Robust Delegation

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Introduction

Delegating decision rights to agents is common in organizations

- Address a mismatch between authority and information:
- The agent is privately informed about the state.

Examples

- Fiscal policy, budget allocation
- Monopoly regulation, pricing regulation
- Healthcare Treatment
- Corporate management, investment opportunity
- WTO, assessing tariff response to dumping

Introduction

Agent's preference may not be perfectly aligned with the principal

- Constraint what the agent can choose may be beneficial

The literature on delegation: what is the optimal delegation set?

Classical Problem in the literature

- In one dimension, when is (simple) interval delegation optimal?
- Optimal delegation can be sophisticated (digging holes)

Delegation sets in reality:

- Often simple delegation rule, sometimes even full delegation

Overview

Our insight: delegation rules are simple because

- Uncertainty in how agents exploit/manipulate rules for private interest
- Sophisticated rules leave more space for manipulation

This paper: the principal has limited information on agent's preference

- Doesn't know how agent trades off among sub-optimal options

The optimal delegation set is simple: no holes, convex, even full delegation

Model

Model

Principal delegates agent to take an action $a \in A$, a compact subset of $\mathbb{R}^n$

Agent privately know state $\theta \in \Theta$, a compact subset of $\mathbb{R}^m$

Principal does not know state θ, only holds a belief $F(\theta)$ with density.

Principal's continuous utility function $v(a, \theta)$: quasi-concave in a

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Principal does not perfectly know agent's utility function $u(a, \theta)$

Model: Max-min Design I

Principal delegates a compact delegation set D to agent.

In state θ, agent with utility u chooses an action

$$\tilde{a}(\theta; u, D) \in argmax_{a \in D} u(a, \theta)$$

Principal's expected payoff from delegation set D with a given u is

$$\mathbb{E}_F[v(\tilde{a}(\theta; u, D), \theta)].$$

Principal is uncertain about agent's utility function u

Model: Max-min Design II

Principal considers every $u \in \mathscr{U}$ possible.

She is ambiguity-averse and evaluates the performance of a set D by its *worst-case expected payoff*

$$\inf_{u \in \mathscr{U}} \mathbb{E}_F[v(\tilde{a}(\theta; u, D), \theta)]$$

The max-min design problem is to find delegation set D that solves

$$\max_{D} \inf_{u \in \mathscr{U}} \mathbb{E}_F[v(\tilde{a}(\theta; u, D), \theta)]$$

Model: Uncertainty in Utility I

What is the structure of $\mathscr{U}$?

Principal knows agent's preferred action is in $A^*(\theta)$, continuous in θ,

$$A^*(\theta) \supseteq \arg\max_{a \in A} u(a, \theta)$$

To fix idea: perfect identification: $A^*(\theta) = a^*(\theta)$

$$\mathscr{U} \equiv \{u \,|\, \text{continuous and strictly quasiconcave in } a, \\ A^*(\theta) \supseteq \arg\max_{a \in A} u(a, \theta)\}$$

Across different θ, no restrictions: pick preference for a in each θ

Model Discussion

Asymmetric treatment on two sources of uncertainty: state + preferences

Think about a long-last institution with agents coming and leaving

Preferences are higher dimensional objective $\mathbb{R}^{A\times\Theta}$, hard to measure (revealed preference) max-min a standard tool for large uncertainty

Better connect with classical delegation literature

Technically hard to do Bayesian design on generic preferences set

Literature Review

Bayesian Delegation in Uni-dimension:

Holmström (1984), Melumad and Shibano (1991), Alonso and Matouschek (2008), Amador and Bagwell (2013) etc.

