Foreign Exchange Intervention with UIP and CIP Deviations[1]

Philippe Bacchetta
University of Lausanne
Swiss Finance Institute
CEPR

Kenza Benhima
University of Lausanne
CEPR

Brendan Berthold
University of Lausanne

March 3, 2025

[1]We would like to thank three referees, Julien Bengui, Paolo Cavallino, Tobias Cwik, Egemen Eren, Xiang Fang, Wenqian Huang, Michael Kumhof, Hanno Lustig, Serra Pelin, Daniele Siena, Kristin Trautmann, Eric van Wincoop and participants at the CEPR-IMF and BdI-BdF-BoE International Macroeconomics meetings in Rome, the 12th workshop on Exchange Rates at BIS, the Women in Macro, Finance and Economic History conference 2024, the AMSE-BdF Workshop 2024, the Salento Macro Meetings, the Qatar Center for Global Banking and Finance Annual Conference 2024, the SNB Research Conference 2024, the EFA Congress in Amsterdam and the AEA Meeting in San Francisco for comments. Philippe Bacchetta gratefully acknowledges financial support from the Swiss National Science Foundation.

Abstract

We examine the welfare-based opportunity cost of foreign exchange (FX) intervention when both CIP and UIP deviations are present. We consider a small open economy that receives international capital flows through constrained international financial intermediaries. Deviations from CIP come from limited arbitrage or through a convenience yield, while UIP deviations are also affected by global risk. We show that the sign of CIP and UIP deviations may differ for safe haven countries. We find that FX reserves may provide a net benefit, rather than a cost, when international intermediaries value the safe-haven properties of a currency more than domestic households. We show that this has been the case for the Swiss franc and the Japanese Yen. We examine the optimal policy of a constrained central bank planner in this context.

1 Introduction

A vast literature examines the optimal level of central bank international reserves (see Bianchi and Lorenzoni (2022) for a recent survey). A recurrent feature is that the accumulation of reserves bears an opportunity cost arising from a return differential between the liabilities and the assets of the central bank. In the recent literature on optimal Foreign Exchange (FX) interventions, some authors focus on Uncovered Interest Rate Parity (UIP) wedges (see Cavallino, 2019; Basu et al., 2020; Fang and Liu, 2021; Maggiori, 2021; Itskhoki and Mukhin, 2022). In contrast, other researchers argue that what matters are deviations from Covered Interest Rate Parity (CIP) (e.g., Amador et al., 2020; Fanelli and Straub, 2021).[1] The distinction between CIP and UIP deviations appears particularly relevant for safe haven countries such as Switzerland and Japan, since these deviations may be of different sign. On the one hand, hedged returns in these currencies (equal to CIP deviations) have been positive since the GFC.[2] On the other hand, investors accept lower unhedged excess return (equal to UIP deviations) for safe haven currencies, given their hedging properties in bad times. UIP deviations measured with survey exchange rate expectation data show systematic negative expected excess returns for these currencies as shown in Appendix A.1.

To clarify these issues, we develop a model where both CIP and UIP deviations are present. We consider a small open economy that receives international capital flows through international financial intermediaries as in Gabaix and Maggiori (2015). The structure of the model is similar to that in recent papers examining the role of international reserves (see Cavallino, 2019; Amador et al., 2020; Fanelli and Straub, 2021; Basu et al., 2020; Maggiori, 2021), but financial intermediaries are risk averse. Fang and Liu (2021) propose a related framework in a two-country model, but they focus on the US dollar and do not analyze FX interventions. The international financial intermediaries are the marginal investors and determine both UIP and CIP deviations through their unhedged and hedged portfolio choices. These deviations typically do not coincide and may even be of different sign.

In this environment, we examine the opportunity costs of FX intervention in terms of welfare. We identify the conditions under which CIP or UIP deviations matter for this cost. We find that there may be no opportunity cost, and that there may even be a benefit, of FX intervention in a safe haven country, even if that country faces a positive CIP deviation. We examine the implications for optimal FX intervention in these cases.

The presence of systematic deviations from CIP in the wake of the Global Financial

[1] Note that in this literature, the cost of reserves is driven by the currency excess return, while in other frameworks the cost of reserves is determined by an external finance premium independent of the currency position, for example due to a government default premium (e.g., Alfaro and Kanczuk, 2009; Bianchi et al., 2018; Samano, 2022).

[2] Rime et al. (2022) show that CIP deviations for the CHF and the JPY with respect to the USD have been the most profitable for financial institutions.

Crisis is a major development in international finance (see Du and Schreger (2022) or Cerutti et al. (2021) for recent surveys). The theoretical literature has provided explanations for CIP deviations, but has devoted limited attention to the link between CIP and UIP deviations. Several papers analyzing interest rate differentials assume complete markets so that either there are no UIP deviations or CIP deviations are equal to UIP deviations. This is not consistent with the data.

The recent literature has followed two main approaches to explain interest rate differentials. First, there may be financial frictions that limit arbitrage, for example, by assuming constrained financial intermediaries. The other approach is to assume differences in convenience yields. The two approaches are present in our model and determine deviations from CIP. However, we do not assume complete markets, so that UIP deviations differ from CIP deviations. A basic result from this analysis is the following relationship between UIP and CIP deviations:

$$devUIP = devCIP - \frac{cov(m^*, X^*)}{\mathbb{E}m^*} \tag{1}$$

where m^* is the stochastic discount factor (SDF) of financial intermediaries and X^* is the foreign currency excess return from the international intermediary perspective. The covariance term in (1) represents the risk premium for a UIP, or carry-trade, investment strategy, which involves exchange-rate risk in contrast with a CIP strategy. This risk premium arises because we assume that international financial intermediaries are risk averse. They could hedge exchange-rate risk on the forward market, but it may not be optimal to fully hedge.

Safe-haven currencies are of particular interest since in that case we have $cov(m^*, X^*) > 0$, that is, the safe-haven currency yields a higher return in bad times. Therefore, it is possible to have a positive CIP deviation with a negative UIP deviation.

We analyze the impact and the welfare cost of accumulating international reserves in this framework. While sterilized FX interventions by the central bank affect the currency composition of a country's assets, it could be undone by domestic households if they are unconstrained. However, we assume that households face short-selling constraints. We show that, when these constraints are binding, FX interventions are effective, and that there is room for welfare improvement. The welfare impact of FX interventions depends on how households value currency risk in comparison to financial intermediaries.

For safe-haven currencies, we show that if domestic households attribute less value to the safe-haven properties of their currency than international financial intermediaries (i.e., the domestic SDF is less correlated to the excess return than for financial intermediaries), then FX reserves may have a benefit, not a cost. We examine this issue empirically by estimating the SDF of financial intermediaries following He et al. (2017). When considering the CHF and JPY with respect to the USD, we find that the SDF of financial intermediaries is more correlated with excess returns than the SDF of domestic households.

This paper complements the literature on the estimation of the opportunity cost of FX reserves. There is a long tradition of estimating the cost and benefits of accumulating FX reserves (e.g., Jeanne and Rancière, 2011). Adler and Mano (2021) estimate the quasi-fiscal cost of interventions for 73 countries using UIP deviations. Using survey expectations or assuming a random walk for the nominal exchange rate, they find that the ex ante cost of intervention is negative for Japan and Switzerland in the period 2002-2013, while it is positive for most other countries.[3] In this paper, we examine the cost of intervention from the welfare point of view, and find that it is also negative for Japan and Switzerland, but that it is not equal to UIP deviations in general.

By focusing on countries like Switzerland or Japan, this paper provides a different perspective on safe haven economies. A growing literature has been analyzing the special role of the US dollar as a reserve currency. In particular, several papers have focused on the role of convenience yields in generating currency movements and expected excess returns (e.g., Jiang et al., 2021b,a; Valchev, 2020; Kekre and Lenel, 2024; Bianchi et al., 2022). We show that convenience yields are not the sole determinant for exchange rate movements and UIP deviations in safe haven economies.

The rest of the paper is organized as follows. Section 2 describes the model and Section 3 discusses the decentralized equilibrium and the impact of FX interventions. Section 4 analyzes the opportunity cost of reserves in this context. Section 5 examines optimal FX intervention and Section 6 concludes.

2 The Model

This section presents a two-period model of a small open economy facing international financial intermediaries in the spirit of Gabaix and Maggiori (2015). These intermediaries buy domestic bonds and are the marginal investors in both the spot and the forward markets, since households are financially constrained. Intermediaries are risk averse, so there is a difference between their covered and uncovered positions. After presenting the financial intermediaries, we describe the households, the government and the central bank, as well as the equilibrium in the asset markets.

We call the foreign currency the dollar and assume that the foreign interest rate i^* is given. The domestic interest rate for period-one bonds is i. S_t is the spot price of dollars in terms of domestic currency for $t = 1, 2$ and F is the forward rate priced in period one for period two. The second-period spot rate, S_2 is exogenous. The price of goods in dollars is normalized to one and purchasing power parity (PPP) is assumed to hold. Let Z^* be the excess return hedged by a forward contract, from the perspective of

[3] In the case of developing or emerging economies, the opportunity cost may be based on the country's borrowing cost, which implies a credit risk (e.g., Edwards, 1985). However, Yeyati and Gómez (2022) argue that when reserves are used for leaning-against-the-wind interventions, it is more appropriate to use UIP deviations.

international financial intermediaries, or the CIP deviation:

$$Z^* \equiv (1+i)\frac{S_1}{F} - (1+i^*) \tag{2}$$

and X^* the domestic currency excess return, expressed in foreign currency:

$$X^* \equiv (1+i)\frac{S_1}{S_2} - (1+i^*) \tag{3}$$

2.1 International Financial Intermediaries

International financial intermediaries value their expected profits with their stochastic discount factor m^*, which we assume to be related to a risky global factor y^* in period 2. More precisely, we assume that $log(y^*) \sim N(\sigma^2/2, \sigma^2)$, with $\sigma > 0$ and that $\log(m^*) = \log(\beta) - log(y^*)$. This implies that m^* and y^* are negatively correlated and that $\mathbb{E}(m^*) = \beta$. This is a small economy so the world interest rate i^* is exogenous. It is set to $\beta^{-1} - 1$ so that $\mathbb{E}[m^*(1+i^*)] = 1$, without loss of generality.

In period 1, intermediaries invest in domestic bonds in quantity b^{H*}, expressed in dollars. They have a zero net position and fund their investments by borrowing in dollars. We also assume that they can use forward contracts in quantity f^* and that they are the only players in the forward market.[4]

Moreover, financial intermediaries may value the liquidity of dollar assets. We assume that investors have operating costs that are increasing in non-dollar assets holdings b^{H*} and that it is a linear function: $\chi \cdot b^{H*}$, with $\chi \geq 0$. Their objective function is in dollars (and equivalently, in terms of goods, since the dollar price is constant):

$$V^* = \mathbb{E}\left\{ m^* \left[b^{H*} \left((1+i)\frac{S_1}{S_2} - (1+i^*) \right) - f^* \left(\frac{1}{S_2} - \frac{1}{F} \right) \right] \right\} - \chi b^{H*}$$

b^{H*} represents the total funds invested in the domestic currency, covered or uncovered. $f^*/(1+i)S_1$ is the covered amount, and $b^{H*} - f^*/(1+i)S_1$ is the uncovered amount.

From the first-order condition with respect to f^*, we find a relationship between the covered and uncovered excess returns:

$$\mathbb{E}(m^* X^*) = \mathbb{E}(m^* Z^*) \tag{4}$$

Equation (4) can be rewritten as a relationship between the UIP deviation, $\mathbb{E}X^*$, and the CIP deviation, Z^* (an equivalent of Equation (1)):

$$\mathbb{E}X^* = Z^* - \frac{cov(m^*, X^*)}{\mathbb{E}m^*} \tag{5}$$

Covered and uncovered carry trades yield the same returns in expectation, up to a covariance term, which corresponds to minus the currency risk premium, because intermediaries

[4] Allowing domestic households to participate in the forward market would not change the analysis, since households positions are limited.

are risk averse. In most of the existing literature, the risk premium is equal to zero. This is the case because of the effective risk neutrality of financial intermediaries, as in Gabaix and Maggiori (2015), Amador et al. (2020), Basu et al. (2020); or in the absence of uncertainty, as in Fanelli and Straub (2021). In these cases, there is no distinction between UIP and CIP deviations. A difference between these two deviations may arise if we introduce additional frictions for CIP arbitrage, as discussed by Fanelli and Straub (2021) and Fang and Liu (2021).

We assume that financial intermediaries are constrained as in Gabaix and Maggiori (2015). Intermediaries can divert a fraction Γb^{H*} of the total invested funds, after the investment decisions are taken, but before shocks are realized. This yields a participation constraint for investors:[5]

$$V^* \geq \Gamma(b^{H*})^2$$

If the participation constraint is binding, we have:

$$\mathbb{E}\left(m^* X^*\right) = \Gamma b^{H*} + \chi \tag{6}$$

This, along with Equations (4) and (5), implies

$$Z^* = \frac{\Gamma b^{H*} + \chi}{\mathbb{E}m^*} \tag{7}$$

and

$$\mathbb{E}X^* = \frac{\Gamma b^{H*} + \chi}{\mathbb{E}m^*} - \frac{cov(m^*, X^*)}{\mathbb{E}m^*} \tag{8}$$

The term $\Gamma b^{H*} + \chi$ in Equations (7) and (8) shows that the impact of limited arbitrage and of the convenience yield, which are typically found in the literature, is the same for CIP and UIP deviations. The new element in the analysis is the covariance term affecting the UIP deviation.

To gain more insight on the covariance term in (5), we consider a specific stochastic distribution for S_2. We assume that $\log(S_2) = \rho[\log(y^*) - (1-\rho)\sigma^2/2]$ for some real ρ. This implies that $\mathbb{E}(1/S_2) = 1$ and that the correlation of S_2 with y^* is ρ. In this case, we can show that $cov(m^*, X^*)/\mathbb{E}m^* = e^{\rho\sigma^2} - 1$. When $\rho > 0$, the domestic currency tends to appreciate in bad times, i.e. when the global factor decreases. This is typical for safe-haven currencies. In this case, $cov(m^*, X^*) > 0$ and $\mathbb{E}X^* < Z^*$: the hedging properties of the currency make it attractive to financial intermediaries, who accept a lower expected return. This effect is stronger when ρ and σ^2 increase, which makes the UIP deviation $\mathbb{E}X^*$ more negative, for a given level of b^{H*}. When $\rho < 0$, we have the opposite effect and the UIP deviation is larger than the CIP deviation and increases with σ^2.

[5] Notice that we assume that the financial constraint applies to the whole foreign exchange investment of financial intermediaries, whether hedged or not.

2.2 Domestic Households

Households receive a real endowment $y_1 = 1$ in period 1. In period 2, they receive y_2, which we assume to be correlated to the global factor: $\log(y_2) = g + \alpha[\log(y^*) - (1-\alpha)\sigma^2/2]$ for some real α. This implies that $\mathbb{E}(1/y_2) = e^{-g}$, where g can be thought of as the average growth rate of the home endowment. They can hold domestic-currency bonds b^H (expressed in real terms) and foreign-currency bonds b^F. They consume c_1 in period 1 and c_2 in period 2. We assume that households do not use the forward exchange market.

Their budget constraint can be written as:

$$c_1 = y_1 - b^H - b^F + t_1 \tag{9}$$

$$c_2 = y_2 + \frac{1+i}{1+\pi} b^H + (1+i^*)b^F + t_2 \tag{10}$$

where $1 + \pi = P_2/P_1$ is the inflation rate, b^H is the real level of domestic bonds and t_1 and t_2 are real transfers.

Households face short-selling constraints :

$$b^H \geq 0, \qquad\qquad b^F \geq 0 \tag{11}$$

Their utility function is:

$$U(c_1) + \beta \mathbb{E} U(c_2) \tag{12}$$

Domestic households choose bond holdings to maximize (12) subject to constraints (11). Using the assumption of PPP ($P_t = S_t$, $t = 1, 2$), the first-order conditions associated with bond portfolio choices are:

$$1 - \mathbb{E}\left(m(1+i^*)\right) \qquad\qquad -\lambda^F = 0 \tag{13}$$

$$\mathbb{E}\left(mX^*\right) \qquad\qquad +\lambda^H - \lambda^F = 0 \tag{14}$$

where $m = \beta U'(c_2)/U'(c_1)$ is the households' intertemporal marginal rate of substitution or stochastic discount factor and λ^H and λ^F are the multipliers associated with the short-selling constraints (11).

Equations (13) and (14) describe whether households are able to achieve, respectively, their optimal intertemporal allocation and their optimal portfolio allocation between currencies. These concepts are defined as follows:

Definition 1 *We introduce the following optimality concepts:*

- *Intertemporal optimality is characterized by* $\mathbb{E}(m(1 + i^*)) = 1$.
- *Portfolio optimality is characterized by* $\mathbb{E}(mX^*) = 0$.

For example, if $\lambda^F = \lambda^H = 0$, then both the intertemporal and portfolio allocations are optimal. If, on the opposite, $\lambda^F > 0$ and $\lambda^H > 0$, then households can neither implement its optimal intertemporal allocation nor its optimal portfolio allocation, except if $\lambda^F = \lambda^H$. In this knife-edge case, despite being unable to borrow, households' exposure to domestic and foreign currency happens to be optimal. If $\lambda^F = 0$ and $\lambda^H > 0$, then households are able to implement their optimal intertemporal allocation through their foreign currency bond holdings, but cannot implement their optimal portfolio allocation because they cannot borrow in domestic currency.

2.3 The Government

At time 1 the government issues a positive amount of domestic debt $b^G > 0$ (expressed in real terms) and transfers the funds to households:

$$b^G = t_1 \tag{15}$$

At time 2, the government receives the central bank profits, τ_2^{CB} and repays its debt :

$$t_2 = -\frac{1+i}{1+\pi} b_t^G + \tau_2^{CB} \tag{16}$$

We assume that the government is passive and that the level of debt b^G is exogenous.

2.4 The Central Bank

In period 1, the central bank uses the domestic nominal interest rate i as an instrument. Because we specify the policy in terms of the interest rate, we do not need to introduce money explicitly. The interest rate policy can follow a Taylor rule or target a fixed exchange rate. At this stage, we do not specify the nominal policy rule, but we assume a general policy function of the form $f(i, S_1) = 0$. This general form can accommodate an interest rate or exchange rate peg, or a Taylor rule that relates the interest rate to the price level (since $S_1 = P_1$). However, notice that in the absence of nominal rigidity, this policy rule is neutral.

