects on the children's utilities. To the extent that they reduce the cost of providing informal care to the children, as in a TU regime, they will be beneficial. However, they can also be detrimental in that they might lead the parents to save less thus shifting a larger share of the burden to informal caregivers.

2 The model

Consider a single generation of parents and children, each treated as a single unit, over two periods of their lifetime. All parents are identical *ex ante* and face two types of risk. First is the risk of becoming dependent when they are old and retired; the second pertains to the informal care they may or may not receive, if they become dependent, from their grown-up children. Provision and the extent of informal care depends on how altruistic the children are.

The sequence of events/actions, described as a game, is as follows. Period 0 constitutes the first stage when the government *formulates and announces* its tax/transfer policy. Period 1 is the second stage when young working parents decide on their savings. Period 2 is when the parents have grown old, are retired, and may or may not be dependent. The game will be over for parents who remain healthy in old age; they simply consume their savings. Dependent parents, on the other hand, move to the third stage where their children, who have by now turned into working adults, decide how much informal care, if any, they want to provide their parents with.[9]

The two sources of uncertainty come into play in the first stage when parents are to make their savings decisions. The probability of their becoming dependent when old, π, is exogenously given and known. The second source of uncertainty, the degree of altruism of their children when grown up, is represented by a random variable $\beta \geq 0$ distributed according to the distribution function $F(\beta)$ with density $f(\beta)$.[10] Altruism is to be understood broadly; it captures not just the willingness to help but also the financial ability to provide informal care. The higher is β the more altruistic a child is. Children with $\beta = 0$ have no altruistic feelings toward their parents. We assume that $F(\cdot)$ is concave; this implies $F(\beta) > \beta f(\beta)$.[11]

Parents have preferences over consumption when young, $c \geq 0$, consumption when old and healthy, $d \geq 0$, and consumption when old and dependent, $e \geq 0$ (which is probabilistic and inclusive of LTC services). Parents associate no disutility to work and supply a fixed amount

[9] In our setup, parents always find it optimal to accept the informal care that their children are willing to provide regardless of their participation in any public scheme.

[10] We rule out $\beta < 0$. A negative β implies that children will become happier if their parents are worse off.

[11] This condition is sufficient (but not always necessary) for most of the second-order condition of the paper to be satisfied and for some comparative statics results. We shall point out explicitly where and how it is used.

of labor when young. Their preferences are quasilinear in c; risk aversion is introduced through the concavity of the state-dependent utilities in the second period. Denote the utility function for consumption when old and healthy by $U(d)$ and when old and dependent by $H(e)$. The parents' life-time expected utility is

$$EU = c + (1-\pi)U(d) + \pi E[H(e)], \tag{1}$$

where $E(\cdot)$ is the expected value operator. Assume that $U' > 0$, $U'' < 0$, $U(0) = 0$, $U'(0) = \infty$, and that the same properties hold for H.

Grown-up children too have quasilinear preferences and their altruism toward their parents comes into play only if the parents become dependent. The children's utility function is represented by

$$u = \begin{cases} y - a + \beta H(e) & \text{if the parent is dependent,} \\ y & \text{if the parent is non-dependent,} \end{cases} \tag{2}$$

where y denotes the children's fixed income and $a \geq 0$ denotes any transfers that they might make to their dependent elderly parents. No transfers are made to the healthy elderly parents regardless of the size of their savings, s.

2.1 Laissez faire—No private insurance markets

Parents' uncertainty regarding the degree of altruism of their grown-up children plays a central role in any potential justification for government intervention. To understand this, it will be helpful to compare the equilibrium solutions that emerge in the laissez faire with and without private insurance markets. We begin with the case that there are no private insurance markets. Proceeding by backward induction, consider the last decision-making stage in our setup.[12] This is when the grown-up children decide on the extent of their help to their parents, if any.

2.1.1 Stage 3: The children's choice

Children supplement the saving s of their dependent parents by an amount $a \geq 0$ from their own income y. The optimal level of assistance, a^*, is found through the maximization of equation (2). The first-order condition with respect to a is, assuming an interior solution,

$$-1 + \beta H'(s+a) = 0. \tag{3}$$

Concavity of $H(\cdot)$ ensures that the second-order condition is satisfied. Equation (3) implies that, with an interior solution, a^* satisfies[13]

$$s + a^* = (H')^{-1}\left(\frac{1}{\beta}\right) \equiv m(\beta), \tag{4}$$

where the concavity of $H(\cdot)$ implies that $m'(\beta) > 0$.

[12] There is no first stage in the laissez faire. However, to be consistent with the sections that follow, we refer to the last and the next-to-last stages as 3 and 2.

[13] A corner solution at $a = y$ cannot be ruled out. To avoid a tedious and not very insightful multiplication of cases we assume throughout the paper that the constraint $a \leq y$ is not binding in equilibrium.

Setting $a = 0$ in (3) gives the minimum level of β for which a child provides a positive level of care to his parent who has saved s. Denoting this level by $\beta_0(s)$, we have

$$\beta_0(s) \equiv \frac{1}{H'(s)}. \tag{5}$$

We shall refer to a child on the verge of providing informal care to his parents as the "marginal child".[14] When $\beta < \beta_0$, we have $a^* = 0$ and the parents' consumption is equal to his own savings. To sum,

$$e = \begin{cases} s & \text{if } \beta < \beta_0(s), \\ s + a^* = m(\beta) & \text{if } \beta \geq \beta_0(s). \end{cases} \tag{6}$$

Differentiating (6) with respect to β yields

$$\frac{de}{d\beta} = \begin{cases} 0 & \text{if } \beta < \beta_0(s), \\ m'(\beta) = \frac{-1}{\beta^2 H''(e)} > 0 & \text{if } \beta \geq \beta_0(s), \end{cases}$$

where the sign of $m'(\beta)$ follows from the concavity of $H(\cdot)$.[15] As expected, a dependent parent's total consumption increases with the degree of altruism of his child. Figure 1 illustrates the relationship between old-age consumption e and β.

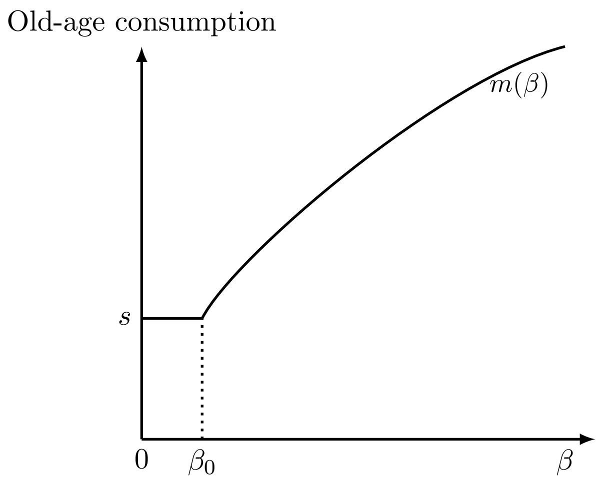

Figure 1: *Laissez faire with no private insurance markets: consumption of dependent parents as a function of the children's degree of altruism.*

Finally, observe that parents' savings crowd out informal care in two ways. The first is crowding out at the intensive margin. Equation (6) indicates that, as long as children continue to provide informal care, its amount is crowded out by parents' savings on a one-to-one basis. Second, differentiating (5) tells us that,

$$\frac{d\beta_0}{ds} = -\frac{H''}{(H')^2} > 0.$$

Hence increasing savings increases β_0 and in this way reduces the likelihood of children assisting their parents in case of dependency (crowding out at the extensive margin).

[14] The "marginal child" will be a different child depending on the considered economic setting.

[15] The function m is not differentiable at $\beta = \beta_0$. To avoid cumbersome notation we use $m'(\beta)$ for the right derivative at this point.

2.1.2 Stage 2: The parents' choice

Recall that, when retire, parents will either be healthy or dependent (with probabilities $1-\pi$ and π). If healthy, their sole means of consumption is their own saving s as they will receive no transfers from their children. If dependent, they may or may not receive a transfer depending on the children's degree of altruism β. Denote the (fixed) labor supply of parents by $\overline{T}$ and their wage by w so that their expected lifetime utility is

$$EU = w\overline{T} - s + (1-\pi)\,U(s) + \pi E\left[H(e)\right], \tag{7}$$

where e is equal to s when $\beta < \beta_0(s)$ and equal to $s + a^*(\beta, s)$ when $\beta \geq \beta_0(s)$. From (4), substitute $m(\beta)$ for $s + a^*(\beta, s)$ into (7) to get

$$EU = w\overline{T} - s + (1-\pi)\,U(s) + \pi\left[H(s)\,F(\beta_0) + \int_{\beta_0}^{\infty} H(m(\beta))\,dF(\beta)\right].$$

Maximizing EU with respect to s, and assuming an interior solution, the optimal value of savings, s^{LF}, satisfies[16]

$$(1-\pi)\,U'\left(s^{LF}\right) + \pi F(\beta_0(s^{LF}))H'\left(s^{LF}\right) = 1, \tag{8}$$

where the second-order condition is also satisfied due to concavity of $F(\beta)$.[17]

Equation (8) states that the expected benefit of saving must be equal to its cost (which is equal to one). The first term on the left-hand side shows the benefit of saving to elderly parents if they remain healthy when old; the second term indicates the benefit of saving to them if they become dependent but would not receive informal care. Saving is of no benefit to dependent parents who would receive assistance from their children: own saving crowds out "free" informal care from children on a one to one basis. Observe also that equation (8) contains no terms relating to $\partial m/\partial s$. This is because, with $m(\beta_0) = s$, the derivatives of EU with respect to β_0 cancel out.

Finally, empirical evidence suggests that $H'(s) > U'(s)$.[18] Under this assumption, equation (8) implies $H'(s^{LF}) > 1$.[19] Consequently, dependent parents who do not receive informal care are under-insured.[20] The question is if private insurance markets can take care of this under-insurance. The important point to bear in mind here is that because of the personalized nature

[16] A corner solution at $s = 0$ can be excluded by the assumption that $U'(0) = \infty$. However, a corner solution at $s = w\overline{T}$, yielding $c = 0$, cannot be ruled out. To avoid a tedious and not very insightful multiplication of cases we assume throughout the paper that the constraint $c \geq 0$ is not binding in equilibrium (even when first period income is taxed to finance social LTC).

[17] The second-order condition is given by

$$(1-\pi)\,U''(s) + \pi F(\beta_0)H''(s) + \pi f(\beta_0)H'(s)\,\frac{\partial \beta_0}{\partial s} < 0.$$

Or, substituting for $d\beta_0/ds$,

$$(1-\pi)\,U''(s) + \pi H''(s)\left[F(\beta_0) - \beta_0 f(\beta_0)\right] < 0,$$

for which the concavity of $F(\beta)$ represents a *sufficient* condition.

[18] See Ameriks et al. (2019) and Lillard and Weiss (1997) who find that "a fall into poor health raises the marginal utility of consumption".

[19] Assume the contrary so that $H'(s^{LF}) \leq 1$. This implies $F(\beta_0(s^{LF}))H'\left(s^{LF}\right) < 1$ resulting in $U'\left(s^{LF}\right) < H'\left(s^{LF}\right) < 1$. Hence the left-hand side of (8), a weighted average of $U'\left(s^{LF}\right)$ and $H'\left(s^{LF}\right)$, must also be less than one. And we have a contradiction.

[20] Full insurance is achieved when $H'(e) = 1$; i.e. when the benefit of one extra dollar of consumption when dependent is equal to its cost.

of informal care, one can never insure himself against lack or insufficient care from one's children. However, private insurance markets *for dependency* can exist—at least in principle. We now examine the implications of such a market for the under-insurance we have discovered under laissez faire.

2.2 Actuarially-fair private insurance markets for dependency

Let δ denote the amount of insurance against old-age dependency that a parent purchases at the actuarially fair premium of $\pi\delta$. Children then expect their parents to have $s+\delta$ resources of their own to consume in case of dependency rather than s. Turning to the parents, their expected utility (7) will change to

$$EU = w\overline{T} - s - \pi\delta + (1-\pi)\,U\,(s) + \pi\left[H\,(s+\delta)\,F(\beta_0) + \int_{\beta_0}^{\infty} H\,(m\,(\beta))\,dF(\beta)\right]. \qquad (9)$$

Maximizing EU with respect to s and δ, and continuing to assume that the solution for s is interior, there are two possible outcomes.

Case (i): The solution for δ is interior. Under this circumstance, the optimal value of δ and of s, denoted by s^{FI} to distinguish it from s^{LF}, satisfy equations[21]

$$(1-\pi)\,U'\left(s^{FI}\right) + \pi F\left(\beta_0\left(s^{FI}+\delta\right)\right)H'\left(s^{FI}+\delta\right) = 1, \qquad (10)$$

$$F(\beta_0\left(s^{FI}+\delta\right))H'\left(s^{FI}+\delta\right) = 1. \qquad (11)$$

Again note the absence of derivatives with respect to β_0 in the first-order conditions (10)–(11). In this case, because $m(\beta_0) = s+\delta$, the derivatives cancel out. Substituting from equation (11) into equation (10), we have[22]

$$U'\left(s^{FI}\right) = 1, \qquad (12)$$

$$H'\left(s^{FI}+\delta\right) = 1/F(\beta_0) > 1. \qquad (13)$$

Equation (12) shows that healthy parents' consumption is at its first-best optimum (i.e., its marginal benefit is equal to its marginal cost); equation (13) shows that dependent parents who would not receive informal care are under-insured.

Case (ii): The problem yields a corner solution for δ. This arises if, at $\delta = 0$,

$$F\left(\beta_0(s)\right)H'\left(s\right) - 1 \leq 0.$$

Under this circumstance $\delta = 0$ and equation (11) is no longer valid. Setting $\delta = 0$ in equation (10) yields

$$(1-\pi)\,U'\left(s\right) + \pi F\left(\beta_0\left(s\right)\right)H'\left(s\right) = 1.$$

[21] We continue to assume that the constraint $c \geq 0$ is not binding in equilibrium.

[22] The second-order conditions are

$$\pi H''\left(s+\delta\right)\left[F(\beta_0) - \beta_0 f(\beta_0)\right] < 0,$$

$$\pi(1-\pi)U''\left(s\right)H''\left(s+\delta\right)\left[F(\beta_0) - \beta_0 f(\beta_0)\right] > 0,$$

which are satisfied due to the concavity of $H(\cdot), U(\cdot)$ and $F(\beta)$.

This is identical to the corresponding first-order condition in the absence of private insurance markets. Consequently, in this case,

$$s = s^{LF},$$
$$H'(s) = H'\left(s^{LF}\right) > 1,$$

and we are back to the laissez faire solution in the absence of private insurance markets. This can arise if $F(\beta_0(s))$ is "sufficiently" small. When almost all parents expects to be able to rely on informal care, the benefit of insurance is small and outweighed by its cost in terms of expected crowding out.

