

Trade Fragmentation, Inflationary Pressures and Monetary Policy ^{*}

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Abstract

How does trade fragmentation affect inflationary pressures? What is the response of monetary policy needed to sustain inflation at target? To answer these questions, we develop a heterogeneous agent, open-economy model featuring imperfect international risk-sharing. The model captures both the demand and supply side effects of fragmentation. It illustrates how the impact of fragmentation on inflationary pressures and the appropriate policy response depend not only on the direct effect of higher import prices on supply but, crucially, on how aggregate demand adjusts in response to lower real incomes and productivity.

Keywords: Monetary policy, trade fragmentation, open economies, inflation, heterogeneity, globalisation

JEL classification: F12, F15, F41, F62

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1 Introduction

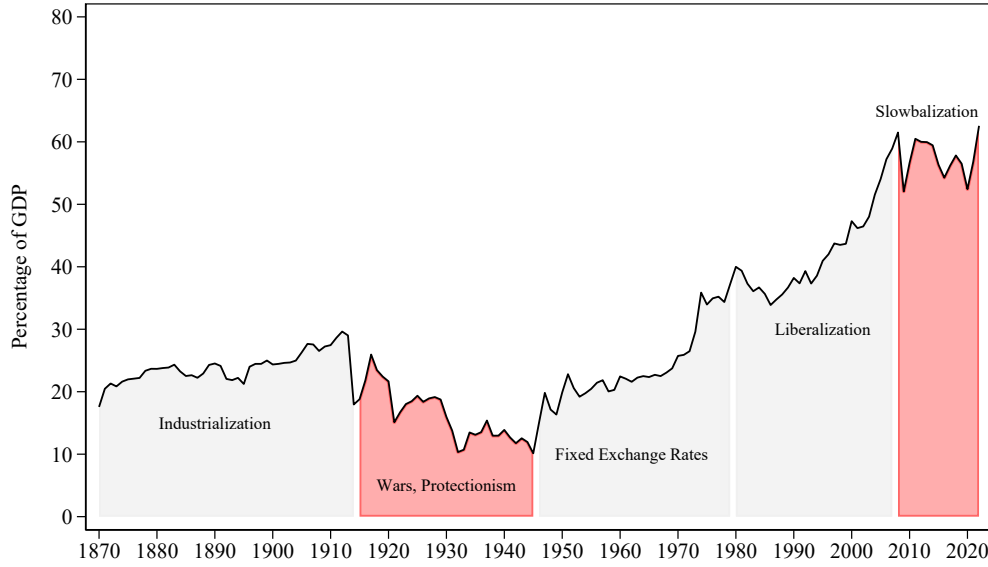
Global trends have shifted noticeably in recent decades. The protracted postwar increase in trade openness has stalled, amidst a resurgence in trade wars and protectionism. This shift is visible in Figure 1, which plots the long-term trajectory of global trade flows relative to world GDP, as well as in Figure 2, which tracks a broad index of economic fragmentation since the 1970s (Fernández-Villaverde, Mineyama, and Song, 2024). Both figures show a turning point around the global financial crisis, with trade openness plateauing and fragmentation steadily increasing, before rising sharply during the pandemic and Russia’s invasion of Ukraine. The geopolitical factors driving these changes are likely to persist. New trade paradigms, such as friendshoring or fragmentation into trading blocs of geopolitically aligned countries are becoming normalized (Yellen, 2022). This reconfiguration of trade patterns raises concerns about potential losses in efficiency and aggregate output (Javorcik, Kitzmueller, Schweiger, and Yildirim, 2022; Georgieva, 2023).

A key question for policymakers is how trade fragmentation will affect inflation dynamics and the optimal monetary policy response. The conventional view suggests that as nations retreat from global integration and supply chains duplicate, production costs will rise, exerting upward pressure on inflation (e.g., Lagarde (2023), Goodhart and Pradhan (2020)). The disinflationary trends of the 1990s and 2000s have often been linked the rapid increase in trade integration during that period, leading to expectations that its reversal will be inflationary. However, this relationship remains contentious. Other forces besides globalisation may have contributed to the era of disinflation, including advances in manufacturing (IMF (2006)), the shift to inflation-targeting regimes (Roberts (2006)), and the lower bound constraint on interest rates in many countries (Attinasi and Balatti (2021)). Taking the United States as an example, estimates of the disinflationary effects of increased trade integration appear modest at best (Yellen (2006)).¹

The conventional view is built around the direct or partial-equilibrium impact of trade integration on supply, abstracting from its indirect impact on aggregate demand. This paper addresses this gap by studying the broader, general equilibrium effects of trade fragmentation in a setting in which aggregate demand is also affected through changes in real incomes. Modelling fragmentation as an increase in the price of imported goods or, alternatively, as a fall in tradable sectors’ productivity, we illustrate how the inflationary impact of fragmentation depends crucially on the adjustment of aggregate demand. Higher import prices or lower productivity in the tradables sector not only constrain supply through higher marginal costs but also demand through lower real incomes and consumption - the general equilibrium effects. Consequently, the net impact on inflation is a priori ambiguous.

¹Kamin, Marazzi, and Schindler (2004), for example, show that the impact of Chinese exports on global prices has been fairly modest. Moreover, these studies do not explicitly take into account real exchange-rate adjustments. As argued by Kohn (2005), during the second half of the 1990s, the dollar experienced a substantial appreciation, driven by increased investment flows drawn to the prospect of higher productivity growth.

Figure 1: Sum of exports and imports,% of GDP



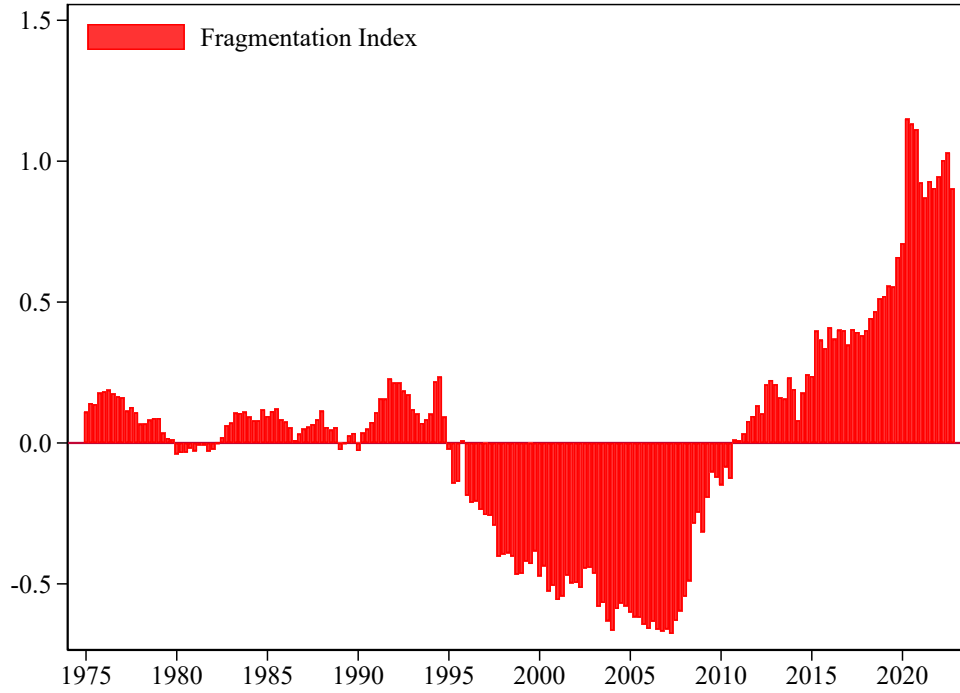
Source: Peterson Institute for International Economics; Jordà-Schularick-Taylor Macrohistory Database; Penn World Table (10.0); World Bank; OWID.

We capture these competing channels in a two-sector, open economy New Keynesian model featuring household heterogeneity and home bias in consumption. Specifically, following [Debortoli and Galí \(2017\)](#), the economy features two types of consumers. The first type consists of *unconstrained* agents, with access to international and domestic security markets. To account for frictions in international financial markets, we introduce imperfect international risk sharing: unconstrained agents trade risk-free foreign bonds, with a convex cost of holding assets in quantities that deviate from some long-run level ([Schmitt-Grohé and Uribe \(2003\)](#)). The second type consists of *constrained* hand-to-mouth households, who consume only out of their labour income and have no access to financial markets. The domestic economy trades with the rest of the world, importing goods for direct consumption, for use as intermediate inputs, or both.

We consider three scenarios to show different forms of fragmentation have distinct macroeconomic implications. First, we consider a *gradual* (and permanent) increase in the price of imported goods. This yields a persistent increase in imported inflation, which lasts until the import price stabilises at a higher level (in the medium-to-longer term). Aggregate consumption falls in response to fragmentation, as both financially constrained and unconstrained households suffer losses in real income: the real disposable income of hand-to-mouth consumers falls as a direct consequence of higher prices, restricting their spending; in turn, financially unconstrained households, who take into account their permanent-income losses also reduce their consumption in anticipation of lower future incomes. This further

This may have amplified the downward trend in dollar prices of U.S. imports.

Figure 2: Fragmentation has increased since 2008.



Source: Fernández-Villaverde, Mineyama, and Song (2024)

accentuates the fall in aggregate demand, spilling over to hand-to-mouth consumers. Real wages fall both because of the negative terms-of-trade effect and because of the fall in domestic demand. Financially constrained households make up for some of the fall in income by increasing their labour supply. The fall in aggregate demand pushes down on domestic inflation. Aggregate CPI (consumer price index) inflation, a composite of domestic and imported goods inflation, falls, given the larger weight of domestic components on the basket. The reduction in demand is reflected in the real natural rate of interest, which decreases with the fragmentation shock. This suggests when demand adjusts, the overall effect is not inflationary. The calibration leads to a long period of stagnation, with low demand and low inflationary pressures. In this setting, monetary policy needs to loosen in order to bring inflation back to target.

Next, we consider a fully *front-loaded*, permanent increase in the price of imported goods.² The shock creates a sharp temporary trade-off, with inflation increasing and aggregate demand falling on impact. Both financially unconstrained and constrained households lower their consumption. The fall in real wages (relative to the price of imported inputs) triggers an increase in labour supply from hand-to-mouth consumers. On impact, the short-term real interest rate increases, which requires a tightening in monetary policy to bring inflation

²This is akin to the recent U.S. and E.U. tariffs on Chinese electric vehicles, reaching up to 100 percent and 38 percent, respectively.

back to target. The result is a temporary overshoot in inflation, with longer-term losses in income and consumption.

Finally, we study a fall in the total factor productivity (TFP) of tradable goods as a potential consequence of increased fragmentation. The fall is persistent, but not permanent, as TFP reverses over time. Real wages fall, alongside the fall in TFP. While financially unconstrained consumers can smooth the impact on consumption, constrained hand-to-mouth households lower their consumption and increase their labour supply in response to lower disposable incomes. In principle, the impact of this shock on the natural real rate is ambiguous, in our calibration, the natural rate falls and demand and supply balance so that the shock is not inflationary.

In summary, all three fragmentation scenarios lead to a contraction in aggregate supply. However, they have different implications for the demand of goods and services. Conventional assessments of the impact of fragmentation on inflation often focus on its adverse supply-side effects, abstracting from the demand-side or general-equilibrium impact that fragmentation can have through lower real incomes. While the direct (or partial equilibrium) effect of fragmentation might be inflationary, the general equilibrium effect could dampen inflation, as lower real incomes weigh on aggregate demand. The effects of fragmentation on inflation dynamics and the direction of monetary policy cannot be decoupled from its impact on the natural real interest rate (r^*). As trade fragmentation affects the desired levels of savings and spending, the balance between these supply and demand forces ultimately determines the sign and size of changes in the natural rate of interest.

To sharpen our understanding of these dynamics, we vary two key parameters in our simulations: the share of hand-to-mouth agents and the degree of home bias in consumption. A higher share of hand-to-mouth households leads to a smaller fall in consumption on impact. This is because fewer forward-looking households anticipate the adjustment in consumption in response to the fall in their permanent income. The demand adjustment is still sufficient to lower domestic inflationary pressures and offset the increase in imported goods inflation. The extent of home bias in consumption seems to play a more important role. More open economies are more exposed to shocks in foreign prices, which is reflected in the responses of consumption and production. In the scenarios with a persistent increase in foreign prices, whether gradual or front-loaded, we see a deeper fall in the natural rate in the more open economy. This reverts in the case of negative TFP shock; this is because although the shock primarily affects the tradable sector, it is a direct shock to domestic production and consumption, affecting the consumption basket of the more closed economy to a greater extent (a more open economy can diversify away the domestic shock).

To build intuition, we analyse a representative-agent New Keynesian (RANK) version of our model as a special case of our TANK baseline model, where there are no constrained households. We also consider an extension of the RANK model with nominal wage rigidities. This friction introduces additional supply-side constraints, leading to a fall in output in both sectors, with implications for the composition of domestic inflationary pressure.

Related Literature We build on a rich literature studying monetary policy in small open economies (SOEs), including the seminal work of [Benigno and Benigno \(2003\)](#) and [Gali and Monacelli \(2005\)](#). Other important contributions to this line of research include but are not limited to, [Santacreu et al. \(2005\)](#) and [De Paoli \(2009\)](#), who study tradable and non-tradable sectors in SOEs, and [Schmitt-Grohé and Uribe \(2003\)](#), who introduce imperfect international risk sharing and price stability.

We also draw on an extensive literature that studies the impact of external shocks on macroeconomic outcomes using structural models, such as [Romero et al. \(2008\)](#), [Catão and Chang \(2013\)](#), [Hevia and Nicolini \(2013\)](#), [Bergholt \(2014\)](#), [Ferrero and Seneca \(2019\)](#), [Wills \(2013\)](#), [Drechsel, McLeay, and Tenreyro \(2019\)](#), [Broadbent, Di Pace, Drechsel, Harrison, and Tenreyro \(2023\)](#), and [Guerrieri, Marcussen, Reichlin, and Tenreyro \(2024\)](#). Recent contributions to this literature have explored the transmission of external shocks in models with household heterogeneity. [Auclert, Rognlie, Souchier, and Straub \(2021\)](#) study the real income channel of exchange rate depreciations, highlighting the importance of trade elasticities for the amplification of this channel in a HANK model. The macroeconomic impact of external shocks depends on their effect on relative factor prices. In this regard, this paper builds on research using structural heterogeneous agent models to understand the effects of the recent energy price shock. In models where labor and imported energy as complementary inputs in production and consumption, [Auclert, Monnery, Rognlie, and Straub \(2023\)](#) and [Chan, Diz, and Kanngiesser \(2024\)](#) show how the demand-side effects of this shock depend on how it redistributes economic resources.³

Finally, we build on the vast literature that has examined the macroeconomic effects of globalisation. While increased competition in import prices has placed downward pressure on prices of manufactured goods, studies have shown that globalisation has had a negative, but economically small (if not negligible) effect on core inflation ([Carluccio, Gautier, and Guilloux-Nefussi \(2023\)](#)). Moreover, there is evidence that global disinflationary forces, such as the shift to inflation targeting regimes ([European Central Bank \(2021\)](#), [Roberts \(2006\)](#), [Attinasi and Balatti \(2021\)](#)) or technological advances in manufacturing ([IMF \(2006\)](#)) may offer a better explanation for the observed disinflationary trends. Theoretical results provide support for these findings. [Sbordone \(2008\)](#) shows that in a model in which firms' desired markup is a function of its relative market share, an increase in the number of traded goods can generate real rigidities that affect the slope of the Philips curve. As the economy reaches a steady state with higher trade, the elasticity of demand that firms face increases, but the elasticity of the desired markup declines. These opposing forces determine how the inflation-marginal cost component of the Phillips curve slope varies. Estimates on trade data from 1960 to 2006 suggest that it remains uncertain whether trade growth observed during the globalisation era is sufficient to have driven a decline in this component of the

³The process of fragmentation, like the green transition, will likely entail a gradual adjustment of relative prices. [Del Negro, Di Giovanni, and Dogra \(2023\)](#) show that climate policies are not necessarily inflationary, as this depends on the relative degree of price stickiness in green and brown sectors.

slope. On the empirical front, [Chen, Imbs, and Scott \(2009\)](#) provide evidence of short-run pro-competitive effects from increased openness. They also show that trade liberalization can have ambiguous effects in the long run, as firms can respond to increased competition by locating to protected markets.

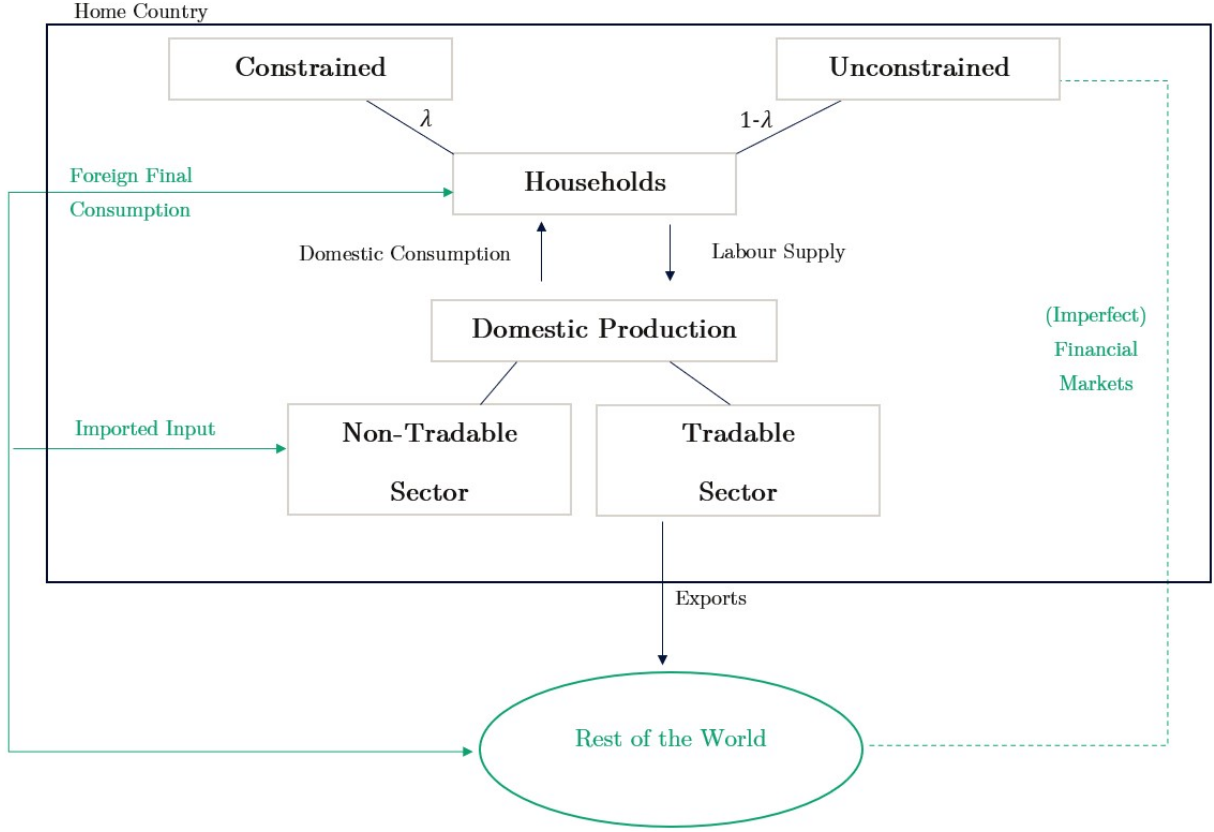
Our paper also relates to the strand of literature pioneered by [Rogoff et al. \(2003\)](#) and [Rogoff et al. \(2006\)](#), which looks at how economic integration affects global inflationary trends. We abstract, however, from the political economy factors studied by [Afrouzi, Halac, Rogoff, and Yared \(2024\)](#), who suggest that globalisation would worsen the trade-offs faced by central banks, leading them to succumb to political pressures and deviate from or abandon their inflation targets. The question we ask in this paper is a different one: what would it take for central banks to bring inflation back to target under different fragmentation scenarios? As we show, in some scenarios, activity and inflation both fall, leading to stagnation (that is, without a trade-off); in others, activity and inflation move in opposite directions, creating short-term trade-offs or temporary stagflation. What is required of monetary policy to return inflation to target depends on how aggregate demand responds to lower incomes in general equilibrium. This is contingent on a number of structural parameters that we consider, as well as on the trajectory of fragmentation, particularly on the extent to which the impact on import prices is gradual or front-loaded.

