Robust Delegation

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Introduction

Organizations face two sources of uncertainty (Cyert and March (1963)).

- External uncertainty: fluctuations in the competitive environment, operational constraints, or technological developments
- Internal uncertainty: heterogeneity in the preferences, skills, and beliefs of the individuals charged with making those decisions.

Delegation literature has placed considerable attention on understanding how external uncertainty shapes internal decision-making processes.

However, internal uncertainty may be equally consequential

Introduction

Study the role of internal uncertainty in the design of delegation rules

Delegation where principal has limited information on agent's preference

- Doesn't know how agent trades off among sub-optimal options

Uncertainty may give more or less discretion to agent

Uncertainty makes delegation simple: no holes, convex, even full delegation

- Uncertainty in how agents exploit/manipulate rules for private interest
- Sophisticated rules leave more space for manipulation

Model

Model

Principal delegates agent to take an action $a \in A$, a compact subset of $\mathbb{R}^n$

Agent privately know state $\theta \in \Theta$, a compact subset of $\mathbb{R}^m$

Principal does not know state θ, only holds a belief $F(\theta)$ with density.

Principal's continuous utility function $v(a, \theta)$: quasi-concave in a

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Principal does not perfectly know agent's utility function $u(a, \theta)$

Model

Principal delegates a compact delegation set D to agent.

In state θ, agent with utility u chooses an action

$$\tilde{a}(\theta; u, D) \in argmax_{a \in D} u(a, \theta)$$

Principal's expected payoff from delegation set D with a given u is

$$\mathbb{E}_F[v(\tilde{a}(\theta; u, D), \theta)].$$

Principal is uncertain about agent's utility function u

Model: Uncertainty in Utility

Principal considers every $u \in \mathscr{U}$ possible.

Principal knows agent's preferred action is in $a^*(\theta)$, continuous in θ,

$$a^*(\theta) = \arg\max_{a \in A} u(a, \theta)$$

Perfect identification, can generalize to partial identification $A^*(\theta)$

$$\mathscr{U} \equiv \{u \mid \text{continuous in } (a, \theta), \text{ strictly quasiconcave in } a, \\ a^*(\theta) = \arg\max_{a \in A} u(a, \theta)\}$$

Our results also hold for other functional forms $\mathscr{U}_{gq} \subset \mathscr{U}_{qc} \subset \mathscr{U}_{ss}$

Model: Max-min Design

Principal considers every $u \in \mathscr{U}$ possible.

She is ambiguity-averse and evaluates the performance of a set D by its *worst-case expected payoff*

There are two possible max-min design framework

$$\sup_D \inf_{u \in \mathscr{U}} \mathbb{E}_F[v(\tilde{a}(\theta; u, D), \theta)]$$

$$\sup_D \mathbb{E}_F[\inf_{u \in \mathscr{U}} v(\tilde{a}(\theta; u, D), \theta)].$$

We prove they are equivalent: focus on the second one from now

Model Discussion

Asymmetric treatment on two sources of uncertainty: state + preferences

Preferences are higher dimensional objective $\mathbb{R}^{A\times\Theta}$ than states Θ

Sampling preferences requires revealed-preference designs— demanding repeated observations of the decision problem

Screening over high-dimensional preferences is not that tractable

Better connect with classical delegation literature

Literature Review

Bayesian Delegation in Uni-dimension:

Holmström (1984), Melumad and Shibano (1991), Alonso and Matouschek (2008), Amador and Bagwell (2013) etc.

Bayesian Delegation in Multi-dimension:

Alonso et al. (2014), Gan et al. (2023), Frankel (2016), Kleiner (2022)

Robust Design:

Robust in A, $F(\theta)$, agents' belief about θ, agents' equilibrium play

Robust Delegation in $v(\theta, a)$: Frankel (2014)

Analysis

Uncertainty in Behaviors

Uncertainty in preference $\rightarrow$ Uncertainty in how agent manipulate rules

For a D, define the *$D-$Admissible Set correspondence*, $A_D(\theta)$, the set of decisions selected by agents with preferences in $\mathscr{U}$ when the state is θ, i.e.,

$$A_D(\theta) \equiv \{a \in D : \exists u \in \mathscr{U}, \forall a' \in D, u(a, \theta) \geq u_A(a', \theta)\}.$$

Principal doesn't know how agent makes tradeoffs when constrained.