The central bank also uses foreign reserves b^{CBF} as an instrument. We assume that foreign reserves are positive: $b^{CBF} \geq 0$. Changes in foreign reserves take the form of sterilized FX interventions, that is, they are compensated by a change in the central bank holdings of domestic currency bonds b^{CB} (expressed in real terms). In period 2, the central bank distributes its profits τ_2^{CB} to the government. The central bank's budget constraints write as follows:

$$b^{CBF} + b^{CB} = 0 \tag{17}$$

$$\tau_2^{CB} = (1+i^*) b^{CBF} + \frac{1+i}{1+\pi} b^{CB} \tag{18}$$

By focusing on a cashless economy, we ignore the money component of the central bank's balance sheet. We also assume no transfers from the government to the central bank in period 1. In an extension, we examine unsterilized interventions, where an increase in b^{CBF} can be financed by a transfer from the government.

3 On the Impact of Foreign Reserves

In this section, we define a set of conditions under which FX interventions are effective in affecting equilibrium cross-border asset holdings and asset prices, and we discuss their impact. For most of the analysis, we focus on the case of safe-haven economies, which is defined more precisely below.

3.1 Gross and Net Foreign Position

For the rest of our analysis, it is convenient to focus on the Home country's period 1 net and gross foreign liabilities. Gross foreign liabilities are the foreign holdings of domestic bonds b^{H*}. They are equal to the amount of domestic bonds not held domestically:

$$b^{H*} = b^G - b^{CB} - b^H = b^G + b^{CBF} - b^H \tag{19}$$

The second equality is obtained by using the central bank's budget constraint. It shows that, everything else equal, sterilized FX interventions affect the supply of domestic bonds to foreigners. The central bank can increase its holding of foreign currency assets only through a balance-sheet expansion, and hence through an increase in its domestic liabilities.

Net foreign liabilities in period one are gross foreign liabilities minus gross foreign assets

$$nfl = b^{H*} - (b^{CBF} + b^F) = b^G - b^H - b^F \tag{20}$$

where gross foreign assets $b^F + b^{CBF}$ are the domestic holding of foreign assets. For given net foreign liabilities nfl, the level of gross foreign liabilities b^{H*} determines gross foreign assets $b^{CBF} + b^F = b^{H*} - nfl$. For a given nfl, a higher b^{H*} is a measure of the economy's gross foreign position.

The second equality is obtained by replacing b^{H*} using (19). It is useful to note that, everything equal, sterilized FX interventions affect b^{H*}, but not nfl. An increase in b^{CBF} increases b^{H*} one for one, through a decrease in b^{CB}, but it also increases gross foreign assets b^{F*} one for one. Therefore, FX interventions affect the gross foreign position of the economy and its currency exposure, but not the net foreign position.

3.2 Equilibrium in Asset Markets

In equilibrium, gross foreign liabilities are equal to the net domestic supply of domestic currency bonds: b^{H*}. From Equations (7) and (8), we have:

$$\mathbb{E}X^* = \underbrace{\frac{1}{\mathbb{E}m^*}\left(\Gamma b^{H*} + \chi\right)}_{Z^*} - \frac{cov(m^*, X^*)}{\mathbb{E}m^*} \tag{21}$$

The net supply of domestic liabilities to foreigners determines the equilibrium expected domestic currency excess return $\mathbb{E}X^*$ (the UIP deviation). A higher domestic supply can only be absorbed by intermediaries if it offers a higher excess return (the same holds for the CIP deviation Z^*). In contrast, an increase in $cov(m^*, X^*)$ leads to a decrease in the excess return of the domestic currency. Intuitively, the increase in covariance makes domestic bonds more attractive to foreigners and generates an excess demand for domestic bonds. The decline in the excess return of the domestic currency clears this excess demand.

How does the excess return in domestic currency adjust in practice? Consider, for instance, an increase in the excess return. Since the foreign interest rate i^* is exogenous, this implies a higher domestic real interest rate $(1+i)S_1\mathbb{E}(1/S_2)$. $\mathbb{E}(1/S_2)$ is exogenous and equal to 1. As a result, a higher b^{H*} increases $(1+i)S_1$. Whether it affects i or S_1 depends on the policy rule $f(i, S_1)$, and does not matter for our analysis.

3.3 When Are Foreign Exchange Reserve Interventions Effective?

As apparent in Equation (19), everything else equal, sterilized FX interventions b^{CBF} affect b^{H*}, the gross foreign liabilities of the economy. However, in equilibrium, households' portfolio adjustments, and especially adjustments in b^H, could offset the central bank intervention. In what follows, we consider FX interventions to be "effective" only if they affect the gross foreign liabilities of the economy b^{H*} in equilibrium. When interventions are effective, they affect $(1+i)S_1$ through the change in b^{H*}, the effect of which is discussed in the previous subsection.

To understand, consider the consolidated household's budget constraints, which are obtained using the equilibrium in the domestic asset markets, and substituting transfers in the household's budget constraint:

$$\begin{aligned} c_1 &= y_1 + nfl \\ c_2 &= y_2 - (1+i^*)nfl - X^* b^{H*} \end{aligned} \tag{22}$$

If households are unconstrained, they can affect the gross foreign position b^{H*} by changing their holdings of domestic and foreign bonds. To study FX intervention effectiveness, we introduce the concept of "desired" gross foreign liabilities, which we denote by b^{max}. b^{max} is the level of gross foreign liabilities that would satisfy portfolio optimality (as introduced in Definition 1). When the amount of publicly-issued domestic bonds $b^G + b^{CBF}$ is below the desired level of gross foreign liabilities b^{max}, households are constrained because they would like to choose a negative b^H.

To characterize b^{max} and solve the model, we now assume a logarithmic utility function: $U(.) = log(.)$ and use an approximation of the model (see Appendix C). We focus on a safe haven economy that is characterized by an exchange rate that is negatively correlated with the global factor y^* (and positively correlated with the foreign intermediaries' SDF) and by an output that is weakly correlated with the global factor. More precisely, we assume:

Condition 1 (Safe haven economy) *A safe haven economy is an economy where* $\rho > 0$ *and* $0 \leq \alpha < 1/(1+b^G)$.

To characterize the solution more precisely, it is useful to introduce the following two conditions:

Condition 2 (Risky environment) *A risky environment is an environment where* Γ *and* χ *are small relative to* σ^2.

This condition ensures that risk is more relevant than financial frictions. CIP deviations will be smaller in magnitude than UIP deviations.

Condition 3 (Technical condition) g, Γ, χ *and* σ^2 *are small relative to* 1.

This technical condition ensures the existence of a unique and well-behaved solution. It implies that the parameters are such that the net foreign position and the excess returns are small, which is realistic.[6]

In Appendix C.4, we show that, under these three conditions, $b^{max} = \frac{(1-\alpha)\rho\sigma^2-\chi}{\Gamma+\rho^2\sigma^2} - \frac{\alpha\rho\sigma^2 \min(nfl^{opt}, b^G)}{\Gamma+\rho^2\sigma^2}$, where nfl^{opt} is the level of net foreign liabilities that holds under both intertemporal and portfolio optimality. nfl^{opt} is independent of fiscal and central bank policy.

The impact of FX interventions can then be described formally in the following proposition (see proof in Appendix C.4):

Proposition 1 *We assume that Conditions 1, 2, and 3 hold. If* $b^{CBF} + b^G < b^{max}$, *then FX interventions are effective:* $b^{H*} = b^G + b^{CBF}$ *and* $b^H = 0$. *FX interventions that increase* b^{CBF} *increase gross foreign liabilities* b^{H*} *one-for-one, and increase* $\mathbb{E}X^*$ *and* Z^*.

According to the Wallace Irrelevance Proposition (Wallace, 1978), if households are free to adjust their portfolio, FX interventions are neutral, because households can offset any central bank open-market operation. In our model though, households face short-selling constraints, which breaks the Wallace Irrelevance under some conditions. In particular, sterilized FX interventions cannot be offset when the sum of government bonds and foreign reserves $b^G + b^{CBF}$ is below b^{max}, because offsetting would require households to issue domestic currency bonds, which they cannot. In that case, $b^{H*} = b^G + b^{CBF}$, and the effective supply of domestic bonds b^{H*} responds directly to FX interventions b^{CBF}. This extra supply of domestic bonds is absorbed by financial intermediaries through an increase in the excess returns $\mathbb{E}X^*$ and Z^* as discussed above. The Wallace Irrelevance result therefore breaks in situations where households are constrained in their ability to

[6] Appendix C provides more details about the model's properties in a linear-quadratic approximation. Sections C.1, C.2 and C.3 derive respectively the asset pricing equations, the households' stochastic discount factor, and the households' first-order conditions.

issue or save domestic currency bonds. This implies that FX interventions are effective when they do not satiate the households' need for gross foreign liabilities.

An interesting question is whether FX interventions can affect net foreign liabilities. Appendix C.5 shows that there is no impact on nfl when b^G is small. When b^G is large, there may be an impact, as households are able to smooth their consumption when FX interventions affect their period-2 income, using their foreign-currency bond holdings.[7]

Under the conditions of Proposition 1, b^{H*} depends only on b^{CBF} and b^G. According to the CIP expression (7), this implies that the intermediation friction parameters Γ and χ directly affect the CIP deviation: an increase in collateral requirements Γ, or an increase in the convenience yield, generate a more positive CIP deviation. Given that $cov(m^*, X^*)/\mathbb{E}m^* = e^{\rho\sigma^2} - 1$ only depends on ρ and σ, shocks to Γ and χ move the UIP deviation in the same direction and with the same magnitude as the CIP deviation, according to the UIP expression (8). Shocks to global risk σ, in contrast, only move the UIP deviation: an increase in global risk makes the UIP deviation more negative, while the CIP deviation remains the same.

4 On the Cost of Foreign Reserves

In this section, we introduce the utility cost of FX reserves and assess its relation to UIP and CIP deviations. We show that the key determinant of this cost is the covariance between excess returns and the SDF of domestic households on the one hand, and the SDF of international financial intermediaries on the other. In the context of safe-haven currencies, it depends on whether international financial intermediaries value the safe-haven properties more than domestic investors. We examine this issue empirically and show that this is the case for Switzerland and Japan.

4.1 Utility Cost of Reserves with UIP and CIP Deviations

The cost of reserves, in the traditional sense of the literature, is the forgone earnings on reserve holdings.[8] In our framework, this corresponds to X^*. One can see this simply by noticing that the central bank's resource constraints (17) and (18) imply that the central bank's profits depend directly on X^*: $\tau_2^{CB} = -X^* b^{CBF}$.

[7] In that case, FX interventions increase nfl (see Appendix C.5). This is because FX interventions increase period-2 income, so that a higher nfl helps households' smooth their consumption. In models where households are allowed to borrow, but are facing a higher interest rate due to an external finance premium, sterilized FX interventions have an impact on the economy's net position through a "spread channel" (e.g., see Bianchi and Lorenzoni, 2022). FX interventions widen the spread, which affects households' borrowing decisions and increases the economy's net foreign assets, through an intertemporal substitution channel.

[8] See for instance Frenkel and Jovanovic (1981), Jeanne and Rancière (2011), and more recently Adler and Mano (2021).

In a safe haven case with $\mathbb{E}X^* < 0$, there is an expected gain of holding reserves, i.e., the central bank can exploit the UIP deviation. However, the increase in reserves also increases the exchange-rate risk faced by the central bank. Since the central bank's profits are eventually distributed to households, the question is whether this could increase households' utility. We therefore introduce a novel concept of the cost of reserves, the marginal *utility* cost of reserves, defined as the "traditional" monetary cost of reserves discounted by the household's SDF:

Definition 2 (The marginal utility cost of FX interventions) *The marginal utility cost of FX interventions is the expected product of the UIP deviation X^* and the SDF of domestic households m, divided by the expected discount factor:*

$$UCFX = \frac{\mathbb{E}(mX^*)}{\mathbb{E}m} \tag{23}$$

The excess return on domestic bonds X^* is valued using the utility-based stochastic discount factor. It is normalized by the expected discount factor so that it coincides with the traditional monetary cost X^* in the absence of risk. In that sense, $UCFX$ can be seen as the certainty-equivalent cost of reserves. It can also be seen as a measure of the deviation from portfolio optimality (see Equation (14) and Definition 1), evaluated in units of future goods.

To see how FX intervention potentially affects utility, consider the resource constraints (22). Everything else equal, FX interventions increase the economy's gross foreign position and increase the economy's exposure to currency risk through changes in b^{H*}.

4.2 A Decomposition of the Utility Cost of Reserves

The marginal utility cost of FX interventions can be rewritten as

$$UCFX = \mathbb{E}X^* + \frac{cov(m, X^*)}{\mathbb{E}m} \tag{24}$$

The utility cost is composed of the expected excess return on foreign bonds, minus the risk premium associated with this excess return. Since $\mathbb{E}X^* < 0$ for safe haven countries, there may be a utility gain.

Substituting $\mathbb{E}X^*$ using Equation (8), we can rewrite the utility cost of FX interventions:

$$UCFX = \underbrace{\frac{\overbrace{\Gamma b^{H*} + \chi}^{Z^*}}{\mathbb{E}m^*} - \frac{cov(m^*, X^*)}{\mathbb{E}m^*}}_{\mathbb{E}X^*} + \frac{cov(m, X^*)}{\mathbb{E}m} \tag{25}$$

Equation (25) shows how CIP and UIP deviations affect the utility cost. Notice that the risk premium terms $\frac{cov(m^*,X^*)}{\mathbb{E}m^*}$ and $\frac{cov(m,X^*)}{\mathbb{E}m}$ enter $UCFX$ for very different reasons. The first risk premium affects the pricing of the excess return, since international financial intermediaries are the marginal investors. The second risk premium only affects its welfare valuation. The role of these risk premia can be summarized in the following proposition

Proposition 2 *Consider the SDF of domestic households, m, and of international financial intermediaries m^* and the excess return in foreign currency, X^*. The utility cost (or benefit) of FX intervention is equal to*

(i) CIP deviations Z^* when $cov(m, X^*)/\mathbb{E}m = cov(m^*, X^*)/\mathbb{E}m^*$.

(ii) UIP deviations $\mathbb{E}X^*$ when $cov(m, X^*) = 0$.

In fact, $UCFX$ is equal to the CIP deviation Z^*, which is a riskless excess return; minus the difference between the foreign and domestic risk premia, which we denote $\Delta Cov = cov(m^*, X^*)/\mathbb{E}m^* - cov(m, X^*)/\mathbb{E}m$:

$$UCFX = Z^* - \Delta Cov \tag{26}$$

The first term is an intermediation wedge and the second is a risk-sharing wedge. If foreign and domestic agents have the same risk premium, only CIP deviations Z^* matter. This is the case in the absence of risk, as in Amador et al. (2020), or when financial intermediaries have the same discount factor as households. In contrast, in the limit case where domestic agents have negligible risk aversion compared to financial intermediaries, the sum of the two wedges is equal to the UIP deviation and the cost of reserves would be equal to the UIP deviations.[9]

In general, the sum of the two wedges does not coincide with either the CIP or the UIP deviations. A safe haven currency may be more desirable for foreign investors as a diversification hedge than for domestic investors so that $\Delta cov > 0$. If the difference is large enough, there may be a utility gain from accumulating reserves, rather than a cost.

4.3 When Does a Utility Cost or Gain of Reserves Arise?

A non-zero utility cost of reserves represents an arbitrage opportunity. We can see that by using the households' FOC (14) and the definition of the utility cost of reserves (23):

$$-UCFX = \frac{\lambda^H - \lambda^F}{\mathbb{E}m} \tag{27}$$

A necessary condition for a negative $UCFX$ is $\lambda^H > 0$, that is, a binding domestic currency bond short-selling constraint. A negative UCFX means that households would be willing to engage in a domestic currency carry trade: go short in domestic currency and long in foreign currency. This arbitrage opportunity is an equilibrium outcome only if households are prevented from going short in domestic currency.[10]

[9] This is what Itskhoki and Mukhin (2021) implicitly assume. In their linear approximation, they take the level of risk to zero but ensure that the risk premium of financial intermediaries remains a first-order object by rescaling their risk aversion, but not that of households. This implies that the intermediary risk aversion is an order of magnitude higher than that of households.

[10] On the opposite, a positive utility cost of reserves can only arise if $\lambda^F > 0$, that is, households cannot go short in foreign currency.

To understand, consider the budget constraints (22), which clearly show that consumption in period 2 depends on the foreign currency excess return $-X^*$ and on the households' carry trade position b^{H*}. Suppose that $UCFX$ is negative. From Equation (24) this means that the expected excess return on foreign currency $-\mathbb{E}X^*$ is high relative to the households' currency risk premium $Cov(m, X^*)/\mathbb{E}m$. Then, a positive b^{H*} is attractive as it increases average consumption in period 2 without excessively exposing households to currency risk. If households are not constrained, they can set b^{H*} high enough by changing their domestic currency position b^H. As a result, the two wedges in Equation (26) adjust so that $UCFX = 0$. On the one hand, the more domestic bonds need to be absorbed by financial intermediaries, the higher the excess return (Z^* is higher). On the other hand, the more household consumption is exposed to currency risk, the higher the household covariance and the smaller the differential ΔCov. In Appendix C.6, we show that ΔCov depends indeed negatively on b^{H*}, in a linear approximation:

$$\Delta Cov = \rho\sigma_y^2 \left[1 - \alpha(1 + nfl) - \rho b^{H*}\right] \tag{28}$$

In contrast, when the domestic short-selling constraint is binding, these adjustments do not take place, so $UCFX < 0$.

The following proposition establishes that a utility gain of reserves appears when the supply of bonds by the central bank and government is insufficient (see proof in Appendix C.7):

Proposition 3 *We assume that Conditions 1, 2, and 3 hold. If $b^{CBF} + b^G < b^{max}$, then $UCFX < 0$ and $UCFX$ is increasing in b^{CBF}.*

When the total supply of domestic bonds by the government and the central bank is below b^{max}, gross foreign liabilities b^{H*} are not sufficient to exhaust the risk-adjusted excess return associated to domestic carry trade $-UCFX$. Households would like to issue domestic bonds so that their total exposure to currency risk b^{H*} reaches b^{max}, but they face a short-selling constraint.

An implication of Propositions 1 and 3 is that a utility gain of reserves appears in situations where FX interventions are effective. In these situations, an increase in b^{CBF} increases b^{H*}. FX intervention can thus generate a gain for the economy. However, these gains are not unlimited. As the central bank accumulates reserves, $UCFX$ becomes less negative, until $UCFX = 0$ when $b^{CBF} = b^{max} - b^G$.

