How Well Does Bargaining Work in Consumer Markets? A Robust Bounds Approach*

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Abstract

This study provides a structural analysis of detailed, alternating-offer bargaining data from eBay, deriving bounds on buyers and sellers private value distributions and the gains from trade using a range of assumptions on behavior. These assumptions range from weak (assuming only that acceptance and rejection decisions are rational) to less weak (e.g., assuming that bargaining offers are weakly increasing in players' private values). We estimate the bounds and show what they imply for consumer negotiation behavior and inefficient breakdown. For the median product, bargaining ends in impasse in 35% of negotiations even when the buyer values the good more than the seller.

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1 Introduction

Bilateral bargaining is one of the oldest and most common forms of trade. A large theoretical literature and growing structural empirical literature examine the topic, but modeling choices of theorists and empiricists diverge widely, especially in how they consider the issue of *impasse*. Theoretical work (e.g., Myerson and Satterthwaite 1983) allows for the possibility that negotiators fail to agree even when gains from trade exist, whereas the workhorse model for empirical studies — Nash bargaining, in various forms — assumes that inefficient impasse *never* occurs. In these empirical models, negotiating agents know the opposing party's value precisely, and hence agents only negotiate over how to split a pie of known size. In many real-world settings, these strong assumptions are immediately rejected by data. In this paper, we analyze a large, detailed dataset of alternating-offer sequences from consumers negotiating online. We propose an approach to bound buyers' and sellers' values and the degree of inefficient impasse. Unlike Nash bargaining, the approach accommodates the presence of incomplete information. Our quantification exercises are aimed at providing motivation for more realistic theoretical empirical bargaining models and determining how well bargaining performs in practice in consumer markets.

The data comes from eBay's Best Offer platform, with thousands of eBay listings, each corresponding to a particular product identifier (such as an iPhone 6 or X-Box 360). For each listing, the seller posts a list (a Buy-It-Now) price and a buyer begins negotiating with a counteroffer. We observe these prices and all subsequent counteroffers between any buyer-seller pair.

We model each bilateral bargaining pair as a buyer with value $B \sim F_B$ negotiating sequentially with a seller of value $S \sim F_S$. The key objects we wish to bound are F_B, F_S, and $P(B \geq S)$, the probability that the buyer values the good more than the seller (the trade probability in a first-best world). Comparing this object to the realized trade probability in the data offers a measure of the inefficient impasse relative to the first-best outcome.

The challenges we face are, first, S and B are not observed in the data, and second, there is no theoretical characterization of equilibria in our game (bilateral negotiations in which both parties potentially have incomplete information and can make offers), and hence no obvious way to identify F_S and F_B from observed bargaining actions.[1] Indeed, unlike *auction* games or *complete-information bargaining* games (e.g. the Rubinstein 1982 model of non-cooperative bargaining with complete information), there is no canonical model of bargaining under *incomplete* information. This dearth is especially pertinent for studying consumer price negotiations, where agents meet and negotiate infrequently and where it is arguably particularly unrealistic to model agents as perfectly informed about the game's structure or opponents' values (as Nash bargaining presumes, for example).

To study this setting empirically, we propose a bounds approach based on an *incomplete* model. We first derive bounds on F_B and F_S. We begin with weak rationality assumptions on agents' behavior. We then propose stronger conditions that appear in previous game theoretic bargaining models: monotonicity (an agent's value being weakly increasing in her first offer) and independence (an agent's value being independent of her opponent's first offer). We demonstrate theoretically that these assumptions can be violated by unobserved game-level heterogeneity, and then propose two weaker conditions, stochastic monotonicity (an agent's value being stochastically increasing in her offer) and positive correlation (an agent's value being stochastically increasing in her opponent's value). Building on these assumptions, we derive bounds on the gains from trade and, in turn, the first-best trade probability, leading to bounds on the degree of inefficient breakdown in the eBay data.

