Micro Responses to Macro Shocks

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Motivation

- Estimates of transmission of aggregate shocks to individual outcomes are key objects.
- Panel local projection (LP) at horizon h:

$$Y_{i,t+h} = \beta(h) s_i X_t + \text{controls} + \xi_{it}(h)$$

with micro outcome Y_{it}, macro shock X_t and micro covariate $s_i \implies$ least squares $\hat{\beta}(h)$.

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- Despite a lot of progress in time series, little is known about the panel data case.

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2. How to compute standard errors/confidence intervals?

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- We allow for DGPs with potentially low macro-micro signal-noise.

- Macro shock of interest X is observed, recovered from SVAR or proxy/IV.

Contributions

1. Estimand of $\hat{\beta}(h)$.
   - Population projection of impulse response β_{ih} on s_i:

$$\beta(h) = \frac{\mathrm{Cov}(\, s_i, \beta_{ih}\,)}{\mathrm{Var}(\, s_i\,)}.$$

Nonparametric in the sense of permitting unrestricted unobserved heterogeneity.

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   Nonparametric in the sense of permitting unrestricted unobserved heterogeneity.

2. Panel LP inference.
   - We propose a simple yet robust recipe: lags + clustering on t.
   - Relative to standard practice, offers both technical and practical advantages.

   Uniform validity over DGPs with different macro-micro signal-noise.

Empirical relevance

- Disagreement in the choice of standard errors in applied work.
  In our review of almost 50 recent empirical papers:
  - (i, t)-clustering (two-way) $\approx$ 50%
  - i-clustering (within units) $\approx$ 33%
  - Driscoll, Kraay (1998) $\approx$ 15%

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- Instead, our recommendation ➡ t-LAHR inference:
  - Lags + clustering on t.
  - Small-sample refinements if T is small; Imbens, Kolesár (2016).
  - Inference is simple and robust to the pervasiveness of micro variation.

- Code: `https://github.com/TinchoAlmuzara/PanelLocalProjections`

Literature

- Time series literature on local projections: Jordà (2005), Stock, Watson (2018), Montiel Olea, Plagborg-Møller (2021), Montiel-Olea, Plagborg-Møller, Qian, Wolf (2024) ...
- Econometrics with aggregate shocks: Hahn, Kuersteiner, Mazzocco (2020), Chang, Chen, Schorfheide (2024), Almuzara, Arellano, Blundell, Bonhomme (2025) ...
- Cross-sectional dependence: Driscoll, Kraay (1998), Andrews (2005), Pesaran (2006)
- Complement to regional exposure/shift-share: Adao, Kolesar, Morales (2019), Arkhangelsky, Korovkin (2023), Majerovitz, Sastry (2023), Hahn, Kuersteiner, Santos, Willigrod (2024), ...

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➡ This paper: panel data + aggregate shocks + robustness to macro signal strength

Outline

1. Panel local projections
   - Main result
   - Synthetic time series representation
   - Additional structure: VAR DGP
2. Extensions: Panel LP-IV
3. Simulation study
4. Empirical illustration
5. Conclusion

Panel local projections

Main result

Panel local projections: estimator

- Panel LP at horizon h with p lags, controls and unit and time FEs:

$$Y_{i,t+h} = \hat{\beta}(h) s_i X_t + \hat{\psi}(h)' W_{it} + \hat{\mu}_i(h) + \hat{\nu}_t(h) + \hat{\xi}_{it}(h),$$

  where $W_{it} = (s_i X_{t-1}, Y_{i,t-1}, \ldots, s_i X_{t-p}, Y_{i,t-p})'$ and $\hat{\xi}_{it}(h)$ are least-squares residuals.

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- Can include additional macro and micro controls.

- Easy to extend to unbalanced panels and time-varying s_i.

- Focus on the dominant case where X_t is observable for simplicity (extensions).

Panel local projections: inference

- Confidence interval based on sandwich formula:

$$\hat{C}_\alpha(h) = \left[\hat{\beta}(h) \pm z_{1-\alpha/2}\hat{\sigma}(h)\right], \qquad \hat{\sigma}(h) = \sqrt{\frac{\hat{V}(h)}{(T-h)\hat{J}^2}},$$

where $\hat{J} = \frac{1}{N(T-h)} \sum_{i=1}^{N} \sum_{t=1}^{T-h} \hat{x}_{it}^2$ and $\hat{x}_{it}$ are residuals from reg. $s_i X_t$ on W_{it} and the FEs.

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- What about $\hat{V}$? Right choice relies on time clustering:

$$\hat{V}(h) = \frac{1}{(T-h)}\sum_{t=1}^{T-h}\left(\frac{1}{N}\sum_{i=1}^{N}\hat{x}_{it}\hat{\xi}_{it}(h)\right)^2.$$

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- We now study the properties of $\hat{\beta}(h)$ and $\hat{C}_\alpha(h)$ in a general class of DGPs.

Model: setup

- DGP:

$$Y_{it} = \mu_i + \sum_{\ell=0}^{\infty} \beta_{i\ell} X_{t-\ell} + v_{it}, \quad t = 1, \ldots, T, \quad i = 1, \ldots, N,$$

$$v_{it} = \sum_{\ell=0}^{\infty} \gamma_{i\ell} Z_{t-\ell} + \kappa \sum_{\ell=0}^{\infty} \delta_{i\ell} u_{i,t-\ell}.$$

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  - Micro-macro Wold representation, more flexible than VAR.
- Macro-micro signal-noise κ.
  - We consider a range of DGPs P_κ where κ might grow as $N \to \infty$.

Model: assumptions

Assumption: stationarity and iidness

$\{X_t, Z_t, \{u_{it}\}_i\}$ stationary given $\{\theta_i, s_i\}_i$.
$\{\theta_i, s_i, \{u_{it}\}\}_i$ i.i.d. over i given $\{X_t, Z_t\}$.

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Regularity cond's: decay of β, γ, δ + moments of X, Z, u + summability of squares.

Model: macro/micro signal-noise

- R^2's of aggregate shocks:

$$\bar{R}^2 = 1 - \frac{\text{Var}\left(\bar{Y}_t \mid \{X_\tau, Z_\tau\}, \{\theta_i\}\right)}{\text{Var}\left(\bar{Y}_t \mid \{\theta_i\}\right)} = 1 - O\left(\frac{\kappa^2}{N}\right),$$

with $\bar{Y}_t = N^{-1} \sum_{i=1}^{N} Y_{it}$.

