Idiosyncratic Labor Risk and Aggregate Risk Sharing with Financial Frictions*

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Abstract

The exposure of firms and financial institutions to aggregate shocks is a key driver behind financial crises. This paper studies how idiosyncratic uninsurable labor income risk faced by lender households influences the concentration of aggregate risk on borrower entrepreneurs' balance sheets. I propose a tractable model of households' idiosyncratic labor risk and embed it into a workhorse business cycle framework with informational asymmetries in entrepreneurial financing and privately optimal contracting. The presence of idiosyncratic labor income risk affects aggregate fluctuations and risk concentration through two explicit channels: *i)* endogenous increases in the share of human wealth in households' total wealth increase realized idiosyncratic consumption risk, given labor income risk; and *ii)* cyclicality in the idiosyncratic labor income risk itself leads to cyclicality in realized consumption risk. In the calibrated model with nominal rigidities, both channels make households less reluctant to bear aggregate risk, resulting in its higher concentration on the balance sheets of entrepreneurs. Quantitatively, the former channel plays a small role, while empirically plausible countercyclicality in idiosyncratic labor income risk can lead to considerable amplification of aggregate volatility, reminiscent of conventional financial accelerator dynamics.

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1 Introduction

Time and again, financial crises roll around and serve as a reminder of how concentrated certain aggregate risks are on the balance sheets of leveraged firms and financial institutions. A central role in the 2008-09 financial crisis was played by financial institutions being exposed to losses from the subprime mortgage bubble bursting, while financing themselves with low-risk, flighty wholesale funding. More recently, similar forces were at play on a smaller scale when the Federal Reserve's rate hikes of 2022 caused various U.S. banks to suffer significant losses on their unhedged positions in long-term U.S. treasuries, leading to turmoil in the U.S. financial system in early 2023, culminating with a bank run on Silicon Valley Bank and the closure of a variety of other mid-sized banks. The macro-finance literature has long developed an established set of theories explaining how such risk concentration on the balance sheets of productive capital-managing "entrepreneurs" (or, "capitalists", "specialists" etc.) can lead to the amplification and propagation of potentially small disturbances into full-blown financial crises (through the *financial accelerator* mechanism), e.g., Kiyotaki and Moore (1997), Bernanke et al. (1999), Brunnermeier and Sannikov (2014), and the vast literature building on these frameworks.

However, it is also by now understood that most theoretical analyses in this literature do not explain *why* the aggregate risk associated with investing in capital should be concentrated on such borrower entrepreneurs' balance sheets, as they tend to impose arbitrary restrictions on the availability of risk-management tools to entrepreneurs by assumption. For example, in the scope of the Bernanke et al. (1999) framework, henceforth BGG, where capital-managing entrepreneurs borrow from households subject to a costly state verification (CSV) friction, the financial accelerator mechanism arises thanks to the exogenously imposed constraint that the lenders must, on aggregate, receive a predetermined constant return, independently of *ex post* aggregate shock realizations observed at the time of repayment. When the economy is hit by an adverse shock to capital returns, the excessive exposure of entrepreneurs causes their wealth share to fall, misaligning incentives and tightening constraints related to frictional external financing going forward. Yet, in the presence of a risk-averse lender and a borrower with time-varying investment opportunities, the counterparties could engage in mutual insurance against aggregate risk, which could be achieved by agreeing on a lender's return which is indexed to observable aggregate outcomes. Carlstrom et al. (2016), henceforth CFP, formalize this idea in the BGG framework[1] with a representative lender household and show that, in the privately optimal one period contract, the *ex post* return paid to the lenders is indexed one-for-one to the return on entrepreneurial capital, adjusted for fluctuations in the borrower's and lender's marginal valuations of wealth. And in the calibrated model, the privately optimal contract significantly dampens financial accelerator dynamics in response to various conventional business cycle shocks.

In this paper, I study whether a high degree of concentration of aggregate risk on borrowing

[1] Early criticism of the assumption on return predeterminacy has also been voiced, for example, by Chari (2003). Analogous results have been formalized by Krishnamurthy (2003) for a stylized three-period version of the Kiyotaki and Moore (1997) model, and by Di Tella (2017) for the Brunnermeier and Sannikov (2014) model.

entrepreneurs' balance sheets, as exogenously assumed in conventional financial accelerator models, might be the endogenous outcome of privately optimal aggregate risk sharing between households and entrepreneurs when households are exposed to uninsurable idiosyncratic risk in their labor income. To do so, I study a general equilibrium model of worker-saver households lending to capital-investor entrepreneurs subject to a CSV friction in the spirit of the BBG framework, allowing for privately optimal contracting following CFP. In my formulation of the model, there is a representative entrepreneur who receives logarithmic utility from consumption and owns a continuum of *firms*, each running individual projects subject to limited liability and firm-specific risk. I show that, to a first order, this reformulation is equivalent to assuming that that there are individual entrepreneurs with linear utility consuming a constant fraction of wealth, as in the original BGG formulation.[2] This establishes that if one were to assume logarithmic utility from consumption for a representative lender household, as BGG and CFP do, one is effectively studying a risk-sharing problem between two agents with identical preferences over consumption.[3]

Such a setting allows to clearly illustrate that the optimal sharing of aggregate risk involved with financing the entrepreneurs' capital investments also depends on the agents' exposure to aggregate risk through other sources of wealth. In this conventional framework, entrepreneurs' total wealth equals their financial wealth invested in capital while households are also endowed with human wealth. This means that optimal risk sharing between households and entrepreneurs requires sharing financial returns in a way that takes into account fluctuations in human wealth. For any positive shock to households' human wealth, they should cede more of their financial returns to the entrepreneurs, all else equal. With relative fluctuations in aggregate financial and human wealth comoving closely in response to many conventional aggregate shocks in a business cycle model with Cobb-Douglas production, a representative household also ends up taking on a considerable share of any realized unexpected financial returns.

Given that idiosyncratic shocks to human wealth are conceivably less diversifiable than the idiosyncratic risk embedded in owning individual assets or financing entrepreneurial projects, I focus my analysis in this paper on how the presence of households' uninsurable idiosyncratic risk in labor income acts an effective source of increased (or decreased) risk aversion to aggregate business cycle fluctuations. I do so by introducing uninsurable idiosyncratic shocks to human capital (or, *labor risk* shocks) in a tractable model of *ex post* heterogeneous households. More specifically, my household model builds on existing models which allow households to individually choose how much human capital to accumulate (e.g., Krebs, 2003a), and extends such a framework by also introducing

[2]Similarly to CFP, Dmitriev and Hoddenbagh (2017) analyze optimal contracting in the BGG framework and consider varying degrees of entrepreneurial risk aversion. However, in their formulation, entrepreneurs are assumed to only consume when they die, which is not optimal for an agent that is not risk neutral. Thus, their underlying entrepreneurial preference structure is different from the one in this paper.

[3]While being a valid theoretical benchmark, it also demonstrates that if one were to instead consider the conventional BGG specification with households that do not have logarithmic utility, the high degree of sharing aggregate financial risk found by CFP might not necessarily follow. Relatively more aggregate risk would trivially be taken on by agents with lower aversion to fluctuations in consumption. Bocola and Lorenzoni (2023) and an earlier version of this paper (Jeenas, 2018) study how the elevated degree of household risk aversion can generate aggregate risk concentration on entrepreneurs' balance sheets.

a simple "spot" labor supply decision of how much of this human capital to supply to the market in the form of efficiency labor in return for a market wage. The tractability of this framework derives from the combination of homothetic preferences, the households' ability to choose human capital accumulation, and the shocks to the stock of human capital being independent across households and time. Assuming that the aggregate supply of human capital is fixed across time, the special case with zero-variance idiosyncratic shocks nests the most conventional business cycle model of a representative household with an active labor supply margin, and no human capital fluctuations.

In such a framework, the total (human + financial) wealth of a household becomes a sufficient state variable for an individual household's choices, and all households' choices of consumption, human wealth and financial wealth investments become linear in total individual wealth. Equivalently, all households consume the same share of current wealth, and choose to invest in human and financial wealth in identical portfolio shares. As a result, the realizations of idiosyncratic shocks to human capital will have permanent effects on individual outcomes, with the elasticity of individual choices, such as consumption, to these shocks being governed by the share of households' total wealth held in the form of human wealth.

In this economy, the presence of uninsurable idiosyncratic labor risk manifests itself as an additional term (wedge) in the Euler equation for aggregate consumption (absent in the representative household case) that arises due to a precautionary saving motive from idiosyncratic consumption risk – a *risk shifter* Debortoli and Galí (2024). Importantly, owing to its tractability, the model allows to derive a simple analytical form for the equilibrium value of this risk shifter and establish it is a function of only the households' share of wealth derived from risky human wealth relative to total household wealth, and the distribution of the exogenous idiosyncratic labor risk shocks. This implies that the presence of idiosyncratic labor income risk affects the dynamics of the risk shifter, and thus aggregate fluctuations in general, through two, and only these two, explicit channels: (i) endogenous fluctuations in households' human vs. financial wealth shares, and (ii) any potential cyclicality in the idiosyncratic labor income risk process itself.

Fluctuations in the risk shifter influence households' equilibrium sharing of aggregate risk with the entrepreneurs, as they directly affect the households' stochastic discount factor and thus their willingness to take on aggregate risk. All else equal, an increase in households' risky human capital share in their overall wealth, or the riskiness of idiosyncratic labor income (e.g., higher variance or left-skewness of the labor risk shock) will increase the realized value of the stochastic discount factor and require the optimal contract to dictate the entrepreneurs pay out a relatively higher *ex post* return to the households. Moreover, persistent fluctuations in the risk shifter will also introduce cyclicality into into the households' incentives for precautionary saving, potentially affecting aggregate business cycle dynamics to a significant degree.

In my calibration of the model, various conventional recessionary business cycle shocks, e.g., negative TFP shocks or capital quality shocks, increase the share of households' wealth held in risky human capital, leading them to value returns in recessions relatively more than a representative household without idiosyncratic risk would. This concentrates relatively more of the aggregate risk related to these shocks on the entrepreneurs' balance sheets and amplifies financial accelerator dynamics. However, the induced fluctuations of the human wealth share are relatively small in magnitude, implying that the differences between a representative household and a heterogeneous household model with acyclical idiosyncratic labor income risk are quantitatively minor. If in addition, recessionary shocks were to induce an empirically plausible increase in idiosyncratic labor income risk, as observed in the data (Guvenen et al., 2014), significantly more of the aggregate risk would be concentrated on the entrepreneurs' balance sheets. And a model specification with nominal rigidities would exhibit considerable amplification of aggregate volatility, reminiscent of conventional financial accelerator dynamics even if agents are allowed to write privately optimal contracts to share aggregate risk

Related Literature. This paper contributes to several strands of the literature. First and foremost, there have been a number of papers studying potential explanations for the high degree of concentration of aggregate risk on levered productive borrowers' balance sheets. Krishnamurthy (2003) points out that it could be that the lenders, who are effectively providing hedging to borrowers, might themselves be subject to limited pledgeability and the supply of hedging may be limited by the aggregate value of collateral in the economy. Asriyan (2021) makes the point that dispersed information and imperfect competition in the secondary markets for macro-contingent claims can give rise to mispricing and misallocation of aggregate risk, and amplification of asset price and output volatility. Di Tella (2017) emphasizes the fact that increases in the volatility of unobservable entrepreneur-specific shocks that cause the moral hazard problem, and thus worsen financial frictions between entrepreneurs and households, can themselves be a source aggregate risk which entrepreneurs optimally choose to be exposed to. Bocola and Lorenzoni (2023) point out that a representative lender household might not be willing to take on aggregate risk if her risk aversion is considerably higher than that of the entrepreneurs, and part of her wealth is also held in the form of labor income. In relation to this literature, I propose and explore another natural and plausible mechanism: the existence and relevance of uninsurable idiosyncratic labor income risk (as a share of total income) that common households face, in contrast to wealthier agents, i.e., entrepreneurs, who tend to have a relatively larger share of their wealth held as financial wealth.

Secondly, my study is a production economy parallel to the strand of the asset pricing literature which studies how countercyclical idiosyncratic risk increases agents' effective aversion towards aggregate fluctuations and explains asset pricing puzzles. The work of Mankiw (1986), Constantinides and Duffie (1996), Krusell and Smith (1997), Storesletten et al. (2007), Constantinides and Ghosh (2017), and Schmidt (2022) are a few prominent examples. In relation to this body of work, my paper focuses on how such forces affect the financing of productive firms and aggregate economic activity in a production economy, and how they interact with the financial frictions faced by firms.

Third, since the seminal work by Krusell and Smith (1998), there has been a recent revival of interest in how household heterogeneity and uninsurable idiosyncratic risk affect aggregate dynamics, such as the work by Gornemann et al. (2016), McKay and Reis (2016), McKay (2017), Kaplan et al. (2018), Bayer et al. (2019), or Debortoli and Galí (2024), just to name a few. In parallel to models which require a computational approach to solving them, researchers have proposed tractable setups of models with household heterogeneity and business cycles, e.g. Krusell et al. (2011), Werning (2015), Bilbiie (2021), or Ravn and Sterk (forthcoming). These frameworks usually derive their tractability from either making assumptions which ensure zero financial wealth held by households in equilibrium, or by the idiosyncratic consumption risk being realized in the form of them hitting a borrowing (or, "liquidity") constraint – conditions that do not apply when studying the effects of idiosyncratic risk on investors who price the assets that finance the capital stock of the productive sector of an economy. One exception is that of Acharya and Dogra (2020) where tractability is achieved thanks to a CARA-normal preference-shock structure. In my model, I use more conventional CRRA (log) preferences and no other restrictions on the distributions of shocks apart from individual shocks to the stock of human wealth being i.i.d. across households and independent across time. In relation to this literature, I propose an alternative framework that yields tractability also when households hold significant amounts of non-human wealth, and I study the aggregate implications of the interaction between households' uninsurable idiosyncratic risk and firms' financing frictions. To do so, I closely follow existing work which achieves tractability in models of *ex post* heterogeneous households by allowing for trade in human capital and uses such models to study the implications of idiosyncratic, uninsurable labor income risk on human capital accumulation and economic growth (Krebs, 2003b), on the growth effects and welfare costs of business cycles (Krebs, 2003a, 2007), on the mathematical properties of cross-sectional distributions in incomplete market economies (Toda, 2014; Toda and Walsh, 2015), or on asset prices (Krebs and Wilson, 2004; Schmidt, 2022). I extend the insights employed in this literature by (i) allowing for asymmetry in the cost of accumulating (i.e., the supply side of) human capital compared to physical capital, and (ii) introducing an explicit labor supply choice made by households, given accumulated human capital. While these extensions are in principle simple, they allow to bring the framework significantly closer to (by *nesting*) conventional representative household models of business cycle dynamics with an endogenous labor supply margin and no endogenous growth nor human capital accumulation in the aggregate.

The rest of the paper is organized as follows. Section 2 describes the model environment and the competitive equilibrium, and illustrates some of its theoretical implications. In Section 3, I calibrate the model and study the quantitative implications of idiosyncratic labor risk for aggregate risk sharing and aggregate dynamics. Section 4 concludes.

2 Model

2.1 The Environment

In the following, I describe the model environment. I thereafter provide a deeper discussion of key modeling assumptions in Section 2.1.2.

For comparability with earlier work in the literature, the model environment closely follows the treatment of BGG and CFP in most parts. Time in the model is discrete and infinite. There is a numeraire final good that is used for consumption and as input in producing new capital goods. The model features two central types of agents, called *households* and *entrepreneurs* – a unit mass of each. Households are *ex ante* identical, but are subject to uninsurable idiosyncratic shocks, and thus *ex post* heterogeneous. All entrepreneurs are identical also *ex post*, and their behavior is characterized by a representative entrepreneur for the remainder of the analysis. There is also a unit mass of *firms* indexed by $j \in [0,1]$, new capital producers, a representative financial intermediary and a representative final goods producer, all discussed below.

Households in the economy have finite lives which end probabilistically in the style of Yaari (1965) and Blanchard (1985), with probability $\theta \in [0,1]$ of survival between any two periods.[4] Each household who is alive at the beginning of period t is identified with a point i in the unit-mass set $\mathcal{E}_t \subset \mathbb{R}_+$. Households derive utility from consumption of the final good and from enjoying leisure. They are endowed with one unit of time that they split between work and leisure activities. Preferences are common to all households and for household i, born in period t, they are given by $\mathbb{E}_t\left[\sum_{s=t}^{\infty}(\beta\theta)^{s-t}u(c_{i,s}, 1-l_{i,s})\right]$ where $\mathbb{E}_t[\cdot]$ is the conditional expectations operator given information available at t, $c_{i,t}$ denotes household i's consumption of the final good in period t, $l_{i,t}$ is the time spent working, $\beta \in (0,1)$ is the time discount factor and, $u(c, 1-l) = \log(c) + \phi\log(1-l)$, with $\phi > 0$ a parameter.

The effective productivity of a household's labor supplied to the labor market is given by its accumulated human capital $\hat{h}_{i,t}$. Letting W_t denote the real wage rate per efficiency units of labor, a household's labor income in t is thus given by $W_t\hat{h}_{i,t}l_{i,t}$. In this benchmark model, human capital does not depreciate and each household alive in t can buy (or sell) units of human capital at price P_t^H, determined in equilibrium. Households' human capital accumulation is subject to idiosyncratic, household-specific (human capital, or *"labor risk"*) shocks $\eta_{i,t} > 0$, where a household i's human capital $h_{i,t}$ chosen at the end of period $t-1$ becomes a stock of $\hat{h}_{i,t} = \eta_{i,t}h_{i,t}$ units of human capital at the beginning of t, when $\eta_{i,t}$ is realized. Each period t, household i's shock $\eta_{i,t}$ is drawn from a distribution with cumulative distribution function $\mathcal{F}_t(\eta)$, independently over time. The draws of $\eta_{i,t}$ are distributed i.i.d. across households, with normalization $\mathbb{E}_{\mathcal{F}_t}[\eta_{i,t}] = 1, \forall t$.[5] The notation allowing for time-variation in the distribution $\mathcal{F}_t$ allows to introduce fluctuations in the degree of idiosyncratic labor income risk experienced by households over the business cycle, taken as given by the agents. In the special case of $\mathcal{F}_t = \mathcal{F}, \forall t$, the draws of $\eta_{i,t}$ are simply i.i.d. across households and time, i.e., idiosyncratic labor income risk is *acyclical*.

At the aggregate level, I assume that the supply of human capital H_t is fixed at an exogenously given level $\bar{H}$. This assumption on the aggregate human capital supply being constant at $\bar{H}$ allows the model's special case with no idiosyncratic labor risk to naturally nest the most conventional

[4]The only conceptual role that the Blanchard-Yaari OLG structure plays in the current analysis is that of ensuring the existence of a stationary equilibrium distribution of households over their idiosyncratic state space (e.g., see Toda, 2014). But as will be seen in the analysis below, $\theta < 1$ will have no bearing on the aggregate equilibrium behavior of the households, other than depressing their effective time discounting to be determined by $\beta\theta$, instead of just β.

[5] I will use the notation $\mathbb{E}_{\mathcal{F}_t}[\cdot]$ to emphasize integration *only* with respect to the idiosyncratic shock $\eta_{i,t}$, given the probability measure induced by $\mathcal{F}_t(\eta)$. That is, for any function f and random vector X, I denote $\mathbb{E}_{\mathcal{F}_t}[f(\eta_{i,t}, X)] \equiv \int_0^\infty f(\eta, X)d\mathcal{F}_t(\eta)$, taking the realization of X as given.

business cycle model of a representative household with an active labor supply margin, and no human capital fluctuations. Endogenous fluctuations in the human capital price P_t^H then ensure that households' aggregate demand for human capital equals exactly $\bar{H}$ in every t. Market clearing will thus require $\bar{H} = \int_{i\in\mathcal{E}_t} h_{i,t}di = \int_{i\in\mathcal{E}_t} h_{i,t+1}di$.[6]

In addition to accumulating human capital $h_{i,t+1}$ subject to idiosyncratic risk, households can save in period t by depositing savings $d_{i,t+1}$ in a financial intermediary. These deposits yield gross real returns R_{t+1}^D in $t+1$. The returns are not predetermined in t and are realized at $t+1$, possibly depending on aggregate shocks.

Dying households leave accidental bequests to newborns. When a household dies between $t-1$ and t, the deposits $d_{i,t}$ and new capital producers' shares $\iota_{i,t}$ (see below) that they left $t-1$ with are distributed evenly across newly born households in t. Each household is born with $\bar{H}$ units of human capital.

As in BGG and CFP, the representative financial intermediary accepts deposits from households and extends loans, between t and $t+1$, to the continuum of firms. Financial markets in the economy are rendered incomplete due to the presence of asymmetric information between firms and the intermediary/households (as detailed below), and by assumption, there are no assets in the economy with payoffs dependent on the realized household-specific shocks $\{\eta_{i,t}\}_{i\in\mathcal{E}_t}$, nor can the financial intermediary write contracts conditional the realizations of $\{\eta_{i,t}\}_{i\in\mathcal{E}_t}$. The intermediary is effectively a pass-through entity that diversifies all idiosyncratic risk arising from lending to firms hit with individual firm-specific shocks (described below). Yet aggregate risk on each extended loan, and on the whole loan portfolio remains. As CFP, I assume that there is free-entry into the financial intermediation market and gross returns to the depositors cannot be negative. This implies that in equilibrium, the gross real returns on households' deposits, R_{t+1}^D will equal the returns on the intermediary's loan portfolio.[7]

In general, a collection of households subject to uninsurable idiosyncratic risk and *ex post* heterogeneity would not necessarily agree on the pricing of assets and on what the intermediary's optimal portfolio should look like. And thus, in general, the assumption of there being a representative financial intermediary would be restrictive – households' in different idiosyncratic states could potentially be catered to by different financial intermediaries with different portfolios. However, as seen in Section 2.2.1 below, due to the assumptions of households' homothetic preferences, their ability to choose human capital accumulation, and the shocks to their stocks of human capital being i.i.d. across households and independent across time, the above-described household setting gives rise to highly tractable equilibrium behavior, in which *all* households agree on the pricing of any asset whose payoffs are dependent on aggregate outcomes. Thus, in this environment, the assumption of there being only *one* financial intermediary who makes portfolio decisions on behalf

[6] With a slight abuse of notation, I use $\int_{i\in\mathcal{E}_t} h_{i,t}di$ to refer to the human capital that all households in $\mathcal{E}_t$ (including the period-t newborns and survivors from $t-1$) hold at the beginning of t, and thus have disposable for production in t. And $\int_{i\in\mathcal{E}_t} h_{i,t+1}di$ refers to the holdings of human capital that these same agents acquire in t to take into $t+1$, conditional on their survival.

[7] Alternatively, one can assume that the households own the financial intermediary and provide frictionless equity financing to arrive at identical equilibrium conditions.

of all households, is without loss of generality.[8]

The representative entrepreneur is infinitely lived, with preferences given by $\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta_e^t \log(C_t^e)\right]$, where C_t^e denotes the entrepreneur's consumption of the final good in period t and $\beta_e \in (0,1)$ is the entrepreneur's discount factor. In the spirit of BGG, it is the only agent assumed to participate in the market for direct ownership (inside equity) of firms. The entrepreneur consumes dividends paid by the firms and it is restricted from participating in any other financial markets.[9] As assumed by CFP, the representative entrepreneur is not endowed with any labor.