The bounds under any given set of assumptions are sharp. We propose nonparametric estimators for the bounds and estimate them separately for each product, limiting to products for which we have at least 200 bargaining sequences. To assess the validity of the underly-

[1]Previous theoretical discussions have emphasized the difficulties of incomplete information in bargaining models. Fudenberg and Tirole (1991) claimed that "the theory of bargaining under incomplete information is currently more a series of examples than a coherent set of results. This is unfortunate because bargaining derives much of its interest from incomplete information." Binmore et al. (1992) observed, "In spite of this progress [in bargaining theory], important challenges are still ahead. The most pressing is that of establishing a properly founded theory of bargaining under incomplete information. A resolution of this difficulty must presumably await a major breakthrough in the general theory of games of incomplete information." These challenges, still unsolved today, arise from the sequential nature of the game: belief updating after off-equilibrium-path actions can sustain an infinite set of on-path behavior. Refinements, such as perfect Bayes equilibrium, do little or nothing to narrow the set of equilibria. See discussion in Gul and Sonnenschein (1988), and Ausubel et al. (2002) for a survey.

ing assumptions, we look for cases where bounds cross. We find evidence that our strongest assumptions (weak monotonicity of the seller's first offer or independence of the buyer's value and seller's first offer) can be violated. Bounds based on our weaker assumptions, such as stochastic monotonicity or weak monotonicity of the buyer's first offer conditional on the seller's, do not cross.

We also exploit *auto-accept* and *auto-decline* thresholds that sellers can report secretly to the platform; sellers respond to price offers that fall between these thresholds, but eBay automatically rejects or accepts prices lying outside of these thresholds. These secret prices are themselves bounds on the true distribution F_S. We confirm that the estimated bounds are consistent with these auto-accept/decline bounds (which are not explicitly used anywhere in computing our bounds).

Having demonstrated the informativeness of these bounds on the marginal distributions, we estimate bounds on the first-best trade probability separately for each product in our sample. Under the weakest assumptions, bounds on this object are uninformative, with the lower bound corresponding to the sale probability observed in the data and the upper bound being 1. Under our strongest assumptions, the bounds can cross. We propose assumptions of intermediate strength that are reasonable, informative, and do not cross.

The lower bound on the first-best trade probability can be compared to the trade probability in the data to infer a bound on the degree of inefficient impasse. For example, for a popular cell phone product in our sample, agents agree in the real-world negotiations 26.6% of the time. Under our preferred assumptions, the counterfactual first-best trade probability is bounded below by 0.465, suggesting that at least 43% of the time $(1-0.266/0.465)$, agents fail to reach an agreement even when the buyer truly values the good more than the seller. For the median product, this lower bound on inefficient impasse is 35.4%. The inefficient impasse lower bound ranges from 13.0% to 53.2% across all products.

We explore features of the negotiation or the agents themselves to study when efficiency appears to improve, which we define as a decrease in the inefficient impasse lower bound. We find that this lower bound decreases when agents communicate with one another via messages on the platform (although this decrease is not statistically significant), when the buyer is in the U.S., or when the seller provides more photos of the item. The largest improvement comes when sellers choose to use eBay's auto-accept/decline feature, which is associated with a decrease in the lower bound on inefficient impasse of 10 percentage points. More seller reviews or additional buyer experience, in contrast, are associated with an increase in the inefficient impasse lower bound.

Our study contributes to the theoretical and empirical bargaining literature. Theory work studies incomplete information bargaining either by explicitly modeling the extensive form of the game or applying mechanism design tools. Even our strongest assumptions (monotonicity and independence) are satisfied in the environments and equilibria of extensive-form games in the literature (e.g. Perry 1986, Grossman and Perry 1986, and Cramton 1992). We demonstrate, however, that in the presence of unobserved game-level heterogeneity (i.e., features of the negotiation that shift or scale the values of both agents in a given instance of the game, but that are unobservable to the econometrician), monotonicity assumptions can fail. This is not an indication that these theoretical equilibria cannot possibly describe real-world bargaining games well, but rather that data limitations (unobserved heterogeneity) can invalidate any attempt to use these existing theoretical results to analyze bargaining, even if the researcher is confident that she knows which of many equilibria generates the data. We show that our milder assumptions, such as stochastic monotonicity, can still be satisfied under unobserved heterogeneity.

In the mechanism design literature, our study is related to Myerson and Satterthwaite (1983), who demonstrated that when agents have independent values and face uncertainty about whether gains from trade exist, no incentive-compatible, individually rational mechanism will realize the first-best surplus without running a deficit. In his extensive-form game, Cramton (1992) showed that the first-best *probability* of trade is attainable if agents burn surplus to signal their values. Our study quantifies how close real-world negotiators in consumer markets get to that first-best trade probability.