- High-signal case $\implies$ κ fixed, $\bar{R}^2 \approx 1$.
- Low-signal case $\implies$ $\kappa \propto \sqrt{N}$, $\bar{R}^2 \in (0, 1)$.
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- But... κ not always estimable.

Main result

- Asymptotics. $T, N \to \infty$ with $T/N \to 0$ holding $p \geq h$ fixed.

Proposition: estimand

$$\beta(h) = \text{Cov}(\, s_i, \beta_{ih}\,) \,/ \text{Var}(\, s_i\,)\,.$$

  - If $s_i = 1$, this is we have $\beta(h) = \text{E}[\,\beta_{ih}\,]$.

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  - Proof: drifting sequences + martingale approximations + uniform bounds.

Panel local projections

Synthetic time series representation

Synthetic time series representation (I)

- By FWL, the panel LP estimator with p lags + unit and time FEs can also be written as

$$Y_{i,t+h} = \hat{\beta}(h)\hat{s}_i\hat{X}_t + \sum_{\ell=1}^{p} \tilde{\varphi}_j(h)s_iX_{t-\ell} + \tilde{\mu}_i(h) + \tilde{\nu}_t(h) + \hat{\xi}_{it}(h),$$

with $\hat{X}_t$ = residual from reg. X_t on $1, X_{t-1}, ..., X_{t-p}$ and $\hat{s}_i = s_i - N^{-1}\sum_{j=1}^{N} s_j$.

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- Letting $\hat{Y}_t(h) = \left(\sum_{i=1}^{N} \hat{s}_i Y_{i,t+h}\right) / \left(\sum_{i=1}^{N} \hat{s}_i^2\right)$, we have

$$\hat{\beta}(h) = \frac{\sum_{t=1}^{T-h}\sum_{i=1}^{N} \hat{s}_i\hat{X}_t Y_{i,t+h}}{\sum_{t=1}^{T-h}\sum_{i=1}^{N} \hat{s}_i^2\hat{X}_t^2} = \frac{\sum_{t=1}^{T-h}\hat{X}_t\hat{Y}_t(h)}{\sum_{t=1}^{T}\hat{X}_t^2},$$

i.e., $\hat{\beta}(h)$ is also the regression coeff. on *synthetic* time series data $\{\hat{Y}_t(h), \hat{X}_t\}_{t=1}^{T-h}$.

Synthetic time series representation (II)

- Similarly, define the synthetic residuals as

$$\hat{\xi}_t(h) = \frac{\sum_{i=1}^{N} \hat{s}_i \hat{\xi}_{it}(h)}{\sum_{i=1}^{N} \hat{s}_i^2} = \hat{Y}_t(h) - \left(\hat{\beta}(h)\hat{X}_t + \sum_{\ell=1}^{p} \tilde{\varphi}_\ell(h) X_{t-\ell} + \tilde{\mu}(h) \right).$$

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- This is a powerful tool that is behind many of our results:
  - Provides guidance for inference and qualifies the role of the cross-sectional dim.
  - A "low-dimensional" representation of the problem ➡ visualization tool.

Macro-micro decomposition

- Representation of estimation error:

$$\hat{\beta}(h) = \underbrace{\frac{\sum_{i=1}^{N} \hat{s}_i \beta_{ih}}{\sum_{i=1}^{N} \hat{s}_i^2}}_{\beta(h)+O_p\left(N^{-1/2}\right)} + \frac{\sum_{t=1}^{T-h} X_t \xi_t(h)}{\mathrm{E}\left[X_t^2\right]} + o_p\left(T^{-1/2}\right)$$

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where

$$\xi_t(h) = \left(\sum_{\ell \notin [h,h+p]} \tilde{\beta}_\ell X_{t+h-\ell} + \sum_{\ell=0}^{\infty} \tilde{\gamma}_\ell Z_{t+h-\ell} \right) + \frac{\kappa}{\sqrt{N}} \left(\frac{\sum_{i=1}^{N} \hat{s}_i \sum_{\ell=0}^{\infty} \delta_{i\ell} u_{i,t+h-\ell}}{\sqrt{N} \mathrm{Var}(\, s_i \,)} \right)$$

- Nature of estimation error depends on κ. Micro noise non-negligible if $\kappa \propto \sqrt{N}$.

Which inference procedures work?

- Regression score:
  - MA(h) with $\mathrm{Cov}(\, X_t \xi_t(h), X_{t-\ell} \xi_{t-\ell}(h) \,) = 0$ if $1 \leq |\ell| \leq p$.
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- A CI works if it captures score's sum of autocovariances.
  - Unit-level clustering ignores the presence of macro shocks.
  - Driscoll-Kraay OK in theory. Tricky in practice (kernel + difficulties with HAC).
    - It also estimates a lot of unnecessary autocovariances.
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- Applied recommendation: t-LAHR + small-sample refinement if T small.
  - Choice of lags p is an important dimension in high- or moderate-signal regimes.

Panel local projections

Additional structure: VAR DGP

Panel VAR model: setup and estimation

- A low-order VAR often provides a reasonable approximation to the data.
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- Vector autoregressive DGP:

$$Y_{it} = m_i + \sum_{\ell=1}^{p} A_\ell Y_{i,t-\ell} + \sum_{\ell=0}^{p} B_{i\ell} X_{t-\ell} + C_{i0} Z_t + \kappa D_{i0} u_{it}.$$

- Estimation: LP augmented with p lags of Y_{it} and $s_i X_t$.

$$Y_{i,t+h} = \hat{\beta}(h) s_i X_t + \sum_{\ell=1}^{p} \left(\hat{\psi}_\ell(h) Y_{i,t-\ell} + \hat{\varphi}_\ell(h) s_i X_{t-\ell} \right) + \hat{\mu}_i(h) + \hat{\nu}_t(h) + \hat{\xi}_{it}(h).$$

Panel VAR model: inference

Proposition: uniformly valid inference (panel VAR)

$$\lim_{T,N\to\infty} \sup_{0\leq h\leq \phi T} \sup_{P_\kappa} \left| P_\kappa(\beta(h) \in \hat{C}_\alpha(h)) - (1-\alpha) \right| = 0.$$

- Time-level aggregation of $\hat{\xi}_{it}(h)$ + Eicker–Huber–White works for $h \propto T$.
  - Dimension reduction if low-order VAR is a good approximation.
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- In practical terms, IC on the synthetic time series can guide lag length choice.

Shocks from SVARs

- t-LAHR inference remains uniformly valid when:
  - The shock X_t is observed with serial correlation: $X_t^* = \sum \alpha_\ell X_{t-\ell}^* + X_t$.
  - Inference uses lag-augmented LPs with controls for lags of X_t^* and Y_{it}.

