AN INTRODUCTION TO VISCOSITY SOLUTIONS WITH APPLICATIONS IN ECONOMICS

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Abstract. This course offers an accessible yet rigorous overview of the theory of viscosity solutions for Hamilton-Jacobi (HJ) equations, with a particular focus on first-order equations. In twelve hours, we explore the fundamental concepts and key properties that make viscosity solutions a powerful framework for analyzing nonlinear partial differential equations across various fields, including mathematical economics.

Beginning with a broad introduction, we establish the core principles of viscosity solutions and demonstrate essential results, such as their pivotal stability property, which underpins their robustness and extensive applicability in both theoretical and applied contexts. The course then shifts focus to first-order HJ equations, emphasizing their significance through selected examples from dynamic optimization, optimal control, and economic modeling.

The primary goal is to provide students with a foundational technical understanding of viscosity solution theory, thereby fostering deeper insights into the mathematical modeling of economic dynamics. Additionally, the course prepares participants for potential research in mathematical economics, optimal control, and applied analysis involving Hamilton-Jacobi equations.

Prerequisites: Arzelà-Ascoli Theorem, Rademacher Theorem for locally Lipschitz functions.

Grading system: One homework assignment and an essay (written in LaTeX) on an agreed-upon topic related to the course.

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