Our work relates to a small but recently growing literature estimating structural models of incomplete-information bargaining. The most closely related studies are Keniston (2011), who studied bargaining for auto-rickshaw rides in India, and Larsen (2021), who analyzed bargaining between used-car businesses. Our study is distinct in several dimensions. First, we study a setting where both agents may be inexperienced negotiators (unlike the drivers in Keniston 2011 or used-car businesses in Larsen 2021). The importance of this distinction is that previous studies assumed more about agents' behavior (such as optimality) or knowledge of game outcomes that, while plausible for the frequent market participants and professionals in those studies, are unlikely to hold when applied to consumers in a marketplace like eBay. Our study develops a new, incomplete-model approach that relies on a series of intuitive (and falsifiable) assumptions, and takes these bounds to real-world consumer negotiation data to estimate private values and the degree of inefficient impasse. Our study is also distinct methodologically. Keniston (2011) relied on inequality bounds generated from a two-step dynamic game method. Larsen (2021) relied on auction (in addition to bargaining) data, and applied a special case of one of the bounds we propose herein (seller independence).[2] In contrast, the methodology we develop does not rely on auction data, only sequential-offer bargaining data, and extends beyond the independence case.

Several structural empirical studies have focused on take-it-or-leave-it-offer bargaining (e.g. Silveira 2017, studying judicial settings) or sequential bargaining with all offers by one party (Ambrus et al. 2018, studying ransom negotiations for Spaniards taken captive by North African pirates in the seventeenth century). Li and Liu (2015) studied incomplete-information bargaining in the form of a k double auction, where each party simultaneously makes a single offer. In our setting, multiple offers from both parties can and frequently do occur, and hence the frameworks of these previous papers do not apply.

Our work also relates to a literature exploiting eBay as a laboratory for studying fundamental questions of price discovery and efficiency. The structural literature examining efficiency of eBay trading mechanisms has largely focused on auctions (e.g. Hendricks

[2]Two structural empirical studies that also examined the used-car setting are Larsen and Zhang (2021) and Larsen et al. (2022). The latter paper relied on the methodology of Larsen (2021) and examined the impact of intermediaries in bargaining, while the former studied bargaining power using incentive compatibility and optimality assumptions that may be too strong for the consumer negotiations we study.

et al. 2021; Bodoh-Creed et al. 2021). Backus et al. (2020), Keniston et al. (2021), and Green and Plunkett (2022) documented a number of patterns in eBay bargaining data consistent with the existence of incomplete information and cognitive limitations, underscoring the benefit of our flexible approach bounding agents' values without assuming a complete model of fully rational equilibrium behavior.

Finally, our work connects to a large literature using partially identified models to study objects of interest, such as distributions of wages or treatment effects. Canonical papers in this literature include Manski (1989, 1990) and Manski and Pepper (2000), among others. Examples that, like ours, begins with weak, uncontroversial assumptions and add stronger assumptions to improve bound informativeness, are Blundell et al. (2007), studying wages, and Frandsen and Lefgren (2021), studying treatment effects. See Ho and Rosen (2017) for additional discussion of this literature.

2 eBay's Best Offer Platform

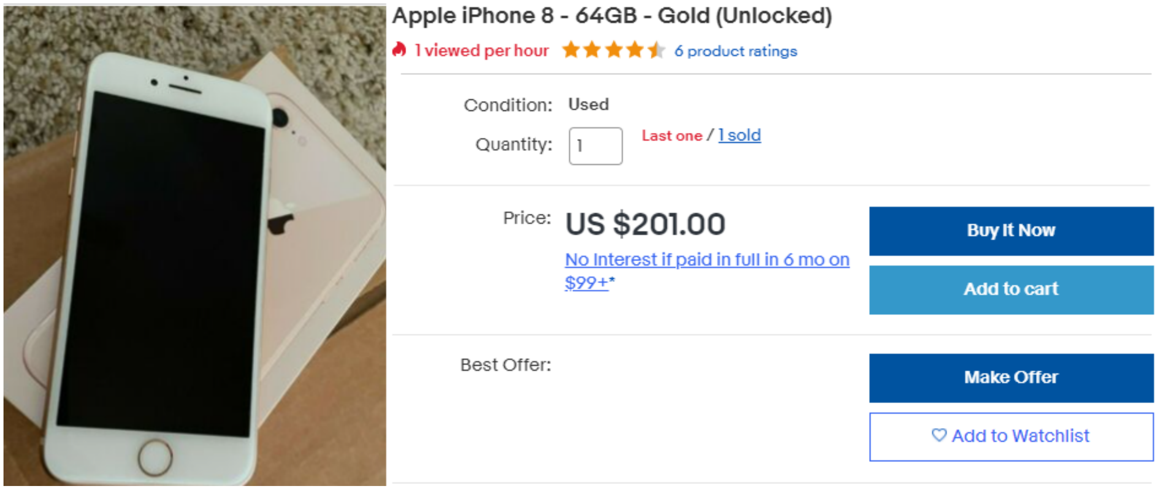
eBay's *Best Offer* option, introduced in 2006, permits a posted-price seller to "allow offers"; a prospective buyer will then see the Buy-It-Now price (which we will refer to as the *list* price) as well as a *Make Offer* button, illustrated for an iPhone 8 in Figure 1.[3] Clicking this button lets the buyer to propose an offer. The seller responds by declining, accepting, or countering. If she counters, it is then the buyer's turn to accept, decline, or counter. If the seller declines, the buyer may still choose to make a counteroffer. Each party is limited to three offers, and the buyer can purchase at the list price at any time.[4] If any agent delays responding more than 48 hours, the offer expires (effectively declining).