We also prove in Appendix A that

$$s^{FI} < s^{LF} < s^{FI} + \delta.$$

This is to say that private insurance markets lower private savings while increasing parents' overall resources for old-age consumption. Put differently, parents' purchases of private insurance more than make up for the decline in their savings. Intuitively, the availability of private insurance markets has two implications. On the one hand, active private insurance markets make it less expensive for a parent to insure himself so that $s^{FI} + \delta > s^{LF}$. On the other hand, when parents are able to insure themselves against dependency, they are effectively buying some insurance against not getting informal care when dependent. This reduces the self-insurance benefits of private savings and with it the amount of savings ($s^{FI} < s^{LF}$).

Two lessons are to be learnt from this discussion. First, actuarially-fair insurance markets for dependency improve the laissez faire outcome. Healthy parents will have equal marginal benefit of consumption in the two periods of their lives, while dependent parents increase their overall self insurance. Second, while private insurance mitigates the under-insurance problem, it does not eliminate it. Private insurance markets notwithstanding, $H'\left(s^{FI} + \delta\right) > 1$ so that dependent parents who would not receive informal care continue to remain under-insured. The reason is, of course, the unavailability of insurance against lack or insufficient informal care. Private markets cannot solve this problem.

We summarize the main results of this section in the following proposition.

Proposition 1 *In the context of a model with uncertain altruism represented by equations (1)–(2), actuarially-fair private insurance markets for dependency:*

(a) Increase the overall amount of self-insurance by parents while reducing the portion due to private savings.

(b) Leave dependent parents who are not getting informal care under-insured.

Having established that uncertainty surrounding children's altruism creates a potential role for the government in providing LTC, we next study the design of LTC policies. Observe that we implicitly assume that informal care, a, is not observable. The only exception is that $a = 0$ can be enforced to implement an *OO* policy. If a were fully observable, we could of course do better by using a nonlinear transfer scheme $g(a)$ to screen for the β's. This would amount to characterizing the optimal incentive-compatible mechanisms of which *TU*, *OO*, and mixed policies are special cases.

3 Topping up

The government provides LTC insurance, g, to all dependent elderly whether or not they receive informal care. This is done in the form of a good which is non-exclusive in the sense that it can be topped up by a and s. The policy is financed through a proportional tax at rate τ on the parents' first-period exogenous income. The parents' expected utility is then given by

$$EU^{TU} = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi E\left[H(s+g+a^*(\beta,s,g))\right], \tag{14}$$

where $a^*(\beta,s,g) \geq 0$ is care provided by children (which is shown in the next subsection to also depend on g). Preferences of grown-up children continue to be represented by equation (2), with $e = s+g+a$. Once again, we proceed by backward induction and start with the last stage.

3.1 Stage 3: The children's choice

Children allocate an amount $a \geq 0$ of their income y to assist their dependent parents (given the parents' savings s and the government's provision of g). The optimal level of transfers, a^*, is found through the maximization of equation (2). The first-order condition with respect to a, assuming an interior solution, is given by

$$-1 + \beta H'(s+g+a) = 0.$$

Setting $a = 0$ in above gives the minimum level of β for which a child provides a positive level of care to his parent who has saved s and receives g from the government. Denote this threshold by $\widetilde{\beta}$ to differentiate it from β_0, the threshold in the Laissez fair when there is no government provision. Thus, define $\widetilde{\beta}(s+g)$ such that

$$\widetilde{\beta}(s+g) \equiv 1/H'(s+g). \tag{15}$$

Observe that, from (15) and (5), $\widetilde{\beta}(\cdot)$ has the same functional form and $\beta_0(\cdot)$. Distinguishing between the two is helpful in keeping track of the solutions in different settings. Clearly, then, $\widetilde{\beta}(s+g) > \beta_0(s)$ for all $g > 0$.

It follows from (15) that, when $\beta \geq \widetilde{\beta}(s+g)$, a^* satisfies

$$e = s+g+a^* = \left(H'\right)^{-1}\left(\frac{1}{\beta}\right) \equiv m(\beta). \tag{16}$$

As depicted by the solid line in Figure 3, for all $\beta \geq \widetilde{\beta}(s+g)$ the consumption of dependent parents $m(\beta)$ is exactly the same as in the laissez faire. Thus, when children's altruism is in this range, government assistance crowds out informal care one to one (at the intensive margin). The crowding out stops when caregivers are brought to a corner solution; i.e. for $\beta = \widetilde{\beta}(s+g)$. When $\beta < \widetilde{\beta}(s+g)$, no informal care is provided, $a^* = 0$ and $e = s+g > m(\beta)$. In this range, g increases the total informal care beyond what parents receive through self-insurance and we have $\partial e/\partial g = 1$. Finally, same as with $d\beta_0(s)/ds > 0$, we have

$$\frac{\partial\widetilde{\beta}(s+g)}{\partial g} = \frac{d\widetilde{\beta}(s+g)}{d(s+g)} = -\frac{H''}{(H')^2} > 0. \tag{17}$$

As the total amount of formal care increases, the degree of altruism necessary to yield a positive level of informal care increases (for a given level of a parents' saving).

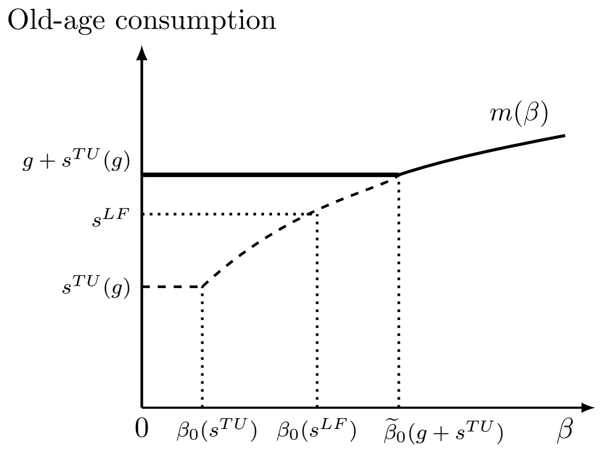

Figure 2: *Topping up – Consumption of dependent parents as a function of the children's degree of altruism*

3.2 Stage 2: The parents' choice

Recall that parents are dependent with probability π and healthy with probability $(1-\pi)$. Substituting for a^* from (16) in the parents' expected utility function (14), we have

$$EU^{TU} = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi\left[H(s+g)F(\widetilde{\beta}(s+g)) + \int_{\widetilde{\beta}(s+g)}^{\infty} H(m(\beta))\, dF(\beta)\right].$$

Parents choose s to maximize EU^{TU}. Again, with $m\left(\widetilde{\beta}(s+g)\right) = s+g$, the derivatives of EU^{TU} with respect to β_0 cancel out. The optimal value of s, assuming an interior solution, satisfies[23]

$$(1-\pi)U'(s) + \pi F(\widetilde{\beta}(s+g))H'(s+g) = 1. \tag{18}$$

Denote the solution to equation (18) by $s^{TU}(g)$. Substituting $s^{TU}(g)$ for s in (18), the resulting relationship holds for all values of g. Totally differentiating this relationship, while making use of (17) and of the concavity of $F(\cdot)$, yields

$$\frac{ds^{TU}}{dg} = -\frac{\pi H''\left(s^{TU}+g\right)\left[F(\widetilde{\beta}) - \widetilde{\beta} f\left(\widetilde{\beta}\right)\right]}{(1-\pi)U''(s^{TU}) + \pi H''(s^{TU}+g)\left[F(\widetilde{\beta}) - \widetilde{\beta} f\left(\widetilde{\beta}\right)\right]} < 0. \tag{19}$$

Consequently, $s^{TU}(g)$ decreases with g. This is not surprising. Savings play a self-insurance role for the dependent parents in case they do not receive informal care in addition to serving as the sole source of consumption for healthy parents. As public LTC becomes available, the expected self-insurance benefits associated with s become less important. Parents will be able to count on g even when their children fail to deliver; consequently the marginal benefit of s decreases in g. The above expression also shows $ds^{TU}/dg > -1$ so that g does not fully crowd out s^{TU}.

[23] The second-order condition, upon substitution for $\partial\widetilde{\beta}/\partial(s+g)$ from (17), is given by

$$(1-\pi)U''(s) + \pi H''(s+g)\left[F(\widetilde{\beta}) - \widetilde{\beta} f\left(\widetilde{\beta}\right)\right] < 0,$$

which is satisfied due to the concavity of $F(\cdot)$.

Finally, substitute $s^{TU}(g)$ for s in (15) to write $\widetilde{\beta}$ as a function of g only:

$$\widetilde{\boldsymbol{\beta}}(g) \equiv \widetilde{\beta}\left(s^{TU}(g) + g\right).$$

Totally differentiating $\widetilde{\boldsymbol{\beta}}$ with respect to g yields

$$\frac{d\widetilde{\boldsymbol{\beta}}(g)}{dg} = \frac{\partial \widetilde{\beta}\left(s^{TU}(g) + g\right)}{\partial g}\left[1 + \frac{ds^{TU}}{dg}\right] < \frac{\partial \widetilde{\beta}}{\partial g}|_s.$$

Because g affects savings negatively, the positive direct effect of g on $\widetilde{\beta}$ diminishes. Substituting the expressions for $\partial\widetilde{\beta}/\ \partial g$ and ds^{TU}/dg, from (17) and (19), into the above and simplifying results in

$$\frac{d\widetilde{\boldsymbol{\beta}}(g)}{dg} = -\frac{H''\left(s^{TU} + g\right)}{\left[H'\left(s^{TU} + g\right)\right]^2} \frac{(1-\pi)\,U''\left(s^{TU}\right)}{(1-\pi)\,U''\left(s^{TU}\right) + \pi H''\left(s^{TU} + g\right)\left[F(\widetilde{\beta}) - \widetilde{\beta} f\left(\widetilde{\beta}\right)\right]} > 0.$$

An increase in public LTC reduces the likelihood of children providing informal care and we have crowding out at the extensive margin.

3.3 Stage 1: The optimal policy

Government determines the optimal values of τ and g in such as way as to maximize the parents' optimized value of EU^{TU} determined in stage 2. The optimization is subject to the government's budget constraint

$$\tau w\overline{T} = \pi g. \tag{20}$$

Substitute $s^{TU}(g)$ for s and $\pi g/w\overline{T}$ for τ into EU^{TU} to rewrite it as a function of g only. The government's Lagrangian associated with the maximization of EU^{TU} with respect to g is

$$\begin{aligned}\pounds^{TU} \equiv\; & w\overline{T} - \pi g - s^{TU}(g) + (1-\pi)\,U\left(s^{TU}(g)\right) + \\ & \pi\left[F(\widetilde{\beta}\left(s^{TU}(g) + g\right))H\left(s^{TU}(g) + g\right) + \int_{\widetilde{\beta}(s^{TU}(g)+g)}^{\infty} H\left(m\left(\beta\right)\right) dF(\beta)\right].\end{aligned}$$

Differentiating $\pounds^{TU}$ with respect to g yields, using the envelope theorem,

$$\frac{d\pounds^{TU}}{dg} = \pi\left[F(\widetilde{\beta})H'\left(s^{TU}(g) + g\right) - 1\right]. \tag{21}$$

The first term in the bracketed expression on the right-hand side reflects the benefits of an increase in g. A dependent parent who does not receive informal care (their children's β is smaller than $\widetilde{\beta}$), gains $H'\left(s^{TU}(g) + g\right)$; the remaining dependent parents gain no benefit due to the crowding out effect of g on informal care. The second term reflects the unit cost of g.

To determine if the government will in fact provide LTC, evaluate the sign of $d\pounds^{TU}/dg$ at $g = 0$. One possibility is to have

$$F\left[\widetilde{\beta}\left(s^{TU}(0)\right)\right] H'\left(s^{TU}(0)\right) - 1 > 0,$$

In this case, there will be an interior solution for g, and τ, characterized by

$$H'\left(s^{TU}\left(g^{TU}\right) + g^{TU}\right) = \frac{1}{F\left[\widetilde{\beta}\left(s^{TU}\left(g^{TU}\right) + g^{TU}\right)\right]} > 1. \tag{22}$$

Consequently, the optimal TU does not provide full insurance. Moreover, substituting from (22) into (18) results in

$$U'(d) = U'\left(s^{TU}\left(g^{TU}\right)\right) = 1,$$

which implies that healthy parents' consumption is at its first-best optimum (i.e., its marginal benefit is equal to its marginal cost).

A different outcome occurs if

$$F\left[\widetilde{\beta}\left(s^{TU}\left(0\right)\right)\right] H'\left(s^{TU}\left(0\right)\right) - 1 \leq 0,$$

In this case, the solution is given by $g = \tau = 0$ and government need not provide any TU insurance. This occurs if $F[\widetilde{\beta}\left(s^{TU}\left(0\right)\right)]$ is sufficiently small. Under this circumstance, the probability that children provide free informal care is "large enough" as to make the benefit of insurance very small and outweighed by its cost. The laissez faire leaves some individuals (those whose children have a $\beta < \widetilde{\beta}$) without LTC benefits other than self-insurance. This is inefficient, but the TU policy we consider here cannot do any better.

We summarize our findings in the following proposition.

Proposition 2 *Consider a topping-up scheme financed by a proportional tax on earnings. Let* $s^{TU}\left(g\right)$ *solve*

$$\left(1-\pi\right)U'\left(s^{TU}(g)\right) + \pi F(\widetilde{\beta})H'\left(s^{TU}(g) + g\right) = 1,$$

where

$$\widetilde{\beta} = \frac{1}{H'\left(s^{TU}(g) + g\right)}.$$

(i) Public LTC insurance is not effective in supplementing informal care and $g^{TU} = 0$ *if*

$$F\left[\widetilde{\beta}\left(s^{TU}\left(0\right)\right)\right] H'\left(s^{TU}\left(0\right)\right) - 1 \leq 0.$$

(ii) Otherwise, there is an interior solution $g^{TU} > 0$ *implicitly defined by*

$$H'\left(s^{TU}\left(g^{TU}\right) + g^{TU}\right) = \frac{1}{F\left[\widetilde{\beta}\left(s^{TU}(g) + g\right)\right]}.$$

(a) This relationship balances insurance benefit against the crowding out cost of informal care.

(b) Public LTC reduces parents' private savings. as well as the likelihood of children providing informal care.

(c) There is crowding out at intensive and extensive margins.

