Insuring Climate Risks in Integrated Markets

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Abstract

We develop a spatial model of climate risks when goods markets across regions are economically integrated but firms can only insure against local climate shocks. We show that firms' insurance demands across regions can be strategic complements or substitutes depending on the correlation of climate shocks across regions. Strategic complementarity can result in equilibrium multiplicity when regions are highly integrated, leading to underinsurance traps. Underinsurance can persist even at actuarially fair insurance premiums and can be Pareto dominated by an economy without insurance markets. We show that insurance market collapse in one region creates contagion effects, potentially contracting insurance markets in other regions. Insurance subsidies can paradoxically worsen the underinsurance problem.

Keywords: Climate risks, economic spillovers, business interruption insurance, commercial property insurance, multiple equilibria, underinsurance
JEL Classifications: D62, D81, G22, G28, G32, G52, Q54, R12, R13

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1 Introduction

Natural disasters and climate change are among the most significant business risks facing firms today (e.g., Allianz, 2025). These events can disrupt firm operations, depress local productivity, and threaten business continuity. According to the National Oceanic and Atmospheric Administration (NOAA), damages from large weather and climate disasters have increased multi-fold—from approximately \$22 billion per year in the 1980s to approximately \$150 billion per year in the last five years (NOAA, 2025). This trend is expected to accelerate, with businesses projected to face annual earnings losses of of up to 7% from climate disasters by 2035 (WEF, 2024). Despite these substantial and growing risks, many businesses remain underinsured against climate risks, despite the widespread availability of insurance products aimed at reducing exposure to these risks, such as commercial property and business interruption insurance. For instance, estimates by the National Association of Insurance Commissioners suggest that only about 30%-40% of small businesses have business interruption insurance (NAIC, 2020).

We develop a spatial model of climate risks to understand the determinants of firms' insurance choices when regions are economically integrated. The key friction is that insurance markets are local while goods markets are integrated across regions. Firms can only insure their production against local climate risks, but households consume goods from all regions. Consequently, households are exposed to climate risks from other regions through economic spillovers.

Our findings demonstrate how economic linkages between regions create strategic interactions in climate insurance demand. First, we show that firms' insurance demands across economically integrated regions can be strategic complements or substitutes. When climate shocks are positively correlated across regions, insurance choices are strategic substitutes. Conversely, when climate shocks are negatively correlated, insurance choices are strategic complements. Second, the strategic complementarity can result in equilibrium multiplicity, creating underinsurance traps. Firms across regions may choose too low a level of insurance due to coordination failures even when a mutually higher level of insurance can be sustained at the same insurance premiums. Third, we demonstrate that the undesirable low-insurance equilibrium can persist even at actuarially fair insurance premiums. While actuarially fair pricing can result in a first-best equilibrium under a utilitarian welfare criterion, coordination failures leading to underinsurance remain possible. In the low-insurance equilibrium, households can be worse off than in a benchmark without insurance markets. Fourth, we show that the collapse of insurance markets in one region creates contagion effects, potentially contracting insurance markets in other regions. Finally, we show that discounted insurance pricing can backfire and worsen the underinsurance problem.

We consider a two-period spatial model of climate risks with two regions. Competitive firms in each region specialize in producing a distinct consumption good using their initial capital endowment and local household labor. Households work locally but consume goods from both regions, creating economic linkages between areas. Households supply labor inelastically and earn wages that they spend on purchasing goods. We assume that households have constant relative risk aversion (CRRA) preferences over a Cobb-Douglas composite of the two regions' goods. This structure allows us to examine how the degree of economic integration affects equilibrium outcomes by varying the expenditure share households allocate to the imported goods.

Production in both regions is subject to climate shocks that we model as productivity losses, consistent with empirical evidence showing that climate change adversely affects firm productivity (e.g., Burke et al., 2015; Moore and Diaz, 2015; Barrot and Sauvagnat, 2016). A key parameter in our model is the correlation between climate shocks across regions, enabling us to examine how the degree of correlation affects insurance decisions. The regions in our model can be broadly interpreted as cities, states, or countries that are economically interconnected. Such economic linkages mean that climate shocks in one region can have spillover effects on others, consistent with evidence from Feng et al. (2023) who document sizable cross-country spillovers in economic performance among trading partners after the flooding of major ports.

Our model is designed to capture climate risks, which differ fundamentally from more idiosyncratic risks such as automobile accidents, and disability, health, or other life events. In our framework, climate risks are perfectly correlated within each region. Since each region specializes in producing a distinct consumption good that households across both regions consume, these shocks become systematic rather than idiosyncratic. Crucially, because agents consume goods from both regions, they care not only about insurance coverage in their own region but also about insurance take-up elsewhere, as this determines their exposure to systematic climate risks. This contrasts sharply with markets for other risks, where spillovers are largely absent. For automobile, health, or life insurance, individuals primarily care about insuring their own exposure, and their utility is not significantly affected by others' insurance choices. The systematic nature of climate risks means that the correlation of shocks across economically connected regions becomes a central determinant of insurance demand—making it a key focus of our analysis.

We first study a baseline model without insurance. We show that household expected utility depends crucially on households' relative preference for the other region's good and on the correlation of climate shocks across regions. When climate shocks are negatively correlated and households value goods from both regions, households benefit from implicit insurance. Because only one region typically experiences a climate shock in a given state of the world, low output in one region is offset by high output in the other region. As shocks become increasingly positively correlated, households' implicit insurance benefit declines.

Next, we introduce an insurance contract that allows firms to protect against climate shocks by reallocating capital from no-disaster states to disaster states. This reallocation is desirable because it smooths consumption for risk-averse households. Insurance premiums determine the cost of this capital reallocation. We assume that firms compete for household labor and therefore choose the insurance contract that maximizes their workers' expected utility. We require that firm capital remains non-negative in all states, enabling firms to take negative insurance positions up to a limit. A negative insurance position means that firms reduce their capital when hit by a disaster, amplifying the decline in production in disaster states.

The insurance contract in our model most closely corresponds to commercial property and business interruption insurance. These insurance products are widely available and designed to replace damaged property, compensate for lost income, and cover continued operating expenses following shocks such as climate disasters. In practice, policies pay for property replacement, ongoing costs (e.g., lease payments and payroll), and extra expenses to maintain production or accelerate reopening. This enables firms to sustain productivity in disaster states even while physical assets are being repaired.

Although insurance markets are local, economic linkages give rise to strategic interactions in insurance demands across regions. When firms insure their production, they increase the output of their local good in disaster states, smoothing the consumption of households in both regions.

This force creates a demand externality, whose nature depends on the correlation of climate shocks across regions.

When climate shocks are positively correlated, insurance demands across regions are strategic substitutes. To see this, consider the extreme case where climate disasters always occur simultaneously in both regions. In this scenario, insurance by firms in one region shifts that region's production from the no-disaster state to the joint-disaster state. This benefits households in the other region during disasters—they can consume more of the insured region's good during these joint disasters. Insurance take-up by firms in one region therefore makes the joint-disaster state less severe for households in the other region, reducing the benefit of insuring the production of the other region's firms.

By contrast, when climate shocks are negatively correlated, insurance demands across regions are strategic complements. To see this, consider the extreme case where climate disasters never occur simultaneously in both regions. In this scenario, insurance by firms in one region shifts that region's production toward its own disaster state, reducing production in its no-disaster state. This creates opposing effects for households in the other region. Their utility is higher when their own region does not experience a climate shock but the other region does, but lower when their region experiences a climate shock and the other region does not. Insurance take-up by firms in one region therefore makes the other region's disaster state more severe, increasing the benefit of insuring the production of the other region's firms. Intuitively, insurance take-up in one region reduces the implicit insurance that the other region receives through economic linkages, increasing the other region's demand for explicit insurance.

This strategic complementarity can give rise to multiple equilibria when economic linkages across regions are sufficiently strong. Building on the logic above, firms face strategic complementarities in insurance demand when climate risks have a sufficiently low correlation: the benefit of insuring their own production increases with the level of insurance in the other region. When households derive substantial utility from imported goods, firms become highly sensitive to insurance decisions in the other region. If firms anticipate high insurance in the other region, they have a strong incentive to insure their own production; conversely, if they anticipate low insurance in the other region, their incentive to insure diminishes. Such interactions can lead to multiple equilibria and an underinsurance trap in which firms in both regions choose low insurance levels due to coordination failures, even when mutually higher levels of insurance can be sustained at the same insurance premiums.

We show that the low-insurance equilibrium persists even when insurance is offered at actuarially fair prices. In particular, we endogenize insurance supply by considering competitive, deep-pocketed insurers, setting prices to break even in expectation in each region. Under these conditions, there is always an equilibrium in which all firms purchase the first-best level of insurance based on a utilitarian welfare criterion. In this equilibrium, firms in both regions are fully insured against climate risks. However, a Pareto dominated low-insurance equilibrium may also arise when strategic complementarities are sufficiently strong. Notably, firms may even choose negative insurance positions, reducing their capital in disaster states and thereby amplifying the productivity shock. In particular, the low-insurance equilibrium can make households in both regions worse off than in an economy without insurance markets.

Insurance companies traditionally benefit from insuring risks with low or negative correlation. By diversifying across regions and climate risks, insurers can more easily finance payouts during climate disasters, reducing the *cost* of insurance provision. However, our analysis demonstrates that the spatial nature of climate risks, and its impact on local economic activity, introduces a novel counter-veiling channel that geographically links insurance demand. When economically connected regions face negatively correlated climate risks, strategic complementarity emerges in insurance demand. This complementarity operates through the *benefit* rather than the *cost* of insurance, and can lead to coordination failures that cause underinsurance traps.

We extend our framework to analyze two relevant economic scenarios. First, we study insurance market collapse in one region. This issue is particularly pertinent given that insurer exit due to climate disasters has been increasing over the past decade and is expected to accelerate (e.g., Sen et al., 2024a; Meredith, 2025). When climate shocks are negatively correlated, strategic complementarity causes firms in the remaining region to underinsure their production—the absence of insurance in one region reduces incentives to insure in the other. Conversely, when climate shocks are positively correlated, strategic substitutability causes firms in the remaining region to overinsure. In both cases, households in the region where insurance markets collapse can be worse off than if insurance markets had collapsed in both regions. Firms may under- or overinsure to such a large extent that they increase consumption risk of the imported good. The strategic responses by firms in economically integrated regions can therefore amplify the negative effects of insurer exit. As such, economic linkages can amplify the collapse of insurance markets because of spillover effects.

Second, we study insurance choices when insurance premiums are offered at a discount to actuarially fair prices. This case is relevant because many government policies cap insurance prices, keeping them at artificially low levels (e.g., Oh et al., 2022). As expected, discounted premiums can give rise to overinsurance. Surprisingly, when multiple equilibria exist, discounted premiums can also amplify the underinsurance problem in the low-insurance equilibrium. Discounted insurance can therefore paradoxically worsen underinsurance.

While our framework is most applicable to commercial property and business interruption insurance, its insights can also be applied to other forms of insurance such as household demand for flood or wildfire insurance. In such an interpretation, households individually decide whether to take-up insurance for their home, making these decisions local, but they benefit from the condition and amenities of neighboring properties.[1] When neighbors' insurance choices affect property values and local amenities, similar strategic interactions arise. Our framework can therefore also help explain underinsurance problems among homeowners (e.g., Cookson et al., 2024; Sen et al., 2024b).

Our model can also be interpreted in terms of supply chain linkages across firms. Households may consume final goods produced locally while firms source inputs from other regions. The degree of economic linkages through supply chains would depend on how critical these imported inputs are for local production. When firms cannot fully insure shocks to their supply chains, our mechanism creates strategic interactions in firm insurance demand across regions. If firms rely heavily on imported inputs and climate risks across regions are negatively correlated, strategic complementarities in insurance choices can create underinsurance traps.

[1] See Gao et al. (2021) for a model of spillover effects in housing markets within a neighborhood.

2 Related Literature

Our paper contributes to the literature studying how climate risks affect insurance provision and take-up. This literature examines several key dimensions: belief heterogeneity and risk misperception (e.g., Kunreuther and Pauly, 2004; Bakkensen and Barrage, 2022; Conell-Price and Mulder, 2024; Mulder, 2024), catastrophe risk characteristics including fat-tailed distributions (e.g., Weitzman, 2009; Raykov, 2015; Louaas and Picard, 2021),[2] adverse selection (e.g., Mulder, 2019; Boomhower et al., 2024), regulatory interventions and pricing distortions (e.g., Kaplow, 1991; Charpentier and Le Maux, 2009; Kousky, 2018; Goussebaïle, 2020; Wu, 2020; Ge et al., 2022; Oh et al., 2022; Sen et al., 2024a,b; Ge et al., 2024; Jia et al., 2025b; Taylor et al., 2025), reinsurance constraints and capital market frictions (e.g., Jaffee and Russell, 1997; Froot, 2001; Keys and Mulder, 2024), contract design including term structure and catastrophe bonds (e.g., Lakdawalla and Zanjani, 2011; Kleindorfer et al., 2012; Borensztein et al., 2017; Altunbas et al., 2024), and household location responses to climate risks (e.g., Frame, 1998). We introduce a novel focus on economic linkages across regions and demonstrate their significant implications for insurance demand. In addition, while existing research focuses primarily on homeowner insurance, we study business insurance—a particularly important area given the substantial climate risks firms face (e.g., WEF, 2024; Allianz, 2025)—though our findings also extend to homeowner insurance markets.