Our data comes from a mix of business-to-consumer and consumer-to-consumer price negotiations: buyers are typically retail consumers while sellers may be businesses or in-

[3]This option has, for many years, been the default, and sellers must opt out to disable it.

[4]This three-offer limit stood during our time period; in more recent years, eBay moved to a five-offer limit. Sellers can specify *auto-accept* and *auto-decline* prices; buyer offers above the auto-accept or below the auto-decline price are automatically accepted or declined by the platform. In our analysis we take advantage of these secretly reported thresholds to examine the validity of our bounds.

Figure 1: Illustration of Best Offer Listing



dividuals. Consumers thus play an important role in this market, in contrast to the professional negotiators studied in Keniston (2011), Ambrus et al. (2018), or Larsen (2021). As some consumers may be particularly inexperienced with the eBay game, our study adopts a robust bounds approach that does not require a complete model of equilibrium behavior.

We use data created for the descriptive analysis of Backus et al. (2020), which studied bargaining on the U.S. eBay site from June 2012 to May 2013. We arrange the data with an observation being a given bargaining sequence, containing the list price and all offers.[5] For this paper, we focus on a subset of items with well-labeled product identifiers (bar codes), such as "Apple iPhone 8 64 GB" or "Xbox 360". We observe each item's condition (used/new), and we consider a *product* to be a combination of the condition type and product identifier. The data includes a *reference price*, the average of all non-Best-Offer posted-price sales of that same product during the sample period.[6] We restrict the sample to products with reference prices constructed from at least ten sales. We limit the sample to listings to which a buyer makes an offer. This means we do not analyze cases where a buyer arrives at a listing page but immediately buys at the Buy-It-Now price or leaves

[5]We use the terms *negotiation* (or *negotiation sequence*) and *bargaining sequence* (or *sequence*) interchangeably.

[6]We use the reference price as a normalization to put each product on a similar scale by dividing prices/offers by its reference price.

without making an offer. Our motivation for focusing on these listings is that we wish to analyze the degree of inefficient impasse conditional on both the buyer and seller indicating an interest in negotiating.

We refer to this sample as our original data. We impose several restrictions to remove a small fraction of incomplete sequences or cases where offers are extreme outliers. Section 3.2 discusses two additional sample restrictions that are best understood after introducing the model. To leave sufficient data to estimate our bounds, we also limit to products for which we observe at least 200 negotiation sequences after imposing all other restrictions. In the end, we are left with 14,557 bargaining sequences corresponding to 44 products. Appendix Table 7 describes each sample restriction.

Table 1: Descriptive Statistics: Highest-Selling Product per Category and Full Sample

Category	Reference Price ($)	n	P(sale)	Final Price Over List (if trade)	Buyer Price Over List (if no trade)	Seller Price Over List (if no trade)
Consumer Electronics	51.44	419	0.43	0.77	0.54	0.97
Video Games/Consoles	80.78	342	0.42	0.84	0.6	0.96
Cell Phones	224.82	1,227	0.26	0.87	0.67	0.99
Computers/Tablets	543.27	321	0.12	0.89	0.63	0.99
All Products	215.71	14,557	0.3	0.83	0.63	0.98

Notes: First four rows show statistics for top-selling product in each category. Final row shows same statistics for estimation sample of 44 products (14,557 observations). n represents number of observations.