(iii) Under both (i) and (ii), $H'\left(\cdot\right) > 1$ *so that there is less than full insurance.*

4 Opting out

Assume now that the government provides LTC on an exclusive basis in the sense that it cannot be topped up by a or s. The policy is only relevant when the amount of the assistance, G, exceeds a parent's private savings, s; otherwise, public assistance would be of no use to the parents. To receive it, one has to give up his own private savings to the government. Its net cost to the government is thus $G - s$ which makes it on a par with providing g in a topping-up policy. The program is voluntary and one can decide not to participate; that is, to opt out and rely instead on his own savings and children's assistance.[24]

4.1 Stage 3: The children's choice

Children's preferences continue to be given by (2). Hence their utility is equal to

$$u = y - a + \beta H\left(s + a\right), \tag{23}$$

if they provide informal care to their parents; and

$$u = y + \beta H\left(G\right), \tag{24}$$

if they decide not to assist their parents (who will then rely exclusively on public LTC insurance). If they provide care, they will do it at a level a^* that maximizes their utility given by (23). Hence a^* satisfies $\beta H'\left(s + a^*\right) = 1$ or $s + a^* = \left(H'\right)^{-1}\left(1/\beta\right) = m\left(\beta\right)$. This implies that with assistance from their children, parents' consumption will be equal to its level in the laissez-faire. Of course, children provide assistance only if it increases their utility above the level they get when they allow the parents to rely exclusively on public assistance.

Each child thus compares (23), evaluated at a^*, with (24) and provides care if

$$\beta\left[H(m(\beta)) - H\left(G\right)\right] > \left(m(\beta) - s\right).$$

In words, children provide informal care only if the utility gain from altruism $\beta[H(m(\beta)) - H\left(G\right)]$ exceeds the cost of care $a^* = m(\beta) - s$. By contrast, parents would prefer to opt out whenever $m(\beta) > G$. This implies that, whenever children decide to assist their parents, parents will definitely opt out from public LTC. Observe that the left-hand side of the above inequality is increasing in β for all $m(\beta) > G$.[25] Consequently, for each value of G and s, there exists a $\widehat{\beta}(G, s)$ such that all children with $\beta > \widehat{\beta}$ provide care and all children with $\beta \leq \widehat{\beta}$ provide no assistance. This threshold level, $\widehat{\beta}(G, s)$, is implicitly defined by

$$\widehat{\beta}\left[H(m(\widehat{\beta})) - H\left(G\right)\right] - \left(m(\widehat{\beta}) - s\right) = 0. \tag{25}$$

[24] In the US, eligibility for Medicaid, whose services include LTC, is based on having minimal private resources. This creates a perverse incentive for "not-quite-rich" people to transfer their savings to their relatives to become eligible. The *OO* policy we are considering allows all parents to participate as long as they are prepared to "transfer" their savings to the government.

[25] The derivative of the left-hand side with respect to β is

$$\left[H(m(\beta)) - H\left(G\right)\right] + \left[\beta H'\left(m(\beta)\right) - 1\right]\frac{\partial m}{\partial \beta}.$$

If children do not provide care, $\partial m/\partial \beta = 0$. If they do, $\beta H'\left(m(\beta)\right) - 1 = 0$. The above expression thus reduces to

$$\left[H(m(\beta)) - H\left(G\right)\right],$$

which is positive for all $m(\beta) > G$.

Differentiating (25) with respect to G and s yields

$$\frac{\partial\widehat{\beta}}{\partial G}=\frac{\widehat{\beta}H'(G)}{\left[H(m(\widehat{\beta}))-H(G)\right]}>0, \tag{26}$$

$$\frac{\partial\widehat{\beta}}{\partial s}=-\frac{1}{\left[H(m(\widehat{\beta}))-H(G)\right]}<0. \tag{27}$$

In the topping-up scheme, the threshold level of β moved positively with $g+s$. The higher either g or s, the less likely it was that children provide assistance (top up). Here a similar logic applies to G. The higher is G, the happier the children are with public LTC and the less inclined they are to provide informal care. Hence the threshold level increases with G. On the other hand, unlike in the topping up case, the threshold with opting out is *decreasing* in s. The reason is that a higher level of s reduces the amount of care the children have to provide to surpass a given level of public LTC, G.

Figure 3 illustrates how, under opting out, the dependent parents' consumption varies with their children's degree of altruism (solid line). If $\beta\leq\widehat{\beta}$, dependent parents consume G; if $\beta>\widehat{\beta}$, they opt out and consume $m(\beta)$ (which is equal to their laissez faire consumption). However, there is now a discontinuity in the level of m at $\widehat{\beta}$ (unlike at $\widetilde{\beta}$ in the topping-up regime). The discontinuity follows from equation (25) which shows that at $\widehat{\beta}$, the marginal child is just indifferent between his parent consuming G or consuming $m(\widehat{\beta})$. The first option costs the children nothing and the second $a^*=m(\beta)-s>0$. This implies, through equation (25), that $m(\widehat{\beta})>G$ so that the parents are strictly better off to the right of $\widehat{\beta}$.[26] In words, under *OO,* children provide care only if $m(\beta)$ is sufficiently larger than G to make up for the cost of care.

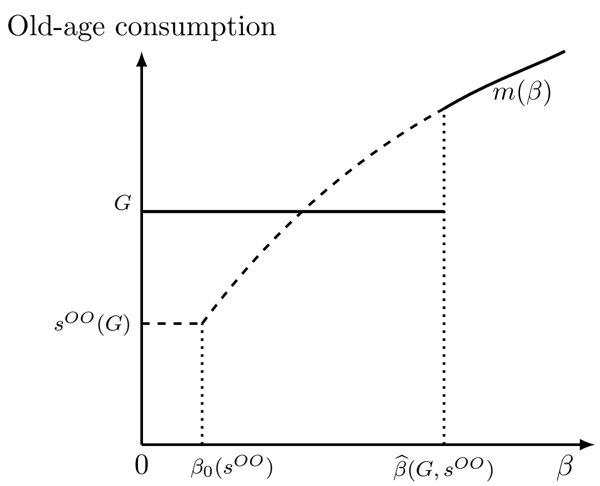

Figure 3: *Opting out: consumption of dependent parents as a function of children's degree of altruism*

[26]This also explains why, whenever children are willing to provide care, their parents accept it and forego G. Intuitively, while the children are altruistic they have to pay for the cost of care that comes free to the parents.

4.2 Stage 2: The parents' choice

The parents' expected utility is

$$EU^{OO} = w\,(1-\tau)\,\overline{T} - s + (1-\pi)\,U\,(s) + \pi\left[H\,(G)\,F(\widehat{\beta}) + \int_{\widehat{\beta}}^{\infty} H\,(m\,(\beta))\,dF(\beta)\right].$$

Maximizing EU^{OO} with respect to s, and assuming an interior solution, the optimal value of s satisfies

$$(1-\pi)\,U'\,(s) - \pi f(\widehat{\beta})\left[H(m(\widehat{\beta})) - H\,(G)\right]\frac{\partial\widehat{\beta}}{\partial s} = 1, \tag{28}$$

Note that in this case the derivatives with respect to $\widehat{\beta}$ do *not* cancel out. The marginal child is indifferent between providing care or not; but not his parent. And it is the children, and not the parents, who incur the cost of informal care. Substituting for $\partial\widehat{\beta}/\partial s$ from (27) into (28), one can rewrite the first-order condition for the maximization of EU^{OO} with respect to s as[27]

$$(1-\pi)\,U'\,(s) + \pi f(\widehat{\beta}\,(G,s)) = 1. \tag{29}$$

The second term on the left-hand side of (29), or equivalently (28), represents the positive effect of s on the likelihood that children provide assistance. Under OO saving is not useful in case of dependency: It is either fully taxed or is fully crowded out by the informal care. Yet saving affects the likelihood that children provide help. We have found that $\partial\widehat{\beta}/\partial s < 0$ under OO, meaning that private savings increase the likelihood of informal care under OO. Now, because parents are always better off under family assistance than under public assistance, this effect enhances the desirability of savings under OO.

Denote the solution to equation (29) by $s^{OO}(G)$ and substitute $s^{OO}\,(G)$ for s in (29). The resulting equation holds for all values of $G > s^{OO}(G)$. Its differentiation with respect to G yields

$$\frac{ds^{OO}}{dG} = \frac{-\pi f'(\widehat{\beta})\frac{\partial\widehat{\beta}}{\partial G}}{(1-\pi)\,U''\,(s^{OO}\,(G)) + \pi f'(\widehat{\beta})\frac{\partial\widehat{\beta}}{\partial s}} < 0, \tag{30}$$

where the sign follows from the concavity of F as long as the second-order condition for maximization of EU^{OO} is satisfied. Consequently, as with TU , $s^{OO}(G)$ decreases with G. This is because the benefit that saving provides in terms of self-insurance decreases as G increases. Saving does not provide any benefit to the parents who receive public LTC because it will be taxed away. However, an increase in G lowers the likelihood that children provide informal care. This is measured by the numerator of (30) and explains the negative sign of ds^{OO}/dG.

Finally, with s^{OO} being determined by G, one can rewrite the threshold level of altruism, $\widehat{\beta}(G,s)$, solely as a function of G (as was the case under TU in terms of g). Thus define

$$\widehat{\boldsymbol{\beta}}(G) \equiv \widehat{\beta}[G, s^{OO}(G)].$$

[27]We assume that the second-order condition for maximizing EU^{OO} with respect to s, found from differentiating the left-hand side of (29),

$$(1-\pi)\,U''\,(s) + \pi f'(\widehat{\beta})\frac{\partial\widehat{\beta}}{\partial s} < 0,$$

is satisfied. Unlike the second-order conditions previously encountered, this is not guaranteed with a concave $F\,(\cdot)$ which implies $f' = F'' < 0$.

Differentiating this relationship totally with respect to G yields,

$$\frac{d\widehat{\boldsymbol{\beta}}}{dG}=\frac{\partial\widehat{\beta}}{\partial G}+\frac{\partial\widehat{\beta}}{\partial s}\frac{ds^{OO}}{dG}=\frac{\widehat{\beta}H'(G)}{\left[H(m(\widehat{\beta}))-H(G)\right]}-\frac{1}{\left[H(m(\widehat{\beta}))-H(G)\right]}\frac{ds^{OO}}{dG}>0. \tag{31}$$

This expression accounts for the direct effect of G and for its indirect impact via the induced variation in s^{OO}; its sign follows from our finding that $ds^{OO}/dG<0$. We have crowding out at the extensive margin.

4.3 Stage 1: The optimal policy

The government's budget constraint in this case is given by

$$\tau wT=\pi F(\widehat{\beta})\left[G-s^{OO}(G)\right].$$

It differs from (20), its counterpart in the TU case, in two ways. First, G is offered only to dependent parents who do not receive informal care (and whose share of the dependent elderly population is $F(\widehat{\beta})$). Second, parents who opt in have to forego their savings. This feature puts the *net* insurance benefit of the OO system, $G-s^{OO}(G)$, on par with that of the TU system, g. It also means that only the net benefit of public LTC will have to be financed by taxing wages.

Substituting the above budget constraint into the parents' optimized value of EU^{OO}, we are left with choosing G to maximize

$$\begin{aligned}\pounds^{OO}\equiv\; & w\overline{T}-\pi F(\widehat{\beta})\left[G-s^{OO}(G)\right]-s^{OO}(G)+(1-\pi)U\left(s^{OO}(G)\right)+ \\ & \pi\left[\int_{\widehat{\beta}}^{\infty}H\left(m\left(\beta\right)\right)dF(\beta)+F(\widehat{\beta})H\left(G\right)\right].\end{aligned} \tag{32}$$

Differentiating $\pounds^{OO}$ with respect to G yields, using the envelope theorem,[28]

$$\begin{aligned}\frac{d\pounds^{OO}}{dG}=\;&\pi\underbrace{F(\widehat{\beta})H'(G)}_{A}\quad-\pi\underbrace{f(\widehat{\beta})\left[H(m(\widehat{\beta}))-H(G)\right]\frac{\partial\widehat{\beta}}{\partial G}}_{B}\\ &-\pi\underbrace{F(\widehat{\beta})\left(1-\frac{ds^{OO}}{dG}\right)+(G-s^{OO})f(\widehat{\beta})\frac{d\widehat{\boldsymbol{\beta}}}{dG}}_{C}\quad.\end{aligned} \tag{33}$$

This expression shows that an increase in G has three different effects, labeled A, B and C. Term A measures the expected insurance benefit that G provides to parents who opt in. The public LTC insurance benefit also affects the extent of informal care through the extensive margin. By increasing $\widehat{\beta}$, it reduces the number of informal caregivers. The cost of this adjustment is measured by term B. Finally, C expresses the impact of an increase in G on first-period consumption. It accounts for the induced adjustments in s^{OO} and $\widehat{\beta}$.

Comparing expression (33) to its counterpart in the TU case given by equation (21), we note the following points. Term A, which indicates insurance benefit, has a similar counterpart in

[28] The derivative of the parents' objective function with respect to s is zero. Consequently, the terms pertaining to the induced variation of s, including $\partial\widehat{\beta}/\partial G$, vanish for the parents' objective function but *not* for the budget constraint. This explains why we have $\partial\widehat{\beta}/\partial G$ in term B but $d\widehat{\boldsymbol{\beta}}/dG$ in term C.

TU (except that $\widehat{\beta}$ replaces $\widetilde{\beta}$). Term B is absent in the TU case because the extensive margin crowding out via $\widetilde{\beta}$ has no first-order effect on parents' utility. Finally, term C, which captures the marginal cost of increasing assistance in terms of first-period consumption, has one as its counterpart under TU (the cost of crowding out at the intensive margin).

Substitute for $\partial\widehat{\beta}/\partial G$ from (26), and for $d\widehat{\boldsymbol{\beta}}/dG$ from (31), into (33) and rearrange the terms. An interior solution for G will then be characterized by

$$\left\{F(\widehat{\beta}) - f(\widehat{\beta})\widehat{\beta}\left[1 + \frac{G^{OO} - s^{OO}}{H(m(\widehat{\beta})) - H\left(G^{OO}\right)}\right]\right\} H'\left(G^{OO}\right) =$$
$$F(\widehat{\beta})\left(1 - \frac{ds^{OO}}{dG}\right) - \left[f(\widehat{\beta})\frac{G^{OO} - s^{OO}}{H(m(\widehat{\beta})) - H\left(G^{OO}\right)}\right]\frac{ds^{OO}}{dG}.$$

The bracketed terms on the left-hand side of this expression is smaller than $F(\widehat{\beta})$. On the other hand, because $ds^{OO}/dG < 0$, the whole of the right-hand side is larger than $F(\widehat{\beta})$. Consequently, $H'(G^{OO}) > 1$; there is less than full insurance for the dependent parents who opt in under an OO policy.

The main results of this section are summarized in the following proposition.