The role of financial frictions What role do financial frictions play to generate a utility gain or cost of reserves? The key friction that makes $UCFX$ strictly negative (or positive) is the households' limited bond market participation. This is evident from Equation (27), where a non-zero $UCFX$ relies on at least one short-selling constraint being binding ($\lambda^H > 0$ or $\lambda^F > 0$), as discussed above. A utility gain or cost of reserves arises from the households' inability to exploit an arbitrage opportunity by themselves. However, for households to face an arbitrage opportunity, it must be that financial intermediaries do not fully exhaust it.

An arbitrage opportunity can have two origins in our framework, as shown by the decomposition of $UCFX$ in (26). First, the CIP deviation Z^* is a return that is not fully arbitraged away by financial intermediaries, thus generating an arbitrage opportunity for households. As Equation (7) shows, this return is positive due intermediation frictions: financial intermediaries face a participation constraint ($\Gamma > 0$) and a foreign currency convenience yield ($\chi > 0$).

Second, the covariance differential ΔCov represents the discrepancy between the households' valuation of currency risk (the households' risk premium) and the price of that risk (the intermediaries' risk premium). Even if the intermediaries' arbitrage opportunity vanishes ($Z^* = \mathbb{E}(m^* X^*) = 0$) – e.g., in the absence of intermediation frictions –, an arbitrage opportunity could still persist from the point of view of households ($UCFX = \mathbb{E}(mX^*) \neq 0$), because households value risk differently from intermediaries. This arbitrage opportunity thus hinges on the risky environment assumption. To see this, note that Equation (28) implies that $\Delta Cov = 0$ if $\sigma = 0$.

The role of finite horizon The two-period assumption of our model allows us to assume that the period-2 exchange rate is exogenous and its correlation with the global factor is given. With more than two periods, anticipated future FX interventions could affect the exchange rate dynamics and its stochastic properties. However, the main properties of the model would still hold, since the exchange rate dynamics is taken as given by households and intermediaries. In particular, Proposition 2 and the equations for $UCFX$ would remain unchanged. However, in a dynamic model, the stochastic properties of the exchange rate would be endogenous, and so would the safe-haven status of a currency. A comprehensive dynamic extension, in which a safe-haven currency emerges endogenously, presents an interesting avenue for future research.[11]

4.4 Numerical illustration

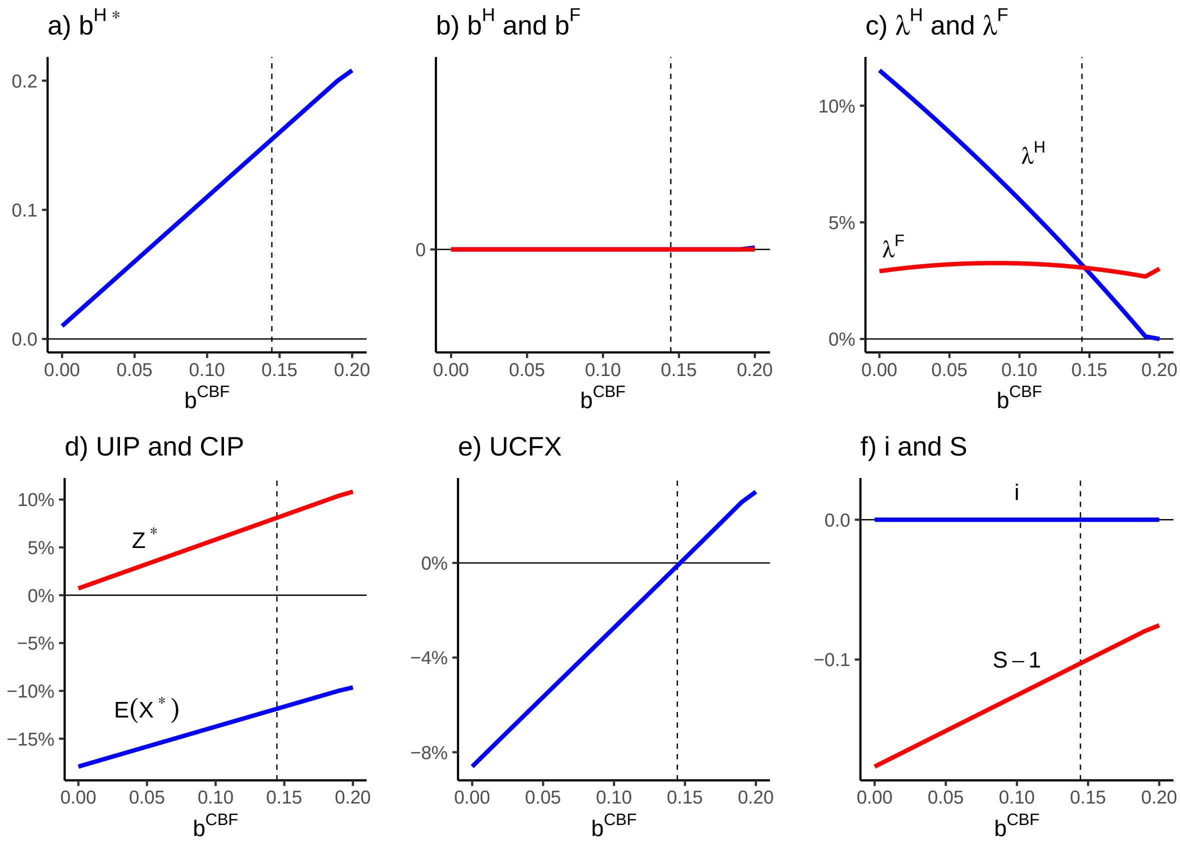
We consider a numerical illustration of the mechanism, setting parameters that are consistent with Conditions 1, 2, and 3. We assume the following policy rule:

$$\begin{aligned} f(i, S_1) &= S_1 - 1 && \text{if } i > 0 \\ &= i && \text{if } S_1 < 1 \end{aligned} \tag{29}$$

This policy rule assumes that the nominal interest adjusts to target a fixed exchange rate, but that it faces a zero lower bound (ZLB).

[11] There is a small literature which provides explanations for safe-haven effects, but the focus is on the US and the mechanisms do not apply to small countries. See Maggiori (2017) or Hassan et al. (2022). Papers that model time-varying safe haven effects include Gourinchas and Rey (2022), Devereux et al. (2022), and Kekre and Lenel (2024).

Figure 1: The Effectiveness of FX Interventions and the Utility Cost of Reserves



Notes: Parameter values : $\beta = 0.98$, $\sigma^2 = 1$, $\chi = 0.002$, $\Gamma = 0.5$, $\alpha = 0.6$, $\rho = 0.2$, $g = 0.05$, $b^G = 0.01$. The dashed vertical lines represent $b^{CBF} = b^{max} - b^G$.

Figure 1 describes the equilibrium effect of FX reserves b^{CBF} when FX intervention is effective, i.e., when the increase in b^{CBF} increases b^{H*}. We consider the simpler case where the constraints are always binding ($\lambda^H > 0$ and $\lambda^F > 0$), which is obtained by setting a low b^G, and where the interest rate is at the ZLB.[12] The UIP deviation $\mathbb{E}X^*$ is negative because the domestic currency has a safe-haven value for intermediaries, and the CIP deviation Z^* is positive because of the intermediaries' participation constraint (see Panel d)). As b^{CBF} increases, the CIP deviation becomes more positive (and the UIP less negative), as the excess supply of domestic bonds arising from FX interventions is absorbed by an increase in the real interest rate. This is achieved in equilibrium through an exchange rate depreciation, because the interest rate is at the ZLB (see Panel f)). Otherwise, it would be achieved through a nominal interest rate increase.

In Panel e), a negative $UCFX$ represents a carry-trade arbitrage opportunity for households. However, households cannot issue domestic bonds (λ^H is positive, as shown in Panel c)). This implies that FX interventions are effective, in the sense that the gross foreign position of the economy b^{H*} increases with b^{CBF} (see Panel a)). This is consistent with Proposition 1.

As b^{CBF} increases, $UCFX$ becomes less negative, due to the increase in Z^*, but also

[12] Figure E.1 considers the more complex case where b^G is higher and constraints are not always binding.

because households become more exposed to currency risk, as explained above. When b^{CBF} reaches $b^{max} - b^G$, $UCFX$ becomes equal to zero.

In this specification, FX interventions continue to be effective beyond $b^{max}-b^G$. This is because, for $b^{CBF} > b^{max} - b^G$, $UCFX$ becomes positive: households become excessively exposed to domestic currency. They want to offset FX interventions by engaging in another form of carry trade: going short in foreign currency, and long in domestic currency. However, households are constrained in their capacity to issue foreign bonds (λ^F is positive in Panel c)). As a result, FX interventions continue to be effective beyond $b^{max} - b^G$.

Here, FX interventions are always effective because households are always constrained for both domestic and foreign bonds. They are constrained because they would like to have a large net foreign position nfl, but b^G is too small to achieve that level of nfl. In Figure E.1 in the Appendix, we represent a specification with a larger b^G. In that case, b^G can accommodate the desired nfl and, for $b^{CBF} = b^{max} - b^G$, households hold a positive amount of foreign bonds. Therefore, if b^{CBF} increases above $b^{max} - b^G$, households can offset FX interventions by liquidating foreign bonds and buying domestic bonds, but only up to the point where it exhausts their stock of foreign bonds.

4.5 Estimating the Utility Cost for Switzerland and Japan

The theoretical analysis has shown that the utility cost of FX interventions depends crucially on the difference between $cov(m^*, X^*)/\mathbb{E}m^*$ and $cov(m, X^*)/\mathbb{E}m$. In this subsection, we provide estimates of these two terms for Switzerland and Japan. First, Appendix A confirms that both countries can be considered safe havens, in the sense that the excess return on their currencies is positively related to global risk variables. Since X^* is priced by financial intermediaries, we expect $cov(m^*, X^*)/\mathbb{E}m^*$ to reflect the safe haven property of these currencies. In contrast, the model has no implications for $cov(m, X^*)/\mathbb{E}m$. For the rest of this section, we introduce time t in our notation.

A key issue is the measurement of stochastic discount factors m_{t+1} and m^*_{t+1}. The former is not reflected in asset prices, while the latter is. For domestic households, we simply assume that $m_{t+1} = \beta(c_{t+1}/c_t)^{-\gamma}$, where $1/\gamma$ is the rate of intertemporal substitution. For international financial intermediaries, we follow the literature on intermediary asset pricing (e.g., He and Krishnamurthy, 2011; Brunnermeier and Sannikov, 2014), and assume that their SDF is proportional to their net worth NW_t:[13]

$$m^*_{t+1} = \beta \left(\frac{NW_{t+1}}{NW_t}\right)^{-\gamma} \tag{30}$$

As in He et al. (2017), we assume that the financial intermediaries' net worth is equal to the aggregate wealth in the economy (denoted by W_t) multiplied by the intermediaries'

[13]Here, we implicitly assume that, on top of the Gabaix-Maggiori constraint on their international arbitrage, financial intermediaries face a borrowing constraint on their overall balance sheet, such that their constraint depends on their net worth, e.g. as in Gertler and Kiyotaki (2010). This gives rise to intermediary asset pricing in the form of Equation (30).

capital ratio (denoted by η_t). This specification implies that the financial intermediaries' marginal utility of wealth rises when either the aggregate wealth in the economy or the equity capital ratio is low. The first term captures the asset pricing effect of weaker fundamentals, while the second captures the idea that the intermediaries' risk-bearing capacity is impaired when the capital ratio is low. As a result, risk aversion increases the marginal value of wealth. Using time-series and cross-sectional asset pricing tests, He et al. (2017) show that this specification captures well the marginal utility of wealth of financial intermediaries, and find supporting evidence that financial intermediaries are indeed marginal investors for a wide class of assets.

In our empirical exercise, we consider two measures of the capital ratio (η_t) and two measures of aggregate wealth (W_t), giving rise to four different possible specifications. For the first capital ratio measure, we consider the equity capital ratio of financial intermediaries (*Primary Dealer* counterparties of the New York Federal Reserve) from He et al. (2017), which we denote by η_{t+1}^{HKM}. The second measure is from Adrian et al. (2014), and is defined as the (inverse of) book leverage of security *Brokers & Dealers*.[14] We denote it as η_{t+1}^{AEM}. For total wealth, we consider a real measure using US GDP (W_t^{GDP}) and a financial measure using the MSCI World Equity Index (W_t^{MSCI}).

As in He et al. (2017), our measure of net worth is obtained by interacting the capital ratio measure with the total wealth measure: $NW_t = \eta_t \times W_t$. To convert net worth into a growth rate (as suggested by (30)), we adopt an approach similar to He et al. (2017). For the capital ratio, we define the intermediary capital risk factor by dividing the residual from a regression of the capital ratio on its lag by the lagged capital ratio. For the financial measure of wealth (W_t^{MSCI}), we compute the excess returns on the equity index, using the 3-month US risk-free rate. For the real measure of wealth (W_t^{GDP}), we simply compute the growth rate. $\frac{NW_{t+1}}{NW_t}$ is then defined by the interaction of the intermediary capital risk factor and the growth rate measure of total wealth. Appendix B provides additional details about the sources of the data, as well as the construction of excess returns and stochastic discount factors.

We consider excess returns using the CHF and the JPY as the domestic currency and the USD as the foreign currency. Let us define the log excess returns of going long in the domestic currency from the international investors perspective:

$$x_{t+1}^* = i_t - i_t^* + s_t - s_{t+1} \tag{31}$$

We use x_{t+1}^* as an approximation of X_{t+1}^*.

Table 1 displays an estimate of $cov(m_{t+1}^*, x_{t+1}^*)/\mathbb{E}_t m_{t+1}^*$ and $cov(m_{t+1}, x_{t+1}^*)/\mathbb{E}_t m_{t+1}$ using either the CHF or the JPY as the domestic currency. We assume that $\beta = 0.99$ and $\gamma = 10$. For each currency, we consider two subsamples (2000M1-2009M12 and 2010M1-2022M12) to highlight potential time-variation in these measures. Columns 2 to 5 display

[14] It is obtained using balance sheet data reported in the Flow of Funds from the Federal Reserve Board. It is computed as the ratio of total equity (total financial assets minus total financial liabilities) to total financial assets.

Table 1: $\frac{Cov(x^*_{t+1}, m^*_{t+1})}{\mathbb{E}_t(m^*_{t+1})}$ and $\frac{Cov(x^*_{t+1}, m_{t+1})}{\mathbb{E}_t(m_{t+1})}$

A) CHF domestic currency, USD foreign currency					
	Fin. Intermediaries				HH
$NW_{t+1} =$	$\eta^{HKM}_{t+1} \times W^{MSCI}_{t+1}$	$\eta^{AEM}_{t+1} \times W^{MSCI}_{t+1}$	$\eta^{HKM}_{t+1} \times W^{GDP}_{t+1}$	$\eta^{AEM}_{t+1} \times W^{GDP}_{t+1}$	C^{CH}_{t+1}
1999-2010	0.2	-0.82	0.2	-0.79	0.25***
2010-2022	5.3***	0.96***	5.0**	0.8**	0.01

B) JPY domestic currency, USD foreign currency					
$NW_{t+1} =$	$\eta^{HKM}_{t+1} \times W^{MSCI}_{t+1}$	$\eta^{AEM}_{t+1} \times W^{MSCI}_{t+1}$	$\eta^{HKM}_{t+1} \times W^{GDP}_{t+1}$	$\eta^{AEM}_{t+1} \times W^{GDP}_{t+1}$	C^{JP}_{t+1}
1999-2010	-2.2	-1.16	-2.0	-1.46	0.7***
2010-2022	6.4***	1.02**	6.2***	1.01**	0.33

Note:
This table estimates $\frac{Cov(x^*_{t+1}, m^*_{t+1})}{\mathbb{E}_t(m^*_{t+1})}$ and $\frac{Cov(x^*_{t+1}, m_{t+1})}{\mathbb{E}_t(m_{t+1})}$ using different proxies of the SDF of (international) financial intermediaries and Swiss and Japanese households. Values are expressed in percentage points. Appendix B provides details on their construction and the source of the data. Statistical significance is assessed by regressing excess returns on the different measures of the SDF using Newey-West standard errors. $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$.

the covariance terms from the perspective of financial intermediaries using the capital ratio measure from He et al. (2017) and Adrian et al. (2014) and the two measures of total wealth to compute the SDF. The last column displays the covariance term for Swiss and Japanese households, using real consumption growth to compute the SDF. Statistical significance is assessed by regressing the excess returns on the different measures of SDF and using Newey-West standard errors.

The results show that, since 2010, the covariance term for financial intermediaries is clearly positive and statistically significant for most of the specifications of the stochastic discount factor, and quantitatively in line with the UIP deviations depicted in Figure A.1, reaching as high as 6.4% for Japan and 5.3% for Switzerland. Interestingly, the covariance term is generally an order of magnitude smaller (or negative) before 2010. Since 2010, being long in CHF or JPY tends to provide higher returns when the marginal utility of the wealth of financial intermediaries is high, which indicates that the CHF and the JPY behave as a hedge for international intermediaries. On the other hand, the covariance term between excess returns and SDF based on real domestic consumption growth tends to be much smaller and statistically not significant since 2010. For Switzerland and Japan, Proposition 2 implies that it is not CIP but UIP deviations that should matter for FX interventions since $cov(m_{t+1}, x^*_{t+1})/\mathbb{E}_t m_{t+1}$ is not significantly different from zero.

5 The Central Bank as a Constrained Planner

To determine how the cost of reserves influences the policy trade-offs of the central bank, we consider a central bank that maximizes households' welfare. We first reframe the central bank problem as that of a constrained central planner. We then show how the resulting optimal allocation can be decentralized using FX interventions.

Before that, we relate the country's consolidated financial liabilities to the households' short-selling constraint (11). Using the definition of b^{H*} in (19), we can show that the households constraint on domestic bond issuance translates into a constraint on gross foreign liabilities:

$$b^{H*} \leq b^{G} + b^{CBF} \tag{32}$$

However, (32) is not an effective constraint since the central bank can change its holding of foreign bonds b^{CBF}.

Similarly, the foreign currency no-borrowing constraint implies:

$$nfl \leq b^{H*} - b^{CBF} \tag{33}$$

This constraint cannot be relaxed by sterilized FX intervention since changes in b^{H*} are offset by changes in b^{CBF}.[15] This constraint is effective except if we allow the central bank to perform unsterilized interventions combined with fiscal transfers, where changes in b^{H*} need not be offset by changes in b^{CBF}. Equations (32) and (33) are equivalent to the no-borrowing constraints (11).