Outline Section 2 develops our theoretical framework. Section 3 calibrates the model and analyses shocks that are linked to trade fragmentation. Next, it studies the relative importance of heterogeneity and home bias in the policy response to trade fragmentation. Section 4 studies an extension with nominal wage rigidity. Section 5 presents concluding remarks and potential directions for future research.

2 Baseline Model

The goal of this section is to deliver qualitative insights into shocks associated with de-globalisation. We present a small open economy model that builds on [Drechsel, McLeay, and Tenreyro \(2019\)](#) and [Ferrero and Seneca \(2019\)](#). To capture a more realistic response of aggregate demand to international shocks, we introduce constrained and unconstrained households as in [Debortoli and Galí \(2017\)](#). Finally, to study the impact of fragmentation, we introduce an imported input used in the production of domestic goods.

Figure 3: Model Structure from Home Country Perspective.



2.1 Households

There is a continuum of households with identical preferences at any given point in time t . They consume C_t and supply labour N_t , at wage W_t , leading to an expected utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - x_\ell \frac{N_t^{1+\phi}}{1+\phi} \right\}.$$

The parameters β , σ , and ϕ capture the discount factor, the inverse intertemporal elasticity of substitution and the inverse Frisch elasticity, respectively. x_ℓ is the disutility weight put on labour, which we set equal to one in the TANK case. Figure 3 presents an illustration of the model described in this section.

Unconstrained households A constant measure $(1-\lambda)$ of households are *unconstrained* (U). These households are able to smooth consumption through their access to international and domestic financial markets. Their period budget constraint is given by:

$$P_t C_t^U + B_t + \mathcal{E}_t B_t^* = B_{t-1}(1 + i_{t-1}) + \mathcal{E}_t B_{t-1}^*(1 + i_{t-1}^*) + W_t N_t^U - \frac{\chi}{2} \mathcal{E}_t P_t^* \left(\frac{B_t^*}{P_t^*} - \bar{b}^* \right)^2 \quad (1)$$

where B_t denotes the holdings of a risk-free one-period nominal bond in domestic currency, which pays the nominal interest rate i_t . B_t^* is the risk-free one-period nominal bond in for-

foreign currency, where i_t^* is the foreign interest rate. \mathcal{E}_t is the nominal exchange rate (expressed in terms of domestic currency relative to foreign currency). Following [Schmitt-Grohé and Uribe \(2003\)](#), we assume that there is a quadratic cost in changing the real bond position relative to a real steady-state value (\bar{b}^*) when trading in the foreign bond market. These costs are a common feature of small open economy models to ensure that the model returns to a unique steady-state net foreign asset position following a transitory shock. χ is a non-negative parameter that measures this cost in terms of units of the consumption index. For simplicity, we can rewrite the budget constraint in real terms as

$$C_t^U + \frac{B_t}{P_t} + \mathcal{S}_t \frac{B_t^*}{P_t^*} = \frac{B_{t-1}}{P_{t-1}} \frac{(1 + i_{t-1})}{(1 + \pi_t)} + \mathcal{S}_t \frac{B_{t-1}^*}{P_{t-1}^*} \frac{(1 + i_{t-1}^*)}{(1 + \pi_t^*)} + \frac{W_t}{P_t} N_t^U - \frac{\chi}{2} \mathcal{S}_t \left(\frac{B_t^*}{P_t^*} - \bar{b}^* \right)^2$$

or, introducing real variables, as;

$$C_t^U + b_t + \mathcal{S}_t b_t^* = b_{t-1} \frac{(1 + i_{t-1})}{(1 + \pi_t)} + \mathcal{S}_t b_{t-1}^* \frac{(1 + i_{t-1}^*)}{(1 + \pi_t^*)} + w_t N_t^U - \frac{\chi}{2} \mathcal{S}_t (b_t^* - \bar{b}^*)^2 \quad (2)$$

where $b_t = \frac{B_t}{P_t}$, $b_t^* = \frac{B_t^*}{P_t^*}$, $w_t = \frac{W_t}{P_t}$, and $\mathcal{S}_t = \frac{\mathcal{E}_t P_t^*}{P_t}$ is the real exchange rate.

Unconstrained households maximise their expected lifetime utility by choosing a sequence $\{C_t^U, N_t^U, b_t, b_t^*\}_{t=0}^\infty$ subject to the sequence of budget constraints (2). For unconstrained households, the first-order conditions with respect to C_t^U, N_t^U, b_t, b_t^* are respectively given by:

$$\begin{aligned} (C_t^U)^{-\sigma} &= \delta_t \\ (N_t^U)^\phi &= \delta_t \frac{W_t}{P_t} \\ \delta_t &= \beta \mathbb{E}_t \left[\frac{(1 + i_t)}{(1 + \pi_{t+1})} \delta_{t+1} \right] \\ \delta_t [\mathcal{S}_t + \mathcal{S}_t \chi (b_t^* - \bar{b}^*)] &= \beta \mathbb{E}_t \left[\frac{(1 + i_t^*)}{(1 + \pi_{t+1}^*)} \mathcal{S}_{t+1} \delta_{t+1} \right] \end{aligned}$$

where δ_t is the Lagrange multiplier on the budget constraint. Consequently, the optimality conditions are as follows,

$$(N_t^U)^\phi = (C_t^U)^{-\sigma} \frac{W_t}{P_t} \quad (3)$$

$$\frac{1}{(1 + i_t)} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1}{(1 + \pi_{t+1})} \right] \quad (4)$$

$$[1 + \chi (b_t^* - \bar{b}^*)] = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1 + i_t^*}{(1 + \pi_{t+1}^*)} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} \right] \quad (5)$$

where $\Pi_{t+1} = (1 + \pi_{t+1}) = \frac{P_{t+1}}{P_t}$. We define $\Lambda_{t,t+1}^U = \beta \left(\frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma}$ as the relevant stochastic discount factor, since only the unconstrained households have access to bonds. The household's optimality condition for labour yields the labour supply relation (3). The first order

condition for b_t implies the Euler equation (4). Finally, households' choices of foreign and domestic bonds give rise to an uncovered interest rate parity condition which links the expected change in exchange rate to the differential between the domestic and foreign interest rate. The conditions on b_t and b_t^* imply equation (5), the deviation from the uncovered interest-rate parity (UIP). According to equation (5), consumption risk will *not* necessarily be shared internationally at all times, as long as $b_t^* \neq \bar{b}^*$. This is due to convex costs associated with adjusting foreign bond holdings, which reflects frictions in international financial markets. While in steady state $b_t^* = \bar{b}^*$, outside of steady state $\mathcal{D}_t = [1 + \chi(b_t^* - \bar{b}^*)]$ can fluctuate inefficiently, contingent on shocks.

$$\chi(b_t^* - \bar{b}^*) = \mathbb{E}_t \left[\Lambda_{t,t+1}^U \left(\frac{(1 + i_t^*)}{(1 + \pi_{t+1}^*)} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} - \frac{(1 + i_t)}{(1 + \pi_{t+1})} \right) \right]$$

Constrained households The remaining fraction (λ) of households are fully *constrained* (C). These households do not have access to financial markets and therefore they cannot smooth their consumption over time. Instead, they consume their labour income and transfers each period:

$$P_t C_t^C = W_t N_t^C$$

$$C_t^C = \frac{W_t}{P_t} N_t^C \quad (6)$$

Aggregate consumption is a weighted average of the consumption of the two types of households, $C_t = (1 - \lambda)C_t^U + \lambda C_t^C$. Similarly, aggregate labour is $N_t = (1 - \lambda)N_t^U + \lambda N_t^C$. Finally, we define the heterogeneity index as the ratio of unconstrained consumption to aggregate consumption,

$$\Gamma_t \equiv \frac{C_t^U}{C_t}. \quad (7)$$

Total consumption is a CES aggregate of domestic and foreign goods:

$$C_t \equiv \left[(1 - \alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $1 - \alpha$ captures the home bias: smaller values of α imply that the economy consumes less foreign goods. As in [Santacreu et al. \(2005\)](#), domestic production includes both *tradable* (T) and *non-tradable* (N) goods. Therefore, $C_{H,t}$ is a CES aggregate of consumption goods produced in the domestic economy,

$$C_{H,t} = \left[(1 - \gamma)^{\frac{1}{\nu}} C_{N,t}^{\frac{\nu-1}{\nu}} + \gamma^{\frac{1}{\nu}} C_{T,t}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \quad (8)$$

where γ is the share of tradable goods in domestic consumption. Tradables and nontradables are given, respectively, by:

$$C_{T,t} \equiv \left(\int_0^1 C_{T,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$C_{N,t} \equiv \left(\int_0^1 C_{N,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

where ϵ is the elasticity of substitution across varieties. We define $C_{F,t}$ and the consumption variables in the foreign economy in an analogous way. The aggregate CPI price level, P_t and the domestic price level, $P_{H,t}$ are respectively given by:

$$P_t \equiv \left[(1 - \alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$P_{H,t} \equiv \left[(1 - \gamma) P_{N,t}^{1-\nu} + \gamma P_{T,t}^{1-\nu} \right]^{\frac{1}{1-\nu}}$$

Aggregate prices depend on domestic prices and foreign prices according to the home bias of the country. In turn, domestic prices depend on traded and non-traded goods prices. The price index for non-tradable goods is given by

$$P_{N,t} \equiv \left(\int_0^1 P_{N,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

The price index for tradable goods and the price variables in the foreign economy are defined in an analogous way.

Total consumption expenditure by households is given by the sum of the expenditures on domestic and foreign goods they consume:

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_{T,t} C_{T,t} + P_{N,t} C_{N,t} + P_{F,t} C_{F,t}$$

The terms of trade are defined as the price of imports in terms of the price of domestic goods.

$$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}} \quad (9)$$

We assume that the law of one price holds for individual goods at all times,

$$\mathcal{S}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t} = \mathcal{T}_t^{1-\alpha}$$

where \mathcal{S}_t is the real exchange rate.

The system of demand functions is hence given by

$$\begin{aligned}
C_{H,t} &= (1 - \alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \\
C_{F,t} &= \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \\
C_{N,t} &= (1 - \gamma) \left(\frac{P_{N,t}}{P_{H,t}} \right)^{-\nu} C_{H,t} \\
C_{T,t} &= \gamma \left(\frac{P_{T,t}}{P_{H,t}} \right)^{-\nu} C_{H,t} \\
C_{N,t}(i) &= \left(\frac{P_{N,t}(i)}{P_{N,t}} \right)^{-\epsilon} C_{N,t}.
\end{aligned}$$

2.2 Firms

Households supply labour to both the tradable and non-tradable sectors, such that

$$N_t = N_{T,t} + N_{N,t} = N_t^C \lambda + N_t^U (1 - \lambda). \quad (10)$$

Labour is completely mobile across sectors, therefore there is only one wage rate in equilibrium. In this version of the model, we do not allow for redistribution of firms' profits to households or across households. Indeed, in the baseline calibration, all real profits Ψ_t are assumed to accrue to foreign households who consume abroad.⁴

2.2.1 Non-tradable goods sector

Final Goods Producers Competitive final goods producers assemble intermediate goods $Y_{N,t}(i)$, where $P_{N,t}(i)$ is the price charged by the individual firm i . Their optimisation problem,

$$\max_{Y_{N,t}(i)} \int_0^1 P_{N,t}(i) Y_{N,t},$$

is subject to an aggregation technology with constant elasticity of substitution,

$$Y_{N,t} \equiv \left(\int_0^1 Y_{N,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $P_{N,t}(i)$ is the price charged by the individual firm i . This results in a downward-sloping demand function for firm i 's product,

$$Y_{N,t}(i) = \left[\frac{P_{N,t}(i)}{P_{N,t}} \right]^{-\epsilon} Y_{N,t} \quad (11)$$

⁴Including profits in the unconstrained agent's budget constraint or allowing for redistribution does not change our qualitative results. Our current assumption is consistent with foreigners owning the companies located in the home country and consuming all proceeds abroad.

Intermediate Goods Producers Intermediate goods firms use labour and an *intermediate imported input* $M_{F,t}$ in production,

$$Y_{N,t}(i) = A_{N,t} M_{F,t}^\kappa(i) N_{N,t}^{1-\kappa}(i) \quad (12)$$

where $A_{N,t} = (A_{N,t}^{ss})^{1-\rho_n} A_{N,t-1}^{\rho_n} \epsilon_{N,t}$, and $\rho_n \in (0, 1]$. $M_{F,t}$ is the foreign country's final good that captures intermediate input utilisation in the production function.

Firms are monopolistically competitive and adjust prices according to [Rotemberg \(1982\)](#), incurring an adjustment cost each time,

$$AC_t(i) = \frac{\xi}{2} \left(\frac{P_{N,t}(i)}{P_{N,t-1}(i)} - \bar{\Pi} \right)^2 Y_{N,t} P_{N,t},$$

where ξ summarises the degree of nominal rigidity in the economy. Let $\bar{\Pi}$ denote steady state inflation. Firm i in the non-tradable sector maximises profits

$$\max_{M_{F,t}(i), N_{N,t}(i), P_{N,t}(i)} P_{N,t}(i) Y_{N,t}(i) - P_{F,t} M_{F,t}(i) - W_t N_{N,t}(i) - AC_t(i)$$

subject to its production technology (12) and the demand function for its product (11). This problem yields the following first-order conditions,

$$W_t = MC_t(i) (1 - \kappa) \frac{Y_{N,t}(i)}{N_{N,t}(i)}, \quad (13)$$

$$P_{F,t} = MC_t(i) \kappa \frac{Y_{N,t}(i)}{M_{F,t}(i)}, \quad (14)$$

where $MC_t(i)$ is the Lagrange multiplier on the technology constraint. We can interpret this multiplier as the shadow cost of producing an additional unit of good $Y_{N,t}(i)$, that is, the marginal cost. Substituting equations (13) and (14) into the production function we obtain demand functions for the two production inputs,

$$N_{N,t}(i) = \frac{Y_{N,t}(i)}{A_{N,t}} \left[\frac{1 - \kappa}{\kappa} \frac{P_{F,t}}{W_t} \right]^\kappa, \quad M_{F,t}(i) = \frac{Y_{N,t}(i)}{A_{N,t}} \left[\frac{\kappa}{1 - \kappa} \frac{W_t}{P_{F,t}} \right]^{1-\kappa}$$

The total cost function is equal to:

$$\begin{aligned} TC_t(i) (W_t, Y_{N,t}(i), P_{F,t}, A_{N,t}) &= W_t N_{N,t} + P_{F,t} M_{F,t} \\ &= W_t \frac{Y_{N,t}(i)}{A_{N,t}} \left[\frac{1 - \kappa}{\kappa} \frac{P_{F,t}}{W_t} \right]^\kappa + P_{F,t} \frac{Y_{N,t}(i)}{A_{N,t}} \left[\frac{\kappa}{1 - \kappa} \frac{W_t}{P_{F,t}} \right]^{1-\kappa} \\ &= \frac{Y_{N,t}(i)}{A_t} W_t^{1-\kappa} P_{F,t}^\kappa \left[\left(\frac{\kappa}{(1 - \kappa)} \right)^{1-\kappa} + \left(\frac{(1 - \kappa)}{\kappa} \right)^\kappa \right] \end{aligned} \quad (15)$$

Differentiation of the total cost function yields marginal cost,

$$MC_{N,t}(i) = \frac{W_t^{1-\kappa} P_{F,t}^\kappa}{A_t} \left[\left(\frac{\kappa}{(1 - \kappa)} \right)^{1-\kappa} + \left(\frac{(1 - \kappa)}{\kappa} \right)^\kappa \right]. \quad (16)$$

Since equation (16) is solely a function of factor prices and productivity, $MC_{N,t}(i) = MC_{N,t} \forall i$. We can rewrite nominal marginal cost in real terms as follows,

$$\begin{aligned} MC_{N,t}^r(i) &= \frac{MC_{N,t}}{P_{N,t}} = \frac{1}{A_t} \frac{P_t}{P_t} \frac{1}{P_{N,t}} W_t^{1-\kappa} P_{F,t}^\kappa \left[\left(\frac{\kappa}{(1-\kappa)} \right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa} \right)^\kappa \right] \\ &= \frac{1}{A_t} \frac{P_t}{P_{N,t}} \left(\frac{W_t}{P_t} \right)^{1-\kappa} \left(\frac{P_{F,t}}{P_t} \right)^\kappa \left[\left(\frac{\kappa}{(1-\kappa)} \right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa} \right)^\kappa \right] \end{aligned} \quad (17)$$

Taking the first order condition with respect to $P_{N,t}(i)$, we obtain the Phillips Curve for non-tradable goods:

$$\Pi_{N,t} (\Pi_{N,t} - \bar{\Pi}) = \beta \mathbb{E}_t \left[\Lambda_{t,t+1}^U \Pi_{N,t+1} (\Pi_{N,t+1} - \bar{\Pi}) \frac{Y_{N,t+1}}{Y_{N,t}} \right] + \frac{\epsilon}{\xi} \left(MC_t - \frac{\epsilon - 1}{\epsilon} \right), \quad (18)$$

where $\bar{\pi}$ is the steady state level of inflation. Using the demand relation and the labour market clearing condition $N_{N,t} = \int_0^1 N_{N,t}(i) di$, we can write the aggregate production function as

$$Y_{N,t} \Delta_t = A_{N,t} M_{F,t}^\kappa N_{N,t}^{1-\kappa},$$

where $\Delta_t = \left(1 - \frac{\xi}{2} (\Pi_N - \bar{\Pi})^2 \right)$ is the index of price dispersion.