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- It also applies to cases where X_t is identified using conventional macro restrictions (recursive, long-run, etc.) in SVARs $\alpha_0 \boldsymbol{R}_t = \sum_{\ell=1}^{p_R} \alpha_\ell \boldsymbol{R}_{t-\ell} + \varepsilon_t$ where $\varepsilon_{jt} = X_t$.
  - LP on $\hat{\alpha}_{0,j\bullet} \boldsymbol{R}_t$ or $\hat{X}_t = \hat{\alpha}_{0,j\bullet} \boldsymbol{R}_t - \sum_{\ell=1}^{p_R} \hat{\alpha}_{\ell,j\bullet} \boldsymbol{R}_{t-\ell}$ with a suitable choice of controls.

- E.g., consider recursive identification:
  - Either project on $s_i R_{jt}$ ($\hat{\beta}_R(h)$) or on estimated $s_i \hat{X}_t$ ($\hat{\beta}_X(h)$).
  - Use rich controls: lags of Y_{it}, macro variables, and fixed effects.
  - Result: Confidence intervals based on $\hat{\beta}_R(h)$ or $\hat{\beta}_X(h)$ are uniformly valid over h and κ.

Extensions

Panel LP-IV

Panel LP-IV model: setup

- General DGP (again):

$$Y_{it} = \mu_i + \sum_{\ell=0}^{\infty} \beta_{i\ell} X_{t-\ell} + \sum_{\ell=0}^{\infty} \gamma_{i\ell} Z_{t-\ell} + \kappa \sum_{\ell=0}^{\infty} \delta_{i\ell} u_{i,t-\ell}.$$

- It is often more appealing to treat observed shocks as contaminated proxies.

- Rather than X_t, the researcher observes endogenous $\tilde{X}_t$ and proxy X_t^*:

$$\begin{aligned} \tilde{X}_t &= \sum_{\ell=0}^{\infty} b_\ell X_{t-\ell} + \sum_{\ell=0}^{\infty} c_\ell Z_{t-\ell}, \\ X_t^* &= a_0 X_t + \nu_t. \end{aligned}$$

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$$X_t^* = a_0 X_t + \nu_t.$$

  - Assumptions: $b_0 = 1$ (normalization) + $a_0 \neq 0$ (IV relevance) + regularity cond's.
  - Panel counterpart to Stock, Watson (2018), Plagborg-Møller, Wolf (2021).

Panel LP-IV model: estimation

- LP-IV regresses $Y_{i,t+h}$ on $\tilde{\boldsymbol{X}}_t = (\tilde{X}_t, \ldots, \tilde{X}_{t-p})'$ using instrument $\boldsymbol{X}_t^* = (X_t^*, \ldots, X_{t-p}^*)'$ both interacted with s_i and controlling for unit/time FEs.
  - We are interested in the first entry $\hat{\beta}_0^{\mathrm{IV}}(h)$ but need to instrument the whole vector.
- Estimand continues to be $\beta(h) = \mathrm{Cov}(\, s_i, \beta_{ih}\,) / \mathrm{Var}(\, s_i\,)$.

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- Estimand continues to be $\beta(h) = \text{Cov}(\, s_i, \beta_{ih}\,) / \text{Var}(\, s_i\,)$.
- Suitably residualized instrument $\hat{\boldsymbol{X}}_t^*$:

$$\hat{\boldsymbol{\beta}}^{\text{IV}}(h) = \left(\sum_{t=1}^{T-h} \hat{\boldsymbol{X}}_t^* \tilde{\boldsymbol{X}}_t' \right)^{-1} \sum_{t=1}^{T-h} \hat{\boldsymbol{X}}_t^* \hat{Y}_{i,t+h},$$

  i.e., it also admits a synthetic time series LP-IV representation.

Panel LP-IV model: inference

- $\hat{C}_\alpha^{\text{IV}}(h) = \left[\hat{\beta}_0^{\text{IV}}(h) \pm z_{1-\alpha/2}\hat{\sigma}_0^{\text{IV}}(h)\right]$ with t-clustered heteroskedasticity-robust SE:

$$\hat{\sigma}_0^{\text{IV}}(h) = \left[\frac{1}{(T-h)} \cdot \left(e_1' \left[\hat{\boldsymbol{J}}^{\text{IV}}\right]^{-1}\right) \hat{\boldsymbol{V}}^{\text{IV}}(h) \left(e_1' \left[\hat{\boldsymbol{J}}^{\text{IV}}\right]^{-1}\right)'\right]^{1/2}$$

where $\hat{\boldsymbol{J}}^{\text{IV}}$ and $\hat{\boldsymbol{V}}^{\text{IV}}(h)$ come from the usual sandwich formula.

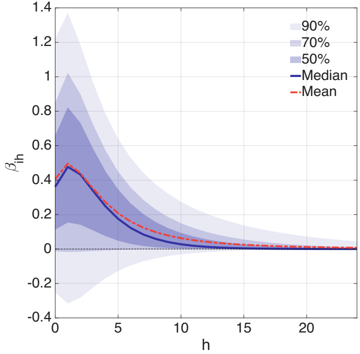
Proposition: uniformly valid inference (panel LP-IV)

$$\lim_{T,N\to\infty} \sup_{P_\kappa} \left| P_\kappa(\beta(h) \in \hat{C}_\alpha^{\text{IV}}(h)) - (1-\alpha) \right| = 0.$$