We also contribute to the literature on spillover effects in insurance markets. One strand of this literature examines peer effects in insurance take-up through information channels (e.g., Mobarak and Rosenzweig, 2012; Karlan et al., 2014; Cai et al., 2015), showing that insurance choices are influenced by neighbors' beliefs and decisions. A second strand studies how insurance take-up by one group affects insurance availability for others through adverse selection and pricing (e.g., Cutler and Reber, 1998; Handel et al., 2015). In contrast, our focus is on spillover effects arising from economic linkages and geographic correlations of climate shocks.

Our paper also relates to the literature on spatial equilibrium models of climate change.[3] This literature examines how spatial heterogeneity in climate risks, migration, and the mobility of capi-

[2] See Rossi et al. (2005) for an overview of modeling features of catastrophe risks and insurance.

[3] See Desmet and Rossi-Hansberg (2024) for a recent review of this literature.

tal and goods shape the economic impact of climate change. Most relevant to our work are papers analyzing economic linkages between regions through trade (e.g., Dingel et al., 2019; Conte et al., 2021; Rudik et al., 2021), which examine trade as an adaptation mechanism that can reallocate production across regions. Our contribution differs by focusing on insurance markets. We show that trade linkages can create strategic interactions in insurance demand, potentially leading to underinsurance traps.[4]

Our paper connects to literature on international trade and production networks that examine how local disasters propagate along supply chains and across space. For example, Barrot and Sauvagnat (2016) show that input specificity amplifies disruption spillovers, while Boehm et al. (2019) and Carvalho et al. (2021) document how climate shocks transmit through supply-chain networks. Like these studies, our analysis recognizes that local shocks have broader consequences due to economic interconnectedness. However, we examine how the propagation of shocks through economic linkages influences firms' insurance choices.

The paper proceeds as follows. Section 3 presents the baseline model without insurance, and Section 4 analyzes this framework. Section 5 introduces and analyzes insurance markets. Section 6 examines insurance market collapse and discounted insurance pricing. The last section concludes.

3 Model

We develop a two-region spatial equilibrium model to analyze how climate disasters affect local economies through productivity shocks. The economy consists of two regions, each specializing in producing a distinct consumption good. Households in each region work locally but consume goods from both regions, creating cross-region economic linkages. Firms in each region produce using labor and capital, with their productivity subject to climate shocks that can be correlated across regions. In the baseline model, climate risks cannot be insured, and all economic decisions occur after shocks are realized. In Section 5, we extend the model to include insurance contracts that are available before shocks occur.

[4] Another strand of this literature (e.g., Jia et al., 2025a) focuses on how climate risks impact business productivity in the absence of trade effects.

Spatial Setup: The economy consists of two regions, A and B. Let m_i denote the mass of households in region $i \in \{A, B\}$, with $m_A + m_B = 1$. Each region specializes in producing a distinct consumption good using household labor and capital, with region A producing good A and region B producing good B. Labor markets are local—households can only work in their region of residence.

Households: Households in region i have constant relative risk aversion (CRRA) preferences with risk aversion parameter $\gamma > 1$ over a Cobb-Douglas composite consumption good

$$C_i = \left(\frac{c_{i,i}}{1-\alpha}\right)^{1-\alpha} \left(\frac{c_{-i,i}}{\alpha}\right)^{\alpha}, \tag{1}$$

where $c_{j,i}$ denotes consumption of good j by a region i household, and the parameter $\alpha \in [0, 1]$ governs the expenditure share on the imported good. Throughout, we use i to index a household's region and $-i$ to denote the other region. Since all households within a region are identical and make identical choices, we omit household-specific subscripts.

Each household supplies one unit of labor inelastically in its region ($l_i = 1$) and receives wage w_i. After climate shocks realize, a household in region i solves

$$V_i = \max_{\{c_{A,i}, c_{B,i}\}} u(C_i) \text{ subject to } p_A c_{A,i} + p_B c_{B,i} = w_i, \tag{2}$$

where p_A and p_B are the prices of goods A and B, respectively.

Firms: Each region contains a unit continuum of identical firms that produce the local consumption good using the production technology

$$Y_i = Z_i L_i K_i,$$

where Z_i is productivity, L_i is labor input, and K_i is capital input. Each firm is endowed with K units of capital. In the baseline model, firms use all their capital for production ($K_i = K$), such that $Y_i = Z_i L_i K$. With insurance, studied in Section 5, firms can allocate the capital endowment to pay for insurance against climate shocks.

Each firm chooses labor to maximize profits

$$\Pi_i = \max_{L_i} p_i Z_i L_i K - w_i L_i. \tag{3}$$

The first-order condition yields the equilibrium wage

$$w_i = p_i Z_i K. \tag{4}$$

In equilibrium, firms employ the entire labor force ($L_i = m_i$) and earn zero profits.

Climate Risks: Production in both regions is subject to climate shocks that reduce firm productivity. Following Burke et al. (2015), Moore and Diaz (2015), and Barrot and Sauvagnat (2016), who document that climate change adversely impacts economic productivity, we model these shocks as productivity losses—for example fractional losses in output from impaired business operations.

Without a climate shock in region i, $Z_i = 1$. In the event of a climate shock in region i, productivity falls to $Z_i = \phi_i$, where $\phi_i \in (0,1)$ represents the fraction of output preserved. From equation (4), wages decline in regions experiencing climate shocks due to reduced labor productivity. This mechanism is consistent with Boustan et al. (2020), who document that the local economic response to natural disasters in U.S. counties is characterized by declining labor productivity.

Table 1 presents the joint distribution of climate shocks. The parameter $q \in (0,1)$ determines each region's marginal shock probability, while ρ controls the correlation between shocks across regions, with the correlation coefficient given by

$$\mathrm{Corr}(Z_A, Z_B) = \frac{\rho - q^2}{q(1-q)}.$$

In the baseline model, these shocks cannot be insured.

This structure allows us to examine how equilibrium outcomes vary with the correlation between climate shocks across regions—a key comparative static that we consider in our analysis. When $\rho = q^2$, climate shocks are independent across regions. When $\rho > q^2$ ($\rho < q^2$), shocks are positively (negatively) correlated. In the analysis, we will often focus on two polar cases. First,

$Z_A \backslash Z_B$	1	ϕ_B	Marginal of Z_A
1	$1-2q+\rho$	$q-\rho$	$1-q$
ϕ_A	$q-\rho$	ρ	q
Marginal of Z_B	$1-q$	q	

Table 1: Joint Distribution of Productivity Shocks Z_A and Z_B

when $\rho = 0$, shocks are negatively correlated and regions are never hit by climate shocks simultaneously. Second, when $\rho = q$, shocks are perfectly positively correlated and regions are always hit by climate shocks simultaneously.

Equilibrium: An equilibrium without insurance consists of prices $\{p_i\}_{i \in A,B}$, wages $\{w_i\}_{i \in A,B}$, and consumption choices $\{c_{A,i}, c_{B,i}\}_{i \in A,B}$, such that household choices solve the household problem (2), wages are determined by firm profit maximization (3), and goods markets clear:

$$m_A c_{A,A} + m_B c_{A,B} = Z_A L_A K, \tag{6}$$

$$m_A c_{B,A} + m_B c_{B,B} = Z_B L_B K. \tag{7}$$

4 Equilibrium without Insurance

In this section, we study the equilibrium of the baseline model in which climate risks cannot be insured. Proposition 1 characterizes the unique equilibrium.

Proposition 1. *There exists a unique equilibrium with household consumption given by*

$$c_{i,i} = (1-\alpha) Z_i K,$$

and

$$c_{-i,i} = \alpha \frac{Z_{-i} K m_{-i}}{m_i},$$

where $Z_i \in \{1, \phi_i\}$ *and* $i \in \{A, B\}$*. The relative price of goods A and B is given by*

$$\frac{p_B}{p_A} = \frac{Z_A m_A}{Z_B m_B}.$$

The expected utility of a household in region $i \in \{A, B\}$ before climate shocks realize, $\mathbb{E}[V_i]$, is given by

$$\frac{\left(\frac{m_{-i}}{m_i}\right)^{\alpha(1-\gamma)} K^{1-\gamma}}{1-\gamma}\left[(1-2q+\rho)+(q-\rho)\left(\phi_i^{(1-\alpha)(1-\gamma)}+\phi_{-i}^{\alpha(1-\gamma)}\right)+\rho\phi_i^{(1-\alpha)(1-\gamma)}\phi_{-i}^{\alpha(1-\gamma)}\right].$$

Proposition 1 shows that, as is standard, a household in either region consumes a fixed fraction $1-\alpha$ of its own region's good and a fraction α of the other region's good. With the Cobb-Douglas composite good, expenditure shares are constant, so consumption shares do not vary with income or output. The relative prices of the two goods depend on relative outputs—the good with greater supply is relatively cheaper.

Household expected utility increases with the productivity of each region, ϕ_i and ϕ_{-i}, when $\alpha \in (0,1)$. Higher productivity in either region raises that region's output, expanding the consumption bundle available to households in both regions. Expected utility also decreases with the size of the local population and increases with the other region's population. The Cobb-Douglas structure implies that households in each region consume shares $1-\alpha$ and α of domestic and imported goods, respectively. When the local population increases, it raises local output but also increases the number of people sharing that output. These two effects exactly offset each other for domestic good consumption. However, a larger local population means each household gets a smaller share of the fixed amount of the imported good, reducing expected utility. Conversely, when the other region's population increases, it increases imported good production without reducing each household's share. Expected utility is therefore increasing in the other region's population.

Households expected utility depends crucially on households' preference for the other region's good, α, and on the correlation of climate shocks, ρ. Figure 2 plots certainty equivalent consumption as a function of α in a symmetric economy with identical populations and productivity shocks. The blue line represents the case when $\rho = 0$ and climate shocks are negatively correlated and never occur simultaneously, while the red line represents the case when $\rho = q$ and climate shocks are positively correlated and always occur simultaneously. When $\alpha = 0$, households only care about their own region's good, so certainty equivalent consumption depends only on their region's unconditional shock probability and is unaffected by the correlation between climate shocks. When

$\rho = 0$, certainty equivalent consumption is inverted U-shaped in α. With negatively correlated climate shocks, households benefit from implicit insurance when $\alpha \in (0,1)$—when one region experiences a shock, the other region's production remains unaffected. In a symmetric economy, this implicit insurance is maximized at $\alpha = 0.5$ where households equally value both goods. When $\rho = q$, households lose this implicit insurance benefit, resulting in flat certainty equivalent consumption across α values.

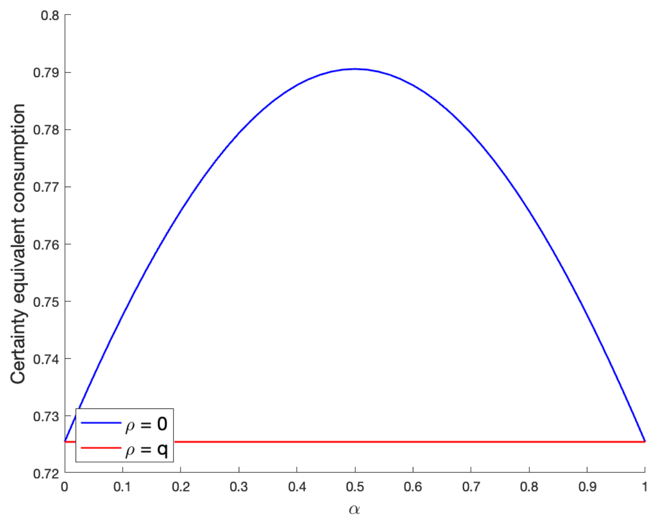

Figure 1: Certainty Equivalent Consumption (Symmetric Economy): This figure plots certainty equivalent consumption of a household as a function of α for $\rho = 0$ (regions are never hit by climate shocks simultaneously), and $\rho = q$ (regions are always hit by climate shocks simultaneously), in the equilibrium without insurance. The figure shows a symmetric economy, with parameters $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

Figure 2 examines an asymmetric economy where $\phi_A > \phi_B$, so region B experiences larger productivity losses than region A during climate shocks. The solid lines plot certainty equivalent consumption for households in region A, while the dashed lines plot that for households in region B. When $\alpha = 0$, households in region A have higher certainty equivalent consumption than those in region B because they consume only the good with lower climate risk exposure. This reverses when $\alpha = 1$. With perfectly correlated shocks ($\rho = q$), certainty equivalent consumption remains linear in α for both regions, but slopes in opposite directions: decreasing for region A households and increasing for region B households. With negatively correlated shocks ($\rho = 0$), both regions exhibit the same inverted U-shape due to implicit insurance, but at different levels reflecting their different risk exposures.

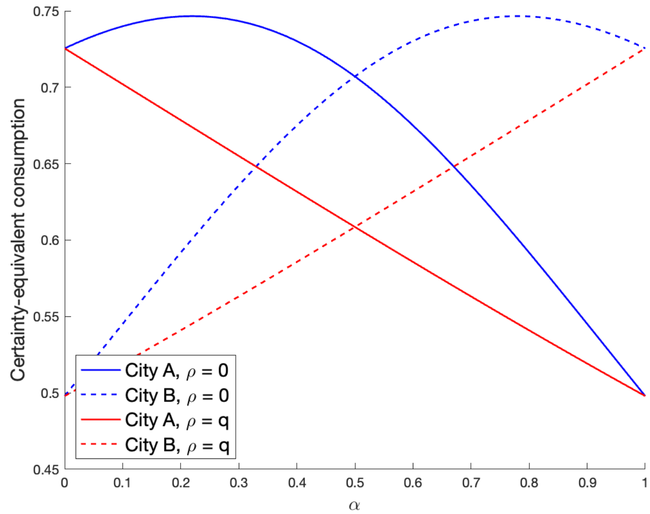

Figure 2: Certainty Equivalent Consumption (Asymmetric Economy): This figure plots certainty equivalent consumption as a function of α for $\rho = 0$ (regions are never hit by climate shocks simultaneously) and $\rho = q$ (regions are always hit by climate shocks simultaneously), in the equilibrium without insurance. The figure shows an asymmetric economy in which region B has a worse productivity shock when hit by a climate disaster relative to region A. The solid (dashed) lines plot certainty equivalent consumption of households in region A (region B), with parameters $\phi_A = 0.5$, $\phi_B = 0.3$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

5 Equilibrium with Insurance

In this section, we introduce insurance that allows firms to protect against climate shocks. In Section 5.1, we discuss how insurance operates in our model. In Section 5.2, we solve for insurance demand by firms for given insurance premiums. In Section 5.3, we endogenize insurance supply and characterize equilibrium when insurance markets are competitive and offer actuarially fair insurance. In Section 5.4, we study welfare implications.