Proposition 3 *Consider an opting out scheme financed by a proportional tax on earnings. Define the degree of altruism of the marginal child $\widehat{\beta}$ implicitly by*

$$\widehat{\beta}\left[H(m(\widehat{\beta})) - H\left(G\right)\right] - \left[m(\widehat{\beta}) - s^{OO}(G)\right] = 0,$$

and let $s^{OO}\left(G\right)$ solve

$$\left(1 - \pi\right)U'\left(s\right) + \pi f(\widehat{\beta}\left(G, s\right)) = 1.$$

(i) It will not be desirable to provide LTC on an OO basis, if at $G = s^{OO}$,

$$\underbrace{F(\widehat{\beta})H'\left(G\right)}_{A} - \underbrace{f(\widehat{\beta})\left[H(m(\widehat{\beta})) - H\left(G\right)\right]\frac{\partial\widehat{\beta}}{\partial G}}_{B} - \underbrace{F(\widehat{\beta})\left(1 - \frac{ds^{OO}}{dG}\right) + (G - s^{OO})f(\widehat{\beta})\frac{d\widehat{\boldsymbol{\beta}}}{dG}}_{C} \leq 0.$$

(ii) Otherwise, the solution is interior and defined by

$$A - B - C = 0,$$

with $G = G^{OO}$.

(a) This relationship balances the expected insurance benefit, A, against the cost of the induced crowding out at the extensive margin, B, plus the budgetary cost C (the reduction in the parents' first-period consumption).

(b) An increase in public LTC reduces parents' private savings.

(c) There is crowding out at the extensive margin.

(iii) Under both (i) and (ii), $H'\left(G\right) > 1$ so that there is less than full insurance.

5 Topping up versus opting out

The previous sections have shown that under both TU and OO policies public provision of LTC crowds out informal care. Under TU, crowding out occurs at both intensive and extensive margins. At the intensive margin, TU crowds out informal care by $-1 < da^*/dg < 0$. At the extensive margin, TU reduces the number of informal caregivers: $d\widetilde{\boldsymbol{\beta}}/dg > 0$. However, since the informal care provided by the marginal child $\widetilde{\beta}$ is equal to zero, this has no first-order impact on the parents' utility. Under OO, crowding out occurs only at the extensive margin. At the intensive margin, there is no *direct* crowding out. Indeed, indirectly, an increase in G leads to an increase in informal care via the reduction in private savings it induces. At the extensive margin, $d\widehat{\boldsymbol{\beta}}/dG > 0$, and there is crowding out as the number of caregivers declines. This crowding out does have a first-order effect on the parents' utility: The parents of marginal children $\widehat{\beta}$ are strictly better off when they receive informal care.

The precise comparison of the TU and OO policies is somewhat complicated. To understand the tradeoffs that are involved, we construct a *sufficient* condition for OO to yield a higher level of welfare than TU. The following proposition is proved in Appendix A.

Proposition 4 *Consider an optimal TU scheme g^{TU} with saving s^{TU} and an optimal OO scheme G^{OO} with saving s^{OO}. Let $\beta^A \equiv \widehat{\beta}(g^{TU}+s^{TU}, s^{TU})$ denote the threshold level of β below which no assistance is provided to the parents under an OO policy with $G = g^{TU}+s^{TU}$ and $s = s^{TU}$. Then $\beta^A > \widetilde{\beta}(g^{TU}+s^{TU})$ and the OO scheme dominates if*

$$\left[1 - F(\beta^A)\right] g^{TU} - \int_{\widetilde{\beta}}^{\beta^A} [H\left(m\left(\beta\right)\right) - H(g^{TU}+s^{TU})]dF(\beta) \geq 0. \tag{34}$$

To arrive at this condition, one starts from an optimal TU policy and provides a condition that ensures its replication under OO will be welfare improving. The first term on the left-hand side of (34) measures the *benefit* of switching to OO while keeping the net transfer per beneficiary, g^{TU}, and savings, s^{TU}, constant. The benefit in switching comes from the fact that under OO children with $\beta > \beta^A$ transfer $G - s^{TU}$ in resources to their parents; but not so under TU. The second term measures the *cost* of switching to OO. The cost arises because children with a β in the interval $[\widetilde{\beta}, \beta^A]$ who transfer $m(\beta) - \left(g^{TU}+s^{TU}\right)$ to their parents under TU would no longer do so under OO. Roughly speaking, OO dominates if the share of children with a high degree of altruism is large enough; that is, if $1 - F(\beta^A)$ is sufficiently large. This makes sense in that, for this population, switching to OO avoids the crowding out at the intensive margin that the TU policy induces.[29]

This tradeoff is illustrated in Figure 4 (where β has an upper-bound β^+). The solid gray and solid black lines represent the consumption of dependent parents under an optimal TU policy and an OO policy with $G = g^{TU}+s^{TU}$ *and* $s = s^{TU}$. The parents of children with β in the interval $[\beta^A, \beta^+]$ have the same level of second-period consumption $m\left(\beta\right)$ under TU and OO policies; yet they incur different costs in attaining this identical consumption level. They save

[29] Generally speaking, not targeting is more wasteful when the targeted group is small because it entails unnecessary transfers to a larger group of people. In our model, $1 - F(\widetilde{\beta})$ is the proportion of the not-targeted group in the population.

the same amount, but the financing of g^{TU} in their old-age consumption falls on different people. Under TU, the financing comes from all of the parents through higher wage taxes. Under OO, it is the children with a β in $[\beta^A, \beta^+]$ who pay for it.[30] Area B in Figure 4 is a representation of what parents gain under OO in comparison to TU. On the negative side, under OO parents whose children have a β in the interval $[\widetilde{\beta}, \beta^A]$ would each lose $m(\beta) - \left(g^{TU} + s^{TU}\right)$ in transfers that they would have received under TU. Area C in Figure 4 represents the loss to these parents.

The size of area B depends on the number of dependent parents receiving family help under OO, $1 - F(\beta^A)$, as well as the level of public insurance, g^{TU}. The size of area C depends on the number of people in the interval $\beta^A - \widetilde{\beta}$ as well as on how much less each parent in this interval consumes under OO, $m(\beta) - \left(g^{TU} + s^{TU}\right)$. The optimal regime depends on the respective sizes of the two areas.[31] Observe that the comparison hinges crucially on the distribution of the altruism parameter, $F(\beta)$, and on the degree of concavity of the utility function $H(\cdot)$.

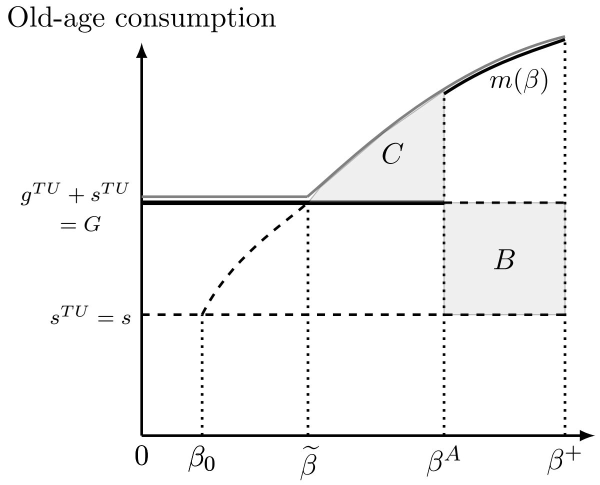

Figure 4: *Topping up vs Opting out*

To illustrate the interplay of the factors that determine the optimality of TU and OO policies, we next turn to a numerical example. Unlike the discussion above, the example compares the two regimes *unconditionally*. Assume, in line with the health economics literature, that old-age dependency is captured by a monetary loss L. Thus write the utility function associated with old-age consumption by $U(x)$ if healthy and $H(x - L)$ if dependent. The specifications used in the example are $U(x) = \ln(x)$ and $H(x - L) = \ln(x - L)$. We assume that β is distributed over the interval $[0, \beta^+]$ according to the cumulative density function $F(\beta) = \mu + (1 - \mu)\beta/\beta^+$ with $\mu \in (0, 1)$. This distribution implies that a fraction μ of the children are not altruistic and the remaining $1 - \mu$ altruistic ones are distributed uniformly over the interval $[0, \beta^+]$. The values we consider are $2.5, 3$, and 3.5 for β^+; and 20% and 30% for μ. We further assume that the

[30] The tax rate is $\tau = \pi g^{TU}/w\overline{T}$ under TU and $\tau = \pi F\left(\beta^A\right) g^{TU}/w\overline{T}$ under OO.

[31] This argument is purely illustrative of the tradeoff as the areas cannot directly be compared. First, area B does not account for the distribution of β. To obtain the effective cost savings one has to multiply area B by $[1 - F(\beta^A)]$. Second, area C represents the loss in consumption and not in utility. Furthermore, the sum is not weighted by the density.

		$\mu = 0.2$			$\mu = 0.3$		
		$\beta^+ = 2.5$	$\beta^+ = 3$	$\beta^+ = 3.5$	$\beta^+ = 2.5$	$\beta^+ = 3$	$\beta^+ = 3.5$
Laissez Faire	β_0	0.231	0.219	0.211	0.313	0.301	0.292
	s^{LF}	1.231	1.219	1.211	1.313	1.301	1.292
	EU^{LF}	0.730	0.797	0.854	0.663	0.720	0.769
Topping up	$\widetilde{\beta}$	**0.294**	0.273	0.259	**0.417**	**0.391**	0.375
	s^{TU}	**1.000**	1.000	1.000	**1.000**	**1.000**	1.000
	g	**0.294**	0.273	0.2592	**0.417**	**0.391**	0.375
	EU^{TU}	**0.744**	0.810	0.866	**0.689**	**0.744**	0.791
Opting out	$\widehat{\beta}$	0.845	**0.915**	**0.976**	0.313	1.138	**1.209**
	s^{OO}	0.595	**0.577**	**0.565**	0.313	0.566	**0.556**
	G	1.193	**1.212**	**1.230**	0.000	1.286	**1.308**
	EU^{OO}	0.742	**0.835**	**0.913**	0.663	0.742	**0.814**

Table 1: *Numerical illustrations with* $U(x) = \ln(x)$*,* $H(x) = \ln(x-1)$*,* $F(\beta) = \mu + (1-\mu)\beta/\beta^+$*,* $w\overline{T} = 2$*,* $\pi = 0.5$*.*

probability of dependency is $\pi = 0.5$, the first-period exogenous income is equal to two, and the monetary loss L is equal to one.

Table 1 reports the laissez-faire allocation and the optimal allocations attainable under TU and OO regimes (for the different distributions of the altruism parameters considered). The regime that yields a higher EU dominates; it is highlighted in bold for each configuration of parameters. Observe that, in our examples, OO always results in more crowding out at the extensive margin than TU (i.e., $\widehat{\beta} > \widetilde{\beta}$). For a given μ, a higher β^+ shifts the mass distribution towards higher levels of altruism, implying that the OO policy becomes cheaper and thus more attractive. Conversely, given β^+, a higher μ implies fewer children with high levels of altruism and makes it less likely that the OO policy dominates. Indeed, the configuration of a high μ and a low β^+ in our example ($\mu = 0.3$, $\beta^+ = 2.5$), implies that there is no OO regime that can improve the parents' welfare over its laissez faire level.

6 Opting-out-cum-transfers

Parents' expected utility in our model is determined not just by how much they decide to save but also by the decision of their children as to whether or not to assist their parents if they become dependent. Under the TU and OO schemes we have been studying, the government attempts to influence both of these decisions through its choice of a public LTC.[32] One would expect that the government should be able to do better (or at least just as well), if it can find another policy instrument to affect these two sets of decisions. What is needed is an instrument that affects parents who would receive informal care and those who would not differently. A tax/transfer policy carried out in period one, when the parents are identical and the children

[32]The wage tax is not an independent instrument; the magnitude of LTC provision determines it through the government's budget constraint.

have not grown up yet, cannot do this. The trick is to fine-tune the OO policy and combine it with a transfer to, or tax on, the parents who opt out.[33] The basic idea is to increase the total size of transfers from children to their parents through the phenomenon of crowding out, both the extensive and the intensive margin.

A positive transfer to parents lowers the children's cost of providing informal care; thus encouraging more children to opt out and assist their parents. This is good for the parents. While children pay for the cost of informal care, the cost of public LTC is borne by the parents. Increasing the number of children who opt out and assist their parents, however, is not the only effect of this policy. There is a downside to it as well which comes in the form of the crowding out at the intensive margin. The children who already assist their parents under a pure OO system, will now be able to cut their current transfers by an amount equal to the government transfers (while keeping the parents' consumption unchanged).[34] However, the cost of transfers is borne by the parents themselves in terms of extra wage taxes in the first period. With the extensive margin and intensive margin effects of transfers on the parents' expected utility going in opposite directions, the second instrument need not necessarily be a positive transfer. It may very well be a tax (paid in the second period by the parents who opt out).[35] The upshot is that the dependent parents who opt out should be given a positive transfer if the extensive margin effect dominates, and be taxed if the intensive margin effect dominates. Either way, to implement it, the first-period tax rate on the parents' wages must be adjusted. The adjustment is upward if the second-period transfer is positive and downward if negative. Effectively, the government is enabled to treat *ex ante* identical parents differently (as far as their tax and transfers are concerned).

The policy we consider thus consists of two instruments: a public and exclusive LTC provision of G to dependent parents whose children do not assist them in exchange for their savings and a positive or negative transfer of g to dependent parents whose children do take care of them. In what follows, for ease in exposition, we shall refer only to a positive transfer. However, as we proceed, it will become clear under what circumstances one should rely on a transfer or on a tax.

6.1 Stage 3: The children's choice

If children decide to provide care, the optimal amount of family assistance a^* is again such that the dependent parents consumption is equal to its laissez faire level, $m(\beta)$. And, as previously, children provide assistance only if it gives them a higher utility than letting parents consume the exclusive LTC that the government provides. There exists a $\overline{\beta}(G, s+g)$ such that all parents with children whose $\beta > \overline{\beta}$ opt out, while parents with children whose $\beta \leq \overline{\beta}$ receive no assistance

[33]Gahvari and Mattos (2007) use a similar argument to rationalize conditional cash transfer programs in developing countries (such as Bolsa-Escola in Brazil and PROGRESA in Mexico). There the transfers are given to those who participate in a "free" publicly-provided program in order to encourage opting in; here it is given to those who opt out in order to encourage opting out.

[34]This second effect is absent in the existing conditional cash transfer programs referred to above.