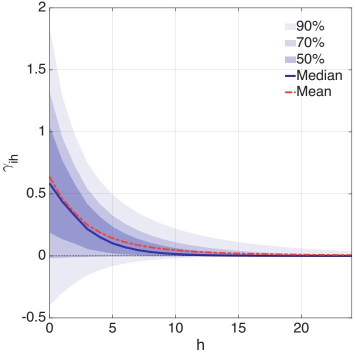
Simulation study

Design

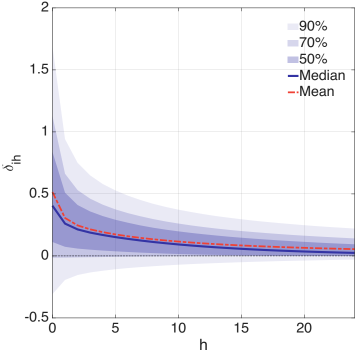
(a) Distribution β_{ih}



(b) Distribution γ_{ih}

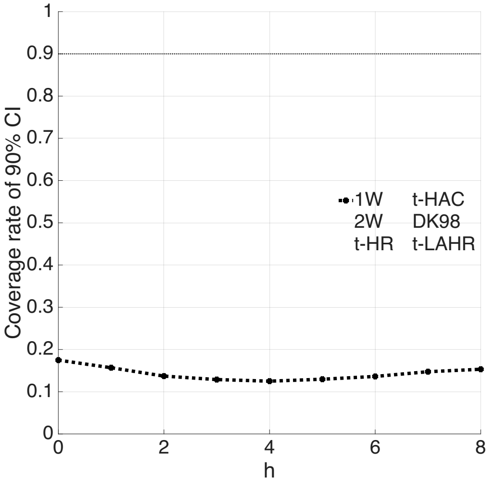


(c) Distribution δ_{ih}

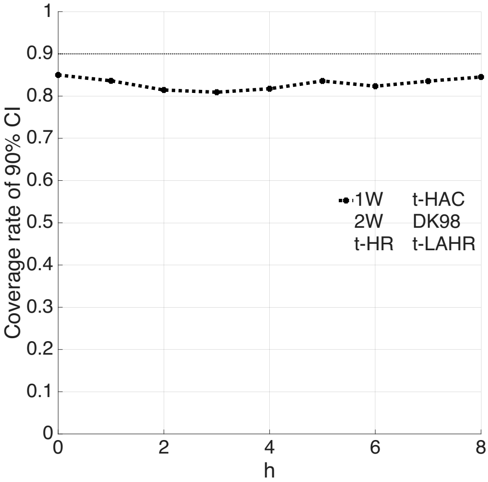


Panel LP with sample size $T = 30$

(a) $\bar{R}^2 = 0.99$



(b) $\bar{R}^2 = 0.33$



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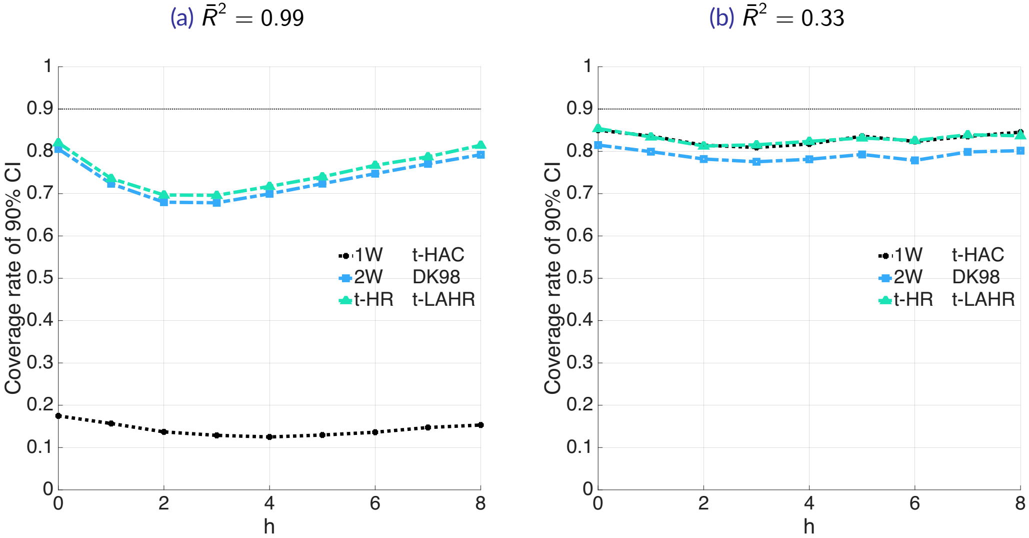

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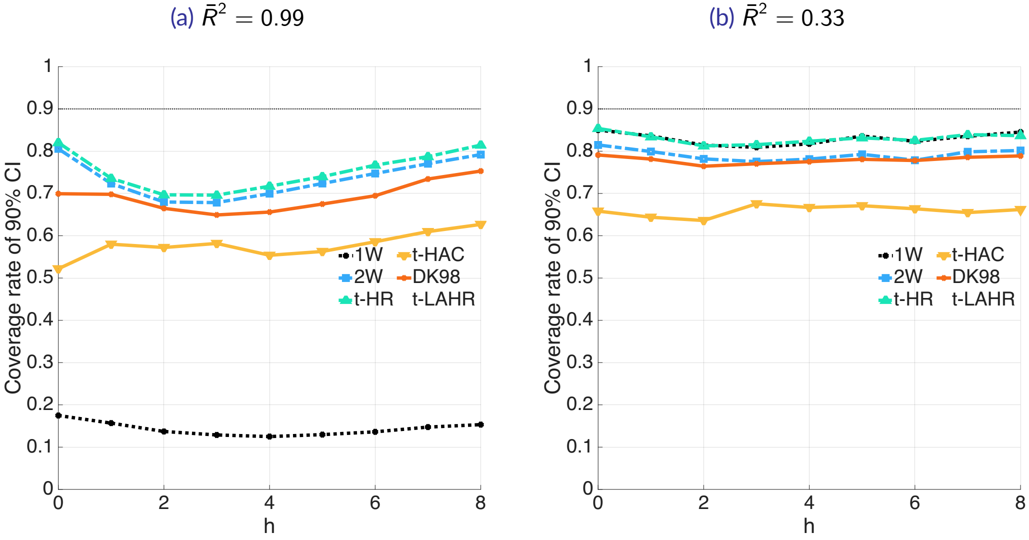

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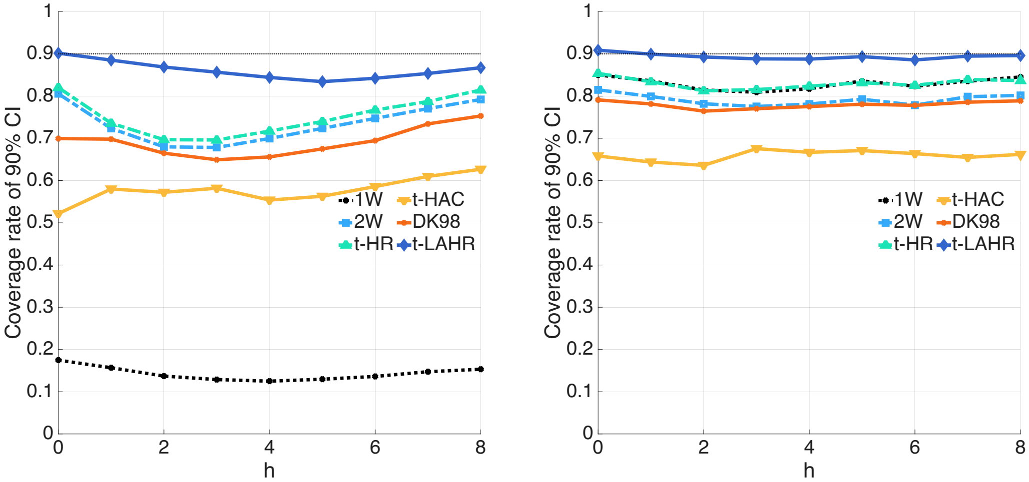