Bayesian Delegation in Multi-dimension:

Alonso et al. (2014), Gan et al. (2023), Frankel (2016), Kleiner (2022)

Robust Design:

Robust in A, $F(\theta)$, agents' belief about θ, agents' equilibrium play

Robust Delegation in θ: Li and Hu (2024)

Robust Delegation in $v(\theta, a)$: Frankel (2014)

Analysis

Two Max-min Formation

The max-min design problem is to find delegation set D that solves

$$\max_{D} \inf_{u \in \mathscr{U}} \int_{\Theta} v(\tilde{a}(\theta; u, D), \theta) dF(\theta)$$

With $\mathscr{U}$ in our framework, we can prove it is equivalent to

$$\max_{D} \int_{\Theta} \inf_{u \in \mathscr{U}} [v(\tilde{a}(\theta; u, D), \theta)] dF(\theta)$$

Ex-post non trivial proof on measurability

Use the second one for analysis starting from now

Uncertainty in Behaviors

Uncertainty in preference $\rightarrow$ Uncertainty in how agent manipulate rules

For a D, define the *$D-$Admissible Set correspondence*, $A_D(\theta)$, the set of decisions selected by agents with preferences in $\mathscr{U}$ when the state is θ, i.e.,

$$A_D(\theta) \equiv \{a \in D : \exists u \in \mathscr{U}, \forall a' \in D, u(a,\theta) \geq u_A(a',\theta)\}.$$

Principal doesn't know how agent makes tradeoffs when constrained.

Principal's worst payoff in state θ when agent selects from D is

$$\underline{V}_D(\theta) \equiv \inf_{a \in A_D(\theta)} v(a,\theta).$$

Rewrite Design Problem

Principal's worst payoff in state θ when agent selects from D is

$$\underline{V}_D(\theta) \equiv \inf_{a \in A_D(\theta)} v(a, \theta).$$

Principal's optimal robust delegation problem can be written as

$$\max_{\text{compact } D \subset A} \int_\Theta \underline{V}_D(\theta) \mathrm{d}F. \tag{1}$$

Lemma (Existence)

The robust delegation problem (1) has a solution.

What are properties of optimal robust delegation?

Visible Set

Define $C(x, D)$ the set of visible points on D from point x

$$C(x, D) \equiv \{a \in D : [a, x] \cap D = a\}$$

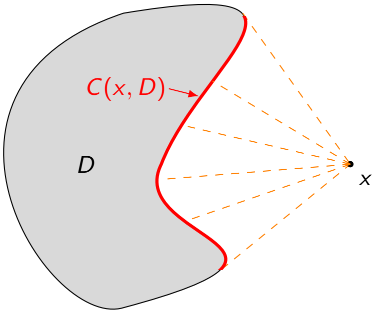
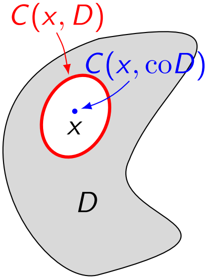


Figure: Illustration of visible set

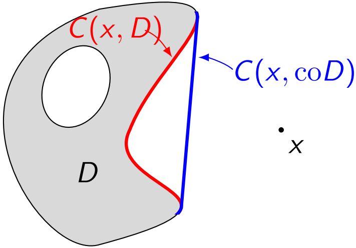
The Geometry of Visible Sets

Lemma

$C(x, \mathrm{co}D) \subseteq \mathrm{co}C(x, D)$.



(a) $x \in \mathrm{co}D \setminus D$



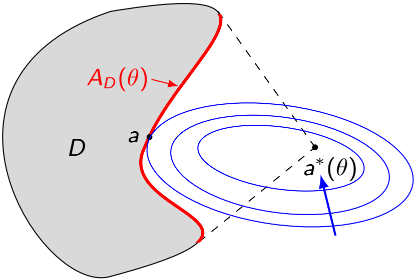
(b) $x \notin \mathrm{co}D$

The Geometry of Admissible Sets

Lemma

For any non-empty and compact $D \subset A$ and $\theta \in \Theta$ we have

$$\mathrm{cl}[A_D(\theta)] = \mathrm{cl}[\cup_{a \in A^*(\theta)} C(a, D)]$$



The Geometry of Admissible Sets

Combining two previous lemmas we get

Lemma

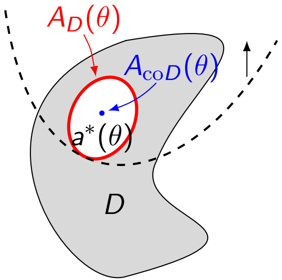
$\mathrm{cl}(A_{\mathrm{co}D}(\theta)) \subseteq \mathrm{co}[\mathrm{cl}(A_D(\theta))]$

$$
\begin{aligned}
\mathrm{cl}(A_{\mathrm{co}D}(\theta)) &= \mathrm{cl}[\cup_{a \in A^*(\theta)} C(a, \mathrm{co}D)] \\
&\subseteq \mathrm{cl}[\cup_{a \in A^*(\theta)} \mathrm{co}\, C(a, D)] \\
&\subseteq \mathrm{cl}[\mathrm{co}[\cup_{a \in A^*(\theta)} C(a, D)]] \\
&= \mathrm{co}[\mathrm{cl}(A_D(\theta))]
\end{aligned}
$$