[35]Since these parents keep their savings, if the policy consists of a tax, they will be paying it from their savings.

and opt in. This threshold $\overline{\beta}(G, s+g)$ is defined by

$$\overline{\beta}\left[H(m(\overline{\beta})) - H(G)\right] - \left(m(\overline{\beta}) - s - g\right) = 0. \quad (35)$$

Differentiating (35) with respect to G, g, and s yields

$$\frac{\partial \overline{\beta}}{\partial G} = \frac{\overline{\beta} H'(G)}{H(m(\overline{\beta})) - H(G)} > 0, \quad (36)$$

$$\frac{\partial \overline{\beta}}{\partial s} = \frac{\partial \overline{\beta}}{\partial g} = -\frac{1}{H(m(\overline{\beta})) - H(G)} < 0. \quad (37)$$

Observe that G and s have the same effects on $\overline{\beta}$ as in the pure OO scheme. The effect of g on $\overline{\beta}$ is identical to the one of s because $s+g$ now plays the role that s did in the pure OO scheme.

6.2 Stage 2: The parents' choice

Parents choose s to maximize their expected utility

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi\left[H(G)F(\overline{\beta}) + \int_{\overline{\beta}}^{\infty} H(m(\beta))\,dF(\beta)\right],$$

which, except for the position of the marginal child, is identical to the EU under the pure OO scheme. Observe that g affects EU only through $\overline{\beta}$. This is because parents' consumption with children's assistance is $m(\beta)$ whether or not they receive a transfer from the government. The derivative of EU with respect to s is

$$\frac{\partial EU}{\partial s} = (1-\pi)U'(s) - \pi f(\overline{\beta})\left[H(m(\overline{\beta})) - H(G)\right]\frac{\partial \overline{\beta}}{\partial s} - 1. \quad (38)$$

Assuming an interior solution and substituting the expression for $\partial\overline{\beta}/\partial s$ in the equation above, one arrives at the following expression for the optimal level of savings[36]

$$(1-\pi)U'(s) + \pi f(\overline{\beta}) = 1. \quad (39)$$

Equations (35) and (39) jointly determine s and $\overline{\beta}$ as functions of G and g: $s^{OC}(G,g)$ and $\overline{\boldsymbol{\beta}}(G,g) \equiv \overline{\beta}(G, s^{OC}(G,g)+g)$. Differentiating this system of equations with respect to G and g, we show in Appendix A that

$$\frac{\partial \overline{\boldsymbol{\beta}}}{\partial G} > 0, \ \frac{\partial s^{OC}}{\partial G} < 0 \quad \text{and} \quad \frac{\partial \overline{\boldsymbol{\beta}}}{\partial g} < 0, \frac{\partial s^{OC}}{\partial g} > 0\,. \quad (40)$$

As one would expect, an increase in G affects the location of the marginal child and the parents' savings similarly as it did under the pure OO scenario. An increase in g, on the other hand, has opposite effects to those found for the TU scheme. There, the prospect of a higher consumption level for dependent parents made it less likely for children to help; it thus increased $\widetilde{\boldsymbol{\beta}}$. Here, in deciding to provide assistance, the children base their decision on what their parents get without

[36] We assume that the second-order condition is satisfied:

$$(1-\pi)U''(s) - \pi\frac{f'(\overline{\beta})}{H(m(\overline{\beta})) - H(G)} < 0.$$

their help, G, and what it costs them to provide help $(m(\beta) - (g + s))$. An increase in g reduces their cost and encourages them to provide assistance. This explains why $\partial\overline{\boldsymbol{\beta}}/\partial g$ is negative. As far as saving is concerned, with g being offered to everyone under TU, an increase in g lowered the self-insurance benefit of savings and thus reduced it. Here, with g being provided only to the parents who receive assistance, it will have no direct effect on parents. However, since $\overline{\beta}$ has a negative feedback effect on savings, the reduction in $\overline{\boldsymbol{\beta}}$ due to an increase in g boosts saving.[37].

In Appendix A we also prove that

$$\overline{\beta}H'(G) = -\frac{\frac{\partial s^{OC}}{\partial G}|_g}{\frac{\partial s^{OC}}{\partial g}|_G} = \left.\frac{dg}{dG}\right|_{s^{OC}} = -\frac{\frac{\partial \overline{\beta}}{\partial G}|_g}{\frac{\partial \overline{\beta}}{\partial g}|_G} = \left.\frac{dg}{dG}\right|_{\overline{\beta}}. \tag{41}$$

These relationships the "marginal rate of substitution" between G and g for a given value of s^{OC}, and for a given value of $\overline{\boldsymbol{\beta}}$; they also show that the two are equal.[38] The effect of a marginal increase in G on $\overline{\boldsymbol{\beta}}$ and s^{OC} can be offset by an increase of $\overline{\boldsymbol{\beta}}H'(G)$ in g.

6.3 Stage 1: The optimal policy

The government's budget constraint is now given by

$$\tau wT = \pi\left\{F(\overline{\beta})\left[G - s^{OO}(G)\right] + \left(1 - F(\overline{\beta})\right)g\right\}.$$

The optimal policy is thus found by choosing g and G to maximize

$$\begin{aligned}\pounds^{OC} \equiv\ & w\overline{T} - \pi F(\overline{\beta})\left[G - s^{OC}(G,g)\right] - \pi\left[1 - F(\overline{\beta})\right]g - s^{OC}(G,g) + \\ & + (1-\pi)U\left(s^{OC}(G,g)\right) + \pi\left[\int_{\overline{\boldsymbol{\beta}}}^{\infty} H(m(\beta))\,dF(\beta) + F(\overline{\beta})H(G)\right],\end{aligned}$$

where $\overline{\boldsymbol{\beta}} = \overline{\boldsymbol{\beta}}\ (G,g)$. Differentiating $\pounds^{OC}$ partially with respect to G and g, and using the envelope theorem, one obtains

$$\frac{\partial \pounds^{OC}}{\partial G} = \pi\left\{\underbrace{F(\overline{\beta})H'(G)}_{A} - \underbrace{f(\overline{\beta})\frac{\partial\overline{\beta}}{\partial G}\Delta H}_{B} - \underbrace{F(\overline{\beta}) - F(\overline{\beta})\frac{\partial s^{OC}}{\partial G} + (G - s^{OC} - g)f(\overline{\beta})\frac{\partial\overline{\boldsymbol{\beta}}}{\partial G}}_{C}\right\}, \tag{42}$$

$$\frac{\partial \pounds^{OC}}{\partial g} = \pi\left\{\underbrace{-f(\overline{\beta})\frac{\partial\overline{\beta}}{\partial g}\Delta H}_{B'} - \underbrace{1 - F(\overline{\beta}) - F(\overline{\beta})\frac{\partial s^{OC}}{\partial g} + (G - s^{OC} - g)f(\overline{\beta})\frac{\partial\overline{\boldsymbol{\beta}}}{\partial g}}_{C'}\right\}, \tag{43}$$

where $\Delta H \equiv H(m(\overline{\boldsymbol{\beta}})) - H(G)$. Condition (42) has the same formulation as its counterpart in the pure OO scheme (except that $G - s^{OC} - g$ appears in place of $G - s^{OO}$). Interpretations of expressions A, B, and C are also the same and bear no repeating. Turning to condition (43),

[37] Differentiating (39) with respect to $\overline{\beta}$ yields

$$\frac{ds}{d\overline{\beta}} = \frac{-\pi f'(\overline{\beta})}{(1-\pi)U''(s)} < 0,$$

where $f'(\overline{\beta}) = F''(\overline{\beta}) < 0$ due to the concavity of $F(\cdot)$.

[38] The equality is due to the fact that neither G nor g appear directly in equation (35).

there is no term corresponding to A because g is given to the dependent parents who receive informal care and whose consumption is $m(\beta)$. As such, and in contrast to G which by virtue of being given only to parents not receiving informal care provides them with insurance, provides no insurance benefit. Term B' corresponds to B and captures the fact that g affects informal care at the extensive margin (the same as with G albeit in the opposite direction). By reducing $\overline{\beta}$, it increases the number of children who provide informal care. This is a benefit so that B' is positive. Term C' corresponds to C, reflecting the effects of an increase in g on first-period consumption. Because g is given to dependent parents who receive informal care, as opposed to G given to those who do not, its direct impact is measured by $1 - F(\overline{\beta})$ instead of $F(\overline{\beta})$. The terms also accounts for the induced adjustments in s^{OO} and $\overline{\beta}$ which are similar to adjustments induced by an increase in G.

To determine the conditions under which parents receiving informal care should be given a transfer or be taxed, one has to evaluate $\partial \pounds^{OC}/\partial g$ at the optimum under a pure OO scheme (when $G = G^{OO}$ and $g = 0$). We show in Appendix A that this is given by

$$\frac{\partial \pounds^{OC}}{\partial g}|_{g=0} = \pi F(\widehat{\beta}) \left[\frac{H'\left(G^{OO}\right) - 1}{\widehat{\beta} H'\left(G^{OO}\right)} \right] - \pi \left[1 - F(\widehat{\beta}) \right]. \quad (44)$$

The first term on the right-hand side measures the marginal benefit of increasing g. An increase in g leaving $\widehat{\beta}$ and s^{OO} unchanged requires a concomitant increase in G which, from (41), is equal to $(dG/dg)_{\widehat{\beta}, s^{OO}} = 1/\widehat{\beta} H'\left(G^{OO}\right)$. The resulting increase in insurance benefits, $H'\left(G^{OO}\right) - 1$, goes to $\pi F(\widehat{\beta})$ dependent parents who do not get informal care.[39] The second term measures the marginal cost of increasing the g transfer going to $\pi\left(1 - F(\widehat{\beta})\right)$ dependent parents who receive informal care. It is in terms of reduced first-period consumption. It immediately follows from (44) that,

$$F(\widehat{\beta}) \left[H'\left(G^{OO}\right) - 1\right] \gtreqless \left[1 - F(\widehat{\beta})\right] \widehat{\beta} H'\left(G^{OO}\right) \Rightarrow g^{OC} \gtreqless 0. \quad (45)$$

Finally, it follows from (44) that at an interior solution for G and for g (positive or negative), it must be the case that

$$F(\overline{\beta}) \left[H'\left(G^{OC}\right) - 1\right] = \left[1 - F(\overline{\beta})\right] \overline{\beta} H'\left(G^{OC}\right). \quad (46)$$

That is, the marginal insurance benefit that the opting-out-cum-transfer policy provides must be equal to its marginal cost in terms of the foregone first-period consumption. Equation (46) also implies that $H'(G^{OC}) > 1$ as long as $1 - F(\overline{\beta}) > 0$. In words, the optimal opting-out-cum-transfer regime too implies less than full insurance for the parents who do not receive family help (except when there is an upper bound to β, say β^+, and $\overline{\beta} = \beta^+$ so that no child provides help).

The main results of this section are summarized in the following proposition.

Proposition 5 *Consider an optimal OO policy with an interior solution $G = G^{OO}$. Replace this policy with an alternative that supplements the provision of G with a positive or negative*

[39] Another way of looking at the gain is that an increase in g reduces $\overline{\beta}$ resulting in a higher number of children assisting their parents. Consequently, with a smaller number of parents on public LTC, one can help the remaining ones more.

Topping up	$\tilde{\beta} = 0.391$	$s^{TU} = 1.000$		$g^{TU} = 0.391$	$EU^{TU} = 0.744$
Opting out	$\hat{\beta} = 1.138$	$s^{OO} = 0.566$	$G^{OO} = 1.286$		$EU^{OO} = 0.742$
Opting-out-cum-transfers	$\bar{\beta} = 1.210$	$s^{OC} = 0.566$	$G^{OC} = 1.900$	$g^{OC} = -0.566$	$EU^{OC} = 0.757$

Table 2: *The three regimes with $U(x) = \ln(x)$, $H(x) = \ln(x-1)$, $F(\beta) = \mu + (1-\mu)\beta/\beta^+$, $w\overline{T} = 2$, $\pi = 0.5$, $\mu = 0.3$, $\beta^+ = 3$.*

transfer g to dependent parents who opt out and rely on their own private savings and family informal care. Parents who opt in receive no transfers and, in exchange for their entire savings, get to consume the publicly-provided G that cannot be topped up.

(a) Such a scheme will never decrease the parents' expected utility attained under a pure OO scheme. The transfer g must be positive (negative) if , at $g = 0$, the insurance benefit of the last dollar spent on public LTC exceeds (falls short of) its marginal cost in terms of the reduced first-period consumption. This is shown by condition (45).

(b) Denote the optimal publicly-provided LTC under this new policy by G^{OC} and the optimal conditional transfer by g^{OC}. Let $\overline{\boldsymbol{\beta}} = \overline{\boldsymbol{\beta}}\left(G^{OC}, g^{OC}\right)$, defined by

$$\overline{\boldsymbol{\beta}}\left[H(m(\overline{\boldsymbol{\beta}})) - H\left(G^{OC}\right)\right] - \left(m(\overline{\boldsymbol{\beta}}) - s^{OC} - g^{OC}\right) = 0,$$

denote the threshold of β below which children will not assist their parents under this policy. Then G^{OC} and g^{OC} satisfy equation (46). That is, they equate the marginal insurance benefit of the last dollar spent on LTC to its marginal cost in terms of the foregone first-period consumption.

(c) The policy implies less than full insurance.

(d) Provision of G leads to crowding out at the extensive margin and transfers g, if positive, to crowding out at the intensive margin and crowding in at the extensive margin.

We make one final observation on combining an *OO* policy with conditional transfers. This scheme can potentially enhance welfare not just over a pure *OO policy* but also over a *TU* policy that dominates its pure *OO* competitor. According to Table 1, when $\mu = 0.3$ and $\beta^+ = 3$, the dominant regime is *TU* with a corresponding $EU^{TU} = 0.744$ (exceeding $EU^{OO} = 0.742$ and thus being the preferable policy). However, combining *OO* with a one-hundred percent tax on the parents' private savings ($g^{OC} = -0.566$), increases the parents' expected utility to $EU^{OC} = 0.757$ thus making the opting-out-cum-transfers the optimal LTC regime. Under this mixed policy, public LTC would increase from $G^{OO} = 1.286$ to $G^{OC} = 1.900$ and the marginal β from $\widehat{\beta} = 1.138$ to $\overline{\beta} = 1.210$. Consequently, the policy entails more crowding out of informal care at the extensive margin as compared to both *OO* and *TU* policies ($\overline{\beta} > \widehat{\beta} > \widetilde{\beta}$). Table 2 illustrates these comparisons.

7 Private insurance markets and public LTC

The three public LTC policy regimes we discussed in previous sections, have ignored the availability of private insurance markets. It is a straightforward exercise to allow for such markets.

The formulation and the analysis of the policy regimes remain very much the same as they were without insurance markets. The derivation of the various expressions and the proofs are also similar under the two scenarios. To avoid what might look like repetitious presentation, we leave the details of this exercise to Appendix B. We limit our discussion in this section to making a few remarks and to presenting a summary of our formally-proved results under Proposition 6.