Firm j maximizes its value to shareholders (i.e., the entrepreneur) by investing in productive physical capital. Firms are assumed to be the only entities who can hold capital between periods t and $t+1$. At the end of each period, they purchase physical capital, financed by their accumulated wealth, referred to as *net worth*, and external financing provided by the financial intermediary. At the beginning of period $t+1$ each firm's capital holdings $K_{j,t+1}$ are scaled by an idiosyncratic shock $\omega_{j,t+1}$ which is observed by the firm, but by the lender only if a monitoring cost is incurred. This idiosyncratic shock is i.i.d. across time and firms and independent of any aggregate realizations, with density $f(\omega)$, cumulative distribution $F(\omega)$ and $\mathbb{E}[\omega] = 1$.[10] Let R_{t+1}^K denote the aggregate gross return to a unit of the final good invested in capital, meaning the average return in the cross-section of firms. R_{t+1}^K is perfectly observed by all agents in the economy. Then, the total return to a unit of the final good invested in firm j's capital project at time t is $\omega_{j,t+1}R_{t+1}^K$, with

$$R_{t+1}^K \equiv \frac{r_{t+1}^K + (1-\delta)Q_{t+1}}{Q_t} \tag{1}$$

with r_{t+1}^K the capital rental rate, δ the depreciation rate and Q_t the relative price of capital in t.

As is conventional in this line of models starting with Carlstrom and Fuerst (1997), I assume that there is enough inter-period anonymity in financial markets that only one-period contracts between the firms and the intermediary are feasible. Firms derive returns to capital from capital gains in the price of capital when selling it and renting it out to a representative final goods producer, as evident in the definition of R_{t+1}^K above. These returns are then used to cover any payments previously contracted to be made to the lender.[11]

As in BGG, I assume that monitoring costs are a proportion μ of the realized gross payoff to a

[8]By an analogous argument, the assumption that there are no other assets in zero-net-supply with payoffs dependent on aggregate outcomes (e.g., risk-free bonds or aggregate-state-specific Arrow securities) traded among the group of households is also without loss of generality. Each household $i \in \mathcal{E}_t$ would acquire the same share of such assets in their wealth portfolio, which in equilibrium would then have to equal zero, $\forall i \in \mathcal{E}_t$. This idea draws a parallel to the "no-trade" result of Constantinides and Duffie (1996) in an endowment economy. In both their and my models, the individual-specific components of income growth rates will be unpredictable, implying the same for individual-specific components in equilibrium consumption growth rates, e.g., see Krebs (2003a) for further discussion. Although, the "no-trade" terminology does not do justice to the model in this paper, as there is active re-trading of assets in response to idiosyncratic shocks among the households to ensure that they leave each period with identical portfolio shares.

[9]Otherwise it might be able to bypass and effectively eliminate the financial frictions faced by the firms it owns.

[10]I will relax this assumption and introduce time-variation in the distribution of $\omega_{j,t}$ in Section 3.

[11]Because of the common assumption of constant returns to scale in final good production, one can equivalently assume that the firms themselves have access to the production technology, hire labor and combine it with their capital to produce output.

given firm's capital: $\mu\omega_{j,t+1}R^K_{t+1}Q_tK_{j,t+1}$. Also, firms have limited liability in that each individual firm's project cannot make payouts in excess of the proceeds $\omega_{j,t+1}R^K_{t+1}Q_tK_{j,t+1}$. That is, even though the firm is owned by an entrepreneur who could inject equity into firm j, equity injections or dividend payments to the owner can only be made *after* the payments with the lender have been settled. This assumption renders each individual contracting problem identical to that in BGG and CFP. The firms are assumed to liquidate all their capital and all capital must be repurchased. This assumption dates back to BGG who make it to ensure that agency problems affect the entire capital stock and not just the marginal investment. Finally, as is a common assumption in the literature to prevent the firms from growing out of their financial constraints and become self-financed (with the entrepreneur's equity) in the long run, I will work with calibrations in which it is ensured that $\beta_e < M_{ss}$, where M_{ss} is the equilibrium stochastic discount factor of the households (integrating out idiosyncratic risk) in the steady state of the economy.[12]

There is a representative final goods producer who runs a Cobb-Douglas production function in aggregate labor L_t and capital K_t, producing $Z_tK^\nu_tL^{1-\nu}_t$. L_t is the aggregate effective labor employed, satisfying $L_t = \int_{i\in\mathcal{E}_t} l_{i,t}h_{i,t}\eta_{i,t}di$. The final goods producer rents capital from firms for rental rate r^K_t, and labor from the households for wage rate W_t per efficiency unit, both in competitive markets. Z_t is exogenous TFP which follows a stationary Markov process. The realization of Z_t is publicly observed at the beginning of period t. For brevity and clarity in the model environment description, let Z_t be the only source of aggregate uncertainty for now. In the quantitative analysis of Section 3, I will introduce a variety of other conventional business cycle shocks to the model.

Finally, there are competitive new capital producers who produce new capital subject to adjustment costs and sell it to firms. Following CFP, they take $I_t\vartheta\left(\frac{I_t}{I_{ss}}\right)$ units of the final good and transform these into I_t investment goods, i.e. gross capital investment. ϑ is convex and I_{ss} is the steady state level of gross investment. These investment goods are sold at price Q_t. I make the standard assumptions that $\vartheta(1) = 1$, $\vartheta'(1) = 0$ and $\vartheta''(1) \equiv \phi_Q \geq 0$. This normalizes the capital price in steady state to 1 and guarantees that at steady state, the elasticity of the capital price to I_t is ϕ_Q, a key calibration target. New capital producers' profits $\Pi^I_t \equiv Q_tI_t - I_t\vartheta\left(\frac{I_t}{I_{ss}}\right)$ are zero in steady state but possibly non-zero outside of it. Households trade new capital producers' equity shares, defined as claims on the future stream of profits $\{\Pi^I_{t+j}\}^\infty_{j=1}$, at equilibrium price P^I_t.

2.1.1 Model Extension with Nominal Rigidities

In order to illustrate the relevance of nominal rigidities in influencing the model's behavior, both qualitative and quantitative, I also study an extension of the framework outlined above by introducing price stickiness using the most conventional, parsimonious New Keynesian approach, similarly as BGG and CFP. In this extension, the firms $j \in [0, 1]$ are assumed to rent their capital to a representative *wholesale producer* that produces a wholesale good using the production function

[12] I will use the *steady state* to refer to the equilibrium that arises when all aggregates and aggregate shocks, and household and firm distributions are constant, and expected to remain constant, while individual agents face idiosyncratic risk.

$Z_t K_t^{\nu} L_t^{1-\nu}$ in capital and labor, rented in competitive markets at real rental rates r_t^K and W_t.

This wholesale goods producer sells the good at nominal price P_t^w in a competitive market to *retailers*. There is a unit mass of retailers $m \in [0,1]$, each with a linear production function that transforms wholesale goods into differentiated intermediate retail goods m: $y_{m,t} = y_{m,t}^w$, where $y_{m,t}^w$ is the amount of wholesale goods employed as input by retailer m in period t. The retailers sell their production for price $p_{m,t}$, taking the demand curve for their retail good as a function of $p_{m,t}$ as given. In setting prices, the retailers face Rotemberg (1982) adjustment costs $\frac{\phi_p}{2}\left(\frac{p_{m,t}}{p_{m,t-1}} - 1\right)^2 Y_t$, e.g., as in Kaplan et al. (2018), in units of the final good. Y_t is aggregate output of the final good.

In this extension, the final good is produced by a perfectly competitive *final good producer* who takes the nominal prices of the final good P_t and the retail goods $\{p_{m,t}\}_{m\in[0,1]}$ as given. It has a constant elasticity of substitution production function, combining the retail goods into the final good with elasticity of substitution $\epsilon > 1$: $Y_t = \left(\int y_{m,t}^{\frac{\epsilon-1}{\epsilon}} dm\right)^{\frac{\epsilon}{\epsilon-1}}$.

There is a monetary authority that sets the gross nominal one period risk-free rate R_{t+1}^n on a nominal zero net supply bond between t and $t+1$ following a standard Taylor rule, in nonlinear form, with $\phi_\pi > 1$:

$$R_{t+1}^n = R_{ss}^n (1+\pi_t)^{\phi_\pi} \tag{2}$$

where R_{ss}^n is the steady state nominal risk-free rate, and π_t is the net inflation rate $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$.

To make sure that the differences between the model versions with nominal rigidities and flexible prices arise only due to how business cycle dynamics are affected by sticky prices, I ensure that the steady states of the two economies are identical by making the conventional assumption that there is a government that subsidizes the retailers' input costs at a rate $\tau_m = \frac{1}{\epsilon}$, simultaneously taxing them lump sum to cover the costs of these subsidies. As a result, in the symmetric equilibrium where all retailers set the same prices across $m \in [0,1]$, they sell their goods at a gross markup of $\mathcal{M}_t \equiv \frac{p_{m,t}}{P_t^w} = \frac{P_t}{P_t^w}$, with $\mathcal{M}_{ss} = 1$, and their profits will be zero in steady state, but possibly non-zero outside of it, as for the new capital producers. Households trade the retailers' equity shares, defined as claims on the future streams of their profits, traded at equilibrium (real) price P_t^m.

2.1.2 Discussion of Key Assumptions

i.i.d. shocks to human capital stock. The above household model features idiosyncratic, household-specific shocks to accumulated human capital that are distributed independently across time and households. If human capital was not tradable across households, this would be equivalent to having permanent labor productivity shocks and a household's (log) labor productivity (defined as $\hat{h}_{i,t} \equiv h_{i,t}\eta_{i,t}$) follow a unit root process. As, for example, discussed by Heathcote et al. (2014), assuming an individual labor efficiency process that consists of a unit root *plus* an independently distributed (one-period) *temporary* shock has a long tradition in the literature on statistical models estimating individual wage dynamics (e.g., MaCurdy, 1982).[13] The presence of the transitory

[13] Approaches that have used an AR(1) specification for labor income tend to estimate a serial correlation coefficient close to one (e.g., Hubbard et al., 1995; Storesletten et al., 2004).

component is usually motivated by the fact that the empirical autocovariance function for individual wage dynamics usually exhibits a sharp drop at the first lag, while the presence of permanent shocks is motivated by within-cohort wage dispersion increasing approximately linearly with age.

While the household model laid out above does not currently feature an explicit temporary shock to labor productivity[14], it *does* feature equilibrium dynamics of individual labor productivity $\hat{h}_{i,t} \equiv h_{i,t}\eta_{i,t}$ where $\eta_{i,t}$ has permanent effects on $\hat{h}_{i,t+j}$ with a discrete drop in the effect from $j \geq 1$ onwards. The reason why this happens is that, trivially, $\frac{\partial \log(\hat{h}_{i,t})}{\partial \log(\eta_{i,t})} = 1$, and thus the $\eta_{i,t}$-shock passes through to the contemporaneous wage one-for-one. Yet the degree of pass-through of $\eta_{i,t}$ to future labor productivities is governed by the effect of $\eta_{i,t}$ on endogenous human capital accumulation $h_{i,t+j}$, $j \geq 1$. Since, in equilibrium, labor income is only a share of overall income, the elasticity of individual wealth and (by the equilibrium household behavior) also the elasticity of future human capital $h_{i,t+j}$ is constant across all $j \geq 1$, but strictly below 1 (see also Section 2.2.1).

No depreciation of human capital and a constant aggregate $\bar{H}$ stock. As mentioned in the model setup above, exogenously imposing that the aggregate human capital supply is constant at $\bar{H}$ is a natural and convenient assumption to allow the model to easily nest the most conventional business cycle model of a representative household with an active labor supply margin and no human capital fluctuations. Assuming that the human capital held by households does not depreciate is then a natural next step to preserve simplicity, allowing one to bypass the need to make assumptions on how new human capital gets created and whether any resources in the economy need to be spent on doing so. Nontheless, the above environment can be extended in a straightforward manner without jeopardizing its tractability by introducing a depreciation rate $\delta_H \in [0, 1]$ on human capital, and making an assumption on where the "replenishment" of $\delta_H \bar{H}$ comes from.[15]

One potentially unsatisfactory feature of such an environment with no human capital depreciation and a constant aggregate human capital stock is the obvious implication that, in equilibrium, the stock of $\bar{H}$ simply gets traded around across households, and thus there will be a considerable share of households in each period that are *actively reducing* their human capital. That is, their *gross* investment into human capital will be negative, making it potentially non-obvious to square with reality. One should view this environment simply as a "detrended" and simplified version of an economy with constant net growth rate $g > 0$ in aggregate human capital and depreciation $\delta_H > 0$. What ultimately matters for the tractable idiosyncratic risk and its implications in this household sector, are the *relative* human capital positions of households compared to the average. In such a world with growth, a household that chooses not to actively acquire nor reduce new human capital would see their human capital stock *relative to the average* fall at the rate of $(\delta_H + g)/(1 + g)$.

[14] Note that the tractability of the model, in terms of the aggregation results on the household sector, would survive even if one additionally introduced such an independent, temporary shock to individual wages. That is, one could assume period t labor income of household i equals $W_t h_{i,t}\eta_{i,t}\xi_{i,t}l_{i,t}$, with $\xi_{i,t}$ independent across households and time, but only $\eta_{i,t}$ affects the value of the remaining human capital $h_{i,t}\eta_{i,t}$.

[15] Possible options include assuming that the "extra" $\delta_H \bar{H}$ is distributed across the cohort of newborn households every period, or assuming that there is a human capital *endowment process* (a "tree") which returns $\delta_H \bar{H}$ units of human capital each period, and the claims on the future stream of these endowments can be traded across all households that are alive at any given point in time.

And the counterparts of households who decumulate human capital in the current simplified model would simply be the ones who accumulate human capital at a relatively slower pace than average.

Entrepreneurs do not have any human wealth. Financial accelerator models in the macro-finance literature commonly feature the assumption that the entrepreneurs (or, "experts", "capitalists" etc.) have no labor income. This assumption is usually not a very consequential one, and it often helps in deriving easy aggregation results on the behavior of the "entrepreneurs + firms block" of the economy. Yet, for the main message of this specific paper, which emphasizes the importance of agents' asymmetric exposure to uninsurable idiosyncratic risk embedded in human wealth, this is not an innocuous assumption. And the assumption of the entrepreneur having *no* human wealth takes the asymmetry to the extreme. For the purposes of the main conceptual point of the paper, I view this simply as an approximation to the more general point about asymmetries in *relative* wealth shares across groups of agents. As illustrated by the analysis in Section 2.2.1, and foreshadowed above, the influence of idiosyncratic labor risk on households' aggregate behavior is fully driven by the relative *share* of household wealth being held in human capital. The individuals and households who correspond to the model's "entrepreneurial sector" in reality, i.e., literal *entrepreneurs* and *inside equity holders* in firms, tend to be, on average, wealthier overall, and hold a relatively larger share of wealth in the form of financial wealth invested in their firms. Thus, the idiosyncratic risk embedded in their human wealth is not as consequential for them as it is for the households.[16]

Absence of binding borrowing (or liquidity) constraints on households. The heterogeneous household framework that I employ is one in which households' borrowing constraints do not bind in equilibrium and all assets are equally liquid, i.e., they can be traded at all times, not subject to any transaction costs. While these assumptions are naturally not satisfied in reality and are relaxed in many studies on household heterogeneity (e.g., Bewley, 1977; Imrohoroglu, 1992; Huggett, 1993; Aiyagari, 1994; Kaplan et al., 2018, and many more), they are unlikely to play a central role on the *aggregate* implications of the relationship between households' idiosyncratic risk and their willingness to finance firms with relatively more or less risky assets (i.e., sharing aggregate risk), which is the focus of this paper. Namely, households who are close to facing a binding borrowing constraint are unlikely to be the marginal investors pricing the assets that finance the capital stock of the productive sector of an economy. As for the households who price firms' liabilities potentially facing non-convex adjustment costs in accessing the wealth invested in these liabilities, and thus at times experiencing binding *liquidity* constraints (e.g., the *wealthy hand-to-mouth* of Kaplan and Violante, 2014), the marginal valuation of holding illiquid assets (and thus their pricing) is determined by asset payouts in the instances when the owner chooses to pay the adjustment cost and access the assets – exactly the instances in which the liquidity constraints are *not* binding. Because of these insights, I believe that the first order effects of idiosyncratic labor risk on firms'

[16] One could extend the model to introduce idiosyncratically risky human capital also to the entrepreneurial sector of the model and calibrate their relative wealth shares. Yet, note that the assumed *mass* of entrepreneurs is inconsequential for any outcomes of the model. I will calibrate the entrepreneurs to be holding half of the financial wealth in the economy. But theoretically, this financial wealth could as well be concentrated among a small mass of entrepreneurs, who in sum would hold a small amount of human wealth (if their *per capita* human wealth was similar to that of the households') and thus it would likely not have a meaningful effect on entrepreneurs' aggregate behavior.

financing are well captured by a framework that abstracts from these additional imperfections.[17]

Exogenously introduced countercyclicality in idiosyncratic risk. As the last part of my quantitative analysis (Section 3.4), I will introduce empirically motivated countercyclicality in uninsurable idiosyncratic labor income risk into the model by exogenously imposing that the realizations of conventional structural business cycle shocks, e.g., to TFP, themselves influence the underlying labor risk distribution $\mathcal{F}_t$. I will not propose a specific endogneous mechanism nor take a stand on why aggregate economic activity might comove with cross-sectional moments of individuals' labor income growth. Rather, I view this as an exercise that aims to quantify the effects of the presence of such empirically observed comovements on aggregate dynamics and risk sharing, while simply taking their presence as given in "reduced form" as often done implicitly in the literature, e.g., by Krusell and Smith (1998) or Storesletten et al. (2007). Also, while one might be tempted to endogenize countercyclicality in labor income risk by explicitly modeling a frictional labor market with endogenous fluctuations in the risk of falling into unemployment, a prevalent view in the literature on earnings risk cyclicality seems to be that the observed fluctuations in earnings risk cannot themselves be explained by only unemployment risk dynamics themselves. For example, Guvenen et al. (2014) make this point with a back-of-the-envelope calculation using individual-level labor income data, and McKay (2017) does it with the help of a structural model of heterogeneous households. Notably, Busch et al. (2022) document that the strong procyclicality in the skewness of individual labor income growth is evident also among continuously employed full-time workers.[18]

2.2 Equilibrium

In this section, I present the agents' problems and derive their equilibrium optimality conditions.

2.2.1 Households

Household $i \in \mathcal{E}_t$ maximizes lifetime utility of streams of consumption $c_{i,t}$ and leisure $1 - l_{i,t}$. I will immediately present the household's problem in recursive form, with the relevant idiosyncratic state of a household at the beginning of t being comprised of incoming deposits $d_{i,t}$, past accumulated human capital $h_{i,t}$, equity shares in the new capital producer $\iota_{i,t}$, and the period t realization of the individual human capital shock $\eta_{i,t}$. For brevity, let us collapse this state into the vector $\mathbf{s}_{i,t} \equiv (d_{i,t}, h_{i,t}, \iota_{i,t}, \eta_{i,t})$. The Bellman equation for a household's value function $\mathcal{V}_t(\mathbf{s}_{i,t})$ then satisfies:

$$\mathcal{V}_t(\mathbf{s}_{i,t}) = \max_{\substack{c_{i,t}, l_{i,t}, d_{i,t+1}, \\ h_{i,t+1}, \iota_{i,t+1}}} \left\{ \log(c_{i,t}) + \phi \log(1 - l_{i,t}) + \beta\theta \mathbb{E}_t \left[\mathcal{V}_{t+1}(\mathbf{s}_{i,t+1}) \right] \right\} \tag{3}$$

$$\text{s.t.} \quad c_{i,t} + d_{i,t+1} + P_t^H (h_{i,t+1} - h_{i,t}\eta_{i,t}) + P_t^I \iota_{i,t+1} \leq W_t h_{i,t} \eta_{i,t} l_{i,t} + R_t^D d_{i,t} + (\Pi_t^I + P_t^I)\iota_{i,t} \tag{4}$$

[17] Examples of works that study the implications of asset trading frictions on firms' financing choices include Kozlowski (2021), Caramp et al. (2023), Jeenas and Lagos (2024). The focus of these studies is first and foremost on the effects of the trading frictions themselves, not so much the uninsurability of investors' idiosyncratic risk.

[18] Huckfeldt (2022) proposes a model, matching empirical facts, where the earnings cost of job loss is concentrated among displaced workers who get reemployed in lower-skill occupations, and the occurrence and earnings losses of such occupation displacement are higher in recessions when hiring becomes endogenously more selective.

and non-negativity constraints on all of $(c_{i,t}, l_{i,t}, d_{i,t+1}, h_{i,t+1}, \iota_{i,t+1})$.[19] I have immediately plugged into budget constraint (4) for the household's expenditures on gross human capital investments $h_{i,t+1} - h_{i,t}\eta_{i,t}$ at price P_t^H. Following the model setup, the appearance of the term $h_{i,t}\eta_{i,t}$ here and in the period t labor income $W_t h_{i,t}\eta_{i,t} l_{i,t}$ captures the idea that $\eta_{i,t}$ is a shock to the household's whole stock of human capital. Although I consider recursive equilibria, for brevity I subsume fluctuations in the aggregate state by allowing for a time-varying household value function $\mathcal{V}_t$.

By noting that the first order necessary condition for labor supply must satisfy[20]

$$\phi c_{i,t} = W_t h_{i,t}\eta_{i,t}(1 - l_{i,t}) \tag{5}$$

one can substitute (5) for $(1 - l_{i,t})$ into problem (3)–(4) above, and rewrite it as:

$$\mathcal{V}_t(\mathbf{s}_{i,t}) = \phi\left[\log\left(\frac{\phi}{W_t}\right) - \log(\eta_{i,t}) - \log(h_{i,t})\right] + \max_{\substack{c_{i,t}, d_{i,t+1}, \\ h_{i,t+1}, \iota_{i,t+1}}} \left\{\log(c_{i,t}) + \beta\theta\mathbb{E}_t\left[\mathcal{V}_{t+1}(\mathbf{s}_{i,t+1})\right]\right\} \tag{6}$$

$$\text{s.t.} \quad (1+\phi)c_{i,t} + d_{i,t+1} + P_t^H h_{i,t+1} + P_t^I \iota_{i,t+1} \leq (W_t + P_t^H)h_{i,t}\eta_{i,t} + R_t^D d_{i,t} + (\Pi_t^i + P_t^I)\iota_{i,t} \tag{7}$$

Given this reformulation, one can notice certain features of the household problem that will give rise to the tractability of households' aggregate behavior. First, by the fact that the household draws $\eta_{i,t}$ from distribution $\mathcal{F}_t(\eta)$ in each period, independently of past realizations of $\eta_{i,t-j}$, the choices of $(c_{i,t}, d_{i,t+1}, h_{i,t+1}, \iota_{i,t+1})$ going forward will only depend on the household's total wealth $\hat{a}_{i,t}$, and not on the full vector $\mathbf{s}_{i,t} = (d_{i,t}, h_{i,t}, \iota_{i,t}, \eta_{i,t})$, with

$$\hat{a}_{i,t} \equiv (W_t + P_t^H)h_{i,t}\eta_{i,t} + R_t^D d_{i,t} + (\Pi_t^I + P_t^I)\iota_{i,t} \tag{8}$$

Also, for notational convenience, let us define $a_{i,t+1} \equiv d_{i,t+1} + P_t^H h_{i,t+1} + P_t^I \iota_{i,t+1} = \hat{a}_{i,t} - (1+\phi)c_{i,t}$ as household i's total wealth reinvested at the end of t.