2.2.2 Tradable goods sector

The tradable sector is internationally competitive, therefore domestic firms in this sector take the prices as given at $P_{T,t} = \mathcal{E}_t P_{T,t}^*$. We assume that foreign price dynamics ($P_{T,t}^*$) are driven by developments in world markets and therefore exogenous for a small open economy. Tradable sector production is given by

$$Y_{T,t} = A_{T,t} N_{T,t}^{1-\zeta}. \quad (19)$$

Therefore, the problem of the tradable firm is to maximise profits

$$\max_{N_{T,t}} P_{T,t} Y_{T,t} - W_t N_{T,t}$$

subject to its production technology (19). This yields

$$\frac{W_t}{P_{T,t}} = (1 - \zeta) A_{T,t} N_{T,t}^{-\zeta} \implies W_t N_{T,t} = (1 - \zeta) Y_{T,t} P_{T,t} \quad (20)$$

which gives us profits $\Psi_{T,t} = (\zeta) P_{T,t} Y_{T,t}$. Finally, similarly to $P_{F,t}^*$, the foreign price index for the tradable sector evolves according to

$$P_{T,t}^* = (P_{T,ss}^*)^{1-\rho_T} (P_{T,t-1}^*)^{\rho_T} \epsilon_{T,t} \quad (21)$$

with $\rho_T \in (0, 1]$.

2.3 Monetary Policy

The monetary authority sets the interest rate according to the following Taylor rule:

$$\frac{R_t}{R_{ss}} = \left(\frac{\Pi}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{Y}{\bar{Y}} \right)^{\phi_y} \exp(\nu_m) \quad (22)$$

where $\nu_m \sim N(0, \sigma_m^2)$ is a monetary policy shock, which follows an AR(1) process, and \bar{Y} is the steady state level of output.

2.4 Equilibrium

Given the tradable and foreign imported prices $P_{T,t}^*, P_{F,t}^*$, the monetary policy rule determining i_t , foreign output, inflation and interest rates Y_t^*, Π_t^*, i_t^* , and an initial condition on price dispersion, equilibrium in the economy is given by a sequence of quantities $\{C_{H,t}, C_{T,t}, C_{N,t}, C_{F,t}, C_t, C_t^U, C_t^C, N_t, B_{t+1}^H, B_{t+1}^*, Y_{N,t}, Y_{T,t}, M_{F,t}, \Psi_t\}_{t=0}^\infty$ and prices $\{\Lambda_{t,t+1}, \Pi_{H,t}, \Pi_{N,t}, \Pi_{T,t}, \Pi_{F,t}, \Pi_t, W_t, \mathcal{T}_t, \mathcal{S}_t, \mathcal{E}_t, \Delta_t\}_{t=0}^\infty$ such that firms and households maximise their objectives, and the goods, labour and financial markets clear, we obtain:

$$\begin{aligned} Y_t &= C_{T,t} + C_{T,t}^* + C_{N,t} \\ Y_{T,t} &= C_{T,t} + C_{T,t}^* \\ C_{N,t} &= Y_{N,t} \\ B_t &= 0 \\ N_t &= N_{T,t} + N_{N,t} = N_t^C \lambda + N_t^U (1 - \lambda) \\ Y_t^* &= C_t^* \end{aligned}$$

We derive demand functions for $Y_{T,t}$ and $Y_{N,t}$ in order to obtain the market clearing conditions. Tradables are by definition consumed both domestically and in the rest of the world,

$$\begin{aligned} C_{T,t} &= \gamma \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu C_{H,t} = \gamma \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu (1 - \alpha) \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t = \gamma(1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t \\ C_{T,t}^* &= \gamma \alpha \left(\frac{P_t}{P_{T,t}} \right)^\eta \mathcal{S}_t C_t^* \\ Y_{T,t} &= \gamma(1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t + \gamma \alpha \left(\frac{P_t}{P_{T,t}} \right)^\eta \mathcal{S}_t C_t^* \end{aligned}$$

where $C_{T,t}^*$, the foreign tradable demand, is derived by assuming symmetric preferences in the rest of the world. For non-tradables, instead, we calculate domestic demand as follows:

$$\begin{aligned} C_{N,t} &= (1 - \gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu C_{H,t} \\ C_{N,t} &= (1 - \gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu (1 - \alpha) \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t \\ C_{N,t} &= (1 - \alpha)(1 - \gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t \end{aligned}$$

Therefore, we can express domestic aggregate output as follows,

$$Y_t = (1 - \alpha)(1 - \gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t + \gamma(1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t + \gamma\alpha \left(\frac{P_t}{P_{T,t}} \right)^\eta S_t C_t^*$$

2.4.1 Natural Level of Output and Interest Rate

To understand the nature of our shocks and to determine the appropriate real policy interest rate, we calculate the natural level of output (Y_t^n) and the natural real interest rate (r_t^n).

Y_t^n is the level of output that would arise under flexible prices. To derive it, we need to determine the profit-maximizing flexible price for the domestic non-tradable good firms - the only sector facing nominal rigidities. Profit-maximising firms set the flexible optimal price in order to equalise the marginal cost to the marginal revenue. This is equivalent to setting the real marginal cost to the inverse of the desired markup:

$$MC_{N,t} = \frac{\epsilon - 1}{\epsilon}$$

Using the real marginal cost in equation (17), we obtain

$$\frac{\epsilon - 1}{\epsilon} = \frac{1}{A_{N,t}} \frac{P_t}{P_{N,t}} \left(\frac{W_t}{P_t} \right)^{1-\kappa} \left(\frac{P_{F,t}}{P_t} \right)^\kappa \left[\left(\frac{\kappa}{(1-\kappa)} \right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa} \right)^\kappa \right] \quad (23)$$

Without loss of generality and assuming $C^U = C^C = C$ in steady state, we can write the labour supply condition as follows:⁵

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\phi$$

Therefore,

$$\frac{\epsilon - 1}{\epsilon} = \frac{1}{A_{N,t}} \frac{P_t}{P_{N,t}} \left(C_t^\sigma N_t^\phi \right)^{1-\kappa} \left(\frac{P_{F,t}}{P_t} \right)^\kappa \left[\left(\frac{\kappa}{(1-\kappa)} \right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa} \right)^\kappa \right]. \quad (24)$$

$$= \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{(1 + \pi_{t+1})} \right], \quad (25)$$

$$\frac{1}{(1 + i_t^*)} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{(1 + \pi_{t+1}^*)} \right]. \quad (26)$$

If $\pi = \pi^* = 1$ in steady state, then $r = r^* = \frac{1}{\beta}$. We can therefore write:

$$[1 + \chi(b_t^* - \bar{b}^*)] = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{C_t^*}{C_{t+1}^*} \right)^{-\sigma} \frac{1}{\beta} \frac{S_{t+1}}{S_t} \right]$$

$$[1 + \chi(b_t^* - \bar{b}^*)] = \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{C_t^*}{C_{t+1}^*} \right)^{-\sigma} \frac{S_{t+1}}{S_t} \right]$$

$$[1 + \chi(b_t^* - \bar{b}^*)] = \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^\sigma \left(\frac{C_{t+1}^*}{C_t^*} \right)^\sigma \frac{S_{t+1}}{S_t} \right]$$

⁵Galí, López-Salido, and Vallés (2007).

$$\mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^\sigma \right] = \mathbb{E}_t \left[\left(\frac{C_t^*}{C_{t+1}^*} \right)^\sigma \frac{\mathcal{S}_t}{\mathcal{S}_{t+1}} \right] [1 + \chi(b_t^* - \bar{b}^*)], \quad (27)$$

which is the resulting international risk-sharing condition. Note that in steady state, we are back to the condition that would arise in a perfect risk-sharing scenario with symmetric countries. Moreover,

$$\begin{aligned} \mathcal{D}_t^{\frac{1}{\sigma}} &= \frac{C_t}{C_t^* \mathcal{S}_t} & \mathcal{D}_t^{\frac{1}{\sigma}} &= (1 + \chi(b_t^* - \bar{b})) \neq 1, \quad D = 1 \\ C_t &= \mathcal{D}_t^{\frac{1}{\sigma}} C_t^* \mathcal{S}_t \\ \frac{C_t}{\mathcal{D}_t^{\frac{1}{\sigma}}} &= C_t^* \mathcal{S}_t \end{aligned}$$

Using the goods market clearing equation, and the assumption that international markets are incomplete, we can write:

$$\begin{aligned} Y_t &= (1 - \alpha)(1 - \gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t + \gamma(1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta C_t + \gamma\alpha \left(\frac{P_t}{P_{T,t}} \right)^\eta \mathcal{S}_t C_t^* \\ Y_t &= \left[(1 - \alpha)(1 - \gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta + \gamma(1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta + \frac{\gamma\alpha}{\mathcal{D}_t^{\frac{1}{\sigma}}} \left(\frac{P_t}{P_{T,t}} \right)^\eta \right] C_t \\ C_t &= \frac{Y_t}{\Sigma_{\alpha\gamma,t}} \end{aligned}$$

where $\Sigma_{\alpha\gamma,t} = \left[(1 - \alpha)(1 - \gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta + \gamma(1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta + \frac{\gamma\alpha}{\mathcal{D}_t^{\frac{1}{\sigma}}} \left(\frac{P_t}{P_{T,t}} \right)^\eta \right]$. The relationship between domestic consumption and aggregate domestic production is described by $\Sigma_{\alpha\gamma,t}$. In an open economy, these two variables do not move in lockstep; instead, their dynamics are influenced by the degree of openness to trade. A higher α widens the wedge between consumption and production, attributable to the increased share of tradable goods destined for export.

In the absence of distortions, we obtain N_t by aggregating labour in each sector:

$$N_{T,t} = \left(\frac{Y_{T,t}}{A_{T,t}} \right)^{\frac{1}{1-\zeta}} \quad (28)$$

$$N_{N,t} = Y_{N,t} \left(\frac{P_{F,t}}{P_{N,t}} \right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \quad (29)$$

Since $N_t = N_{N,t} + N_{T,t}$, we can write:

$$\begin{aligned} N_t &= \left(\frac{Y_{T,t}}{A_{T,t}} \right)^{\frac{1}{1-\zeta}} + Y_{N,t} \left(\frac{P_{F,t}}{P_{N,t}} \right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \\ &= A_{T,t}^{-\frac{1}{1-\zeta}} \left(\left(\gamma(1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta \mathcal{D}_t^{\frac{1}{\sigma}} + \gamma\alpha \left(\frac{P_t}{P_{T,t}} \right)^\eta \right) \mathcal{S}_t C_t^* \right)^{\frac{1}{1-\zeta}} \\ &\quad + (1-\alpha)(1-\gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta \mathcal{D}_t^{\frac{1}{\sigma}} \mathcal{S}_t C_t^* \left(\frac{p_{F,t}}{p_{N,t}} \right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \end{aligned}$$

Substituting back in equation (24), we obtain

$$\begin{aligned} \frac{\epsilon - 1}{\epsilon} \frac{A_{N,t} p_{N,t}}{\Gamma} p_{F,t}^{-\kappa} &= \left(C_t^\sigma N_t^\phi \right)^{1-\kappa} \\ C_t^\sigma N_t^\phi &= \left(\frac{\epsilon - 1}{\epsilon} \frac{A_{N,t} p_{N,t}}{\Gamma} p_{F,t}^{-\kappa} \right)^{\frac{1}{1-\kappa}} \\ Y_t^n &= \Sigma_{\alpha\gamma,t} \left[\left(\frac{\epsilon - 1}{\epsilon} \frac{A_{N,t} p_{N,t}}{\Gamma} p_{F,t}^{-\kappa} \right)^{\frac{1}{1-\kappa}} N_t^{-\phi} \right]^{\frac{1}{\sigma}} \end{aligned}$$

where $\Gamma = \left[\left(\frac{\kappa}{(1-\kappa)} \right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa} \right)^\kappa \right]$. Substituting $N_t, Y_t^* = C_t^*$,

$$\begin{aligned} Y_t^n &= \Sigma_{\alpha\gamma,t} \left[\left(\frac{\epsilon - 1}{\epsilon} \frac{A_{N,t} p_{N,t}}{\Gamma} p_{F,t}^{-\kappa} \right)^{\frac{1}{1-\kappa}} \right. \\ &\quad \left(A_{T,t}^{-\frac{1}{1-\zeta}} \left(\left(\gamma(1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta \mathcal{D}_t^{\frac{1}{\sigma}} + \gamma\alpha \left(\frac{P_t}{P_{T,t}} \right)^\eta \right) \mathcal{S}_t C_t^* \right)^{\frac{1}{1-\zeta}} \right. \\ &\quad \left. \left. + (1-\alpha)(1-\gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta \mathcal{D}_t^{\frac{1}{\sigma}} \mathcal{S}_t C_t^* \left(\frac{p_{F,t}}{p_{N,t}} \right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \right)^{-\phi} \right]^{\frac{1}{\sigma}} \\ Y_t^n &= \Sigma_{\alpha\gamma,t} \left(\mu^{\frac{1}{1-\kappa}} A_{N,t}^{\frac{1}{1-\kappa}} p_{N,t}^{\frac{1}{1-\kappa}} \Gamma^{-\frac{1}{1-\kappa}} p_{F,t}^{-\frac{\kappa}{1-\kappa}} \right)^{\frac{1}{\sigma}} \\ &\quad \left(A_{T,t}^{-\frac{1}{1-\zeta}} \left(\left(\gamma(1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta \mathcal{D}_t^{\frac{1}{\sigma}} + \gamma\alpha \left(\frac{P_t}{P_{T,t}} \right)^\eta \right) \mathcal{S}_t C_t^* \right)^{\frac{1}{1-\zeta}} \right. \\ &\quad \left. + (1-\alpha)(1-\gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta \mathcal{D}_t^{\frac{1}{\sigma}} \mathcal{S}_t C_t^* \left(\frac{p_{F,t}}{p_{N,t}} \right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \right)^{-\phi} \right]^{\frac{1}{\sigma}}, \end{aligned} \tag{30}$$

where μ is the gross markup in steady state. Equation (30) describes the natural output, which can vary with technology in both sectors, relative prices of input and foreign demand. Finally, the natural real interest rate is the risk-free real interest rate consistent with the Euler

equation when output is at its natural level at all times,

$$(C_{t+1}^n)^\sigma = \beta(1 + r_t^n)(C_t^n)^\sigma$$

$$(1 + r_t^n) = \frac{1}{\beta} \left(\left(\frac{Y_{t+1}^n}{Y_t^n} \right)^\sigma \frac{\Sigma_{\alpha\gamma,t}}{\Sigma_{\alpha\gamma,t+1}} \right)$$

3 Calibration

We calibrate the model at a quarterly frequency. Our aggregate baseline calibration is standard, as several preferences and technology parameters are shared with the standard New Keynesian literature. For our baseline scenario, we set the share of hand-to-mouth consumers in the population equal to 30%, following [Kaplan, Violante, and Weidner \(2014\)](#), [Kaplan, Moll, and Violante \(2018\)](#), [Kaplan and Violante \(2022\)](#). To allow for a more realistic representation of the households' optimisation problem, we set the elasticity of intertemporal substitution $\sigma = 4$.⁶ This aligns with recent literature ([Jones \(2023\)](#), [Kimball, Reck, Zhang, Ohtake, and Tsutsui \(2024\)](#), etc.) that discusses the limitations of log-utility assumptions in economics models. For simplicity, we consider unitary elasticities of substitution between foreign and domestic goods (η), between tradable and non-tradable goods (ν), and the Frisch elasticity.⁷ In the baseline model, α equals 0.25, which implies a home bias equal

Parameter	Benchmark Model	Parameter	Benchmark Model
β	0.99	χ	0.00001
α	0.25	ϕ_π	2
ϵ	6	ϕ_y	0
η	1	ρ_s	0.9
ν	1	ξ	28.003
λ	0.3	ζ	0.7
γ	0.2	ϕ	1
σ	4	κ	≈ 0

Table 1: Notes: ρ_s where $s \in \{T, a_N, a_T, i, y^*\}$.

to 0.75, following [Harrison and Oomen \(2010\)](#). We set κ , the income share of foreign primitive input in the production of non-tradable goods, to 0 in the baseline. However a positive κ (around 0.3 to match the labour share of income) amplifies the quantitative impact of the fragmentation scenarios we study, while leaving the qualitative interpretation unchanged. Finally, we calibrate γ , the share of tradable goods in the consumption basket to be 0.2.

⁶The results are robust for other values of $\sigma \neq 1$.

⁷However, results are robust to usual variation in these parameters.

3.1 Special Case: RANK

To establish basic intuition for our results, we conduct our three main exercises in a representative agent model. This corresponds to a special case of our baseline model, where the share of constrained households is $\lambda = 0$ (implying a heterogeneity index of $\Gamma_t = 1$).

Gradual Fragmentation To simulate a gradual shift towards a more restrictive trade environment, Figure 4 plots the impulse response functions (IRFs) of various macroeconomic aggregates to a gradual increase in import prices ($P_{F,t}$). In this scenario, import prices stabilise in the medium term, with a cumulative increase of 100 percent. The price of imported goods, $P_{F,t}$, will affect demand directly, both through the consumption basket $C_{F,t}$ and through imported inputs ($M_{F,t}$) in non-tradable goods production. Additionally, it indirectly affects demand through real wages.

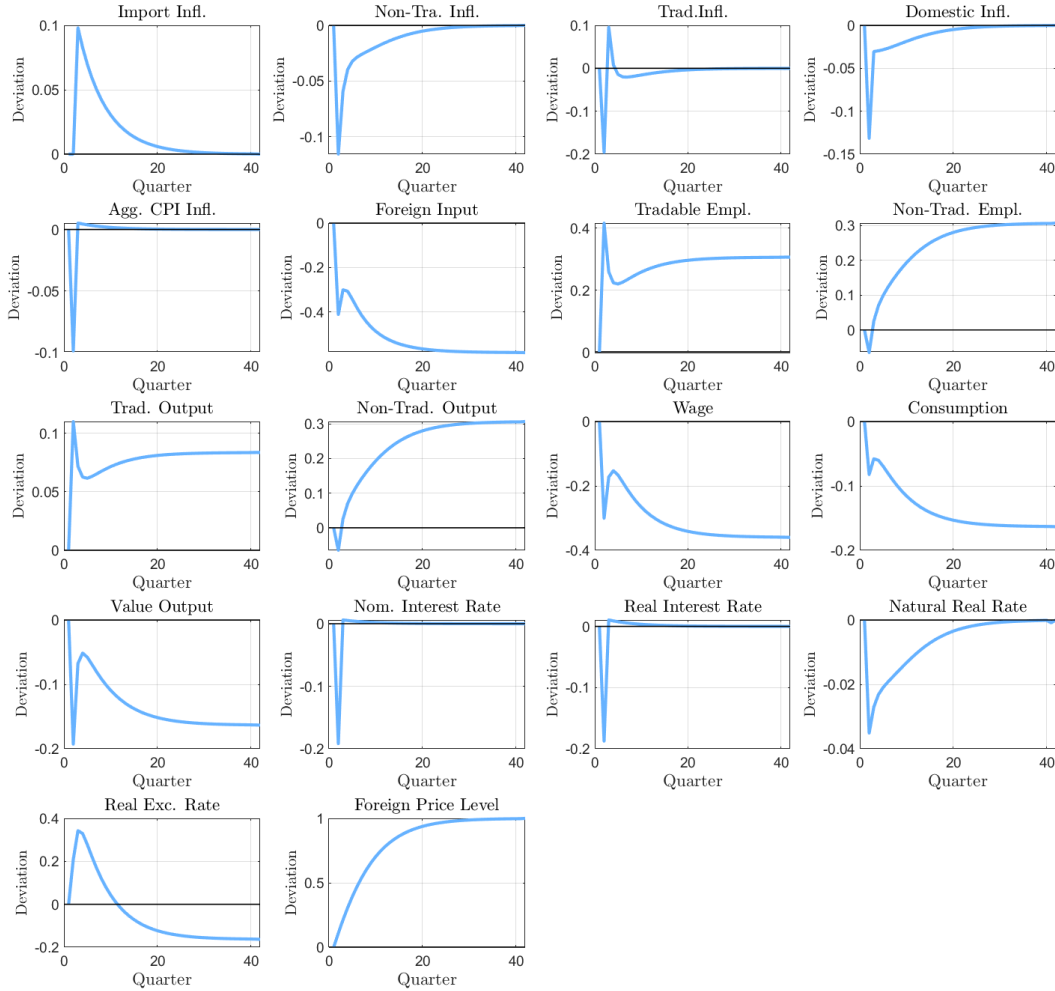
An increase in imported inflation follows from the gradual increases in the price of foreign goods. This places upward pressure on CPI inflation, but is more than offset by the fall in domestic inflation, which declines due to the falls in both non-tradable and tradable inflation. This fall in domestic inflation is driven by the fall in consumption, which responds to the drop in households' permanent real income. Households partly compensate for the fall in real wages by increasing labour supply, which mitigates the impact of higher import prices on aggregate supply. Overall, the anticipation effect of lower real incomes leads to demand falling more than supply, and consequently, to a fall in domestic inflation. CPI inflation, which is a composite of domestic and imported good inflation, falls on balance. This is reflected in a decrease in the natural real rate of interest, indicating that when demand materially adjusts in anticipation, the effect can be disinflationary. This prompts the central bank to ease policy, by lowering the nominal interest rate, in line with the rule characterising its reaction function.

