Game Theory (Théorie des jeux)

Course title - Intitulé du cours	Game Theory – Théorie des jeux
Level / Semester - Niveau /semestre	M1 / S1
School - Composante	Toulouse School of Economics – Ecole d'Economie de Toulouse
Teacher - Enseignant responsable	Bertrand GOBILLARD
Other teacher(s) - Autre(s) enseignant(s)	
Other teacher(s) - Autre(s) enseignant(s)	
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Other teacher(s) - Autre(s) enseignant(s)	
Lecture Hours - Volume Horaire CM	30
TA Hours - Volume horaire TD	12
TP Hours - Volume horaire TP	0
Course Language - Langue du cours	English (international track) French (standard track)
TA / TD Language - Langue des TD	Anglais

Teaching staff contacts - Coordonnées de l'équipe pédagogique :

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Teaching assistants: Amirreza Ahmadzadeh, Maxim Sandiumenge, Thi Hien Pham, Yusuke Yamaguchi, Xin Zhang.

Course's Objectives - Objectifs du cours :

Objectives. Brodadly speaking the game theory class is an introductory course to Economic theory (namely the modelisation and analysis of economic problems using rigorous theoretical reasoning) when the focus concerns situations of strategic interaction. Its prime objective is to familiarize students with basic, key concepts and techniques developped in modern microeconomic / equilibrium / game theory, and learn when and how to use them. On a more concrete level it aims to provide students with a solid skills base in game theory, which aims at developing a theoretical framework for investigating circumstances where interdependent decision-makers intereract strategically using mathematical models; that have become an important area in modern microeconomics. To this end it builds on a precise description of foundations to the framework of game theory, enabling students to acquire knowledge and expertise on why and how apply the approach and, ultimately, be more confortable with standard techniques with rigour of argument used in economics to investigate a wide variety of situations involving strategic decision making.

At the end of the class it is expected students will be able to represent an economic situation as a game they can solve using appropriate solution concepts and method. In that respect, besides the ability to understand a problem and formalize and study it as a game, that of developing a clear and precise argument justifying a suggested solution is to the least as important as finding a right solution; even more here since the class won't focus on arduous technical (mathematical) aspects that quickly arise when conceiving and studying games (with no compromise on accurate formalism, logic and rigourous reasoning, nonetheless) to restrict attention to thoroughly understanding core

concepts and methods. This also means, in should be added, it is central not to be passive in the learning process for this class; practice is key to get an in-depth understanding of core notions and concepts presented and studied in the course of the semester, which includes (individual and collective) home study and class participation; shall you be a couch or field game theory player.

Course outline. Main fundamental notions and tools of game theory are studied in turn (more or less extensively): from definitions of games to solution concepts; from static games under complete information to dynamic games under incomplete information.

The course is illustrated with examples (such as bargaining games, signalling games, coordination games, voting games, repeated games…) and economic applications (including simultaneous and sequential models of competition,tacit collusion, auctions, bank runs, investment races, and so on). Precisely, to develop the theory and help students be more familiarized with the core concepts, specific, first (very) basic, examples will be used. More advanced instances of games are then studied, to further understanding. Unfortunately we won't have time, during the lectures, to develop in greater details economic aspects of general classes of problems such as those mentionned above (it is in fact common in class to introduce very stylised variants; for illustration purposes mainly). Nonetheless, to evidence how these tools can be used to approach standard economic issues, typical such examples will be included in problem sets. Lectures might nonetheless also be the opportunity for brief discussions on specific applications and references to economics.

The theoretical material covered over the semester is outlined in a collection of slides, entitled *Main slides*, already at the disposal of the students on the moodle platform. Their content is summarized in the following, and these notes will be complemented with other notes used during the lectures, containing examples, illustrations, and suggestions on how to proceed to define and solve (examples of) games (a more detailed description of the structure of the course, combining different material contents, and a tentative schedule of the lectures, is given afterwards).

O. Defining and representing games (Normal form game; Mixed extension of a normal form game; Extensive form game, Extensive form and normal form). 1 – 2 weeks

I. Static games of complete information (Games under normal form; The notion of strict dominance; The best response correspondence and Nash equilibria in pure strategies; The mixed extension of a normal form game; The best response correspondence and Nash equilibria in mixed strategies; Link between Nash equilibria and strict dominance). 2 – 3 weeks.

II. Dynamic games of complete information (Games under extensive form; Extensive form and normal form; Nash equilibria and backward induction; Subgame perfect Nash equilibria). 2 – 3 weeks

III. Repeated games (The framework; Finitely repeated games; Infinitely repeated games), 2 weeks

IV. Games with incomplete information . 1 – 2 weeks

<u>Prerequisites - Prérequis :</u>

There is no prerequisite, apart from most basic mathematical tools (derivatives and basic calculus; expected utility), and a taste for rigorous reasoning.

