Martingales theory and applications CM

Course title - Intitulé du cours	Martingales theory and applications CM
Level / Semester - Niveau /semestre	M1 / S2
School - Composante	Ecole d'Economie de Toulouse
Teacher - Enseignant responsable	FAUGERAS Olivier
Lecture Hours - Volume Horaire CM	15
TA Hours - Volume horaire TD	12
TP Hours - Volume horaire TP	0
Course Language - Langue du cours	Anglais
TA and/or TP Language - Langue des TD et/ou TP	Anglais

Teaching staff contacts - Coordonnées de l'équipe pédagogique :

See website/moodle

Course's Objectives - Objectifs du cours :

OBJECTIVES

This course is an introduction to a large class of stochastic processes called martingales, which originated from gambling ideas. Such processes are fundamental in probability theory and are useful in modeling, e.g. the price of a stock on a financial market or the surplus process for insurance companies.

The aim of this course is to give an indepth introduction to such a vast topic, as well as present some applications s.t. optimal gambling theory, mathematical methods in insurance, stochastic optimization, modeling Ponzi schemes and viral marketing in economics, etc...

Prerequisites - Pré requis :

Prerequisite: good background on Measure theory, Lebesgue's integration and measure-theoretic probability covered in any decent probability theory book such as Resnick's *A probability path*, chapters 1-5 or Barbe & Ledoux *Probabilité* chap 1-4.

Having followed the course Markov chains in Semester 1 is not mandatory but recommended.

Grading system - Modalités d'évaluation :

Final exam: 100%.

Bibliography/references - Bibliographie/références :

Some notes and references will be provided. Other relevant books are:

- Williams. *Probability with martingales*.

- Resnick, *A probability path*.

- Baldi, Mazliak, Priouret, *Martingales et chaînes de Markov*.

<u>Session planning - Planification des séances :</u>

COURSE OUTLINE

1. Complements of Probability Theory: understanding what is a stochastic process Sigma-algebras and filtrations as modeling of “information”. (Conditional) quantile transform and canonical construction of a stochastic process with given law.

2. Conditional expectations w.r.t. a sigma algebra Conditional expectation w.r.t. a sigma field, w.r.t. a random variable, Jensen inequality. Conditional expectation as orthogonal projection, linear conditional expectation. Conditional expectation of Gaussian Vectors.
Application 1: conditional expectation and regression model in econometrics. What does the epsilon really stand for?

3. Martingales in discrete time: basic properties and examples Gambling games, sub-, super-martingales, examples, transformations, properties Doob’s decomposition, Stopping times, Optional stopping theorems

4- The 3 pillars of martingale theory – Doob's Optionnal stopping theorems – Doob's maximal inequalities - (sub/super) Martingale convergence theorems (in L2, a.s.)
Application 2: The Gambler's ruin
Application 3: Optimal play in repeated Gambles (Kelly’s criterion): how to become rich. Epistemological consequence: the mathematical and historical origins of the utility function and why it is a flawed concept.
Application 4: Branching processes and the modeling of population dynamics/ Ponzi schemes in Economics
Application 5: Polya's urns and the modeling of reinforcement learning in economics (Viral marketing)