5.1 The Constrained Planner Program

Based on the previous equations, we can examine the planner's optimal choices.

Definition 3 (Constrained planner equilibrium) *A constrained planner equilibrium is an equilibrium in which a planner maximizes objective* (12) *subject to the economy's resource constraints* (22)*; the asset pricing equation* (6)*; the policy rule* $f(i, S_1) = 0$*; the foreign liability constraints* (32) *and* (33)*; and the definition of UIP* (3)*. The planner's choice variables are* $(i, S_1, b^{H*}, nfl, b^{CBF})$*.*

[15] When capital controls are in place, however, Bacchetta et al. (2013) show that sterilized interventions can affect the country's intertemporal allocation.

The central bank's program is:

$$
\begin{aligned}
&\max \mathbb{E}\Big\{ U(c_1) + \beta U(c_2) \\
&+\eta_1 \left(y_1 - c_1 + nfl\right) \\
&+\eta_2 \left[y_2 - c_2 - (1+i^*)nfl - \left[(1+i)\tfrac{S_1}{S_2} - (1+i^*)\right] b^{H*}\right] \\
&+\xi f(i, S_1) \\
&+\Lambda \left(b^{H*} - b^{CBF} - nfl\right) \\
&+\tilde{\Lambda} \left(b^{G} + b^{CBF} - b^{H*}\right) \\
&+\alpha_0 \left(\mathbb{E}\left(m^* \left[(1+i)\tfrac{S_1}{S_2} - (1+i^*)\right]\right) - \Gamma b^{H*} - \chi\right) \Big\}
\end{aligned}
$$

Consider the first-order conditions for assets:

$$
\begin{aligned}
&/nfl: && \eta_1 - \mathbb{E}\left(\eta_2(1+i^*)\right) && -\Lambda && = 0 && (34)\\
&/b^{H*}: && -\mathbb{E}\left(\eta_2 X^*\right) && +\Lambda - \tilde{\Lambda} - \alpha_0\Gamma && = 0 && (35)\\
&/b^{CBF}: && && -\Lambda + \tilde{\Lambda} && = 0 && (36)
\end{aligned}
$$

Equation (36) implies that $\tilde{\Lambda} - \Lambda = 0$. This means that the central bank equalizes the marginal benefit of relaxing the foreign-currency and domestic-currency debt constraints by adjusting its assets and liabilities and going shorter in the asset whose shadow cost is higher and longer in the asset whose shadow cost is lower. Also note that $\eta_1 = U'(c_1)$, $\eta_2 = U'(c_2)$, and that $m = \eta_2/\eta_1$ is the central bank's discount factor, which coincides with the household's (see Appendix D.1).

5.2 Optimal foreign exchange interventions

Equation (35), with $\Lambda - \tilde{\Lambda} = 0$, and using other FOCs, can be rewritten as follows (see Appendix D.2):

$$
\underbrace{\overbrace{-\mathbb{E}X^* - \frac{cov(m, X^*)}{\mathbb{E}m}}^{-UCFX} - \overbrace{\Gamma b^{H*} \frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}}^{\mu}}_{MBFX} = 0 \tag{37}
$$

The left-hand side, $MBFX$, corresponds to the marginal benefit of sterilized FX interventions, that is, of expanding the central bank's balance sheet by going long in foreign bonds and short in domestic bonds. This marginal benefit is composed of the marginal utility benefit of FX interventions $-UCFX$ minus a dynamic terms-of-trade externality μ (as in Costinot et al., 2014). If, in the absence of interventions, $MBFX$ is positive, then it would be optimal for the central bank to accumulate FX reserves. These interventions can drive the marginal benefit to zero, achieving an optimal central bank balance sheet.[16]

[16] $MBFX$, the wedge addressed by the central bank, is a portfolio allocation wedge: issuing domestic-currency bonds to acquire foreign-currency bonds helps achieve the optimal currency composition for the country's portfolio. This is the case in other papers, like Basu et al. (2020), Maggiori (2021), Itskhoki and Mukhin (2022), where a similar wedge is called "UIP wedge", or "risk-sharing wedge".

Sterilized FX interventions cannot address intertemporal optimality. This can be seen from Equation (34), which implies that $\Lambda = 1 - \mathbb{E}[m(1+i^*)] \geq 0$. If $\Lambda > 0$, then $nfl = b^G$ whatever is the level of reserves, because households cannot change their net position. If $\Lambda = 0$, then households are not constrained in their intertemporal allocation and, as we have seen, interventions can affect nfl, but there is no intertemporal wedge to address $(\mathbb{E}[m(1+i^*)]) = 1)$.[17]

The dynamic terms-of-trade externality μ arises from the distortions in the prices (interest rate and exchange rate) implied by the central bank's interventions. μ depends on Γ, because FX interventions have an impact on equilibrium prices only if $\Gamma > 0$. In that case, FX interventions decrease the excess return on foreign currency. If, additionally, the country is short in domestic currency ($b^{H*} > 0$), then reducing this excess return has a cost ($\mu > 0$). This means that (i) the social benefit of FX interventions $MBFX$ is strictly higher than the private benefit $-UCFX$, and (ii) the central bank has an incentive not to fully shut down its risk-adjusted foreign currency excess return in order to maximize its profit: $-UCFX = \mu > 0$. This term reflects the central bank's rent as a monopolistic issuer of domestic bonds.

If Equation (37) is satisfied, then Equation (27) implies that $\lambda^H - \lambda^F = \mu > 0$. This means that the central bank policy does not satisfy the households' portfolio optimality. In fact, because $MBFX > -UCFX$, the level of gross foreign liabilities desired by the central bank is lower than the one desired by households (b^{max}). By limiting the issuance of reserves, the central bank can keep the effective supply of domestic bonds by the economy below b^{max}.

The following lemma sets explicit expressions for optimal FX interventions.

Lemma 1 *Suppose that Conditions 1, 2 and 3 are satisfied. Let $\widehat{nfl}^{opt}$ be the level of net foreign liabilities that holds under both intertemporal optimality and $MBFX = 0$, and denote by $\widehat{b}$ the optimal gross foreign liabilities set by the central bank acting as a constrained planner. Then:*

(i) $\widehat{b}$ is defined by

$$\widehat{b} = \frac{(1-\alpha)\rho\sigma^2 - \chi}{2\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2 \min(\widehat{nfl}^{opt}, b^G)}{2\Gamma + \rho^2\sigma^2} \tag{38}$$

(ii) $\widehat{b} < b^{max}$ if $\Gamma > 0$ and $\widehat{b} = b^{max}$ if $\Gamma = 0$.

(iii) The optimal level of FX reserves is given by

$$\widehat{b}^{CBF} = \widehat{b} - b^G \tag{39}$$

[17] In other papers, the central bank addresses an intertemporal wedge due to financial constraints and pecuniary externalities. In Bianchi and Lorenzoni (2022), sterilized interventions can address the intertemporal wedge through a spread channel. In Arce et al. (2019); Davis et al. (2023); Kim and Zhang (2023), households are constrained, as in our framework, but the central bank is allowed to perform unsterilized interventions combined with fiscal transfers. In Appendix D.5, we consider an extension of our model where the central bank is allowed to perform these fiscally-backed interventions. We show that, in that case, the central bank achieves both intertemporal optimality and $MBFX = 0$.

See the proof in Appendix D.3. Point (i) derives from the optimality condition with respect to b^{H*}, Equation (37). The level of gross domestic liabilities desired by the central bank $\widehat{b}$ can be implemented because first, the central bank is not constrained in its gross portfolio composition and, second, because, as argued above, the marginal benefit of accumulating reserves $MBFX$ is higher than the households' utility gain $UCFX$. It is strictly higher when FX interventions affect the excess currency return, that is, when $\Gamma > 0$. This explains point (ii). The optimum is then easily implementable for the central bank by supplying just the right amount of domestic liabilities to complement the existing public domestic supply and reach $\widehat{b}$, through FX interventions $\widehat{b}^{CBF}$ (point (iii)). For that level of domestic currency bonds, households would like to issue more domestic currency bonds ($b^H < 0$), but they are prevented from doing so by their short-selling constraints.

In what follows, we suppose that $b^G < \widehat{nfl}^{opt}$ so that $nfl = b^G$, which makes the problem more tractable. In that case, the optimal level of intervention is given by

$$\widehat{b}^{CBF} = \frac{\rho\sigma^2[1 - \alpha(1 + b^G)] - \chi}{2\Gamma + \rho^2\sigma^2} - b^G$$

where we used equations (38) and (39).

The comparative statics for optimal FX intervention is given in the following proposition:

Proposition 4 *Suppose that Conditions 1, 2 and 3 are satisfied and that $b^G < \widehat{nfl}^{opt}$. Then optimal FX interventions, $\widehat{b}^{CBF}$:*

(i) are increasing in global risk σ;

(ii) are decreasing in intermediaries financial frictions Γ and χ;

(iii) are decreasing in the domestic output exposure to global risk α;

(iv) are decreasing in the supply of government bonds b^G.

Points (ii) to (iv) can be shown by taking the derivatives of $\widehat{b}^{CBF}$ with respect to Γ, χ, α, and b^G. Point (i) is obtained by taking the derivative with respect to σ, but we need that Γ is small for the derivative to be negative, which is the case under Condition 3. Risk tends to increase the covariance differential ΔCov, which generates an excess benefit of FX interventions, while the intermediation frictions generate a cost. The exposure of domestic output to global risk decreases the covariance differential and generates a cost. Point (iv) arises from the substitutability between government bonds and the central bank liabilities (see Equation (39)). If the government issues more bonds, then this reduces the need for the central bank to issue liabilities through FX interventions.

Interestingly, an increase in global risk, which increases the optimal b^{H*}, typically generates both a more negative UIP deviation and a more positive CIP deviation, as the financial intermediaries have to absorb the excess domestic currency bonds. This is established formally in the following proposition:

Proposition 5 *Suppose that Conditions 1, 2 and 3 are satisfied and that* $b^G < \widehat{nfl}^{opt}$. *Then, under optimal FX intervention:*

(i) Z^* *is increasing in* σ *(it becomes more positive);*

(ii) $\mathbb{E}X^*$ *is decreasing in* σ *(it becomes more negative).*

(iii) $UCFX$ *is decreasing in* σ *(it becomes more negative).*

See the proof in Appendix D.4. As risk increases, the UIP deviation becomes more negative, because it affects positively the foreigners risk premium. $UCFX$ also becomes more negative (the utility gain of reserves increases). The planner responds by increasing the stock of reserves through FX interventions. However, it is not in the planner's interest to completely offset the impact of risk on $UCFX$, because the monopoly rent increases with risk as the foreign intermediaries have more appetite for the domestic currency. The CIP deviation becomes more positive when risk increases, as financial intermediaries need to absorb more capital inflows to finance the excess domestic liabilities b^{H*} resulting from the interventions.

5.3 Numerical Illustration

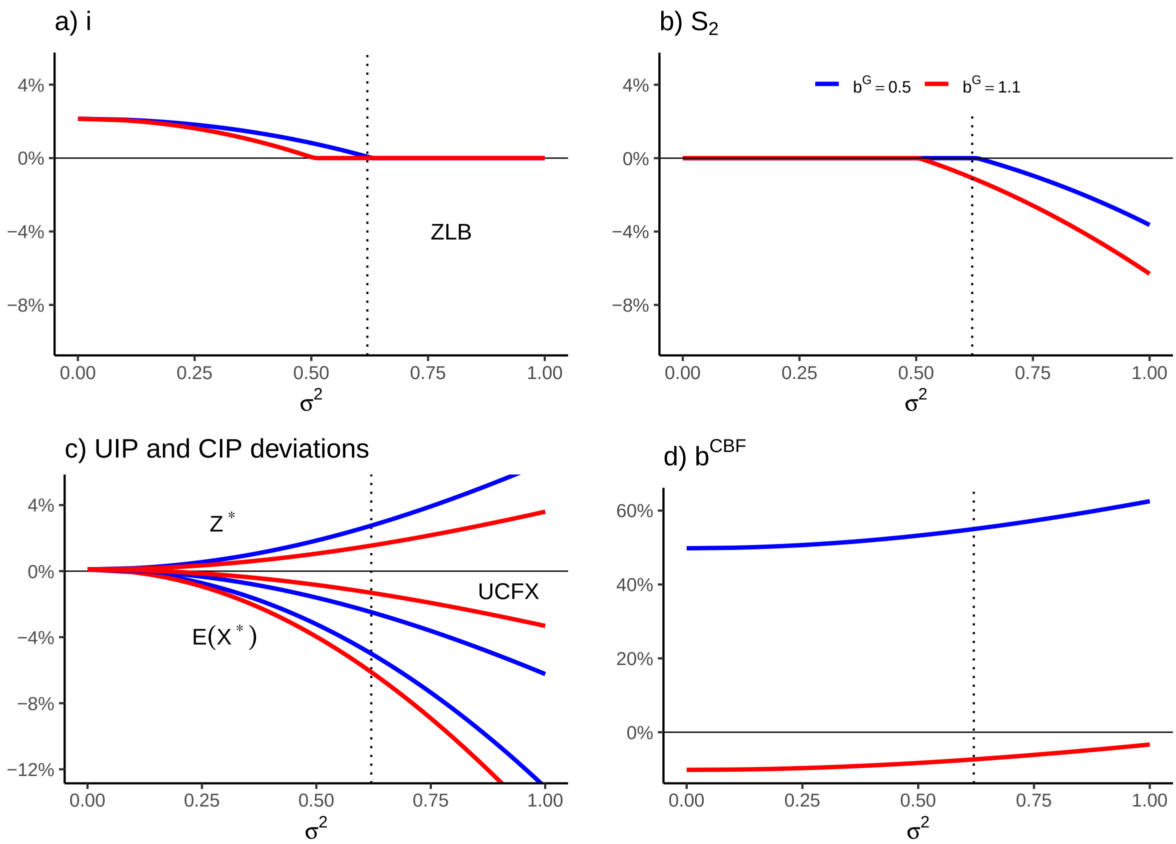
We examine in more detail how the level of global risk σ^2 affects the economy and optimal policy through a numerical example. We assume policy rule (29) is in place. Figure 2 shows the comparative statics of σ^2 under a baseline specification of parameters similar to Figure 1. We also consider two levels of b^G: 0.5 and 1.1.

Panel a) shows the negative relationship between the domestic interest rate and global risk: the higher demand for domestic bonds is accommodated through a decline in the domestic nominal interest rate. The ZLB is attained at $\sigma^2 \geq 0.62$. Outside the ZLB, the interest rate declines to accommodate the higher demand for domestic bonds (see Panel b)), while at the ZLB, the domestic currency appreciates. Panel c) displays the deviations from CIP (Z^*) and from UIP ($\mathbb{E}X^*$), as well as the utility cost of reserves ($UCFX$). As we can see, an increase in risk leads to a more positive CIP deviation, and a more negative UIP deviation and cost of reserves, as explained in Proposition 5.

As shown in Panels a) to c), a higher public debt b^G reduces the domestic currency excess return (generating a less positive CIP deviation and a more negative UIP deviation) through a lower interest rate or an appreciated currency. As both short-selling constraints are binding, a higher level of public debt increases the level of net foreign liabilities nfl. With a higher nfl, households have a higher marginal utility in period 2, which makes them more risk averse. This tends to reduce the benefits of carry-trade, so the central bank targets lower domestic gross liabilities b^{H*}. Then, the lower equilibrium interest rate (or more appreciated exchange rate) results from the smaller supply of domestic assets.

Panel d) shows that $\widehat{b}^{CBF}$ increases with global risk, as stated in Proposition 5. An increase in risk raises the benefit of FX interventions, which the central bank takes advantage of by buying FX reserves. However, the level of $\widehat{b}^{CBF}$ is only positive when $b^G = 0.5$. When b^G is large, the central bank is long in domestic bonds rather than foreign bonds, and short in foreign bonds rather than domestic bonds. In that case, an increase in risk pushes the central bank to sell domestic bonds and decrease its foreign currency leverage. However, this is possible only if the central bank is allowed to be short in foreign currency. Otherwise, the central bank cannot exploit its advantage.

Figure 2: Comparative statics of σ^2



Notes: Baseline parameters : $\beta = 0.98$, $\chi = 0.002$, $\Gamma = 0.5$, $\alpha = 0.6$, $\rho = 0.2$, $g = 0.05$.

6 Conclusion

The GFC was followed by significant changes in the international monetary system. We have been observing systematic deviations from CIP, an increased demand for safe assets, and an expansion in central banks balance sheets. There has been a stronger demand for safe-haven currencies and more FX intervention by these countries' central banks. For example, the spectacular increase in the balance sheet of the Swiss National Bank has occurred exclusively through the purchase of foreign assets.

The objective of this paper is to provide a simple framework to clarify some aspects of these developments. To explain UIP and CIP deviations, we follow the recent literature that gives a key role to constrained international financial intermediaries. However, we assume that these intermediaries face exchange-rate risk, so that the UIP deviation incorporates a risk premium that is not present in the CIP deviation. In this context, financial intermediaries value the hedging properties of safe haven currencies.

We examine the opportunity cost of FX intervention when CIP and UIP deviations differ. We show that whether CIP or UIP deviations matter depends on how domestic residents value the hedging property of their currency compared to international investors. If they give no value to its hedging property, UIP deviations should matter. This may imply a benefit, and thus a higher incentive, for FX accumulation in a safe-haven economy. We show that incentives to accumulate FX reserves in safe haven countries increase with the level of global risk or of effective risk aversion of international intermediaries. In contrast, the incentives decrease with the level of debt.

We also attempt to estimate the opportunity cost of intervention for Switzerland and Japan. We find that in both countries, domestic households value the hedging properties of their currency less than international investors. Overall, the incentives for intervention are stronger for Switzerland as its public debt is much smaller than in Japan.