<u>Practical information about the sessions - Modalités pratiques de gestion du cours :</u>

(Practical organisation of the classes). Lectures: equivalent to 30 hours; language: English (1 class + small groups) and French (1 class + small groups). Tutorials: 12 hours (7 "Standard sessions" and one "Question session"); language: English. The organisation of the lectures is as follows. Two lectures are

taught each week during the first 5 weeks (Monday afternoon and Wednesday afternoon). During the remaining 5 weeks there is one lecture taught on Saturday (morning, normally). Each week, there are additional "small group" sessions, more dedicated to discussion / Q&A / details and precisions related to the material covered so far. This will take place on Thursdays, and each student will attend a class lasting around 40 minutes; meaning two groups will be taught during a "standard class schedule" of one hour and a half.

(Pedagological organisation and usage of Teaching material). A mixed mode of teaching articulating "more standard" lectures one the one hand, with "more interaction based" classes in smaller groups.

The primary idea is that students should work on the content material of a given class before going to class so that lessons can be more dedicated to "concerns and questions" students may have on a specific topic. Following experience, and a first trial in recent past based on "flipped classroom" kind of mode of teaching, the purpose is to experience an alternative protocole this year, combining more standard lectures (that student can still prepare using course material provided in advance) and "small group / interaction based" sessions (relying on what has been done during main lectures and the different notes provided to students), with the idea that I won't "teach per se" in these sessions. Instead they will be driven by students participation, contributions, interpretations, suggestions, questioning, concerns… based on their own (individual and group) preparatory work. The idea is really to open discussion leading to a more thorough study of specific aspects of the content of the course. For these classes, interaction between students will be as important as interaction between "the teacher" and students.

In more pratcial terms the lectures will be organized as follows. It will be announced in advance which part of the theoretical material will be taught during a lecture. To prepare for the class students will have access to a set of slides with guidelines entitled *Student material* (or, Material for student preparation), that corresponds to problems and questions precisely defined to primarily investigate general notions and concepts outlined in the *Main slides*. The study of these questions will then be pursued in class (in a quite condensed manner) and students will have access to the content of their analysis in the format of slides I will provide; of which is referred to as *Complementary slides*. These will provide a comprehensive examination of the main concepts (presented in general slides) integrating detailed analysis of specific games; including games introduced in *Student material*, with suggestions on how to study these games in addition to detailed answers and explanations.

Finally additional classes might be added in the end of the semester if needed.

Students will have access to a lot a material that can be used for training and understanding of the main notions and concepts of game theory studied in class, including past mid-term and final exams with solutions, and a collection of problem sets consisting of a variety of exercises and problems with detailed solutions. These problems build the material for tutorials but teaching assistants will only (are asked to) study in quite detail a few of those each session (there are too many of them). Students will however have detailed solutions to all of them (each being a potential problem for home study), and are free to ask questions about problems not solved in class. Problem sets primarily contain "classic or typical games" for practice and preparation of the exams; although many examples of such "typical games (exams)" will also be included in the material used during the lectures. An additional objective of the problem sets is to include (at the end) some "economic applications" (written is less stylised game theory format); by reason that we won't have time to cover thoroughly such applications in class, but that although it is important to point out how useful game theory can be to economists aiming at modelling (and studying) various economic questions.

Students will also have access to help desk sessions, with two formats provided. *(i) One to one meetings* (or one to small groups meetings) with a teaching assistant. Each meeting has to be scheduled and lasts fifteen minutes, where students ask for discussion of a (possibly several) specific point related to what has been done in class (including tutorials). This is also the opportunity to approach difficulties related to a lack of knowledge in students' background required to study the course material. *(ii) Group meetings*. A session of two or three hours, with two teaching asssitants, when a specific problem (that students can study in advance) is discussed and analysed; usually a problem from a past exam. There, TAs will bring suggestions on how to understand the problem, how to proceed, and how to approach and solve questions asked; in addition to providing right answers (complete, detailed written solutions being also provided). Primarily here the objective is to build on questions students will have (bust also on issues arising during the lecture from discussions between/with TAs).

(Organisation under Covid measures). If due do Covid measures it is not possible to organize the teaching as usual, we shall aim to proceed as follows. The prime idea would be to have lectures taught on site (if possible; in smaller groups if need be), with the possibility for students to follow the lectures "100% on line" and switch from "online" to "onsite" mode of teaching depending on their personnal condition or situation. The overall principle is that we will adapt, in the most appropriate manner given all possible kinds of constraints and possibilities.

Grading system - Modalités d'évaluation :

Midterm exam (20%) and final exam (80%).

Bibliography/references - Bibliographie/références :

There is no compulsory textbooks, but we would recommend:

- Robert Gibbons, "A primer in Game Theory", Wheatsheaf Books, 1992; this book can also be found under the title "Game Theory for Applied Economists", Princeton University Press.
- Steven Tadelis, "Game Theory: An Introduction", Princeton University Press.

More detailed and advanced material can be found in:

- Martin Osborne and Ariel Rubinstein, "A course in Game Theory", The MIT Press,
- Martin Osborne, "Introduction to Game Theory: International Edition", OUP Oxford,
- Drew Fudenberg and Jean Tirole, "Game Theory", The MIT Press.

Lectures notes (provided in the format of slides) contain all the material necessary to study this course:

1. *Main (blue) slides*
2. *Student material*
3. *Complementary slides*

which is complemented with *Problem sets* and *Past Final* and *Mid-term exams*.