Principal's worst payoff in state θ when agent selects from D is

$$\underline{V}_D(\theta) \equiv \inf_{a \in A_D(\theta)} v(a, \theta).$$

Rewrite Design Problem

Principal's worst payoff in state θ when agent selects from D is

$$\underline{V}_D(\theta) \equiv \inf_{a \in A_D(\theta)} v(a, \theta).$$

Principal's optimal robust delegation problem can be written as

$$\max_{\text{compact } D \subset A} \int_\Theta \underline{V}_D(\theta) \mathrm{d}F. \tag{1}$$

To study the property of $\underline{V}_D$, we need geometric property of $A_D(\theta)$

Visible Set

Define $C(x, D)$ the set of visible points on D from point x

$$C(x, D) \equiv \{a \in D : [a, x] \cap D = a\}$$

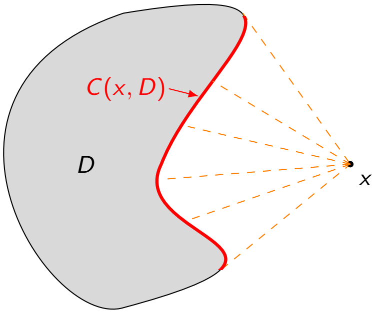


Figure: Illustration of visible set

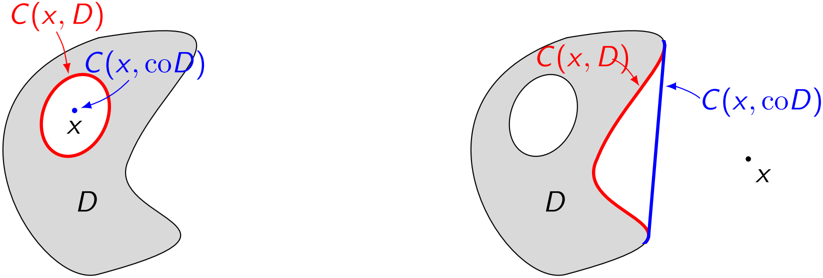
The Geometry of Visible Sets

Lemma

$C(x, D)$ is lower hemi-continuous in D.

Lemma

$C(x, \mathrm{co}D) \subseteq \mathrm{co}C(x, D)$.



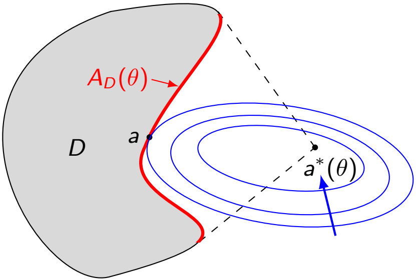
(a) $x \in \mathrm{co}D \setminus D$ (b) $x \notin \mathrm{co}D$

Admissible Sets = Visible Sets

Lemma

For any non-empty and compact $D \subset A$ and $\theta \in \Theta$ we have

$$\mathrm{cl}[A_D(\theta)] = \mathrm{cl}[C(a^*(\theta), D)]$$



Existence of Optimal Solution

First, we use the equivalence to prove existence

Lemma

The max-min optimal delegation set exists.

$$\max_{\text{compact } D \subset A} \int_\Theta \underline{V}_D(\theta)\mathrm{d}F = \max_{\text{compact } D \subset A} \int_\Theta \min_{a \in \mathrm{cl}[C(a^*(\theta), D)]} v(a,\theta)\mathrm{d}F.$$

Endow D with Hausdorff metric

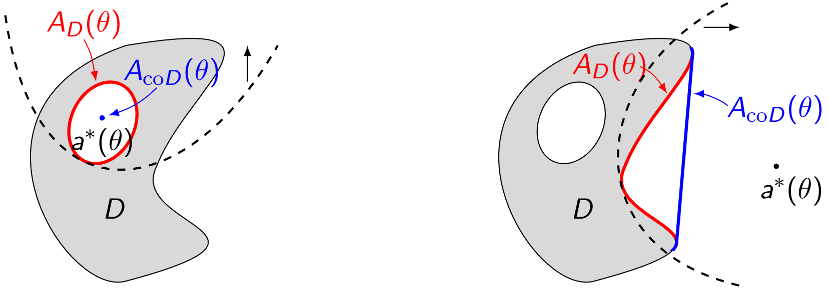
Prove $\mathrm{cl}[C(a, D)]$ is lower hemi-continuous in D

Prove $\underline{V}_D(\theta)$ is upper semi-continuous in D

The Optimality of Convex Set

Proposition

Any delegation set D is weakly out-performed by its convex hull $\mathrm{co}D$.



(a) $a^*(\theta) \in \mathrm{co}D \setminus D$ (b) $a^*(\theta) \notin \mathrm{co}D$

The Optimality of Convex Set

For the reverse direction, denote $A^* = \{a^*(\Theta)\}$

Proposition

Suppose A^* is convex, $a^*(\theta)$ admits positive density in A^*, and the principal's payoff function $v(\cdot, \theta)$ is strictly quasi-concave for each θ. Then, any optimal delegation set is convex.