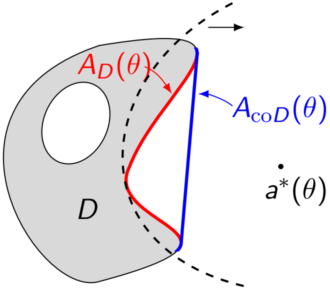
The Optimality of Convex Set

Proposition

Any delegation set D is weakly out-performed by its convex hull $\text{co}D$.



(a) $a^*(\theta) \in \text{co}D \setminus D$



(b) $a^*(\theta) \notin \text{co}D$

The Optimality of Convex Set

For any optimal set to be convex, suppose $a^*(\theta) = A^*(\theta)$, and denote $A^* = \{a^*(\Theta)\}$

Proposition

Suppose A^* is convex, $a^*(\theta) = A^*(\theta)$, $a^*(\theta)$ admits positive density in A^*, and the principal's payoff function $v(\cdot, \theta)$ is strictly quasi-concave for each θ. Then, any optimal delegation set is convex.

Counterexample if A^* is not convex: aligned agent

Properties of Optimal Convex Set I

A set is strictly convex if all boundary points are extreme points

Proposition

Suppose $A^* \subset \mathbb{R}^2$ is strictly convex, $a^*(\theta) = A^*(\theta)$, $a^*(\theta)$ admits positive density in A^*, and the principal's payoff function $v(\cdot, \theta)$ is strictly concave for each θ. Then, any optimal delegation set is strictly convex.

With large uncertainty about how agent makes trade-offs along different dimensions, one linear quota is not optimal

Properties of Optimal Convex Set II

There has to be sufficient discretion for delegation to be valuable

Lemma

Suppose that $a^*(\theta) = A^*(\theta)$, $a^*(\theta)$ admits positive density in A^*, and let the delegation set D be low-dimensional in the sense that $\dim(\mathit{aff}(D)) < \dim(\mathit{aff}(A^*))$. Then D is worse than no delegation.

all of D is visible to any a^* outside $\text{aff}(D)$

Examples

After establishing some general properties, more insights from examples

- One dimension: Interval Delegation (skip)
- Disk Example: Full delegation or No delegation
- Cube Example: Full delegation and High dimensionality

Simplest Delegation: Disk Example

The state space Θ is a unit disk $B(0,1)$ in $\mathbb{R}^2$

θ is distributed radially symmetrically with full support, meaning its density f satisfies: $f(\theta) = f(\theta') > 0$ if $|\theta| = |\theta'|$

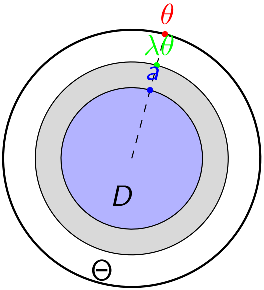
Agent wants to match the state: $a^*(\theta) = \theta$

Principal's most preferred action at state θ is $\lambda\theta$ for some $\lambda \in [0,1]$

$v(a,\theta)$ is given by $g(|a - \lambda\theta|)$, where $g : \mathbb{R}_+ \to \mathbb{R}$ is a concave, strictly decreasing loss function. For example, when $g(z) = -z^2$

Disk: Known Preference

Suppose the agent's preference is known and is given by $h(|a - \lambda\theta|)$



P and A are aligned in "direction", misaligned in "distance"

When $\lambda < 1$, Optimal delegation D is a disk with radius $r^S(\lambda)$

How Optimal Radius Changes

1) Delegation is tight with known preference $r^S(\lambda) \leq \lambda$
2) Optimal radius $r^S(\lambda)$ is strictly increasing