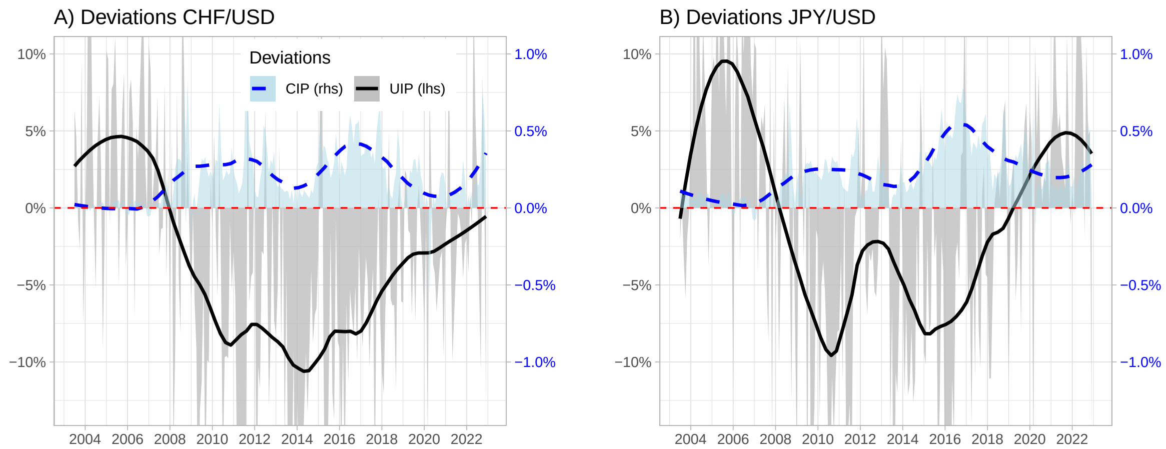
A CHF and JPY as Safe-Haven Currencies

In this Appendix, we show evidence on CIP and UIP deviations for Switzerland and Japan, as well as their safe haven properties.

A.1 UIP and CIP deviations

Figure A.1 shows CIP and UIP deviations for Switzerland and Japan, computed in percentage points as defined in Equations (2) and (3), taking the USD as the foreign currency and considering a 3-month horizon. Panel A) and B) consider the CHF and the JPY as the domestic currencies. For the short-term risk-free rates, we rely on the 3-month Libor before 2020M1, and on the OIS rate the subsequent period. The UIP deviations are computed using monthly data from Datastream for the risk-free rates and from Consensus Economics for the exchange rate forecasts and the spot exchange rates. The CIP deviations are monthly averages of daily observations and are computed using 3-month risk-free rates, spot exchange rates and forward rates with a 3-month maturity from Datastream. All returns are annualized.

Figure A.1: UIP and CIP Deviations



Notes: This figure shows UIP and CIP deviations. The smoothed lines are obtained by applying the LOESS method with a bandwidth parameter set to 0.3.

A.2 Safe-Haven Properties

The safe-haven properties of the Swiss franc and the Japanese yen have been documented by various authors, for example, Stavrakeva and Tang (2021), Ranaldo and Söderlind (2010), Grisse and Nitschka (2015), or Fink et al. (2022). We confirm this by relating expected excess returns to various sources of risk.

We compute UIP deviations using short-term rates from Datastream and survey data from Consensus Economics.[18] Table A.1 shows the correlation between expected excess returns in CHF and JPY ($\mathbb{E}X^*$) and different measures of risk. Since 2010, this correlation is systematically positive, suggesting that agents tend to expect the CHF and JPY to yield excess returns at times of heightened uncertainty. When considering the entire sample (from 1999 to 2022), the correlation is systematically weaker or negative, which suggests that the CHF and JPY have reinforced their perceived safe-haven properties since 2010.

Table A.1: Correlation between UIP deviations and (global) risk variables

$Corr(RiskVariables, \mathbb{E}X^*)$						
	A) CHF/USD			B) JPY/USD		
Sample	USEPU	GEPU	WUI	USEPU	GEPU	WUI
1999-2022	-0.20	-0.25	-0.26	-0.01	0.16	0.12
2010-2022	0.48	0.68	0.40	0.41	0.70	0.37

Notes: This table displays the correlation between $\mathbb{E}_t x^*_{t+1}$ (at a 3-month horizon) and different risk variables for the whole sample and a subsample starting in 2010. Panel A) displays this correlation taking the CHF as the domestic currency and the USD as the foreign one. Similarly, Panel B) considers the JPY as the domestic currency. USEPU is the US Economic Policy Uncertainty index developed in Baker et al. (2016). GEPU is the Global EPU. WUI is the World Uncertainty Index developed in Ahir et al. (2022). Since WUI is only available at a quarterly frequency, we take the quarterly mean of UIP deviations when computing the correlations.

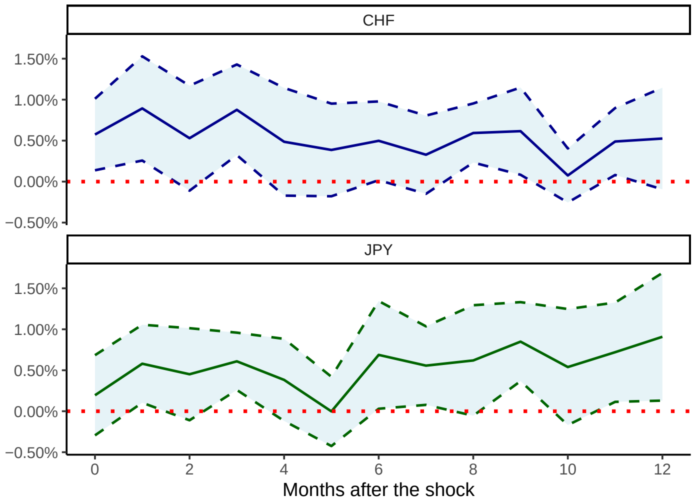
To examine the dynamic impact of uncertainty shocks, Figure A.2 runs a local-projection regression (Jordà, 2005) of a Global Economic Policy Uncertainty (EPU) shock on $\mathbb{E}_t x^*_{t+1}$ for the period 2010-2022. The results show that, following an unanticipated shock to the Global EPU, $\mathbb{E}_t x^*_{t+1}$ tends to increase both for the CHF and the JPY. In other words, the CHF and the JPY are generally expected to appreciate following an uncertainty shock.

B Computing excess returns and stochastic discount factors

In this section, we discuss the construction of $cov(x^*_{t+1}, m^*)/\mathbb{E}_t m^*$ and $cov(x^*_{t+1}, m_{t+1})/\mathbb{E}_t m_{t+1}$ considering either the CHF or the JPY as the domestic currency, and keeping the USD as the foreign currency.

[18] See Kalemli-Özcan and Varela (2021) for a recent analysis of UIP deviations using Consensus Economics survey.

Figure A.2: Local Projections to a Global EPU shock



Notes: This figure shows the results from the local projection of a Global EPU shock on the UIP deviations over the sample 2010-2022, using the CHF and the JPY as the domestic currency, respectively. Formally, we identify an uncertainty shock ($shock_t$) outside of the system by taking the residual of an AR(1) on our Global EPU variable in the spirit of Stock and Watson (2012) who uses the VIX. We then run $\mathbb{E}_t(x^*_{t+h}) = \alpha^h + \beta_h shock_t + \phi^h x_t + u^h_{t+h}$ for $h = 0, ..., 12$ where x_t are control variables made of $p = 3$ lags of the dependent variable. We then report β^h at each horizon as well as the 90% confidence intervals using the Newey-West estimator.

B.1 Excess returns

For i_t, we rely on the domestic (CHF or JPY) 3-month risk-free rate, while i^*_t is the US 3-month risk-free rate. For s_t we use nominal spot exchange rate data expressed in the amount of domestic currency per unit of USD. All data is from Datastream and is retrieved at the daily frequency. The daily data is aggregated to the monthly or quarterly frequency by taking the mean within each quarter. To compute excess returns, we first compute quarterly excess returns according to (31). We assume that what matters for financial intermediaries is the moving excess returns of this carry-trade over the past year by taking a moving sum of excess returns over that of the current and last three quarters. This allows to have a smoother version of excess returns.

B.2 Stochastic discount factors

International Financial Intermediaries We now discuss the construction of the SDF of financial intermediaries, which is defined as $m^* = \beta\left(NW_{t+1}/NW_t\right)^{-\gamma}$. Similarly to He et al. (2017), we define $NW_{t+1} = \eta_{t+1} \times W_{t+1}$, where η_{t+1} is a measure of the capital ratio of financial intermediaries and W_t is a measure of total wealth. The SDF is obtained by interacting a measure of the growth rate of the capital ratio and total wealth. We discuss below the construction of these growth rates.

We consider two measures of the capital ratio. The first specification (HKM) relies on the capital ratio measure from He et al. (2017) which is retrieved from Zhiguo He's website at a daily frequency and aggregated at a monthly or quarterly frequency by taking the mean. The second specification (AEM) is based on Adrian et al. (2014) and is computed using quarterly balance sheet data from the Federal Reserve Flow Of Funds (Table L.130). To obtain an annual growth rate, we divide the residual of a regression of the capital ratio in t on its one-year lagged value by the one-year lagged value of the capital ratio. This gives rise to the intermediary capital risk factor. The two resulting measures are defined as $\Delta\eta_{t+1}^{HKM}$ and $\Delta\eta_{t+1}^{AEM}$, respectively.

For total wealth growth, we rely on a financial measure (MSCI US Equity Index) and a real measure (US GDP). For the financial measure, we consider moving annual excess returns. Every quarter, they are obtained by summing up daily excess returns over the past 4 quarters and subtracting the 3-month US risk-free rate. The resulting series is defined as ΔW_{t+1}^{MSCI}. For the real measure, we compute moving annual growth every quarter. The resulting series is defined as ΔW_{t+1}^{GDP}.

The SDF of financial intermediaries is then computed as $m_{t+1}^* = \beta((1+\Delta\eta_{t+1}^i)\times(1+\Delta W_{t+1}^j))^{-\gamma}$ for $i \in \{AEM, HKM\}$ and $j \in \{MSCI, GDP\}$, with $\beta = 0.99$ and $\gamma = 10$. This gives rise to 4 potential specifications of the SDF of financial intermediaries. Following the approach of He et al. (2017), we confirm that these measures significantly explain FX returns in currency portfolios.

Domestic Households For Households (HH), the SDF is defined as $m_{t+1} = \beta\left(c_t/c_{t+1}\right)^{-\gamma}$. Real consumption for Switzerland and Japan is retrieved from the FRED website at a quarterly frequency. As for the SDF of financial intermediaries, we compute a moving annual growth rate and assume $\gamma = 10$ and $\beta = 0.99$.

C Decentralized Model Properties

In this section, we derive some key properties of the decentralized model. We first develop a preliminary analysis of asset prices (C.1), the households' stochastic discount factor (C.2), and the households' currency positions (C.3). Then we prove Proposition 1 (C.4), the impact of FX interventions on nfl (C.5), the expression for the covariance differential (28) (C.6), and Proposition 3 (C.7).

We use a second-order approximation to solve the model. We denote the variables in log with a tilde. For instance: $\tilde{y} = \log(y)$ and $\tilde{y}^* = \log(y^*)$. We also define $\tilde{i}^* = \log(1+i^*)$ and $\tilde{i} = \log(1+i)$.

C.1 Asset Pricing by Foreign Intermediaries

In this sub-section, we derive the asset pricing equations. The foreign interest rate is set exogenously in a small open economy. We have assumed that $i^* = \beta^{-1} - 1$, which implies

$$e^{\log(\beta)+\tilde{i}^*} = 1. \tag{40}$$

Equation (6) determines the domestic asset prices $\tilde{i} + \tilde{S}_1$:

$$
\begin{aligned}
& & \mathbb{E}(e^{\tilde{m}^* - \tilde{S}_2 + \tilde{i} + \tilde{S}_1}) &= 1 + \chi + \Gamma b^{H*} \\
&\Leftrightarrow & e^{\mathbb{E}(\tilde{m}^* - \tilde{S}_2) + \frac{1}{2}V(\tilde{m}^* - \tilde{S}_2) + \tilde{i} + \tilde{S}_1} &= 1 + \chi + \Gamma b^{H*} \\
&\Leftrightarrow & e^{\log(\beta) - \mathbb{E}((1+\rho)\tilde{y}_2^*) + \rho(1-\rho)\sigma^2/2 + \frac{1}{2}V((1+\rho)\tilde{y}_2^*) + \tilde{i} + \tilde{S}_1} &= 1 + \chi + \Gamma b^{H*} \\
&\Leftrightarrow & e^{\log(\beta) - (1+\rho)\sigma^2/2 + \rho(1-\rho)\sigma^2/2 + (1+\rho)^2\sigma^2/2 + \tilde{i} + \tilde{S}_1} &= 1 + \chi + \Gamma b^{H*} \\
&\Leftrightarrow & e^{\log(\beta) + \rho\sigma^2 + \tilde{i} + \tilde{S}_1} &= 1 + \chi + \Gamma b^{H*}
\end{aligned}
$$

This implies

$$
e^{\rho\sigma^2 + \tilde{i} + \tilde{S}_1 - \tilde{i}^*} = 1 + \chi + \Gamma b^{H*} \tag{41}
$$

C.2 Households Stochastic Discount Factor

We derive here the households' stochastic discount factor $\tilde{m}$. To obtain $\tilde{m}$, we can rewrite the resource constraints (22) as

$$
\begin{aligned}
c_1 &= y_1 \left(1 + \tfrac{nfl}{y_1}\right) \\
c_2 &= y_2 \left(1 - \tfrac{nfl}{y_1}\tfrac{1+i^*}{1+g_2} - \tfrac{b^{H*}}{y_1}\tfrac{X^*}{1+g_2}\right)
\end{aligned}
$$

with $1 + g_2 = y_2/y_1$. Taking logs and using a second-order approximation (assuming $\tilde{y}_2$, nfl/y_1, b^{H*}/y_1, X^* and g are small), we obtain

$$
\begin{aligned}
\tilde{c}_1 &= \tilde{y}_1 + \tfrac{nfl}{y_1} - \tfrac{1}{2}\left(\tfrac{nfl}{y_1}\right)^2 \\
\tilde{c}_2 &= \tilde{y}_2 - \tfrac{nfl}{y_1}(1 + i^* - g_2) + \tfrac{1}{2}\left(\tfrac{nfl}{y_1}\right)^2 (1 + i^*) - \tfrac{b^{H*}}{y_1}X^*
\end{aligned}
$$

Finally, we use the approximation $g_2 = \tilde{y}_2 - \tilde{y}_1$ along with the assumption that $y_1 = 1$ and hence $\tilde{y}_1 = 0$ to obtain the approximated household's budget constraints:

$$
\begin{aligned}
\tilde{c}_1 &= nfl - \frac{1}{2}nfl^2 \\
\tilde{c}_2 &= \tilde{y}_2\,(1 + nfl) - \left(nfl - \frac{1}{2}nfl^2\right)(1 + i^*) - b^{H*}X^*
\end{aligned} \tag{42}
$$

Using (42), we get

$$
\begin{aligned}
\tilde{m} \;&= \log(\beta) + \tilde{c}_1 - \tilde{c}_2 \\
&= \log(\beta) - \tilde{y}_2\,(1 + nfl) + (nfl - nfl^2/2)(2 + \tilde{i}^*) + b^{H*}(\tilde{i} - \tilde{i}^* + \tilde{S}_1 - \tilde{S}_2) \\
&= \log(\beta) - \alpha\tilde{y}^*\,(1 + nfl) + (nfl - nfl^2/2)(2 + \tilde{i}^*) + b^{H*}(\tilde{i} - \tilde{i}^* + \tilde{S}_1 - \rho\tilde{y}^*) \\
&\quad + [\alpha(1-\alpha)(1 + nfl) + \rho(1-\rho)b^{H*}]\sigma^2/2 - g(1 + nfl) \\
&= \log(\beta) - [\alpha\,(1 + nfl) + \rho b^{H*}]\tilde{y}^* + (nfl - nfl^2/2)(2 + \tilde{i}^*) + b^{H*}(\tilde{i} - \tilde{i}^* + \tilde{S}_1) \\
&\quad + [\alpha(1-\alpha)(1 + nfl) + \rho(1-\rho)b^{H*}]\sigma^2/2 - g(1 + nfl),
\end{aligned} \tag{43}
$$

using $\tilde{y}_2 = g + \alpha\tilde{y}^* - \alpha(1-\alpha)\sigma^2/2$, $X^* = \tilde{i} - \tilde{i}^* + \tilde{S}_1 - \tilde{S}_2$ and $\tilde{S}_2 = \rho\tilde{y}^* - \rho(1-\rho)\sigma^2/2$.

Using (41) and its linear approximation $\tilde{i} + \tilde{S}_1 - \tilde{i}^* = \chi + \Gamma b^{H*} - \rho\sigma^2$, we thus have

$$\begin{aligned}&\mathbb{E}(\tilde{m}) + \tfrac{V(m)}{2} = \\ &\log(\beta) + (nfl - nfl^2/2)(2+\tilde{i}^*) + b^{H*}(\chi + \Gamma b^{H*} - \rho\sigma^2) - g(1+nfl) \\ &+[\alpha^2(1+nfl)nfl + \rho^2 b^{H*}(b^{H*}-1) + 2\alpha\rho(1+nfl)b^{H*}]\tfrac{\sigma^2}{2}\end{aligned} \tag{44}$$

and

$$\begin{aligned}&\mathbb{E}(\tilde{m} - \tilde{S}_2) + \tfrac{V(m-\tilde{S}_2)}{2} = \\ &\log(\beta) + (nfl - nfl^2/2)(2+\tilde{i}^*) + b^{H*}(\chi + \Gamma b^{H*} - \rho\sigma^2) - g(1+nfl) \\ &+[\alpha^2(1+nfl)nfl + \rho^2 b^{H*}(1+b^{H*}) + 2\alpha\rho(1+nfl)(1+b^{H*})]\tfrac{\sigma^2}{2}\end{aligned} \tag{45}$$

C.3 Households Currency Positions

We determine here how the households' FOC are affected by the net and gross foreign positions nfl and b^{H*}. We use the household's FOCs (13) and (14), which we rewrite as follows

$$1 - \mathbb{E}\left(e^{\tilde{m}+\tilde{i}^*}\right) = \lambda^F \tag{46}$$

$$\mathbb{E}\left(e^{\tilde{m}+\tilde{i}^*} - e^{\tilde{m}+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right) = \lambda^H - \lambda^F \tag{47}$$

Combining these two equations with the pricing equations (40) and (41) and Equations (44) and (45), we obtain

$$\begin{aligned}&(nfl - nfl^2/2)(2+\tilde{i}^*) + b^{H*}(\chi + \Gamma b^{H*} - \rho\sigma^2) - g(1+nfl) \\ &+[\alpha^2(1+nfl)nfl + \rho^2 b^{H*}(b^{H*}-1) + 2\alpha\rho(1+nfl)b^{H*}]\frac{\sigma^2}{2} \\ &= \log(1-\lambda^F)\end{aligned} \tag{48}$$

$$[\rho^2 b^{H*} + \alpha\rho(1+nfl)]\sigma^2 + \chi + \Gamma b^{H*} - \rho\sigma^2 = \log\left(1 - \frac{\lambda^H - \lambda^F}{1-\lambda^F}\right) \tag{49}$$

C.4 Proof of Proposition 1

To prove Proposition 1 (the effectiveness of FX interventions), it is useful to lay down some preliminary steps. We first characterize the solution that satisfies both intertemporal and portfolio optimality. We then introduce our main assumption of domestic bond scarcity, before characterizing some important properties of intertemporal and portfolio optimality. Finally, we derive our results.

