A first glance at "static" Optimization

Prerequisite: Most notions are recalled during the course but some familiarity with basic facts is preferable.

- Basic Calculus
- Elementary notions of linear algebra
- Elementary differential calculus
- Basic notions in real analysis : supremum/infimum, limits, continuity of functions, closed/open sets, compact sets

In such a short time formal proofs of our results cannot be in general provided[1]. Instead we shall insist a lot on intuition and on the geometry of optimization which is not generally well understood (even by some of the best students). Interested students would be then able to dig further by themselves – a bibliography is provided at the end of the syllabus.

Students already familiar with optimization but who need to refresh their memory might also find this course profitable.

Contents:[2]

I Unconstrained Optimization
- Presentation of the general problem
- Differentiability, Gradient, First-order condition / Fermat's Rule
- Convex case
- Taylor formula, Second-order conditions, the dimension 2 case

II Constrained optimization
- Examples
- Existence/uniqueness theorem
- Convex problems
- General notions: normal cones, tangent cones
- First-order and second-order conditions in terms of normal/tangent cones

[1]Such results as fundamental as the implicit function theorem or Farkas Lemma cannot be treated in such a short time and with a so diverse audience.

[2]The list of results that are announced is in general too optimistic, but facts and results of basic Optimization are generally evoked during this course.

III Optimization with inequality/equality constraints

- Qualification conditions: Slater, Mangasarian-Fromovitz
- Lagrange/KKT Conditions & Examples

Keywords for online resources: Nonlinear programming, Convex minimization, Optimization, Optimization for economics, Lagrange conditions, KKT (Karush-Kuhn-Tucker) conditions

Some accessible references:

S. Boyd and Vandenberghe, "Convex Optimization", Cambridge University Press, 2004. Available online, MOOC available after registration.

Chong E.K.P., Zak S.H., "An introduction to optimization" (Second Edition), Wiley Inter-Science in Discrete Mathematics an Optimization, 2001.

Luenberger, D. G. Optimization by vector space methods". John Wiley & Sons, Inc., New York-London-Sydney 1969 xvii+326 pp

Sundaram, R.K. "A first course in optimization theory". Cambridge University Press, Cambridge, 1996. xviii+357 pp.

More involved material:

Borwein, J., Lewis, A.S., "Convex Analysis and Nonlinear Optimization", Springer-Verlag 2000.

Ciarlet, P.-G, "Introduction à l'analyse matricielle et à l'optimisation", Masson, Paris, 1988.

Ruszczynski, A., "Nonlinear Optimization", Princeton University Press, 2006.

Fixed point theory

Border, K.C., "Fixed point theorems with applications to economics and game theory", Cambridge University Press, 1985.