First, actuarially fair insurance markets render the TU policy redundant as one replicates the other. The result should not be surprising. Under TU, the government is not more efficient than a perfectly competitive insurance market. There is nothing a public insurer can do that markets cannot; public and private insurance are equivalent.

Second, actuarially fair insurance markets do not obviate the usefulness of an OO policy. Intuitively, public insurance extends insurance to dependent parents whose children do not assist them. Private insurance companies can replicate this only through an insurance contract to parents in exchange for their savings with a provision that forbids children to help their dependent parents. Such a contract is highly unlikely to be enforceable. Public insurance under OO, by effectively offering insurance against the failure of children to help, *may* make public intervention desirable.

Third, if private insurance is purchased, there is no need to supplement an opting-out system with conditional transfers. This makes sense as private insurance serves the same purpose as conditional transfers. Fourth, private insurance is of no use if the optimal transfer under an opting-out-cum-transfer policy, in the absence of private insurance markets, is negative (i.e., a tax is required). Under this circumstance, nobody will purchase insurance even if one allows for it. And with no private insurance being purchased, it will be desirable to supplement an OO system with negative transfers.

Proposition 6 *Assume actuarially fair insurance markets exist. Then:*

(i) A TU policy effectively replicates the market solution with insurance purchases. It offers full insurance against dependency but does nothing by way of providing insurance against the default of altruism.

(ii) An OO policy may or may not do better than actuarially fair insurance markets for dependency. The nature of the solution depends on the size of savings under an optimal OO policy in the absence of private insurance markets, s^{OO}.

a. If $U'\left(s^{OO}\right) \geq 1$ *nobody purchases private insurance and we are back to the OO solution without private insurance.*

b. If $U'\left(s^{OO}\right) < 1$ *parents buy private insurance. Then there are two possibilities. One in which an OO policy is desirable; this will be the case if G has an interior solution as characterized by (B3)). In the other, an interior solution for G does not exist and the OO policy is not useful. In this case, the solution will be the same as the laissez faire solution with insurance markets for dependency (and identical to a TU policy). In both cases, the parents remain under-insured.*

(iii) An opting-out-cum-transfer policy may or may not do better than actuarially fair insurance markets for dependency. In particular,

a. If $U'\left(s^{OC}\right) \geq 1$ *nobody purchases any private insurance and we are back to the opting-out-cum-transfer solution without private insurance.*

b. If $U'\left(s^{OC}\right) < 1$ *parents buy private insurance. Then* $g = 0$ *with two possibilities identical to those under (ii)–b.*

8 Incorporating children's utility in the social welfare function

This section briefly examines if and how incorporating the children's utility in the social welfare function might affect our results. To simplify the exposition, we consider a utilitarian social welfare function where parents and children have the same weight. Different weights would affect the results in a straightforward way by either mitigating or reinforcing any new effects that may show up.

That the children's utility is equal to y with the probability $1-\pi$ and $y-a+\beta H(e)$ with the probability of π results in an expected utility of $y-\pi a+\pi\beta H(e)$ for the children. However, fully including the altruistic term of the expected utility, i.e. $\beta H(e)$, in the social welfare function raises some philosophical questions. With $H(e)$, the parents' utility from their consumption when dependent, already appearing in the social welfare function, including $\beta H(e)$ can be considered as double counting. One may reasonably argue that this term should be excluded on the grounds that it is already reflected in the social welfare function—a construct that right from the start incorporates whatever is good for the society as a whole; see Hammond (1987) and Diamond (2006). To defer to both schools of thought, we discount $\beta H(e)$ by a factor $0 \leq \gamma \leq 1$ when including the children's utility in the social welfare function. That is, we will augment our previously stipulated social welfare function by the expression $y-\pi a+\pi\gamma\beta H(e)$. This runs the gamut from the pure utilitarian approach ($\gamma = 1$) to when one completely "launders out" the altruistic term ($\gamma = 0$).

8.1 Topping up

The government's problem is now summarized by the Lagrangian:

$$\begin{aligned}
\pounds^{TU} &= w\overline{T} - \pi g - s^{TU}(g) + (1-\pi)U\left(s^{TU}(g)\right) \\
&+ \pi\left[F(\widetilde{\beta}\left(s^{TU}(g)+g\right))H\left(s^{TU}(g)+g\right) + \int_{\widetilde{\beta}(s^{TU}(g)+g)}^{\infty} H(m(\beta))\,dF(\beta)\right] \\
&+ y - \pi\int_{\widetilde{\beta}}^{\infty}\left[m(\beta) - s^{TU}(g) - g\right]dF(\beta) \\
&+ \pi\gamma\left[H\left(s^{TU}(g)+g\right)\int_{0}^{\widetilde{\beta}}\beta dF(\beta) + \int_{\widetilde{\beta}}^{\infty}\beta H(m(\beta))dF(\beta)\right].
\end{aligned}$$

Differentiating $\pounds^{TU}$ with respect to g and simplifying, using the envelope theorem and the fact that $m(\widetilde{\beta}) = s^{TU} + g$, yields

$$\begin{aligned}\frac{\partial \pounds^{TU}}{\partial g} &= \left[\pi F(\widetilde{\beta})H'\left(s^{TU}(g) + g\right) - \pi\right] \\ &+ \pi \left[\left(1 - F(\widetilde{\beta})\right) + \gamma H'\left(s^{TU}(g) + g\right) \int_0^{\widetilde{\beta}} \beta dF(\beta)\right] \left(1 + \frac{\partial s^{TU}(g)}{\partial g}\right).\end{aligned}$$

The first bracketed expression on the right-hand side of the above reflects the utility of the parents and is identical to the terms in equation (21). The second bracketed expression represents the additional terms associated with the children's utility. It consists of two components both of which are positive. This implies that extending the social welfare function to include children's utility, unambiguously increases the optimal LTC transfer g above its otherwise optimal value of g^{TU}. The first component of this expression relates to care-givers: increasing g above g^{TU} allows them to lower their own contribution by an equal amount (due to the full crowding-out at the intensive margin effect of the public transfers). The second component relates to the children who do not provide care to their parents: expanding g beyond g^{TU} increases the parents' utility and with it the children's utility as well. Observe that this effect vanishes if $\gamma = 0$ (i.e., if the altruistic component of the child's utility is laundered out). In sum, the case for public LTC is strengthened when children's utility is incorporated in the social welfare function.

8.2 Opting out

Incorporating the expected welfare of the children in (32), the social welfare function becomes

$$\begin{aligned}\pounds^{OO} &\equiv w\overline{T} - \pi F(\widehat{\beta})\left[G - s^{OO}(G)\right] - s^{OO}(G) + (1-\pi)\, U\left(s^{OO}(G)\right) \\ &+ \pi \left[\int_{\widehat{\beta}}^{\infty} H\left(m\left(\beta\right)\right) dF(\beta) + F(\widehat{\beta}) H\left(G\right)\right] \\ &+ y - \pi \int_{\widehat{\beta}}^{\infty} \left[m(\beta) - s^{OO}\left(G\right)\right] dF(\beta) \\ &+ \pi\gamma \left[H\left(G\right) \int_0^{\widehat{\beta}} \beta dF(\beta) + \int_{\widehat{\beta}}^{\infty} \beta H(m(\beta)) dF(\beta)\right].\end{aligned}$$

Differentiating $\pounds^{OO}$ with respect to G and simplifying, using the envelope theorem and the definition of $\widehat{\beta}$, yields

$$\begin{aligned}\frac{d\pounds^{OO}}{dG} =& \pi\left[A - B - C\right] \\ &+ \pi\left(1 - F(\widehat{\beta})\right) \frac{\partial s^{OO}(G)}{\partial G} + \pi\gamma H'\left(G\right) \int_0^{\widehat{\beta}} \beta dF(\beta) \\ &+ \pi(1-\gamma)\widehat{\beta} f(\widehat{\beta}) \left[H\left(\widehat{\beta}\right) - H\left(G\right)\right] \frac{d\widehat{\boldsymbol{\beta}}}{dG},\end{aligned}$$

where A, B, and C are defined in equation (33). The first bracketed expression on the right-hand side of above is identical to the one (33). It reflects, as previously, the terms pertaining to the parents. The other three expressions are associated with the children. The first relates to the initial caregivers who will remain as caregivers. They prefer to see G decline from its G^{OO}

level. Such a reduction would lead to an increase in the parents' savings which in turn lowers the share of the private cost of care the children have to cover. The second component relates to the children who do not help their parents the children. They want to see G increased. This will increase their parents' utility and with it their own utility as well. This effect vanishes if the altruistic component of the utility is laundered out in full ($\gamma = 0$). The third component relates to the initial marginal caregivers. They too want G to increase because this would allow them to switch out of providing informal care ($d\widehat{\boldsymbol{\beta}}/dG > 0$). This effect remains as long as at least part of the altruistic component of the children's utility is laundered out ($\gamma < 1$). These conflicting interests imply that including children's utility in the social welfare function will have an ambiguous effect on the optimal value of G.

8.3 Opting-out-cum-transfers

Incorporating the expected welfare of the children in the social welfare function changes the government's Lagrangian expression to:

$$
\begin{aligned}
\pounds^{OC} \equiv\ & w\overline{T} - \pi F(\overline{\beta})[G - s^{OC}(G,g)] - \pi[1 - F(\overline{\beta})]g - s^{OC}(G,g) + \\
& + (1-\pi)U\left(s^{OC}(G,g)\right) + \pi\left[\int_{\overline{\boldsymbol{\beta}}}^{\infty} H\left(m\left(\beta\right)\right)dF(\beta) + F(\overline{\beta})H\left(G\right)\right] \\
& + y - \pi\int_{\overline{\boldsymbol{\beta}}}^{\infty}\left[m(\beta) - s^{OC}\left(G,g\right) - g\right]dF(\beta) \\
& + \pi\gamma\left[H\left(G\right)\int_0^{\overline{\boldsymbol{\beta}}}\beta dF(\beta) + \int_{\overline{\boldsymbol{\beta}}}^{\infty}\beta H(m(\beta))dF(\beta)\right],
\end{aligned}
$$

where $\overline{\boldsymbol{\beta}} = \overline{\boldsymbol{\beta}}\ (G,g)$. Differentiating $\pounds^{OC}$ partially with respect to G and g, and using the envelope theorem, one obtains

$$
\begin{aligned}
\frac{\partial \pounds^{OC}}{\partial G} &= \pi\left[A - B - C\right] \\
&\quad + \pi(1 - F(\overline{\beta}))\frac{\partial s^{OC}\left(G,g\right)}{\partial G} + \pi\gamma H'(G)\int_0^{\overline{\boldsymbol{\beta}}}\beta dF(\beta) \\
&\quad + \pi(1-\gamma)\overline{\beta}f(\overline{\beta})[H(\overline{\beta}) - H(G)]\frac{d\overline{\boldsymbol{\beta}}}{dG} \\
\frac{\partial \pounds^{OC}}{\partial g} &= \pi[B' - C'] \\
&\quad + \pi(1 - F(\overline{\beta}))(1 + \frac{\partial s^{OC}(G,g)}{\partial g}) + \pi(1-\gamma)\overline{\beta}f(\overline{\beta})[H(\overline{\beta}) - H(G)]\frac{d\overline{\boldsymbol{\beta}}}{dg},
\end{aligned}
$$

where A, B, C B' and $C'$are defined in equations (42)–(43) and pertain to the parents' utility.

The effect of a change in G on the children's utility mirrors our discussion above for the OO regime and is ambiguous. Similarly, and in contrast to our discussion above regarding the TU policy, the effect of a change in g on the children's utility is also ambiguous. First, caregivers prefer a higher level of g for otherwise they would have to replace it with their own informal care (due to the full crowding out at the intensive margin). Second, children who do not provide care would not care either way. With only the parents of caregivers receiving g, changing it will have no effect on the children who do not provide care. Third, marginal children prefer a lower level of g. A reduction in g increases $\overline{\beta}$ and allows them to switch out of providing informal care (as long as the government launders out part of the altruistic component of their utility; i.e. if $\gamma < 1$).[40] In sum, including children's utility in the social welfare increases g above g^{OC} if there is no "laundering out"; otherwise another effect in opposite direction surfaces due to the impact on the marginal children.

9 Summary and conclusion

This paper has studied the role of private and public insurance programs in a world in which family assistance is uncertain. It began by arguing that private insurance markets, even if they exist and are actuarially fair, can offer insurance only against dependency and not against the default of altruism. It then considered three public programs, topping up, opting out, and opting-out-cum-transfers to find which, and under what circumstances, would do better than private insurance. In doing so, the paper also studied the crowding out implication of each policy both at the intensive and extensive margin.

The main takeaways of this study are: First, a *TU* policy offers full insurance against dependency only; it does nothing by way of providing insurance against the default of altruism. As such, it performs the same function as an actuarially fair private insurance market against dependency does. Second, an *OO* policy in contrast *may* improve upon what private insurance markets offer; sufficient conditions for which have been derived. Third, opting-out-cum-transfer policies are even more likely to do better than private insurance markets. Distortions arising from opting-out and transfer components can, to some degree, offset one another. Fourth, none of these policies can achieve full insurance against altruism default.

The paper has also found that the three public LTC policies have different crowding-out implications for the informal care that children provide. Whereas a *TU* policy entails crowding out at both intensive and extensive margins, an *OO* policy leads only to crowding out at the extensive margin. In the case of an opting-out-cum-transfer policy, its opting-out component leads to crowding out at the extensive margin. In contrast, its transfer component leads to crowding out at the intensive margin while it also entails crowding in at the extensive margin as it induces more children to assist their parents.

These results were derived for a social welfare function that depends solely on the parents' utilities. They were re-examined in Section 8 by incorporating the utility of the grown-up children in the social welfare function. In this reformulation, crowding out is no longer only a cost to the society but it may also serve as a source of benefit (by reducing caregivers' costs). Interestingly, the desired level of public assistance under *TU* will become higher because the children's expected utility increases with public provision. However, the expansion of public LTC programs may not be warranted for the other two policies. The *OO* will and the *OC* might reduce the parents' private savings thus shifting a larger share of the burden to informal

[40] When $\gamma = 1$ this term vanishes because of the envelope theorem.

caregivers. The various tradeoffs have been described in Section 8.