C.4.1 Intertemporal and portfolio optimality

We characterize here the solution that satisfies both intertemporal and portfolio optimality. According to Equations (48) and (49), the couple (nfl, b^{H*}) that satisfies intertemporal and portfolio optimality is characterized by the following:

$$nfl = nfl^{opt}(b^{H*}) \tag{50}$$
$$b^{H*} = b^{opt}(nfl) \tag{51}$$

where

$$b^{opt}(nfl) = \frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl \tag{52}$$

and $nfl^{opt}(b^{H*})$ is the smallest solution of the quadratic equation in nfl at the right hand side of Equation (48). $nfl^{opt}(b^{H*})$ satisfies $P[nfl^{opt}(b^{H*}), b^{H*}] = 0$ with

$$\begin{aligned} P[nfl^{opt}(b^{H*}), b^{H*}] = \\ [nfl^{opt}(b^{H*}) - nfl^{opt}(b^{H*})^2/2](2 + \tilde{i}^*) + b^{H*}(\chi + \Gamma b^{H*} - \rho\sigma^2) - g[1 + nfl^{opt}(b^{H*})] \\ +[\alpha^2[1 + nfl^{opt}(b^{H*})]nfl^{opt}(b^{H*}) + \rho^2 b^{H*}(b^{H*} - 1) + 2\alpha\rho[1 + nfl^{opt}(b^{H*})]b^{H*}]\frac{\sigma^2}{2} \end{aligned} \tag{53}$$

Suppose that there exists a couple (nfl, b^{H*}) that jointly satisfies (50) and (51). We denote this couple (nfl^{opt}, b^{opt}). nfl^{opt} is thus the value of nfl that holds under both intertemporal and portfolio optimality. It is characterized by $P(nfl^{opt}, b^{opt}(nfl^{opt})) = 0$. $P(nfl^{opt}, b^{opt}(nfl^{opt}))$ is a second-order polynomial in nfl^{opt}. We denote it by P^{opt}.

$$\begin{aligned} P^{opt}(nfl) = [nfl - nfl^2/2](2 + \tilde{i}^*) \\ + \left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right)\left[\chi + \Gamma\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right) - \rho\sigma^2\right] - g(1 + nfl) \\ + \left\{\alpha^2(1 + nfl)nfl + \rho^2\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right)\left[\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right) - 1\right]\right. \\ \left. + 2\alpha\rho(1 + nfl)\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right)\right\}\frac{\sigma^2}{2} \end{aligned} \tag{54}$$

If if $g = \Gamma = \chi = \sigma^2 = 0$, then this equation boils down to

$$P^{opt}(nfl) \simeq [nfl - nfl^2/2](2 + \tilde{i}^*)$$

In that case, there exists two solutions to $P^{opt}(x) = 0$. The existence of these two solutions extends by continuity to the case where Condition 3 is satisfied, that is, g, Γ, χ and σ^2 are small compared to $2 + \tilde{i}^*$.

We focus on the smallest solution because it is well behaved. It corresponds to $nfl^{opt} = 0$ when $g = \Gamma = \chi = \sigma^2 = 0$ (while the larger solution is 2), which is the solution that achieves consumption smoothing. In what follows, we refer to nfl^{opt} as the smallest solution to $P^{opt}(x) = 0$.

We then derive b^{opt} as $b^{opt}(nfl^{opt})$, which is uniquely defined. We summarize these results in the following Lemma

Lemma 2 (Intertemporal and portfolio optimality) *Under Condition 3, $P^{opt}(nfl) = 0$ admits two solutions, where $P^{opt}(nfl)$ is defined by (54). Let nfl^{opt} be the smallest solution to $P^{opt}(nfl) = 0$. Define b^{opt} as $b^{opt}(nfl^{opt})$, where $b^{opt}(nfl)$ is defined by (49). Then (nfl^{opt}, b^{opt}) satisfy intertemporal and portfolio optimality, defined respectively by (48) and (49).*

We can now define $b^{max} = b^{opt}(min\{nfl^{opt}, b^G\}) = \frac{(1-\alpha)\rho\sigma^2-\chi}{\Gamma+\rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma+\rho^2\sigma^2} min(nfl^{opt}, b^G)$.

C.4.2 Assumption of domestic bond scarcity

We assume in what follows that $b^{CBF} + b^G < b^{max}$.

C.4.3 Properties of $n^{opt}(.)$ and $b^{opt}(.)$

It is also useful to characterize how $n^{opt}(b^{H*})$ and $b^{opt}(nfl)$ vary:

Lemma 3 (Monotonicity) *Under Conditions 1, 2 and 3, and if $b^{H*} \leq b^{max}$, $nfl^{opt}(b^{H*})$ is strictly increasing in b^{H*}. Under Condition 1, $b^{opt}(nfl)$ is decreasing in nfl. If, additionally, $\alpha > 0$, then $b^{opt}(nfl)$ is strictly decreasing in nfl.*

Proof.

It is straightforward that, if $\alpha \geq 0$ and $\rho > 0$, which is the case under Condition 1, then $b^{opt\prime}(nfl) \leq 0$. If, additionally, $\alpha = 0$, then $b^{opt}(nfl) = \frac{(1-\alpha)\rho\sigma^2-\chi}{\Gamma+\rho^2\sigma^2}$ and $b^{opt\prime}(nfl) = 0$. If instead $\alpha > 0$, then $b^{opt\prime}(nfl) > 0$.

We can also show that $nfl^{opt\prime}(b^{H*}) > 0$ under some sufficient condition, by differentiating (53) with respect to b^{H*}:

$$\begin{aligned}[1 - nfl^{opt}(b^{H*})](2 + \tilde{i}^*)nfl^{opt\prime}(b^{H*}) + \chi + 2\Gamma b^{H*} - \rho\sigma^2 - g \cdot nfl^{opt\prime}(b^{H*}) \\ + \left\{\alpha^2[1 + 2nfl^{opt}(b^{H*})]nfl^{opt\prime}(b^{H*}) + \rho^2(2b^{H*} - 1) + 2\alpha\rho[1 + nfl^{opt}(b^{H*}) + b^{H*} \cdot nfl^{opt\prime}(b^{H*})]\right\}\frac{\sigma^2}{2} \\ = 0\end{aligned}$$

$$\Leftrightarrow$$

$$\begin{aligned}nfl^{opt\prime}(b^{H*})\left\{[1 - nfl^{opt}(b^{H*})](2 + \tilde{i}^*) - g + [\alpha^2[1 + 2nfl^{opt}(b^{H*})] + 2\alpha\rho \cdot b^{H*} \cdot nfl^{opt\prime}(b^{H*})]\frac{\sigma^2}{2}\right\} \\ + \chi + 2\Gamma b^{H*} - \rho\sigma^2 + [\rho^2(2b^{H*} - 1) + 2\alpha\rho[1 + nfl^{opt}(b^{H*})]]\frac{\sigma^2}{2} \\ = 0\end{aligned}$$

$$\Leftrightarrow$$

$$nfl^{opt\prime}(b^{H*})\left\{\underbrace{[1-nfl^{opt}(b^{H*})](2+\tilde{i}^{*})-g+[\alpha^2[1+2nfl^{opt}(b^{H*})]+2\alpha\rho\cdot b^{H*}\cdot nfl^{opt\prime}(b^{H*})]\frac{\sigma^2}{2}}_{P'[nfl^{opt}(b^{H*})]>0}\right\}$$

$$+\underbrace{\chi+2\Gamma b^{H*}-\rho\sigma^2+[\rho^2 b^{H*}+\alpha\rho[1+nfl^{opt}(b^{H*})]]\sigma^2-\rho^2\sigma^2/2}_{<0 \text{ under Condition 1, 2, and 3, and if } b^{H*}\leq b^{max}}$$

$$=0$$

$P'[nfl^{opt}(b^{H*})]>0$ because $nfl^{opt}(b^{H*})$ is the smallest solution to $P[nfl^{opt}(b^{H*})]=0$ and P opens downwards.

We show in what follows that under Condition 1, Condition 2 (σ^2 is large compared to Γ and χ) and Condition 3 ($nfl^{opt}(b^{H*})$ is close to zero), and if $b^{H*}\leq b^{max}$, the second line in the above equation is strictly negative. Indeed, if $b^{H*}<b^{max}$

$$\begin{aligned}&\chi+2\Gamma b^{H*}-\rho\sigma^2+[\rho^2 b^{H*}+\alpha\rho[1+nfl^{opt}(b^{H*})]]\sigma^2-\rho^2\sigma^2/2\\ < \;&\underbrace{\chi+2\Gamma\frac{(1-\alpha)\rho\sigma^2-\chi}{\Gamma+\rho^2\sigma^2}-\rho\sigma^2+[\rho^2\frac{(1-\alpha)\rho\sigma^2-\chi}{\Gamma+\rho^2\sigma^2}+\alpha\rho[1+nfl^{opt}(b^{H*})]]\sigma^2-\rho^2\sigma^2/2-(2\Gamma+\rho^2\sigma^2)\frac{\alpha\rho\sigma^2}{\Gamma+\rho^2\sigma^2}min(nfl^{opt},b^G)}_{X}\end{aligned}$$

We now use the Conditions to find an approximation for X and show that X is strictly negative. Using Condition 2, we can neglect the terms in Γ and χ:

$$X\simeq-\rho\sigma^2+[\rho(1-\alpha)+\alpha\rho[1+nfl^{opt}(b^{H*})]]\sigma^2-\rho^2\sigma^2/2-\alpha\rho\sigma^2 min(nfl^{opt},b^G)$$

Besides, using the fact that, under Condition 3, $nfl^{opt}(b^{H*})$ is small relative to 1:

$$X\simeq-\rho\sigma^2+[\rho(1-\alpha)+\alpha\rho]\sigma^2-\rho^2\sigma^2/2-\frac{\alpha}{\rho}min(nfl^{opt},b^G)=-\rho^2\sigma^2/2-\alpha\rho\sigma^2 min(nfl^{opt},b^G)$$

Finally, we consider two cases. First, suppose that $min(nfl^{opt},b^G)=b^G$. Noting that b^G is strictly positive, and using Condition 1, according to which $\alpha\geq 0$ and $\rho>0$, we can write

$$X\simeq-\rho^2\sigma^2/2-\alpha\rho\sigma^2 b^G<-\rho^2\sigma^2/2<0$$

Second, suppose that $min(nfl^{opt},b^G)=nfl^{opt}$. Using the fact that, under Condition 3, $nfl^{opt}(b^{H*})$ is close to zero:

$$X\simeq-\rho^2\sigma^2/2-\alpha\rho\sigma^2 nfl^{opt}\simeq-\rho^2\sigma^2/2<0$$

In that case, $nfl^{opt\prime}(b^{H*})$ is strictly positive.

■

C.4.4 Equilibrium nfl

We establish the following Lemma:

Lemma 4 (Equilibrium nfl) *Under Conditions 1, 2 and 3, and if $b^{H*}\leq b^{max}$ in equilibrium, because of the household's domestic bond short-selling constraint, we must have:*

$$nfl(b^{H*})=\min\{nfl^{opt}(b^{H*}),b^G\} \tag{55}$$

Proof.

We ignore all situations where $nfl > 1$, because our approximation is valid only for a small nfl.

First, note that $b^{CBF} + b^G < b^{max}$ implies that $b^{H*} < b^{max}$. To see this, note that, because $b^{H*} = b^G - b^{CB} - b^H$, and $b^{CB} = -b^{CBF}$, then $b^{H*} = b^G + b^{CBF} - b^H$. Since $b^H \geq 0$, then this implies $b^{H*} \leq b^G + b^{CBF}$. Now remember we are assuming $b^{CBF} + b^G < b^{max}$. Since $b^{CBF} + b^G < b^{max}$, then $b^{H*} < b^{max}$.

Second, note that $nfl = b^G - b^H - b^F$, so the short-selling constraints imply $nfl \leq b^G$.

- If $b^G < nfl^{opt}(b^{H*})$, then $nfl = b^G$.

  Indeed, in that case, since $nfl \leq b^G$, then $nfl < nfl^{opt}(b^{H*})$, which implies $\lambda^F > 0$. This implies $b^F = 0$.

  Suppose that $b^H > 0$. Then, $\lambda^H = 0$, so that $\lambda^H - \lambda^F < 0$. This would imply $b^{H^*} > b^{opt}(nfl)$, according to (49) and the definition of $b^{opt}(nfl)$.

  Now, note that, since $nfl \leq b^G$ and $nfl < nfl^{opt}(b^{H*})$, then $nfl \leq min(nfl^{opt}(b^{H*}), b^G)$. Therefore, $b^{max} = b^{opt}[min(nfl^{opt}(b^{H*}), b^G)] \leq b^{opt}(nfl)$, because, according to Lemma 3, $b^{opt}(.)$ is a decreasing function. As a result, because $b^{H*} < b^{max}$, we would have $b^{H*} < b^{opt}(nfl)$. There is a contradiction.

  Therefore, we must have $b^H = b^F = 0$ and $nfl = b^G$.

- If $b^G \geq nfl^{opt}(b^{H*})$, then $nfl = nfl^{opt}(b^{H*})$.

  Indeed, suppose that $nfl > nfl^{opt}(b^{H*})$. In that case, we would have $P(nfl) > P[nfl^{opt}(b^{H*})]$ (because for $nfl^{opt}(b^{H*}) < nfl < 1$, $P(nfl)$ is increasing in nfl, and we ignore cases where $nfl > 1$). As a result, $\lambda^F < 0$, according to (48) and the definition of $P(.)$ and $nfl^{opt}(b^{H*})$. This is not possible.

  Suppose that $nfl < nfl^{opt}(b^{H*})$, then $\lambda^F > 0$. As a consequence, $b^F = 0$.

  Additionally, since $nfl \leq b^G$, then $nfl \leq min(nfl^{opt}(b^{H*}), b^G)$. using the same argument as above, this would imply $b^{max} \leq b^{opt}(nfl)$, and hence $b^{H*} < b^{opt}(nfl)$. This implies $\lambda^H - \lambda^F > 0$ and therefore $\lambda^H > 0$. This means $b^H = 0$.

  $b^F = b^H = 0$ implies that $nfl = b^G \geq nfl^{opt}(b^{H*})$. This leads to a contradiction, since we assume that $nfl < nfl^{opt}(b^{H*})$.

  Therefore, the only possible solution is $nfl = nfl^{opt}(b^{H*})$.

■

C.4.5 Effectiveness of FX interventions

We show, by contradiction, that, if $b^{CBF} + b^G < b^{max}$, then $b^{H^*} = b^{CBF} + b^G$.

(a) Suppose that $b^{H*} > b^{CBF} + b^G$. Then $b^H = b^{CBF} + b^G - b^{H*} < 0$. This would violate the household's domestic bond short-selling constraint.

(b) Suppose that $b^{H*} < b^{CBF} + b^G$. Then $b^H = b^{CBF} + b^G - b^{H*} > 0$. It must then be that $\lambda^H = 0$. Equation (49) implies that

$$b^{H*} \geq b^{opt}(nfl) \tag{56}$$

Replacing nfl using (55) in inequality (56), we obtain

$$b^{H*} \geq b^{opt}(\min\{nfl^{opt}(b^{H*}), b^G\}) \tag{57}$$

Then, combining $b^{H*} < b^{CBF} + b^G$ with the assumption that $b^{CBF} + b^G < b^{max}$, it must be that

$$b^{H*} < b^{max} \tag{58}$$

Therefore, using the definition of b^{max}: $b^{H*} < b^{opt}(\min\{nfl^{opt}, b^G\})$. Combining with inequality (57), we obtain the following inequality:

$$b^{opt}(\min\{nfl^{opt}(b^{H*}), b^G\}) < b^{opt}(\min\{nfl^{opt}, b^G\}) \tag{59}$$

If $\alpha = 0$, then this is already a contradiction, because $b^{opt}(nfl)$ is a constant term and is invariant in nfl.

If $\alpha > 0$, then $b^{opt}(nfl)$ is strictly decreasing in nfl. Then (59) implies that

$$\min\{nfl^{opt}, b^G\} < \min\{nfl^{opt}(b^{H*}), b^G\}$$

Then, necessarily:

$$nfl^{opt} < nfl^{opt}(b^{H*}) \text{ and } nfl^{opt} < b^G \tag{60}$$

Note that, according to Lemma 2, $nfl^{opt} = nfl^{opt}(b^{opt})$. Therefore, the first inequality in (60) yields

$$nfl^{opt}(b^{opt}) < nfl^{opt}(b^{H*}) \tag{61}$$

On the other hand, remember that, according to (58), $b^{H*} < b^{max}$. Therefore, the conditions of Lemma 3 apply, and $nfl^{opt}(.)$ is strictly increasing on the interval $[b^{H*}, b^{max}]$, so that $nfl^{opt}(b^{H*}) < nfl^{opt}(b^{max})$. Combining with (61),

$$nfl^{opt}(b^{opt}) < nfl^{opt}(b^{max}) \tag{62}$$

Note that $b^{opt} \leq b^{max}$, so the conditions of Lemma 3 apply, and $nfl^{opt}(.)$ is strictly increasing on the interval $[b^{opt}, b^{max}]$. Consequently,

$$b^{opt} < b^{max} \tag{63}$$

By definition, $b^{opt} = b^{opt}(nfl^{opt})$ and $b^{max} = b^{opt}(\min\{nfl^{opt}, b^G\})$. Since $b^{opt}(nfl)$ is strictly decreasing in nfl, we must have

$$\min\{nfl^{opt}, b^G\} < nfl^{opt}$$

This implies that

$$b^G < nfl^{opt}$$

which contradicts (60).

Therefore, by contradiction, we must have that, if $b^{CBF} + b^G < b^{max}$, then $b^{H^*} = b^{CBF} + b^G$. as a consequence, $b^H = b^{CBF} + b^G - b^{H*} = 0$, and b^{CBF} affect b^{H*} one-for-one.

The impact of FX interventions on the CIP deviation Z^* and on the UIP deviation $\mathbb{E}(X^*)$ derive immediately from Equations (7) and (8) and the effect of interventions on b^{H*}.