5.1 Setup

We extend the baseline model to introduce insurance that allows firms to protect against climate shocks. The economy now has a two-period structure where firms choose insurance coverage at date 0 before climate shocks realize, and production and consumption decisions occur at date 1 after shocks realize.

Each firm in region i can purchase insurance coverage f_i at a premium rate $\pi_i \in (0,1)$ per unit of coverage, where $f_i > 0$ corresponds to purchasing insurance and $f_i < 0$ corresponds to a negative insurance position. The total premium $f_i \pi_i$ is paid at date 0 from the firm's capital endowment regardless of whether a climate disaster subsequently occurs. If a climate disaster occurs in region i at date 1, the insurance contract pays out f_i. This insurance mechanism allows firms to reallocate capital from the no-disaster state to the disaster state through a state-contingent contract. We solve for insurance demand for given insurance premiums π_i. In Section 5.3, we endogenize insurance supply and premiums.

Insurance affects a firm's capital available for production. Without a disaster in region i, the firm's productive capital is $K_i = K - f_i \pi_i$, where K is the initial capital endowment and $f_i \pi_i$ represents the premium paid. With a disaster, the firm receives an insurance payout of f_i, yielding productive capital of $K_i = K - f_i \pi_i + f_i = K + f_i(1 - \pi_i)$. We define the indicator variable $\delta_i \in \{0,1\}$ to represent the insurance payout per unit of coverage, with $\delta_i = 1$ if a disaster occurs and $\delta_i = 0$ otherwise. Productive capital for a firm in region i is therefore $K_i = K + f_i(\delta_i - \pi_i)$.

Capital must remain non-negative in all states, so insurance choices are constrained by $f_i \leq \frac{K}{\pi_i}$ and $f_i \geq -\frac{K}{1-\pi_i}$. The first constraint ensures that firms can afford the insurance premium in the absence of a disaster, while the second constraint ensures non-negative capital when firms take negative insurance positions.

Insurance in our setting is best interpreted as commercial property and business interruption insurance. These insurance products are widely available and are designed to replace damaged property, compensate for lost net income, and cover continued operating expenses following a shock to the firm's productivity, such as climate disasters. In practice, policies pay for property repair or replacement, ongoing costs (e.g., rent, lease, and payroll), and extra expenses to maintain production or accelerate reopening, enabling firms to sustain their productivity during disasters even while physical assets are being repaired.

While we model insurance through capital reallocation, we could alternatively model direct productivity insurance by allowing firms to spend resources on restoring productivity Z_i in disaster states. The key feature we capture is that firms can insure their production, a feature of commercial property or business productivity insurance. This differs from insurance that provides households lump-sum payments, which would not directly insure firm production.

Firms in region i maximize profits by choosing labor input to solve

$$\Pi_i = \max_{L_i} p_i Z_i L_i K_i - w_i L_i,$$

where K_i depends on both the insurance choice f_i at date 0 and the realization of climate shocks at date 1. The first-order condition yields the equilibrium wage

$$w_i = p_i Z_i K_i = p_i Z_i (K + f_i(\delta_i - \pi_i)). \tag{8}$$

Due to the linearity of firms profits in L_i, firms earn zero profits in equilibrium. We assume firms compete for workers, so they choose their insurance coverage f_i to maximize the expected utility of their workers—the households in their region—$\mathbb{E}[V_i]$, given in Equation (2), taking goods prices as given. This captures the idea that firms must internalize the welfare effects on local households to attract labor: a firm offering lower expected utility cannot attract any workers.

5.2 Insurance Demand

We begin by analyzing insurance demand by firms in both regions for given insurance premiums. Proposition 2 characterizes equilibrium with insurance.

Proposition 2. *There exists an equilibrium with household consumption given by*

$$c_{i,i} = (1-\alpha) Z_i K_i,$$

and

$$c_{-i,i} = \alpha \frac{Z_{-i} K_{-i} m_{-i}}{m_i},$$

where $Z_i \in \{1, \phi_i\}$, $K_i = K + f_i(\delta_i - \pi_i)$, and $i \in \{A, B\}$. The relative price of goods A and B is *given by*

$$\frac{p_B}{p_A} = \frac{Z_A K_A m_A}{Z_B K_B m_B}.$$

The insurance position of a firm in region $i \in \{A, B\}$ before climate shocks realize, f_i, as a function of the insurance position of a firm in the other region, f_{-i}, is given implicitly by

$$\left(\frac{\phi_i (K + f_i (1-\pi_i))}{K - f_i \pi_i}\right)^{(1-\gamma)(1-\alpha)-1} = \frac{\pi_i}{1-\pi_i} \frac{1}{\phi_i} \frac{1 - 2q + \rho + (q-\rho)\left(\frac{\phi_{-i}(K+f_{-i}(1-\pi_{-i}))}{K-f_{-i}\pi_{-i}}\right)^{(1-\gamma)\alpha}}{\left(q - \rho + \rho\left(\frac{\phi_{-i}(K+f_{-i}(1-\pi_{-i}))}{K-f_{-i}\pi_{-i}}\right)^{(1-\gamma)\alpha}\right)}.$$

The expected utility of a household in region $i \in \{A, B\}$ before climate shocks realize, $\mathbb{E}[V_i]$, is given by

$$\frac{\left(\frac{m_{-i}}{m_i}\right)^{\alpha(1-\gamma)}}{1-\gamma} \mathbb{E}\left[(Z_i K_i)^{(1-\alpha)(1-\gamma)} (Z_{-i} K_{-i})^{\alpha(1-\gamma)}\right],$$

where the expectation is taken with respect to the joint distribution of climate shocks (Z_i, Z_{-i}).

Proposition 2 shows that insurance alters consumption by changing the state-contingent output within each region. Firms purchasing insurance reallocate production across states, reducing output in normal times to increase it during disasters. This reallocation smooths consumption for risk-averse households. When firms in a region insure their production against climate risks, they increase output in the disaster state, lowering the relative price of their good and increasing the consumption of households in both regions. Thus, even though insurance markets are local, households can benefit from insurance take-up in the other region.

Proposition 2 reveals strategic interactions between regions in firms' insurance take-up. When $\alpha \in (0,1)$, insurance choices by firms in one region (f_i) depend on insurance choices by firms in the other region (f_{-i}). We examine the nature of this interaction by analyzing how f_i varies with f_{-i} under negatively correlated shocks ($\rho < q^2$) and positively correlated shocks ($\rho > q^2$).

Corollary 1. *Let $\alpha \in (0,1)$. Then, optimal insurance positions exhibit the following strategic relationships: When $\rho = q^2$ (uncorrelated climate shocks), f_i is independent of f_{-i}. When $\rho < q^2$ (negatively correlated climate shocks), f_i is increasing in f_{-i} (strategic complements). When $\rho > q^2$ (positively correlated climate shocks), f_i is decreasing in f_{-i} (strategic substitutes).*

Corollary 1 shows that, although insurance markets are local, economic linkages give rise to strategic interactions in insurance demands across regions. When firms insure their production, they increase the output of their local good in disaster states, smoothing the consumption of households of that good in both regions. This force creates a demand externality, whose nature depends the correlation of climate shocks across regions.

When climate shocks are positively correlated ($\rho > q^2$), insurance demands across regions are strategic substitutes. To see this, consider the extreme case where climate disasters always occur simultaneously in both regions ($\rho = q$). In this scenario, insurance by firms in one region shifts that region's production from the no-disaster state to the joint-disaster state. This benefits households in the other region during disasters—they can consume more of the insured region's good during these joint disasters. Insurance take-up by firms in one region therefore makes the joint-disaster state less severe for households in the other region, reducing the benefit for firms in the other region of insuring their own production.

By contrast, when climate shocks are negatively correlated ($\rho < q^2$), insurance demands across regions are strategic complements. To see this, consider the extreme case where climate disasters never occur simultaneously in both regions ($\rho = 0$). In this scenario, insurance by firms in one region shifts that region's production toward its own disaster state, reducing production in its no-disaster state. This creates opposing effects for households in the other region. Their utility is higher when their own region does not experience a climate shock but the other region does, but lower when their region experiences a climate shock and the other region does not. Insurance take-up by firms in one region therefore makes the other region's disaster state more severe, increasing the benefit of insuring the production of firms in the other region. Intuitively, insurance take-up in one region reduces the implicit insurance that the other region receives through economic linkages, increasing the other region's demand for explicit insurance.

Figure 3 plots region A firms' best response function for insurance take-up. When $\rho = 0$, f_A increases with f_B, demonstrating strategic complementarity in insurance demand. Conversely, when $\rho = q$, f_A decreases with f_B, demonstrating strategic substitution in insurance demand.

Strategic complementarity in insurance take-up can lead to multiple equilibria, and coordination failures in insurance markets. The following proposition establishes necessary conditions for

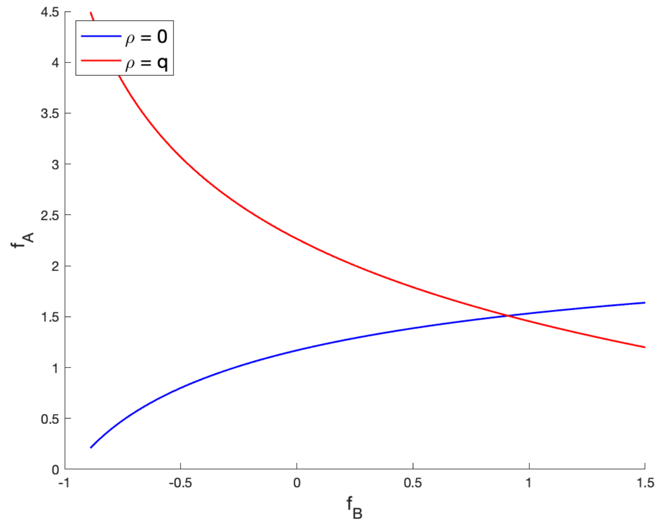

Figure 3: Best Response of Firms in Region A to Insurance Take-up by Firms in Region B: This figure plots region A firms' optimal insurance choice, f_A as a function of region B firms' insurance choice, f_B, for $\rho = 0$ (regions are never hit by climate shocks simultaneously) and $\rho = q$ (regions are always hit by climate shocks simultaneously), with parameters $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, $\alpha = 0.5$, and $\gamma = 3$.

equilibrium multiplicity.

Proposition 3. *There exists a threshold $\rho^* < q$ such that necessary conditions for multiple equilibria are that $\alpha > \frac{1}{2}\frac{\gamma}{\gamma-1}$ and $\rho \leq \rho^*$.*

Our model exhibits multiplicity because of strategic complementarities in insurance demands across economically integrated regions. The first condition requires that α—the expenditure share on the other region's good—is sufficiently large, meaning that households highly value the other regions good. The second condition requires that ρ—the correlation of climate shocks across regions—is sufficiently low.

Figure 4 plots equilibrium insurance levels as a function of the expenditure share on the other region's good, α, in a symmetric economy. With negatively correlated shocks ($\rho = 0$), represented by the blue and red lines, strategic complementarity creates interesting patterns. For low values of α, firms increase their insurance as households value the other region's good more. This result occurs because strategic complementarity makes insurance mutually reinforcing across regions. As α increases, households depend more heavily on the other region's good. Insurance take-up in that region then reduces the implicit insurance these households receive by reducing output of the imported good in their region's disaster state, making explicit insurance in their own region more valuable.

The most striking result emerges when $\alpha > \frac{1}{2}\frac{\gamma}{\gamma-1}$. At this threshold, households' strong preference for the other region's good creates a coordination problem. Recall from the intuition above that when shocks are negatively correlated, insurance take-up in one region reduces the implicit insurance available to the other region. With high α, this effect becomes so strong that firms face two possible outcomes: if they anticipate high insurance in the other region, they find it optimal to purchase high insurance themselves (blue line); but if they anticipate low insurance in the other region, they optimally choose low insurance as well (red line). This coordination problem arises because firms cannot directly insure against the other region's productivity shock, yet households' welfare depends critically on it.

By contrast, with positively correlated shocks ($\rho = q$), strategic substitutability eliminates this multiplicity, yielding a unique equilibrium (green line).