Panel LP with sample size $T = 100$

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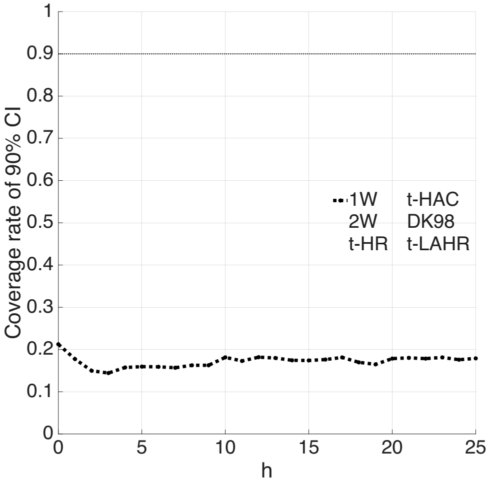

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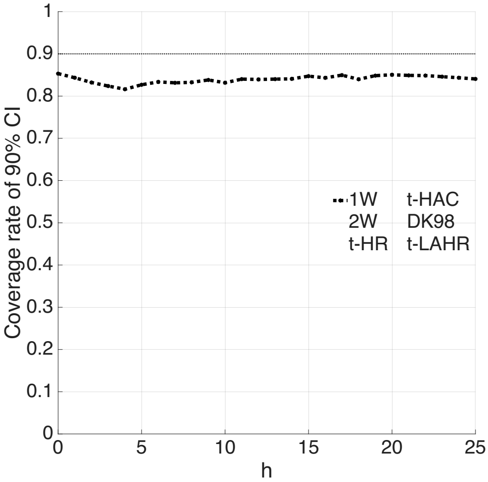

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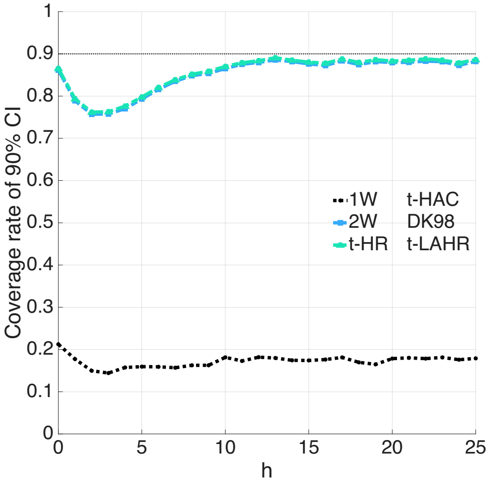

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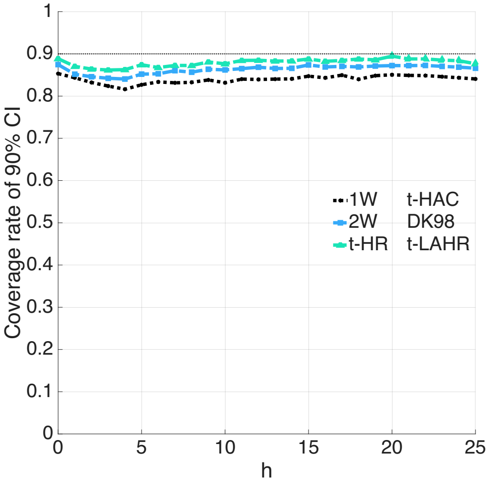

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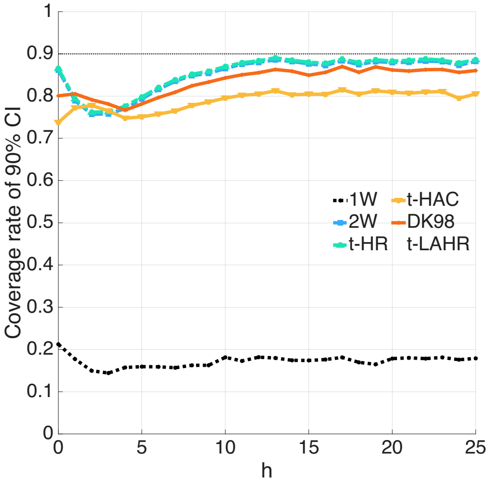

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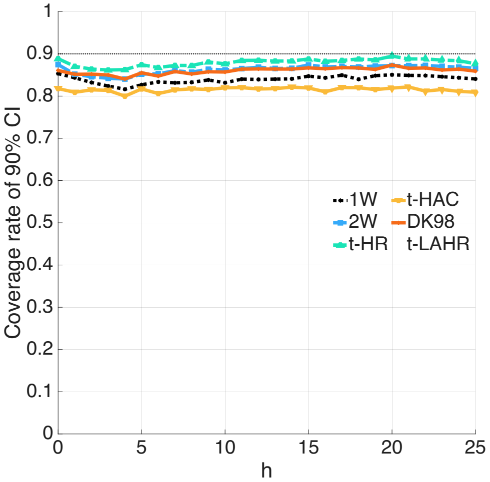

