The basics of "static" Optimization

Prerequisite: Most notions are recalled during the course but some familiarity with basic facts is preferable.

- Basic Calculus
- Elementary notions of linear algebra
- Elementary differential calculus
- Basic notions in real analysis : supremum/infimum, limits, continuity of functions, closed/open sets, compact sets

Goals: It is a basic course oriented to operational methods which is meant for (static) optimization users. We shall insist a lot on intuition and on the geometry of optimization problems which is not generally well understood. Some classical exercises will illustrate the standard methods of optimization (eg using second order conditions, solving simple equality/inequality constrained maximization, graphical solutions for linear programs)

Students willing to deepen the subject may use the bibliographical references provided at the end of the syllabus.

Students already familiar with optimization but who need to refresh their memory might also find this course profitable.

Contents:

I Unconstrained Optimization

- Presentation of the general problem: terminology, what is an infimum/supremum? a minimum/maximum? Minimizing and maximizing.
- Graph of functions and level lines. Case of basic functions in $\mathbb{R}^2$: affine and quadratic functions.
- Differentiability, Gradient, First-order condition / Fermat's Rule
- Convex sets and functions, sufficiency of Fermat's rule
- Taylor formula of order 2, Illustrations in dimension 2 (paraboloid and saddle)
- Second-order conditions, examples and counter-examples.

II Optimization with inequality/equality constraints

II-1 Fundamental tools

- A basic optimization problem: the projection problem.
- Fundamental tools: separation theorem, Farkas lemma.

II-2 Constrained minimization
- The fundamental example of linear programming
- Qualification conditions: Slater, Mangasarian-Fromovitz. Discuss the failures of QC and their consequences
- Lagrange/KKT Conditions
- Examples & exercises

Keywords for online resources: Nonlinear programming, Convex minimization, Optimization, Optimization for economics, Lagrange conditions, KKT (Karush-Kuhn-Tucker) conditions

Elementary references

Optimization in Economic Theory, A.K. Dixit, 1990.

Mathematic Optimization and Economic Theory, M. Intriligator, SIAM, 2002.

Some accessible references [1]:

Chong E.K.P., Zak S.H., "An introduction to optimization" (Second Edition), Wiley Inter-Science in Discrete Mathematics an Optimization, 2001.

Luenberger, D. G. Optimization by vector space methods". John Wiley & Sons, Inc., New York-London-Sydney 1969 xvii+326 pp

Sundaram, R.K. "A first course in optimization theory". Cambridge University Press, Cambridge, 1996. xviii+357 pp.

More involved material:

Bonnans F., "Optimisation continue", Dunod, 2006.

Boyd S. and Vandenberghe L., "Convex Optimization", Cambridge University Press, 2004. Available online, MOOC available after registration.

Borwein, J., Lewis, A.S., "Convex Analysis and Nonlinear Optimization", Springer-Verlag 2000.

Ruszczynski, A., "Nonlinear Optimization", Princeton University Press, 2006.

[1] Only chapters on static optimization are related to this course