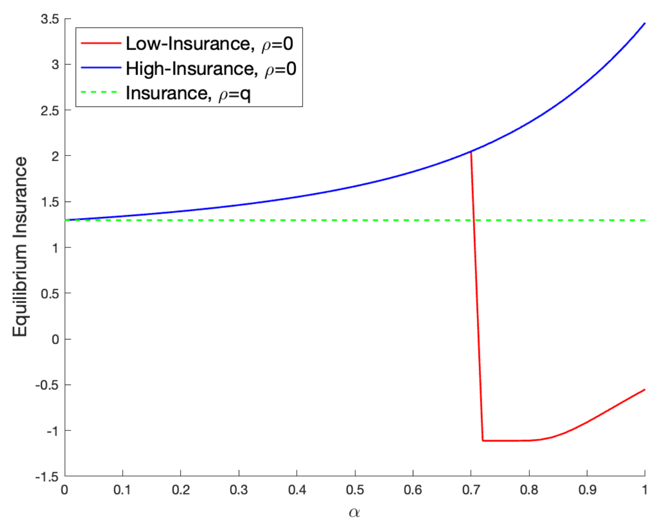

Figure 4: Equilibrium Insurance (Symmetric Economy): This figure plots equilibrium insurance take-up of firms in region A and region B for $\rho = 0$ (regions are never hit by climate shocks simultaneously) and $\rho = q$ (regions are always hit by climate shocks simultaneously), with parameters $\pi_A = \pi_B = 0.1$, $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

Figure 5 plots equilibrium insurance levels in both region for an asymmetric economy where $\phi_A > \phi_B$ when $\rho = 0$. The same patterns emerge—when α is sufficiently high, the economy exhibits both high and low-insurance equilibria. Region B's insurance take-up is higher than region

A's because region B experiences a larger productivity loss during climate shocks, making insurance more valuable.

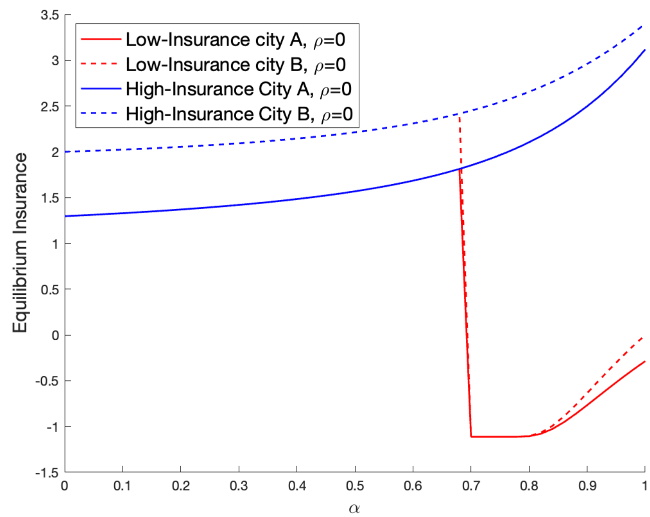

Figure 5: Equilibrium Insurance (Asymmetric Economy): This figure plots the equilibrium insurance take-up of firms in region A and region B for $\rho = 0$ (the regions are never hit by climate shocks simultaneously). The solid (dashed) line plots insurance take-up of firms in region A (region B), with parameters $\pi_A = \pi_B = 0.1$, $\phi_A = 0.5$, $\phi_B = 0.3$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

5.3 Insurance Supply

We endogenize insurance supply by assuming competitive insurance markets where deep-pocketed insurers set premiums to earn zero expected profits in each region. Under competition, insurance is priced actuarially fairly at $\mathbb{E}\left[Z_i\left(\delta_i - \pi_i\right)\right]$=0, where $\delta_i = 1$ if a disaster occurs in region i and $\delta_i = 0$ otherwise, which gives $\pi_i = \frac{q\phi_i}{1-q+q\phi_i}$. The productivity term Z_i appears in this pricing formula because all agents in a region—including insurers—value capital at the same productivity-adjusted rate. This ensures that the marginal value of capital is equated across all uses in equilibrium. In Section 6.2, we examine insurance offered at a discount to actuarially fair prices.

When insurance is competitive and actuarially fair, there always exists a full-insurance equilibrium in which output in both regions is identical across all states. This full-insurance equilibrium corresponds to the utilitarian optimum. However, a Pareto-inferior equilibrium with imperfect insurance can still exist for sufficiently high values of α and sufficiently low values of ρ.

Corollary 2. *With actuarially fair insurance in both regions ($\pi_A = \frac{q\phi_A}{1-q+q\phi_A}$ and $\pi_B = \frac{q\phi_B}{1-q+q\phi_B}$), there exists a full-insurance equilibrium with*

$$f_i^{FI} = \frac{(1-\phi_i)\,K}{\pi_i + (1-\pi_i)\,\phi_i},$$

in region $i \in \{A, B\}$. The full-insurance equilibrium maximizes utilitarian welfare. If $\alpha > \frac{1}{2}\frac{\gamma}{\gamma-1}$ and ρ is sufficiently small, a second equilibrium with imperfect insurance may exist, which is Pareto-inferior.

When insurance is actuarially fair, firms may insure their production completely. However, because insurance markets are local, they cannot directly insure against the climate shocks of the other region. When households have sufficiently strong preferences for the other region's good and climate shocks across regions have a sufficiently low correlation, two equilibria are possible. If firms expect the other region to insure fully, they face only their own productivity risk and will insure fully. However, if firms expect firms in the other region to underinsure, then they benefit from increased implicit insurance—production of the imported good is higher in their disaster state—and therefore may choose to underinsure as well.[5]

Figure 6 plots equilibrium insurance take-up with actuarially fair pricing in a symmetric economy. A full-insurance equilibrium always exists regardless of ρ. However, when ρ is sufficiently low and α is sufficiently high, strategic complementarities create a second low-insurance equilibrium, illustrated by the red lines for $\rho = 0$. By contrast, Figure 4, showed insurance choices at discounted prices. There, the high-insurance equilibrium featured overinsurance while the low-insurance equilibrium featured underinsurance. We discuss discounted pricing in more detail in Section 6.2.

Our mechanism suggests that small changes in economic linkages can trigger large shifts in firms' insurance take-up, echoing results in other literatures about how small changes in underlying fundamentals can lead to severe market responses (e.g., Brunnermeier and Pedersen, 2009). Specifically, low insurance take-up in one region can spill over and cause underinsurance in other regions when economic integration increases even modestly.

[5] Note that the second equilibrium may feature over- or underinsurance.

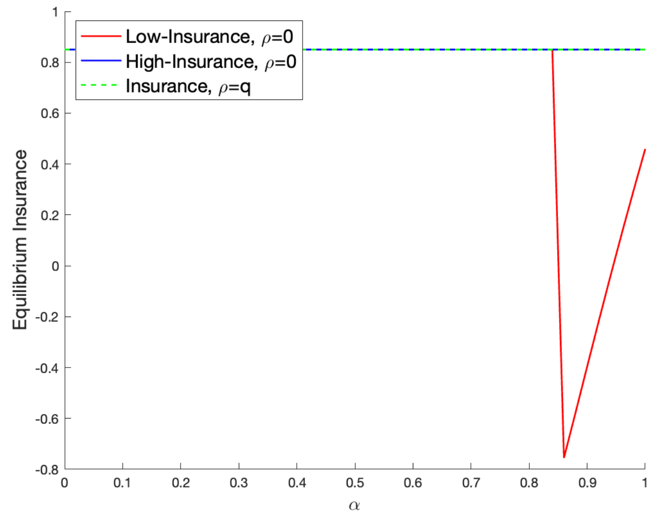

Figure 6: Equilibrium Insurance at Actuarially Fair Insurance Premiums (Symmetric Economy): This figure plots equilibrium insurance take-up of firms in region A and region B when insurance is actuarially for $\rho = 0$ (regions are never hit by climate shocks simultaneously) and $\rho = q$ (regions are always hit by climate shocks simultaneously), with parameters $\pi_A = \pi_B = 0.1765$, $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

5.4 Welfare

Corollary 2 shows that full insurance always constitutes an equilibrium under actuarially fair pricing and that from a utilitarian perspective, full insurance is optimal. However, a Pareto inferior low-insurance equilibrium can also arise when economic linkages between regions are sufficiently strong and climate shocks have sufficiently low correlation.

Figure 7 plots certainty equivalent consumption under actuarially fair pricing as a function of households' expenditure share on the other region's good in a symmetric economy. Under full insurance, certainty equivalent consumption remains constant across correlation levels ($\rho = 0$ and $\rho = q$). The welfare gain from introducing insurance (relative to an economy without insurance markets) is larger when $\rho = q$ than when $\rho = 0$ because positively correlated shocks eliminate the implicit insurance that economic integration provides with negatively correlated shocks.

Most strikingly, households can be worse off in the low-insurance equilibrium than without insurance entirely. This is illustrated by the red line falling below the blue line, showing that the low-insurance equilibrium can generate lower certainty equivalent consumption than the no-insurance benchmark. This counterintuitive result occurs because firms can take negative insurance positions, amplifying climate shocks rather than mitigating them. When negative insurance positions are prohibited, firms in the low-insurance equilibrium may choose no insurance, making welfare identical to the no-insurance benchmark in this case.

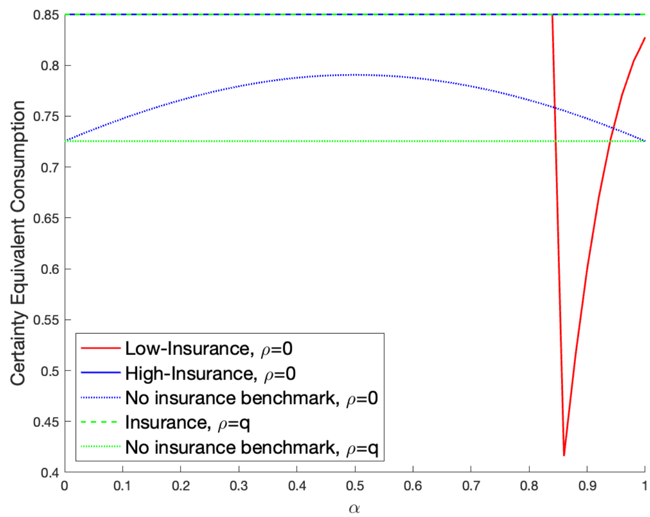

Figure 7: Certainty Equivalent Consumption at Actuarially Fair Insurance Premiums (Symmetric Economy): This figure plots certainty equivalent consumption of households in region A and region B for $\rho = 0$ (regions are never hit by climate shocks simultaneously) and $\rho = q$ (regions are always hit by climate shocks simultaneously), with parameters $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

Figure 8 plots certainty equivalent consumption under actuarially fair pricing in an asymmetric economy. As in the symmetric case, certainty equivalent consumption remains constant across correlation levels ($\rho = 0$ and $\rho = q$) under full insurance. Households can again be worse off in the low-insurance equilibrium than without insurance markets. Note that we do not plot the benchmarks without insurance in this plot but those are shown in Figure 2. Because region B experiences larger productivity losses during climate shocks, asymmetric welfare patterns emerge even under full insurance. When $\alpha < 0.5$, households in region B are worse off than those in region A, and vice versa for $\alpha > 0.5$. Notably, because the low-insurance equilibrium occurs at high values of α, households in region A—who highly value region B's good—suffer most in this equilibrium despite region B experiencing the worse productivity shock. This result highlights how economic linkages can redistribute the welfare costs of climate risks across regions, making insurance market coordination failures particularly damaging for regions that depend heavily on climate-vulnerable trading partners.

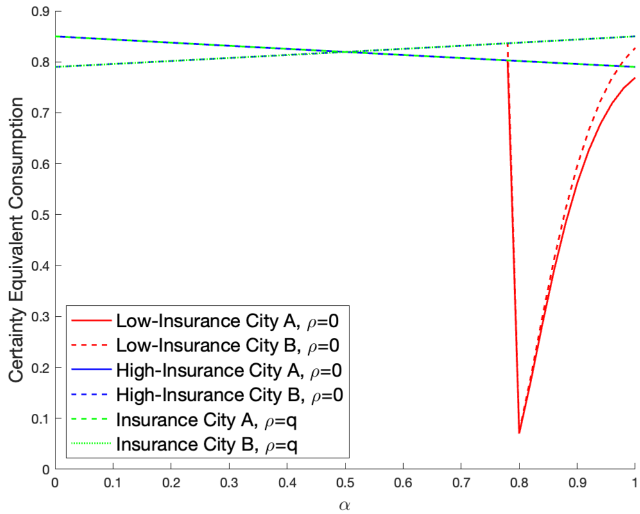

Figure 8: Certainty Equivalent Consumption at Actuarially Fair Insurance Premiums (Asymmetric Economy): This figure plots certainty equivalent consumption of households in region A and region B for $\rho = 0$ (regions are never hit by climate shocks simultaneously) and $\rho = q$ (regions are always hit by climate shocks simultaneously), with parameters $\phi_A = 0.5$, $\phi_B = 0.3$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

6 Extensions and Additional Analyses

In this section, we analyze two extensions. In Section 6.1, we examine insurance market collapse in one region. In Section 6.2, we examine insurance offered at discounted prices relative to actuarially fair prices.

6.1 Collapse of Insurance Markets

Insurers are exiting regions with high climate risks, and such exists are expected to accelerate (e.g., Sen et al., 2024a; Meredith, 2025). We examine how insurer exit in one region affects insurance take-up in other regions by modeling the collapse of insurance in region B (setting the insurance position to $f_B = 0$) while maintaining competitive insurance markets in region A, where firms choose their optimal insurance position f_A.