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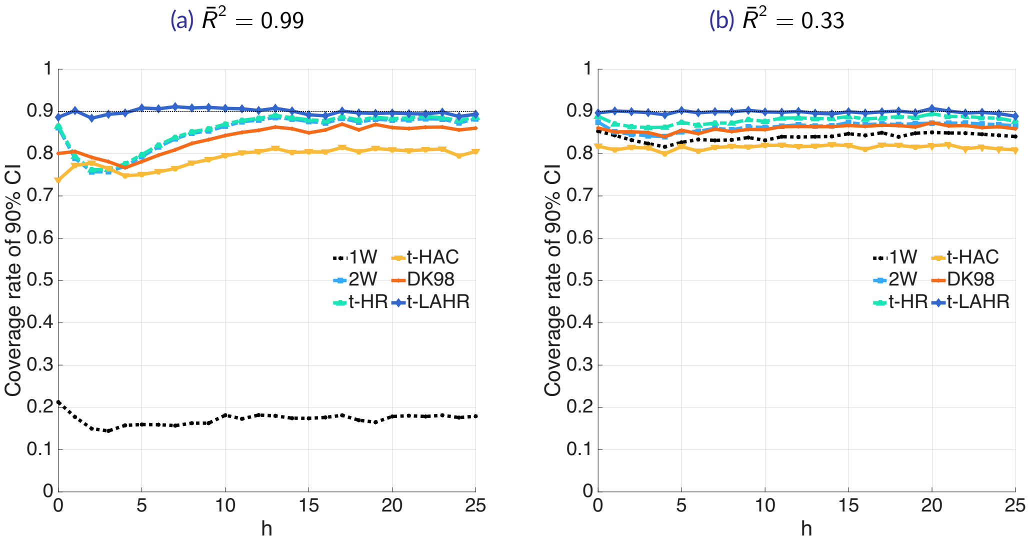
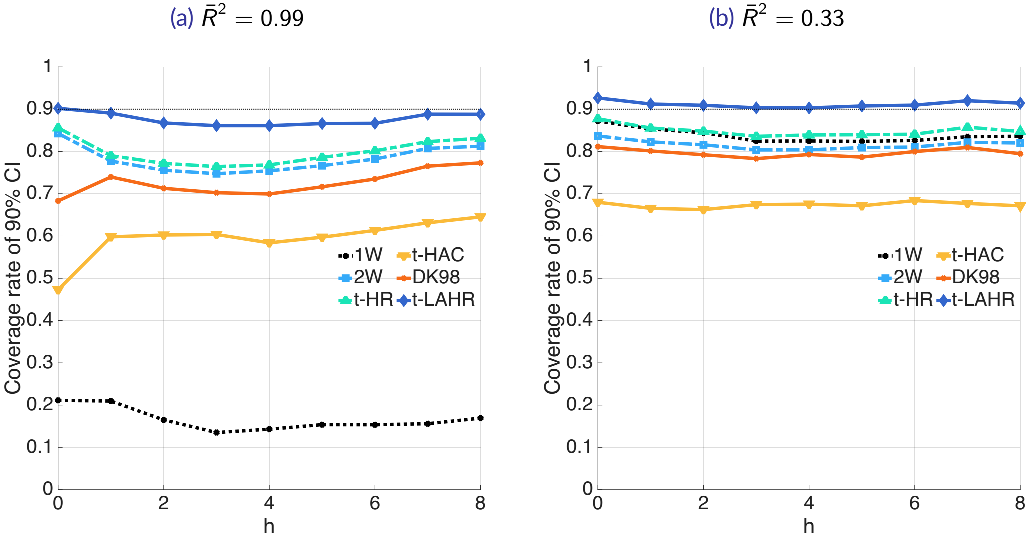


Panel LP-IV with sample size $T = 30$



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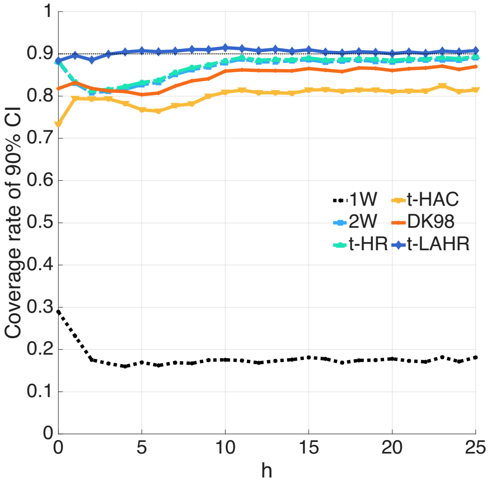

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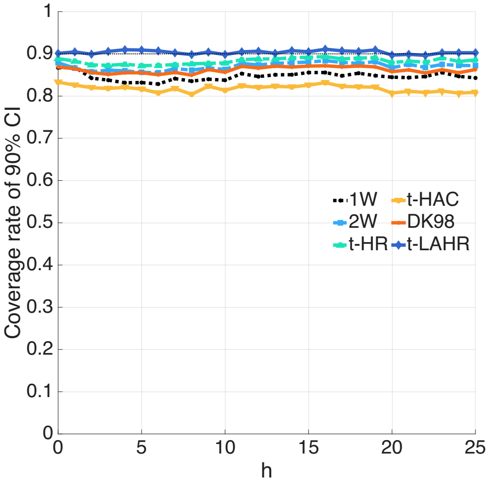

Empirical illustration

Based on Ottonello, Winberry (2020, ECTA)

Data and background

- Goal: Quantify firm-level responses to exogenous changes in mon. policy:
  - Ottonello, Winberry (2020): Default risk.
- Literature: Role of financial het. in shaping the transmission of mon. policy:
  - Crouzet, Mehrotra (2020): Firm size.
  - Cloyne, Ferreira, Froemel, Surico (2023): Firm age and dividends policy.
  - Jeenas, Lagos (2024): Stock turnover.
- Dataset:
  - Based on *Compustat/CRSP* and high-frequency monetary policy shocks.
  - $N = 5,992$ firms and $T = 72$ quarters (1990Q1–2007Q4) ➡ 335,878 observations.

Empirical framework

- Dependent variable is the cumulative investment change:

$$Y_{i,t+h} = \log\left(\frac{k_{i,t+h}}{k_{i,t-1}}\right)$$

  where k_{it} is the capital stock for firm i during quarter t.

- Policy shocks X_t interacted with s_{it} (distance to default or leverage).

- Controls: s_{it}, firm size, sales growth, GDP growth, firm FE and sector $\times$ quarter FE.