The data does not specify product titles (only anonymous product identifiers) but does specify product categories. Table 1 displays statistics for the top-selling product in each of the four categories in our final sample. These four products are all used products (only 3 of the 44 products in our sample are new). The products in Table 1 vary in reference price, ranging from $51 for the consumer electronics product to $543 for computer/table product. When trade occurs, the final price as a fraction of the list price ranges from 0.77 to 0.89. When trade fails, the highest price offered by the buyer as a fraction of the list price ranges from 0.54 to 0.67, whereas the lowest price offered by the seller in these disagreement cases ranges from 0.97 to 0.99. A key object of interest in this study is the fourth column, the sale probability ($P(sale)$), which varies widely — from 0.12 for the computers/tablets product (which was the subject of 321 negotiations) to 0.43 for the consumer electronics product (419 negotiations). Our empirical approach allows us to quantify inefficiency by constructing bounds on the *counterfactual first-best* trade probability.

3 Bounds on Values in a Bargaining Game

In this section we present bounds on buyers' and sellers' values in an alternating-offer bargaining game, the protocol used on eBay.[7] We begin with bounds under minimal assumptions, using revealed preference arguments only. We then introduce assumptions on strategic behavior and the dependence of buyer and seller values to tighten bounds.

3.1. Bargaining Game Setup and Notation. A seller has value $S \sim F_S$ and a buyer has value $B \sim F_B$.[8] If the buyer and seller agree on a price P, the buyer's payoff is $B - P$ and the seller's is P. If they break up the bargaining (i.e., some agent quits), the seller gets S and the buyer gets 0. Throughout the paper, we maintain the assumption that, in a given instance of the game, S and B are known to the seller and buyer, respectively, before any actions take place and are held fixed throughout the negotiation.[9] To match the structure of eBay's protocol, we treat the first bargaining offer as coming from the seller (the list price). The buyer makes the second offer. The seller can then choose to accept, counter, or quit. We refer to each turn as a *period*, beginning with the seller at $t = 1$.

Denote the offers of the seller in odd periods t by P_t^S and the offer by the buyer in even periods t by P_t^B. Denote the decision of the buyer in even t by $D_t^B \in \{a, c, q\}$ (representing "accept", "counter", and "quit"). Similarly, let D_t^S be the decision of the seller in odd t. Either player accepting or quitting ends the game. For a given instance of the game, the data consists of the sequence of offers and any decisions to accept or quit.[10]

[7] The bounds we derive can be modified to allow for protocol other than alternating offers.

[8] Throughout, we use uppercase letters to denote random variables and lowercase to denote realizations.

[9] We allow for the buyer and seller to be learning about their opponents' values during the game, but not for agents to learn any additional information about *their own* values (such as learning about the quality of the good), which Desai and Jindal (2020) show in laboratory experiments is certainly a possibility.

[10] If the buyer has not reached the three-offer limit, she can make an additional offer following a seller choice to decline; we reclassify such sequences as consisting of the seller having not declined but rather

We define the following random variables at the *sequence*- (rather than period-) level: Let $X_{AC}^S = \min\{\{P_t^B : D_{t+1}^S = a\}, \min_t\{P_t^S : D_t^S = c\}\}$ be the smallest offer the seller makes or accepts in a given sequence (where "AC" stands for "accepting or countering"). Note that X_{AC}^S is always defined because the seller always makes the first offer (so $D_1^S = c$). Also, let $X_Q^S = \max\{\{P_t^B : D_{t+1}^S = q\}, 0\}$ be the offer at which a seller quits, if she does quit, and 0 otherwise. There can be at most one offer in a sequence at which an agent quits. The definition ensures that X_Q^S is well defined even if $D_t^S \neq q$ for all t. Let $X_{AC}^B = \max\{\{P_t^S : D_{t+1}^B = a\}, \max_t\{P_t^B : D_t^B = c\}\}$ be the largest price a buyer accepts or offers in a given sequence. This is always defined because we focus on cases where buyers make offers P_2^B (so $D_2^B = c$). Let $X_Q^B = \min\{\{P_t^S : D_{t+1}^B = q\}, \infty\}$ be the offer at which a buyer quits, if indeed the buyer quits, and ∞ if $D_t^B \neq q$ for all t.[11]

A number of the arguments we derive below rely on the following identities, which are representations for F_S and F_B applying the law of iterated expectations:

$$P(S \leq x) = \int P(S \leq x \mid P_1^S = y) dF_{P_1^S}(y) \tag{1}$$

$$P(B \leq x) = \int P(B \leq x \mid P_1^S = y, P_2^B = z) dF_{P_1^S, P_2^B}(y,z) \tag{2}$$

where $F_{P_1^S}$ is the CDF of P_1^S and $F_{P_1^S,P_2^B}$ is the joint distribution of P_1^S, P_2^B. When written as a function, $P(\cdot)$ represents the probability of a given event.