In addition, by the fact that the implicit household preferences over $c_{i,t}$ and $h_{i,t+1}$ in (6) are homothetic, and at any point in time, all households face identical stochastic processes for the returns on their investments in $(d_{i,t+1}, h_{i,t+1}, \iota_{i,t+1})$ going forward (although with possibly heterogeneous *ex post* realizations), one can conjecture and verify that the optimal portfolio choice of allocating total wealth $\hat{a}_{i,t}$ as shares across consumption and the various assets becomes independent of individual wealth $\hat{a}_{i,t}$. Or, put differently, that the choices of $c_{i,t}$, $d_{i,t+1}$, etc. become linear in $\hat{a}_{i,t}$, with slopes that are identical across households, yet possibly time-varying, meaning that households' optimal policies take the form: $c_t(\mathbf{s}_{i,t}) = \tilde{c}_t \cdot \hat{a}_{i,t}$, $d_{t+1}(\mathbf{s}_{i,t}) = \tilde{d}_t \cdot \hat{a}_{i,t}$, $P_t^H \cdot h_{t+1}(\mathbf{s}_{i,t}) = \tilde{h}_t \cdot \hat{a}_{i,t}$, and $P_t^I \cdot \iota_{t+1}(\mathbf{s}_{i,t}) = \tilde{\iota}_t \cdot \hat{a}_{i,t}$, with the unknowns $\tilde{c}_t$, $\tilde{d}_t$, $\tilde{h}_t$, and $\tilde{\iota}_t$ determined by the solution to the

[19] Note that imposing non-negativity constraints on households' holdings of the assets will not be restrictive and is without loss of generality in equilibrium, as long as one would have to either way impose natural debt limits to rule out Ponzi schemes. As foreshadowed in the Introduction and detailed below, all households will choose to allocate the same share of their total wealth to the various "assets" $(d_{i,t+1}, h_{i,t+1}, \iota_{i,t+1})$. By the aggregate supply of all these assets being strictly positive in equilibrium, these non-negativity constraints cannot be binding for any household.

[20] I conjecture and verify in the quantitative applications of the model that no household violates the non-negativity constraint $l_{i,t} \geq 0$ in equilibrium.

households' problem (see Appendix A.1). Note that the budget constraint (7) holding at equality implies, $(1+\phi)\tilde{c}_t + \tilde{d}_t + \tilde{h}_t + \tilde{\iota}_t = 1$, and also, $a_{t+1}(\mathbf{s}_{i,t}) = [1-(1+\phi)\tilde{c}_t] \cdot \hat{a}_{i,t} \equiv \tilde{a}_t \cdot \hat{a}_{i,t}$.[21]

Appendix A.1 provides further details on the characterization of households' idiosyncratic behavior in optimum. Most importantly, it contains the derivations necessary to conclude that the aggregate behavior of the household sector in partial equilibrium, taking prices and returns as given, is characterized by the processes for $C_t \equiv \int_{i\in\mathcal{E}_t} c_{i,t}di$, $H_{t+1} \equiv \int_{i\in\mathcal{E}_t} h_{i,t+1}di$, $D_{t+1} \equiv \int_{i\in\mathcal{E}_t} d_{i,t+1}di$, $\iota_{t+1} \equiv \int_{\mathcal{E}_t} \iota_{i,t+1}di$, and the households' aggregate effective labor supply defined as $L_t \equiv \int_{i\in\mathcal{E}_t} l_{i,t}h_{i,t}\eta_{i,t}di$ that satisfy the following recursive system of 5 equations (9)–(12):[22]

$$1 = \mathbb{E}_t\left[\beta\theta\left(\frac{C_{t+1}}{C_t}\right)^{-1} S_{t+1}\cdot R^X_{t+1}\right], \qquad \text{for } X\in\{D,I\} \tag{9}$$

$$1 = \mathbb{E}_t\left[\beta\theta\left(\frac{C_{t+1}}{C_t}\right)^{-1} S^H_{t+1}\cdot R^H_{t+1}\right] - \beta\theta\frac{\phi C_t}{P^H_t H_{t+1}} \tag{10}$$

$$\phi C_t = W_t(H_t - L_t) \tag{11}$$

$$C_t + A_{t+1} = W_t L_t + P^H_t H_t + R^D_t D_t + (\Pi^I_t + P^I_t)\iota_t \tag{12}$$

where
$$S_{t+1} = \mathbb{E}_{\mathcal{F}_{t+1}}\left[\left(1+\alpha^H_{t+1}(\eta_{i,t+1}-1)\right)^{-1}\right] \tag{13}$$

$$S^H_{t+1} = \mathbb{E}_{\mathcal{F}_{t+1}}\left[\left(1+\alpha^H_{t+1}(\eta_{i,t+1}-1)\right)^{-1}\cdot\eta_{i,t+1}\right] \tag{14}$$

$$A_{t+1} = P^H_t H_{t+1} + D_{t+1} + P^I_t\iota_{t+1} \tag{15}$$

$$\alpha^H_{t+1} = \frac{P^H_t H_{t+1}\cdot R^H_{t+1}}{P^H_t H_{t+1}\cdot R^H_{t+1} + (A_{t+1} - P^H_{t+1}H_{t+1})\cdot R^{\sim H}_{t+1}} \tag{16}$$

$$R^{\sim H}_{t+1} = \frac{1}{A_{t+1} - P^H_t H_{t+1}}\cdot\left[R^D_{t+1}\cdot D_{t+1} + R^I_{t+1}\cdot P^I_t\iota_{t+1}\right] \tag{17}$$

$$R^H_{t+1} = \frac{W_{t+1}+P^H_{t+1}}{P^H_t}, \quad R^I_{t+1} = \frac{\Pi^I_{t+1}+P^I_{t+1}}{P^I_t} \tag{18}$$

Equation (9) captures conventional Euler equations for household held assets whose payoffs are dependent only on aggregate outcomes (i.e., the deposits and the equity in new capital producers). Applying the law of iterated expectations and conditioning on all information that the households have observed up to and including in period t, alongside the information on the realization of

[21]Such a setting and solution to the household problem, although without the endogenous labor supply decision, have been studied, e.g., by Krebs (2003a), Toda (2014), and many others.

[22]Note that because of the subtlety that in the BGG-CFP framework, new capital producers' profits Π^I_t are normalized to zero in steady state and fluctuate around zero outside of it, also the implied equity prices P^I_t fluctuate around zero (as the equity holders' are implicitly assumed to make equity injections to cover any losses that the producers make). Because of this, the definition of "$R^I_{t+1} = \left(\Pi^I_{t+1}+P^I_{t+1}\right)/P^I_t$" can lead to mathematically nonsensical operations, e.g., division by zero. I have simply introduced R^I_{t+1} here as an abuse of notation for expositional purposes to illustrate the symmetry of the various assets that the household can acquire in describing the optimal portfolio decisions. In the actual solution of the model, I will instead make sure to replace the new capital producers' equity pricing equation implicit in (9) with the correct, $P^I_t = \mathbb{E}_t\left[\beta\theta\left(\frac{C_{t+1}}{C_t}\right)^{-1} S_{t+1}\cdot(\Pi^I_{t+1}+P^I_{t+1})\right]$. And I will replace any appearances of $R^I_{t+1}\cdot P^I_t = \Pi^I_{t+1}+P^I_{t+1}$ to ensure that the analysis is mathematically correct.

the *aggregate* state in $t+1$, but not the individual draws of $\{\eta_{i,t+1}\}_{i\in\mathcal{E}_{t+1}}$ (see Footnote 5 for the definition of $\mathbb{E}_{\mathcal{F}_t}[\cdot]$), allows to isolate the effect of idiosyncratic (consumption) risk on households' pricing of assets and collapse it into the "wedge" S_{t+1}, relative to aggregate consumption growth. Thanks to the fact that the stochastic processes for all households' individual consumption growths are the same in equilibrium, this wedge is identical across households and they agree on the pricing of any assets whose payoffs are dependent on aggregate outcomes (more on this below).

Equation (10) is an Euler equation for households' accumulation of (idiosyncratically risky) human capital. Because of idiosyncratic shock realizations $\eta_{i,t+1}$ affecting both individual consumption growth and the realized returns to human capital, the presence of idiosyncratic risk gives rise to an "idiosyncratic risk premium" in the pricing of human capital, and the corresponding wedge S^H_{t+1} is slightly different than the S_{t+1} applied whenever pricing assets whose payoffs are dependent only on aggregate outcomes, seen by comparing (13) versus (14).[23]

Aggregating the individual labor supply conditions (5) across $i\in\mathcal{E}_t$ yields the supply condition (11) for the households' aggregate effective labor L_t.[24] Note that this labor supply condition is identical to the one that would arise in the case of there being a representative household with preferences $\log(C_t)+\phi\log(1-l_t)$ and H_t units of human capital, with the corresponding definition of $L_t = H_t l_t$. Alongside the Euler equations for assets (9) and the aggregate household sector budget constraint (12) this means that, in this model of the household sector, the presence of idiosyncratic labor income risk affects the aggregate dynamics of the household sector – relative to the corresponding representative household model (with $\eta_{i,t}=1,\forall(i,t)$) with constant human wealth $\bar{H}$ – *only* through fluctuations in the wedge S_{t+1}.

The wedge S_{t+1} arises due to the presence of uninsurable idiosyncratic labor risk, driven by the precautionary saving motive from idiosyncratic consumption risk over and above the risk in aggregate consumption. I will refer to S_{t+1} as a *risk shifter*, following Debortoli and Galí (2024). Thanks to the fact that the stochastic processes for all households' individual consumption growths are the same in equilibrium, this wedge is identical across households and they agree on the pricing of any assets whose payoffs are dependent on aggregate outcomes. And all households will effectively employ the stochastic discount factor (SDF) M_{t+1} when valuing $t+1$ payoffs in t, with:

$$M_{t+1} \equiv \beta\theta\left(\frac{C_{t+1}}{C_t}\right)^{-1} S_{t+1} \tag{19}$$

The dependence of this SDF on the risk shifter S_{t+1} is the key link through which the presence of idiosyncratic labor income risk will affect the sharing of aggregate risk in the contracting between

[23]Note that whenever $\alpha^H_{t+1} > 0$ and $\eta_{i,t+1}$ is stochastic, the presence of an "idiosyncratic risk premium" is naturally illustrated by the fact that: $S^H_{t+1} = \mathbb{E}_{\mathcal{F}_{t+1}}\left[\left(1+\alpha^H_{t+1}(\eta_{i,t+1}-1)\right)^{-1}\right]\cdot \mathbb{E}_{\mathcal{F}_{t+1}}[\eta_{i,t+1}] + \text{cov}_{\mathcal{F}_{t+1}}\left(\left(1+\alpha^H_{t+1}(\eta_{i,t+1}-1)\right)^{-1}, \eta_{i,t+1}\right) < S_{t+1}$ which follows by $\mathbb{E}_{\mathcal{F}_{t+1}}[\eta_{i,t+1}] = 1$ and $\text{cov}_{\mathcal{F}_{t+1}}\left(\left(1+\alpha^H_{t+1}(\eta_{i,t+1}-1)\right)^{-1}, \eta_{i,t+1}\right) < 0$.

[24]The aggregation employs the fact that $\int_{i\in\mathcal{E}_t} h_{i,t}\eta_{i,t}di = \left(\int_{i\in\mathcal{E}_t} h_{i,t}di\right)\cdot\left(\int_{i\in\mathcal{E}_t}\eta_{i,t}di\right) = H_t$ by the draws of $\eta_{i,t}$ being mean one, independent of previously accumulated $h_{i,t}$, and applying a law of large numbers.

households and the entrepreneur, as seen below.

Given that in this model, all households choose to invest in assets using the same portfolio shares and their consumption is an identical share of individual wealth, the derivations in Appendix A.1 illustrate further how *ex post* deviations in individual consumption growth from aggregate consumption growth simply arise due to the realized individual portfolio returns deviating from the average households' portfolio returns because of the idiosyncratic shock realizations $\eta_{i,t+1}$. That is, equivalently, one can write $S_{t+1} = \mathbb{E}_{\mathcal{F}_{t+1}}\left[\left(\frac{\hat{R}_{i,t+1}}{\hat{R}_{t+1}}\right)^{-1}\right]$, with $\hat{R}_{i,t+1} \equiv \frac{\tilde{h}_t}{\tilde{a}_t} \cdot R^H_{t+1}\eta_{i,t+1} + \left(1 - \frac{\tilde{h}_t}{\tilde{a}_t}\right) \cdot R^{\sim H}_{t+1}$ and $\hat{R}_{t+1} \equiv \frac{\tilde{h}_t}{\tilde{a}_t} \cdot R^H_{t+1} + \left(1 - \frac{\tilde{h}_t}{\tilde{a}_t}\right) \cdot R^{\sim H}_{t+1}$. The realized individual return to wealth $\hat{R}_{i,t+1}$ is the natural weighted average of the (individual-specific) return to human capital $R^H_{t+1}\eta_{i,t+1}$ and the "composite return" to the remaining wealth portfolio, denoted $R^{\sim H}_{t+1}$, all weighted by the share of the corresponding assets in the total wealth portfolio, as chosen in t.

α^H_{t+1} measures the share of households' beginning of $t+1$ total wealth being derived from human capital (subject to idiosyncratic risk), in contrast to the other sources of wealth (not subject to idiosyncratic risk), and it parsimoniously captures the link between idiosyncratic risk and aggregate household behavior in this economy. If α^H_{t+1} is small, and the exposure of households' individual portfolio returns $\hat{R}_{i,t+1}$ to idiosyncratic risk embedded in human capital $\eta_{i,t+1}$ is low, the exposure of individual consumption growth to idiosyncratic labor risk is low, and the risk shifter S_{t+1} falls due to a weaker precautionary saving mechanism, conditional on a given $\eta_{i,t+1}$ distribution $\mathcal{F}_{t+1}$.[25] Importantly, as illustrated by (13), fluctuations in the risk shifter arise through two, and only these two, explicit channels: endogenous fluctuations in households' human wealth share α^H_{t+1}, and any (assumed) cyclicality in the idiosyncratic labor income risk process $\mathcal{F}_{t+1}(\eta)$ itself.

Note also that the importance of α^H_t governing the dynamics of the risk shifter can be tied to the fact that it measures the degree of pass-through of households' idiosyncratic labor risk shocks to their realized wealth $\hat{a}_{i,t}$ and thus to consumption growth in t (at the mean $\eta_{i,t} = 1$):

$$\begin{aligned}
\left.\frac{\partial \log(c_{i,t})}{\partial \log(\eta_{i,t})}\right|_{\eta_{i,t}=1} &= \left.\frac{\partial \log(\tilde{c}_t \cdot \hat{R}_{i,t} \cdot a_{i,t})}{\partial \log(\eta_{i,t})}\right|_{\eta_{i,t}=1} = \left.\frac{\partial \log(\hat{R}_{i,t})}{\partial \log(\eta_{i,t})}\right|_{\eta_{i,t}=1} \\
&= \left.\frac{\frac{\tilde{h}_{t-1}}{\tilde{a}_{t-1}} \cdot R^H_t \eta_{i,t}}{\hat{R}_{i,t}}\right|_{\eta_{i,t}=1} = \frac{\tilde{h}_{t-1} \cdot R^H_t}{\tilde{h}_{t-1} \cdot R^H_t + (\tilde{a}_{t-1} - \tilde{h}_{t-1}) \cdot R^{\sim H}_t} = \alpha^H_t
\end{aligned}$$

Similarly, by the households' asset accumulation policies being linear in individual wealth $\hat{a}_{i,t}$, it thus follows that α^H_t also captures the strength of the permanent effect that idiosyncratic labor risk shocks have on future household behavior, by noting that $\frac{\partial \log(\hat{a}_{i,t+j})}{\partial \log(\eta_{i,t})} = \frac{\partial \log(\hat{R}_{i,t})}{\partial \log(\eta_{i,t})}$, for $j \geq 0$.[26] This allows to, for example, illustrate the drop in the autocovariance function of individual labor

[25] This follows by the convexity of marginal utility and Jensen's inequality logic, observing that $\mathbb{E}_{\mathcal{F}_{t+1}}[1 + \alpha^H_{t+1}(\eta_{i,t+1} - 1)] = 1$ independently of α^H_{t+1}, by the normalization $\mathbb{E}_{\mathcal{F}_{t+1}}[\eta_{i,t+1}] = 1$.

[26] See equation (A.4) in Appendix A.1.

productivity at lags greater than 1 (as discussed in Section 2.1.2), whenever $\alpha_t^H < 1$, for $j \geq 1$:

$$\frac{\partial \log(\hat{h}_{i,t+j})}{\partial \log(\eta_{i,t})} = \frac{\partial \log(h_{i,t+j})}{\partial \log(\eta_{i,t})} = \frac{\partial(\log(\tilde{h}_{t+j-1}) + \log(\hat{a}_{i,t+j-1}))}{\partial \log(\eta_{i,t})} = \frac{\partial \log(\hat{R}_{i,t})}{\partial \log(\eta_{i,t})} < 1 = \frac{\partial \log(\hat{h}_{i,t})}{\partial \log(\eta_{i,t})}$$

2.2.2 Entrepreneurs

The representative entrepreneur maximizes its lifetime utility over streams of consumption C_t^e:

$$\mathcal{V}_t^e\left(\{s_{j,t}\}_{j\in[0,1]}\right) = \max_{C_t^e, \{s_{j,t+1}\}_j} \left\{\log(C_t^e) + \beta_e \mathbb{E}_t\left[\mathcal{V}_{t+1}^e\left(\{s_{j,t+1}\}_{j\in[0,1]}\right)\right]\right\}$$

subject to the budget constraint

$$C_t^e + \int_0^1 q_{j,t} s_{j,t+1} dj \leq \int_0^1 (q_{j,t} + div_{j,t})\, s_{j,t} dj$$

where $s_{j,t+1}$ denotes the share of firm j's net worth acquired by the entrepreneur at the end of period t, $div_{j,t}$ are the dividends paid by firm j in t, and $q_{j,t}$ is the time t ex-dividend price of firm j's equity. The entrepreneur's first order necessary conditions which price the firms' equity are:

$$q_{j,t} = \mathbb{E}_t\left[M_{t+1}^e (q_{j,t+1} + div_{j,t+1})\right], \quad j \in [0,1]$$
$$\text{with } M_{t+1}^e \equiv \beta_e \left(\frac{C_{t+1}^e}{C_t^e}\right)^{-1}$$

with equality in equilibrium because the firms must be held by the entrepreneur. The key take-away is that, in equilibrium, the firms will thus use the entrepreneur's stochastic discount factor M_t^e when maximizing their value and discounting future dividend streams. Also, market clearing for firms' shares requires $s_{j,t} = 1, \forall(j,t)$, by normalization, verifying that the only source of the entrepreneur's consumption are dividends paid by the firms: $C_t^e = \int_0^1 div_{j,t} dj$. And since the entrepreneur holds no other assets apart from shares in firms, the firms' aggregate net worth is equivalently also equal to the entrepreneur's net worth.

2.2.3 Final Goods and New Capital Producers

The representative final goods producer's optimization yields the demand for labor and capital:

$$W_t = (1-\nu) Z_t K_t^{\nu} L_t^{-\nu} \tag{20}$$
$$r_t^K = \nu Z_t K_t^{\nu-1} L_t^{1-\nu} \tag{21}$$

New capital producers' profits are given by:

$$\Pi_t^I = Q_t I_t - I_t \vartheta\left(\frac{I_t}{I_{ss}}\right) \tag{22}$$

Their optimization with respect to I_t yields that the equilibrium capital price follows:

$$Q_t = \vartheta\left(\frac{I_t}{I_{ss}}\right) + \frac{I_t}{I_{ss}}\vartheta'\left(\frac{I_t}{I_{ss}}\right) \tag{23}$$

The law of motion for aggregate capital is:

$$K_{t+1} = I_t + (1-\delta)K_t \tag{24}$$

2.2.4 Firms and the Loan Contract

Let us denote firm j's accumulated internal wealth, i.e., net worth, after paying dividends in period t by $N_{j,t}$. This net worth is accumulated by purchasing capital $K_{j,t}$ in $t-1$, earning rental returns and capital gains on $\omega_{j,t}K_{j,t}$, paying back the contracted upon payment to the lender in t, and paying dividends to the owner. Because of the imperfect obsevability of firm j's idiosyncratic capital shock $\omega_{j,t+1}$, the costly state verification problem arises. Firm j's investment of $K_{j,t+1}$ units of capital yields $\omega_{j,t+1}K_{j,t+1}$ units in $t+1$ which generates an income flow of $\omega_{j,t+1}\left[r^K_{t+1} + (1-\delta)Q_{t+1}\right]K_{j,t+1} = \omega_{j,t+1}R^K_{t+1}Q_tK_{j,t+1}$. Following Townsend (1979) and Williamson (1986), one can show that if payoffs are linear in the project outcome $\omega_{j,t+1}K_{j,t+1}$, and there is no random monitoring, the optimal contract is risky debt.[27] Since idiosyncratic firm risk is fully diversified in the financial intermediary's portfolio, this is true on the lender's side. As for firm j, I show in Appendix A.2 that if risky debt is the optimal contract, then the firm's value function is linear in net worth, closing the logical circle, as demonstrated by CFP.

By risky debt we mean that monitoring only occurs for low realizations of $\omega_{j,t+1}$. More specifically, in the absence of aggregate uncertainty, i.e., when r^K_{t+1} and Q_{t+1} are known at the time of signing the contract, the borrower and lender agree on a cutoff $\bar{\omega}_{j,t+1}$ and an implied promised repayment to the lender: $\bar{\omega}_{j,t+1}R^K_{t+1}Q_tK_{j,t+1}$. If $\omega_{j,t+1} < \bar{\omega}_{j,t+1}$, the borrower does not have sufficient funds to pay the lender. She declares bankruptcy, the lender incurs the monitoring cost and gets all of the remaining funds, which yields her an income flow of $(1-\mu)\omega_{j,t+1}R^K_{t+1}Q_tK_{j,t+1}$. If $\omega_{j,t+1} \geq \bar{\omega}_{j,t+1}$, no monitoring occurs, the borrower repays the promised amount $\bar{\omega}_{j,t+1}R^K_{t+1}Q_{t+1}K_{j,t+1}$ and holds on to the remaining income flow of $(\omega_{j,t+1} - \bar{\omega}_{j,t+1})R^K_{t+1}Q_tK_{j,t+1}$. Note that $\bar{\omega}_{j,t+1}$ implicitly determines an interest rate $R^{def}_{j,t+1}$ earned by the lender that is subject to default risk, defined by: $R^{def}_{j,t+1}(Q_tK_{j,t+1} - N_{j,t}) = \bar{\omega}_{j,t+1}R^K_{t+1}Q_tK_{j,t+1}$.

In the presence of aggregate uncertainty, however, the optimal contract involves the lender and borrower agreeing upon a schedule of $\{\bar{\omega}_{j,t+1}\}$, with a specific value of the cutoff for each possible realization of the aggregate state in $t+1$. Conditional on having observed aggregate outcomes and thus knowing the implied $\bar{\omega}_{j,t+1}$, the optimality of risky debt, now for each realization of the aggregate state, remains. The CSV problem takes as exogenous the aggregate returns on capital and the opportunity cost of the lender.

[27] The proof is exactly as for the conventional CSV problem without aggregate uncertainty, only applied for each realization of the aggregate state separately.