Properties of Optimal Convex Set I

There has to be sufficient discretion for delegation to be valuable

Lemma

Suppose that $a^*(\theta)$ admits positive density in A^*, and let the delegation set D be low-dimensional in the sense that $\dim(\mathit{aff}(D)) < \dim(\mathit{aff}(A^*))$. Then D is worse than no delegation.

All of D is visible to any a^* outside $\text{aff}(D)$

Properties of Optimal Convex Set II

A set is strictly convex if all boundary points are extreme points

Proposition

Suppose $A^* \subset \mathbb{R}^2$ is strictly convex, $a^*(\theta)$ admits positive density in A^*, and the principal's payoff function $v(\cdot, \theta)$ is strictly concave for each θ. Then, any optimal delegation set is strictly convex.

With large uncertainty about how agent makes trade-offs along different dimensions, one linear quota is not optimal

Examples

After establishing some general properties, more insights from examples

Disk Example: Simplest Delegation, Discretion can be more or less

Cube Example: No Micro-management

One dimension: Interval Delegation, Robustness to more knowledge

Simplest Delegation: Disk Example

The state space Θ is a unit disk $B(0,1)$ in $\mathbb{R}^2$

θ is distributed radially symmetrically with full support, meaning its density f satisfies: $f(\theta) = f(\theta') > 0$ if $|\theta| = |\theta'|$

Agent wants to match the state: $a^*(\theta) = \theta$

Principal's most preferred action at state θ is $\lambda\theta$ for some $\lambda \in [0,1]$

$v(a,\theta)$ is given by $g(|a - \lambda\theta|)$, where $g : \mathbb{R}_+ \to \mathbb{R}$ is a concave, strictly decreasing loss function. For example, when $g(z) = -z^2$

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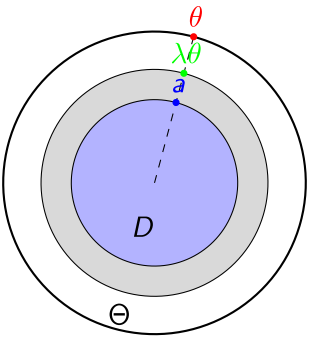
Disk: Known Preference

Suppose the agent's preference is known and is given by $h(|a - \lambda\theta|)$



When $\lambda < 1$, Optimal delegation D is a disk with radius $r^S(\lambda)$

How Optimal Radius Changes

1) Delegation is tight with known preference $r^S(\lambda) \leq \lambda$

2) Optimal radius $r^S(\lambda)$ is strictly increasing

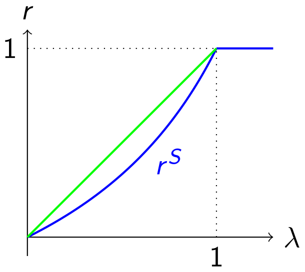


Figure: $F \sim U(B(0,1))$: $r^S = \frac{\lambda}{2-\lambda}$

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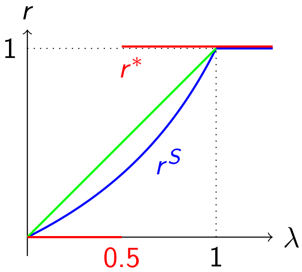
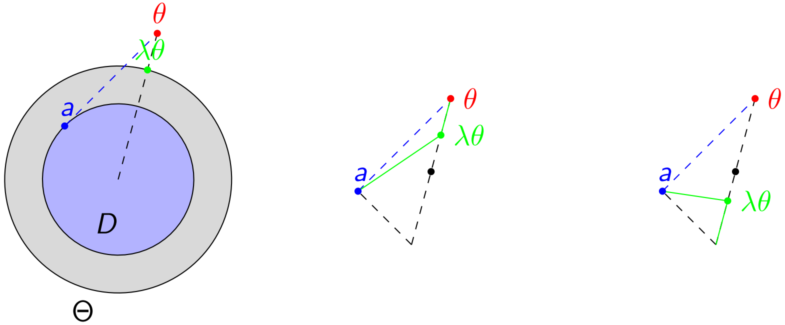


Figure: $F \sim U(B(0,1))$: $r^S = \frac{\lambda}{2-\lambda}$

Disk: Unknown Preference

What is the worst action?



Disk: Robust Design

The max-min solution is bang-bang:

Proposition

if $\lambda < \frac{1}{2}$, $D^* = \{0\}$ *is uniquely optimal;*

if $\lambda > \frac{1}{2}$, *full delegation* $D^* = A^*$ *is uniquely optimal;*

if $\lambda = \frac{1}{2}$, D^* *is optimal if and only if* $D^* = B(0, r)$ *for some* $r \in [0, 1]$.