As a final piece of notation, for given random variables X and Y, let $\overline{\text{supp}}(Y \mid X \geq a)$ and $\underline{\text{supp}}(Y \mid X \geq a)$ be the maximum and minimum of the support of Y given $X \geq a$, respectively. Define $X_{AC}^{S*}(y) \equiv \underline{\text{supp}}(X_{AC}^S : P_1^S \geq y)$ and $X_Q^{S*}(y) \equiv \overline{\text{supp}}(X_Q^S : P_1^S \leq y)$. Thus, $X_{AC}^{S*}(y)$ is the smallest accept/counter price of sellers conditional on the first offer of sellers being at least y. $X_Q^{S*}(y)$ has a similar interpretation. For all (y,z) on the support of (P_1^S, P_2^B), define $X_{AC}^{B*}(y,z) \equiv \overline{\text{supp}}(X_{AC}^B : P_2^B \leq z, P_1^S = y)$ and $X_Q^{B*}(y,z) \equiv \underline{\text{supp}}(X_Q^B : P_2^B \geq z, P_1^S = y)$. Conditional on P_1^S, $X_{AC}^{B*}(y,z)$ is the largest accept/counter price of buyers conditional on events where the second offer of buyers is at most z. $X_Q^{B*}(y,z)$ has a similar interpretation.

having countered at her previous offer. Thus, sequences ending with a seller *quitting* only occur if a seller declines an offer and there is no further action by the buyer.

[11] The support points ∞ and 0 are conservative and can easily be replaced with other assumptions.

3.2. Model Discussion Before deriving bounds, we discuss dynamics *across* negotiations, an issue not explicitly modeled but allowed for in our framework. While we refer to B and S as "values" for brevity, more precise terms would be "net values," or "willingness to pay" and "willingness to sell." Throughout the paper, we focus on one negotiating pair at time, but our framework allows for the possibility that, in a given negotiation, the buyer may receive gross utility V from trading and a nonzero outside option μ from not trading, where μ is a continuation value in a broader game where the buyer returns to the platform or looks for the item elsewhere. The buyer's willingness to pay in this negotiation is $V - \mu$, and we refer to this quantity as the buyer's value, $B \equiv V - \mu$. For our purposes, when studying buyers, we do not need to separately identify bounds on V and μ (nor can we using our method); we only seek bounds on F_B.

Similarly, the seller's willingness to sell S is the utility the seller receives if trade does not occur, which, in a broader continuation game, represents the value she receives from re-entering the eBay market, attempting to sell the item elsewhere, or keeping the good. Thus, in this broader game, the objects S and B are not primitives. They are nonetheless the initial objects we are interested in bounding because doing so will allow us to study several properties of bargaining offers and a notion of efficiency that does not require separately identifying V and μ or unpacking S into more primitive objects.[12]

This notion of B representing a buyer's willingness to pay suggests that, in the eBay context, an upper bound on B for a given item will be P_1^S (the list/Buy-It-Now price), because an eBay buyer can choose to purchase at that price at any time, even after a breakdown in negotiations. We can incorporate this information into our notation above by modifying the definition of X_Q^B (which, as we will show, provides an upper bound on B) to be $\min\{\{P_t^S : D_{t+1}^B = q\}, P_1^S, \infty\}$. In our analysis, we will say that we use the *list-price-recall condition* when taking advantage of this alternative definition of X_Q^B (see Section 5).

As highlighted above, we treat B and S as fixed during a given negotiation and do not consider the possibility of these changing (such as through the buyer's outside option chang-

[12] Appendix D shows how several forms of bargaining costs would also fit into this framework.

ing). Such changes are certainly possible if a buyer negotiates with multiple sellers at once or vice versa. To minimize this possibility, we limit our sample to negotiations in which neither party negotiates with multiple opponents in an overlapping time window.[13] This restriction drops 48.6% of negotiations. Among the remaining, non-overlapping sequences, we limit to the first seller of a given product with whom a given buyer negotiates, and, among these, the first buyer with whom a given seller negotiates. This drops 16% of negotiations (see Table 7 in the Appendix) and yields a unique set of negotiating pairs: no seller appears twice for the same listing and no buyer appears twice for the same product.