Front-loaded Fragmentation Figure 5 shows the effect of a permanent and immediate increase in foreign prices $P_{F,t}$. This type of shock is intended to capture rapid 'fragmentation events' like price increases or – indirectly – tariff increases.⁸

The increase in the price of the foreign good leads to a sharp increase in imported inflation, which quickly reverts to its steady-state level. Aggregate consumption falls on impact, and following some short-term volatility reflecting movements in interest rates and employment, stabilises at a lower steady state. Non-tradable inflation falls at first, following

⁸A caveat is in order: our analysis does not account for the use of fiscal proceeds from tariffs. One way to justify this would be to assume that proceeds from tariffs are used to stimulate supply and demand in equal amounts, without affecting the output gap or inflation. Alternatively, we can assume that import restrictions take the form of non-tariff barriers (which comprise the majority of trade restrictions), in which case there is no tax revenue to be rebated. More broadly, this exercise is intended to capture the realignment of trade, whereby geopolitics forces domestic firms to switch from low-cost to geopolitically friendly suppliers, leading to efficiency losses.

Figure 4: IRFs to a Gradual Increase in Foreign Price Level.



Notes: IRFs to a 100% gradual positive foreign price shock. The results are generated under a RANK calibration ($\lambda = 0$). All the other parameters are calibrated according to Table 1.

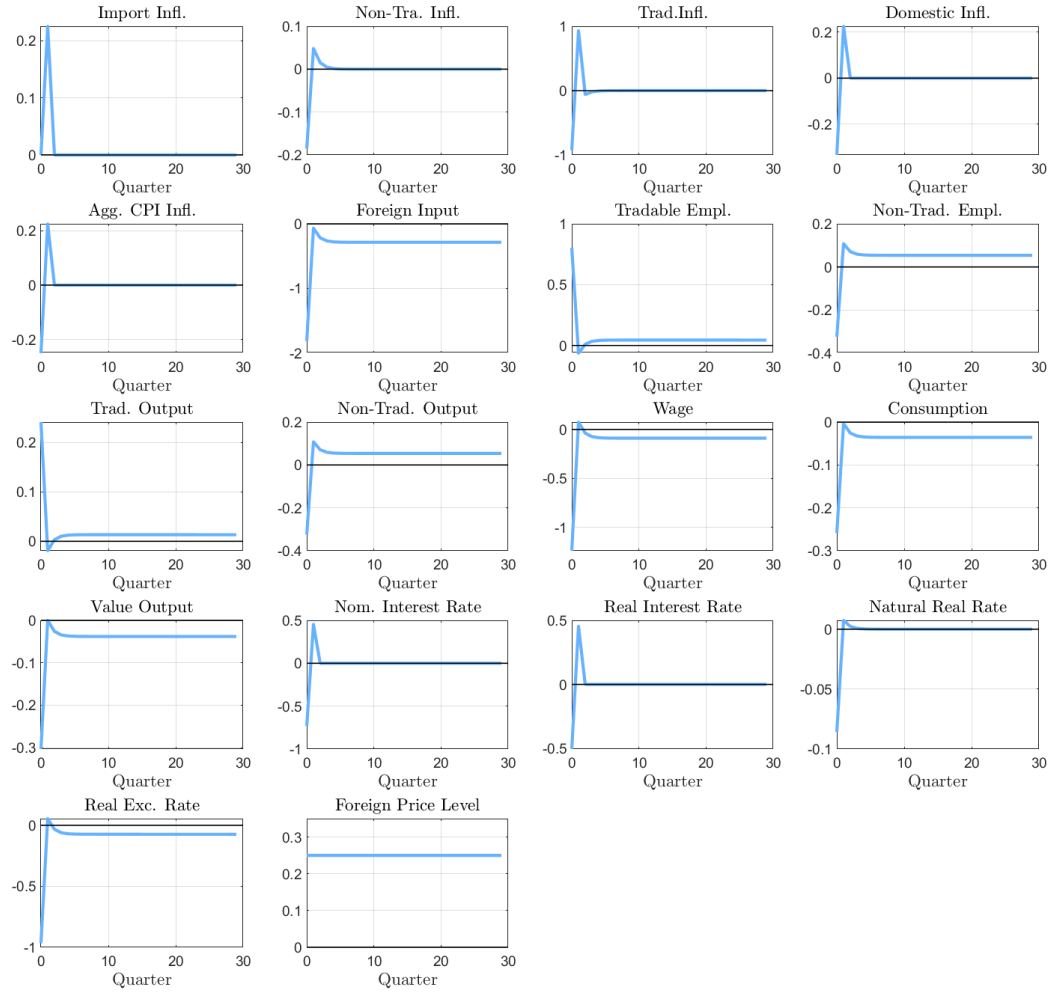
the fall in consumption. As non-tradable firms gradually pass on higher marginal costs, non-tradable inflation increases temporarily. Tradable inflation also falls initially, following the fall in consumption. Domestic inflation rises sharply, driven by the surge in tradables inflation. Combined with imported inflation, this leads to a significant spike in aggregate CPI inflation. Non-tradable output falls temporarily as the higher price of the foreign input restricts supply, but then recovers as households increase labour supply to compensate for losses in real income. The economy enters a temporary period of stagflation, with prices increasing and consumption decreasing. Overall, the natural rate decreases on impact and then slightly increases before going back to the steady state. The Taylor rule followed by the

central bank, along with the volatility in CPI inflation, leads to volatility in the nominal rate, which eventually increases to return inflation to target. On the external side of the economy, the real exchange rate reflects the nature of the shock and the policy response, depreciating on impact and then appreciating as the interest rate tightens.

Fall in Tradables Productivity $A_{T,t}$ We consider an additional shock that can result from “trade fragmentation”: a persistent decrease in the productivity of the tradable goods sector, which makes domestic production of tradable goods less competitive in the global market. In Figure 6, we show the response to a negative one standard deviation shock of total factor productivity in the tradable sector, $A_{T,t}$ (equivalent to 100 percentage-point deviation from the steady state). The shock is persistent but not permanent, as it reverts back to steady state in the medium-to-long term.

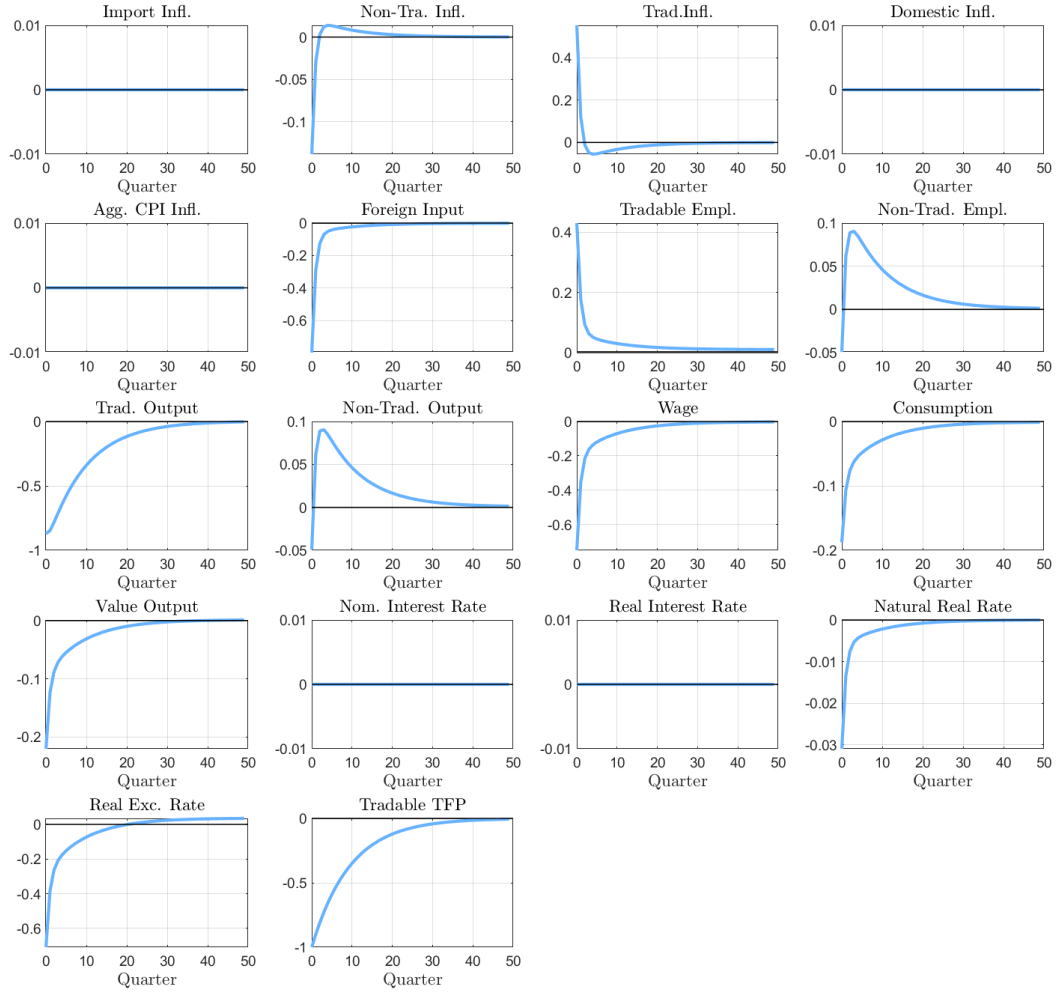
As in the standard New Keynesian model, this constraint on supply results in a fall in tradable output and an increase in tradable goods inflation as marginal costs rise. The fall in tradable output leads to a decrease in labour demand in this sector. Households cut consumption in response to the negative income effect of lower incomes. They also increase their labour supply to compensate for losses in real income. For tradable goods firms, the fall in nominal wages reduces production costs, offsetting the loss in productivity. The increase in household labour supply is, however, not enough to counteract the negative income effect. The fall in consumption also leads to a decrease in demand for non-tradable goods and inflation. Domestic and aggregate CPI inflation remain unchanged as the fall in non-tradable goods prices offset the increase in tradable goods prices. Since monetary policy reacts only to changes in inflation, real and nominal rates do not move. On impact, the natural rate decreases temporarily, reflecting lower demand, but it increases moderately over time as the economy recovers.

Figure 5: IRFs to a Front-loaded Increase in Foreign Price Level.



Notes: IRFs to a 25% positive foreign price shock. The results are generated under a RANK calibration ($\lambda = 0$). All the other parameters are calibrated according to Table 1.

Figure 6: IRFs to a Negative Shock to Tradable TFP.



Notes: IRFs to a negative TFP shock in the tradable sector. The results are generated under a RANK calibration ($\lambda = 0$). All the other parameters are calibrated according to Table 1.

To summarise, since all three scenarios consider shocks that constrain supply capacity, the supply side effects are unambiguous. Marginal costs increase initially and are passed through only gradually to prices due to price rigidities. In partial equilibrium, as nominal expenditures are fixed, this leads to a fall in household demand for goods from the unaffected sector. To capture general equilibrium effects, this section uses a simple framework to demonstrate how aggregate demand adjusts differently in various scenarios that model aspects of trade fragmentation. In the gradual fragmentation scenario, a steady increase in import prices reduces the purchasing power of labour income, through an increase in the price of imported consumption goods as well as through a fall in nominal wages. If this change is expected to be permanent, then households also expect a permanent fall in purchasing power, leading to a fall in consumption spending. Therefore, a fall in permanent labour income leads to a fall in demand, which affects the price level response to the initial increase in import prices. This scenario leads to stagnation, with lower real incomes and low inflationary pressures. In contrast, a front-loaded fragmentation scenario, which takes the form of a sharp permanent increase in import prices, may create a short-term tradeoff for policymakers. Finally, persistent falls in tradable sector productivity may be neutral for inflation.

These results suggest that the form in which fragmentation materialises, the extent to which it is anticipated by households, and households' ability to smooth consumption over time, all matter. The next section will consider the case where a proportion of households are unable to smooth consumption in response to changes in their permanent income.

3.2 TANK Case

An important factor in gauging how inflation will respond to these trade-related shocks is the degree of forward-looking behaviour in demand. Specifically, it hinges on the extent to which households can effectively smooth consumption in the presence of a shock. Therefore, this section considers a more general framework that allows for household heterogeneity.

Relative to the previous section, the presence of constrained households introduces agents who cannot smooth consumption in response to the shock, although they can adjust their labour supply. The fall in real incomes affects these households directly, while the fall in labour demand and aggregate demand by unconstrained households affects them indirectly. As in the previous section, we consider three scenarios to show how the form of fragmentation will affect the demand-side adjustment.

We also explore how much the extensive margin of household heterogeneity matters by considering two cases: an economy in which roughly one-third of households are constrained (hand-to-mouth) ($\lambda = 0.3$) compared with an economy where roughly three-quarters of households are constrained (hand-to-mouth) ($\lambda = 0.8$). We find that for this degree of variation, heterogeneity does not alter the main results. Financially unconstrained households, even in small shares, can trigger a fall in demand, which spills over to the financially

constrained households.⁹

The fall in permanent income highlighted in the previous section will be mitigated by the proportion of constrained households. While all households consume out of permanent income in a RANK model, only a proportion λ of households do so in a TANK model. The presence of constrained households lessens the adverse demand side effect since they cannot cut consumption in anticipation of the shock.¹⁰

Gradual Fragmentation To simulate a gradual shift towards a more restricted trade environment, Figure 7 plots the impulse response functions to an increase in the price of imported goods, which stabilises at a 100 percent higher level in the medium term. As in the RANK case, the price of imported goods, $P_{F,t}$, affects demand directly, both through the consumption basket $C_{F,t}$ and through imported inputs $M_{F,t}$ in production. Additionally, it indirectly affects demand through real wages.

The response of real and nominal variables with a higher share of constrained agents is similar to the RANK case (Figure 4). However, the response of both households' consumption when $\lambda = 0.8$ differs. In particular, constrained households' consumption falls slightly more over time, while unconstrained households are able to borrow to smooth consumption. Unconstrained household consumption increases initially, but then falls below steady state and never recovers. In the case of $\lambda = 0.8$, fewer unconstrained consumers anticipate the permanent income losses and adjust consumption in response, mitigating the fall in aggregate demand.

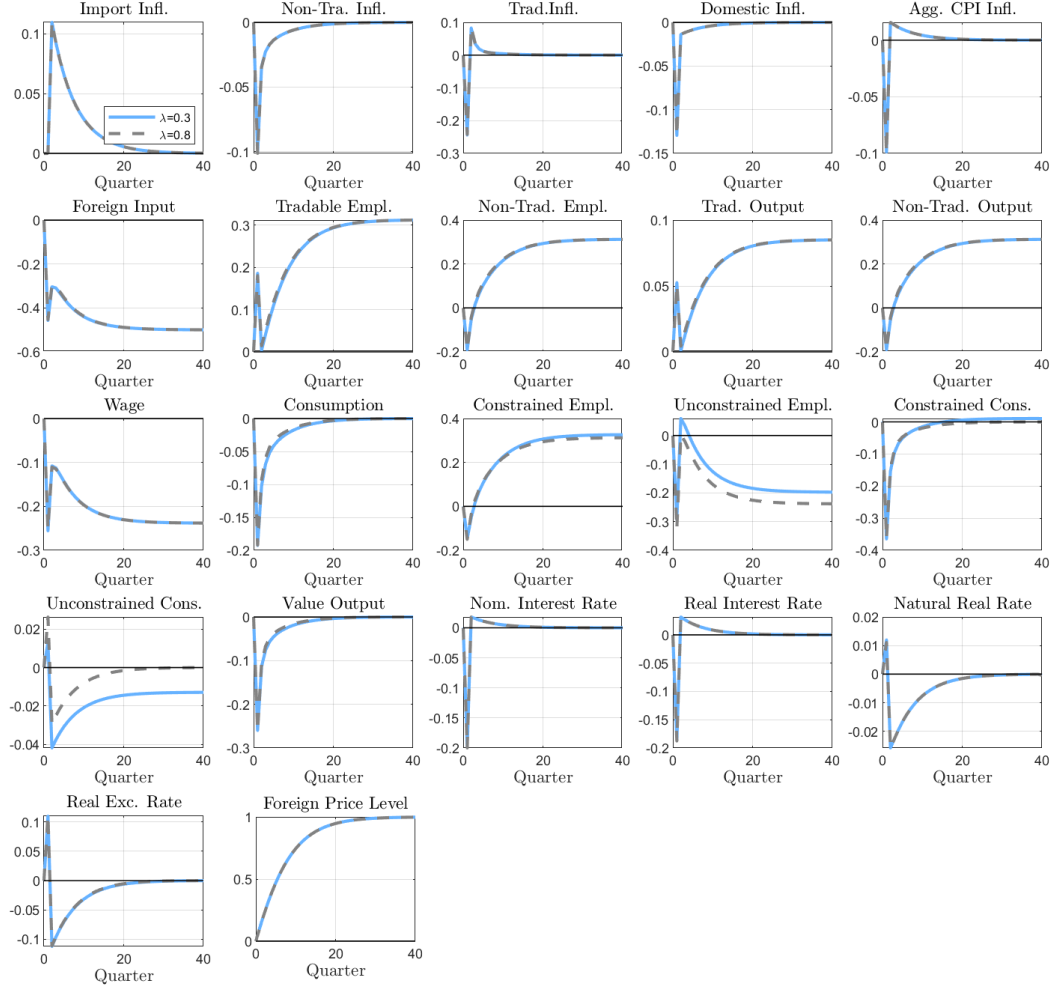
As unconstrained households cut consumption in response to the fall in their permanent real income, demand spillovers lead to lower labour demand and lower wages for constrained households. In addition, constrained households cut consumption due to a fall in their real disposable income, which decreases due to higher prices. These households partly compensate for the loss in real wages by increasing labour supply, mitigating the impact of higher import prices on aggregate supply. Overall, the anticipation effect of lower real incomes on demand by unconstrained households leads to demand falling more than supply, and consequently, to a fall in domestic inflation. The fall in domestic inflation more than

⁹The standard transfer rule assumed in the literature is not innocuous. As a technical assumption intended to simplify the steady state solution, this redistribution mutes the effect of household heterogeneity in our model. We therefore neutralise the effects of transfers by assuming profits are "lost at sea".