With internal uncertainty, agent may get more or less discretion

Sophisticated constrained delegation is suboptimal

Full Delegation: Cube Example

The state space Θ is the cube $[0,1]^{n_1+n_2}$, with density f

Every state happens with a minimum possibility: $\exists \gamma \in (0,1),\ f(\theta) \geq \gamma\ \forall \theta$

Agent wants to match the state: $a^*(\theta) = \theta$.

Principal's most preferred action $a_P(\theta)$ satisfies:

$$a_{iP}(\theta) = \theta_i = a_i^*(\theta), \quad \forall i = n_1+1, n_1+2, ..., n_1+n_2$$

Principal's utility $v(a,\theta) = -\|a - a_P(\theta)\|_q^q$

Full Delegation: Cube Example

n_1 dimension: conflicts of interests n_2 dimension: superficially aligned

Proposition

Full delegation is uniquely robust optimal if n_2 is sufficiently larger than n_1:

$$\frac{n_2}{n_1} > \frac{(1+q)2^q}{\gamma},$$

Imposing constraints on n_1 dimension triggers distortion on n_2 dimension

Principal refrains from micromanaging agent through restrictions on misaligned decisions when aligned decisions becomes more complex

The Optimality of Intervals

Proposition

Suppose $\Theta = [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$*, and* $v(\cdot, \theta)$ *is strictly quasiconcave, then any max min optimal delegation set is an interval.*

Finding the optimal interval: optimization over two numbers, simple FOC

Optimal solution coincides with optimal intervals as Holmstrom (1984)

Justification for interval delegation independent of u, F

The Optimality of Intervals

If $\mathscr{U} = \{u\}$, interval delegation may not be optimal

If $\mathscr{U} = \mathscr{U}_{qc}$, interval delegation is always optimal

What if the principal information of u is in between?

Proposition

For any $\mathscr{U}_1 \subseteq \mathscr{U}_2 \subseteq \mathscr{U}_{qc}$. If interval delegation is optimal under $\mathscr{U}_1$, then interval delegation is also optimal under $\mathscr{U}_2$.

Simple delegation is more desirable if there is more uncertainty.

Interval Delegation: Supermodularity

If principal knows agent's utility $u(a, \theta)$ is supermodular (denoted as $\mathscr{U}_s$).

Proposition

If the principal's utility $u(a, \theta)$ is super-modular in (a, θ) and concave in a, and if the uncertainty set is $\mathscr{U}_s$, then interval delegation is max min optimal.

Extensions I: Money

Money is useless, even if we allow monetary incentives:

$$u(a, \theta, t) = u(a, \theta) + t, \quad u(a, \theta) \in \mathscr{U}$$

If monetary incentive is provided: in the worst case, agent just cares about money and maximizes transfer, which is worse than no delegation

Extensions II: Partial Identification

We assumed that principal perfectly identifies agent's favorite action $a^*(\theta)$ in the absence of any constraints.

Now: principal only has partial identification: she only knows agent's favorite action is within $A^*(\theta)$ in the absence of any constraints.

The characterization of admissible sets $A_D(\theta)$, and consequently the optimality of the convex delegation set, generalizes

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The characterization of admissible sets $A_D(\theta)$, and consequently the optimality of the convex delegation set, generalizes straightforwardly

Extensions II: Partial Identification

Lemma

Under partial identification, for any non-empty compact $D \subset A$ and $\theta \in \Theta$,

$$\mathrm{cl}(A_D(\theta)) = \mathrm{cl}(\cup_{a^*(\theta) \in A^*(\theta)} C(a^*(\theta), D)).$$

Consequently,

$$\mathrm{cl}[A_{\mathrm{co}D}(\theta)] \subset \mathrm{co}[\mathrm{cl}(A_D(\theta))].$$

Proposition

Under partial identification, for any delegation set D, $\underline{V}_D \leq \underline{V}_{\mathrm{co}D}$ and hence $\int_\Theta \underline{V}_D(\theta)\mathrm{d}F \leq \int_\Theta \underline{V}_{\mathrm{co}D}(\theta)\mathrm{d}F$. Consequently, there is always a convex max min optimal delegation set.

Conclusion

This paper: the principal has limited information on agent's preference

- Doesn't know how agent trades off among sub-optimal options

The optimal delegation set is simple: no holes, convex, even full delegation

Our insight: delegation rules are simple because

- Uncertainty in how agents exploit/manipulate rules for private interest
- Sophisticated rules leave more space for manipulation

Thank you!

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