Let $\Gamma(\bar{\omega})$ denote the expected gross share of the returns to a firm's held capital going to the lender, given cutoff $\bar{\omega}$: $\Gamma(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega = \int_0^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega}[1 - F(\bar{\omega})]$. And let $\mu G(\bar{\omega})$ be the expected monitoring costs: $\mu G(\bar{\omega}) \equiv \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega$. Noting that $\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) > 0$ and $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) = [1 - F(\bar{\omega})][1 - \mu \bar{\omega} h(\bar{\omega})] > 0$ if $\bar{\omega} < \bar{\omega}^*$, we have that the firm's expected net share $[1 - \Gamma(\bar{\omega})]$ is decreasing in $\bar{\omega}$ and that of the lender, $[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]$ increasing.[28]

Appendix A.2 provides further details in setting up a firm's dynamic problem and deriving the implications for optimal behavior. In the following, I will simply focus on stating the key conditions necessary for characterizing aggregate equilibrium outcomes. Importantly, by the linearity of the firms' investment technology and their objective function, all firms will choose to operate at the same leverage ratio $\kappa_{j,t} \equiv \frac{Q_t K_{j,t+1}}{N_{j,t}}$ (i.e., their acquired capital $K_{j,t+1}$ scales with their net worth $N_{j,t}$) and the same schedule for $\{\bar{\omega}_{t+1}\}$, implying simple aggregation of the behavior of the firm sector, requiring keeping track only of the firms' aggregate net worth N_t.

Since there are no frictions in the flow of dividends (or if negative, equity injections) between the entrepreneur and the firms, the marginal value of a unit of wealth in the hands of a firm is equal to the marginal utility of the entrepreneur's consumption $(C_t^e)^{-1}$. Also, the fact that the firms' return to investing a unit of net worth in capital, integrating out $\omega_{j,t+1}$, equals $[1 - \Gamma(\bar{\omega}_{t+1})] R_{t+1}^K \kappa_t$ means that in equilibrium, the entrepreneur's consumption must satisfy the Euler equation:

$$(C_t^e)^{-1} = \beta_e \mathbb{E}_t \left[(C_{t+1}^e)^{-1} [1 - \Gamma(\bar{\omega}_{t+1})] R_{t+1}^K \right] \kappa_t \tag{25}$$

The aggregate net worth N_t of firms evolves as

$$N_t = [1 - \Gamma(\bar{\omega}_t)] R_t^K \kappa_{t-1} N_{t-1} - C_t^e \tag{26}$$

$$\text{where } \kappa_t \equiv \frac{Q_t K_{t+1}}{N_t} \tag{27}$$

Note that the leverage ratio κ_t is simultaneously the inverse of the firms' (and thus the entrepreneur's) share of financial wealth in the economy. Because each firm needs a positive amount of net worth to operate its project, I assume that the entrepreneur provides transfers from other firms to any firms who default and must pay out all returns to the lender. These transfers are inconsequential as the distribution of wealth across firms is irrelevant for aggregate outcomes.

The key optimality condition in the contracting problem is the first order condition for $\bar{\omega}_{t+1}$

[28] In the above, $h(\omega) \equiv f(\omega)/[1 - F(\omega)]$ is the hazard rate and $\bar{\omega}^*$ is the cutoff value at which the lender's net share is maximized. Assuming that $\frac{\partial[\omega h(\omega)]}{\partial \omega} > 0$ and $\lim_{\omega \to +\infty} \omega h(\omega) > \frac{1}{\mu}$, as will be satisfied by the log-normal distribution employed in the computations, there exists a unique such $\bar{\omega}^*$. At the optimum, it cannot be the case that for any realization of aggregate shocks $\bar{\omega}^j > \bar{\omega}^*$. Because then, $\bar{\omega}^j$ can be reduced, the borrower made better off and the participation constraint slackened. In the calibration and simulations employed, $\bar{\omega}_t$ will be significantly below $\bar{\omega}^*$.

which characterizes how aggregate risk is shared and which can be summarized as:

$$\frac{\Gamma'(\bar{\omega}_{t+1})}{\Gamma'(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})} = \frac{M_{t+1}}{M^e_{t+1}} \tag{28}$$

This condition holds state-by-state, for each realization of the aggregate state in $t+1$. The equilibrium lender return is:

$$R^L_{t+1} = [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] R^K_{t+1} \frac{\kappa_t}{\kappa_t - 1} \tag{29}$$

How R^L_{t+1} fluctuates in response to realized shocks to R^K_{t+1} naturally captures the degree of aggregate risk sharing embedded in the contract. BGG imposed that R^L_{t+1} is predetermined in t and thus constant across realizations of aggregate uncertainty. CFP showed that under the privately optimal behavior of $\bar{\omega}_{t+1}$ implied by (28), R^L_{t+1} comoves significantly with the *ex post* realizations of R^K_{t+1}.

A thorough analysis of the properties of the privately optimal contract and its implications in the standard BGG framework are presented by Carlstrom et al. (2016), with all the insights extending to the setup presented above.[29] To reiterate, the key optimality condition governing aggregate risk sharing is (28). Given the assumptions in Footnote 28, the left hand side is strictly increasing in $\bar{\omega}_{t+1}$. Therefore, naturally, whenever the households value wealth relatively more, meaning M_{t+1} is high, all else equal, also $\bar{\omega}_{t+1}$, and thus the lender's net share $[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]$ and the lender's return R^L_{t+1} are high, to provide consumption insurance to the households. Conversely, when the value of firms' internal net worth, equal to $(C^e_{t+1})^{-1}$ embedded in M_{t+1} is high, the contract calls for a lower $\bar{\omega}_{t+1}$ allowing the borrowers to hold on to more net worth, all else equal. Or alternatively, one can just see this as the outcome of optimal risk sharing between the household and the entrepreneur, aiming to equalize marginal rates of substitution M_{t+1} and M^e_{t+1}, subject to the marginal cost of redistribution implied by the presence of the monitoring cost $\mu > 0$.

As a final sidenote, because the entrepreneur has log-utility and all her wealth is invested in firm equity, perfectly diversifying the firm-specific risk, we have the conventional result that she ends up always consuming a constant fraction $(1-\beta_e)$ of her incoming wealth in every period, with:

$$C^e_t = (1-\beta_e)[1-\Gamma(\bar{\omega}_t)] R^K_t \kappa_{t-1} N_{t-1} \tag{30}$$

$$N_t = \beta_e [1-\Gamma(\bar{\omega}_t)] R^K_t \kappa_{t-1} N_{t-1} \tag{31}$$

[29] Note that while my setup of the "entrepreneur + firm block" of the model features a representative entrepreneur with logarithmic utility who owns *ex post* heterogeneous firms investing in capital projects, I show in Appendix A.3 that it is virtually equivalent, to a first order approximation, to the standard approach used by BGG and CFP where a unit mass of *ex post* heterogeneous entrepreneurs are assumed to themselves invest in capital subject to the CSV friction and have *linear* utility from consumption. The key insight behind this result is that in both models, entrepreurs' equilibrium aggregate consumption C^e_t behavior and their marginal valuation of a unit of wealth V_t must satisfy the same conceptual equilibrium conditions. In my representative entrepreneur model, logarithmic utility implies that C^e_t is a constant fraction of entrepreneurial net worth. In the BGG-CFP model, the same happens because all linear utility entrepreneurs postpone consumption of their whole net worth until death, and a constant fraction of them are assumed to die in each period. In both models, V_t is determined recursively by an Euler equation that features the equilibrium returns from investing in capital, $[1-\Gamma(\bar{\omega}_{t+1})]\, R^K_{t+1}\kappa_t$.

2.2.5 Market Clearing and Equilibrium Definition

In equilibrium, market clearing requires that the households' deposits fund the firms' projects:

$$D_{t+1} \equiv \int_{i \in \mathcal{E}_t} d_{i,t+1} di = Q_t K_{t+1} - N_t \tag{32}$$

Also, as already stated in Section 2.1, market clearing also requires that households hold the aggregate supply of human capital:

$$\bar{H} = \int_{i \in \mathcal{E}_t} h_{i,t+1} di = \int_{i \in \mathcal{E}_t} h_{i,t} di = H_t \tag{33}$$

By the normalization of there being one unit of new capital producers' equity shares, the corresponding market clearing condition requires:

$$1 = \int_{i \in \mathcal{E}_t} \iota_{i,t+1} di = \int_{i \in \mathcal{E}_t} \iota_{i,t} di = \iota_t \tag{34}$$

In both of the equations (33)–(34), the first equality indicates the clearing of the respective asset markets as a result of trade in t by households contained in $\mathcal{E}_t$. The second equality indicates the fact that the pool of dying households' assets and human capital get reallocated to newborns, and thus any assets acquired in $t-1$ by $\mathcal{E}_{t-1}$ must remain in the hands of $\mathcal{E}_t$ at the beginning of t.

Combining these conditions, the households' and entrepreneurs' budget constraints, the definition of leverage and the rental and labor market equilibrium conditions with new capital producers' profits, one arrives at the aggregate resource constraint of the final good, implied by Walras' law:

$$C_t + C_t^e + I_t \vartheta \left(\frac{I_t}{I_{ss}} \right) + \mu G(\bar{\omega}_t) R_t^K Q_{t-1} K_t = Z_t K_t^{\nu} L_t^{1-\nu}$$

Given that all the necessary equilibrium conditions were imposed in the derivations above, we can immediately define a competitive equilibrium of the model in the case of an acyclical labor risk distribution $\mathcal{F}$ as follows.

Definition 1. *A* competitive equilibrium of the model *is a collection of stochastic processes for: a price system* $\{r_t, W_t, R_t^K, R_t^L, Q_t, R_t^H, R_t^E, R_t^I, P_t^H, P_t^I\}$*, households' consumption, stochastic discount factor, asset accumulation* $\{C_t,\ M_t,\ D_{t+1},\ H_{t+1},\ \iota_{t+1}\}$*, entrepreneurial consumption, net worth and leverage quantities and contractual cutoffs* $\{C_t^e,\ N_t,\ \kappa_t,\ \bar{\omega}_t\}$*, aggregate labor, investment and capital quantities, and new capital producer profits* $\{L_t,\ I_t,\ K_{t+1},\ \Pi_t^I\}$*, such that equations: (1), (9)–(12), (18), (19), (20)–(24), (25)–(27), (28)–(34), with* $R_t^D = R_t^L$*, where applicable, are satisfied, given a labor risk distribution* $\mathcal{F}_t = \mathcal{F}, \forall t$*, a stochastic process for* $\{Z_t\}$*, and initial conditions* (K_0, n_0, Z_0)*.*

2.2.6 Equilibrium Conditions in Extension with Nominal Rigidities

As for the extension of the model with nominal rigidities, one can follow standard steps (e.g., see Galí, 2015) and show that the set of aggregate variables from Definition 1 must be extended by the gross markup $\mathcal{M}_t$, the net inflation rate π_t, and the nominal interest rate R^n_{t+1} between t and $t+1$. These three variables will be determined by the three additional equilibrium conditions which are: (i) the Taylor rule of the monetary authority (2), (ii) the households' Euler equation for the nominal rate, $\mathbb{E}_t\left[M_{t+1}R^n_{t+1}(1+\pi_{t+1})^{-1}\right] = 1$, and (iii) the New Keynesian Phillips Curve which I will immediately introduce in conventional (log-)linearized form:

$$\pi_t = -\kappa_p \log(\mathcal{M}_t) + M_{ss}\mathbb{E}_t[\pi_{t+1}]$$

where $\kappa_p = \frac{\epsilon-1}{\phi_p}$ is the slope of the Phillips curve.

In addition, the gross inverse markup of the retailers now appears in the equilibrium capital rental and wage rates paid by the wholesale goods producer, replacing (20) and (21) with:

$$\begin{aligned} W_t &= (1-\nu)\mathcal{M}_t^{-1} Z_t K_t^{\nu} L_t^{-\nu} \\ r_t^K &= \nu \mathcal{M}_t^{-1} Z_t K_t^{\nu-1} L_t^{1-\nu} \end{aligned}$$

And finally, the households' financial portfolio now also includes the value of the retailers' equity shares, and the implied final good resource constraint of the economy also features the retailers' price adjustment costs. Since there are no nominal rigidities between the financial intermediary and the firms, the contracting problem, and all other equilibrium conditions, remain unchanged.

2.3 Theoretical Implications from the Model

2.3.1 The Relevance of Human and Financial Wealth Dynamics in Aggregate Risk Sharing

An important determinant of aggregate financial risk sharing in the economy is the behavior of human and financial wealth dynamics. To illustrate this idea clearly, let us consider the representative household specification of the model, i.e., $\eta_{i,t} = 1, \forall(i,t)$. First, note that as derived in Appendix A.1, the heterogeneous log-utility households in the model laid out above consume a constant fraction of their total wealth $\hat{a}_{i,t}$, with the fraction equaling a constant $\tilde{c}_t = \frac{1-\beta\theta}{1+(1-\beta\theta)\phi} \equiv \tilde{c}, \forall t$, as defined in Section 2.2.1 Alternatively, one can also *not* substitute out for the households' optimal labor supply in the analysis and instead show that if one were to define an alternative measure of a household's wealth $\hat{a}^l_{i,t}$, replacing the current value of the *total* labor endowment $W_t h_{i,t}\eta_{i,t}$ with the actual equilibrium labor income $W_t h_{i,t}\eta_{i,t} l_{i,t}$ of the household[30] the more "conventional" form of the consumption policy function for a log-utility agent, $c_{i,t} = (1-\beta\theta)\hat{a}^l_{i,t}$ also holds, naturally applying to the representative household case as well.

For notational brevity, let us define the human wealth related terms in the representative house-

[30]That is, define: $\hat{a}^l_{i,t} \equiv (W_t l_{i,t} + P^H_t)h_{i,t}\eta_{i,t} + R^D_t d_{i,t} + (\Pi^I_t + P^I_t)\iota_{i,t}$.

hold's total wealth $\hat{A}^l_t \equiv \int_{i\in\mathcal{E}_t} \hat{a}^l_{i,t} di$ as:

$$\mathcal{W}^H_t \equiv W_t \bar{H} l_t + P^H_t \bar{H} = \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \left(\prod_{s=1}^{j} M_{t+s} \right) W_{t+j} L_{t+s} \right\}$$

where I have used the definition of $L_t = \bar{H} l_t$ and the convention that $\prod_{s=1}^{0} M_{t+s} = 1$. In the last equality, I have employed the households' pricing equation of human capital (10) alongside the optimal labor supply condition (11) to write the present value of human capital as the present discounted value of the future stream of labor income.

Finally, note that in general equilibrium, the value of the households' returns from deposits equals the lenders' share from the incoming total value of physical capital investments, defined for brevity here, as $\mathcal{W}^K_t \equiv R^K_t Q_{t-1} K_t$:

$$R^D_t D_t = R^L_t (Q_{t-1} K_t - N_{t-1}) = [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)]\, R^K_t Q_{t-1} K_t \equiv [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)]\, \mathcal{W}^K_t$$

Also, for brevity and symmetry, let us denote the households' financial wealth coming from the value of ownership of new capital producers as $\mathcal{W}^I_t \equiv \Pi^I_t + P^I_t$. Therefore,

$$C_t = (1 - \beta\theta) \left\{ \mathcal{W}^H_t + [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)]\, \mathcal{W}^K_t + \mathcal{W}^I_t \right\}$$

As the entrepreneur has log-utility, it also consumes a constant fraction, $(1 - \beta_e)$, of its total (financial) wealth in any period: $C^e_t = (1 - \beta_e)[1 - \Gamma(\bar{\omega}_t)] R^K_t \kappa_{t-1} N_{t-1} = (1 - \beta_e)[1 - \Gamma(\bar{\omega}_t)] \mathcal{W}^K_t$.

Now, suppose that the economy is shocked in period t, while previously having been in steady state. Imposing the optimal consumption policies in the risk sharing condition (28) yields:

$$\frac{\Gamma'(\bar{\omega}_t)}{\Gamma'(\bar{\omega}_t) - \mu G'(\bar{\omega}_t)} = \frac{[1 - \Gamma(\bar{\omega}_t)] \mathcal{W}^K_t}{[\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)]\, \mathcal{W}^K_t + \mathcal{W}^H_t + \mathcal{W}^I_t} \cdot \frac{1 - \beta_e}{1 - \beta\theta} \frac{\beta\theta}{\beta_e} \frac{C_{ss}}{C^e_{ss}}$$

Therefore, given that the left hand side is increasing and the right hand side decreasing in $\bar{\omega}_t$, this establishes a negative relationship between $(\mathcal{W}^H_t + \mathcal{W}^I_t)/\mathcal{W}^K_t$ and $\bar{\omega}_t$.

Abstracting from households' (financial) wealth $\mathcal{W}^I_t$ in the ownership of new capital producers for a moment, since their profits are zero in steady state and one can verify quantitatively that the magnitude of their fluctuations is small around it, this becomes a negative relationship between $\mathcal{W}^H_t/\mathcal{W}^K_t$ and $\bar{\omega}_t$. That is, whenever the human wealth in the economy increases more than the financial wealth in productive capital, it is thus optimal to leave a larger share of this financial wealth, implied by a lower $\bar{\omega}_t$, to the firms. For example, if $\mathcal{W}^H_t/\mathcal{W}^K_t = \mathcal{W}^H_{ss}/\mathcal{W}^K_{ss}$, then $\bar{\omega}_t = \bar{\omega}_{ss}$ and the aggregate financial risk is shared perfectly, meaning that $R^L_t = R^D_t$ responds to the shock by the same relative amount as R^K_t. Of course, $\mathcal{W}^H_t$ and $\mathcal{W}^K_t$ are themselves equilibrium objects, dependent on $\bar{\omega}_t$ itself, but this note emphasizes that it is important to keep in mind that shocks which affect human and financial wealth differently, could have markedly different implications for how the aggregate financial risk embedded in productive capital returns is to be shared even in the case of a representative household. And vice versa, since $\mathcal{W}_t^H$ equals the present discounted value of future labor income, whereas $\mathcal{W}_t^K$ is closely tied to the present discounted value of future capital rental returns, and the Cobb-Douglas production function implies constant labor and capital income shares, many conventional business cycle shocks are likely to imply financial risk sharing that is close to "perfect".

The introduction of idiosyncratic labor risk in my model then adds a wedge, the risk shifter S_t, in the relation between aggregate household consumption C_t and the effective marginal utility with which the households value an additional unit of wealth paid out by the entrepreneur. If, for whatever reason, the risk shifter S_t were to increase at shock impact in t, it would, all else equal, dictate that the firms pay out relatively more to the households. One reason why the risk shifter might fluctuate in response to an aggregate shock, is the fact that S_t *itself* depends on the relative fluctuations of households' wealth coming from human capital, as already discussed in Section 2.2.1. A higher human wealth share exposes households to more idiosyncratic risk, increasing the risk shifter, and requiring the entrepreneur to pay out *more*, all else equal.

Finally, reincorporating fluctuations in $\mathcal{W}_t^I$ allows to emphasize the relevance of households' financial wealth invested in assets *other* than the financing of productive firms. In this benchmark model, the only one other such asset is the equity in new capital producers. More generally, one could imagine that, for example, households' ownership of real estate would in reality constitute a large share of their overall wealth portfolio, and contribute considerably to the fluctuations in its value. It is clear from the above optimal risk sharing condition that if $\mathcal{W}_t^I$ falls, all else equal, it would be optimal for the entrepreneur to pay out a larger share of productive capital wealth to the representative household (a higher $\bar{\omega}_t$). Moreover, when introducing idiosyncratic labor risk, such a fall in $\mathcal{W}_t^I$ would, all else equal, increase the share of households' wealth coming from (risky) human capital, exposing the households to relatively more idiosyncratic risk, increasing the risk shifter, and adding *another* reason for the contract to dictate the entrepreneur pay out more to the household. This discussion illustrates that procyclical fluctuations in the value of households' financial wealth held outside of the financing of firms could constitute a relevant force generating the concentration of aggregate risk involved in productive capital investments on firms' balance sheets. I leave such extensions, introducing other sources of household financial wealth (apart from ownership of new capital producers) to future research.

2.3.2 The Importance of Precautionary Saving for Aggregate Dynamics

The above discussion has illustrated how the presence of idiosyncratic risk directly affects aggregate risk sharing in the economy through the appearance of the risk shifter in the households' SDF, M_t, dictating how much gets paid out to the households through the optimality condition (28).

However, the appearance of the risk shifter S_t in M_t and condition (28), is not the only way that the presence of idiosyncratic risk affects aggregate equilibrium dynamics in this economy. Namely, as will be seen in the quantitative results below, a key role will also be played by the *precautionary saving mechanism*, induced by any fluctuations in the future path of the risk shifter S_{t+j}, for $j \geq 1$, after a shock in t has hit the economy. This is best seen from the households' Euler equation (9) for the choice of deposits made in t, rewritten here:

$$(C_t)^{-1} = \mathbb{E}_t\left[\beta\theta(C_{t+1})^{-1}S_{t+1}\cdot R^D_{t+1}\right]$$

In general equilibrium, the households' increased desire to save due to a higher S_{t+1} tends to lower the interest rate R^D_{t+1} at which they are willing to save, i.e., lend to the firms. This lower required return allows firms to borrow cheaper, build net worth faster, relax financial constraints, and improve aggregate investment and capital prices in upcoming periods. Since capital prices will be forward-looking in equilibrium, higher future capital prices create a force to increase capital prices already in t, improving the returns to capital and firms' net worth already at shock impact in t. If S_{t+1} is higher in recessions, this mechanism introduces a *stabilizing* effect on aggregate fluctuations through investment.

On the contrary, a higher S_{t+1} also tends to lower current aggregate consumption C_t. This increases the realization of the households' SDF M_t, which in turn induces the optimal contract to pay out a higher return R^L_t to the households following (28), reducing the firms' net worth and worsening their ability to invest. Thus, the added drop in C_t due to precautionary saving in recessions would introduce a *destabilizing* effect on aggregate fluctuations through investment.

Moreover, analogous stabilizing and destabilizing effects of the precautionary saving mechanism appear even in a conventional business cycle model without financial frictions in the financing of firms, as has been studied, e.g., by Challe et al. (2017) and many others. In a real business cycle framework with flexible prices, a higher precautionary saving motive in recessions tends to stabilize the economy for an analogous reason as mentioned above: lower induced interest rates limit the fall in investment, the capital stock, and ultimately in output. In a model which also features nominal rigidities, the induced fall in household consumption decreases aggregate demand, increasing markups and lowering output. These two countervailing forces are discussed and quantified in a conventional estimated DSGE model without firm financing frictions by Challe et al. (2017), but one must keep in mind they are implicitly also at play in the quantitative analysis of this paper.

All in all, whether the general equilibrium effect of an increase in households' precautionary saving operates relatively more through reducing R^L_{t+1} or through reducing C_t, and thus has a marginal expansionary or recessionary effect, respectively, depends on the model specifics (e.g., flexible vs. sticky prices), the calibrated parameter values, and the persistence of the increase in the risk shifter. Section 3.2 below shows these forces at work in the calibrated model set up above.

3 Quantitative Analysis

3.1 Calibration

In the calibration of model parameters I pursue targets from the earlier literature, following BGG and CFP wherever possible for comparability. One time period t is considered to be a quarter. As

CFP, I set the capital share in production to be $\nu = 0.35$, capital price elasticity with respect to investment $\phi_Q = 0.5$ and the depreciation rate $\delta = 0.025$.