Corollary 3. *Let $\alpha \in (0,1)$. When insurance markets collapse in region B ($f_B = 0$) and region A has actuarially fair insurance ($\pi_A = \frac{q\phi_A}{1-q+q\phi_A}$), the optimal insurance position of firms in region A is given by*

$$f_A = \frac{\left(\frac{q}{1-q}\frac{1-2q+\rho+(q-\rho)\phi_B^{(1-\gamma)\alpha}}{q-\rho+\rho\phi_B^{(1-\gamma)\alpha}}\right)^{\frac{1}{(1-\gamma)(1-\alpha)-1}} - \phi_A}{(1-q)\,\phi_A + \left(\frac{q}{1-q}\frac{1-2q+\rho+(q-\rho)\phi_B^{(1-\gamma)\alpha}}{q-\rho+\rho\phi_B^{(1-\gamma)\alpha}}\right)^{\frac{1}{(1-\gamma)(1-\alpha)-1}} q\phi_A}\,(1-q+q\phi_A)\,K.$$

When $\rho = q^2$ (uncorrelated climate shocks), f_A equals the full insurance value $f_A^{FI} = \frac{(1-\phi_A)K}{\pi_A+(1-\pi_A)\phi_A}$ for all α. When $\rho < q^2$ (negatively correlated climate shocks), $f_A < f_A^{FI}$ and $\frac{\partial f_A}{\partial \alpha} < 0$. When $\rho > q^2$ (positively correlated climate shocks), $f_A > f_A^{FI}$ and $\frac{\partial f_A}{\partial \alpha} > 0$.

Corollary 3 characterizes insurance demand by firms in region A when insurance is unavailable in region B. When $\rho = q^2$ (uncorrelated shocks), region A's insurance demand is independent of region B's choices, so firms choose full insurance at actuarially fair prices. When $\rho < q^2$ (negatively correlated shocks), firms in region A underinsure relative to the full insurance benchmark. This occurs because economic integration provides implicit insurance—when region A experiences a disaster, region B typically does not and maintains high production of the good that region A households import. However, as α increases and region A households depend more heavily on region B's good, firms in region A face a dilemma: they want to maintain high production precisely when region B experiences disasters (and region A does not) to compensate for region B's uninsured losses. This leads them to reduce their own insurance coverage as α increases. When $\rho > q^2$ (positively correlated shocks), firms in region A overinsure relative to full insurance. Because disasters typically occur simultaneously in both regions, region A's insurance can partially substitute for region B's missing coverage by increasing production during joint disasters. As α increases, this substitution becomes more valuable since region A households depend more on region B's good, leading firms to purchase even more insurance. The key insight is that region A's optimal insurance depends on whether it complements or substitutes for the missing insurance in region B, which in turn depends on the correlation of climate shocks across regions.

Figure 9 plots insurance levels by firms in region A when insurance is unavailable in region B ($f_B = 0$) for an otherwise symmetric economy. The black dotted line represents the full-insurance benchmark. When $\rho = 0$ (regions are never hit by climate shocks simultaneously), firms in region A underinsure relative to this benchmark, with insurance decreasing in α (blue line). At $\alpha = 1$, firms in region A choose negative insurance positions, reducing capital during their own disasters to maintain higher production when region B experiences uninsured climate shock. Conversely, when $\rho = q$ (regions are always hit by climate shocks simultaneously), firms in region A overinsure relative to the full-insurance benchmark, increasing insurance as α rises. This allows region A to partially compensate for region B's missing insurance during joint disasters.

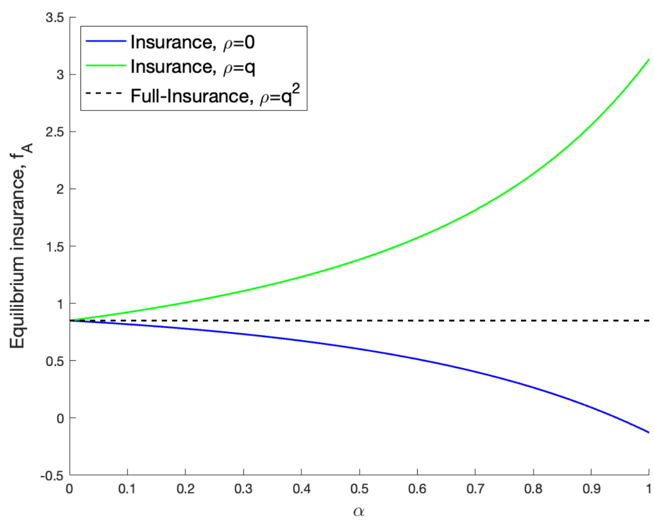

Figure 9: Insurance in Region A when Insurance Markets Collapse in Region B: This figure plots equilibrium insurance take-up of firms in region A at actuarially fair prices when insurance in region B is unavailable for $\rho = 0$ (regions are never hit by climate shocks simultaneously) and $\rho = q$ (regions are always hit by climate shocks simultaneously), with parameters $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

Economic linkages can amplify insurance market collapse through spillover effects. When insurance markets collapse in one region, this affects insurance demand in economically connected regions. When insurance choices are strategic complements, firms in linked regions also reduce their insurance coverage. This creates a contagion effect where insurance market collapse in one region causes insurance markets to contract in other regions.

Figure 10 plots certainty equivalent consumption when $\rho = 0$ and insurance markets collapse in region B. As α increases, region A's welfare (solid blue line) declines because households cannot insure against region B's uninsured shock. Region B households (dashed blue line) initially benefit from higher α because they value region A's good, and firms in region A continue to insure their production at moderate levels of α. However, at high values of α, region B households become worse off than if insurance markets had collapsed in both regions. This occurs because firms in region A choose negative insurance positions, reducing their capital during their own disasters to maintain higher production when region B experiences shocks. This reallocation amplifies rather than mitigates climate risks, leaving region B households worse off than without any insurance markets.

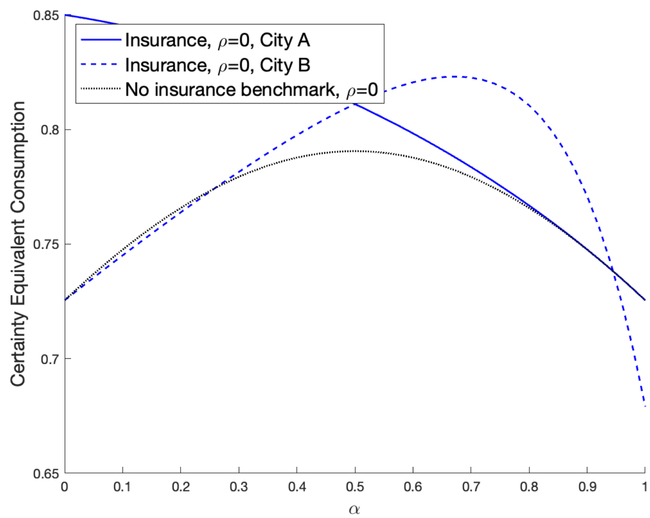

Figure 10: Certainty Equivalent Consumption when Insurance Markets Collapse in Region B ($\rho = 0$): This figure plots certainty equivalent consumption of households in region A and region B when insurance in region B is unavailable for $\rho = 0$ (regions are never hit by climate shocks simultaneously). The solid (dashed) line plots certainty equivalent consumption of households in region A (region B), with parameters $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

Figure 11 plots certainty equivalent consumption when $\rho = q$ and insurance markets collapse in region B. The pattern mirrors the $\rho = 0$ case: as α increases, region A households' welfare declines, and at high values of α, region B households become worse off than without any insurance markets. However, the underlying mechanism differs. While region A firms underinsured when $\rho = 0$, they now overinsure. This overinsurance reduces region A's production during normal times when no shock occurs, making region B households worse off than if region A had no access to insurance at all.

Our results demonstrate that insurer exit imposes multiple costs on affected regions. House-

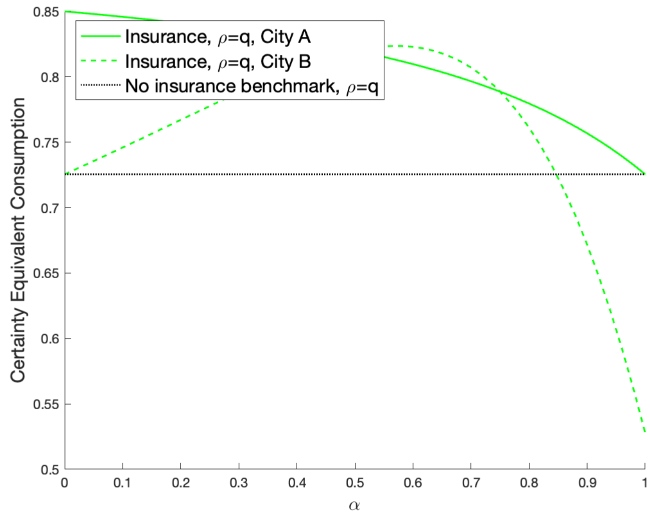

Figure 11: Certainty Equivalent Consumption when Insurance Markets Collapse in Region B ($\rho = q$): This figure plots certainty equivalent consumption of households in region A and region B when insurance in region B is unavailable for $\rho = q$ (regions are always hit by climate shocks simultaneously). The solid (dashed) line plots certainty equivalent consumption of households in region A (region B), with parameters $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

holds suffer not only from the direct loss of insurance coverage but also from spillover effects as firms in other regions adjust their insurance demand in response. Asymmetric insurance availability can amplify the negative effects of insurer exit.

6.2 Discounted Insurance

Insurance against climate risks is often offered at a discount due to government policy (e.g., Oh et al., 2022). We examine how such discounts affect insurance choices and welfare in our framework. We first characterize how insurance demand responds to changes in premiums.

Corollary 4. *In the high-insurance equilibrium, optimal insurance in region i, f_i, is decreasing in the premium π_i. In the low-insurance equilibrium, optimal insurance in region i, f_i, can be increasing in the premium π_i.*

When insurance is discounted—available at prices below actuarially fair levels, it creates opposing effects across equilibria. In the high-insurance equilibrium, firms increase their insurance as premiums decrease. With sufficiently deep discounts, firms overinsure to such an extent that their output is higher during disasters than in normal times. Conversely, in the low-insurance equilibrium, firms may actually decrease their insurance as premiums fall. This counterintuitive result occurs because firms must take even larger negative insurance positions when insurance is cheap to achieve their desired capital transfer across states. Discounted insurance can therefore paradoxically worsen underinsurance.

Figure 12 plots insurance take-up under discounted pricing compared to actuarially fair pricing. With positively correlated shocks when regions are always hit by climate shocks simultaneously ($\rho = q$), discounted insurance leads to overinsurance relative to actuarially fair pricing, as shown by the green solid line lying above the green dashed line. With negatively correlated shocks when regions are never hit by climate shocks simultaneously ($\rho = 0$), the effects are more complex. Strategic complementarities amplify overinsurance in the high-insurance equilibrium under discounted pricing. More strikingly, in this specific parameterization, underinsurance worsens in the low-insurance equilibrium despite the price discount.

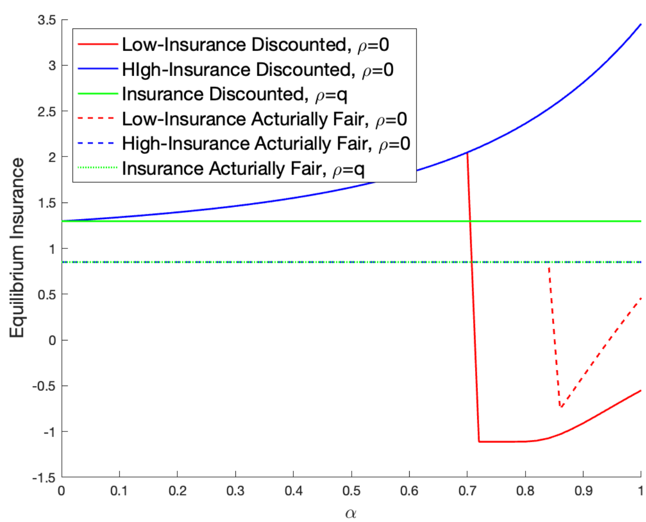

Figure 12: Equilibrium Insurance with Discounted and Actuarially Fair Insurance Premiums: This figure plots equilibrium insurance take-up of firms in region A and region B for $\rho = 0$ (regions are never hit by climate shocks simultaneously) and $\rho = q$ (regions are always hit by climate shocks simultaneously), comparing discounted insurance premiums ($\pi_A = \pi_B = 0.1$) and actuarially fair premiums, with parameters $\phi_A = \phi_B = 0.5$, $m_A = m_B = 0.5$, $q = 0.3$, $K = 1$, and $\gamma = 3$.

Our analysis demonstrates that well-intentioned government policies that cap insurance prices can have unintended consequences. When insurance premiums are kept below actuarially fair levels through regulatory price controls, these policies can paradoxically backfire and worsen underinsurance problems in regions with strong economic linkages. The mechanism operates through coordination failures: in the low-insurance equilibrium, firms may respond to cheaper insurance by taking even more negative insurance positions, amplifying rather than mitigating climate risks. This counterintuitive result highlights the importance of considering strategic interactions across regions when designing insurance market interventions.

7 Conclusion

With businesses facing increasing exposure to climate risks, understanding how they insure against these risks becomes critical. We analyze such insurance using a spatial model where insurance markets are local but create spillovers across regions through economic linkages.

Our analysis reveals that economic linkages generate strategic interactions in insurance choices across regions. Insurance demand by firms across regions exhibits strategic complementarity when climate shocks have negative correlation and strategic substitutability when correlation is positive. Strategic complementarity can create underinsurance equilibria even under actuarially fair pricing. Economic linkages also generate contagion effects: insurance market collapse in one region can cascade to other regions. Moreover, government policies designed to increase insurance take-up, such as price caps, can paradoxically backfire and exacerbate underinsurance.