There are other more specific results that have been reported in Propositions 1–6. Some of these are based on the simplifying assumptions we have made. For instance, the equivalence between TU and (fair) private insurance markets is due to the assumption that individuals are ex-ante identical. However, this should not distract from the fundamental message of the paper; namely, that uncertain altruism and the infeasibility of offering private insurance against its default create a role for LTC public insurance. If properly designed, it can provide the parents with insurance against the risk of not being able to count on adequate informal care if they become dependent. In fact, if anything, ex-ante heterogeneity of parents will only strength the case for public insurance. Private insurance cannot redistribute resources among parents whose children are likely to be more or less altruistic; only social insurance can. Similarly, poor parents may be unable to afford LTC insurance even if it were available at fair rates. Assuming parents to be ex-ante identical divests social insurance from its redistributive advantages and allows one to compare it with private insurance solely in terms of their efficiency properties.

Another simplifying assumption is the quasi-linearity of children's utility in income while expressing informal care in monetary terms (and thus a perfect substitute to income). This lies behind the full crowding out at the intensive margin result under a TU policy regime. As alternatives, one can postulate a concave utility of income net of the monetary cost of care, quasi-linearity but a convex cost of care, or quasi-linearity with a concave benefit of care to the parents.[41] These specifications too will lead to crowding out; thought no longer in full. To be more precise, under suitable concavity/convexity assumptions, crowding out under TU persists albeit not on a one-to-one basis. Similarly, there will be less crowding out of the parents' savings but it will not disappear. Again, these alternative formulations only strengthen the case for public insurance (while complicating the analysis significantly). From a social welfare perspective, crowding out represents a cost; consequently, the less of it there is the stronger will be the justification for having a public LTC.

We conclude by observing that, while the tradeoffs we have highlighted should inform policy makers in their quest for finding the "right" public LTC scheme, a lot more needs to be done. A number of our simplifying assumptions should be relaxed in future research (but not all at the same time). First, our analysis is based on a particular type of altruism. It is one-sided and ascending; namely, it reflects only the concerns of children towards their parents and not vice-versa. It is also restricted, meaning that it is triggered by the state of dependency of the parents (it disappears if parents remain healthy). Implications of dropping one or both of these assumptions are worth exploring. Related to this, is the issue of strategic bequests which, with one-sided altruism, we have ignored.

Second, there is the assumption that parents are unable to influence their children's degree of altruism or its distribution. Introducing such a possibility will be an interesting extension. However, it is too complex to be added to the current paper (but it is on our research agenda). For one, it is not a given that investing time and/or money in the education of children increases the likelihood of receiving informal care from them. Quite the opposite; the relationship between

[41] By writing children's utility as $U_c(c-a)+\beta H(s+g+a)$, $y-\phi(a)+\beta H(s+g+a)$ or $y-a+\beta H(s+g+\varphi(a))$.

the two is likely to be rather complex and possibly inverse U-shaped. Children who feel neglected may not be very inclined to help their dependent parents; but highly-educated children are likely to move away to pursue their career so that they may not be in a position to provide informal care.

Third, a lot more needs to be done with respect to the financing of LTC. We have simply assumed that the financing comes from a proportional income tax levied on the parents. We have additionally assumed that the government is able to tax away the dependent parents' resources (savings and any private insurance they may have purchased). These are rather strong assumptions. It is not clear that the labor supply or private assets are observable at no cost. In future research, it would be important to introduce a richer fiscal tool-box including non-linear taxes.

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Appendix A

Proof of $s^{FI} < s^{LF} < s^{FI} + \delta$: Substitute $s^{FI}(\delta)$ for s^{FI} in equation (10), differentiate it totally with respect to δ, and simplify to get

$$(1-\pi) U''\left(s^{FI}\right) \frac{ds^{FI}}{d\delta} + \pi \left(\frac{ds^{FI}}{d\delta} + 1\right) \left[-f(\beta_0) \frac{H''\left(s^{FI}+\delta\right)}{H'\left(s^{FI}+\delta\right)} + F\left(\beta_0\left(s^{FI}+\delta\right)\right) H''\left(s^{FI}+\delta\right)\right] = 0.$$

Then collect the terms and "solve" for $ds^{FI}/d\delta$. This results in

$$\frac{ds^{FI}}{d\delta} = -\frac{\pi H''\left(s^{FI}+\delta\right)\left[F(\beta_0) - f(\beta_0)\beta_0\right]}{(1-\pi) U''\left(s^{FI}\right) + \pi H''\left(s^{FI}+\delta\right)\left[F(\beta_0) - f(\beta_0)\beta_0\right]} < 0, \tag{A1}$$

where the sign of (A1) follows from the concavity of $H(\cdot), U(\cdot)$ and $F(\beta)$. Now observe that setting $\delta = 0$ in (10) simplifies it to equation (8) in the *laissez faire* so that $s^{FI}(0) = s^{LF}$. This allows us to deduce, for $\delta > 0$,

$$s^{FI} < s^{LF}.$$

Next, $s^{FI} < s^{LF}$ implies that $U'\left(s^{FI}\right) > U'\left(s^{LF}\right)$. Comparing (10) with (8) then tells us that

$$F\left(\beta_0\left(s^{FI}+\delta\right)\right) H'\left(s^{FI}+\delta\right) < F(\beta_0(s^{LF}))H'\left(s^{LF}\right). \tag{A2}$$

But,

$$\begin{aligned} \frac{d}{ds} F(\beta_0(s)) H'(s) &= f(\beta_0) \frac{d\beta_0(s)}{ds} H'(s) + F(\beta_0(s)) H''(s) \\ &= \left[F(\beta_0(s)) - \beta_0(s) f(\beta_0)\right] H''(s) < 0. \end{aligned}$$

so that $F(\beta_0(s)) H'(s)$ is a decreasing function of s. It then follows from (A2) that

$$s^{FI} + \delta > s^{LF}.$$

Proof of Proposition 4: We first prove that $\widetilde{\beta} = \widetilde{\beta}\left(s^{TU} + g^{TU}\right) < \widehat{\beta}\left(g^{TU} + s^{TU}, s^{TU}\right) \equiv \beta^A$. Start from the optimal policy under TU and examine under what conditions it can be replicated under OO. Consider the optimal policy under TU, g^{TU}, which yields s^{TU} and an expected utility for the parent given by

$$EU^{TU} \equiv w\overline{T} - \pi g^{TU} - s^{TU} + (1-\pi) U\left(s^{TU}\right) + \pi \left[\int_{\widetilde{\beta}}^{\infty} H(m(\beta))\, dF(\beta) + F(\widetilde{\beta}) H\left(s^{TU} + g^{TU}\right)\right], \tag{A3}$$

Replace this policy by an OO policy in which in which G is set equal to $g^{TU}+s^{TU}$ and s is set equal to s^{TU}. The expected utility of parents under this alternative policy is, from (32),

$$EU^{OO}\left(g^{TU}+s^{TU},s^{TU}\right)=w\overline{T}-s^{TU}+(1-\pi)\,U\left(s^{TU}\right)+\\ \pi F(\beta^{A})\left[H\left(g^{TU}+s^{TU}\right)-g^{TU}\right]+\pi\int_{\beta^{A}}^{\infty}H\left(m\left(\beta\right)\right)dF(\beta). \tag{A4}$$

Subtracting (A3) from (A4) and simplifying

$$EU^{OO}\left(g^{TU}+s^{TU},s^{TU}\right)-EU^{TU}=\pi\left[\int_{\beta^{A}}^{\infty}H\left(m\left(\beta\right)\right)dF(\beta)-\int_{\beta^{A}}^{\infty}H\left(m\left(\beta\right)\right)dF(\beta)\right]+\\ \pi g^{TU}\left[1-F(\beta^{A})\right]+\pi H\left(g^{TU}+s^{TU}\right)\left[F(\beta^{A})-F(\widetilde{\beta})\right]. \tag{A5}$$

Next compare $\widetilde{\beta}$ with β^{A} to determine if a child with $\beta=\widetilde{\beta}$ provides assistance under this alternative OO policy. Recall from (25) that, under OO and for a given G and s, the threshold level of β below which no assistance is provided is implicitly defined by $\widehat{\beta}\left[H(m(\widehat{\beta}))-H\left(G\right)\right]-\left(m(\widehat{\beta})-s\right)=0$. Hence at $G=g^{TU}+s^{TU}$ and $s=s^{TU}$, this threshold level, $\beta^{A}\equiv\widehat{\beta}\left(g^{TU}+s^{TU},s^{TU}\right)$, is given by

$$\widehat{\beta}\left[H(m(\beta^{A}))-H\left(g^{TU}+s^{TU}\right)\right]-\left(m(\beta^{A})-s^{TU}\right)=0.$$

But, from the definition of $\widetilde{\beta}$, we have that $m(\widetilde{\beta})=g^{TU}+s^{TU}$. Hence at $\beta=\widetilde{\beta}$, the left-hand side of the above expression is

$$\widetilde{\beta}\left[H(m(\widetilde{\beta}))-H\left(g^{TU}+s^{TU}\right)\right]-\left(m(\widetilde{\beta})-s^{TU}\right)=-g^{TU}<0,$$

implying that

$$\widetilde{\beta}<\beta^{A}.$$

Hence a child with $\beta=\widetilde{\beta}$ will not provide aid under this alternative *OO policy.*

With $\widetilde{\beta}<\beta^{A}$, we rewrite equation (A5) as

$$EU^{OO}\left(g^{TU}+s^{TU},s^{TU}\right)-EU^{TU}=\\ \pi\left\{\left[1-F(\beta^{A})\right]g^{TU}-\int_{\widetilde{\beta}}^{\beta^{A}}\left[H\left(m\left(\beta\right)\right)-H\left(g^{TU}+s^{TU}\right)\right]dF(\beta)\right\}, \tag{A6}$$

which is non-negative if the right-hand side of (A6) is non-negative. Now since the optimal OO values of G and s are generally different from $g^{TU}+s^{TU}$ and s^{TU}, it must be the case that

$$EU^{OO}\geq EU^{OO}\left(g^{TU}+s^{TU},s^{TU}\right)\geq EU^{TU},$$

if the right-hand side of (A6) is non-negative.

Proof of (40) and (41): To prove (40), differentiate (38)–(39) partially with respect to G and g, and "solve" using Cramer's rule:

$$\frac{\partial s^{OC}}{\partial G}|_{g}=\frac{-\pi f'(\overline{\beta})\overline{\beta}H'(G)}{(1-\pi)U''(s^{OC})\Delta H-\pi f'(\overline{\beta})}<0, \tag{A7}$$

$$\frac{\partial s^{OC}}{\partial g}|_{G}=\frac{\pi f'(\overline{\beta})}{(1-\pi)U''(s^{OC})\Delta H-\pi f'(\overline{\beta})}>0, \tag{A8}$$

$$\frac{\partial\overline{\boldsymbol{\beta}}}{\partial G}|_{g}=\frac{(1-\pi)U''(s^{OC})\overline{\beta}H'(G)}{(1-\pi)U''(s^{OC})\Delta H-\pi f'(\overline{\beta})}>0, \tag{A9}$$

$$\frac{\partial\overline{\boldsymbol{\beta}}}{\partial g}|_{G}=\frac{-(1-\pi)U''(s^{OC})}{(1-\pi)U''(s^{OC})\Delta H-\pi f'(\overline{\beta})}<0, \tag{A10}$$

where $\Delta H = H(m(\overline{\beta})) - H(G)$. The signs follow from the negativity of the denominator in each of the equations (second-order condition of the parents' optimization problem with respect to s), and the concavity of $F(\cdot)$ which implies $f'(\cdot) = F''(\cdot) < 0$. To prove (41), divide (A7) by (A8) and (A9) by (A10).

Proof of (44): Rearrange equations (42)–(43) and divide the former by the latter to get

$$\frac{\frac{1}{\pi}\frac{\partial £^{OC}}{\partial G} - F(\overline{\beta})H'(G) + f(\overline{\beta})\frac{\partial\overline{\beta}}{\partial G}\Delta H + \left[F(\overline{\beta}) - F(\overline{\beta})\frac{\partial s^{OC}}{\partial G}\right]}{\frac{1}{\pi}\frac{\partial £^{OC}}{\partial g} + f(\overline{\beta})\frac{\partial\overline{\beta}}{\partial g}\Delta H + \left[1 - F(\overline{\beta}) - F(\overline{\beta})\frac{\partial s^{OC}}{\partial g}\right]} = \frac{\frac{\partial\overline{\beta}}{\partial G}}{\frac{\partial\overline{\beta}}{\partial g}} = -\overline{\beta}H'(G),$$

where we have used (41). ex Multiplying through

$$\frac{1}{\pi}\frac{\partial £^{OC}}{\partial G} - F(\overline{\beta})H'(G) + f(\overline{\beta})\frac{\partial\overline{\beta}}{\partial G}\Delta H + \left[F(\overline{\beta}) - F(\overline{\beta})\frac{\partial s^{OC}}{\partial G}\right] =$$
$$-\overline{\beta}H'(G)\left\{\frac{1}{\pi}\frac{\partial £^{OC}}{\partial g} + f(\overline{\beta})\frac{\partial\overline{\beta}}{\partial g}\Delta H + \left[1 - F(\overline{\beta}) - F(\overline{\beta})\frac{\partial s^{OC}}{\partial g}\right]\right\}.$$

Or

$$\frac{1}{\pi}\frac{\partial £^{OC}}{\partial G} - F(\overline{\beta})H'(G) + f(\overline{\beta})\frac{\partial\overline{\beta}}{\partial G}\Delta H + \left[F(\overline{\beta}) - F(\overline{\beta})\frac{\partial s^{OC}}{\partial G}\right] +$$
$$\left\{\frac{1}{\pi}\frac{\partial £^{OC}}{\partial g} + f(\overline{\beta})\frac{\partial\overline{\beta}}{\partial g}\Delta H + \left[1 - F(\overline{\beta}) - F(\overline{\beta})\frac{\partial s^{OC}}{\partial g}\right]\right\}\overline{\beta}H'(G) = 0.$$

or,

$$\frac{1}{\pi}\frac{\partial £^{OC}}{\partial G} + \frac{1}{\pi}\overline{\beta}H'(G)\frac{\partial £^{OC}}{\partial g} = -f(\overline{\beta})\Delta H\left[\frac{\partial\overline{\beta}}{\partial G} + \frac{\partial\overline{\beta}}{\partial g}\overline{\beta}H'(G)\right]$$
$$+F(\overline{\beta})\left[\frac{\partial s^{OC}}{\partial G} + \overline{\beta}H'(G)\frac{\partial s^{OC}}{\partial g}\right] + F(\overline{\beta})\left[H'(G) - 1\right] - \left[1 - F(\overline{\beta})\right]\overline{\beta}H'(G).$$

But we have, from (41),

$$\frac{\partial\overline{\beta}}{\partial G} + \overline{\beta}H'(G)\frac{\partial\overline{\beta}}{\partial g} = \frac{\partial s^{OC}}{\partial G} + \overline{\beta}H'(G)\frac{\partial s^{OC}}{\partial g} = 0.$$

Substituting in the expressions above results in

$$\frac{1}{\pi}\frac{\partial £^{OC}}{\partial G} + \frac{1}{\pi}\overline{\beta}H'(G)\frac{\partial £^{OC}}{\partial g} = F(\overline{\beta})\left[H'(G) - 1\right] + \left[1 - F(\overline{\beta})\right]\left[-\overline{\beta}H'(G)\right].$$

Evaluating this expression at the optimal solution to the OO scheme (assuming it has an interior solution) and $g = 0$, we arrive at equation (44).