As is common since Carlstrom and Fuerst (1997), the idiosyncratic entrepreneurial capital shock is log-normal: $\log(\omega) \sim N\left(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2\right)$. Following the discussion in Footnote 29 (and Appendix A.3) and the targets set by CFP, the parameters (μ, β_e, σ) pertaining to the entrepreneurial financial frictions are pinned down, to yield in steady state: (i) a spread of 200 basis points (annualized) between the borrowing rate R_{ss}^{def} subject to default risk and the riskless lender return R_{ss}^L, both as defined in Section 2.2.4, (ii) a quarterly bankruptcy rate $F(\bar{\omega}_{ss})$ of 0.75%, (iii) a leverage ratio of $\kappa_{ss} = 2$. Exactly as in CFP, this results in $(\mu, \beta_e, \sigma_\omega) = (0.63, 0.94, 0.28)$.

Because the value of the survival rate θ matters only insofar it affects the product $\beta\theta$, I will simply set $\theta = 0.995$. As for β, I calibrate its value depending on the degree of idiosyncratic risk that households face to ensure that in any considered specification, the implied steady state households' SDF is $M_{ss} = 0.99$, following the value of the representative households' β from CFP, yielding an annual risk free rate of 4%. In the benchmark heterogeneous household case, $\beta = 0.926$.

As for households' utility of leisure, ϕ, I calibrate it to match the common target of 1/3 of individuals' time spent engage in market activities (e.g., see Hansen, 1985). Log-linearizing the aggregate labor supply condition (11) around the steady state implies $\hat{C}_t + \dfrac{L_{ss}}{1 - L_{ss}}\hat{L}_t = \hat{W}_t$, with $\hat{X}_t \equiv \log(X_t) - \log(X_{ss})$, implies that to a first order, the elasticity of aggregate labor supply equals $\epsilon_l \equiv \frac{1-L_{ss}}{L_{ss}} = 2$, given $L_{ss} = 1/3$. This requires a value of labor disutility $\phi = 1.916$.

As for the extension with nominal rigidities, I set the price adjustment cost ϕ_p so that the slope of the NKPC is $\kappa_p = 0.025$, and I set the Taylor rule responsiveness to $\phi_\pi = 1.5$, both as in CFP.

3.1.1 Idiosyncratic Labor Risk

What is left to calibrate, are the characteristics of the distribution of the idiosyncratic labor risk shock $\eta_{i,t}$, and its possible cyclicality, i.e., the distribution $\mathcal{F}_t(\eta)$ and any possible cyclical variation in it. Empirical work, e.g., by Geweke and Keane (2000), has found that individual-level earnings risk in the U.S. (in terms of the innovations in log earnings) features a notable degree of left-skewness. Moreover, this left-skewness *increases* in recessions, while the *variance* of earnings risk seems rather acyclical (Guvenen et al., 2014).[31] To explicitly introduce an idiosyncratic labor risk distribution that is flexible enough to match the skewness of emprical earnings risk, yet remains as parsimonious as possible otherwise, I will suppose that $\log(\eta_{i,t}) \equiv \epsilon_{i,t}$, conditional on the aggregate state in t, follows a skew normal distribution. The skew normal is a generalization of the normal distribution that allows for non-zero skewness, first introduced by O'Hagan and Leonard (1976). The variable $\epsilon_{i,t}$ is a skew normal variable with location parameter $\mu_{\eta,t} \in \mathbb{R}$, scale parameter $\sigma_{\eta,t} \in \mathbb{R}_+$ and shape parameter $\alpha_{\eta,t} \in \mathbb{R}$ if its probability density function follows:

[31] Also, empirical work by Brav et al. (2002) using CEX data illustrates that a SDF calculated as an equally weighted mean of individual households' marginal rates of substitution (or its Taylor expansion that captures the variance and skewness of cross-sectional consumption growth) does considerably better at explaining the equity premium than a Taylor expansion of the SDF that captures only the mean and variance of cross-sectional consumption growth, or a SDF derived based on an assumption of idiosyncratic consumption growth following an i.i.d. log-normal distribution.

$f_t(\epsilon) = \frac{2}{\sigma_{\eta,t}}\phi\left(\frac{\epsilon-\mu_{\eta,t}}{\sigma_{\eta,t}}\right)\Phi\left(\alpha_{\eta,t}\frac{\epsilon-\mu_{\eta,t}}{\sigma_{\eta,t}}\right)$, where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal pdf and cdf, respectively. In the specific case of $\alpha_{\eta,t} = 0$, this distribution is simply the normal distribution $\varepsilon_{i,t} \sim \mathcal{N}(\mu_{\eta,t}, \sigma_{\eta,t})$ with zero skewness. Values of $\alpha_{\eta,t} > 0$ imply that the distribution has positive skewness (i.e., is right-skewed) and vice versa for $\alpha_{\eta,t} < 0$. Note that, equivalently, assuming $\log(\eta_{i,t})$ follows a skew normal with parameters $(\mu_{\eta,t}, \sigma_{\eta,t}, \alpha_{\eta,t})$ means that the cdf of $\eta_{i,t}$ is given by

$$\mathcal{F}_t(\eta) = \int_0^\eta \frac{1}{x}\frac{2}{\sigma_{\eta,t}}\phi\left(\frac{\log(x)-\mu_{\eta,t}}{\sigma_{\eta,t}}\right)\Phi\left(\alpha_{\eta,t}\frac{\log(x)-\mu_{\eta,t}}{\sigma_{\eta,t}}\right)dx$$

Following Azzalini (1985), it can be shown that the (conditional) mean of $\eta_{i,t}$ then equals: $\mathbb{E}_{\mathcal{F}_t}[\eta_{i,t}] = 2e^{\mu_{\eta,t}+\sigma^2_{\eta,t}/2}\Phi\left(\sigma_{\eta,t}\alpha_{\eta,t}/\sqrt{1+\alpha^2_{\eta,t}}\right)$. In accordance with the normalization that $\mathbb{E}_{\mathcal{F}_t}[\eta_{i,t}] = 1$, I will at all times impose: $\mu_{\eta,t} = -\sigma^2_{\eta,t}/2 - \log\left(2\Phi\left(\sigma_{\eta,t}\alpha_{\eta,t}/\sqrt{1+\alpha^2_{\eta,t}}\right)\right)$. I then calibrate the remaining two free parameters of the distribution in steady state, $\sigma_{\eta,ss}$ and $\alpha_{\eta,ss}$, so that the model matches two empirical targets from the observed individual-level labor income dynamics for the U.S. More specifically, I match the variance and Kelley skewness of the one-year log annual labor earnings changes for U.S. males aged 25–60, averaged over 1978–2011, as implied by the IRS data used by Guvenen et al. (2014).[32] The respective target values are 0.283 for the variance and 0.0013. for the Kelley skewness The required values ($\mu_{\eta,ss}$, $\sigma_{\eta,ss}$, $\alpha_{\eta,ss}$) are (-0.336, 0.449, 1.141).

As for any cyclicality in the $\mathcal{F}_t$ distribution, I will first, in Section 3.3, study the dynamic properties of the model under the assumption that the labor risk process is acyclical. As discussed in Section 2.2.1, this means that any fluctuations in the risk shifter S_t, and thus, any differences in the dynamics between a heterogeneous vs. a representative household model will arise "endogenously" due to any fluctuations in the households' human wealth share α^H_t. Thereafter, in Section 3.4, I will exogenously assume that aggregate shocks to the economy will also affect $(\sigma_{\eta,t}, \alpha_{\eta,t})$ and the implied idiosyncratic shock distribution. I will calibrate the exposure of $(\sigma_{\eta,t}, \alpha_{\eta,t})$ to the considered aggregate shocks by matching empirically observed cyclical fluctuations in the variance and Kelley skewness of households' idiosyncratic labor income growth, as detailed in Section 3.4 below.

3.1.2 Aggregate Shocks

I assume that aggregate TFP follows $\log(Z_t) = \rho_Z \log(Z_{t-1}) + \varepsilon^Z_t$, with $\rho_Z = 0.9$ and ε^Z_t i.i.d. mean-zero. For computing impulse responses around the steady state, the other properties of ε^Z_t are irrelevant. In the presentation and analysis of the model above, I have assumed that TFP shocks are the only source of aggregate uncertainty. However, to analyze the broader implications of the presence of idiosyncratic labor income risk, I will in the following analysis also consider shocks employed previously in the literature to other parts and parameters of the model, as follows.

1. A one-time *capital quality shock* which shifts the stock of previously invested capital K_t to

[32] Kelley skewness is defined as $\mathbb{S}_K \equiv \frac{(P90-P50)-(P50-P10)}{P90-P10}$. Since the model is calibrated quarterly, whereas individual income data is empirically measured as annual earnings, I employ a simulated method of moments. I simulate labor incomes $W_t\hat{h}_{i,t}l_{i,t}$ for a large panel of households in the steady state of the model and compute the model implied moments from the implied annual earnings dynamics of these households.

$K_t \cdot e^{\varepsilon_t^K}$, and correspondingly affects the returns to capital in t, with ε_t^K again i.i.d. mean-zero. A negative realization of ε_t^K is meant as a simple way to introduce exogenous variation in the value of installed capital, coming from economic obsolescence, as has been widely considered in structural models since the Global Financial Crisis (e.g., see Gertler and Kiyotaki, 2011).

2. A *risk shock* to the standard deviation σ_ω of the firm-specific shocks $\omega_{j,t}$, worsening the CSV problem. Such a shock has been suggested by Christiano et al. (2014) to be a key driver of U.S. aggregate business cycle fluctuations in the BGG framework. I consider an AR(1) process for $\sigma_{\omega,t}$ in logs: $\log(\sigma_{\omega,t+1}) = (1-\rho_\omega)\log(\bar{\sigma}_\omega) + \rho_\omega \log(\sigma_{\omega,t}) + \varepsilon_t^\omega$, with $\bar{\sigma}_\omega = 0.28$ calibrated in steady state, ε_t^ω is i.i.d. mean-zero, revealed in t. I employ $\rho_\omega = 0.9$, as the estimation by Christiano et al. (2014) suggests the shock is highly persistent.

3. A shock to the capital share ν in the economy's aggregate Cobb-Douglas production function. As, for example, suggested by Young (2004), such shocks could explain a significant share of U.S. GDP volatility. I consider an AR(1) process for ν_t in logs: $\log(\nu_t) = (1-\rho_\nu)\log(\bar{\nu}) + \rho_\nu \log(\nu_{t-1}) + \varepsilon_t^\nu/\bar{\nu}$, with $\bar{\nu} = 0.35$ calibrated in steady state, ε_t^ν is i.i.d. mean-zero. A shock of $\varepsilon_t^\nu = 0.01$ constitutes a 1 pp increase in the capital share.

4. A conventional monetary policy shock to the Taylor rule: $R_{t+1}^n = R_{ss}^n (1+\pi_t)^{\phi_\pi} e^{\zeta_t^m}$, where $\zeta_t^m = \rho_m \zeta_{t-1}^m + \varepsilon_t^m$, with ε_t^m i.i.d. mean-zero and $\rho_m = 0.5$.

3.2 Marginal Effects of Risk Shifter Fluctuations

The focus of my analysis below will be contrasting the differences in aggregate dynamics between an economy with a representative household and the benchmark with idiosyncratic labor risk, laid out in Section 2. As discussed therein, the *only* reason these two economies' aggregate dynamics differ from each other is due to fluctuations in the risk shifter S_t. Because of this, it is insightful for the analysis to follow, to first study the behavior of the benchmark economy in response to exogenously introduced fluctuations in S_t. Understanding the economy's responses to changes in the risk shifter allows to more easily interpret the differences in the dynamic behavior of economies where S_t fluctuates in response to aggregate shocks (i.e., the heterogeneous household benchmark), as compared to where it does not (i.e., the representative household special case).

I will introduce exogenous fluctuations in S_t by shocking the underlying distribution of idiosyncratic labor risk $\mathcal{F}_t$. More specifically, I will assume that the shape parameter, $\alpha_{\eta,t}$, follows an AR(1) process: $\alpha_{\eta,t} = (1-\rho_\eta)\alpha_{\eta,ss} + \rho_\eta \alpha_{\eta,t-1} + \varepsilon_t^\eta$, with ε_t^η i.i.d. mean-zero. A negative shock to $\alpha_{\eta,t}$ increases the left-skewness (and variance) of the $\log(\eta_{i,t})$ distribution, increasing S_t due to precautionary reasons, all else equal.[33]

[33] To be precise, the variance and skewness (i.e., the third standardized moment) of a skew normal variable $\log(\eta_{i,t})$ with parameters $(\mu_{\eta,t}, \sigma_{\eta,t}, \alpha_{\eta,t})$ are given by $\mathbb{V}_{\mathcal{F}_t}[\log(\eta_{i,t})] = \sigma_{\eta,t}^2\left(1 - \frac{2}{\pi}\delta_{\eta,t}^2\right)$ and $\mathbb{S}_{\mathcal{F}_t}[\log(\eta_{i,t})] = \frac{4-\pi}{2}\frac{(\delta_{\eta,t}\sqrt{2/\pi})^3}{(1-2\delta_{\eta,t}^2/\pi)^{3/2}}$, respectively, where $\delta_{\eta,t} \equiv \alpha_{\eta,t}/\sqrt{1+\alpha_{\eta,t}^2}$.

Figure 1 depicts the impulse responses of key balance sheet and real variables to a fully transitory 0.1 unit negative shock of $\alpha_{\eta,t}$ over 20 quarters, both for the model versions with flexible prices and with nominal rigidities. As the shock is transitory ($\rho_\eta = 0$), there is no exogenous shock to the households' precautionary saving incentives discussed in Section 2.3.2. In both models, the sudden and fully transitory drop in $\alpha_{\eta,t}$ leads to a drop in the skewness of the $\log(\eta_{i,t})$ distribution, and a corresponding spike in the risk shifter S_t, of about 0.25%. This pushes up the households' SDF M_t and induces entrepreneurs to pay out a higher share of their capital returns as per the optimal risk sharing condition (28). This is evident in the spike of more than 0.2% (annualized) in the return to the lenders $R^L_t = R^D_t$. This effective transfer of wealth from the entrepreneur to the households directly decreases the firms'/entrepreneur's net worth N_t and correspondingly increases the leverage κ_t at which they operate going forward. The (precautionary) transfer of wealth to the households increases aggregate household consumption C_t. The worsening of the firms' financial conditions increases the external finance premium $R^K_{t+1} - R^L_{t+1}$ and reduces investment by more than 0.1%, which reduces capital prices and the realized return to capital R^K_t. The wealth effect in households' labor supply ends up reducing output. The key difference between the flexible and sticky price models is the fact that the reduction in investment induces a destabilizing aggregate demand effect in the latter, reducing inflation, increasing markups, and generating a drop in output almost twice as large.

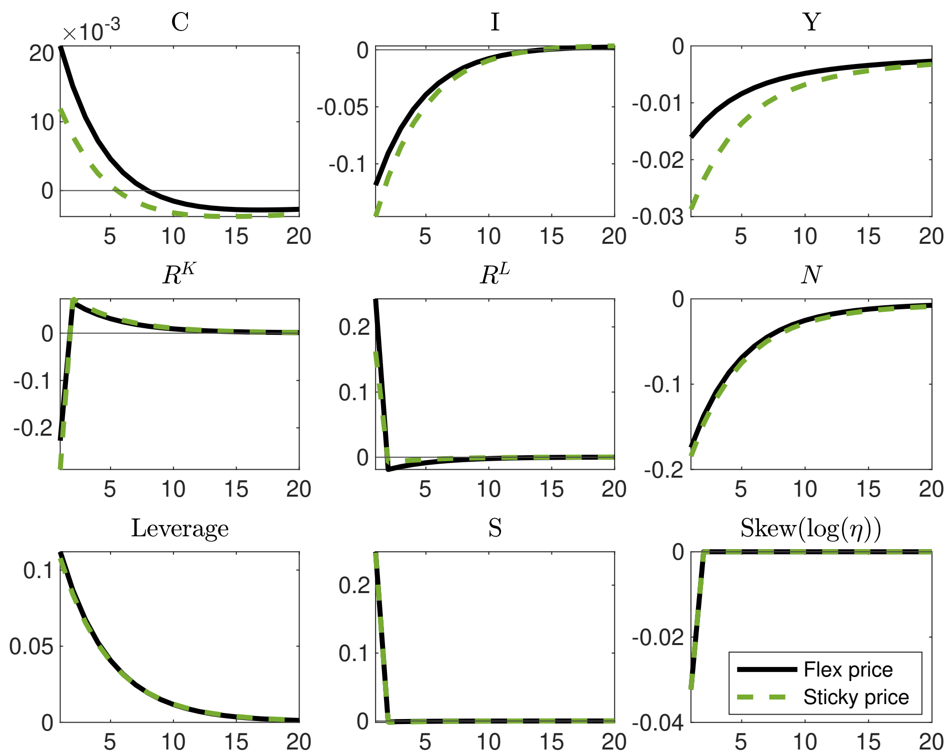


Figure 1: Impulse responses to transitory drop in $\alpha_{\eta,t}$, in flexible and sticky price models

Notes: 0.1 unit fully transitory drop in $\alpha_{\eta,t}$, with $\rho_\eta = 0$; (100×) log-deviations from steady state, returns annualized, Skew(log(η)) as units of (standardized) third moment. Horizontal axis: quarters. Black solid: flexible price model, green dashed: nominal rigidities.

Figure 2 repeats the exercise for a persistent ($\rho_\eta = 0.5$) shock to $\alpha_{\eta,t}$, allowing to illustrate the effects of the precautionary saving mechanism and its strikingly different influence on dynamics in the flexible vs. sticky price economies. In both economies, the persistent increase in S_t, and the implied desire of households to precautionarily save more leads to a notable fall in R^L_{t+j}, for $j \geq 1$. As a result, it becomes relatively easier for the firms/entrepreneur to invest and rebuild their net worth N_{t+j} after the payout to the households. However, in the flexible price economy, the drop in R^L_{t+j} is so large that even though leverage and the external finance premium $R^K_{t+j} - R^L_{t+j}$ have increased, the required equilibrium return to capital R^K_{t+j} also falls, meaning that at impact, the firms actually end up investing *more* than in steady state. And, as is evident from the overall impulse response paths, the variation in household consumption, investment, and aggregate output caused by a persistent shock to S_t is relatively modest, due to the inherent stabilizing effect of the precautionary saving mechanism already discussed in Section 2.3.2. In stark contrast, in the sticky price economy, the (real) cost of borrowing going forward R^L_{t+j} falls by considerably less, inducing an increase in R^K_{t+j} after an increase in the external finance premium (due to the increase in leverage). This means that there is a large, persistent drop in investment, creating a strong negative aggregate demand effect. Moreover, since the real return to households' saving R^L_{t+j} remains high while S_{t+j} is high, consumption must also drop significantly and further reduce aggregate demand. And as a result, inflation falls, markups increase, and aggregate output drops by almost 0.2%.

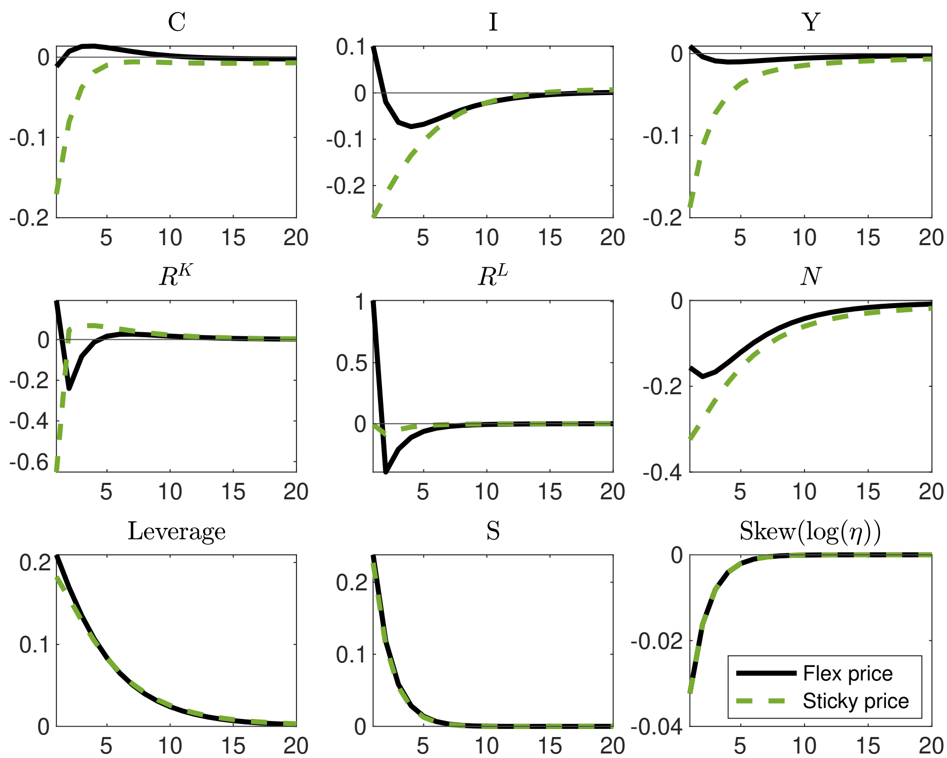


Figure 2: Impulse responses to persistent drop in p_t, in flexible and sticky price models

Notes: 0.1 unit negative shock to $\alpha_{\eta,t}$, with $\rho_\eta = 0.5$; (100×) log-deviations from steady state, returns annualized, Skew(log(η)) as units of (standardized) third moment. Horizontal axis: quarters. Black solid: flexible price model, green dashed: nominal rigidities.

To sum up, in the analysis of aggregate dynamics of the model economy that is to follow, both the endogenous fluctuations in the human wealth share α_t^H (in Section 3.3) and the exogenously introduced cyclicality in the underlying idiosyncratic labor risk distribution $\mathcal{F}_t$ (in Section 3.4) will lead to persistent fluctuations in the risk shifter S_t in the aftermath of the various considered business cycle shocks hitting the economy. The impulse responses seen in Figure 2 suggest that in a world with flexible prices, such fluctuations in the risk shifter do not seem to have significant effects on aggregate outcomes, as the destabilizing net worth effects are significantly dampened by the stabilizing effect of precautionary saving on equilibrium interest rates and investment. And thus, differences across the heterogeneous vs. representative household specifications will be rather minor. Yet, on the contrary, in the specification with sticky prices, the stabilizing effect is instead overshadowed by the aggregate demand effect of precautionary saving, suggesting potentially non-trivial effects of the presence of idiosyncratic labor income risk on aggregate dynamics. In what is to follow, I will therefore exclusively focus on the model specification with nominal rigidities.

3.3 Sharing Aggregate Risk with Acyclical Idiosyncratic Labor Risk

In the following, I will illustrate the dynamics of the economy, and most importantly, the effects of idiosyncratic labor income risk on the degree of risk sharing embedded in the optimal equilibrium contract in response to the various aggregate shocks outlined in Section 3.1.2. To do so, I will compare the dynamics of the heterogeneous household calibration from Section 3.1 to the corresponding representative household benchmark, which sets the variation in $\eta_{i,t}$ to zero (by setting $\sigma_{\eta,t} = \mu_{\eta,t} = 0, \forall t$) and is recalibrated to match all the same targets, except the ones related to idiosyncratic income risk. In this Section, I consider the heterogeneous household specification with an acyclical labor risk process $\mathcal{F}_t = \mathcal{F}$, meaning the parameters of the $\eta_{i,t}$-shock distribution are constant at their steady state calibrated values.