C.5 The Impact of FX Interventions on nfl

Lemma 5 *If Conditions 1, 2 and 3 are satisfied, and $b^{CBF}+b^G < b^{max}$, then the following holds:*

(i) If b^G is small, so that $b^G < nfl^{opt}(b^{CBF} + b^G)$, then $nfl = b^G$ and the households are constrained not only in their capacity to issue domestic-currency bonds ($b^H = 0$), but also in their capacity to issue foreign-currency bonds ($b^F = 0$). In that case, nfl is invariant in b^{CBF}.

(ii) If b^G is large, so that $b^G \geq nfl^{opt}(b^{CBF} + b^G)$, then $nfl = nfl^{opt}(b^{CBF} + b^G)$ and the households are only constrained in their capacity to issue domestic-currency bonds ($b^H = 0$ and $b^F \geq 0$). In that case, nfl is increasing in b^{CBF}.

Proof. We assume that Conditions 1, 2 and 3 are satisfied, and that $b^{CBF} + b^G < b^{max}$.

According to Proposition 1, since $b^{CBF} + b^G < b^{max}$, then $b^{H*} = b^{CBF} + b^G$, and households are constrained in their capacity to issue domestic-currency bonds ($b^H = 0$).

As a consequence, we have $b^{H*} < b^{max}$. According to Lemma 4, this condition implies, when combined with Conditions 1, 2 and 3, that $nfl = nfl(b^{H*}) = \min\{nfl^{opt}(b^{H*}), b^G\}$. Since $b^{H*} = b^{CBF} + b^G$, this implies $nfl = \min\{nfl^{opt}(b^{CBF} + b^G), b^G\}$. This means that, if b^G is small, so that $b^G < nfl^{opt}(b^{CBF} + b^G)$, then $nfl = b^G$ and the households are constrained not only in their capacity to issue domestic-currency bonds, but also in their capacity to issue foreign-currency bonds ($b^F = b^G - nfl = 0$).

If b^G is large, so that $b^G \geq nfl^{opt}(b^{CBF} + b^G)$, then $nfl = nfl^{opt}(b^{CBF} + b^G)$. In that case, intertemporal optimality is satisfied as households desire a positive amount of foreign-currency bonds ($b^F = b^G - nfl \geq 0$). According to Lemma 3, in that case, $nfl^{opt}(b^{CBF} + b^G)$ is increasing in $b^{CBF} + b^G$. Therefore, nfl is increasing in b^{CBF}. ■

C.6 Covariance differential

The difference in risk premia can be written as follows

$$\Delta Cov = \frac{cov(m^*, X^*)}{\mathbb{E}m^*} - \frac{cov(m, X^*)}{\mathbb{E}m} = \frac{1}{\beta}(1 + \chi + \Gamma b^{H*}) \left(1 - e^{cov(\tilde{S}_2, \tilde{m}^*) - cov(\tilde{S}_2, \tilde{m})}\right)$$

We used

$$\begin{aligned} cov(m^*, X^*) &= cov\left(m^*, (1+i)\frac{S_1}{S_2}\right) - \underbrace{cov(m^*, (1+i^*))}_{=0} \\ &= \mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right) - \mathbb{E}\left(m^*\right)\mathbb{E}\left((1+i)\frac{S_1}{S_2}\right) \\ &= \mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right) - \mathbb{E}\left(e^{\tilde{m}^*}\right)\mathbb{E}\left(e^{\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right) \\ &= \underbrace{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}_{1+\chi+\Gamma b^{H*}}\left[1-e^{cov(\tilde{S}_2,\tilde{m}^*)}\right] \end{aligned}$$

where we used (6), and

$$\mathbb{E}m^* = \beta$$

which yields

$$\frac{cov(m^*, X^*)}{\mathbb{E}m^*} = \frac{1}{\beta}(1+\chi+\Gamma b^{H*})\left[1-e^{cov(\tilde{S}_2,\tilde{m}^*)}\right] \tag{64}$$

Similarly:

$$\begin{aligned} &\frac{cov(m,X^*)}{\mathbb{E}m} = \frac{cov\left(m,(1+i)\frac{S_1}{S_2}\right) - \underbrace{cov(m,(1+i^*))}_{=0}}{\mathbb{E}m} \\ &= \frac{\mathbb{E}\left(m(1+i)\frac{S_1}{S_2}\right)}{\mathbb{E}m} - \mathbb{E}\left((1+i)\frac{S_1}{S_2}\right) \\ &= \frac{\mathbb{E}\left(e^{\tilde{m}+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}(e^{\tilde{m}})} - \mathbb{E}\left(e^{\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right) \\ &= e^{-\log(\beta)+\tilde{i}+\tilde{S}_1-\mathbb{E}(\tilde{S}_2)+\frac{V(\tilde{S}_2)}{2}-cov(\tilde{S}_2,\tilde{m})}\left[1-e^{cov(\tilde{S}_2,\tilde{m})}\right] \\ &= \frac{1}{\beta}e^{\tilde{i}+\tilde{S}_1-\mathbb{E}(\tilde{S}_2)+\frac{V(\tilde{S}_2)}{2}+\mathbb{E}(\tilde{m}^*)+\frac{V(\tilde{m}^*)}{2}-cov(\tilde{S}_2,\tilde{m}^*)}\left[e^{cov(\tilde{S}_2,\tilde{m}^*)-cov(\tilde{S}_2,\tilde{m})} - e^{cov(\tilde{S}_2,\tilde{m}^*)}\right] \\ &= \frac{1}{\beta}\underbrace{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}_{1+\chi+\Gamma b^{H*}}\left[e^{cov(\tilde{S}_2,\tilde{m}^*)-cov(\tilde{S}_2,\tilde{m})} - e^{cov(\tilde{S}_2,\tilde{m}^*)}\right] \end{aligned}$$

where we used $-\log(\beta) = \mathbb{E}(\tilde{m}^*) + \frac{V(\tilde{m}^*)}{2}$. This yields

$$\begin{aligned} \Delta Cov &= \frac{cov(m^*, X^*)}{\mathbb{E}m^*} - \frac{cov(m, X^*)}{\mathbb{E}m} \\ &= \frac{1}{\beta}(1+\chi+\Gamma b^{H*})\left[1-e^{cov(\tilde{S}_2,\tilde{m}^*)-cov(\tilde{S}_2,\tilde{m})}\right] \end{aligned} \tag{65}$$

Now, remember that, by assumption, $\tilde{m}^* = \log(\beta) - \tilde{y}^*$, $\tilde{S}_2 = \rho[\tilde{y}^* - (1-\rho)\sigma^2/2]$, and consider Equation (43). This implies

$$cov(\tilde{S}_2, \tilde{m}^*) = -\rho\sigma^2$$

$$cov(\tilde{S}_2, \tilde{m}) = -\rho[\alpha(1+nfl) + \rho b^{H*}]\sigma^2$$

Therefore,

$$\Delta Cov = \frac{1}{\beta}(1+\chi+\Gamma b^{H*})\left[1-e^{-\rho\sigma^2[1-\alpha(1+nfl)-\rho b^{H*}]}\right]$$

A linear approximation of this equation yield Equation (28).

C.7 Proof of Proposition 3

According to Equation (27), $UCFX = -(\lambda^H - \lambda^F)/\mathbb{E}m$. Note that $(\lambda^H - \lambda^F)/\mathbb{E}m = \beta(\lambda^H - \lambda^F)/(1 - \lambda^F)$. According to Equation (49), $(\lambda^H - \lambda^F)/(1 - \lambda^F)$ is decreasing in b^{H*}. Therefore, $UCFX$ is increasing in b^{H*}. On the other hand, since Conditions 1, 2 and 3 hold, and $b^{CBF} + b^G < b^{max}$, Proposition 1 holds. Therefore, $b^{H*} = b^{CBF} + b^G$ and b^{H*} is increasing in b^{CBF}. As a result, $UCFX$ is increasing in b^{CBF}.

According to Equation (49), if the equilibrium (nfl, b^{H*}) is characterized by $b^{H*} < b^{opt}(nfl)$, then $\lambda^H - \lambda^F > 0$, which is equivalent to $UCFX < 0$. We show below, by contradiction, that, under Conditions 1, 2 and 3, and $b^{CBF} + b^G < b^{max}$, the equilibrium solution satisfies $b^{H*} < b^{opt}(nfl)$.

Note first that under Conditions 1, 2 and 3, and $b^{CBF} + b^G < b^{max}$, Proposition 1 holds, so that $b^{H*} = b^{CBF} + b^G$. As a consequence,

$$b^{H*} < b^{max} \tag{66}$$

Besides, according to Lemma 4, we have $nfl = min\{b^G, nfl^{opt}(b^{H*})\}$. We now rule out $b^{H*} = b^{opt}(nfl)$ and $b^{H*} > b^{opt}(nfl)$.

(a) Suppose that $b^{H*} = b^{opt}(nfl)$.

Since $nfl = min\{b^G, nfl^{opt}(b^{H*})\}$, there are two possibilities. First, $nfl = b^G$. Second, $nfl = nfl^{opt}(b^{H*})$. In that case, since $b^{H*} = b^{opt}(nfl)$, then the solution is (nfl^{opt}, b^{opt}) (see Lemma 2). Therefore, $nfl = min\{b^G, nfl^{opt}\}$, and $b^{H*} = b^{opt}(min\{b^G, nfl^{opt}\}) = b^{max}$. This contradicts inequality (66).

(b) Suppose that $b^{H*} > b^{opt}(nfl)$. Combining with inequality (66), we obtain

$$b^{opt}(nfl) < b^{max}$$

We replace in the above inequality nfl with $nfl = min\{b^G, nfl^{opt}(b^{H*})\}$ and $b^{max} = b^{opt}(min\{b^G, nfl^{opt}\})$:

$$b^{opt}(min\{b^G, nfl^{opt}(b^{H*})\}) < b^{opt}(min\{b^G, nfl^{opt}\}) \tag{67}$$

This is the same as (59) in the proof of Proposition 1. Noting that, according to (66), $b^{H*} < b^{max}$ and following the same steps as in the proof of Proposition 1, we show that this leads to a contradiction.

Therefore, we must have $b^{H*} < b^{opt}(nfl)$ in an equilibrium where $b^{CBF} + b^G < b^{max}$. Consequently, $UCFX < 0$.

D Proofs - Constrained Planner Program

In this section, we derive some key properties of the constrained planner equilibrium. In D.1, we derive the other FOCs of the constrained planner and the planner's stochastic discount factor. In D.2, we derive the planner's optimality condition (37). D.3 proves Lemma 1 and D.4 proves Proposition 5 .

D.1 Other FOCs and the Planner's Stochastic Discount Factor

We take the first-order conditions of the constrained planner's program with respect to prices:

$$/i: \quad -\mathbb{E}\left[\eta_2(1+i)\frac{S_1}{S_2}b^{H*}\right] + \xi f_1(i,S_1)(1+i) \quad +\alpha_0\mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right) = 0 \tag{68}$$

$$/S_1: \quad -\mathbb{E}\left[\eta_2(1+i)\frac{S_1}{S_2}b^{H*}\right] + \xi f_2(i,S_1)S_1 \quad +\alpha_0\mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right) = 0 \tag{69}$$

These two equations imply that $\xi f_1(i,S_1)(1+i) = \xi f_2(i,S_1)S_1$. This is true in the general case only if $\xi = 0$ (you can see that by setting $f(i,S_1) = S_1 - 1$ for instance). The shadow cost of the "policy constraint" is zero, because of monetary neutrality.

This leads to

$$\mathbb{E}\left[\eta_2(1+i)\frac{S_1}{S_2}b^{H*}\right] = \alpha_0\mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right) \tag{70}$$

Finally, we derive with respect to consumption:

$$/C_1: \quad U'(C_1) - \eta_1 = 0 \tag{71}$$

$$/C_2: \quad \mathbb{E}\left[\beta U'(C_2) - \eta_2\right] = 0 \tag{72}$$

These equations imply that $m^{CB} = \eta_2/\eta_1 = \beta U'(C_2)/U'(C_1) = m$.

D.2 Optimal foreign exchange interventions

Equation (70) yields

$$\frac{\alpha_0}{\eta_1} = b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} \tag{73}$$

where we have used $\mathbb{E}(\eta_2/\eta_1) = m$. α_0 is of the same sign as b^{H*}, the gross external position in domestic currency. In that case, if the country is short in domestic currency, then α_0 is positive.

Dividing Equation (35) by η_1, replacing η_2/η_1 with m and α_0/η_1 using the above expression, and using $\Lambda - \tilde{\Lambda} = 0$ (Equation (36)), then finally dividing by $\mathbb{E}m$, we obtain

$$-\frac{\mathbb{E}(mX^*)}{\mathbb{E}m} - \Gamma b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} = 0$$

This yields equation (37).

D.3 Proof of Lemma 1

Another way to write Equation (37) is:

$$\begin{aligned}
\mathbb{E}\left(m(1+i^*)\right) - \mathbb{E}\left(m(1+i)\frac{S_1}{S_2 2}\right) - \Gamma b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} &= 0\\
\underbrace{(1+i^*)}_{\frac{1}{\mathbb{E}m^*}}\mathbb{E}\left(m\right) - \underbrace{(1+i)}_{\frac{1+\chi+\Gamma b^{H*}}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}}\mathbb{E}\left(m\frac{S_1}{S_2}\right) - \Gamma b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} &= 0\\
\frac{1}{\mathbb{E}m^*} - \frac{1+\chi+\Gamma b^{H*}}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m} - \Gamma b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} &= 0\\
1 - (1+\chi+\Gamma b^{H*})\frac{\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m}}{\frac{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}{\mathbb{E}m^*}} - \Gamma b^{H*}\frac{\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m}}{\frac{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}{\mathbb{E}m^*}} &= 0\\
1 - (1+\chi+2\Gamma b^{H*})\frac{\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m}}{\frac{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}{\mathbb{E}m^*}} &= 0
\end{aligned}$$

Besides,

$$\frac{\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m}}{\frac{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}{\mathbb{E}m^*}} = e^{cov(\tilde{m}^*,\tilde{S}_2)-cov(\tilde{m},\tilde{S}_2)} = e^{-\rho\sigma^2[1-\alpha(1+nfl)-\rho b^{H*}]} \tag{74}$$

Therefore, Equation (37) can be written as

$$e^{\rho\sigma^2[1-\alpha(1+nfl)-\rho b^{H*}]} = (1+\chi+2\Gamma b^{H*})$$

which can be approximated as

$$\rho\sigma^2[1-\alpha(1+nfl)-\rho b^{H*}] = \chi + 2\Gamma b^{H*}$$

This yields an optimal level of gross foreign liabilities $\widehat{b}(nfl)$:

$$\widehat{b}(nfl) = \frac{(1-\alpha)\rho\sigma^2 - \chi}{\rho^2\sigma^2 + 2\Gamma} - \frac{\alpha\rho\sigma^2 nfl}{\rho^2\sigma^2 + 2\Gamma} \tag{75}$$

This optimal solution is conditional on nfl. nfl itself satisfies $nfl(\widehat{b}) = \min\{b^G, nfl^{opt}(\widehat{b})\}$. $nfl^{opt}(b)$ is the level of net foreign liabilities that satisfies intertemporal optimality for a given b. It is the smallest solution to $P(nfl^{opt}(b), b) = 0$, where P is described in (53).

Suppose that there exists a couple (nfl, b) that jointly satisfies $b = \widehat{b}(nfl)$, where $\widehat{b}(nfl)$ is given by (75), and $P(nfl, b) = 0$. We denote this couple $(\widehat{nfl}^{opt}, \widehat{b}^{opt})$. $\widehat{nfl}^{opt}$ is thus the value of nfl that holds under both intertemporal optimality and $MBFX = 0$. It is characterized by $P(\widehat{nfl}^{opt}, \widehat{b}(\widehat{nfl}^{opt})) = 0$. $P(\widehat{nfl}^{opt}, \widehat{b}(\widehat{nfl}^{opt}))$ is a second-order polynomial in $\widehat{nfl}^{opt}$. We denote it by $\widehat{P}^{opt}$.

Following similar arguments as in Appendix C.4, we can show that under Conditions 1, 2 and 3, two solutions to $\widehat{P}^{opt}(nfl) = 0$ exist. We define the solution $\widehat{nfl}^{opt}$ as the smallest of the two polynomial solutions, as argued above. Then $\widehat{b}^{opt}$ is simply defined as $\widehat{b}^{opt} = \widehat{b}(\widehat{nfl}^{opt})$.

Denote by $(\widehat{nfl}, \widehat{b})$ the solution under the constrained planner. In the constrained planner equilibrium, $MBFX = 0$, so that $\widehat{b} = \widehat{b}(\widehat{nfl})$. On the other hand, $\widehat{nfl} = \min\{b^G, nfl^{opt}(\widehat{b})\}$. If $b^G < nfl^{opt}(\widehat{b})$, then $\widehat{nfl} = b^G$ and $\widehat{b} = \widehat{b}(b^G)$. If $nfl^{opt}(\widehat{b}) \leq b^G$, then $\widehat{nfl} = nfl^{opt}(\widehat{b})$ and $\widehat{b} = \widehat{b}(\widehat{nfl})$. In that case, according to the above analysis, $\widehat{nfl} = \widehat{nfl}^{opt}$ and $\widehat{b} = \widehat{b}(\widehat{nfl}^{opt})$. Combining these two cases, $\widehat{b} = \widehat{b}(\min\{b^G, \widehat{nfl}^{opt}\})$. This yields point (i) of Lemma 1.

To prove point (ii), note that

$$(\rho^2\sigma^2 + \Gamma)b^{max} + \alpha\rho\sigma^2 nfl(b^{max}) = (1-\alpha)\rho\sigma^2 - \chi$$

$$(\rho^2\sigma^2 + 2\Gamma)\widehat{b} + \alpha\rho\sigma^2 nfl(\widehat{b}) = (1-\alpha)\rho\sigma^2 - \chi$$

with $nfl(b) = \min\{b^G, nfl^{opt}(b)\}$.