APPENDIX: MORE DETAILED PRESENTATION OF THE STRUCTURE OF THE COURSE

A/ DETAILED DESCRIPTION OF THE CONTENTS OF THE CLASS AND HOW IT IS COVERED IN THE MATERIAL MADE AVAILABLE TO STUDENTS

INTRODUCTION

- Student material (I): Questions for an introduction
- Complementary slide (I): Introductory notes

CHAPTER 0 / DEFINING AND REPRESENTING GAMES

Part 1: Games under normal form (or strategic form)

- Student material (II) and Complementary slide (II): Normal form games

Part 2: The mixed extension of a normal form game

- Student material (III) and Complementary slide (III): The mixed extension

Part 3: Games under extensive form

- Student material (IV) and Complementary slides (IV): The extensive form

Part 4: Extensive form and normal form

- Student material (V) and Complementary slides (V): From extensive form to normal form
- Student material (VI) and Complementary slides (VI): Playing at random in extensive form games

Part 5: Introducing uncertainty

- Student material (VII) and Complementary slide (VII): Uncertainty and nature player

CHAPTER 1 / STATIC GAMES UNDER COMPLETE INFORMATION (STUDYING NORMAL FORM GAMES)

Part 1: Important preliminaries

Part 2: Games under normal form (or strategic form)

Part 3: The notion of strict dominance

- Student material (VIII) and Complementary slides (VIII): (A first discussion) On Strictly Dominated and Best Response strategies
- Student material (IX) and Complementary slides (IX): Dominance in pure strategies

Part 4: The Best-Response correspondence and Nash equilibria (in pure strategies)

- Student material (VIII) and Complementary slide (VIII): (A first discussion) On Strictly Dominated and Best Response strategies
- Student material (X) and Complementary slide (X): Nash equilibria in pure strategies
    - *Take home exercise*

Part 5: Analysis of games in mixed strategies

- Student material (XI) and Complementary slide (XI): (On) Solutions under mixed strategies in normal form games
    - Student material (XI Bis) and Complementary slide (XI Bis): Typical exam problems on Mixed strategies / Simultaneous games; with solutions

CHAPTER 2 / DYNAMIC GAMES UNDER COMPLETE INFORMATION

Part 1: Representing dynamic games

Part 2: Nash equilibria, Backward Induction, Subgame-Perfection

- Student material (XII) and Complementary slide (XII): Solutions in dynamic games I/ Nash Equilibria
- Student material (XIII) and Complementary slide (XIII): Solutions in dynamic games II/ Backward induction and Subgame perfect Nash equilibrium in perfect information games
- Student material (XIV) and Complementary slide (XIV; Part I, On the meaning / understanding / usage of the concept using previous basic games; Part II, Two standard games / Finite horizon and pure strategies): Solutions in dynamic games III/ Subgame perfect Nash Equilibria

CHAPTER 3 / DYNAMIC GAMES UNDER COMPLETE INFORMATION

Part 1: The model

- Complementary slide (XV): On repeated games

Part 2: Finitely repeated games

- Student material (XVI) and Complementary slide (XVI): Solutions in repeated games I/ First equilibrium constructions and solution in finite horizon games

Part 3: Infinitely repeated games

- Student material (XVII) and Complementary slide (XVII): Solutions in repeated games II/ Infinite horizon games

CHAPTER 4 / GAMES WITH INCOMPLETE INFORMATION

- Student material (XVIII) and Complementary slide (XVII): On incomplete information games

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B/ TENTATIVE SCHEDULE OF THE LECTURES

Week 1: Introduction and Practicalities; Normal form game and its mixed extension (Ch 0; SM & CS I-III)

Week 2: Extensive form game; Links between extensive and normal form; Mixed and behavioral strategies in dynamic games; Nature player (Ch 0; SM & CS IV-VII); On Best Response (BR) and Strict Dominance (SD) (Ch 1; SM & CS VIII)

Week 3: Strict dominance in pure strategies (Ch 1; SM & CS IX) BR and Nash equilibria (NE) in pure strategies; SD; (Ch 1; SM & CS X);

Week 4: Summary of fundamentals learned so far (based on take home exam); BR and NE in mixed strategies (Ch 1; SM & CS XI); End of Chapter 1;

Week 5: NE in dynamic games (Ch 2; SM & CS XII); Backward Induction (BI) and Subgame perfect Nash Equilibria (SPNE) in perfect information games (Ch 2; SM & CS XIII); Preparation for mid-terms (Based on a past mid-term)

Week 6: Discussion of what has been done so far based on the mid-term exam; On solving for SPNE [Chapter 2; CS & SM XIV)

Week 7: SPNE (Ch 2; SM & CS XIV'); Introduction to repeated games and finite horizon games (Ch 3; SM & CS XV and XVI)

Week 8: Infinite horizon repeated games (Ch 2; SM & CS XVII)

Week 9: End of the analysis of dynamic and repeated games; Introduction to incomplete information games (Ch 4; SM XVII)

Week 10: Incomplete information games (Ch 2; CS XVIII)