

Pitfalls of First-Stage Diagnostics and Valid Inference in IV Regression with Heavy Tailed Variables

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Abstract

This paper investigates instrumental variable (IV) estimation in linear instrumental variable regressions with heavy-tailed distributions. We allow for the possibility of weak instruments in the conventional sense, meaning that the first-stage regression coefficients may converge to zero as the sample size increases. Within this framework, we generalize the notion of weak instruments by defining *vague instruments* as those for which the first-stage F -statistic remains stochastically bounded. Notably, under certain heavy-tailed distributions, instruments can be vague even when the first-stage coefficients are constant. We show that the IV estimator can remain consistent even if the IV is vague. However, standard inference procedures—such as the IV t -test and weak-instrument robust methods—may fail to provide valid inference regardless of the vagueness of the instruments. To address this, we propose a valid inference procedure based on the second-stage F -statistic, incorporating a directional transformation of the instruments. We establish that the proposed test controls the size correctly and achieves consistency, provided the instruments are not severely vague.

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