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A Proofs

Proof of Proposition 1. First, we solve the optimization problem of a household in region A. The Lagrangian for region A is given by

$$\mathscr{L}_A = \frac{\left(\left(\frac{c_{A,A}}{1-\alpha}\right)^{1-\alpha}\left(\frac{c_{B,A}}{\alpha}\right)^{\alpha}\right)^{1-\gamma}}{1-\gamma} - \lambda_A\left(p_A c_{A,A} + p_B c_{B,A} - w_A\right),$$

with the first-order conditions

$$(1-\alpha)C_A^{1-\gamma} = \lambda_A p_A c_{A,A},$$

and

$$\alpha C_A^{1-\gamma} = \lambda_A p_B c_{B,A},$$

from which follows

$$\frac{p_B c_{B,A}}{p_A c_{A,A}} = \frac{\alpha}{1-\alpha}. \tag{9}$$

For a household in region B, we get the first-order conditions

$$(1-\alpha)C_B^{1-\gamma} = \lambda_B p_B c_{B,B},$$

and

$$\alpha C_B^{1-\gamma} = \lambda_B p_A c_{A,B},$$

from which follows

$$\frac{p_A c_{A,B}}{p_B c_{B,B}} = \frac{\alpha}{1-\alpha}. \tag{10}$$

Using Equations (9) and (10), the households' budget constraints, and the equilibrium wages from Equation (4), we get

$$c_{A,A} = (1-\alpha)Z_A K, \tag{11}$$

$$c_{B,B} = (1-\alpha)Z_B K. \tag{12}$$

Using $L_i = m_i$ and the market-clearing conditions (6) and (7), we get

$$c_{A,B} = \alpha \frac{Z_A K m_A}{m_B}, \tag{13}$$

$$c_{B,A} = \alpha \frac{Z_B K m_B}{m_A}. \tag{14}$$

Substituting Equations (11) and (14) into Equation (1), we arrive at

$$C_A = (Z_A K)^{1-\alpha} (Z_B K)^{\alpha} \left(\frac{m_B}{m_A}\right)^{\alpha},$$

and similarly by substituting Equations (12) and (13) into Equation (1), we get

$$C_B = (Z_A K)^{\alpha} (Z_B K)^{1-\alpha} \left(\frac{m_A}{m_B}\right)^{\alpha}.$$

Finally, from Equations (9), (11), and (14), we have that

$$\frac{p_B}{p_A} = \frac{Z_A K m_A}{Z_B K m_B}.$$

The utility of a household in region A after climate shocks have realized is given by

$$u(C_A) = \frac{(Z_A K)^{(1-\alpha)(1-\gamma)} (Z_B K)^{\alpha(1-\gamma)} \left(\frac{m_B}{m_A}\right)^{\alpha(1-\gamma)}}{1-\gamma},$$

and for a household in region B is given by

$$u(C_B) = \frac{(Z_A K)^{\alpha(1-\gamma)} (Z_B K)^{(1-\alpha)(1-\gamma)} \left(\frac{m_A}{m_B}\right)^{\alpha(1-\gamma)}}{1-\gamma}.$$

Thus, the expected utility of a household in region A before the realization of climate shocks, $\mathbb{E}[u(C_A)]$, is

$$\frac{\left(\frac{m_B}{m_A}\right)^{\alpha(1-\gamma)} K^{1-\gamma}}{1-\gamma} \left[(1-2q+\rho) + (q-\rho)\left(\phi_A^{(1-\alpha)(1-\gamma)} + \phi_B^{\alpha(1-\gamma)}\right) + \rho\, \phi_A^{(1-\alpha)(1-\gamma)} \phi_B^{\alpha(1-\gamma)}\right].$$

Similarly, for $\mathbb{E}\left[u(C_B)\right]$, we get

$$\frac{\left(\frac{m_A}{m_B}\right)^{\alpha(1-\gamma)} K^{1-\gamma}}{1-\gamma}\left[(1-2q+\rho)+(q-\rho)\left(\phi_A^{\alpha(1-\gamma)}+\phi_B^{(1-\alpha)(1-\gamma)}\right)+\rho\phi_A^{\alpha(1-\gamma)}\phi_B^{(1-\alpha)(1-\gamma)}\right],$$

which completes the proof. ■

Proof of Proposition 2. **Insurance Demand:** We begin by taking the optimal insurance profile from date 0 as given and solve for the date 1 equilibrium state by state. First, we solve the optimization problem of a household in region A. The Lagrangian for region A is given by

$$\mathscr{L}_A = \frac{\left(\left(\frac{c_{A,A}}{1-\alpha}\right)^{1-\alpha}\left(\frac{c_{B,A}}{\alpha}\right)^{\alpha}\right)^{1-\gamma}}{1-\gamma} - \lambda_A\left(p_A c_{A,A} + p_B c_{B,A} - w_A\right),$$

with the first-order conditions

$$\begin{aligned}(1-\alpha)C_A^{1-\gamma} &= \lambda_A p_A c_{A,A},\\ \alpha C_A^{1-\gamma} &= \lambda_A p_B c_{B,A},\end{aligned}$$

from which follows

$$\frac{p_B c_{B,A}}{p_A c_{A,A}} = \frac{\alpha}{1-\alpha}. \tag{15}$$

Using Equation (15), the households' budget constraints, and the equilibrium wage from Equation (8), we get

$$\begin{aligned}c_{A,A} &= (1-\alpha)Z_A(K+f_A(\delta_A-\pi_A)),\\ c_{B,A} &= \alpha\frac{p_A}{p_B}Z_A(K+f_A(\delta_A-\pi_A)).\end{aligned}$$

The utility of a household in region A after the climate shocks have realized is therefore given by

$$u(C_A) = \frac{\left(Z_A\left(K+f_A(\delta_A-\pi_A)\right)\left(\frac{p_A}{p_B}\right)^{\alpha}\right)^{1-\gamma}}{1-\gamma}.$$

Consequently, a firm in region A chooses f_A to maximize

$$\mathbb{E}\left[u(C_A)\right] = (1-2q+\rho)\frac{\left(\left(K-f_A\pi_A\right)\left(\frac{p_A}{p_B}\right)^{\alpha}\right)^{1-\gamma}}{1-\gamma} + (q-\rho)\frac{\left(\left(K-f_A\pi_A\right)\left(\frac{p_A}{p_B}\right)^{\alpha}\right)^{1-\gamma}}{1-\gamma} + (q-\rho)\frac{\left(\phi_A\left(K+f_A(1-\pi_A)\right)\left(\frac{p_A}{p_B}\right)^{\alpha}\right)^{1-\gamma}}{1-\gamma} + \rho\frac{\left(\phi_A\left(K+f_A(1-\pi_A)\right)\left(\frac{p_A}{p_B}\right)^{\alpha}\right)^{1-\gamma}}{1-\gamma}.$$

Note that the price ratio $\frac{p_A}{p_B}$ is different in each state.

The first-order necessary condition for the optimal f_A is then

$$0 = -\left(K-f_A\pi_A\right)^{-\gamma}\left((1-2q+\rho)\left(\frac{p_A}{p_B}\right)^{(1-\gamma)\alpha} + (q-\rho)\left(\frac{p_A}{p_B}\right)^{(1-\gamma)\alpha}\right)\pi_A + \left(\phi_A\left(K+f_A(1-\pi_A)\right)\right)^{-\gamma}\left((q-\rho)\left(\frac{p_A}{p_B}\right)^{(1-\gamma)\alpha} + \rho\left(\frac{p_A}{p_B}\right)^{(1-\gamma)\alpha}\right)\phi_A\left(1-\pi_A\right),$$

which we can write more succinctly as

$$\mathbb{E}\left[\left(Z_A\left(K+f_A\left(\delta_A-\pi_A\right)\right)\right)^{-\gamma}\left(\frac{p_A}{p_B}\right)^{(1-\gamma)\alpha} Z_A\left(\delta_A-\pi_A\right)\right] = 0.$$

Following the same arguments as in the proof of Proposition 1, replacing K with region specific capital K_i, we get the equilibrium price ratio $\frac{p_A}{p_B} = \frac{Z_B K_B m_B}{Z_A K_A m_A}$. Substituting this, and recognizing that m_A and m_B are constants, we arrive at

$$\mathbb{E}\left[\left(Z_A\left(K+f_A\left(\delta_A-\pi_A\right)\right)\right)^{(1-\gamma)(1-\alpha)-1}\left(Z_B\left(K+f_B\left(\delta_B-\pi_B\right)\right)\right)^{(1-\gamma)\alpha} Z_A\left(\delta_A-\pi_A\right)\right] = 0. \quad (16)$$

From Table 1, we can expand Equation (16) to

$$\frac{1-\pi_A}{\pi_A}\phi_A\left(\frac{\phi_A(K+f_A\left(1-\pi_A\right))}{K-f_A\pi_A}\right)^{(1-\gamma)(1-\alpha)-1}\left(q-\rho+\rho\left(\frac{\phi_B(K+f_B\left(1-\pi_B\right))}{K-f_B\pi_B}\right)^{(1-\gamma)\alpha}\right) = 1-2q+\rho+(q-\rho)\left(\frac{\phi_B(K+f_B\left(1-\pi_B\right))}{K-f_B\pi_B}\right)^{(1-\gamma)\alpha}.$$

Similarly, for region B, we get

$$\frac{1-\pi_B}{\pi_B}\phi_B\left(\frac{\phi_B(K+f_B(1-\pi_B))}{K-f_B\pi_B}\right)^{(1-\gamma)(1-\alpha)-1}\left(q-\rho+\rho\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^{(1-\gamma)\alpha}\right)$$
$$=1-2q+\rho+(q-\rho)\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^{(1-\gamma)\alpha}.$$

The two first-order conditions can be rewritten as

$$\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^{(1-\gamma)(1-\alpha)-1}=\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{1-2q+\rho+(q-\rho)\left(\frac{\phi_B(K+f_B(1-\pi_B))}{K-f_B\pi_B}\right)^{(1-\gamma)\alpha}}{q-\rho+\rho\left(\frac{\phi_B(K+f_B(1-\pi_B))}{K-f_B\pi_B}\right)^{(1-\gamma)\alpha}}, \tag{17}$$

and

$$\left(\frac{\phi_B(K+f_B(1-\pi_B))}{K-f_B\pi_B}\right)^{(1-\gamma)(1-\alpha)-1}=\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-2q+\rho+(q-\rho)\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^{(1-\gamma)\alpha}}{q-\rho+\rho\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^{(1-\gamma)\alpha}}.$$

Define $\zeta=\frac{(1-\gamma)\alpha}{(1-\gamma)(1-\alpha)-1}>0$ and $x_i=\left(\frac{\phi_i(K+f_i(1-\pi_i))}{K-f_i\pi_i}\right)^{(1-\gamma)(1-\alpha)-1}$. We then get

$$x_A=\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{1-2q+\rho+(q-\rho)x_B^{\zeta}}{q-\rho+\rho x_B^{\zeta}}=\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{1-q+qx_B^{\zeta}}{q-\rho+\rho x_B^{\zeta}}-\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A},$$

and

$$x_B=\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-2q+\rho+(q-\rho)x_A^{\zeta}}{q-\rho+\rho x_A^{\zeta}}=\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-q+qx_A^{\zeta}}{q-\rho+\rho x_A^{\zeta}}-\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}. \tag{18}$$

We thus get

$$x_A=\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{1-q+q\left(\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-q+qx_A^{\zeta}}{q-\rho+\rho x_A^{\zeta}}-\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\right)^{\zeta}}{q-\rho+\rho\left(\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-q+qx_A^{\zeta}}{q-\rho+\rho x_A^{\zeta}}-\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\right)^{\zeta}}-\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}, \tag{19}$$

which is a fixed-point equation in x_A that identifies x_A, and consequently f_A. Equation (18) then identifies the corresponding f_B.

Equilibrium Existence: As a next step, we show that an equilibrium exists. Let the right-hand side of Equation (19) be $g(x_A)$. Notice that g is defined on $[0,\infty)$ and is a continuous function of x_A. Notice further that $g \geq 0$.

Case 1: $\rho > 0$. If $g(0) = 0$, then $x_A = 0$ is a solution to the fixed point problem and an equilibrium exists.

Consider next the case $g(0) > 0$. Using $\zeta > 0$ and applying L'Hospital's Rule, we have

$$\lim_{x_A \to \infty} x_B = \frac{\pi_B}{1-\pi_B} \frac{1}{\phi_B} \frac{q-\rho}{\rho}.$$

Thus, $\lim_{x_A \to \infty} x_B^{\zeta}$ converges, and so does $\lim_{x_A \to \infty} g(x_A)$. Since the left-hand side of Equation (19) goes to infinity, there must be a crossing point and an equilibrium exists.[6]

Case 2: $\rho = 0$. Then, we can write $g(x_A)$ as

$$g(x_A) = \frac{\pi_A}{1-\pi_A} \frac{1}{\phi_A} \left(\frac{1-2q}{q} + \left(\frac{\pi_B}{1-\pi_B} \frac{1}{\phi_B} \left(\frac{1-2q}{q} + x_A^{\zeta} \right) \right)^{\zeta} \right). \tag{21}$$

Again, if $g(0) = 0$, then $x_A = 0$ is a solution to the fixed point problem and an equilibrium exists. Thus, we consider the case $g(0) > 0$ in the following.

Notice that $\zeta > 0$ since $\gamma > 1$. If $\zeta \in (0,1)$, then asymptotically $g(x_A)$ grows according to $x_A^{\zeta^2} < x_A$ as $x_A \to \infty$. We conclude that there must be a crossing point and an equilibrium exists.

