

# Advanced Analysis Syllabus

## Objective

Transition from measure theory to the structural analysis of infinite-dimensional vector spaces.

- **Banach Spaces:** Completeness in normed linear spaces. Examples including  $L^p$  spaces and  $C(K)$ . Bounded linear operators and the operator norm.
- **Hilbert Spaces:** Inner products, Cauchy-Schwarz inequality, and the Parallelogram Law. Orthogonality, projection theorem, and orthonormal bases (Bessel's inequality, Parseval's identity).
- **Riesz Representation Theorem:** The unique correspondence between a Hilbert space and its dual.
- **Hahn-Banach Theorem:**
  - *Analytic form:* Extension of linear functionals.
  - *Geometric form:* Separation of convex sets by hyperplanes. item **Weak Topologies:** Definition of weak and weak-\* convergence. The **Banach-Alaoglu Theorem** regarding the compactness of the unit ball in the dual space.

## Grading

The final grade is computed based on rigorous proof-based assessments.

Component	Weight	Description
Midterm Exam	30%	Covering Banach and Hilbert spaces.
Final Exam	70%	