Optimization

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1   Objectives

The goal of these series of lectures is to provide M2 students and future PhD with a solid background in Optimization. In particular the course will emphasize rigorous methodologies, namely: importance of proofs, assumption checking, heuristics ("ideas genesis"). Such tasks become easier when the notions at stake are associated to concrete situations, that is why we shall, whenever possible, attach to each introduced notion geometrical ideas or concrete examples. Applications to Economics will be provided.

The outline of the course given below is quite ambitious in terms of length. It corresponds to an ideal situation in which most of the students are already familiar with elementary concepts of Calculus/Analysis such as: closed/compacts sets, continuity, convexity, etc. In other words, some mathematical maturity from the students will help them to better appreciate the course and will allow us to go further.

2   Outline

1. Optimization in Hilbert spaces

   - Theoretical tools : examples, projection theorem, separation results, Riesz representation theorem, convex cones, Farkas lemma
   - Convex functions : continuity issues, characterizations of convexity, existence and characterization of minimizers, second-order conditions for minimization
   - Convex programming : Lagrange method and KKT conditions, Lagrange multipliers,
   - Lagrangian duality
   - Sensitivity and duality
   - Nonlinear programming : KKT conditions

2. Introduction to the calculus of variations

   - Necessary conditions : Euler-Lagrange conditions, transversality condition
   - Regularity of the solutions: continuously differentiable solutions, piecewise continuously differentiable solutions, Erdmann-Weierstrass corner condition
   - Second order conditions and Legendre theorem
   - Infinite-horizon problem
   - Examples

3. Introduction to optimal control

- Pontryagine maximum principle
- Sufficient conditions
- Dynamic programming : Bellman principle
- Infinite-horizon problems

3 Requirements

A background in elementary linear algebra and calculus is strongly advised

4 Bibliographical suggestions

• Most accessible references
∗ Advanced books for rather experienced students or for advanced questions

Convex Analysis and Optimization
– * Bauschke, Heinz H.; Combettes, Patrick L. Convex analysis and monotone operator theory in Hilbert spaces. CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC. Springer, New York, 2011. xvi+468 pp.
– Berge, C., "Espaces topologiques et fonctions multivoques", Dunod, Paris, 1959. (One of the most original and motivating book in this list)
– Barvinok, A first course in Convex Analysis[1] AMS
– Bonnans F., "Optimisation continue", Dunod, 2006.
– Borwein, J., Lewis, A.S., "Convex Analysis and Nonlinear Optimization", Springer-Verlag 2000.
– • Ciarlet, P.-G, "Introduction à l'analyse matricielle et à l'optimisation", Masson, Paris, 1988.
– * Hiriart-Urruty, J.-B, Lemaréchal, C., "Convex analysis and minimization algorithms", Springer-Verlag, 1996. [algorithmic oriented]
– Rockafellar, R.T., "Convex analysis", Princeton University Press, 1970. [2]
– • Sundaram, R. K. "A first course in optimization theory". Cambridge University Press, Cambridge, 1996. xviii+357 pp.

Fixed point theory
– Border, K.C., "Fixed point theorems with applications to economics and game theory", Cambridge University Press, 1985. (A "must" for fixed-point like results)

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[1]A must for duality theory in infinite dimensional spaces
[2]THE reference in convex analysis

Calculus of variations and Optimal control

– Gelfand I.M. and Fomin S.V., "Calculus of variations and Optimal control", Dover publications, 1963.

– * Clarke, Frank H., "Functional Analysis, Calculus of Variations and Optimal Control", Springer, 2013.

– * Clarke, Frank H. Optimization and nonsmooth analysis. Canadian Mathematical Society Series of Monographs and Advanced Texts. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1983. xiii+308 pp.

– Dacorogna, B., "Introduction to the calculus of variations". Translated from the 1992 French original. Second edition. Imperial College Press, London, 2009. xiv+285 pp.

– • Demange, G., Rochet, J.-C, "Méthodes mathématiques de la finance", Economica, 1992.

– Evans, L.C, (Berkeley), Online course on Optimal Control at:

http://math.berkeley.edu/ evans/control.course.pdf

– • Luenberger, D. G. "Optimization by vector space methods". John Wiley & Sons, Inc., New York-London-Sydney 1969 xvii+326 pp

– • Liberzon, D., Calculus of Variations and Optimal Control Theory: A Concise Introduction, Princeton university press, 2012.

– • Weber, Thomas A., Optimal Control Theory with Applications in Economics, MIT Press, 2011.

5 Grading policy

There will be a 2 hours final exam which is meant to check fundamental knowledge and basic proficiencies. More difficult questions requiring some originality or ingenuity will be also included, they represent between 20 and 30% of the total mark.