For brevity, I will focus the main analysis on the TFP shock, depicting the analogous impulse responses to the other considered shocks in Appendix A.5 and commenting on them below. Figure 3 presents impulse responses to a 1% negative TFP shock. Focusing first on the representative household case, one sees similar findings as in the existing literature, e.g., the work by CFP. In response to the large drop in capital returns R_t^K of about 2% (annualized) suffered by the firms and the entrepreneur, the optimal contract dictates that the representative household also takes on a significant loss through a fall in $R^L = R_t^D$. Quantitatively speaking, about 61.2% of the relative drop in R_t^K is passed through to R_t^L.[34] The 1% fall in TFP also induces a similarly-sized fall in investment, and a slightly smaller drop in output and household consumption. In the New Keynesian model, negative TFP shocks are inflationary, causing markups to fall and aggregate labor supply to increase, dampening the drop in output. Since the firms do not share 100% of the capital losses with the lenders, leverage, which is the inverse of the entrepreneur's wealth share in economy-wide financial wealth held in productive capital goes up slightly, while their net worth N_t

[34] Note that I find a slightly lower pass-through to R_t^L than CFP in response to TFP shocks because I am considering a slightly lower persistence of $\rho_Z = 0.90$, instead of 0.95.

falls. The representative household's risk shifter S_t is trivially constant at 1.

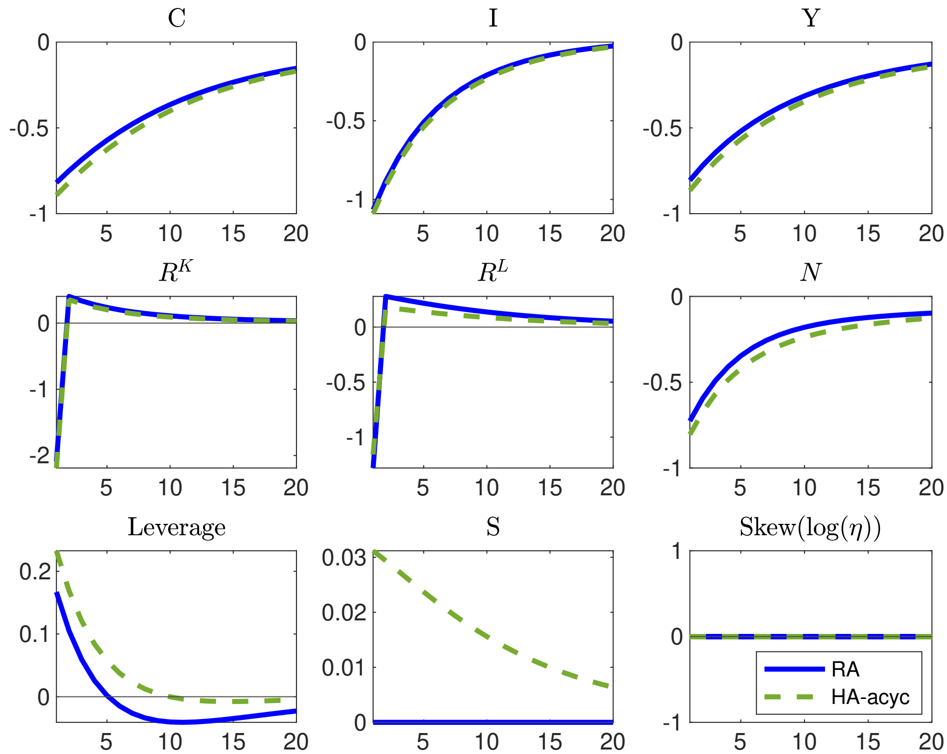


Figure 3: Impulse responses to TFP shock in representative and heterogeneous household models with sticky prices

Notes: 1% negative TFP shock $\varepsilon_t^Z = -0.01$; (100×) log-deviations from steady state, returns annualized, Skew(log(η)) as units of (standardized) third moment. Horizontal axis: quarters. Blue solid: representative agent specification (*RA*), green dashed: heterogeneous agent benchmark (*HA-acyc*).

Moving to the heterogeneous household case, the relevance of idiosyncratic labor risk can be gauged most directly through the equilibrium fluctuations in the risk shifter. In response to the negative TFP shock, the share of households' human wealth share α_t^H ends up rising, and this increases the exposure of their consumption to idiosyncratic risk and pushes up the risk shifter. As can then qualitatively be expected based on the preceding discussion in Section 3.2, the increase in S_t at shock impact t induces the optimal contract to pay households slightly more and thus concentrate more of the TFP shock risk on the entrepreneur's balance sheet – evident in the fact that R_t^L falls slightly less, while R_t^K falls slightly more than in the representative household case. This then leads to a relatively larger jump in entrepreneurial leverage. The fact that the increase in S_t is persistent, further introduces the precautionary saving mechanism which destabilizes the economy, leading to a larger drop in household consumption, investment, and output. However, quantitatively speaking, the effects of acyclical idiosyncratic labor income risk are small. Since the persistent TFP shock moves human and financial wealth in the economy to a similar degree, the induced change in households' human wealth share α_t^H is relatively small, leading to an increase in the risk shifter of about only 0.03%, in turn causing output to fall by about 0.05% more in the heterogeneous household case (-0.86% vs. -0.71%). In this case, about 52.9% of the fall in R_t^K is passed through to R_t^L, as also reported in Table 1 below.

Table 1: Share of aggregate risk in R_t^K passed through to R_t^L, in percentages

Shock	RA	HA-acyc	HA-ccyc
ε_t^Z	61.2	52.9	36.7
ε_t^K	133.8	129.6	73.6
ε_t^ω	152.8	148.3	33.0
ε_t^ν	64.9	49.8	25.5
ε_t^m	114.4	118.2	103.4

Notes: Share of aggregate risk in R_t^K passed through to R_t^L, computed as the ratio of $\hat{R}_t^L/\hat{R}_t^K$ in response to shock ε_t^X, reported in percentages. *RA* – representative agent specification, *HA-acyc* – heterogeneous agent benchmark with acyclical labor risk, *HA-ccyc* – heterogeneous agent model with counter-cyclical labor risk (as quantified in Section 3.4), all with nominal rigidities.

Figures A.1–A.4 in Appendix A.5 and Table 1 report the corresponding results for the remaining four types of business cycle shocks discussed in Section 3.1.2. For all shocks except the monetary one, a drop in output is accompanied by an persistent increase in the risk shifter, leading to a slight amplification of aggregate fluctuations through the mechanisms discussed above. However, as in the case of the TFP shock, the (endogenous) fluctuations in the households' human wealth share α_t^H generated by all of these shocks are relatively small. And as a result, the fluctuations in the risk shifter and any differences between the dynamics of the representative household and heterogeneous household acyclical labor risk cases are quantitatively small. As one might expect, the largest fluctuations in the risk shifter and implied differences in the representative and heterogeneous household cases appear for a negative shock to the capital share ν_t. As this shock directly shifts income shares from capital towards labor, reducing financial and increasing human wealth, it increases households' exposure to the idiosyncratic risk embedded in human capital. Quantitatively speaking, after a drop in the capital share, the heterogeneous household specification features an immediate fall in output that is about 1/4 stronger than the representative household case.

3.4 Sharing Aggregate Risk with Countercyclical Idiosyncratic Labor Risk

Finally, to illustrate the effects of *counter*cyclical idiosyncratic labor income risk on aggregate risk sharing and macroeconomic dynamics in this model, I will now consider a model specification where I assume that the aggregate business cycle shocks hitting the economy also affect the $\mathcal{F}_t$ distribution. More specifically, expanding on Section 3.2, I will now assume that *both* of the free parameters of the idiosyncratic labor income risk distribution, $\sigma_{\eta,t}$ and $\alpha_{\eta,t}$, follow separate AR(1) processes, with "exposure" to each of the business cycle shocks discussed in Section 3.1.2:

$$\begin{bmatrix} \log(\sigma_{\eta,t}) \\ \alpha_{\eta,t} \end{bmatrix} = (1-\rho_\eta) \begin{bmatrix} \log(\sigma_{\eta,ss}) \\ \alpha_{\eta,ss} \end{bmatrix} + \rho_\eta \begin{bmatrix} \log(\sigma_{\eta,t-1}) \\ \alpha_{\eta,t-1} \end{bmatrix} + \begin{bmatrix} \Upsilon'_\sigma \\ \Upsilon'_\alpha \end{bmatrix} \boldsymbol{\varepsilon}_t, \quad \text{where} \tag{35}$$

where $\boldsymbol{\varepsilon}_t \equiv (\varepsilon_t^Z, \varepsilon_t^K, \varepsilon_t^\omega, \varepsilon_t^\nu, \varepsilon_t^m)$, and $\Upsilon_\sigma \equiv (\Upsilon_{\sigma Z}, \Upsilon_{\sigma K}, \Upsilon_{\sigma\omega}, \Upsilon_{\sigma\nu}, \Upsilon_{\sigma m})$ and Υ_α, defined analogously as Υ_σ, are 5×1 vectors of coefficients that capture the exposure to each of these shocks, respectively.

I calibrate the parameters $(\rho_\eta, \Upsilon'_\sigma, \Upsilon'_\alpha)$ in (35) so as to replicate empirically observed business cycle fluctuations in the variance and the Kelley skewness of the one-year log annual labor earnings growth of U.S. males, exactly as I did for calibrating the idiosyncratic labor income risk process in steady state. I employ the cyclical moments implied by the IRS data used by Guvenen et al. (2014). More specifically, I identify the parameters $(\rho_\eta, \Upsilon'_\sigma, \Upsilon'_\alpha)$ by matching the empirically observed autocorrelation of the Kelley skewness of 0.21 in the annual empirical timeseries over 1979–2011, and the empirically observed business cycle comovement of both the variance and Kelley skewness with aggregate U.S. output. To do so, I separately regress the variance and the Kelley skewness series from the Guvenen et al. (2014) database on (linearly detrended) contemporaneous output year-by-year and recover the OLS coefficients of -0.06 and 0.993, respectively, capturing the weak countercyclicality in the variance and the strong procyclicality of the skewness of idiosyncratic labor income risk emphasized by Guvenen et al. (2014). And then, separately and analogously for each individual shock $\varepsilon_t^X \in \boldsymbol{\varepsilon}_t$, I generate model simulated data conditional on *only* ε_t^X hitting the economy, and identify the corresponding entries $\Upsilon_{\sigma X} \in \Upsilon_\sigma$ and $\Upsilon_{\alpha X} \in \Upsilon_\alpha$ by ensuring that the model-simulated annualized data yields the same OLS coefficient when regressing the the variance and Kelley skewness on aggregate output.[35] While surely, the implied comovement between idiosyncratic labor risk and output would very likely differ across various structural shocks in reality, and the (unconditional) empirical comovement between variance and the Kelley skewness with output of -0.06 and 0.993 arises as a combination of all conceivable shocks operating simultaneously, I view this as a clean and straightforward exercise to illustrate the model's quantitative properties without requiring to delve into empirical identification of the separate structural shocks in $\boldsymbol{\varepsilon}_t$ or a full-blown dynamic estimation of the structural model.

The calibrated values for Υ_σ and Υ_α imply that for all the types of considered structural shocks, a contractionary shock requires a fall in $\alpha_{\eta,t}$, which induces left-skewness and increases the variance of the idiosyncratic (log) labor shock distribution. In order to undo some of this increase in the variance and match the relatively weak negative empirical comovement of the variance with output over the cycle, $\sigma_{\eta,t}$ must also fall with contractionary shocks. In the specific case of the TFP shock, the resulting coefficients are $\rho_\eta = 0.66$, $\Upsilon_{\sigma Z} = 12.41$ and $\Upsilon_{\alpha Z} = 89.57$.[36]

Figure 4 depicts the implied impulse responses to the TFP shock, alongside the representative and heterogeneous household acyclical labor risk cases already reported in Figure 3. Now, when the negative 1% TFP shock is realized, the implied decrease in $\alpha_{\eta,t}$ widens the left tail of the idiosyncratic shock distribution and as a result increases the risk shifter by more than 0.5% at im-

[35] To be precise, I also recalibrate ρ_η conditional on each type of shock, so (35) contains an abuse of notation when it comes to the persistence of the $\sigma_{\eta,t}$ and $\alpha_{\eta,t}$ processes. However, since the autocorrelation target is identical across all the five shocks considered, the different calibrated values of ρ_η are very similar, and results would change little if ρ_η were to be fixed at one of the five calibrated values for all the other shocks as well.

[36] The full calibrated vectors take the values $\Upsilon_\sigma = (12.41, 9.84, -9.38, 9.02, -9.81)$ and $\Upsilon_\alpha = (89.57, 84.65, -147.15, 104.17, -66.04)$. And, following Footnote 35 the corresponding recalibrated values of ρ_η are 0.66, 0.66, 0.63, 0.64, and 0.64, respectively.

pact, inducing the firms to pay out a larger share to the households, increasing R_t^L, all else equal. The firms'/entrepreneur's net worth and leverage thus exhibit significantly more volatile dynamics. Following the discussion in Section 3.2, this worsening of the firms'/entrepreneur's financial conditions alongside the destabilizing precautionary saving mechanism introduced by the persistent increase in S_t gives rise to significant amplification of the effect of the TFP shock, reminiscent of the conventional financial accelerator dynamics of BGG. The privately optimal contract does still feature the households' R_t^L taking on some of the drop in capital returns R_t^K, ∼36.7% to be precise (see Table 1), and thus the contract does not look exactly like the non-indexing contract imposed by BGG. Yet the combination of precautionary saving and nominal rigidities in this environment generates such strong amplification and a large fall in R_t^K, that this off-loading of part of the risk is not enough to protect the firms/entrepreneur from a collapse in their (relative) financial position.

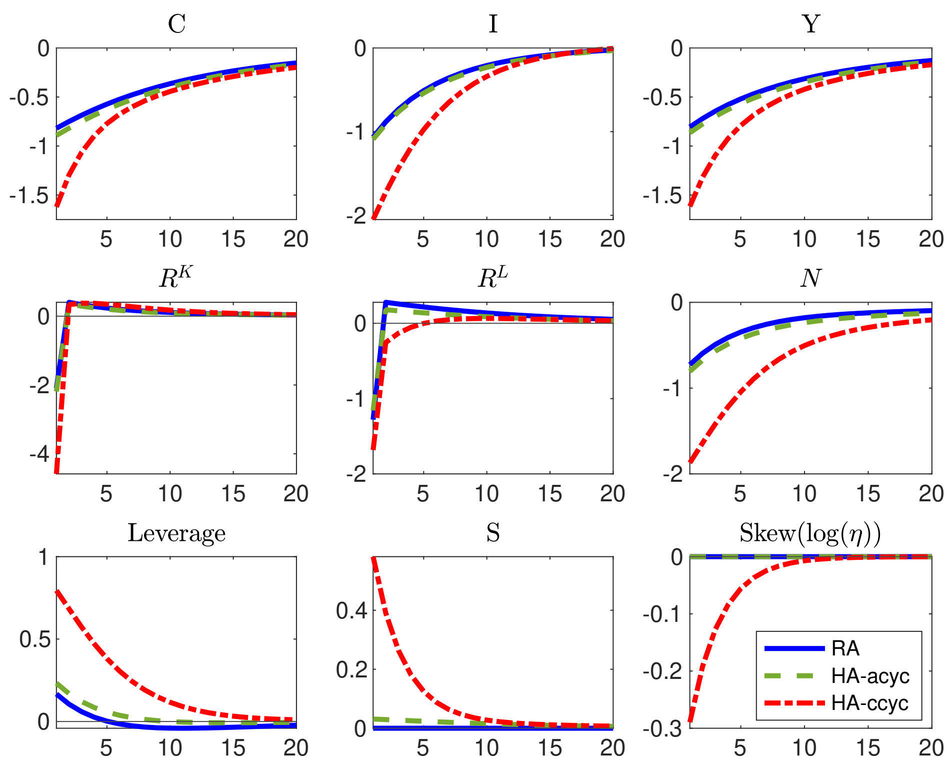


Figure 4: Impulse responses to TFP shock in representative and heterogeneous household (acyclical and countercyclical risk) models with sticky prices

Notes: 1% negative TFP shock $\varepsilon_t^Z = -0.01$; (100×) log-deviations from steady state, returns annualized, Skew(log(η)) as units of (standardized) third moment. Horizontal axis: quarters. Blue solid: representative agent specification (*RA*), green dashed: heterogeneous agent benchmark with acyclical labor risk (*HA-acyc*), red dash-dotted: heterogeneous agent benchmark with countercyclical labor risk (*HA-ccyc*).

Figures A.1–A.4 in Appendix A.5 and Table 1 again report the corresponding results for the remaining four types of business cycle shocks. For each shock, the quantification of Υ_σ and Υ_α implies that the recessionary effect of the shock also induces a persistent increase in the risk shifter, considerably amplifying the size and persistence of the recession, as in the case of the negative TFP shock. And while the privately optimal contract still implies households taking on a nontrivial share of the fall in capital returns induced by these shocks, as seen in Table 1, this is in most part still enough to set in motion countercyclical fluctuations in the firms'/entrepreneur's wealth share (leverage), as implied by the non-indexing contracts assumed by BGG and the existing financial accelerator literature.

Finally, given the relevance of households' precautionary saving incentives influencing the aggregate dynamics seen above, and the fact that they could lead to amplification even in a sticky-price economy without firm financing frictions, one might raise the question whether the amplification caused by countercyclical labor risk observed in Figure 4 indeed arises due to the effects on the privately optimal contract, the implied worsening of firms' financial conditions, and the financial accelerator dynamics set in motion, and not simply due to the conventional aggregate demand effects (e.g., see Challe et al., 2017, and the discussion in Section 2.3.2). To explore this issue, I compare the effects of the TFP shock on aggregate investment seen in Figure 4 for the BGG-CFP model to a version of the model in which the CSV friction is absent in firm financing.[37]

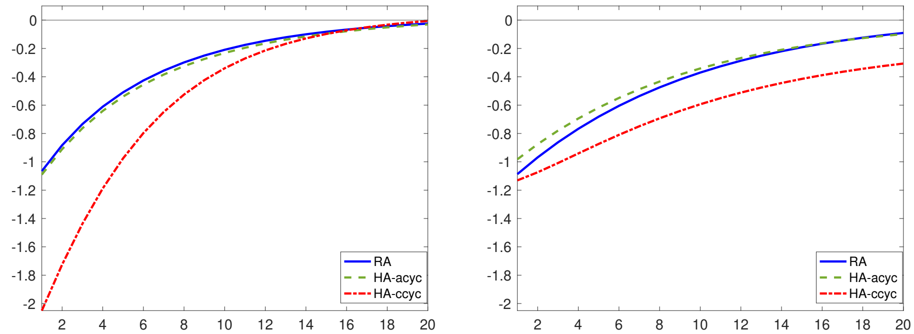
The results can be seen in Figure 5. Panel 5a repeats the finding on the considerable amplification of aggregate investment fluctuations seen already in Figure 4. Yet Panel 5b shows that the amplification arising purely from countercyclical precautionary savings is considerably weaker, although still visible and persistent. In this specification without the CSV friction, the relative financial position of the firms'/entrepreneur is not directly relevant for aggregate investment. While the sudden increase in households' idiosyncratic labor risk after a negative TFP shock induces the firms to pay out a relatively larger return R_t^L to the households, the implied fall in their net worth does not cause a widening of the spread $R_{t+1}^K - R_{t+1}^L$, which remains zero (in expectation).[38] And therefore, the increased households' savings can flow frictionlessly back into aggregate investment without setting in motion the contractionary financial accelerator dynamics. Convergence of investment back to steady state is slowed down because the implied increase in household wealth and initial drop in consumption keep the households' wealth share high for considerably longer than the increase in the risk shifter. As a result, household consumption is relatively higher at longer horizons in the case of countercyclical idiosyncratic risk, decreasing labor supply through the wealth effect and decreasing the economy's resources available for investment.

4 Conclusion

Privately optimal aggregate risk sharing is affected if one party is exposed to uninsurable idiosyncratic labor income risk. This paper exemplifies this idea in a reformulation of the workhorse

[37]To be precise, as this "no CSV friction model", I consider the special case of the benchmark model studied in this paper with zero monitoring costs, $\mu = 0$. To ensure the existence of a non-trivial steady state of this economy with both the continuum of households and the representative entrepreneur, I recalibrate β and β_e so that the equilibrium annual risk free rate remains 4%, i.e., $M_{ss} = \beta_e = 1/R_{ss}^K = 1/R_{ss}^L = 0.99$, while the entrepreneur's wealth share is still the inverse of the previously calibrated steady state leverage $\kappa_{ss} = 2$. I also recalibrate ϕ so that the households' labor supply elasticity around the steady state remains unchanged. For brevity, I do not to recalibrate the underlying steady state labor risk distribution parameters ($\mu_{\eta,ss}$, $\sigma_{\eta,ss}$, $\alpha_{\eta,ss}$), nor the "exposure" parameters ρ_η, Υ_σ, or Υ_α.

[38]More precisely, in the $\mu = 0$ case, we have $\mathbb{E}[M_{t+1}(R_{t+1}^K - R_{t+1}^L)] = 0$, while $M_{t+1} = M_{t+1}^e$ at all times.



(a) Benchmark CSV friction model (b) No CSV friction model

Figure 5: Impulse responses of aggregate investment to TFP shock in benchmark and no CSV friction models with sticky prices

Notes: 1% negative TFP shock $\varepsilon_t^Z = -0.01$ on I_t in benchmark BGG-CFP model (Panel a) and in special case without firm financing frictions, $\mu = 0$ (Panel b); (100×) log-deviations from steady state. Horizontal axis: quarters. Blue solid: representative agent specification (*RA*), green dashed: heterogeneous agent benchmark with acyclical labor risk (*HA-acyc*), red dash-dotted: heterogeneous agent benchmark with countercyclical labor risk (*HA-ccyc*).

Bernanke et al. (1999) model with financial frictions and analyzes its quantitative relevance for the implied aggregate dynamics by proposing a tractable model of household heterogeneity. In response to a variety of conventional business cycle shocks, households exposed to uninsurable idiosyncratic labor income risk are willing to take on less aggregate risk, leading to stronger cyclicality in leveraged borrowers' financial conditions and amplification of aggregate dynamics. While the quantitative effects are relatively small when the idiosyncratic labor risk is acyclical, empirically plausible countercyclicality in idiosyncratic household labor risk can lead to considerable amplification reminiscent of the financial accelerator mechanism, even if agents are allowed to write privately optimal contracts to share aggregate risk.

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Appendix

A Model Appendix

A.1 Additional Derivations on Households' Optimal Behavior

This Appendix provides the derivations behind the results on households' optimal behavior discussed in Section 2.2.1.

