

# Optimization refresher: advanced camp

**Prerequisite:** Some familiarity with the following notions is preferable:

- Linear algebra
- Differential calculus
- Basic notions in real analysis : supremum/infimum, limits, continuity of functions, closed/open sets, compact sets
- Elementary results in optimization: first and second-order conditions, Lagrange conditions.

**Goals:** Recall/introduce the basic knowledge in optimization required to enter the ETE program both in Economics and Mathematics.

In such a short time formal proofs of our results cannot be in general provided. Instead we shall insist a lot on intuition and informal ideas. Students concerned by thorough theoretical justifications will dig further by themselves – a bibliography is provided at the end of the syllabus.

## Contents:<sup>1</sup>

### I Introduction

- closed/open/compact sets, continuity
- differentiability, gradient, Hessian, Fermat's rule and second-order conditions.

### II Constrained optimization

- Examples: projection theorem, existence and variational characterization
- Compactness and Weierstrass theorem
- General notions: normal cones (tangent cones)
- First-order (and second-order) conditions in terms of normal/tangent cones

### III Optimization with inequality/equality constraints

- Fundamental tools: separation theorem, (Farkas lemma).
- Qualification conditions: Slater, (Mangasarian-Fromovitz).
- Lagrange/KKT Conditions, Discuss the failures of QC and their consequences.

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<sup>1</sup>The list of results that are announced is in general too optimistic, but major facts are generally evoked during this course. Items between parenthesis will only be treated if time allows.

#### IV A glance at dynamic programming

- Definition : finite horizon problems e.g. shortest path problems in a graph, a consumer problem
- Optimal policy, existence result finite states case
- Bellman's principle
- Functional characterization of the value function à la Shapley
- Resolution by backward induction.
- Infinite horizon, examples, Ramsey's problem.

**Keywords for online resources:** Discrete dynamic programming, Normal cones, Non-linear programming, Convex minimization, Optimization, Optimization for economics, Lagrange conditions, KKT (Karush-Kuhn-Tucker) conditions

#### **Some references:**

- N. Stokey, R. Lucas, Recursive Methods In Economic Dynamics, Topics Economics, Dynamic Programming, Deterministic vs Stochastic Dynamics, opensource, 1989
- Leonard, D. Van Long N., Optimal Control Theory and Static Optimization in Economics, Cambridge University Press, 1992.
- Luenberger, David G. Optimization by vector space methods. John Wiley & Sons, Inc., New York-London-Sydney 1969 xvii+326 pp