¹⁰The rise in import prices could potentially affect aggregate demand through distributional effects if the two households differ in their sources of income. The contractionary demand-side effect of fragmentation may be larger if labour income falls and profit income increases. Elasticities of substitution between production inputs determine the magnitude of the demand-side effect of energy price shocks in Auclert, Monnery, Rognlie, and Straub (2023) and Chan, Diz, and Kanngiesser (2024), while trade elasticities affect the real income channel of exchange rate depreciations in Auclert, Rognlie, Souchier, and Straub (2021). We abstract from these effects here in assuming unitary elasticity of substitution between inputs in nontradables production and the same between home and foreign goods in consumption (in addition to profits "lost at sea"). This is consistent with the view that trade fragmentation entails a shift from low-cost suppliers to friendly suppliers, or that elasticities of substitution are high in the medium to long run (if fragmentation is a permanent, phased-in process).

offsets the persistent increase in imported inflation, leading to a fall in aggregate CPI. The natural real rate of interest falls, suggesting that when demand adjusts in anticipation, the effect can be disinflationary. The nominal interest rate falls in response, in line with the rule characterising its reaction function.

Figure 7: IRFs to a Gradual Increase in Foreign Price Level.



Notes: $\lambda = 0.3$ in blue, $\lambda = 0.8$ in grey dashed line. All the other parameters are calibrated according to Table 1.

Front-loaded Fragmentation Figure 8 shows the effect of a permanent and immediate increase in foreign prices $P_{F,t}$. The responses of real and nominal variables with a higher share of constrained agents ($\lambda = 0.8$, dashed grey lines) are similar to the RANK case (Figure 5). The immediate impact of the shock suggests that the share of constrained households mat-

ters less, as the trough in unconstrained consumption is only slightly higher in the case of $\lambda = 0.8$. For this range of variation in the share of hand-to-mouth consumers, there is no significant difference in the transmission of the import price shock.

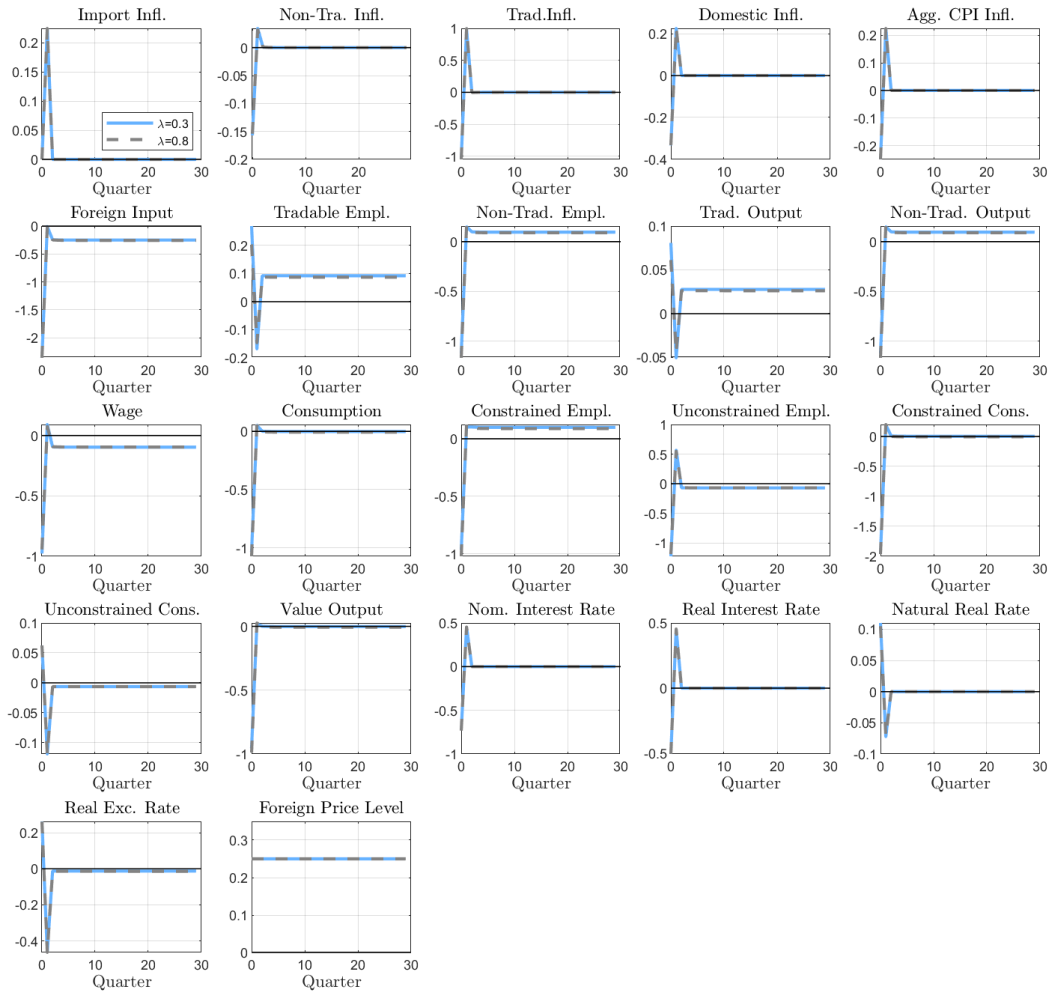
This scenario leads to a sharp increase in imported inflation, which quickly reverts back to the steady-state level. This frontloaded price shock adversely affects the consumption of constrained households immediately, as they are unable to smooth consumption. The initial sharp fall in consumption is driven by the constrained households who react to the fall in purchasing power by increasing labour supply. However, as constrained households respond by increasing labour supply to compensate for lower incomes, constrained consumption does not fall as much and it recovers to a new steady state level. On impact, consumption goes below steady state, before settling at a lower steady state level for both constrained and unconstrained consumers. Aggregate consumption therefore falls on impact and stabilises at a lower steady state value. Tradable and non-tradable output fall temporarily as the higher price of the foreign input restricts supply, but both recover as households increase their labour supply to compensate for income losses. Aggregate and domestic inflation increase and the economy enters a temporary period of stagflation. The monetary policy response to inflation volatility leads to volatility in the nominal rate, which eventually increases to return inflation to target. Overall, the natural rate increases on impact and then falls. Due to the nature of the shock and the policy response, the real exchange rate depreciates on impact and then appreciates as policy tightens.

Fall in Tradables Productivity Finally, we consider a persistent decrease in the productivity of the tradable goods sector, which makes domestic production of tradable goods less competitive in the global market. Figure 9 plots the responses of all variables to a one standard deviation negative shock to total factor productivity in the tradable sector, $A_{T,t}$. The shock is persistent but not permanent, as it reverts back to steady state in the medium-to-long term.

Tradable output decreases, while tradable inflation increases. This is due to the rise in the marginal cost of production caused by the fall in productivity. Consumption decreases, almost entirely driven by the constrained agents. Financially unconstrained households also adjust their consumption downwards, but because they can smooth over time and the shock is not permanent, the adjustment in consumption is very small. The adjustment in consumption also reflects a negative income effect coming from lower wages, as constrained households increase their labour supply in response to the losses on real income.

For tradable goods firms, higher labour supply partially compensates for the loss of productivity. As a result, labour demand in this sector increases, leading to an increase in tradable employment. While constrained agents substantially increase their labour supply, this is not enough to counteract the negative income effect. The drop in consumption also temporarily affects non-tradable demand and inflation adversely. Tradable output recovers over time as productivity improves. Inflation falls in the non-tradable sector to offset the increase

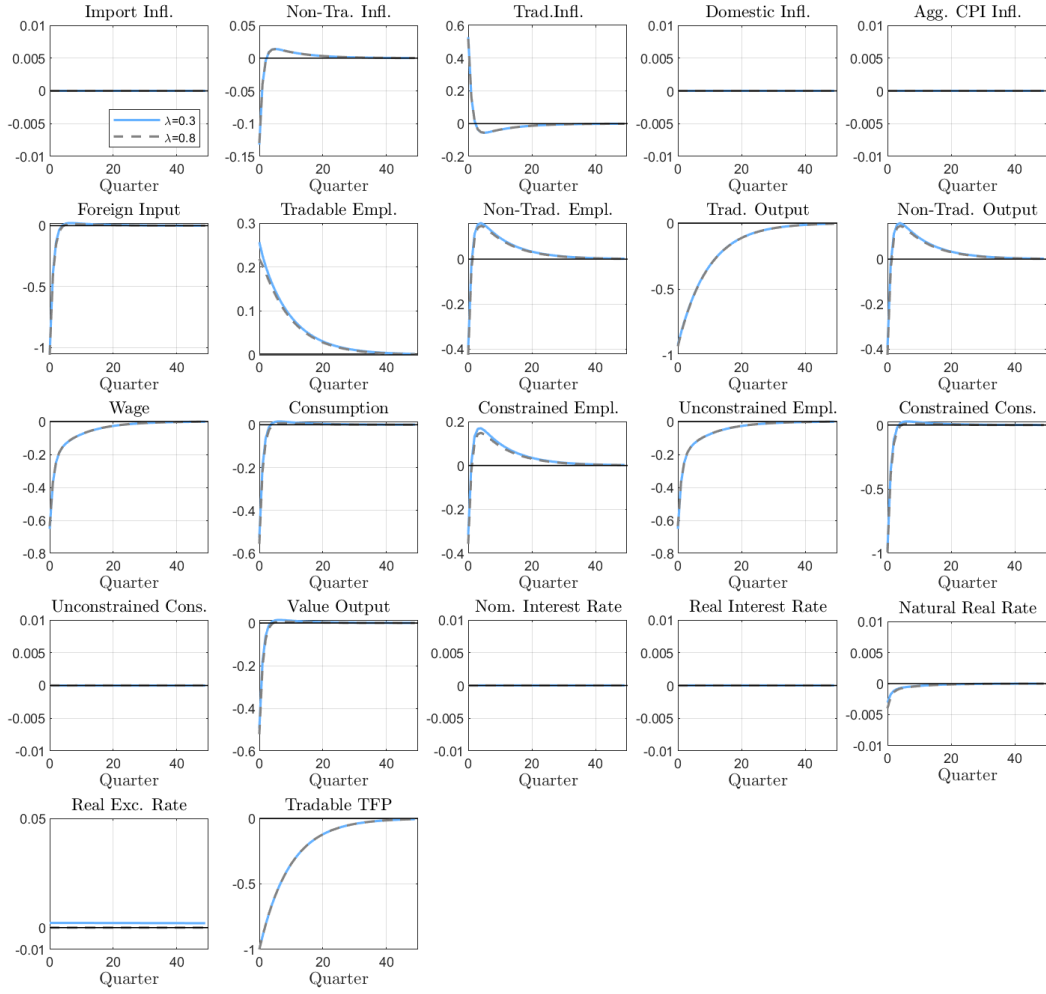
Figure 8: IRFs to a Front-loaded Increase in Foreign Price Level.



Notes: $\lambda = 0.3$ in blue, $\lambda = 0.8$ in grey dashed line. All the other parameters are calibrated according to Table 1.

in inflation in the tradable sector. Consequently, domestic and aggregate inflation remains unchanged. Real and nominal rates remain constant as a result. On impact, the natural rate decreases temporarily, reflecting lower demand, but it increases moderately as the economy recovers.

Figure 9: IRFs to a Negative Shock to Tradable TFP.



Notes: $\lambda = 0.3$ in blue, $\lambda = 0.8$ in grey dashed line. All the other parameters are calibrated according to Table 1.

In summary, the form in which fragmentation takes place will matter for the balance of supply and demand. The sudden implementation of tariffs will have different effects compared to a gradual implementation or an adverse shock to tradables productivity. The presence of fewer forward-looking agents mitigates the fall in aggregate demand, and more so in the scenario where the permanent increase in foreign prices takes place gradually.

3.3 Different Degrees of Openness

This section explores the impact of the initial level of trade openness on the macroeconomic response to fragmentation. We therefore consider the same set of fragmentation scenarios while varying the level of home bias, denoted by α . This adjustment allows us to capture how different economies experience trade shocks with varying degrees of intensity, and how policy responses may differ depending on the level of openness. A lower value of α indicates a less open economy: the consumption basket and price index are *less* influenced by foreign prices and goods. Consequently, consumption relies more heavily on domestic production.

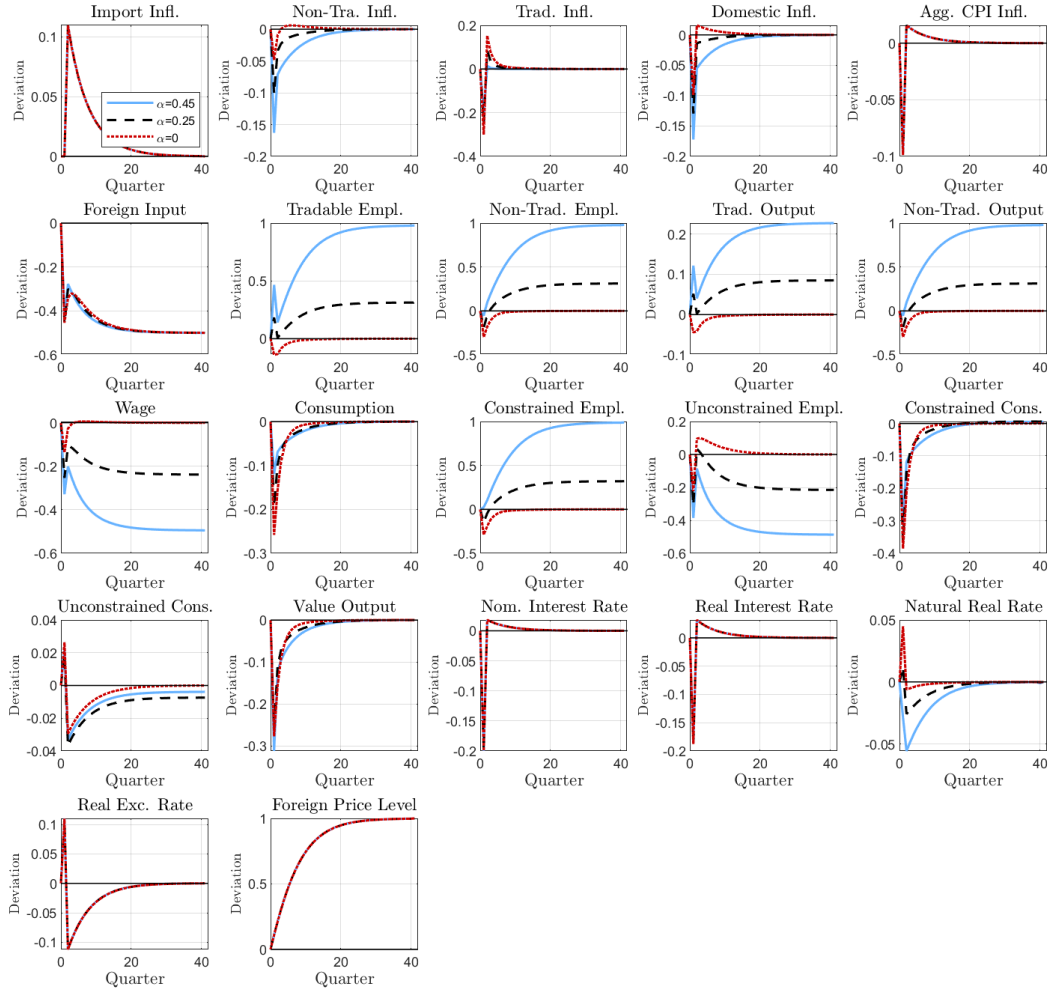
The degree of home bias in consumption appears to play a crucial role. Economies that are more open tend to be more exposed to fluctuations in foreign prices, which affects their consumption and production responses. In the scenarios involving sustained increases in foreign prices, whether gradual or front-loaded, the more open economy experiences a more pronounced decline in the natural rate of interest. Conversely, this trend reverses in the event of a negative TFP shock. Although the shock primarily affects the tradable sector, it directly affects domestic production and consumption, particularly the consumption basket of less open economies. In contrast, more open economies can mitigate the impact by diversifying away from domestic shocks through trade with foreign suppliers (and buyers).

Gradual Fragmentation Figure 10 shows that the open economies ($\alpha = 0.25$ and $\alpha = 0.45$) respond differently from the closed economy ($\alpha = 0$), in a gradual fragmentation scenario. Aggregate inflation has the same behaviour in all three cases; however, the underlying mechanisms differ quantitatively.

A useful benchmark to start with is the closed economy case ($\alpha = 0$, red dotted lines). Here, imported inputs are only used as an intermediate input for the production of non-tradable goods and the consumption basket of domestic households consists solely of domestically produced tradable and non-tradable goods. As the price of imported intermediate inputs increases, imported inflation increases and the use of the imported input falls. Non-tradable employment falls as non-tradable output falls. Employment falls for all households and real wages fall. The adverse effect on labour income leads to a fall in consumption for both types of households. The fall in aggregate consumption, which is almost entirely attributable to the fall in constrained households' consumption, leads to downward pressure on tradable output, tradable goods inflation, and domestic inflation.

Next, consider the case of an open economy ($\alpha = 0.25$ and $\alpha = 0.45$, dashed black lines and solid blue lines), where imported goods are not only used as intermediate inputs in the production of non-tradable goods but also comprise part of the consumption basket. More openness leads to a larger domestic adjustment in response to the import price shock. The additional fall in non-tradable goods inflation relative to the case of $\alpha = 0$ is due to the fall in domestic consumption, which now exerts additional downward pressure on domestic inflation. More openness also leads to a larger domestic adjustment in response to the import price shock. The import price shock leads households to substitute towards relatively cheaper domestically produced goods and production factors, which stimulates domestic demand and labour demand. Employment and output in both the tradable and non-tradable sectors increase. As exposure to foreign shocks intensifies with a lower home bias (higher α), the natural rate of interest falls by more in the open economies.

Figure 10: IRFs to a Gradual Increase in Foreign Price Level.