Consider next the case in which $\zeta > 1$. Then $g(x_A)$ is convex in x_A. Then there are three cases: (i) it intersects the 45-degree line twice, (ii) it is tangent for one value, (iii) it never intersects. In the first two cases, there is a solution to the fixed point problem and an equilibrium exists. If it never intersects, the only solution is $x_A = \infty$, which is equivalent to $f_A = -\frac{K}{1-\pi_A}$ because $(1-\gamma)(1-\alpha) - 1 < 0$.

[6] We can also construct an algorithm for finding fixed points. Notice that $g(x)$ is nondecreasing in x_A because

$$\frac{dg(x_A)}{dx_A} = \frac{\pi_A}{1-\pi_A} \frac{1}{\phi_A} \frac{\pi_B}{1-\pi_B} \frac{1}{\phi_B} \left(\frac{\zeta (q^2 - \rho)}{\left(q - \rho + \rho y(x_A)^{\zeta}\right)\left(q - \rho + \rho x_A^{\zeta}\right)} \right)^2 y(x_A)^{\zeta - 1} x_A^{\zeta - 1} \geq 0, \tag{20}$$

where $y(x_A) = \frac{\pi_B}{1-\pi_B} \frac{1}{\phi_B} \frac{1-2q+\rho+(q-\rho)x_A^{\zeta}}{q-\rho+\rho x_A^{\zeta}}$. We can then apply the Knaster-Tarksi fixed-Point theorem because g is bounded to not only conclude an equilibrium exists, but also to provide an algorithm for finding them when there are multiple. For instance, see Eisenberg and Noe (2001).

Finally, consider the case in which $\zeta = 1$. Then $g(x_A)$ is linear in x_A. If the slope is lower than one, there is a crossing point and an equilibrium exists. If the slope is larger or equal to one, then $x_A = \infty$ is a solution.

Household Expected Utility: Having characterized equilibrium insurance choices, we can compute the other equilibrium quantities. Total output in each region is given by

$$\begin{aligned} Z_A K_A L_A &= Z_A K_A m_A, \\ Z_B K_B L_B &= Z_B K_B m_B. \end{aligned}$$

The ex-post utility of households in region A is

$$\frac{C_A^{1-\gamma}}{1-\gamma} = \frac{(Z_A K_A)^{(1-\alpha)(1-\gamma)} (Z_B K_B)^{\alpha(1-\gamma)} \left(\frac{m_B}{m_A}\right)^{\alpha(1-\gamma)}}{1-\gamma}.$$

The ex-post utility of households in region B is

$$\frac{C_B^{1-\gamma}}{1-\gamma} = \frac{(Z_A K_A)^{\alpha(1-\gamma)} (Z_B K_B)^{(1-\alpha)(1-\gamma)} \left(\frac{m_A}{m_B}\right)^{\alpha(1-\gamma)}}{1-\gamma}.$$

The expected utility of households in region A depending on insurance take-up in regions A and B is given by

$$\begin{aligned} \mathbb{E}\left[\frac{C_A^{1-\gamma}}{1-\gamma}\right] = \frac{\left(\frac{m_B}{m_A}\right)^{(1-\gamma)\alpha}}{1-\gamma} \Bigg[&(1-2q+\rho)(K - f_A\pi_A)^{(1-\gamma)(1-\alpha)}(K - f_B\pi_B)^{(1-\gamma)\alpha} \\ &+ (q-\rho)(K - f_A\pi_A)^{(1-\gamma)(1-\alpha)}(\phi_B(K + f_B(1-\pi_B)))^{(1-\gamma)\alpha} \\ &+ (q-\rho)(\phi_A(K + f_A(1-\pi_A)))^{(1-\gamma)(1-\alpha)}(K - f_B\pi_B)^{(1-\gamma)\alpha} \\ &+ \rho(\phi_A(K + f_A(1-\pi_A)))^{(1-\gamma)(1-\alpha)}(\phi_B(K + f_B(1-\pi_B)))^{(1-\gamma)\alpha} \Bigg]. \end{aligned} \tag{22}$$

We can rewrite the expected utility of a households in region i as

$$\mathbb{E}\left[\frac{C_i^{1-\gamma}}{1-\gamma}\right] = \frac{\left(\frac{m_{-i}}{m_i}\right)^{\alpha(1-\gamma)}}{1-\gamma}\mathbb{E}\left[(Z_iK_i)^{(1-\alpha)(1-\gamma)}(Z_{-i}K_{-i})^{\alpha(1-\gamma)}\right],$$

which completes the proof. ∎

Proof of Corollary 1. We can write Equation (17) as $G(f_A, f_B) = 0$, where

$$G(f_A, f_B) \equiv \left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^\theta - \frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{1-2q+\rho+(q-\rho)y}{q-\rho+\rho y},$$

$\theta \equiv (1-\gamma)(1-\alpha)-1$, and $y \equiv \left(\frac{\phi_B(K+f_B(1-\pi_B))}{K-f_B\pi_B}\right)^{(1-\gamma)\alpha}$.

By the Implicit Function Theorem,

$$\frac{\partial f_A}{\partial f_B} = -\frac{\partial G/\partial f_B}{\partial G/\partial f_A}.$$

Because $\theta < 0$ for $\gamma > 1$ and $\alpha \in (0,1)$, and $\frac{\phi_A(K+(1-\pi_A)f_A)}{K-\pi_A f_A}$ is strictly increasing in f_A, we have $\partial G/\partial f_A = \theta\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^{\theta-1}\left(\partial\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}/\partial f_A\right) < 0$. Hence, we have $\text{sign}(\partial f_A/\partial f_B) = \text{sign}(\partial G/\partial f_B)$.

Only the second term of G depends on f_B. Write

$$H(y) \equiv \frac{1-2q+\rho+(q-\rho)y}{q-\rho+\rho y}.$$

Then $\partial G/\partial f_B = -\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}H'(y)\frac{\partial y}{\partial f_B}$. We further have

$$\frac{\partial y}{\partial f_B} = (1-\gamma)\alpha\left(\frac{\phi_B(K+f_B(1-\pi_B))}{K-f_B\pi_B}\right)^{(1-\gamma)\alpha-1}\frac{\partial\left(\frac{\phi_B(K+f_B(1-\pi_B))}{K-f_B\pi_B}\right)}{\partial f_B} < 0,$$

since $(1-\gamma)\alpha < 0$ and $\frac{\phi_B(K+f_B(1-\pi_B))}{K-f_B\pi_B}$ is strictly increasing in f_B.

We also have

$$H'(y) = \frac{(q-\rho)^2-\rho(1-2q+\rho)}{(q-\rho+\rho y)^2} = \frac{q^2-\rho}{(q-\rho+\rho y)^2}.$$

Therefore $\text{sign}(\partial G/\partial f_B) = \text{sign}(q^2 - \rho)$.

When $\rho = q^2$, and the shocks are uncorrelated, $\frac{\partial f_A}{\partial f_B} = 0$. When $\rho < q^2$, and the shocks are negatively correlated, $\frac{\partial f_A}{\partial f_B} > 0$. When $\rho > q^2$, and the shocks are positively correlated, $\frac{\partial f_A}{\partial f_B} < 0$. ■

Proof of Proposition 3. Note that when $\rho = q$ it is straightforward to show that $x_A = g(x_A)$ has a unique solution. We therefore consider the case $\rho < q$ below. We first recognize that $g(x)$ is a continuous, increasing function of x_A because

$$\frac{dg(x_A)}{dx_A} = \frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\left(\frac{\zeta\left(q^2-\rho\right)}{\left(q-\rho+\rho y(x_A)^{\zeta}\right)\left(q-\rho+\rho x_A^{\zeta}\right)}\right)^2 y(x_A)^{\zeta-1}x_A^{\zeta-1} \geq 0, \quad (23)$$

where $y(x_A) = \frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-2q+\rho+(q-\rho)x_A^{\zeta}}{q-\rho+\rho x_A^{\zeta}}$.

In addition, notice that because $\zeta > 0$ and $\rho < q$, we have

$$g(0) = \frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{1-2q+\rho+(q-\rho)\left(\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-2q+\rho}{q-\rho}\right)^{\zeta}}{q-\rho+\rho\left(\frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-2q+\rho}{q-\rho}\right)^{\zeta}} > 0,$$

from which follows that $g(x_A)$ is above x_A at $x_A = 0$ and is above 0 for all $x_A \geq 0$.

Notice that when $g(x_A)$ first crosses x_A it must be the case that $\frac{dg(x_A)}{dx_A} < 1$. It is therefore sufficient that $\frac{d^2g(x_A)}{dx_A^2} \leq 0$ globally for there to be a unique equilibrium because $g(x_A)$ then never subsequently increases sufficiently to intersect x_A again.

Differentiating Equation (23), with some manipulation we arrive at

$$\begin{aligned}\frac{x_A}{\frac{dg(x_A)}{dx_A}}\frac{d^2g(x_A)}{dx_A^2} &= \zeta\left(\zeta-1-2\zeta\frac{\rho y(x_A)^{\zeta}}{q-\rho+\rho y(x_A)^{\zeta}}\right)\frac{q^2-\rho}{1-2q+\rho+(q-\rho)x_A^{\zeta}}\frac{x_A^{\zeta}}{q-\rho+\rho x_A^{\zeta}} \\ &\quad +\zeta-1-2\zeta\frac{\rho x_A^{\zeta}}{q-\rho+\rho x_A^{\zeta}}.\end{aligned}$$

Notice that if $\zeta \leq 1$ and $\rho \leq q^2$, $\frac{d^2g(x_A)}{dx_A^2} \leq 0$.

In the case that $\rho > q^2$, we recognize that since $q \geq \rho$ and $y(x_A)^\zeta \geq 0$, we have

$$\frac{\rho y(x_A)^\zeta}{q-\rho+\rho y(x_A)^\zeta} \leq 1,$$

because generically $\frac{z}{a+z} \leq 1$ for $a, z \geq 0$. This property, Equation (24), and $\rho > q^2$ then imply

$$\begin{aligned}
\frac{x_A}{\frac{dg(x_A)}{dx_A}} \frac{d^2 g(x_A)}{dx_A^2} &\leq \frac{\zeta(\zeta+1)\left(\rho-q^2\right)}{1-2q+\rho+(q-\rho)x_A^\zeta} \frac{x_A^\zeta}{q-\rho+\rho x_A^\zeta} - 2\zeta \frac{\rho x_A^\zeta}{q-\rho+\rho x_A^\zeta} + \zeta - 1 \\
&\leq 2\zeta \frac{\rho-q^2}{1-2q+\rho+(q-\rho)x_A^\zeta} \frac{x_A^\zeta}{q-\rho+\rho x_A^\zeta} - 2\zeta \frac{\rho x_A^\zeta}{q-\rho+\rho x_A^\zeta} + \zeta - 1 \qquad (26) \\
&= -2\zeta \frac{(q-\rho)^2+(q-\rho)\rho x_A^\zeta}{1-2q+\rho+(q-\rho)x_A^\zeta} \frac{x_A^\zeta}{q-\rho+\rho x_A^\zeta} + \zeta - 1,
\end{aligned}$$

where the second step follows because $\zeta \leq 1$. It is then immediate from Inequality (26) that

$$\frac{x_A}{\frac{dg(x_A)}{dx_A}} \frac{d^2 g(x_A)}{dx_A^2} \leq \zeta - 1.$$

Again, if $\zeta \leq 1$, we have $\frac{d^2 g(x_A)}{dx_A^2} \leq 0$.

Consequently, if $\zeta \leq 1$, there is a unique equilibrium. The trivial exception is when $\zeta = 1$, $\rho = q$, and $\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A} = \frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}$, in which case there are infinitely-many solutions because from Equation (18),

$$x_A x_B = \frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{1-q}{q} = \frac{\pi_B}{1-\pi_B}\frac{1}{\phi_B}\frac{1-q}{q}.$$

Consequently, it is necessary that $\zeta > 1$ for there to be multiple equilibria because of demand externalities. Given that $\zeta = \frac{(1-\gamma)\alpha}{(1-\gamma)(1-\alpha)-1}$, this imposes

$$\alpha > \frac{1}{2}\frac{\gamma}{\gamma-1}.$$

Consider the case in which $\zeta > 1$. With respect to ρ, notice that when $\rho = 0$ and $\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\frac{1-2q}{q}$ and $\frac{\pi_A}{1-\pi_A}\frac{1}{\phi_A}\left(\frac{1}{\phi_B}\frac{\pi_B}{1-\pi_B}\right)^\zeta$ are sufficiently small but nonzero, then there are multiple equilibria from

Equation (21). When $\rho = q$, we can solve Equation (19) when $\zeta \neq 1$ to find

$$x_A = \left(\frac{\pi_A}{1-\pi_A} \frac{1}{\phi_A} \left(\frac{1-\pi_B}{\pi_B} \phi_B \right)^{\zeta} \right)^{\frac{1}{1-\zeta^2}} \left(\frac{1-\rho}{\rho} \right)^{\frac{1}{1+\zeta}},$$

and there exists a unique optimal choice of insurance. Thus, there exists a threshold $\rho^* < q$ such that $\rho \leq \rho^*$ is a necessary condition for multiple equilibria. ■

Proof of Corollary 2. If a firm in region A fully insures its production, then it chooses its insurance position such that its production is the same in all states. This requires that

$$K - f_A \pi_A = \phi_A \left(K + f_A \left(1 - \pi_A \right) \right), \tag{27}$$

where the left-hand side of Equation (27) is output when there is no climate disaster in region A, and the right-hand side is output when there is a climate disaster in region A. Equation (27) implies that

$$f_A^{FI} = \frac{(1-\phi_A) K}{\pi_A + \phi_A (1-\pi_A)}, \tag{28}$$

and an analogous expression for f_B in region B,

$$f_B^{FI} = \frac{(1-\phi_B) K}{\pi_B + \phi_B (1-\pi_B)}. \tag{29}$$

Substituting Equations (28) and (29) into Equation (17), we arrive at

$$\frac{\pi_A}{1-\pi_A} = \frac{q}{1-q} \phi_A,$$

which holds when $\pi_A = \frac{q\phi_A}{1-q+q\phi_A}$, and analogously for $\pi_B = \frac{q\phi_B}{1-q+q\phi_B}$. Consequently, if insurance is actuarially fair, full insurance is an equilibrium. In particular, insured output in region A is $(1-q+q\phi_A)K$ and in region B is $(1-q+q\phi_B)K$ in all states.