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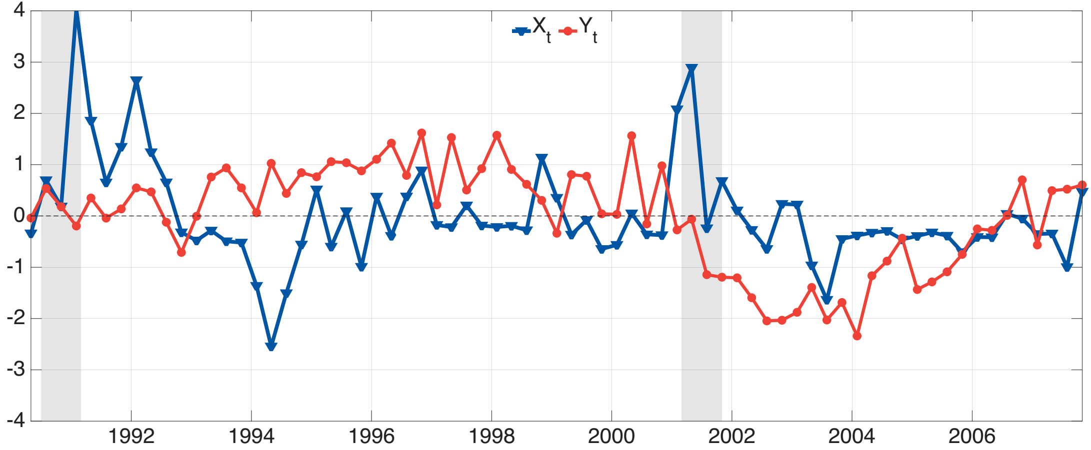
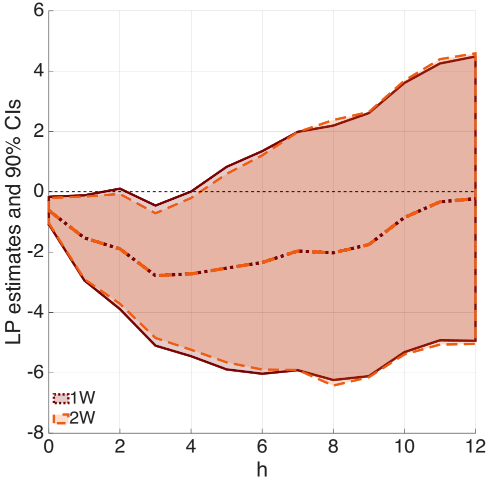
- Interpretation:
  - Linear projection of firm-level impulse responses on s_{it}.
  - How does the average semi-elasticity of investment to mon. pol. change along s_{it}?

Synthetic time series

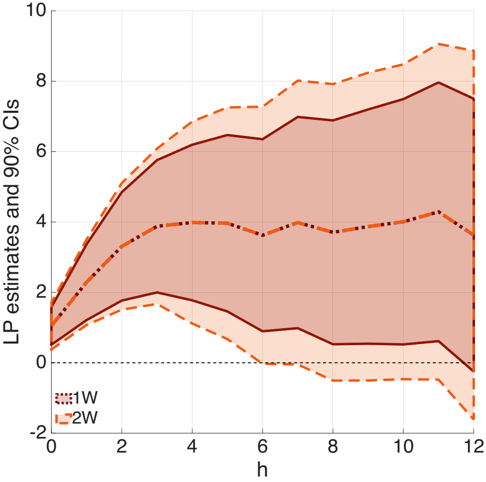


Confidence intervals

(a) Leverage



(b) Distance to default



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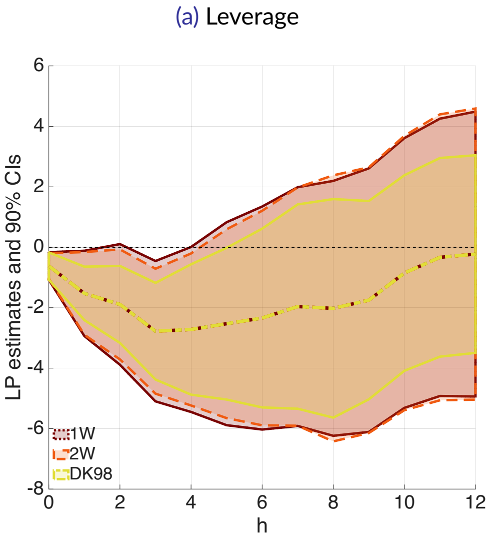
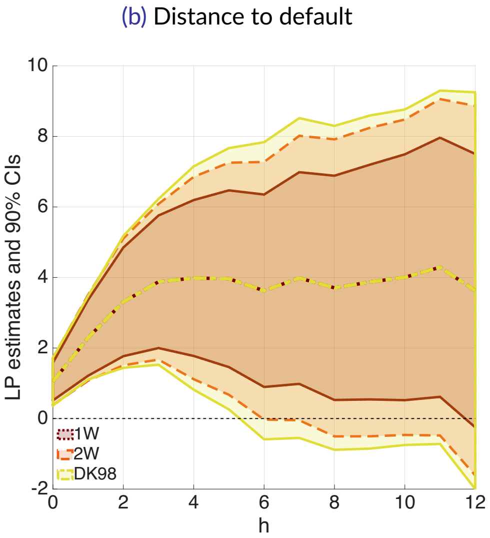
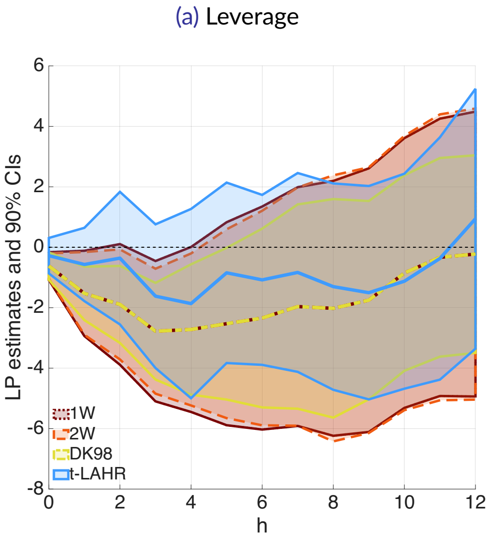
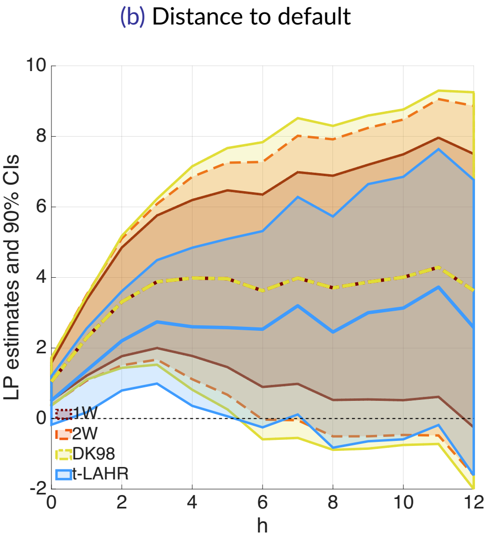




Confidence intervals





Conclusion

Conclusion and practical recommendations

- Explosion of empirical work using panel local projections with aggregate shocks.
- Estimand under unrestricted heterogeneity $=$ population regression.
- Inference is simple and robust:
  - Lags $+$ t-clustering.
  - Reliable even in low-signal environments.
  - Easy to refine in small samples and tractable over moderate horizons.
  - Synthetic time series representation as a guiding principle.
- Code: `https://github.com/TinchoAlmuzara/PanelLocalProjections`