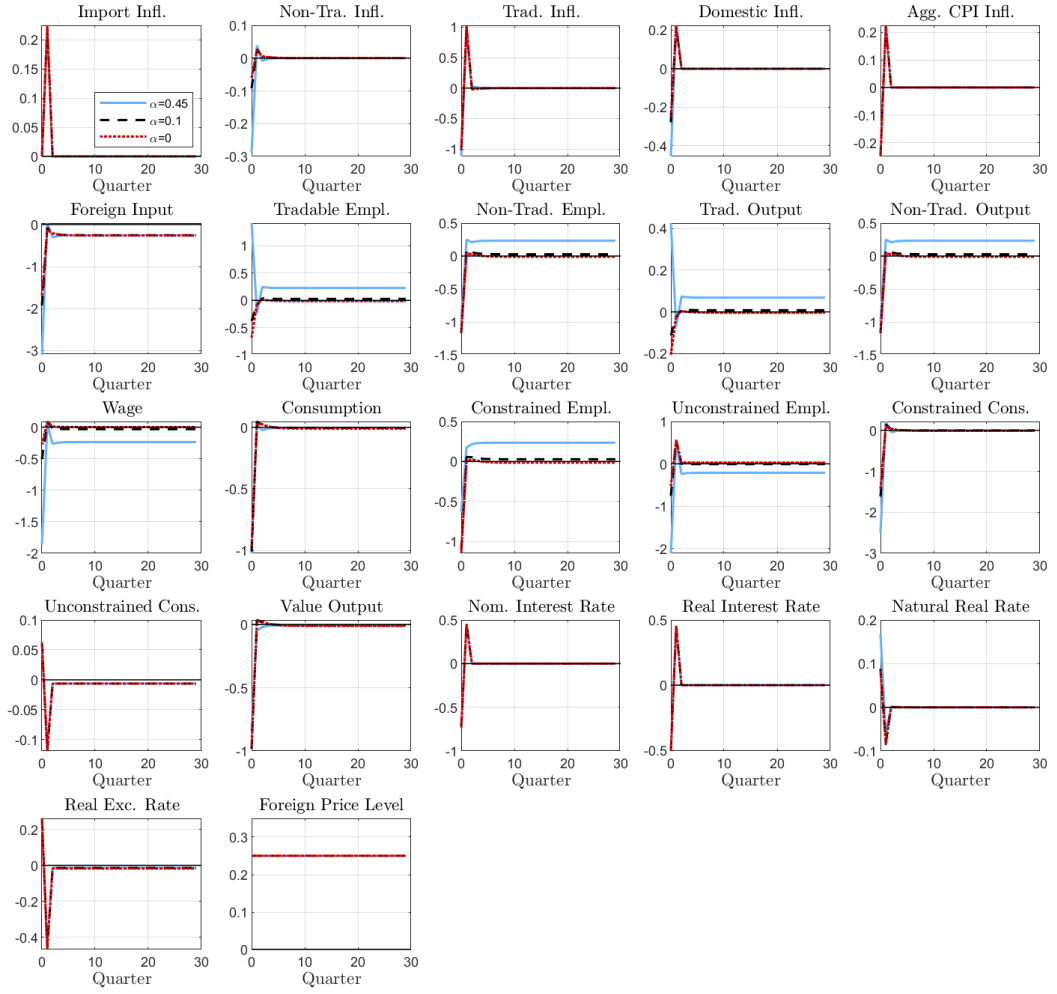
Notes: $\alpha = 0$ in red dotted line, $\alpha = 0.25$ in black dashed line, $\alpha = 0.45$ in blue line.

Front-loaded Fragmentation In the front-loaded fragmentation scenario, openness also plays a significant role in the responsiveness to changes in the level of $P_{F,t}$ (Figure 11). Notably, in the more open economy, non-tradable inflation exhibits a more pronounced decline immediately following the shock, contributing to a greater fall in domestic inflation.

Consider the case of a closed economy, where the import price shock only affects intermediate inputs in the production of non-tradable goods ($\alpha = 0$). As the price of foreign inputs rises, the economy experiences a decrease in demand for these inputs, leading to a fall in non-tradable output, a fall in non-tradable employment, a fall in real wages, and a fall in consumption. The fall in aggregate consumption is mostly attributable to the fall in constrained households' consumption. Tradable output and employment also fall, with tradable inflation falling as a result.

The same dynamics exist in the open economies ($\alpha = 0.25$ and $\alpha = 0.45$, dashed black lines and solid blue lines). However, imported goods are now also part of the consumption basket. The import price shock therefore stimulates the domestic economy, and to a greater degree in the more open economy, as households substitute towards relatively cheaper domestically produced goods. Output and employment increase in both the tradable and non-tradable sectors. The fall in consumption, which would occur as a result of more expensive foreign goods, is mitigated by the increase in domestic demand and labour income.

Figure 11: IRFs to a Front-loaded Increase in Foreign Price Level.



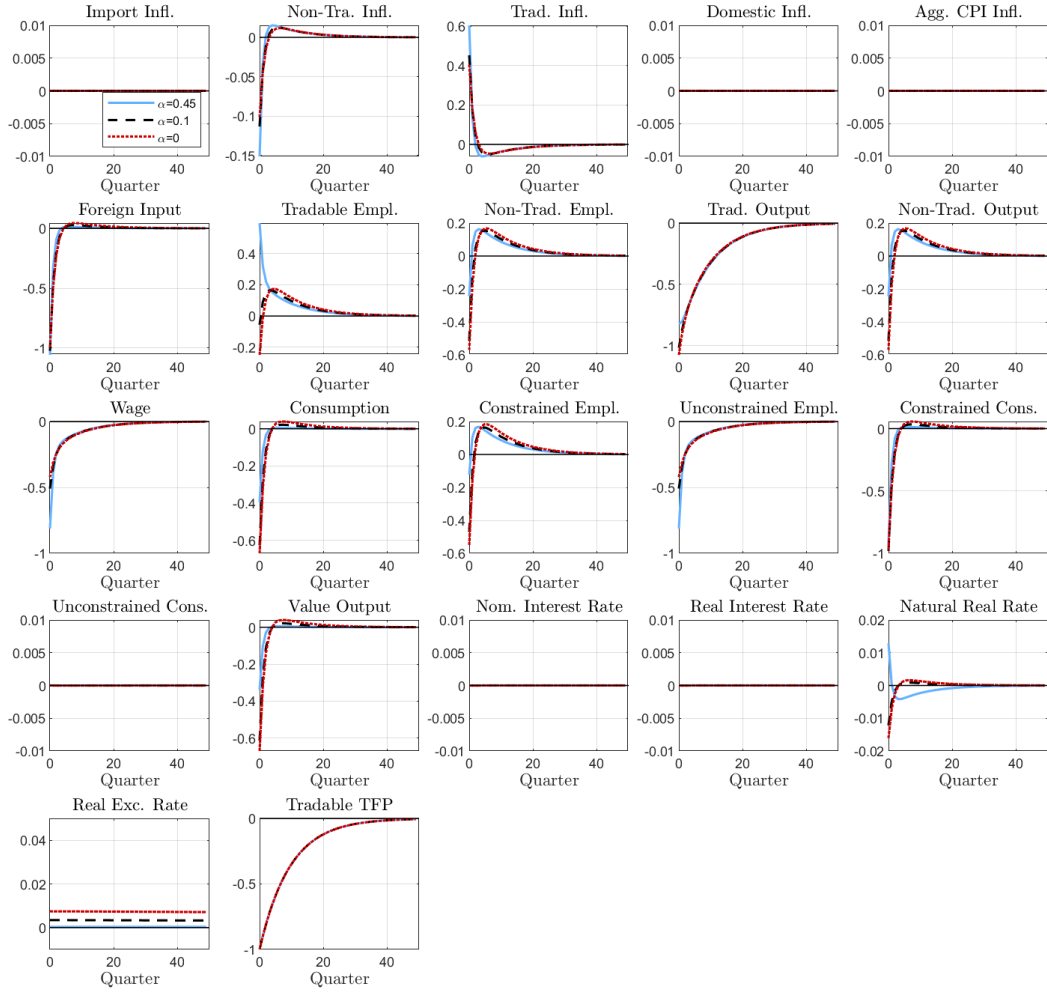
Notes: $\alpha = 0$ in red dotted line, $\alpha = 0.25$ in black dashed line, $\alpha = 0.45$ in blue line.

Fall in Tradables Productivity Figure 12 shows the impulse response functions for a one-standard deviation fall in TFP for the tradable sector. As before, consider first the $\alpha = 0$ specification (red dotted lines). In this case, imported inputs are only used in the production of non-tradable goods. An adverse productivity shock in the tradables sector leads to a fall in tradable output. Each unit of tradable output now requires more labour to produce, and hours worked increase gradually due to higher labour demand. The adverse TFP shock leads to an increase in marginal costs in the tradable sector, which initially leads to upward pressure on tradable inflation. Labour supply increases, leading to a fall in real wages through a fall in the nominal wage. The fall in real wages leads to a fall in consumption, which is entirely driven by a fall in constrained households' consumption, as unconstrained households can borrow to smooth over the transitory shock. Constrained households are unable to borrow to smooth consumption, increasing their labour supply instead.

The fall in consumption affects the non-tradable sector, where output falls. However, since the non-tradable sector is unaffected by the adverse productivity shock, it can increase output by substituting towards labour as real wages fall. Non-tradable output rises as a result, through an increase in the intensive margin of labour and a fall in the use of the foreign input. The increase in tradable inflation counteracts the fall in non-tradable inflation and domestic inflation remains unchanged as a result. As this is purely a domestic shock, there is no change in imported inflation or in aggregate CPI inflation as a result.

In a more open economy (and $\alpha = 0.45$), the adverse effect of the productivity shock on overall consumption is smaller on impact, and tradable output falls by less as the use of the foreign input falls by less, relative to the closed economy. Employment in the tradable sector increases significantly, compensating for part of the losses in productivity (given the Cobb-Douglas specification, the scope for labour substitutability is limited). The fall in productivity leads to a large fall in the real wage on impact. As in the $\alpha = 0$ case, the adverse productivity shock in the tradables sector has some spillovers for the non-tradables sector, although this effect is not as large. Despite a significant fall in real wages, non-tradable employment and output increase by roughly the same magnitude as in the $\alpha = 0$ case. Changes in tradable inflation are balanced by countervailing changes in non-tradable inflation, with domestic and aggregate CPI inflation unchanged as a result.

Figure 12: IRFs to a Negative Shock to Tradable TFP.



Notes: $\alpha = 0$ in red dotted line, $\alpha = 0.25$ in black dashed line, $\alpha = 0.45$ in blue line.

4 Extensions

In this section, we extend the RANK model introduced in Section 3.1 to incorporate nominal wage rigidities.

4.1 Households

The setting is similar to the RANK model we discuss before. The key difference we introduce in this section is that, once, households have chosen how much labour to supply in a given period, $N_t(j)$, this labour is supplied to a union, in return for a nominal wage W_t . The union allocates the workers into categories or varieties indexed by j , $N_t(j)$, $j \in [0, 1]$ and sells these units of labour varieties at wage $W_t(j)$. Due to imperfect substitutability among labour types, the union can act as a monopolist over each variety.

Labour Packers Varieties $N_t(j)$ are in turn combined by labour “packers” according to a CES production function:

$$N_t = \left[\int_0^1 N_t(j)^{1-\frac{1}{\epsilon_w}} dj \right]^{\frac{1}{1-\frac{1}{\epsilon_w}}}$$

$N_t(j)$ denotes the demand for a specific labour variety j and N_t denotes aggregate labour demand. The elasticity of substitution between labour varieties is ϵ_w . After the packers have assembled the labour bundle, they supply it at wage W_t to firms that then use it in the production process. Labour packers maximise the following objective:

$$\max_{N_t(j)} \left\{ W_t N_t - \int_0^1 N_t(j) W_t(j) dj \right\}$$

subject to the CES production function. The first-order condition is

$$W_t N_t^{\frac{1}{\epsilon_w}} N_t(j)^{-\frac{1}{\epsilon_w}} - W_t(j) = 0.$$

Therefore labour demand is given by

$$N_t(j)^d = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t$$

As the labour packers are perfectly competitive we can use the zero-profit condition to calculate the wage index:

$$\begin{aligned} W_t N_t - \int_0^1 N_t(j) W_t(j) dj &= 0 \implies \\ W_t N_t &= \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t W_t(j) dj \\ W_t &= \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}} \end{aligned}$$

Labour Unions The labour union maximises:

$$\max_{W_t(j)} \delta_t N_t^d W_t(j) - \frac{N_t^{1+\phi}}{1+\phi}$$

subject to:

$$N_t(j)^d = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t^d$$

This is equivalent to the maximisation of household utility subject to the budget constraint and the demand for labour. The FOC becomes:

$$\delta_t \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} (1 - \epsilon_w) N_t^d + \epsilon_w \left[\left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t^d \right]^\phi \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w - 1} \frac{N_t^d}{W_t} = 0,$$

which yields

$$W_t(j) = \frac{\epsilon_w}{\epsilon_w - 1} \frac{(N_t^d)^\phi}{C_t^\sigma},$$

where $\frac{\epsilon_w}{\epsilon_w - 1} = \mathcal{M}^w$ and $mc_t^w = \frac{(N_t^d)^\phi}{C_t^\sigma}$. This represents the flexible wage setting. However, as we introduce *nominal and real wage stickiness*, we have that with probability θ_w a union is stuck with its previous-period wage indexed to a composite such that:

$$W_t(j) = \begin{cases} W_{t-1}(j) ((\Pi_{ss}^w)^{1-\xi_w} (\Pi_{t-1}^w)^{\xi_w}) & \text{with prob } \theta_w \\ W_t^*(j) & \text{with prob } 1 - \theta_w \end{cases}$$

Therefore,

$$W_{t+s}(j) = W_t^*(j) (\Pi_{ss}^w)^{s(1-\xi_w)} \left(\prod_{g=0}^{s-1} \left((\Pi_{t+g}^w)^{\xi_w} \right) \right) = W_t^*(j) \left[(\Pi_{ss}^w)^{s(1-\xi_w)} \left(\frac{W_{t+s-1}}{W_{t-1}} \right)^{\xi_w} \right]$$

Given this demand constraint and assuming that a union j always meets the demand for its labour at the current wage labour unions solve the following optimisation problem

$$\max_{W_t^*(j)} E_t \sum_{s=0}^{\infty} (\theta_w)^s \Lambda_{t,t+s} P_{t+s} \left[\left(\frac{W_t^*(j)}{P_{t+s}} - mc_{t+s}^w \right) \left(\frac{W_t^*(j)}{W_{t+s}} \right)^{-\frac{M_{tw}}{M_w - 1}} N_{t+s} \right] \quad (31)$$

Taking the derivative with respect to $W_t^*(j)$ delivers the familiar wage inflation schedule

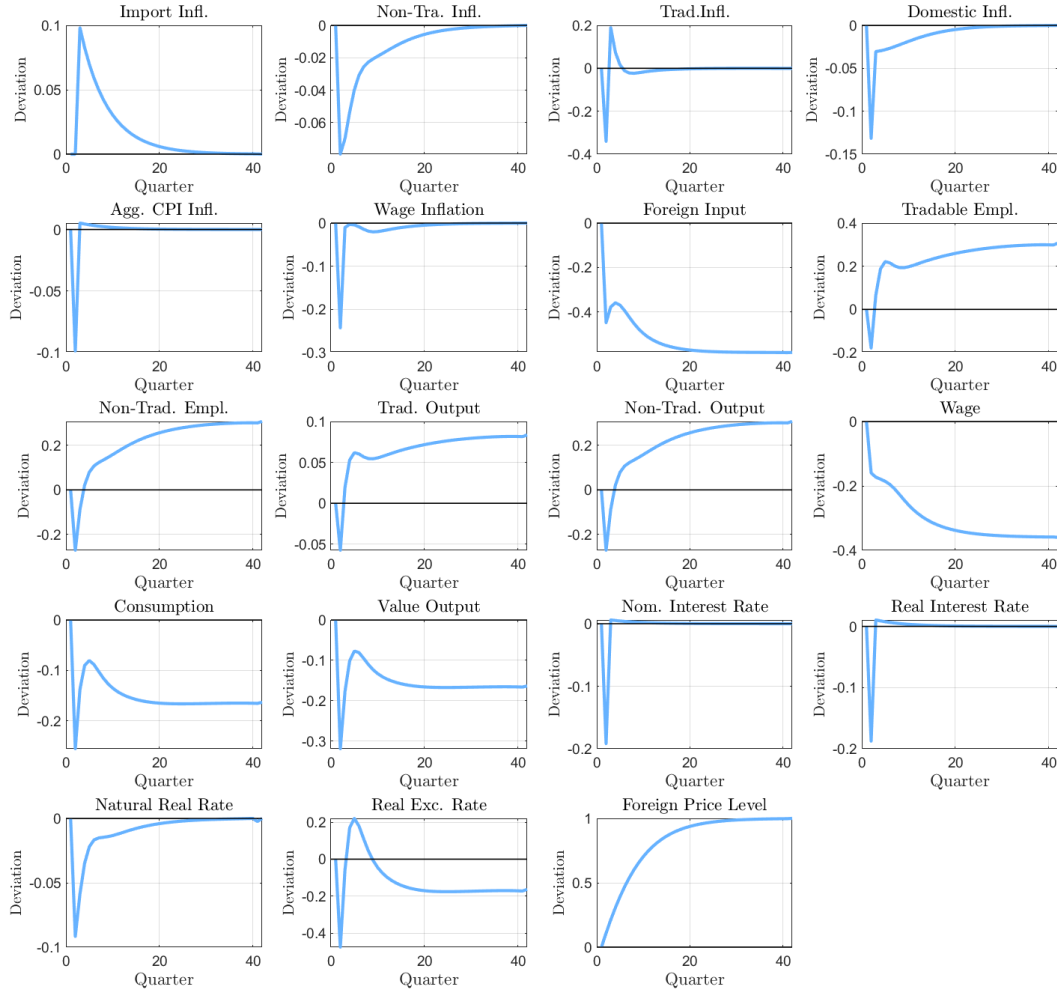
$$\begin{aligned}
\frac{f_t^{W,1}}{f_t^{W,2}} \mathcal{M}_w &= w_t^* = \frac{W_t^*}{W_t} = \left(\frac{1 - \theta_w (\zeta_t^W)^{\frac{1}{M_w-1}}}{1 - \theta_w} \right)^{1 - \mathcal{M}_w} \\
f_t^{W,1} &= N_t \frac{mc_t^W}{w_t} + \theta_w E_t \left[(\Lambda_{t,t+1}) \left(\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}} \right) \left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W} \right)^{\frac{M_w}{M_w-1}} f_{t+1}^{W,1} \right] \\
f_t^{W,2} &= N_t + \theta_w \beta E_t \left[(\Lambda_{t,t+1}) \left(\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}} \right) \left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W} \right)^{\frac{1}{M_w-1}} f_{t+1}^{W,2} \right] \\
\zeta_t^W &= \Pi_t^W / \Pi_{ss}^W \\
w_t &= \Pi_t^W / \Pi_t w_{t-1} \\
\mathcal{D}_t^W &= (1 - \theta_w) \left(\frac{1 - \theta_w (\zeta_t^W)^{\frac{1}{M_w-1}}}{1 - \theta_w} \right)^{\mathcal{M}_w} + \theta_w (\zeta_t^W)^{\frac{M_w}{M_w-1}} \mathcal{D}_{t-1}^W
\end{aligned}$$

Where D_t^W is wage dispersion. We calibrate $\theta_w = 0.92$, $\xi_w = 0$, $\epsilon_w = 11$ as in [Schmitt-Grohé and Uribe \(2006\)](#) and [Chan, Diz, and Kanngiesser \(2024\)](#). All the other features of the model stay unchanged.