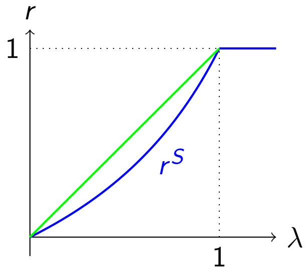


Figure: $F \sim U(B(0,1))$: $r^S = \frac{\lambda}{2-\lambda}$

How Optimal Radius Changes

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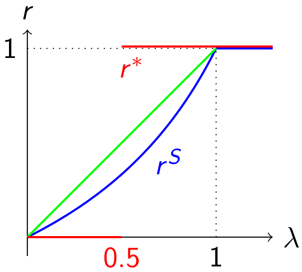
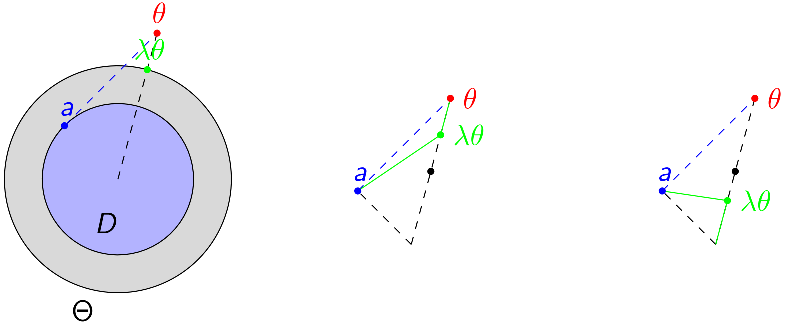


Figure: $F \sim U(B(0,1))$: $r^S = \frac{\lambda}{2-\lambda}$

Disk: Unknown Preference

What is the worst action?



Disk: Robust Design

The max-min solution is bang-bang:

Proposition

if $\lambda < \frac{1}{2}$, $D^* = \{0\}$ *is uniquely optimal;*

if $\lambda > \frac{1}{2}$, *full delegation* $D^* = A^*$ *is uniquely optimal;*

if $\lambda = \frac{1}{2}$, D^* *is optimal if and only if* $D^* = B(0, r)$ *for some* $r \in [0, 1]$.

P and A are superficially aligned in "direction", misaligned in "distance"

Constraining only "distance" is not robustly optimal

as it triggers unpredicted deviation in "direction"

Full Delegation: Cube Example

The state space Θ is the cube $[0,1]^{n_1+n_2}$, with density f

Every state happens with a minimum possibility: $\exists\gamma \in (0,1),\ f(\theta) \geq \gamma\ \forall\theta$

Agent wants to match the state: $a^*(\theta) = \theta$.

Principal's most preferred action $a_P(\theta)$ satisfies:

$$a_{iP}(\theta) = \theta_i = a_i^*(\theta), \quad \forall i = n_1+1, n_1+2, ..., n_1+n_2$$

Principal's utility $v(a,\theta) = -\|a - a_P(\theta)\|_q^q$

Full Delegation: Cube Example

n_1 dimension: conflicts of interests n_2 dimension: superficially aligned

Proposition

Full delegation is uniquely robust optimal if n_2 is sufficiently larger than n_1:

$$\frac{n_2}{n_1} > \frac{(1+q)2^q}{\gamma},$$

Imposing constraints on n_1 dimension triggers distortion on n_2 dimension

Principal refrains from micromanaging agent through restrictions on misaligned decisions when aligned decisions becomes more complex

Extensions I

Money is useless, even if we allow monetary incentives:

$$u(a, \theta, t) = u(a, \theta) + t, \quad u(a, \theta) \in \mathscr{U}$$

If monetary incentive is provided: in the worst case, agent just cares about money and maximizes transfer, which is worse than no delegation

Conclusion

This paper: the principal has limited information on agent's preference

- Doesn't know how agent trades off among sub-optimal options

The optimal delegation set is simple: no holes, convex, even full delegation

Our insight: delegation rules are simple because

- Uncertainty in how agents exploit/manipulate rules for private interest
- Sophisticated rules leave more space for manipulation

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