Another way to see that full insurance is an equilibrium, which corresponds to $x_A = 1$, is to recognize that Equation (21) simplifies in the case of actuarially fair insurance to

$$x_A = g\left(x_A\right) = 1 - \frac{q}{1-q} + \frac{q}{1-q}\left(1 - \frac{q}{1-q} + \frac{q}{1-q}x_A^{\zeta}\right)^{\zeta},$$

from which it is clear that $x_A = 1$ is indeed an equilibrium.

Further, recall that equilibrium multiplicity is a generic feature of our framework when $\alpha > \frac{1}{2}\frac{\gamma}{\gamma-1}$ and $\rho = 0$. From the proof of Proposition 2, if $\rho = 0$ and $\zeta > 1$, then $g\left(x_A\right)$ is convex in x_A, and there are either zero, one, or two intersections with the 45-degree line. Because we have found one equilibrium, $x_A = 1$, we can rule out the zero solutions case. If there is only one solution, then $g\left(x_A\right)$ must be tangent to x_A at $x_A = 1$, that is, $\left.\frac{dg(x_A)}{dx_A}\right|_{x_A=1} = 1$. However, notice that when there is actuarially fair insurance and $\rho = 0$, we have

$$\left.\frac{dg\left(x_A\right)}{dx_A}\right|_{x_A=1} = \left.\left(\zeta\frac{q}{1-q}\right)^2\left(1 - \frac{q}{1-q} + \frac{q}{1-q}x_A^{\zeta}\right)^{\zeta-1} x_A^{\zeta-1}\right|_{x_A=1} = \left(\zeta\frac{q}{1-q}\right)^2.$$

By continuity, it must be the case that there exists a second equilibrium for ρ sufficiently small if $\zeta \neq \frac{1-q}{q}$.

Finally, because insurers make zero profits in expectation, the optimal choice of f_A that maximizes utilitarian welfare across both regions, where each region is weighted by its mass of households, satisfies from Equation (22) the first-order necessary and sufficient condition

$$\begin{aligned} 0 = m_A\left(1-\alpha\right)\left(\frac{m_B}{m_A}\right)^{(1-\gamma)\alpha} &\mathbb{E}\left[\left(Z_A K_A\right)^{(1-\gamma)(1-\alpha)-1}\left(Z_B K_B\right)^{(1-\gamma)\alpha} Z_A\left(\delta_A - \pi_A\right)\right] \\ + m_B\alpha\left(\frac{m_A}{m_B}\right)^{(1-\gamma)\alpha} &\mathbb{E}\left[\left(Z_B K_B\right)^{(1-\gamma)(1-\alpha)}\left(Z_A K_A\right)^{(1-\gamma)\alpha-1} Z_A\left(\delta_A - \pi_A\right)\right]. \end{aligned} \tag{30}$$

That it is sufficient, and the socially optimal choice of insurance is unique, follows from the strict concavity of the utility functions of households in both regions in their insurance positions.

When π_A and π_B are set to their actuarially fair values, Equation (30) is satisfied at full insurance. This is because production is then constant across states of the world and Equation (30) reduces to

$$0 = m_A (1-\alpha) \left(\frac{m_B}{m_A}\right)^{(1-\gamma)\alpha} (Z_A K_A)^{(1-\gamma)(1-\alpha)-1} (Z_B K_B)^{(1-\gamma)\alpha} \mathbb{E}\left[Z_A (\delta_A - \pi_A)\right] \\ + m_B \alpha \left(\frac{m_A}{m_B}\right)^{(1-\gamma)\alpha} (Z_B K_B)^{(1-\gamma)(1-\alpha)} (Z_A K_A)^{(1-\gamma)\alpha-1} \mathbb{E}\left[Z_A (\delta_A - \pi_A)\right],$$

and when insurance is actuarially fair, $\mathbb{E}\left[Z_A (\delta_A - \pi_A)\right] = 0$. By contrast, only the first term in Equation (30) is zero (by construction from Equation (16)) at the other insurance equilibrium when it exists, regardless of whether it is a high or low insurance equilibrium. A similar argument applies to the socially optimal choice of f_B.

This second insurance equilibrium must therefore deliver lower utilitarian welfare than that under full insurance. In addition, the second insurance equilibrium must be Pareto inferior. ■

Proof of Corollary 3. In the special case that $f_B = 0$, $x_B = \phi_B^{(1-\gamma)(1-\alpha)-1}$, and Equation (19) simplifies to

$$x_A = \frac{\pi_A}{1-\pi_A} \frac{1}{\phi_A} \frac{1-2q+\rho+(q-\rho)\phi_B^{(1-\gamma)\alpha}}{q-\rho+\rho\phi_B^{(1-\gamma)\alpha}}.$$

In the case of actuarially fair insurance in region A, $\pi_A = \frac{q\phi_A}{1-q+q\phi_A}$, this equation further simplifies to

$$x_A = \frac{q}{1-q} \frac{1-2q+\rho+(q-\rho)\,\phi_B^{(1-\gamma)\alpha}}{q-\rho+\rho\phi_B^{(1-\gamma)\alpha}}. \tag{31}$$

Using $x_A = \left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^{(1-\gamma)(1-\alpha)-1}$, it follows that

$$f_A = \frac{\left(\frac{q}{1-q}\frac{1-2q+\rho+(q-\rho)\phi_B^{(1-\gamma)\alpha}}{q-\rho+\rho\phi_B^{(1-\gamma)\alpha}}\right)^{\frac{1}{(1-\gamma)(1-\alpha)-1}} - \phi_A}{(1-q)\,\phi_A + \left(\frac{q}{1-q}\frac{1-2q+\rho+(q-\rho)\phi_B^{(1-\gamma)\alpha}}{q-\rho+\rho\phi_B^{(1-\gamma)\alpha}}\right)^{\frac{1}{(1-\gamma)(1-\alpha)-1}} q\phi_A} (1-q+q\phi_A)\,K. \tag{32}$$

When $\rho = q^2$, $\left(\frac{q}{1-q}\frac{1-2q+\rho+(q-\rho)\phi_B^{(1-\gamma)\alpha}}{q-\rho+\rho\phi_B^{(1-\gamma)\alpha}}\right)^{\frac{1}{(1-\gamma)(1-\alpha)-1}} = 1$, and Equation (32) reduces to

$$f_A = \frac{1-\phi_A}{\pi_A + (1-\pi_A)\,\phi_A} K,$$

that is, its full insurance value.

Define $N(\rho) = 1-2q+\rho+(q-\rho)\phi_B^{(1-\gamma)\alpha}$ and $D(\rho) = q-\rho+\rho\phi_B^{(1-\gamma)\alpha}$. Then, differentiating x_A w.r.t. ρ and using $N'(\rho) = 1-\phi_B^{(1-\gamma)\alpha}$ and $D'(\rho) = -1+\phi_B^{(1-\gamma)\alpha}$,

$$\frac{\partial}{\partial\rho}\left(\frac{N}{D}\right) = \frac{N'D-ND'}{D^2} = \frac{(1-\phi_B^{(1-\gamma)\alpha})[D+N]}{D^2}.$$

Using the fact that $D+N = 1-q+q\phi_B^{(1-\gamma)\alpha} > 0$ and that $\phi_B^{(1-\gamma)\alpha} > 1$ since $\gamma > 1$, $\alpha > 0$, and $\phi_B \in (0,1)$, it follows that

$$\frac{\partial x_A}{\partial\rho} < 0.$$

Because x_A is decreasing in f_A, we get that $\frac{\partial f_A}{\partial\rho} > 0$. It follows that when $\rho < q^2$, $f_A < f_A^{FI}$, and when $\rho > q^2$, $f_A > f_A^{FI}$.

Differentiating (31) w.r.t α we have

$$\frac{\partial x_A}{\partial\alpha} = \frac{q}{1-q}\cdot\frac{q^2-\rho}{\left(q-\rho+\rho\phi_B^{(1-\gamma)\alpha}\right)^2}(1-\gamma)\ln(\phi_B)\phi_B^{(1-\gamma)\alpha}.$$

It is clear that $\frac{\partial x_A}{\partial\alpha} = 0$ when $\rho = q^2$, $\frac{\partial x_A}{\partial\alpha} > 0$ when $\rho < q^2$, $\frac{\partial x_A}{\partial\alpha} < 0$ when $\rho > q^2$.

Also differentiating $x_A = \left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)^{(1-\gamma)(1-\alpha)-1}$ w.r.t α gives

$$\frac{\partial x_A}{\partial\alpha} = x_A\left[(\gamma-1)\ln\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right) + \left((1-\gamma)(1-\alpha)-1\right)\left(\frac{1-\pi_A}{K+f_A(1-\pi_A)}+\frac{\pi_A}{K-f_A\pi_A}\right)\frac{\partial f_A}{\partial\alpha}\right].$$

Dividing both sides by x_A and solving for $\frac{df_A}{d\alpha}$ yields

$$\frac{\partial f_A}{\partial\alpha} = \frac{\frac{1}{x_A}\frac{\partial x_A}{\partial\alpha} - (\gamma-1)\ln\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right)}{\left((1-\gamma)(1-\alpha)-1\right)\left(\frac{1-\pi_A}{K+f_A(1-\pi_A)}+\frac{\pi_A}{K-f_A\pi_A}\right)}.$$

When $\rho = q^2$, $\ln\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right) = 0$ because $\phi_A(K+f_A(1-\pi_A)) = K-\pi_A f_A$ (full insurance), and the numerator is 0. When $\rho < q^2$, $\ln\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right) < 0$ because $\phi_A(K+f_A(1-\pi_A)) < K-f_A\pi_A$ (underinsurance), and the numerator is positive. When $\rho > q^2$, $\ln\left(\frac{\phi_A(K+f_A(1-\pi_A))}{K-f_A\pi_A}\right) > 0$

because $\phi_A(K+(1-\pi_A)f_A) > K-\pi_A f_A$ (overinsurance), and the numerator is negative. The denominator is always negative.

Therefore,

$$\frac{\partial f_A}{\partial \alpha}\begin{cases} <0, & \text{if } \rho < q^2, \\ =0, & \text{if } \rho = q^2, \\ >0, & \text{if } \rho > q^2, \end{cases}$$

which completes the proof. ■

Proof of Corollary 4. Let the right-hand side of Equation (19) be $g(x_i)$. In the equilibrium with high insurance (low x_i), $\partial_{x_i}(x_i - g(x_i)) > 0$ because the right-hand side intersects the 45-degree line from above. In the equilibrium with low insurance (high x_i), $\partial_{x_i}(x_i - g(x_i)) < 0$ because it intersects the 45-degree line from below.

In the high insurance equilibrium, by the Implicit Function Theorem with some manipulation

$$\frac{\partial x_i}{\partial \pi_i} = \frac{1}{\pi_i(1-\pi_i)}\frac{x_i}{\partial_{x_i}(x_i - g(x_i))} > 0.$$

Given that $f_i = \frac{K}{\pi_i + \left(\frac{1}{\phi_i}x_i^{\frac{1}{(1-\gamma)(1-\alpha)-1}} - 1\right)^{-1}}$ and $(1-\gamma)(1-\alpha)-1<0$, we have that

$$\frac{\partial f_i}{\partial \pi_i} = -\frac{f_i^2}{K}\left(1 + \frac{1}{1+(\gamma-1)(1-\alpha)}\frac{\frac{1}{\phi_i}x_i^{\frac{1}{(1-\gamma)(1-\alpha)-1}}}{\left(\frac{1}{\phi_i}x_i^{\frac{1}{(1-\gamma)(1-\alpha)-1}} - 1\right)^2}\frac{1}{\pi_i(1-\pi_i)}\frac{1}{\partial_{x_i}(x_i - g(x_i))}\right).$$

In the high-insurance equilibrium $\partial_{x_i}(x_i - g(x_i)) > 0$ such that $\frac{\partial f_A}{\partial \pi_A} < 0$. In the low-insurance equilibrium, $\partial_{x_i}(x_i - g(x_i)) < 0$. As such, if

$$\frac{1}{1+(\gamma-1)(1-\alpha)}\frac{\frac{1}{\phi_i}x_i^{\frac{1}{(1-\gamma)(1-\alpha)-1}}}{\left(\frac{1}{\phi_i}x_i^{\frac{1}{(1-\gamma)(1-\alpha)-1}} - 1\right)^2}\frac{1}{\pi_i(1-\pi_i)}\frac{1}{\partial_{x_i}(x_i - g(x_i))} < -1,$$

then $\frac{\partial f_A}{\partial \pi_i} > 0$. This condition is satisfied only when $|\partial_{x_i}(x_i - g(x_i))|$ is sufficiently small. Otherwise, $\frac{\partial f_A}{\partial \pi_A} < 0$. ■