Under Conditions 1, 2 and 3, $b^{max} > 0$, which implies that

$$(\rho^2\sigma^2+2\Gamma)b^{max}+\alpha\rho\sigma^2 nfl(b^{max}) \geq (\rho^2\sigma^2+2\Gamma)b^{max}+\alpha\rho\sigma^2 nfl(b^{max}) = (\rho^2\sigma^2+\Gamma)\widehat{b}+\alpha\rho\sigma^2 nfl(\widehat{b})$$

if $\Gamma \geq 0$, with a strict inequality if $\Gamma > 0$. Under Conditions 1, 2 and 3, $nfl(b)$ is weakly increasing in b for $b \leq b^{max}$ (Lemma 3). We can also show, that under the same conditions, $nfl(b)$ is weakly increasing in b for $b \leq \widehat{b}$ (using similar arguments as for Lemma 3). Therefore, $nfl(b)$ is weakly increasing in b for $b \in [b^{max}, \widehat{b}]$. As a result, $(\rho^2\sigma^2 + 2\Gamma)b + \alpha\rho\sigma^2 nfl(b)$ is strictly increasing in b for $b \in [b^{max}, \widehat{b}]$. Therefore, $\widehat{b} \leq b^{max}$. If $\Gamma > 0$, this inequality is strict.

Point (iii) derives directly from point (i) and $b^{H*} = b^G + b^{CBF}$.

D.4 Proof of Proposition 5

Note that the CIP deviation, as defined in (7), is increasing in b^{H*} (hence (i)), since $\mathbb{E}m^* = \beta$ is fixed. Moreover, note that the UIP deviation can be written as (we use Equations (8), (64) and $\mathbb{E}m^* = \beta$):

$$\begin{aligned} \mathbb{E}X^* &= \tfrac{1}{\beta}\left[\chi + \Gamma b^{H*} - (1+\chi+\Gamma b^{H*})(1-e^{-\rho\sigma^2})\right] \\ &= -\tfrac{1}{\beta}\left[1 - (1+\chi+\Gamma b^{H*})e^{-\rho\sigma^2}\right] \end{aligned}$$

It can be approximated as follows:

$$1 = \frac{1}{\beta}\left(\chi + \Gamma b^{H*} - \rho\sigma^2\right)$$

Since $b^G < \widehat{nfl}^{opt}$ and $\widehat{nfl}^{opt} = nfl^{opt}(\widehat{b})$, then $b^G < nfl^{opt}(\widehat{b})$. Note that, according to Lemma 4, $nfl = min(b^G, nfl^{opt}(\widehat{b}))$. As a result, $nfl = min(b^G, nfl^{opt}(\widehat{b})) = b^G$.

Replacing b^{H*} with $\widehat{b}$ and nfl with b^G, we obtain

$$\mathbb{E}X^* = \chi + \Gamma\frac{\rho\sigma^2[1-\alpha(1+b^G)] - \chi}{\rho^2\sigma^2 + 2\Gamma} - \rho\sigma^2$$

It is decreasing in σ if Γ is small, which is satisfied under Condition 3.

D.5 Fiscally-backed Interventions

We consider here an extension where we allow the central bank to take a net position. We remove the constraint on the first-period portfolio decision of the central bank (17), and we do not impose $b^{CBF} + b^{CB} = 0$. Instead, we assume

$$b^{CBF} + b^{CB} + \tau_1^{CB} = 0 \tag{76}$$

This means that the central bank can set b^{CB} independently from b^{CBF}. The central bank's net borrowing is $-(b^{CBF} + b^{CB})$ and is not necessarily equal to zero. A net position is financed via a government subsidy $-\tau_1^{CB}$. We therefore replace the period-1 government budget constraint (15) with

$$b^G + \tau_1^{CB} = t_1 \tag{77}$$

As a consequence, the equilibrium in the bond market is not directly affected by b^{CBF} as it depends on b^{CB}: $b^{H*} = b^G - b^{CB} - b^H$. As a result, the constraint $b^H \geq 0$ results in the effective constraint for the central bank

$$b^{H*} \leq b^G - b^{CB} \tag{78}$$

instead of (32). This implies that the central bank can change the gross foreign liabilities of the country by issuing domestic bonds (setting a more negative b^{CB}).

The effective constraint on nfl, which results from $b^F \geq 0$, remains unchanged:

$$nfl \leq b^{H*} - b^{CBF}$$

However, a FX intervention (an increase in b^{CBF}) is not necessarily offset by an increase in the gross foreign liabilities (an increase in b^{H*}), because $b^{CBF} + b^{CB}$ is not necessarily zero. To see this, note that $b^{H*} - b^{CBF} = b^G - (b^{CB} + b^{CBF}) - b^H \leq b^G - (b^{CB} + b^{CBF})$. In fact, by increasing it net borrowing $-(b^{CB} + b^{CBF})$, the central bank can relax the constraint on the economy's net borrowing constraint nfl.

A constrained planner that performs both sterilized and fiscally-backed intervention has thus an additional choice variable, b^{CB}, and faces the constraint (78) instead of (32). We define the new modified constrained planner equilibrium as follows:

Definition 4 (Modified Constrained planner equilibrium) *A modified constrained planner equilibrium is an equilibrium where a planner maximizes objective (12) subject to the economy's resource constraints (22); the asset pricing equation (6); the policy rule $f(i, S_1) = 0$; the foreign liability constraints (78) and (33); and the definition of UIP (3). The planner's choice variables are $(i, S_1, b^{H*}, nfl, b^{CBF}, b^{CB})$.*

The central bank's program in that case is:

$$
\begin{aligned}
&\max \mathbb{E}\Big\{ U(c_1) + \beta U(c_2) \\
&+\eta_1 \left(y_1 - c_1 + nfl\right) \\
&+\eta_2 \left[y_2 - c_2 - (1+i^*)nfl - \left[(1+i)\tfrac{S_1}{S_2} - (1+i^*)\right] b^{H*}\right] \\
&+\xi f(i, S_1) \\
&+\Lambda \left(b^{H*} - b^{CBF} - nfl\right) \\
&+\tilde{\Lambda} \left(b^{G} - b^{CB} - b^{H*}\right) \\
&+\alpha_0 \left(\mathbb{E}\left(m^* \left[(1+i)\tfrac{S_1}{S_2} - (1+i^*)\right]\right) - \Gamma b^{H*} - \chi\right) \Big\}
\end{aligned}
$$

Consider the first order conditions for assets:

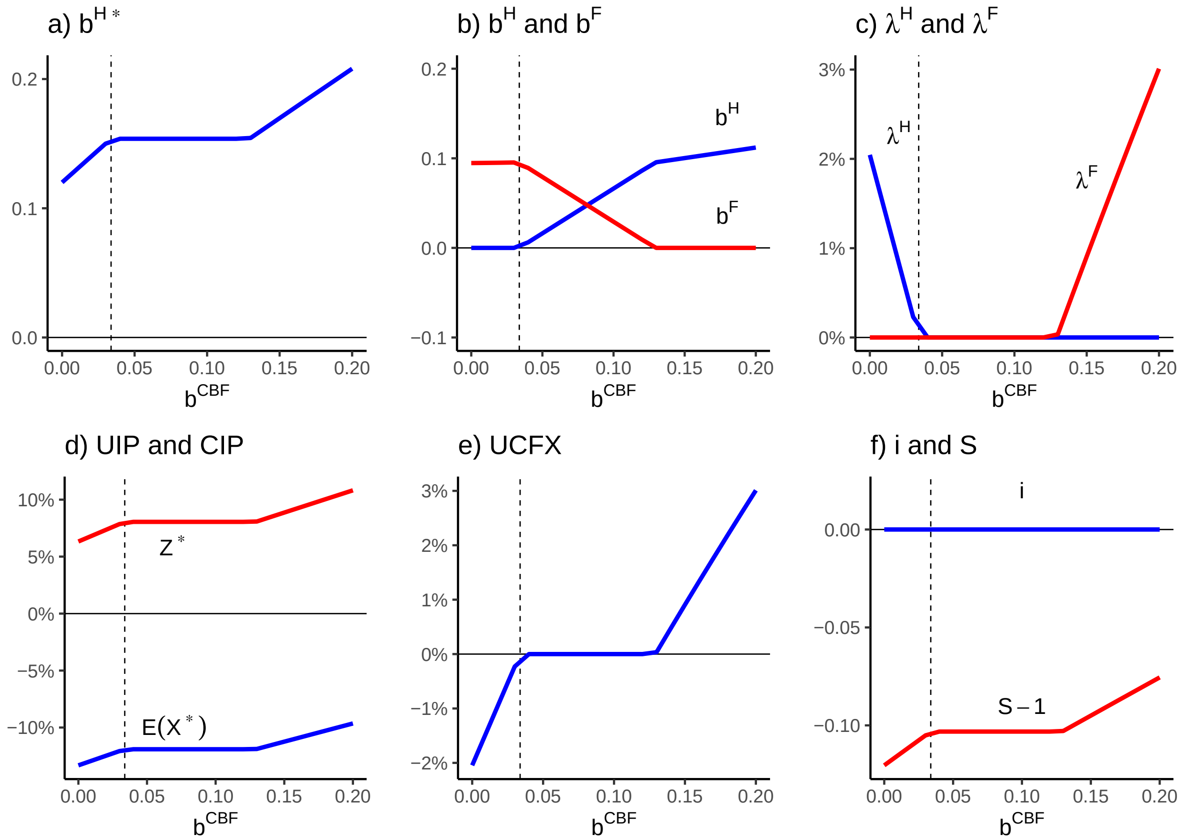
$$
\begin{aligned}
&/nfl: && \eta_1 - \mathbb{E}\left(\eta_2(1+i^*)\right) && -\Lambda && = 0 && (79) \\
&/b^{H*}: && - \mathbb{E}\left(\eta_2 X^*\right) && +\Lambda - \tilde{\Lambda} - \alpha_0 \Gamma && = 0 && (80) \\
&/b^{CBF}: && && -\Lambda && = 0 && (81) \\
&/b^{CB}: && && -\tilde{\Lambda} && = 0 && (82)
\end{aligned}
$$

Equations (81) and (82) imply that $\tilde{\Lambda} = \Lambda = 0$. This means that the central bank is able to relax both its foreign-currency and domestic-currency debt constraints by adjusting its assets b^{CBF} and liabilities b^{CB}. Also, note that, as in our baseline analysis, $\eta_1 = U'(c_1)$, $\eta_2 = U'(c_2)$, and that $m = \eta_2/\eta_1$ is the central bank's discount factor, which coincides with the household's following similar arguments.

As a result, Equation (80) implies that optimal FX intervention follow the same rule (37) as in the baseline, that is, $MBFX = 0$, and Equation (79) implies that intertemporal optimality is now always satisfied: $\mathbb{E}[m(1+i^*)] = 1$.

E Additional Figures

Figure E.1: The Effectiveness of FX Interventions and the Utility Cost of Reserves - Alternative specification



Notes: Parameter values : $\beta = 0.98$, $\sigma^2 = 1$, $\chi = 0.002$, $\Gamma = 0.5$, $\alpha = 0.6$, $\rho = 0.2$, $g = 0.05$, $b^G = 0.12$. The dashed lines represent $b^{CBF} = b^{max} - b^G$.

References

Adler, G. and R. C. Mano (2021). The cost of foreign exchange intervention: Concepts and measurement. *Journal of Macroeconomics 67*, 103045.

Adrian, T., E. Etula, and T. Muir (2014). Financial intermediaries and the cross-section of asset returns. *The Journal of Finance 69*(6), 2557–2596.

Ahir, H., N. Bloom, and D. Furceri (2022). The world uncertainty index. Technical report, National Bureau of Economic Research.

Alfaro, L. and F. Kanczuk (2009). Optimal reserve management and sovereign debt. *Journal of International Economics 77*(1), 23–36.

Amador, M., J. Bianchi, L. Bocola, and F. Perri (2020). Exchange Rate Policies at the Zero Lower Bound. *Review of Economic Studies 87*, 1605–1645.

Arce, F., J. Bengui, and J. Bianchi (2019, September). A macroprudential theory of foreign reserve accumulation. Working Paper 26236, National Bureau of Economic Research.

Bacchetta, P., K. Benhima, and Y. Kalantzis (2013). Capital Controls with International Reserve Accumulation: Can This Be Optimal? *American Economic Journal: Macroeconomics 5*(3), 229–62.

Baker, S. R., N. Bloom, and S. J. Davis (2016). Measuring economic policy uncertainty. *The quarterly journal of economics 131*(4), 1593–1636.

Basu, S., E. Boz, G. Gopinath, F. Roch, and F. Unsal (2020). A Conceptual Model for the Integrated Policy Framework. Working Paper 20/121, International Monetary Fund.

Bianchi, J., S. Bigio, and C. Engel (2022). Scrambling for Dollars: International Liquidity, Banks and Exchange Rates. Mimeo.

Bianchi, J., J. C. Hatchondo, and L. Martinez (2018, September). International reserves and rollover risk. *American Economic Review 108*(9), 2629–70.

Bianchi, J. and G. Lorenzoni (2022). The Prudential Use of Capital Controls and Foreign Currency Reserves. In G. Gopinath, E. Helpman, and K. Rogoff (Eds.), *Handbook of International Economics*, pp. forthcoming. Amsterdam: North Holland.

Brunnermeier, M. K. and Y. Sannikov (2014, February). A macroeconomic model with a financial sector. *American Economic Review 104*(2), 379–421.

Cavallino, P. (2019). Capital Flows and Foreign Exchange Intervention. *American Economic Journal: Macroeconomics 11*(2), 127–70.

Cerutti, E. M., M. Obstfeld, and H. Zhou (2021). Covered interest parity deviations: Macrofinancial determinants. *Journal of International Economics 130*, 103447.

Costinot, A., G. Lorenzoni, and I. Werning (2014). A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation. *Journal of Political Economy 122*(1), 77–128.

Davis, J. S., M. B. Devereux, and C. Yu (2023). Sudden stops and optimal foreign exchange intervention. *Journal of International Economics 141*, 103728.

Devereux, M. B., C. Engel, and S. P. Y. Wu (2022). Collateral Advantage: Exchange Rates, Capital Flows, and Global Cycles. Mimeo.

Du, W. and J. Schreger (2022). CIP Deviations, the Dollar, and Frictions in International Capital Markets. In G. Gopinath, E. Helpman, and K. Rogoff (Eds.), *Handbook of International Economics*, pp. forthcoming. Amsterdam: North Holland.

Edwards, S. (1985). On the interest-rate elasticity of the demand for international reserves: Some evidence from developing countries. *Journal of International Money and Finance 4*, 287–295.

Fanelli, S. and L. Straub (2021). A Theory of Foreign Exchange Interventions. *The Review of Economic Studies 88*(6), 2857–2885.

Fang, X. and Y. Liu (2021). Volatility, intermediaries, and exchange rates. *Journal of Financial Economics 141*(1), 217–233.

Fink, F., L. Frei, and O. Gloede (2022). Global Risk Sentiment and the Swiss Franc: A Time-varying Daily Factor Decomposition Model. *Journal of International Money and Finance 122*, 102539.

Frenkel, J. A. and B. Jovanovic (1981). Optimal international reserves: A stochastic framework. *The Economic Journal 91*(362), 507–514.

Gabaix, X. and M. Maggiori (2015). International Liquidity and Exchange Rate Dynamics. *The Quarterly Journal of Economics 130*(3), 1369–1420.

Gertler, M. and N. Kiyotaki (2010). Chapter 11 - financial intermediation and credit policy in business cycle analysis. Volume 3 of *Handbook of Monetary Economics*, pp. 547–599. Elsevier.

Gourinchas, P.-O. and H. Rey (2022). Exorbitant Privilege and Exorbitant Duty. CEPR Discussion Paper 16944, Centre for Economic Policy Research.

Grisse, C. and T. Nitschka (2015). On Financial Risk and the Safe Haven Characteristics of Swiss Franc Exchange Rates. *Journal of Empirical Finance 32*, 153–164.

Hassan, T. A., T. M. Mertens, and T. Zhang (2022). A risk-based theory of exchange rate stabilization. *The Review of Economic Studies 90*, 879–911.

He, Z., B. Kelly, and A. Manela (2017). Intermediary asset pricing: New evidence from many asset classes. *Journal of Financial Economics 126*(1), 1–35.

He, Z. and A. Krishnamurthy (2011). A Model of Capital and Crises. *The Review of Economic Studies 79*(2), 735–777.

Itskhoki, O. and D. Mukhin (2021). Exchange Rate Disconnect in General Equilibrium. *Journal of Political Economy 129*(8), 2183–2231.

Itskhoki, O. and D. Mukhin (2022). Optimal Exchange Rate Policy. mimeo.

Jeanne, O. and R. Rancière (2011). The optimal level of international reserves for emerging market countries: A new formula and some applications. *The Economic Journal 121*, 905–930.

Jiang, Z., A. Krishnamurthy, and H. Lustig (2021a). Dollar Safety and the Global Financial Cycle. Mimeo.

Jiang, Z., A. Krishnamurthy, and H. Lustig (2021b). Foreign Safe Asset Demand and the Dollar Exchange Rate. *The Journal of Finance 76*, 1049–1089.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. *American economic review 95*(1), 161–182.

Kalemli-Özcan, S. and L. Varela (2021). Five Facts about the UIP Premium. NBER Working Paper 29238, National Bureau of Economic Research.

Kekre, R. and M. Lenel (2024). The Flight to Safety and International Risk Sharing. *American Economic Review 114*(6), 1650–91.

Kim, Y. J. and J. Zhang (2023). International capital flows: Private versus public flows in developing and developed countries. *International Economic Review 64*(1), 225–260.

Maggiori, M. (2017). Financial Intermediation, International Risk Sharing, and Reserve Currencies. *American Economic Review 107*(10), 3038–71.

Maggiori, M. (2021). FX Policy When Financial Markets are Imperfect. Working Paper 942, BIS.

Ranaldo, A. and P. Söderlind (2010). Safe Haven Currencies. *Review of Finance 14*(3), 385–407.

Rime, D., A. Schrimpf, and O. Syrstad (2022). Covered Interest Parity Arbitrage. *The Review of Financial Studies 35*(11), 5185–5227.

Samano, A. (2022). International reserves and central bank independence. *Journal of International Economics 139*, 103674.

Stavrakeva, V. and J. Tang (2021). The Dollar During the Great Recession: The Information Channel of US Monetary Policy and the Flight to Safety. manuscript.

Stock, J. H. and M. W. Watson (2012). Disentangling the channels of the 2007-2009 recession. Technical report, National Bureau of Economic Research.

Valchev, R. (2020). Bond Convenience Yields and Exchange Rate Dynamics. *American Economic Journal: Macroeconomics 12*(2), 124–66.

Wallace, N. (1978). The overlapping-generations model of fiat money. *Staff Report*.

Yeyati, E. L. and J. F. Gómez (2022). Leaning-against-the-wind intervention and the "carry-trade" view of the cost of reserves. *Open Economies Review*, forthcoming.