Imposing that the non-negativity constraints on $(d_{i,t+1}, h_{i,t+1}, \iota_{i,t+1})$ will not bind in equilibrium (see Footnote 19), the household's optimal choices solving (6)–(7) must satisfy the first order necessary conditions (Euler equations):[39]

$$1 = \mathbb{E}_t\left[\beta\theta\left(\frac{c_{i,t+1}}{c_{i,t}}\right)^{-1} R^X_{t+1}\right], \qquad \text{for } X \in \{D, I\} \tag{A.2}$$

$$1 = \mathbb{E}_t\left[\beta\theta\left(\frac{c_{i,t+1}}{c_{i,t}}\right)^{-1} R^H_{t+1}\eta_{i,t+1}\right] - \beta\theta\frac{\phi}{(c_{i,t})^{-1}\cdot P^H_t h_{i,t+1}} \tag{A.3}$$

where R^H_{t+1}, and R^I_{t+1} are as defined in (18). Following the discussion in Section 2.2.1, I will conjecture that households' optimal policies take the form: $c_t(\mathbf{s}_{i,t}) = \tilde{c}_t \cdot \hat{a}_{i,t}$, $d_{t+1}(\mathbf{s}_{i,t}) = \tilde{d}_t \cdot \hat{a}_{i,t}$, $P^H_t \cdot h_{t+1}(\mathbf{s}_{i,t}) = \tilde{h}_t \cdot \hat{a}_{i,t}$, and by implication, imposing the budget constraint (7) at equality, $P^I_t \cdot \iota_{t+1}(\mathbf{s}_{i,t}) = [1-(1+\phi)\tilde{c}_t - \tilde{d}_t - \tilde{h}_t]\cdot\hat{a}_{i,t} \equiv \tilde{\iota}_t \cdot \hat{a}_{i,t}$, with the unknowns $\tilde{c}_t$, $\tilde{d}_t$, $\tilde{h}_t$, and $\tilde{\iota}_t$ determined below. Also, by the budget constraint, we must have $a_{t+1}(\mathbf{s}_{i,t}) = [1-(1+\phi)\tilde{c}_t]\cdot\hat{a}_{i,t} \equiv \tilde{a}_t \cdot \hat{a}_{i,t}$. Under these conjectured policies, the implied law of motion for household i's wealth $\hat{a}_{i,t}$ becomes:

$$\begin{aligned}
\hat{a}_{i,t} &\equiv [W_t + P^H_t]h_{i,t}\eta_{i,t} + R^D_t d_{i,t} + (\Pi^I_t + P^I_t)\iota_{i,t} \\
&= \left[R^H_t \eta_{i,t}\cdot\frac{P^H_{t-1}h_{i,t}}{a_{i,t}} + R^D_t\cdot\frac{d_{i,t}}{a_{i,t}} + R^I_t\cdot\frac{P^I_{t-1}\iota_{i,t}}{a_{i,t}}\right]\cdot a_{i,t} \\
&= \hat{R}_{i,t}\cdot\tilde{a}_{t-1}\cdot\hat{a}_{i,t-1}
\end{aligned}$$

$$\text{Or,} \qquad \log(\hat{a}_{i,t}) = \log(\hat{a}_{i,0}) + \sum_{j=0}^{t-1}\left[\log(\hat{R}_{i,t-j}) + \log(\tilde{a}_{t-j-1})\right] \tag{A.4}$$

$$\begin{aligned}
\text{where} \qquad \hat{R}_{i,t} &\equiv R^H_t\eta_{i,t}\cdot\frac{P^H_{t-1}h_{i,t}}{a_{i,t}} + R^D_t\cdot\frac{d_{i,t}}{a_{i,t}} + R^I_t\cdot\frac{P^I_{t-1}\iota_{i,t}}{a_{i,t}} \\
&= R^H_t\eta_{i,t}\cdot\frac{\tilde{h}_{t-1}}{\tilde{a}_{t-1}} + R^D_t\cdot\frac{\tilde{d}_{t-1}}{\tilde{a}_{t-1}} + R^I_t\cdot\frac{\tilde{\iota}_{t-1}}{\tilde{a}_{t-1}}
\end{aligned} \tag{A.5}$$

Thus, idiosyncratic shocks to $\eta_{i,t}$ will have permanent effects (through $\hat{R}_{i,t}$) on households' wealth

[39] Note that, using the optimal labor supply condition (5) in $t+1$, the Euler equation for $h_{i,t+1}$ can alternatively be written as

$$1 = \mathbb{E}_t\left[\beta\theta\left(\frac{c_{i,t+1}}{c_{i,t}}\right)^{-1}\frac{W_{t+1}l_{i,t+1} + (1-\delta_H)P^H_{t+1}}{P^H_t}\eta_{i,t+1}\right] \tag{A.1}$$

accumulation, and as a result, their consumption and asset accumulation choices. Moreover, the implied individual consumption growth, conditional on survival in $t+1$, thus behaves as:

$$
\begin{aligned}
c_{i,t+1} = \tilde{c}_{t+1} \cdot \hat{a}_{i,t+1} = \tilde{c}_{t+1} \cdot \hat{R}_{i,t+1} \cdot \tilde{a}_t \cdot \hat{a}_{i,t} = \tilde{c}_{t+1} \cdot \hat{R}_{i,t+1} \cdot \tilde{a}_t \cdot \frac{c_{i,t}}{\tilde{c}_t} \\
\Rightarrow \frac{c_{i,t+1}}{c_{i,t}} = \frac{\tilde{c}_{t+1}}{\tilde{c}_t} \cdot \hat{R}_{i,t+1} \cdot \tilde{a}_t
\end{aligned} \tag{A.6}
$$

Because $\eta_{i,t+1}$ is i.i.d. across households, it must therefore be the case that, under the conjectured policies, the distributions of the "composite return" $\hat{R}_{i,t+1}$, of $\hat{R}_{i,t+1}/\eta_{i,t}$, and of individual consumption growth $\frac{c_{i,t+1}}{c_{i,t}}$, conditional on information in t, are identical across all households $i \in \mathcal{E}_t$.

Plugging these implications into the Euler equations (A.2)–(A.3) and employing the implied budget constraint allows to verify that the conjectured policy functions satisfy the households' first order optimality conditions, with $(\tilde{c}_t, \tilde{d}_t, \tilde{h}_t, \tilde{\imath}_t)$ determined recursively as the solution to the following system of 4 equations (A.7)–(A.9):

$$
(\tilde{c}_t)^{-1} = (1+\phi) + \mathbb{E}_t\left[\beta\theta(\tilde{c}_{t+1} \cdot \hat{R}_{i,t+1})^{-1} R^X_{t+1}\right], \qquad \text{for } X \in \{D, I\} \tag{A.7}
$$

$$
(\tilde{c}_t)^{-1} = (1+\phi) + \mathbb{E}_t\left[\beta\theta(\tilde{c}_{t+1} \cdot \hat{R}_{i,t+1}/\eta_{i,t+1})^{-1} R^H_{t+1}\right] - \beta\theta\frac{\phi[1-(1+\phi)\tilde{c}_t]}{\tilde{h}_t} \tag{A.8}
$$

$$
1 = (1+\phi)\tilde{c}_t + \tilde{d}_t + \tilde{h}_t + \tilde{\imath}_t \tag{A.9}
$$

$$
\text{where} \quad \hat{R}_{i,t+1} = \frac{R^H_{t+1}\eta_{i,t+1} \cdot \tilde{h}_t + R^D_{t+1} \cdot \tilde{d}_t + R^I_{t+1} \cdot \tilde{\imath}_t}{1-(1+\phi)\tilde{c}_t} \tag{A.10}
$$

What is more, one can multiply each of the three Euler equations in (A.7)–(A.8) with the corresponding shares $\tilde{d}_t$, $\tilde{\imath}_t$ and $\tilde{h}_t$, respectively (all known at time t), sum them up and employ (A.9) and (A.10) to get that $\tilde{c}_t$ must satisfy the recursion:

$$
\tilde{c}^{-1}_{t+1} - (1+\phi) + \beta\theta\phi = \beta\theta\mathbb{E}_t[\tilde{c}^{-1}_{t+1}]
$$

which is solved by the constant $\tilde{c}_t = \tilde{c} \equiv \frac{1-\beta\theta}{1+\phi(1-\beta\theta)}, \forall t$. Plugging this into (A.7)–(A.10), we have that $(\tilde{d}_t, \tilde{h}_t, \tilde{\imath}_t)$ is determined recursively as the solution to the following system of three equations:

$$
\begin{aligned}
1 &= [1+\phi(1-\beta\theta)] \cdot \mathbb{E}_t\left[R^X_{t+1}/\hat{R}_{i,t+1}\right], \qquad \text{for } X \in \{D, I\} \\
1 + \frac{\beta\theta\phi(1-\beta\theta)}{1+\phi(1-\beta\theta)} \cdot \frac{1}{\tilde{h}_t} &= [1+\phi(1-\beta\theta)] \cdot \mathbb{E}_t\left[R^H_{t+1}\eta_{i,t+1}/\hat{R}_{i,t+1}\right] \\
\text{where} \quad \hat{R}_{i,t+1} &= \frac{R^H_{t+1}\eta_{i,t+1} \cdot \tilde{h}_t + R^D_{t+1} \cdot \tilde{d}_t + R^I_{t+1} \cdot \tilde{\imath}_t}{\beta\theta/[1+\phi(1-\beta\theta)]}
\end{aligned}
$$

Finally, note that aggregating the labor supply condition (5) across $i \in \mathcal{E}_t$ yields:

$$
\phi C_t = W_t(H_t - L_t) \tag{A.11}
$$

where $C_t \equiv \int_{i\in\mathcal{E}_t} c_{i,t}di$ and I have used the definition of aggregate effective labor supply $L_t = \int_{i\in\mathcal{E}_t} l_{i,t}h_{i,t}\eta_{i,t}di$, and the fact that $\int_{i\in\mathcal{E}_t} h_{i,t}\eta_{i,t}di = \left(\int_{i\in\mathcal{E}_t} h_{i,t}di\right)\cdot\left(\int_{i\in\mathcal{E}_t}\eta_{i,t}di\right) = H_t$ follows by the draws of $\eta_{i,t}$ being mean one, independent of previously accumulated $h_{i,t}$, and applying a law of large numbers.

Having established the properties of individual household behavior that yield tractability, it is helpful to finally relate individual household behavior to aggregate household consumption. First, note that aggregate consumption in t, denoted C_t is naturally given by:

$$C_t = \int_{i\in\mathcal{E}_t} c_{i,t}di = \tilde{c}_t\cdot\int_{i\in\mathcal{E}_t}\hat{a}_{i,t}di = \tilde{c}_t\cdot\left(\int_{i\in\mathcal{E}_t^S}\hat{a}_{i,t}di + \int_{i\in\mathcal{E}_t^{NB}}\hat{a}_{i,t}di\right)$$

where $\mathcal{E}_t^S \equiv \mathcal{E}_t\cap\mathcal{E}_{t-1}$ is the set of surviving households between $t-1$ and t, and $\mathcal{E}_t^{NB}\equiv\mathcal{E}_t\setminus\mathcal{E}_{t-1}$ the set of newborns in t. Recall that the financial assets holdings of households dying between $t-1$ and t get distributed evenly across the newborns $\mathcal{E}_t^{NB}$ and the newborns each have $\bar{H}$ units of human capital. Since the set of dying agents is a random draw from $\mathcal{E}_{t-1}$, by a law of large numbers, it must be that also $\int_{i\in\mathcal{E}_{t-1}\setminus\mathcal{E}_t} h_{i,t}di = (1-\theta)\bar{H}$, i.e., the average dying household also leaves $\bar{H}$ units of human capital behind. All of this is to simply say that the total wealth of newborns in t must be equal to the total wealth (with identical wealth shares across the different assets) that dying agents whom they replace would have had if they had survived:

$$\int_{i\in\mathcal{E}_t^{NB}}\hat{a}_{i,t}di = \int_{i\in\mathcal{E}_t^{NB}}\hat{R}_{i,t}a_{i,t}di = \hat{R}_t\cdot\int_{i\in\mathcal{E}_t^{NB}}a_{i,t}di = \hat{R}_t\cdot\int_{i\in\mathcal{E}_{t-1}\setminus\mathcal{E}_t}a_{i,t}di \tag{A.12}$$

$$\text{where}\qquad \hat{R}_t \equiv R_t^H\cdot\frac{\tilde{h}_{t-1}}{\tilde{a}_{t-1}} + R_t^D\cdot\frac{\tilde{d}_{t-1}}{\tilde{a}_{t-1}} + R_t^I\cdot\frac{\tilde{\iota}_{t-1}}{\tilde{a}_{t-1}} \tag{A.13}$$

and recall that $\tilde{a}_{t-1} = 1-(1+\phi)\tilde{c}_{t-1}$. The second equality in (A.12) follows because the realization of $\eta_{i,t}$, and thus of $\hat{R}_{i,t}$ is independent of $a_{i,t}$, and by applying a law of large numbers. We can then apply it also for the surviving agents $i\in\mathcal{E}_t^S$ to write:

$$\begin{aligned}
C_t &= \tilde{c}_t\cdot\hat{R}_t\cdot\left(\int_{i\in\mathcal{E}_t^S}a_{i,t}di + \int_{i\in\mathcal{E}_t^{NB}}a_{i,t}di\right) = \tilde{c}_t\cdot\hat{R}_t\cdot\left(\int_{i\in\mathcal{E}_t^S}a_{i,t}di + \int_{i\in\mathcal{E}_{t-1}\setminus\mathcal{E}_t}a_{i,t}di\right)\\
&= \tilde{c}_t\cdot\hat{R}_t\cdot\int_{i\in\mathcal{E}_{t-1}}a_{i,t}di = \tilde{c}_t\cdot\hat{R}_t\cdot\tilde{a}_{t-1}\cdot\int_{i\in\mathcal{E}_{t-1}}\hat{a}_{i,t-1}di\\
&= \tilde{c}_t\cdot\hat{R}_t\cdot\tilde{a}_{t-1}\cdot\frac{1}{\tilde{c}_{t-1}}\cdot\int_{i\in\mathcal{E}_{t-1}}c_{i,t-1}di = \frac{\tilde{c}_t}{\tilde{c}_{t-1}}\cdot\hat{R}_t\cdot\tilde{a}_{t-1}\cdot C_t
\end{aligned}$$

Combining this with (A.6) allows to simply relate individual consumption growth, conditional on survival, and aggregate consumption growth as:

$$\frac{c_{i,t+1}}{c_{i,t}} = \frac{\hat{R}_{i,t+1}}{\hat{R}_{t+1}}\cdot\frac{C_{t+1}}{C_t}$$

Plugging this into the household's Euler equations (A.2) and applying the law of iterated expectations allows to explicitly illustrate that the heterogeneous household Euler equations feature a wedge relative to those implied by aggregate consumption dynamics:

$$1 = \mathbb{E}_t \left[\beta\theta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{\hat{R}_{i,t+1}}{\hat{R}_{t+1}} \right)^{-1} \cdot R^X_{t+1} \right] \Longrightarrow 1 = \mathbb{E}_t \left[\beta\theta \left(\frac{C_{t+1}}{C_t} \right)^{-1} S_{t+1} \cdot R^X_{t+1} \right] \tag{A.14}$$

$$\text{where} \qquad S_{t+1} \equiv \mathbb{E}_{\mathcal{F}_{t+1}} \left[\left(\frac{\hat{R}_{i,t+1}}{\hat{R}_{t+1}} \right)^{-1} \right] \tag{A.15}$$

I have isolated S_{t+1} by applying the law of iterated expectations, conditioning on all information that the households have observed up to and including in period t, plus the information on the realization of the *aggregate* state in $t+1$, but not the individual draws of $\{\eta_{i,t+1}\}_{i \in \mathcal{E}_{t+1}}$ (see Footnote 5 for the definition of $\mathbb{E}_{\mathcal{F}_t}[\cdot]$).

To elaborate further on the behavior of S_{t+1}, let us, for notational brevity, reformulate the definition of $\hat{R}_{i,t+1}$ (and $\hat{R}_{t+1}$ analogously), as

$$\hat{R}_{i,t+1} = \frac{\tilde{h}_t}{\tilde{a}_t} \cdot R^H_{t+1} \eta_{i,t+1} + \left(1 - \frac{\tilde{h}_t}{\tilde{a}_t} \right) \cdot R^{\sim H}_{t+1} \tag{A.16}$$

$$\text{where} \qquad R^{\sim H}_{t+1} \equiv R^D_{t+1} \cdot \frac{\tilde{d}_t}{\tilde{a}_t - \tilde{h}_t} + R^I_{t+1} \cdot \frac{\tilde{\iota}_t}{\tilde{a}_t - \tilde{h}_t} \tag{A.17}$$

That is, the realized individual return to wealth $\hat{R}_{i,t+1}$ is the natural weighted average of the (individual-specific) return to human capital $R^H_{t+1}\eta_{i,t+1}$ and the "composite return" to the remaining wealth portfolio, denoted $R^{\sim H}_{t+1}$, all weighted by the share of the corresponding assets in the total wealth portfolio, as chosen in t. This implies that S_{t+1} can further be rewritten as:

$$S_{t+1} = \mathbb{E}_{\mathcal{F}_{t+1}} \left[\left(\frac{1 + \frac{\tilde{h}_t}{\tilde{a}_t} \left(\frac{R^H_{t+1}}{R^{\sim H}_{t+1}} \eta_{i,t+1} - 1 \right)}{1 + \frac{\tilde{h}_t}{\tilde{a}_t} \left(\frac{R^H_{t+1}}{R^{\sim H}_{t+1}} - 1 \right)} \right)^{-1} \right] \tag{A.18}$$

$$\text{Or, alternatively,} \qquad S_{t+1} = \mathbb{E}_{\mathcal{F}_{t+1}} \left[\left(1 + \alpha^H_{t+1} (\eta_{i,t+1} - 1) \right)^{-1} \right] \tag{A.19}$$

$$\text{where} \qquad \alpha^H_{t+1} \equiv \frac{\tilde{h}_t \cdot R^H_{t+1}}{\tilde{h}_t \cdot R^H_{t+1} + (\tilde{a}_t - \tilde{h}_t) \cdot R^{\sim H}_{t+1}} \tag{A.20}$$

α^H_{t+1} measures the share of households' beginning of $t+1$ total wealth being derived from human capital (subject to idiosyncratic risk), in contrast to the other sources of wealth (not subject to idiosyncratic risk).

A.2 Additional Derivations on Firms' Optimal Behavior and Contracting with Intermediary

This Appendix provides additional details and derivations behind the results on the optimal behavior of firms and the contract written with the representative intermediary in equilibrium, as discussed in Section 2.2.4.

Adding to the terms defined in Section 2.2.4, let us denote firm j's accumulated net worth after paying the lender yet *before* paying dividends in period t by $n_{j,t}$. Then, integrating out the realization of $\omega_{j,t+1}$, conditional on the aggregate realizations of (r^K_{t+1}, Q_{t+1}), one can write the expected $t+1$ equity of the firm and the lender's return $R^L_{j,t+1}$ conditional on aggregates and j's leverage $\kappa_{j,t}$ as:

$$
\begin{aligned}
n_{j,t+1} &\equiv [1-\Gamma(\bar{\omega}_{j,t+1})]R^K_{t+1}Q_tK_{j,t+1} = [1-\Gamma(\bar{\omega}_{j,t+1})]R^K_{t+1}\kappa_{j,t}N_{j,t} \\
R^L_{j,t+1} &\equiv \frac{[\Gamma(\bar{\omega}_{j,t+1}) - \mu G(\bar{\omega}_{j,t+1})]R^K_{t+1}Q_tK_{j,t+1}}{Q_tK_{j,t+1} - N_{j,t}} = [\Gamma(\bar{\omega}_{j,t+1}) - \mu G(\bar{\omega}_{j,t+1})]R^K_{t+1}\frac{\kappa_{j,t}}{\kappa_{j,t}-1}
\end{aligned}
$$

The relation between $N_{j,t}$ and $n_{j,t}$ is naturally: $N_{j,t} = n_{j,t} - div_{j,t}$. Since all firms are identical, apart from their net worth, the relevant idiosyncratic state variable for firm j when making decisions in t will just be $n_{j,t}$. Let us denote the value of a firm with period t net worth $n_{j,t}$, before paying dividends by $\tilde{V}_t(n_{j,t})$. Given net worth, the contracting problem is to choose $K_{j,t+1}$ and the schedule $\{\bar{\omega}_{j,t+1}\}$ subject to the lender's participation constraint. Or equivalently, one can choose $\kappa_{j,t}$ and $\{\bar{\omega}_{j,t+1}\}$. Because firm j cannot raise external financing without any internal finance $N_{j,t}$, dividends necessarily cannot exceed $div_{j,t} \le n_{j,t}$, and to continue operating a capital project, the inequality must be strict. $div_{j,t} < 0$ is understood as equity injections by the owner into the firm. Firm j's value function will thus satisfy the Bellman equation:

$$
\tilde{V}_t(n_{j,t}) = \max_{\{\bar{\omega}_{j,t+1}\},\kappa_{j,t},div_{j,t}} \left\{ div_{j,t} + \mathbb{E}_t\left[M^e_{t+1}\tilde{V}_{t+1}(n_{j,t+1})\right]\right\} \tag{A.21}
$$

$$
\text{s.t. } \mathbb{E}_t\left[M_{t+1}R^L_{j,t+1}\right] = \mathbb{E}_t\left\{M_{t+1}[\Gamma(\bar{\omega}_{j,t+1}) - \mu G(\bar{\omega}_{j,t+1})]R^K_{t+1}\frac{\kappa_{j,t}}{\kappa_{j,t}-1}\right\} \ge \mathbb{E}_t[M_{t+1}R^L_{t+1}] = 1
$$

$$
n_{j,t+1} = \max\{\omega_{j,t+1} - \bar{\omega}_{j,t+1}, 0\}R^K_{t+1}\kappa_{j,t}\left(n_{j,t} - div_{j,t}\right), \quad div_{j,t} \le n_{j,t}
$$

As mentioned in Section 2.2.2, in equilibrium the firms apply the entrepreneur's stochastic discount factor M^e_{t+1}. The lender's participation constraint arises as the result of the intermediary being a pass-through entity, combining with the facts that in equilibrium all contracts will offer the same expected return to the lender $R^L_t = R^L_{j,t}, \forall j$ and as elaborated above $R^L_t = R^D_t$ in equilibrium, and finally employing the households' shared stochastic discount factor M_{t+1}.