Appendix B

Let δ denote the amount of private insurance against dependency purchased and $\pi\delta$ its actuarially fair premium.

B1 Topping up

We have previously examined the implications of actuarially fair insurance markets for the laissez faire solution in Subsection 2.2. Comparing the market outcome there with the TU solution engineered by the government in Section 3, one immediately observes that the two solutions are identical. The implication of this result is that a TU policy offers only full insurance against dependency and does nothing by way of providing insurance against the default of altruism.

B2 Opting out

Start with a pure OO policy and examine the changes that private insurance may lead to in each stage of our model. As far as the children are concerned, equation (25) changes to

$$\widehat{\beta}\left[H(m(\widehat{\beta}))-H\left(G\right)\right]-\left(m(\widehat{\beta})-s-\delta\right)=0,$$

where $\widehat{\beta}(G,s+\delta)$ has replaced $\widehat{\beta}(G,s)$. Yet partial differentiation of $\widehat{\beta}(G,s+\delta)$ with respect to G and s yields equations for $\partial\widehat{\beta}/\partial G$ and $\partial\widehat{\beta}/\partial s$ identical to (26)–(27). Partial differentiation of $\widehat{\beta}(G,s+\delta)$ with respect to δ results in $\partial\widehat{\beta}/\partial\delta=\partial\widehat{\beta}/\partial s$.

The parents' expected utility now includes a term for the cost of purchasing insurance:

$$EU=w\left(1-\tau\right)\overline{T}-s-\pi\delta+\left(1-\pi\right)U\left(s\right)+\pi\left[H\left(G\right)F(\widehat{\beta})+\int_{\widehat{\beta}}^{\infty}H\left(m\left(\beta\right)\right)dF(\beta)\right],$$

which they maximize with respect to s and δ. The first-order condition with respect to s, assuming an interior solution as previously, yields an equation identical to (28), and then, upon substituting for $\partial\widehat{\beta}/\partial s$, an equation identical to (29), except for $s+\delta$ replacing s in $\widehat{\beta}$. To determine δ, consider the partial derivative of EU with respect to δ,

$$\frac{\partial EU}{\partial\delta}=-\pi-\pi f(\widehat{\beta})\left[H(m(\widehat{\beta}))-H\left(G\right)\right]\frac{\partial\widehat{\beta}}{\partial\delta}.$$

Substitute for $\partial\widehat{\beta}/\partial\delta$, from the expression for $\partial\widehat{\beta}/\partial s$ in (27), evaluate at $\delta=0$, and use (29). This yields

$$\frac{\partial EU}{\partial\delta}|_{\delta=0}=-\pi+\pi f(\widehat{\beta})=\left(1-\pi\right)\left[1-U'\left(s^{OO}\right)\right].$$

Two possibilities arise:

Case (i): $U'\left(s^{OO}\right)\geq 1$ so that $\delta=0$ and nobody purchases any private insurance for dependency even if offered at an actuarially fair premium. This lead us back to the pure OO solution.

Case (ii): $U'\left(s^{OO}\right)<1$. Under this circumstance $\delta>0$ so that at the optimum $U'\left(s\right)=1$. Consequently, the solution for savings is the same we had under *laissez faire* with insurance markets: $s=s^{FI}$. Substituting this value in the first-order condition for s (which continues to be represented by (29)), we have[42]

$$f(\widehat{\beta}(G,s^{FI}+\delta))=1. \tag{B1}$$

This condition implies that $\widehat{\beta}$ only depends on the shape of the distribution function $F(\beta)$, and not on the public policy. Solving for δ then results in $\delta\left(G\right)$. Substituting $\delta\left(G\right)$ in (B1), differentiating the resulting identity with respect to G, and simplifying results in

$$\frac{d\delta}{dG}=-\frac{\frac{\partial\widehat{\beta}}{\partial G}}{\frac{\partial\widehat{\beta}}{\partial\delta}}=-\frac{\frac{\widehat{\beta}H'(G)}{H(m(\widehat{\beta}))-H(G)}}{-\frac{1}{H(m(\widehat{\beta}))-H(G)}}=\widehat{\beta}H'\left(G\right). \tag{B2}$$

We can now study the government's optimal choice of G. The government maximizes the parents' optimized value of EU subject to its budget constraint,

$$\tau wT=\pi F(\widehat{\beta})\left[G-s^{FI}-\delta\right].$$

This leads to the maximization of the following welfare function,

$$\pounds=EU\left(G\right)-\pi F(\widehat{\boldsymbol{\beta}})\left[G-s^{FI}-\delta\left(G\right)\right],$$

[42] There can only exist a unique β that satisfies condition (B1). This follows from the concavity of $F(\cdot)$. Obviously, if there does not exist any β that satisfies (B1), only Case (i) can arise.

where $\widehat{\boldsymbol{\beta}} \equiv \widehat{\beta}(G, s^{FI} + \delta(G))$. Maximizing $\pounds$ with respect to G, using the envelope theorem, we have

$$\frac{d\pounds}{dG} = \pi H'(G) F(\widehat{\beta}) - \pi f(\widehat{\beta}) \left[H(m(\widehat{\beta})) - H(G) \right] \frac{\partial \widehat{\beta}}{\partial G}|_{s,\delta} - \pi \left[F(\widehat{\boldsymbol{\beta}}) \left(1 - \frac{d\delta}{dG} \right) + (G - s^{FI} - \delta(G)) f(\widehat{\beta}) \frac{d\widehat{\boldsymbol{\beta}}}{dG} \right].$$

Observe that the three terms on the right-hand side correspond to terms A, B, and C in the pure opting out solution (equation (33)). The first two terms have identical formulations. In term C, $d\delta/dG$ has replaced ds^{OO}/dG and $(G - s^{FI} - \delta(G))$ has replaced $(G - s^{OO})$. Moreover, we have $\widehat{\beta} = \widehat{\beta}(G, s^{FI} + \delta(G))$ rather than $\widehat{\beta} = \widehat{\beta}(G, s^{OO}(G))$.

Next substitute for $\partial\widehat{\beta}/\partial G, d\delta/dG$, and $d\widehat{\boldsymbol{\beta}}/dG$ in the expression for $d\pounds/dG$ to get

$$\frac{d\pounds}{dG} = \pi \left\{ \left[F(\widehat{\beta}) - f(\widehat{\beta})\widehat{\beta} + F(\widehat{\beta})\widehat{\beta} \right] H'(G) - F(\widehat{\beta}) \right\},$$

where $F(\widehat{\beta}) - f(\widehat{\beta})\widehat{\beta} > 0$ due to the concavity of $F(\cdot)$. Two possibilities arise depending on the sign of $d\pounds/dG$ at $G = s^{FI} + \delta(G)$. If $d\pounds/dG \leq 0$, there is no interior solution for G and an OO policy is not helpful. Under this circumstance, we have the *laissez faire* solution with insurance markets for dependency (as in Subsection 2.2) which is equivalent to the TU solution. Otherwise, if $d\pounds/dG > 0$, there is an interior solution for G given by

$$H'(G) = \frac{F(\widehat{\beta})}{F(\widehat{\beta}) + \left(1 - F(\widehat{\beta})\right)\left(-\widehat{\beta}\right)} > 1, \tag{B3}$$

where, from (B1), $f(\widehat{\beta})$ has been set equal to one. An OO policy is desirable but it still does not offer full insurance.

B3 Opting-out-cum-transfers

The presence of private insurance markets lead to :the following changes. As far as the children are concerned, their threshold level of β, $\overline{\beta}(G, s + \delta + g)$, changes to

$$\overline{\beta} \left[H(m(\overline{\beta})) - H(G) \right] - \left(m(\overline{\beta}) - s - \delta - g \right) = 0, \tag{B4}$$

Partial differentiation of $\overline{\beta}(G, s + \delta + g)$ with respect to G, s, and g yields identical equations to (36)–(37) for $\partial\overline{\beta}/\partial G$ and $\partial\overline{\beta}/\partial g = \partial\overline{\beta}/\partial s$; partial differentiation of $\overline{\beta}(G, s + \delta + g)$ with respect to δ results in $\partial\overline{\beta}/\partial\delta = \partial\overline{\beta}/\partial g = \partial\overline{\beta}/\partial s$.

Turning to the parents' expected utility, it is now given by

$$EU = w(1 - \tau)\overline{T} - s - \pi\delta + (1 - \pi) U(s) + \pi \left[H(G) F(\overline{\beta}) + \int_{\overline{\beta}}^{\infty} H(m(\beta)) dF(\beta) \right],$$

which they maximize with respect to s and δ. The first-order condition with respect to s, assuming an interior solution as previously, yields an equation identical to (38), and then upon substituting for $\partial\overline{\beta}/\partial s$ an equation identical to (39), except for $s + \delta$ replacing s in $\overline{\beta}$. To determine δ, consider the partial derivative of EU with respect to δ,

$$\frac{\partial EU}{\partial \delta} = -\pi - \pi f(\overline{\beta}) \left[H(m(\overline{\beta})) - H(G) \right] \frac{\partial \overline{\beta}}{\partial \delta}.$$

Substitute for $\partial\overline{\beta}/\partial\delta$, from the expression for $\partial\overline{\beta}/\partial s$ in (37), evaluate at $\delta = 0$, and use (39) to get

$$\frac{\partial EU}{\partial \delta}|_{\delta=0} = -\pi + \pi f(\overline{\beta}) = (1 - \pi) \left[1 - U'\left(s^{OC}\right) \right].$$

Two possibilities arise:

Case (i): $U'\left(s^{OC}\right) \geq 1$ so that $\delta = 0$ and nobody purchases any private insurance even if offered at an actuarially fair premium. This leads us back to the opting-out-cum-transfer solution.

Case (ii): $U'\left(s^{OC}\right) < 1$. Under this circumstance $\delta > 0$ so that at the optimum $U'(s) = 1$. Again, the solution for savings will be the same as we had under *laissez faire* with insurance markets: $s = s^{FI}$. Substituting in the first-order condition for s, which continues to be represented by (39), we have

$$f(\overline{\beta}(G, s^{FI} + \delta + g)) = 1. \tag{B5}$$

The system of equations (B4)–(B5) jointly determines the values of δ and $\overline{\beta} = \overline{\beta}(G, s^{FI} + \delta + g)$ as functions of G and g: $\delta^{OC}(G, g)$ and $\overline{\boldsymbol{\beta}}(G, g) \equiv \overline{\beta}(G, s^{FI} + \delta^{OC}(G, g) + g)$. Differentiating this system of equations with respect to G and g, we have

$$\frac{\partial \overline{\boldsymbol{\beta}}}{\partial G}|_g = 0, \frac{\partial \delta}{\partial G}|_g = \overline{\beta} H'(G); \text{ and } \frac{\partial \overline{\boldsymbol{\beta}}}{\partial g}|_G = 0, \frac{\partial \delta}{\partial g}|_G = -1.$$

Next is the determination of the government's optimal choice of G. The government maximizing the parents' optimized value of EU subject to its budget constraint

$$\tau w T = \pi\left\{F(\overline{\beta})\left[G - s^{FI} - \delta\right] + \left(1 - F(\overline{\beta})\right) g\right\}.$$

This leads to the maximization of the following welfare function

$$\pounds = EU(G, g) - \pi\left\{F(\overline{\boldsymbol{\beta}})\left[G - s^{FI} - \delta\right] + \left(1 - F(\overline{\boldsymbol{\beta}})\right) g\right\},$$

where $\overline{\boldsymbol{\beta}} = \overline{\boldsymbol{\beta}}(G, g)$. Differentiating $\pounds$ partially with respect to G and g, and using the envelope theorem, one obtains

$$\begin{aligned}\frac{\partial \pounds}{\partial G} &= \pi H'(G) F(\overline{\beta}) - \pi f(\overline{\beta})\left[H(m(\overline{\beta})) - H(G)\right] \frac{\partial \overline{\beta}}{\partial G}|_{s,\delta,g} - \\ &\pi\left[\left(G - s^{FI} - \delta\right) f(\overline{\beta}) \frac{\partial \overline{\boldsymbol{\beta}}}{\partial G} + F(\overline{\beta})\left(1 - \frac{\partial \delta}{\partial G}\right) - g f(\overline{\beta}) \frac{\partial \overline{\boldsymbol{\beta}}}{\partial G}\right], \\ &= \pi\left\{H'(G) F(\overline{\beta}) - f(\overline{\beta})\overline{\beta} H'(G) - F(\overline{\beta})\left[1 - \overline{\beta} H'(G)\right]\right\},\end{aligned}$$

$$\begin{aligned}\frac{\partial \pounds}{\partial g} &= -\pi f(\overline{\beta})\left[H(m(\overline{\beta})) - H(G)\right] \frac{\partial \overline{\beta}}{\partial g}|_{s,\delta,G} - \\ &\pi\left[\left(G - s^{FI} - \delta\right) f(\overline{\beta}) \frac{\partial \overline{\boldsymbol{\beta}}}{\partial g} - F(\overline{\beta}) \frac{\partial \delta}{\partial g} - g f(\overline{\beta}) \frac{\partial \overline{\boldsymbol{\beta}}}{\partial g} + \left(1 - F(\overline{\beta})\right)\right] \\ &= \pi\left[f(\overline{\beta}) - 1\right] = 0.\end{aligned}$$

Observe again that the three terms on the right-hand side of $\partial \pounds / \partial G$ correspond to terms A, B, and C in the opting-out-cum-transfer solution (equation (42)), with a slightly different formulation for term C. The terms continue to have the same interpretations. There continue to be two possibilities depending on the sign of $\partial \pounds / \partial G$ at $G = s^{FI} + \delta(G) + g$. If $\partial \pounds / \partial G \leq 0$, there is no interior solution for G and an opting out policy is not desirable (not even a pure one). The solution will then be the same as the *laissez faire* solution with insurance markets. If $\partial \pounds / \partial G > 0$, there is an interior solution for G characterized by (B3). Either way parents are under-insured.

Similarly, the two terms on the right-hand side $\partial \pounds / \partial g$ correspond to terms B', and C' in the opting-out-cum-transfer solution (equation (43)), with a slightly different formulation for C'. As before, and for the same reason, there is no term corresponding to A. They continue to have the same interpretations.