Gradual Fragmentation The impact of a gradual import price shock on aggregate CPI inflation and domestic inflation with wage rigidities (Figure 13) is similar to the case without wage rigidities (Figure 4). However, the composition of domestic inflation differs. There is a deeper fall in tradables inflation, and a shallower trough for non-tradables inflation. A tt feature of this extension is the initial fall in tradable output. Higher marginal costs lead both tradable and non-tradable goods firms to cut production. Hours worked fall in both the tradables sector and the non-tradables sector. Aggregate hours worked fall as a result and to a much greater degree than in the flexible wage case. Aggregate consumption falls to a greater extent initially, despite a more moderate fall in the real wage than the flexible wage case. Despite the fall in supply capacity for tradable goods, tradable inflation falls significantly, as consumption also falls to a greater degree than in the case without wage rigidities.

Front-loaded Fragmentation The differences between the flexible-wage and the sticky-wage cases are more salient in the front-loaded fragmentation scenario. Relative to the flexible-wage case (Figure 5), nominal wage rigidities mitigate the fall in wage inflation (Figure 14). As aggregate price inflation falls to a greater degree, real wages increase. The increases in both labour costs and intermediate input costs lead to a fall in tradable and non-tradable output. Hours worked fall in both the tradables sector and the non-tradables sector, leading to a fall in aggregate hours worked (and to a much greater degree than in the flexible-wage case). Aggregate consumption falls despite an increase in the real wage.

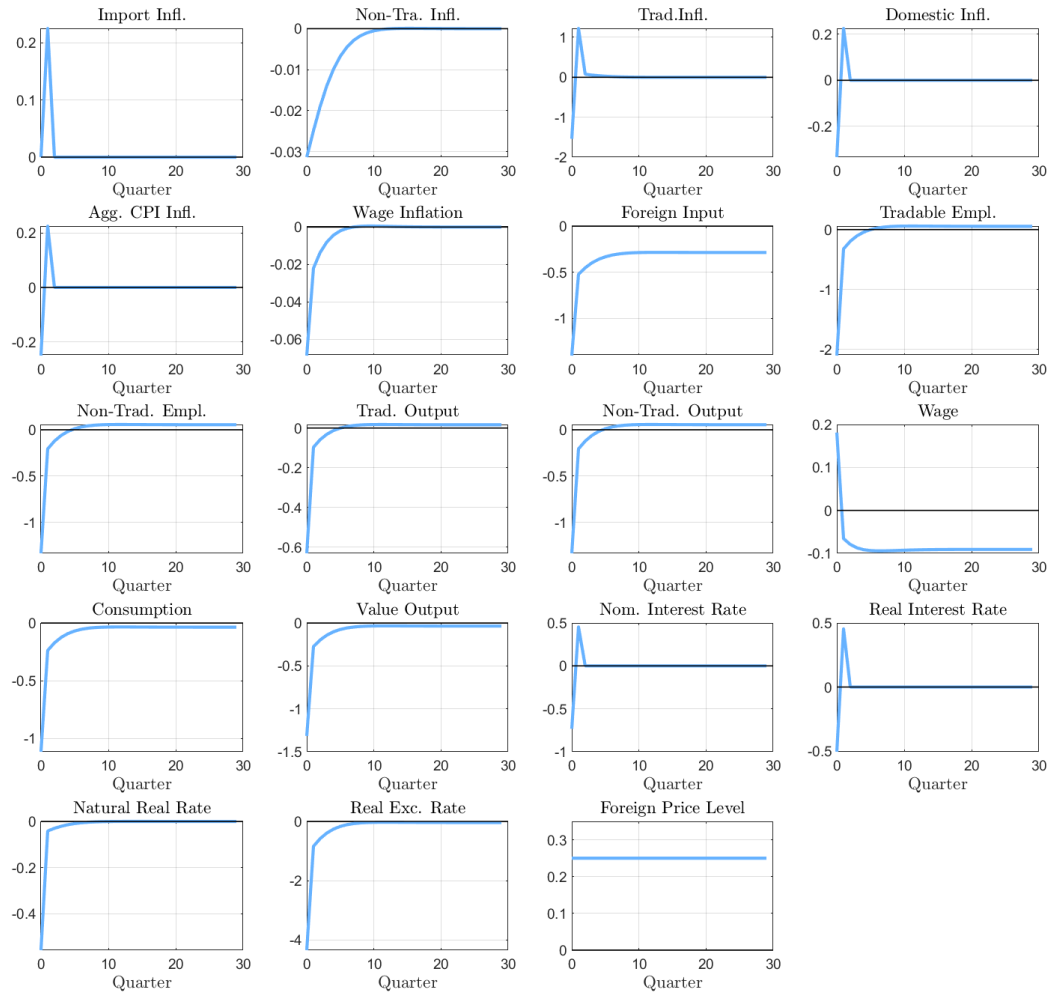
Figure 13: IRFs to a Gradual Increase in Foreign Price Level.



Notes: IRFs to a 100% gradual positive foreign price shock.

While the impact of a front-loaded import price shock on aggregate CPI inflation and domestic inflation is similar to the case without wage rigidities, the fall in domestic inflation is mostly due to the fall in tradables inflation instead, as non-tradable inflation falls by a negligible amount. Again, the fall in tradable output is a notable feature of this extension, relative to the baseline model. Higher labour costs lead tradable goods firms to cut production. Despite the fall in supply capacity for tradable goods, tradable inflation falls significantly, as consumption also falls to a greater degree than in the case without wage rigidities. The real exchange rate appreciates, resulting in a significant fall in exports and a smaller fall in imports, relative to the flexible wage case.

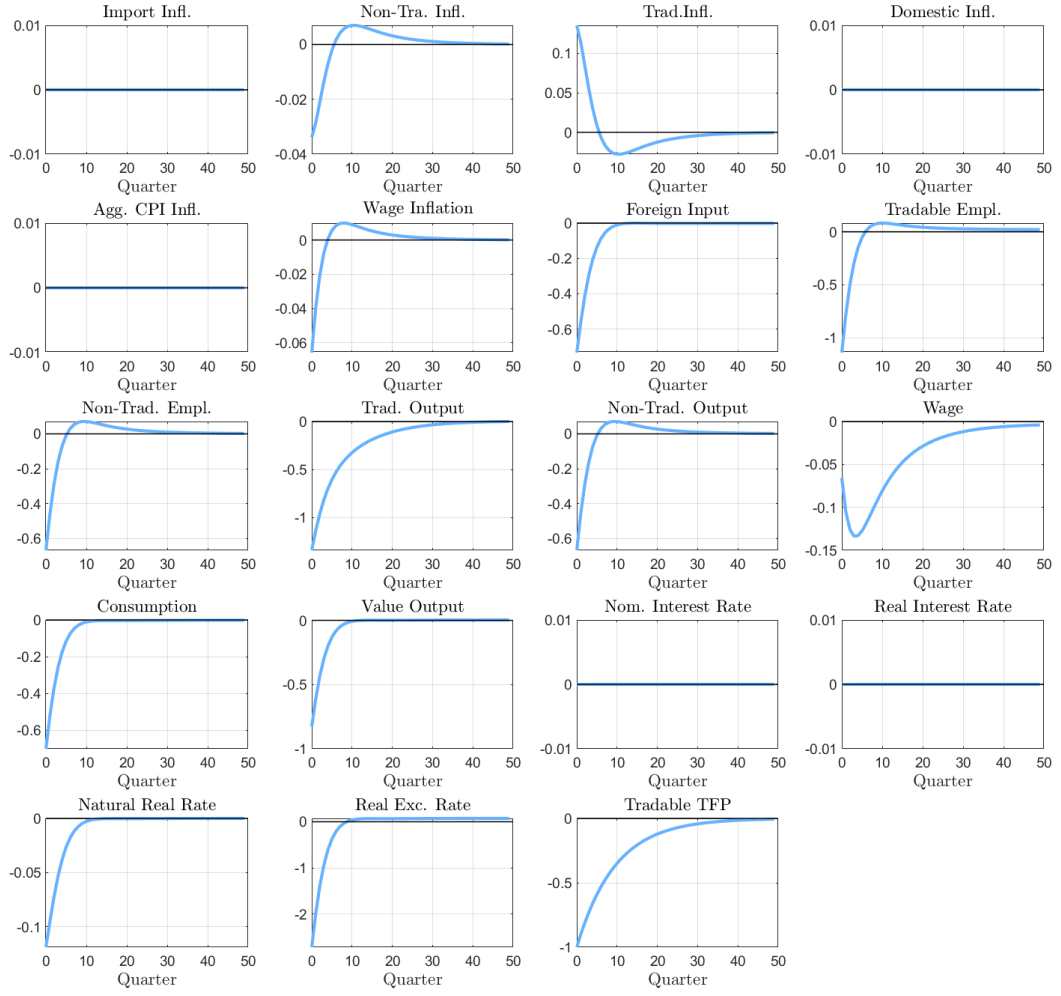
Figure 14: IRFs to a Front-loaded Increase in Foreign Price Level.



Notes: IRFs to a 25% positive foreign price shock.

Fall in Tradables Productivity Figure 15 shows the responses to an adverse productivity shock in the tradable goods sector. As in the case of an import price shock, nominal wage rigidities mitigate the fall in wages that would occur in the presence of flexible wages, resulting in a smaller decrease in the real wage. The increase in marginal costs leads to a sharper fall in tradable output. Relative to the flexible-wage case, tradable employment falls instead. In addition, non-tradable output now falls, as labour costs have increased for both sectors. Aggregate consumption falls by more in the sticky-wage case, despite a much shallower fall in the real wage. Tradable inflation still increases due to supply constraints in this sector, yet the substantial fall in aggregate demand in this extension mitigates its rise.

Figure 15: IRFs to a Negative Shock to Tradable TFP.



Notes: IRFs to a negative TFP shock in the tradable sector.

5 Conclusion

There is a broad consensus that a realignment of trading patterns is taking place. While trade fragmentation is likely to result in higher imported goods prices and lower real incomes, we show that the inflationary impact and the appropriate monetary policy response depend crucially on how demand adjusts to lower incomes, which in turn, depends on the form that fragmentation takes.

We study the macroeconomic effects of fragmentation using a two-sector, open economy New Keynesian model featuring imperfect risk sharing and heterogeneous households. We consider different fragmentation scenarios by varying the speed at which foreign prices adjust to a permanently higher level, as well as a negative shock to tradable sector productivity. The scenarios we consider capture the usual supply-side channels whereby higher input prices affect inflation, but emphasize how the overall effect on inflation depends on the adjustment of demand to lower real incomes. The balance between supply and demand differs in each of the scenarios, which has implications for inflation, and hence, monetary policy.

Our results suggest that trade fragmentation is not necessarily inflationary. In our first scenario, the gradual and permanent increase in foreign prices yields a persistent increase in imported inflation; the pass-through to aggregate inflation is counteracted by a fall in domestic inflation. The key mechanism for this result is that forward-looking households reduce their spending in anticipation of more restrictive future supply, as they respond to lower permanent income by smoothing consumption. The natural rate of interest decreases, suggesting that, when we allow for demand to adjust, the overall effect is not inflationary. The economy enters a period of stagnation, with lower real incomes, lower demand and lower inflationary pressures. In our second scenario, a front-loaded and persistent increase in the price of imported goods leads to a temporary period of stagflation, with lower consumption and higher inflationary pressures. Finally, a fall in the productivity of tradable goods leads to lower tradable output and an increase in tradable inflation, countervailed by a fall in non-tradable inflation driven by a fall in consumption and an increase in labour supply.

To understand the various channels shaping the response of demand and inflation to fragmentation, we vary the proportion of constrained households and the home bias in these three scenarios. We find that as the share of hand-to-mouth agents in the economy increases, demand falls by less in the gradual fragmentation scenario, as fewer unconstrained households anticipate the future fall in real incomes. More open economies are more exposed to shocks in foreign prices, which is reflected in the response of consumption and production. In both the front-loaded and gradual increase in imported prices scenarios, we see an amplification of these responses. However, this reverts in the case of a negative TFP shock, which is a domestic shock. In this scenario, more openness allows the economy to diversify away the impact of a domestic shocks on the economy, dampening the impact of the shock. Wage

rigidities introduce additional supply-side constraints through the domestic labour market, leading to a fall in output in both sectors, with implications for the composition of domestic inflation.

In future work, we plan to formalise the policy tradeoffs posed by fragmentation, study optimal monetary policy, and explore settings with higher inertia in price setting and lags in the transmission of monetary policy.

References

- Afrouzi, Hassan, Marina Halac, Kenneth S Rogoff, and Pierre Yared (2024). *Changing Central Bank Pressures and Inflation*. Tech. rep. National Bureau of Economic Research.
- Attinasi, Maria Grazia and Mirco Balatti (2021). “Globalisation and its implications for inflation in advanced economies”. In: *Economic Bulletin Articles* 4.
- Auclert, Adrien, Hugo Monneray, Matthew Rognlie, and Ludwig Straub (Aug. 2023). *Managing an Energy Shock: Fiscal and Monetary Policy*. Working Paper 31543. National Bureau of Economic Research.
- Auclert, Adrien, Matthew Rognlie, Martin Souchier, and Ludwig Straub (2021). *Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel*. Tech. rep. National Bureau of Economic Research.
- Benigno, Gianluca and Pierpaolo Benigno (2003). “Price stability in open economies”. In: *The Review of Economic Studies* 70.4, pp. 743–764.
- Bergholt, Drago (2014). “Monetary policy in oil exporting economies”. In.
- Broadbent, Ben, Federico Di Pace, Thomas Drechsel, Richard Harrison, and Silvana Tenreyro (Sept. 2023). “The Brexit Vote, Productivity Growth, and Macroeconomic Adjustments in the U.K.” In: *The Review of Economic Studies*, rdad086. ISSN: 0034-6527.
- Carluccio, Juan, Erwan Gautier, and Sophie Guilloux-Nefussi (2023). “Dissecting the impact of imports from low-wage countries on inflation”. In: *European Economic Review* 160, p. 104613. ISSN: 0014-2921.
- Catão, Luis and Roberto Chang (2013). “Monetary rules for commodity traders”. In: *IMF Economic Review* 61.1, pp. 52–91.
- Chan, Jenny, Sebastian Diz, and Derrick Kanngiesser (2024). “Energy Prices and Household Heterogeneity: Monetary Policy in a Gas-TANK”. In: *Journal of Monetary Economics*, p. 103620. ISSN: 0304-3932.
- Chen, Natalie, Jean Imbs, and Andrew Scott (2009). “The dynamics of trade and competition”. In: *Journal of International Economics* 77.1, pp. 50–62. ISSN: 0022-1996.
- De Paoli, Bianca (2009). “Monetary policy and welfare in a small open economy”. In: *Journal of international Economics* 77.1, pp. 11–22.
- Debortoli, Davide and Jordi Galí (2017). “Monetary policy with heterogeneous agents: Insights from TANK models”. In.
- Del Negro, Marco, Julian Di Giovanni, and Keshav Dogra (2023). “Is the green transition inflationary?” In: *FRB of New York Staff Report* 1053.
- Drechsel, Thomas, Michael McLeay, and Silvana Tenreyro (2019). “Monetary policy for commodity booms and busts”. In.

European Central Bank (2021). *The implications of globalisation for the ECB monetary policy strategy*. report 263. European Central Bank.

Fernández-Villaverde, Jesús, Tomohide Mineyama, and Dongho Song (June 2024). *Are We Fragmented Yet? Measuring Geopolitical Fragmentation and Its Causal Effect*. NBER Working Papers 32638. National Bureau of Economic Research, Inc.

Ferrero, Andrea and Martin Seneca (2019). “Notes on the Underground: Monetary Policy in Resource-Rich Economies”. In: *Journal of Money, Credit and Banking* 51.4, pp. 953–976.

Gali, Jordi and Tommaso Monacelli (2005). “Monetary policy and exchange rate volatility in a small open economy”. In: *The Review of Economic Studies* 72.3, pp. 707–734.

Galí, Jordi, J David López-Salido, and Javier Vallés (2007). “Understanding the effects of government spending on consumption”. In: *Journal of the European Economic Association* 5.1, pp. 227–270.

Georgieva, Kristalina (2023). “Confronting fragmentation where it matters most: trade, debt, and climate action”. In: *IMF Blog* 16.

Goodhart, Charles Albert Eric and Manoj Pradhan (2020). *The great demographic reversal: Ageing societies, waning inequality, and an inflation revival*. Vol. 10. Springer.

Guerrieri, Veronica, Michala Marcussen, Lucrezia Reichlin, and Silavna Tenreyro (2024). “The Art and Science of Patience: Relative Prices and Inflation”. In: *Geneva Report 2024*. URL: <https://cepr.org/publications/books-and-reports/geneva-26-art-and-science-patience-relative-prices-and-inflation>.

Harrison, Richard and Ozlem Oomen (2010). “Evaluating and estimating a DSGE model for the United Kingdom”. In.

Hevia, Constantino and Juan Pablo Nicolini (2013). “Optimal devaluations”. In: *IMF Economic Review* 61.1, pp. 22–51.

IMF (Apr. 2006). *How Has Globalization Affected Inflation?* Chap. 3.

Javorcik, Beata, Lucas Kitzmueller, Helena Schweiger, and Muhammed A Yildirim (2022). “Economic costs of friendshoring”. In: *The World Economy*.

Jones, Charles I (2023). *The AI dilemma: Growth versus existential risk*. Tech. rep. National Bureau of Economic Research.

Kamin, Steven, Mario Marazzi, and John Schindler (Jan. 2004). *Is China ‘Exporting Deflation’?* International Finance Discussion Paper 2004-791. Federal Reserve Board of Governors.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante (2018). “Monetary policy according to HANK”. In: *American Economic Review* 108.3, pp. 697–743.

Kaplan, Greg and Giovanni L Violante (2022). “The marginal propensity to consume in heterogeneous agent models”. In: *Annual Review of Economics* 14, pp. 747–775.