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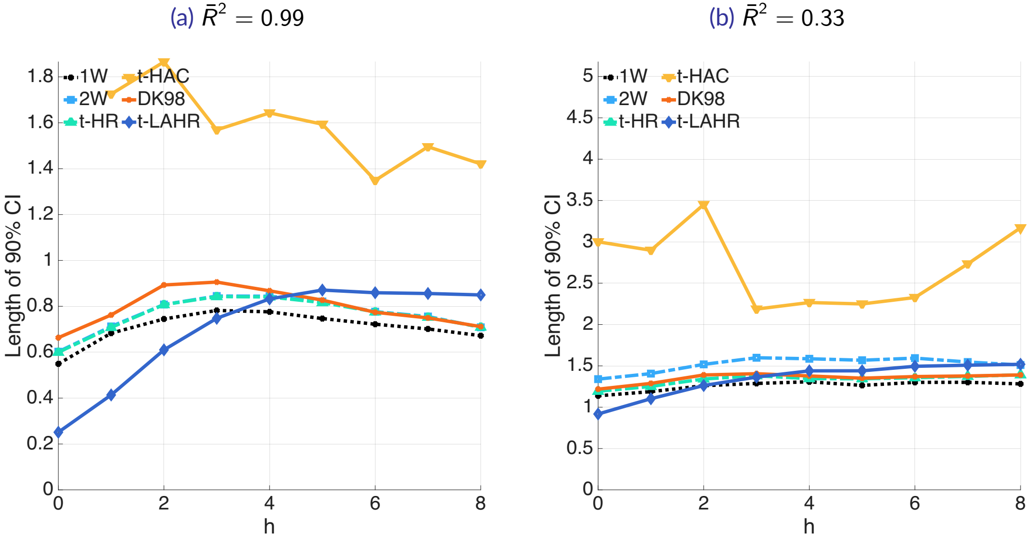
Thank you!

Appendix

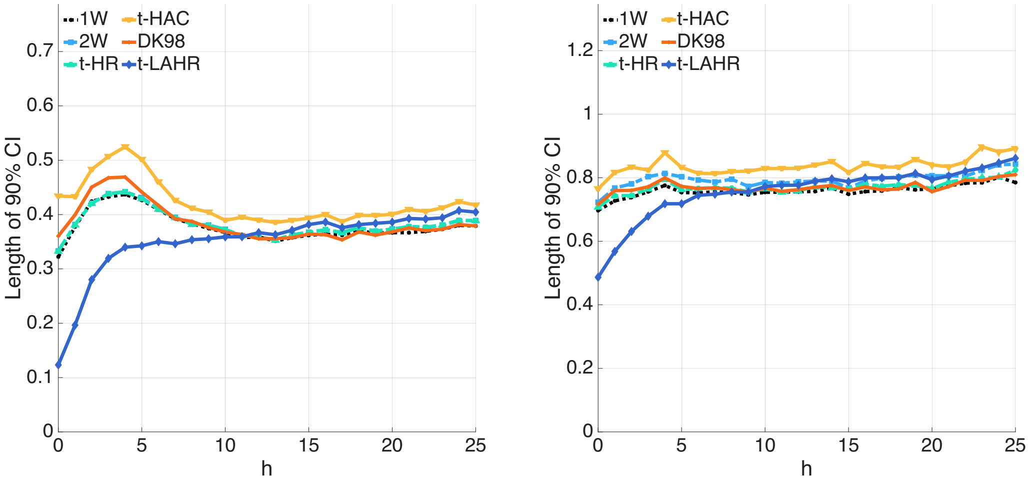
Additional results on the simulations

Panel LP with sample size $T = 30$



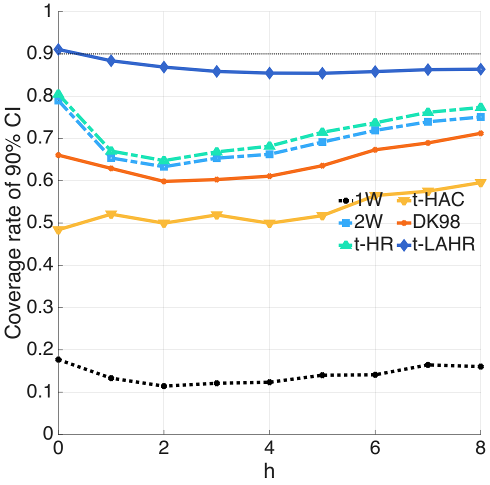
Panel LP with sample size $T = 100$

(a) $\bar{R}^2 = 0.99$ (b) $\bar{R}^2 = 0.33$

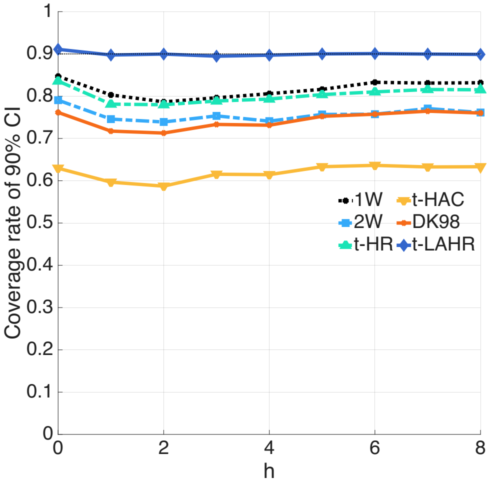


Panel LP with sample size $T = 30$ (cond. volatility)

(a) $\bar{R}^2 = 0.99$

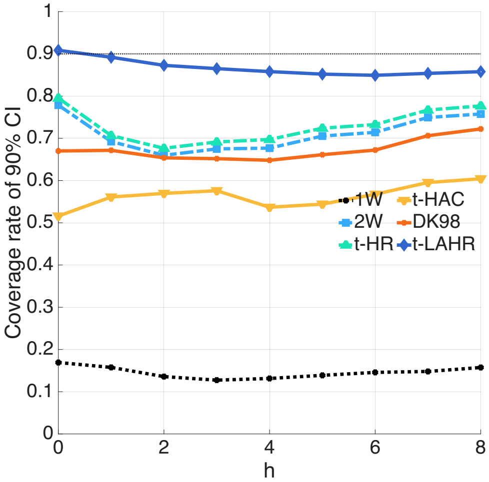


(b) $\bar{R}^2 = 0.33$



Panel LP with sample size $T = 30$ (non-Gaussianity)

(a) $\bar{R}^2 = 0.99$



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