As is commonly done in computational models of firm heterogeneity, for example by Khan and Thomas (2008), one can also redefine the firm's value measured in units of the owner's marginal utility, taken as given by firm j, as $V_t(n_{j,t}) \equiv (C_t^e)^{-1}\tilde{V}_t(n_{j,t})$, and rewrite (A.21) as:[40]

$$V_t(n_{j,t}) = \max_{\{\bar{\omega}_{j,t+1}\},\kappa_{j,t},div_{j,t}} \left\{ (C_t^e)^{-1}\, div_{j,t} + \beta_e \mathbb{E}_t \left[V_{t+1}(n_{j,t+1}) \right] \right\}$$

One can guess that the continuation value function is linear, i.e. $V_{t+1}(n_{j,t+1}) = V_{t+1} \cdot n_{j,t+1}$, where, with an abuse of notation, V_{t+1} is now understood to be a *variable* that measures the marginal valuation of an additional unit of net worth $n_{j,t+1}$ to the firm. Plugging in the law of motion for $n_{j,t+1}$ and applying the law of iterated expectations to integrate out the realization of $\omega_{j,t+1}$, one can define the dividends-net worth ratio as $\widehat{div}_{j,t} \equiv \frac{div_{j,t}}{n_{j,t}}$ and rewrite the Bellman equation as:

$$\begin{aligned} V_t(n_{j,t}) = n_{j,t} \cdot \max_{\substack{\{\bar{\omega}_{j,t+1}\},\kappa_{j,t},\\ \widehat{div}_{j,t}\leq 1}} \Big\{ & \widehat{div}_{j,t} \left((C_t^e)^{-1} - \beta_e \mathbb{E}_t \left[V_{t+1}[1-\Gamma(\bar{\omega}_{j,t+1})] R_{t+1}^K \right] \kappa_{j,t} \right) + \\ & + n_{j,t} \cdot \beta_e \mathbb{E}_t \left[V_{t+1}[1-\Gamma(\bar{\omega}_{j,t+1})] R_{t+1}^K \right] \kappa_{j,t} \Big\} \end{aligned} \tag{A.22}$$

$$\text{s.t. } \mathbb{E}_t \left\{ M_{t+1}[\Gamma(\bar{\omega}_{j,t+1}) - \mu G(\bar{\omega}_{j,t+1})] R_{t+1}^K \right\} \kappa_{j,t} \geq \kappa_{j,t} - 1 \tag{A.23}$$

In equilibrium, the constraint $div_{j,t} \leq n_{j,t}$ could not be binding as, by linearity, it would have to be binding for all firms $j \in [0,1]$, implying no net worth were to be left for the firms and no capital K_{t+1} could be acquired, violating capital rental market clearing in $t+1$.[41] The individual $\widehat{div}_{j,t}$ across $j \in [0,1]$ are thus not uniquely determined, and in equilibrium it must be the case that:

$$(C_t^e)^{-1} = \beta_e \mathbb{E}_t \left[V_{t+1}[1-\Gamma(\bar{\omega}_{t+1})] R_{t+1}^K \right] \kappa_{j,t}$$

Since the participation constraint was already initially written independently of $n_{j,t}$, the above verifies the guess that the firm's value function $V_t(n_{j,t})$ is linear in $n_{j,t}$ and the problem of choosing $\kappa_{j,t}$ and $\{\bar{\omega}_{j,t+1}\}$ is independent of firm j's incoming net worth. Thus, given that the optimal choices of $\kappa_{j,t}$ and $\{\bar{\omega}_{j,t+1}\}$ are unique in the equilibria considered, which can be shown rigorously, each firm chooses the same leverage ratio κ_t and cutoff schedule $\{\bar{\omega}_{t+1}\}$. The envelope condition of the firm's problem implies:

$$V_t = \beta_e \mathbb{E}_t \left[V_{t+1}[1-\Gamma(\bar{\omega}_{t+1})] R_{t+1}^K \right] \kappa_t$$

further implying that $(C_t^e)^{-1} = V_t$. Given that all firms choose the same κ_t and $\{\bar{\omega}_{t+1}\}$, the distribution of internal wealth across the firms does not matter for aggregates and we need to only track the aggregate level of firms' internal wealth. And although the distribution of dividend payments is not pinned down in equilibrium, we have established in Section 2.2.2 that it must

[40] This redefinition of the value function is not directly useful here for solving the firm's equivalent problem, but defining $V_t(n_{j,t})$ in such a way makes it easy to point out the close similarities between this setup and that used by CFP. See Appendix A.3 for details.

[41] To be precise, one can first establish that the value function is *affine*, and given an affine value function, it must be the case that in equilibrium $(C_t^e)^{-1} = \beta_e \mathbb{E}_t \left[V_{t+1}[1-\Gamma(\bar{\omega}_{t+1})] R_{t+1}^K \right] \kappa_t$, yielding *linearity*.

necessarily be the case that $C_t^e = \int_0^1 div_{j,t} dj$. Plugging for $(C_t^e)^{-1} = V_t$ in the above envelope condition, we have that the entrepreneur's consumption must satisfy an Euler equation, as stated in (25). And aggregating the law of motion of net worth across firms $j \in [0,1]$ implies (26) in Section 2.2.4.

Taking the first order conditions with respect to $\bar{\omega}_{j,t+1}$ and $\kappa_{j,t}$ in (A.22), and combining them with $(C_t^e)^{-1} = V_t$ and the binding participation constraint (A.23) one can summarize the key optimality condition for $\bar{\omega}_{t+1}$ as:

$$\frac{\Gamma'(\bar{\omega}_{t+1})}{\Gamma'(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})} = \frac{M_{t+1}}{M_{t+1}^e}$$

which holds state-by-state, for each realization of the aggregate state in $t+1$, also stated as (28) in Section 2.2.4.

A.3 Entrepreneurs' Problem in the CFP Model and Equivalence to the log-Utility Representative Entrepreneur

In this Appendix, I will argue that the setup of the "entrepreneur + firm block" in the model analyzed in Section 2, with a representative entrepreneur with logarithmic utility who owns firms that are subject to idiosyncratic shocks is virtually equivalent, to a first order approximation, to the standard approach used by BGG and CFP where a unit mass of *ex post* heterogeneous entrepreneurs are assumed to themselves invest in capital subject to the CSV friction and have *linear* utility from consumption. To do so, I compare first order approximations of the equilibrium conditions pertaining to the entrepreneurs' choices and the optimal contract across the two models.

A.3.1 CFP Model

In the benchmark setup employed by BGG and CFP, a unit mass of entrepreneurs are assumed to invest in capital subject to the CSV friction directly, have linear utility from consumption and a time discount factor identical to that of the (representative) household. To be precise, let us denote this time discount factor as β_e^{CFP}. In each period, a fraction $1-\gamma$ of entrepreneurs dies each period and gets replaced by an equal mass of entering ones who get a small transfer from the survivors to start operations.

Entrepreneurs' equilibrium conditions Since the entrepreneurs are financially constrained in the equilibrium of the BGG-CFP model, it is optimal for them to postpone consumption indefinitely. Entrepreneurs thus only consume when they die and assumption of $1-\gamma$ of the entrepreneurs dying each period ensures that they do not save themselves out of financial constraints. Thus, in each period, a fraction $1-\gamma$ of incoming entrepreneurial net worth is consumed and the remaining fraction is invested as internal financing (net worth) going forward. For more details on the entrepreneurs' problem in the CFP model see Carlstrom et al. (2016). The bottom line is that in the CFP model, the optimality conditions for an entrepreneur's problem can be combined into the following Bellman equation, laws of motion and first order condition in the equilibrium variables $\{V_t, C_t^e, N_t, \bar{\omega}_t, \kappa_t\}$:

$$V_t = (1-\gamma) + \gamma\beta_e^{CFP}\mathbb{E}_t\left\{V_{t+1}[1-\Gamma(\bar{\omega}_{t+1})]R_{t+1}^K\right\}\kappa_t \tag{A.24}$$

$$C_t^e = (1-\gamma)[1-\Gamma(\bar{\omega}_t)]R_t^K\kappa_{t-1}N_{t-1} \tag{A.25}$$

$$N_t = [1-\Gamma(\bar{\omega}_t)]R_t^K\kappa_{t-1}N_{t-1} - C_t^e \tag{A.26}$$

$$\frac{\Gamma'(\bar{\omega}_t)}{\Gamma'(\bar{\omega}_t) - \mu G(\bar{\omega}_t)} = \left(\gamma\beta_e^{CFP}\frac{V_t}{V_{t-1}-(1-\gamma)}\right)^{-1} M_t \tag{A.27}$$

Plus the participation constraint (A.23), which effectively determines κ_t. Since these and all remaining equilibrium conditions (pertaining to the behavior of households and final goods and new capital producers) are independent of the setup of the "entrepreneur + firm block", I will not focus those. For brevity, let us denote the left hand side of (A.27) with the increasing function $\Psi(\bar{\omega}_t)$.

Steady state In the non-stochastic steady state, combining (A.25) and (A.26) yields:

$$1 = \gamma[1-\Gamma(\bar{\omega}_{ss})]R_{ss}^K\kappa_{ss} \tag{A.28}$$

And using this in (A.24) yields:

$$V_{ss} = (1-\gamma) + \gamma\beta_e^{CFP}V_{ss}[1-\Gamma(\bar{\omega}_{ss})]R_{ss}^K\kappa_{ss} \Rightarrow V_{ss} = \frac{1-\gamma}{1-\beta_e} \tag{A.29}$$

And (A.27), combined with (A.24) yields:

$$\Psi(\bar{\omega}_{ss}) = M_{ss}\left(\frac{\gamma\beta_e^{CFP}V_{ss}}{V_{ss}-(1-\gamma)}\right)^{-1} = M_{ss}\left(\frac{\gamma\beta_e^{CFP}V_{ss}}{\beta_e^{CFP}V_{ss}}\right)^{-1} = M_{ss}\gamma^{-1} \tag{A.30}$$

Thus, (A.29) separately determines V_{ss}, and (A.28) and (A.30) alongside the remaining equilibrium conditions determine the rest of the steady state values.

First order dynamics Comparing (A.25) and (A.26) to (30) and (31) in Section 2.2.2 shows that entrepreneurs' consumption and net worth dynamics, conditional on the contract details, are identical in the two models whenever $\gamma = \beta_e$, with β_e being the representative entrepreneur's time discount factor in the model of Section 2, so their equivalence follows trivially. Also, to save on notation, I will denote $X_t^r \equiv [1-\Gamma(\bar{\omega}_t)]R_t^K\kappa_{t-1}$ in log-linearizing (A.24), as this product shows up in the same manner in both of the models. Log-linearizing (A.24) gives, using the fact that in steady state $\gamma X_{ss}^r = 1$:

$$v_t = \beta_e^{CFP}\mathbb{E}_t\left\{v_{t+1} + x_{t+1}^r\right\} \tag{A.31}$$

And log-linearizing (A.27), using the fact that in steady state $V_{ss} - (1-\gamma) = \beta_e^{CFP} V_{ss}$, yields:

$$\frac{\Psi'(\bar{\omega})\bar{\omega}}{\Psi(\bar{\omega})}\hat{\omega}_t = m_t - \left(v_t - \frac{1}{\beta_e^{CFP}} v_{t-1}\right) \tag{A.32}$$

A.4 Representative Entrepreneur Model

Entrepreneurs' equilibrium conditions Following the analysis in Sections 2.2.2 and 2.2.4, one can write the equilibrium conditions determining $\{V_t, C_t^e, N_t, \bar{\omega}_{t+1}, \kappa_t\}$ as:

$$V_t = \frac{1}{C_t^e} \tag{A.33}$$

$$C_t^e = (1-\beta_e)[1-\Gamma(\bar{\omega}_t)]R_t^K \kappa_{t-1} N_{t-1} \tag{A.34}$$

$$N_t = [1-\Gamma(\bar{\omega}_t)]R_t^K \kappa_{t-1} N_{t-1} - C_t^e \tag{A.35}$$

$$\Psi(\bar{\omega}_t) = M_t \left(\beta_e \frac{V_t}{V_{t-1}}\right)^{-1} \tag{A.36}$$

Plus the participation constraint (A.23), which again effectively pins down κ_t. As discussed in Section 2.2.4, the result that under log-utility, consumption is a constant fraction of equity can be reached by employing (A.35) and the entrepreneurs' Euler equation (25), with the latter now being replaced by (A.34). As mentioned, (A.34) and (A.35) are identical across the two models.

Steady state In steady state, combining (A.34) and (A.35) implies:

$$1 = \beta_e[1-\Gamma(\bar{\omega}_{ss})]R_{ss}^K \kappa_{ss} \tag{A.37}$$

And (A.36) implies

$$\Psi(\bar{\omega}_{ss}) = M_{ss}\beta_e^{-1} \tag{A.38}$$

which are identical to (A.28) and (A.30) whenever $\gamma = \beta_e$, so the two models have *exactly* the same non-stochastic steady states, apart from the value of V_{ss} which in this case is pinned down by

$$V_{ss} = \frac{1}{C_{ss}^e} \tag{A.39}$$

First order dynamics Log-linearizing (A.36) directly yields:

$$\frac{\Psi'(\bar{\omega})\bar{\omega}}{\Psi(\bar{\omega})}\hat{\omega}_t = m_t - (v_t - v_{t-1}) \tag{A.40}$$

which is equivalent to (A.32) whenever $\beta_e^{CFP} \to 1$. And finally, because the Euler equation for the entrepreneur must still be satisfied by V_t, even though now redundant, it is necessarily the case that V_t satisfies

$$V_t = \beta_e \mathbb{E}_t \left\{ V_{t+1}[1 - \Gamma(\bar{\omega}_{t+1})] R^K_{t+1} \right\} \kappa_t$$
$$\Rightarrow v_t = \mathbb{E}_t \left\{ v_{t+1} + x^r_{t+1} \right\} \tag{A.41}$$

which is equivalent to (A.31) whenever $\beta_e^{CFP} \to 1$.

We have thus established the equivalence of the five equilibrium conditions relevant for determining the outcome of the entrepreneurs'+firms' problem in these two (log-linearized) models whenever $\beta_e^{CFP} \to 1$ and $\gamma = \beta_e$.

A.5 Additional Model Impulse Responses

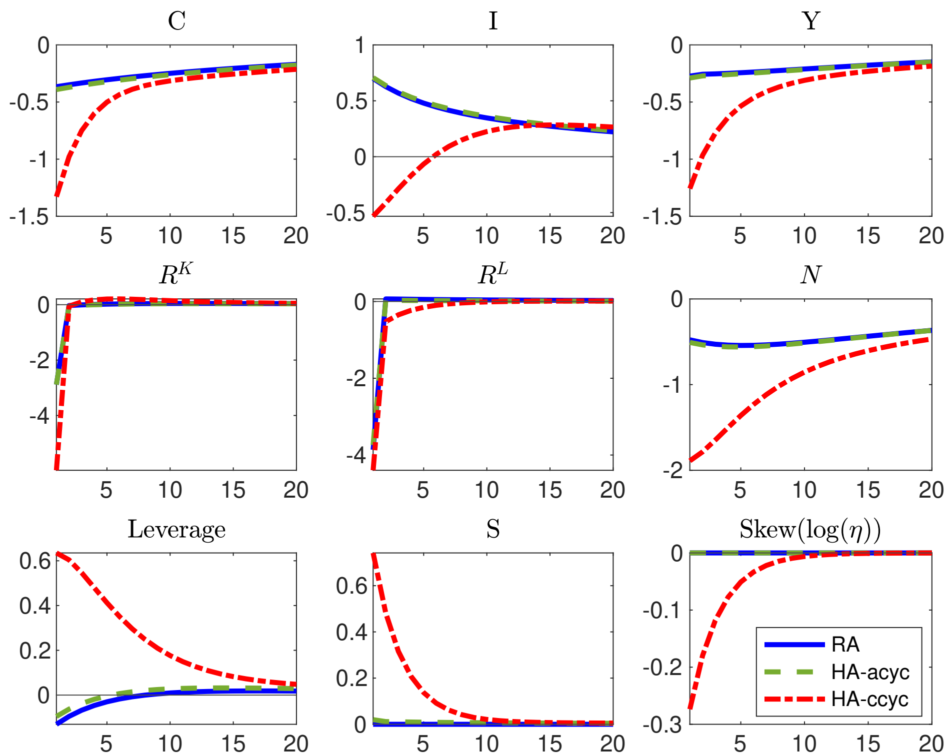

Figure A.1: Impulse responses to capital quality shock in representative and heterogeneous household (acyclical and countercyclical risk) models with sticky prices

Notes: 1% negative capital quality shock $\varepsilon_t^K = -0.01$; (100×) log-deviations from steady state, returns annualized, Skew(log(η)) as units of (standardized) third moment. Horizontal axis: quarters. Blue solid: representative agent specification (*RA*), green dashed: heterogeneous agent benchmark with acyclical labor risk (*HA-acyc*), red dash-dotted: heterogeneous agent model with countercyclical labor risk (*HA-ccyc*).

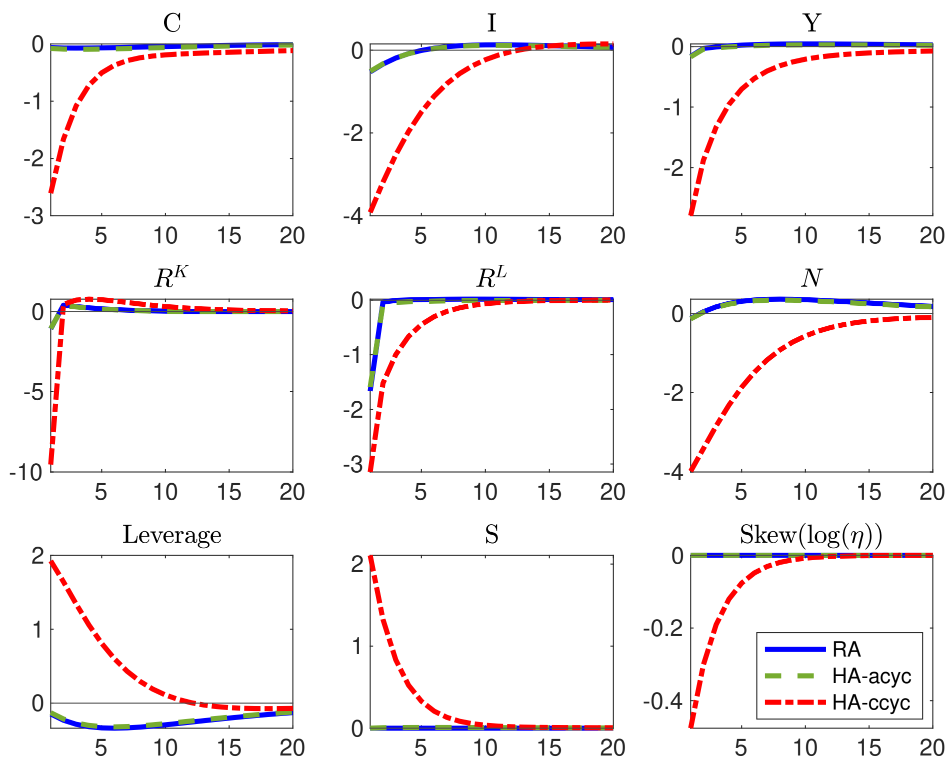

Figure A.2: Impulse responses to risk shock in representative and heterogeneous household (acyclical and countercyclical risk) models with sticky prices

Notes: 0.01 unit positive risk shock $\varepsilon_t^{\omega} = 0.01$; (100×) log-deviations from steady state, returns annualized, Skew(log(η)) as units of (standardized) third moment. Horizontal axis: quarters. Blue solid: representative agent specification (*RA*), green dashed: heterogeneous agent benchmark with acyclical labor risk (*HA-acyc*), red dash-dotted: heterogeneous agent model with countercyclical labor risk (*HA-ccyc*).

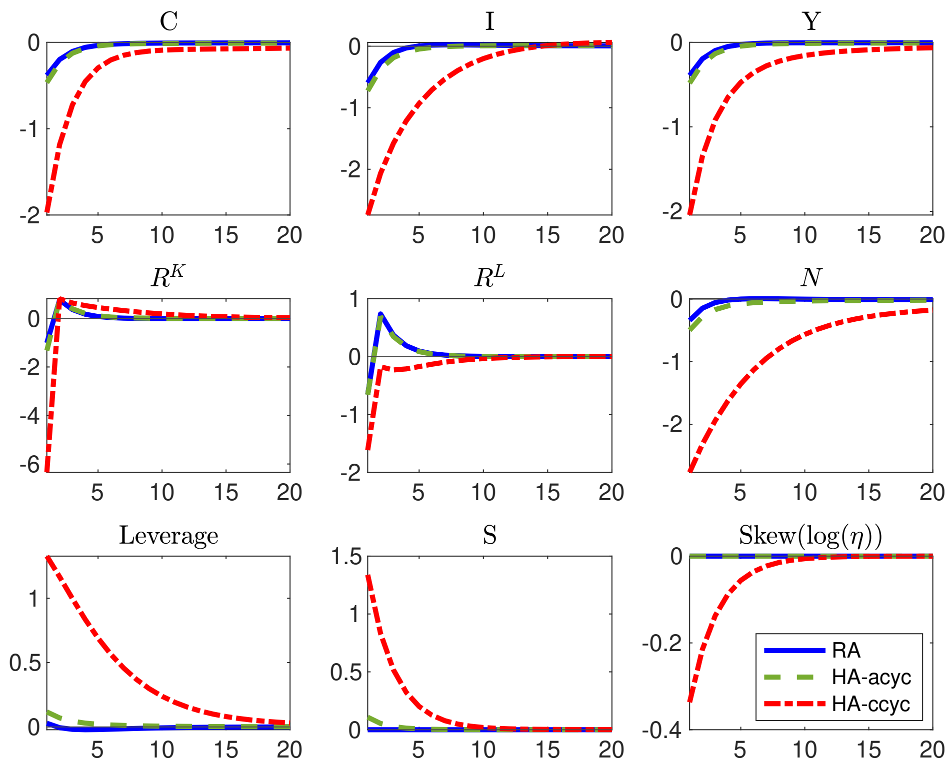

Figure A.3: Impulse responses to capital share shock in representative and heterogeneous household (acyclical and countercyclical risk) models with sticky prices

Notes: 1 pp negative capital share shock $\varepsilon_t^\nu = -0.01$; (100×) log-deviations from steady state, returns annualized, Skew(log(η)) as units of (standardized) third moment. Horizontal axis: quarters. Blue solid: representative agent specification (*RA*), green dashed: heterogeneous agent benchmark with acyclical labor risk (*HA-acyc*), red dash-dotted: heterogeneous agent model with countercyclical labor risk (*HA-ccyc*).

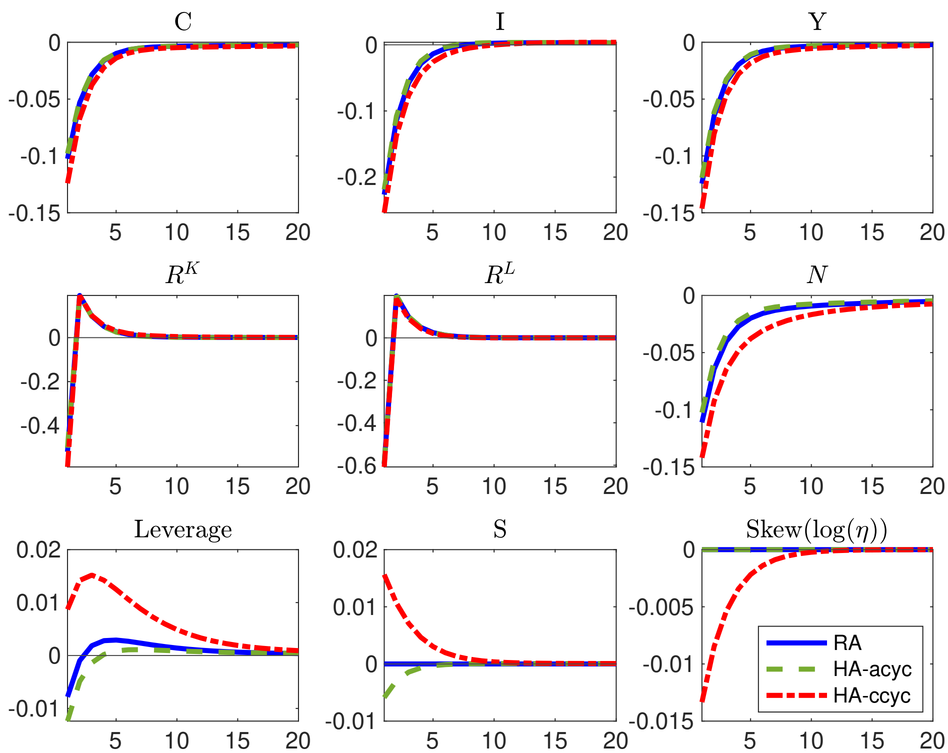

Figure A.4: Impulse responses to monetary policy shock in representative and heterogeneous (acyclical and countercyclical risk) household models with sticky prices

Notes: 25 bp (annualized) contractionary monetary policy shock $\varepsilon_t^\nu = 0.0025/4$; (100×) log-deviations from steady state, returns annualized, Skew(log(η)) as units of (standardized) third moment. Horizontal axis: quarters. Blue solid: representative agent specification (*RA*), green dashed: heterogeneous agent benchmark with acyclical labor risk (*HA-acyc*), red dash-dotted: heterogeneous agent model with countercyclical labor risk (*HA-ccyc*).