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- Kaplan, Greg, Giovanni L Violante, and Justin Weidner (2014). *The wealthy hand-to-mouth*. Tech. rep. National Bureau of Economic Research.
- Kimball, Miles S, Daniel Reck, Fudong Zhang, Fumio Ohtake, and Yoshiro Tsutsui (2024). *Diminishing marginal utility revisited*. Tech. rep. National Bureau of Economic Research.
- Kohn, Donald L. (2005). *Globalization, Inflation, and Monetary Policy*. Remarks delivered at the James R. Wilson Lecture Series, The College of Wooster, Wooster, OH, October 11.
- Lagarde, Christine (2023). “Breaking the persistence of inflation”. In: *Speech by President of the ECB, at the ECB Forum on Central Banking 2023 on “Macroeconomic stabilisation in a volatile inflation environment” in Sintra, Portugal* June.
- Roberts, John M. (Sept. 2006). “Monetary Policy and Inflation Dynamics”. In: *International Journal of Central Banking* 2.3.
- Rogoff, Kenneth et al. (2003). “Globalization and global disinflation”. In: *Economic Review-Federal Reserve Bank of Kansas City* 88.4, pp. 45–80.
- (2006). “Impact of globalization on monetary policy”. In: *a symposium sponsored by the Federal Reserve Bank of Kansas City on “The new economic geography: effects and policy implications”, Jackson Hole, Wyoming*, pp. 24–26.
- Romero, Roman E et al. (2008). *Monetary policy in oil-producing economies*. Citeseer.
- Rotemberg, Julio J (1982). “Sticky prices in the United States”. In: *Journal of political economy* 90.6, pp. 1187–1211.
- Santacreu, Ana Maria et al. (2005). *Reaction functions in a small open economy: What role for non-traded inflation?* Tech. rep. Reserve Bank of New Zealand.
- Sbordone, Argia M. (Apr. 2008). *Globalization and Inflation Dynamics: The Impact of Increased Competition*. Staff Report 324. Federal Reserve Bank of New York.
- Schmitt-Grohé, Stephanie and Martin Uribe (2006). *Comparing two variants of Calvo-type wage stickiness*. Tech. rep.
- Schmitt-Grohé, Stephanie and Martin Uribe (2003). “Closing small open economy models”. In: *Journal of international Economics* 61.1, pp. 163–185.
- Wills, Samuel (2013). “Optimal monetary responses to news of an oil discovery”. In.
- Yellen, Janet L (2022). “Remarks by Secretary of the Treasury Janet L. Yellen on Way Forward for the Global Economy”. In: *US Department of the Treasury* 13.
- Yellen, Janet Louise (2006). “Monetary policy in a global environment”. In: *Federal Reserve Bank of San Francisco*.

A Appendix: Model details

Appendix A.1 contains the derivation of some of the main equations under specific assumptions of the parameters. Appendix A.2 contains the relevant model equations for our exercise.

A.1 Derivations

Assuming $\sigma = \eta = \nu = 1$, we obtain

$$C_t \equiv \frac{C_{H,t}^{1-\alpha} C_{F,t}^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \quad (32)$$

$$C_{H,t} \equiv \frac{C_{N,t}^{1-\gamma} C_{T,t}^\gamma}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \quad (33)$$

$$P_t \equiv P_{H,t}^{1-\alpha} P_{F,t}^\alpha \quad (34)$$

$$P_{H,t} \equiv P_{N,t}^{1-\gamma} P_{T,t}^\gamma \quad (35)$$

We can write the aggregate domestic prices as follows:

$$P_t = P_{H,t}^{1-\alpha} P_{F,t}^\alpha = (P_{N,t}^{1-\gamma} P_{T,t}^\gamma)^{1-\alpha} P_{F,t}^\alpha \quad (36)$$

Finally, we can derive the necessary demand functions:

$$\begin{aligned} C_{H,t} &= (1-\alpha) \left(\frac{P_t}{P_{H,t}} \right) C_t = (1-\alpha) \mathcal{T}_t^\alpha C_t \\ C_{F,t} &= \alpha \left(\frac{P_t}{P_{F,t}} \right) C_t = \alpha \mathcal{T}_t^{\alpha-1} C_t \\ C_{n,t} &= (1-\gamma) \left(\frac{P_{H,t}}{P_{n,t}} \right) C_{H,t} \\ C_{T,t} &= \gamma \left(\frac{P_{H,t}}{P_{T,t}} \right) C_{H,t} \\ C_{N,t}(i) &= \left(\frac{P_{N,t}(i)}{P_{N,t}} \right)^{-\epsilon} C_{N,t} \end{aligned}$$

For clarity, we derive all the price dependencies below. From equation (34), we obtain

$$\begin{aligned} P_{H,t} &= \left(\frac{P_t}{P_{F,t}^\alpha} \right)^{\frac{1}{1-\alpha}} \\ P_{F,t} &= \left(\frac{P_t}{P_{H,t}^{1-\alpha}} \right)^{\frac{1}{\alpha}} \\ \mathcal{T}_t^{-\alpha} &= \frac{P_{H,t}}{P_t} \\ \mathcal{T}_t^{1-\alpha} &= \frac{P_{F,t}}{P_t} \end{aligned}$$

Now using equation (35):

$$\begin{aligned}\mathcal{T}_t^{-\alpha} &= \frac{P_{H,t}}{P_t} = \frac{P_{N,t}^{1-\gamma} P_{T,t}^\gamma}{P_t} \\ \mathcal{T}_t^{-\alpha} &= \frac{P_{N,t} P_{N,t}^{-\gamma} P_{T,t}^\gamma}{P_t} = \frac{P_{N,t}}{P_t} \left(\frac{P_{T,t}}{P_{N,t}} \right)^\gamma \implies \frac{P_{N,t}}{P_t} = \mathcal{T}_t^{-\alpha} \left(\frac{P_{N,t}}{P_{T,t}} \right)^\gamma \\ \mathcal{T}_t^{-\alpha} &= \frac{P_{N,t}^{1-\gamma} P_{T,t}^\gamma}{P_t} = \frac{P_{N,t}^{1-\gamma} P_{T,t}^\gamma}{P_t} \frac{P_{T,t}}{P_{T,t}} = \frac{P_{N,t}^{1-\gamma} P_{T,t}^{\gamma-1}}{P_t} P_{T,t} = \left(\frac{P_{N,t}}{P_{T,t}} \right)^{1-\gamma} \frac{P_{T,t}}{P_t} \implies \frac{P_{T,t}}{P_t} = \mathcal{T}_t^{-\alpha} \left(\frac{P_{T,t}}{P_{N,t}} \right)^{1-\gamma}\end{aligned}$$

Tradable goods are by definition consumed both at home and abroad,

$$\begin{aligned}C_{T,t} &= \gamma \left(\frac{P_{H,t}}{P_{t,t}} \right) C_{H,t} = \gamma \left(\frac{P_{H,t}}{P_{t,t}} \right) (1-\alpha) \left(\frac{P_t}{P_{H,t}} \right) C_t = \gamma(1-\alpha) \left(\frac{P_t}{P_{T,t}} \right) C_t \\ C_{T,t}^* &= \gamma\alpha \left(\frac{P_t^*}{P_{t,t}^*} \right) C_t^* = \gamma\alpha \left(\frac{P_t^* \mathcal{E}_t P_t}{P_{T,t} P_t} \right) C_t^* = \gamma\alpha \left(\frac{P_t}{P_{T,t}} \right) \mathcal{S}_t C_t^* \\ Y_{T,t} &= C_{T,t} + C_{T,t}^* = \gamma \left(\frac{P_t}{P_{T,t}} \right) [(1-\alpha)C_t + \alpha \mathcal{S}_t C_t^*]\end{aligned}$$

For non-tradable goods,

$$\begin{aligned}C_{N,t} &= (1-\gamma) \left(\frac{P_{H,t}}{P_{n,t}} \right) C_{H,t} \\ C_{N,t} &= (1-\gamma) \left(\frac{P_{H,t}}{P_{n,t}} \right) (1-\alpha) \left(\frac{P_t}{P_{H,t}} \right) C_t \\ C_{N,t} &= (1-\alpha)(1-\gamma) \left(\frac{P_t}{P_{n,t}} \right) C_t\end{aligned}$$

Therefore,

$$Y_t = (1-\alpha)(1-\gamma) \left(\frac{P_t}{P_{n,t}} \right) C_t + \gamma \left(\frac{P_t}{P_{T,t}} \right) [(1-\alpha)C_t + \alpha \mathcal{S}_t C_t^*].$$

Next, to derive the natural level of output, we use our assumption on $\sigma = \eta = 1$, we obtain

$$\begin{aligned}\mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right) \right] &= \left(\frac{C_t^*}{C_{t+1}^*} \right) \frac{\mathcal{S}_t}{\mathcal{S}_{t+1}} [1 + \chi(b_t^* - \bar{b}^*)], \\ Y_t^n &= \Sigma_{\alpha\gamma,t} \mu^{\frac{1}{1-\kappa}} A_{N,t}^{\frac{1}{1-\kappa}} p_{N,t}^{\frac{1}{1-\kappa}} \Gamma^{-\frac{1}{1-\kappa}} p_{F,t}^{-\frac{\kappa}{1-\kappa}} \left[\left(A_{T,t}^{-\frac{1}{1-\zeta}} \left(\gamma \left(\frac{P_t}{P_{T,t}} \right) ((1-\alpha)\mathcal{D}_t + \alpha) \mathcal{S}_t C_t^* \right)^{\frac{1}{1-\zeta}} \right. \right. \\ &\quad \left. \left. + (1-\alpha)(1-\gamma) \left(\frac{P_t}{P_{N,t}} \right) \mathcal{D}_t \mathcal{S}_t C_t^* \left(\frac{p_{F,t}}{p_{N,t}} \right)^{\frac{1}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \right)^{-\phi} \right]\end{aligned}$$

A.2 Appendix: Summary of Model Equations – TANK

Throughout this appendix and in the Dynare code, all expressions are in terms of aggregate domestic CPI (P_t). This implies the following,

$$p_{j,t} = \frac{P_{j,t}}{P_t} \quad \forall j = \{T, F, N, H\}$$

$$w_t = \frac{W_t}{P_t}$$

Rewriting the exchange rate in terms of foreign prices,

$$p_{F,t} = \frac{P_{F,t}}{P_t}$$

Households For unconstrained households, we have the marginal utility of consumption (A.1), the budget constraint (A.2), the intertemporal substitution of consumption conditions (A.3), the foreign bonds condition (A.4), and the intratemporal substitution between labour and consumption (A.5). For the constrained household, we only have the budget constraint (A.6), because they do not have access to financial markets, so they can't smooth consumption. Finally, we have the heterogeneity index (A.7) and total consumption (A.8).

$$\delta_t = (C_t^U)^{-\sigma} \quad (\text{A.1})$$

$$C_t^U + b_t + S_t b_t^* = b_{t-1} \frac{(1 + i_{t-1})}{(1 + \pi_t)} + S_t b_{t-1}^* \frac{(1 + i_{t-1}^*)}{(1 + \pi_t^*)} + \frac{W_t}{P_t} N_t^U - \frac{\chi}{2} S_t (b_t^* - \bar{b}^*)^2 \quad (\text{A.2})$$

$$\frac{1}{(1 + i_t)} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1}{(1 + \pi_{t+1})} \right] \quad (\text{A.3})$$

$$[1 + \chi(b_t^* - \bar{b}^*)] = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1 + i_t^*}{(1 + \pi_{t+1}^*)} \frac{S_{t+1}}{S_t} \right] \quad (\text{A.4})$$

$$(N_t^U)^\phi (C_t^U)^\sigma = w_t \quad (\text{A.5})$$

$$C_t^C = w_t N_t^C \quad (\text{A.6})$$

$$\Gamma_t = \frac{C_{u,t}}{C_t} \quad (\text{A.7})$$

$$C_t^U = \frac{1}{(1 - \lambda)} (C_t - \lambda C_t^C) \quad (\text{A.8})$$

Non-tradable goods firms We have a technology equation (A.9), production factor demand, respectively demand for foreign input (A.10) and for non-tradable labour (A.11), the

non-tradable Phillips Curve (A.12) and the Non-Tradable Domestic Demand (A.13).

$$Y_{N,t} = A_{N,t} M_{F,t}^\kappa N_{N,t}^{1-\kappa} \quad (\text{A.9})$$

$$p_{F,t} M_{F,t} = mc_t \kappa Y_{N,t} \quad (\text{A.10})$$

$$w_t N_{N,t} = mc_t (1 - \kappa) Y_{N,t} \quad (\text{A.11})$$

$$\Pi_{N,t} (\Pi_{n,t} - \bar{\pi}) = \beta \mathbb{E}_t \left[\frac{\delta_{t+1}}{\delta_t} \Pi_{n,t+1} (\Pi_{N,t+1} - \bar{\pi}) \frac{Y_{N,t+1}}{Y_{n,t}} \right] + \frac{\epsilon}{\xi} \left(mc_t - \frac{\epsilon - 1}{\epsilon} \right) \quad (\text{A.12})$$

$$Y_{N,t} = (1 - \alpha)(1 - \gamma) \left(\frac{C_t}{p_{N,t}} \right) \quad (\text{A.13})$$

Tradable goods firms We have a technology equation (A.14), employment demand (A.15), and total tradable demand (A.16). We do not have a price-setting equation because tradable goods firms take the price as given by the international market.

$$Y_{T,t} = A_{T,t} N_{T,t}^{1-\zeta} \quad (\text{A.14})$$

$$w_t N_{T,t} = (1 - \zeta) Y_{T,t} p_{T,t} \quad (\text{A.15})$$

$$Y_{T,t} = \left(\frac{\gamma}{p_{T,t}} \right) [(1 - \alpha) C_t + \alpha \mathcal{S}_t C_t^*] \quad (\text{A.16})$$

Total profits are given by:

$$\Psi_t = p_{T,t} Y_{T,t} - w_t N_{T,t} + p_{N,t} Y_{N,t} \left(1 - \frac{\xi}{2} (\Pi_n - \bar{\pi})^2 \right) - p_{F,t} M_{F,t} - w_t N_{N,t} \quad (\text{A.17})$$

Market Clearing The goods market clearing condition is given by (A.18), labour market clearing by (A.19), bond market clearing by (A.20), and foreign demand by (A.21).

$$Y_t = p_{T,t} Y_{T,t} + p_{N,t} Y_{N,t} \left(1 - \frac{\xi}{2} (\Pi_n - \bar{\pi})^2 \right) \quad (\text{A.18})$$

$$N_t^C \lambda + N_t^U (1 - \lambda) = N_{N,t} + N_{T,t} \quad (\text{A.19})$$

$$b_t^H = 0 \quad (\text{A.20})$$

$$Y_t^* = C_t^* \quad (\text{A.21})$$

Prices CPI is given by (A.22), inflation for non-tradables by (A.23) and tradables by (A.24), imported inflation by (A.25) and finally real depreciation by (A.28).

$$1 = (p_{N,t}^{1-\gamma} p_{T,t}^\gamma)^{1-\alpha} p_{F,t}^\alpha \quad (\text{A.22})$$

$$\pi_{N,t} = \frac{p_{N,t}}{p_{N,t-1}} \pi_t \quad (\text{A.23})$$

$$\pi_{T,t} = \frac{p_{T,t}}{p_{T,t-1}} \pi_t \quad (\text{A.24})$$

$$\pi_{F,t} = \frac{p_{F,t}}{p_{F,t-1}} \pi_t \quad (\text{A.25})$$

$$p_{H,t} = p_{N,t}^{(1-\gamma)} p_{T,t}^\gamma \quad (\text{A.26})$$

$$\pi_{H,t} = \frac{p_{H,t}}{p_{H,t-1}} \pi_t \quad (\text{A.27})$$

$$\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \Delta \mathcal{E}_t \frac{\Pi_t^*}{\Pi_t} \quad (\text{A.28})$$

Policy Rule Note that $1 + i_t = R_t$.

$$\frac{R_t}{R_{ss}} = \left(\frac{\Pi}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{Y}{\bar{Y}} \right)^{\phi_y} \exp(\nu_m) \quad (\text{A.29})$$

Natural real rate

$$(1 + r_t^n) = \frac{1}{\beta} \left(\frac{Y_{t+1}^n}{Y_t^n} \frac{\Sigma_{a\gamma,t}}{\Sigma_{a\gamma,t+1}} \right) \quad (\text{A.30})$$

AR (1) processes For non-tradable productivity, tradable productivity, foreign demand, foreign tradable prices and inflation and foreign interest rate, the exogenous processes are given by

$$A_{N,t} = (A_{n,ss})^{1-\rho_{an}} (A_{N,t-1})^{\rho_{an}} \exp(\nu_{an,t}) \quad (\text{A.31})$$

$$A_{T,t} = (A_{T,ss})^{1-\rho_{at}} (A_{T,t-1})^{\rho_{at}} \exp(\nu_{at,t}) \quad (\text{A.32})$$

$$C_t^* = (C_{t,ss}^*)^{1-\rho_{y^*}} (C_{t-1}^*)^{\rho_{y^*}} \exp(\nu_{y^*,t}) \quad (\text{A.33})$$

$$\log \Pi_{F,t} = \rho_F (\log p_{F,t-1} - \log p_{F,t-2} - \log p_{F,ss}) + \nu_{\Pi_f,t} \quad (\text{A.34})$$

$$i_t^* = (i_{ss}^*)^{1-\rho_i} (i_{t-1}^*)^{\rho_i} \exp(\nu_{i,t}) \quad (\text{A.35})$$

$$\Pi_t^* = 1. \quad (\text{A.36})$$

Frontloaded fragmentation The foreign price ($P_{F,t}$) for the front-loaded fragmentation scenario is an MA(53) process,

$$\log(P_{F,t}) = \sum_{j=1}^{53} \theta_j \nu_{P_{F,t-j}} \quad (\text{A.37})$$

Gradual Fragmentation The foreign price ($P_{F,t}$) for the gradual fragmentation scenario is

$$\log(P_{F,t}) = \log(P_{F,t-1}) + \rho_{P_F}(\log(P_{F,ss}(-\log(P_{F,t-1}))) + \nu_{P_{F,t}} \quad (\text{A.38})$$

Auxiliary Variables: The real Interest rate is given by (17), nominal imports by (A.40) and exports by s(A.41), natural level of output by (A.42).

$$RR_t = \frac{(1 + i_t)}{\Pi_{t+1}} \quad (\text{A.39})$$

$$IM_t = p_{F,t}(M_{f,t} + C_{f,t}) \quad (\text{A.40})$$

$$X = p_{T,t}\gamma\alpha\mathcal{S}_t\mathcal{C}_t^* \quad (\text{A.41})$$

$$\begin{aligned} Y_t^n = & \Sigma_{\alpha\gamma,t} \left(\mu^{\frac{1}{1-\kappa}} A_{N,t}^{\frac{1}{1-\kappa}} p_{N,t}^{\frac{1}{1-\kappa}} \Gamma^{-\frac{1}{1-\kappa}} p_{F,t}^{-\frac{\kappa}{1-\kappa}} \right)^{\frac{1}{\sigma}} \\ & \left(A_{T,t}^{-\frac{1}{1-\zeta}} \left(\left(\gamma(1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta \mathcal{D}_t^{\frac{1}{\sigma}} + \gamma\alpha \left(\frac{P_t}{P_{T,t}} \right)^\eta \right) \mathcal{S}_t\mathcal{C}_t^* \right)^{\frac{1}{1-\zeta}} \right. \\ & \left. + (1-\alpha)(1-\gamma) \left(\frac{P_{H,t}}{P_{N,t}} \right)^\nu \left(\frac{P_t}{P_{H,t}} \right)^\eta \mathcal{D}_t^{\frac{1}{\sigma}} \mathcal{S}_t\mathcal{C}_t^* \left(\frac{p_{F,t}}{p_{N,t}} \right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \right)^{-\phi} \right]^{\frac{1}{\sigma}} \end{aligned} \quad (\text{A.42})$$