It is tempting to retain data on cases where a given seller negotiates with multiple (non-overlapping) buyers over time (for example, one buyer today and another buyer a week later), under the additional assumption that S stays constant *across* negotiations, not just within a negotiation. If this stronger assumption were true, future and past negotiations between a given seller and different buyers would contain information about S in the current negotiation. We do not adopt this assumption and instead limit our data to a single buyer per seller and vice versa. Our motivation is twofold: many of the bounds we derive do not immediately apply to cases where an agent negotiates with multiple opponents over time, and attempts to extend these assumptions quickly become unwieldy (see Section 3.4), making a set of unique pairs appealing. This set of unique pairs could be formed using the last (or a random) seller among those with whom a given buyer negotiates, rather than the *first*. Our motivation for selecting the *first* is to reduce a type of selection that could occur in the bargaining. Specifically, if, after an agreement, an agent exits the market, then among multiple sequences involving a given agent, the later sequences are more likely to involve agreement. This can lead to buyer and seller values being *negatively correlated* in later

[13]We define this time window for a given negotiation as the time from the buyer's first offer to the last action taken. The 25th and 75th percentile of window length are 9 seconds and 6.29 hours; the 10th and 90th percentiles are zero seconds (which can happen if the sequence ends immediately through the auto accept/decline mechanism) and 48 hours. We say a sequence has *overlapping buyers* if the window overlaps that of any other sequence for that same seller/item and some other buyer (or if some other buyer bought the item through the Buy-It-Now option on a day that overlaps the window). A sequence has *overlapping sellers* if the window for a given buyer/product with a given seller overlaps the window for that same buyer/product and some other seller. 44.4% of negotiations in the original data have overlapping buyers, 9.0% have overlapping sellers, and some have both, leading to 48.6% overall that have either.

sequences, with a higher-value buyer negotiating with a lower-value seller (and hence the parties agree to trade).[14]

3.3. Sharpness In our presentation of bounds below, we will say a CDF, F, is in the identified set of buyer values under a given set of assumptions if there exists a data generating process (DGP) satisfying those assumptions such that F is the CDF of buyer values. If no such DGP exists, F is not in the identified set under those assumptions. Our sharpness notion is one sided: under the assumptions and data used to derive a given lower bound, that bound and any CDF lying above it are in the identified set. Similarly, under the assumptions used to derive a given upper bound, that bound and any CDF lying below it are in the identified set.[15] For some bounds, our proofs do not imply that the lower and upper bounds would constitute the sharp identified set if we were to consider the upper and lower bound assumptions jointly. These issues are related to sharpness discussions in Chesher and Rosen (2017) and Molinari (2020).[16]

3.4. Unconditional Bounds on Value Distributions. We now describe a range of assumptions about equilibrium behavior yielding sharp bounds on marginal value distributions. Our first and weakest assumption is the following:

Assumption A1 (Revealed Preferences)**.** *The seller (i) never accepts (or counters) at a price $P < S$ and (ii) never quits at a price $P > S$. The buyer (iii) never accepts (or counters) at a price $P > B$ and (iv) never quits at a price $P < B$.*

These assumptions are similar to those in Haile and Tamer (2003) for English auctions.[17]

[14]While this is our motive for focusing on the first seller a buyer engages with and vice versa, in practice this restriction has a negligible effect on our results, which are similar when using the last (or a random) seller a buyer negotiates with (and vice versa); see Appendix Table 8.

[15]Throughout the paper we will use the term "lower bound" on a CDF to refer to a bound lying graphically below that CDF (and vice-versa for "upper bound"), although a graphical lower bound is in fact an upper bound on the random variable in the stochastic dominance sense.

[16]For example, with interval data, it is not necessarily true that a CDF will be in the identified set even if it lies between the CDF of the lower bounds of intervals and the CDF of the upper bounds of the intervals.

[17]There, the authors assume that a bidder (i) never bids above her value and (ii) never lets another agent win at a price she is willing to beat. An important distinction is that upper and lower bounds exist *for each* observation in Haile and Tamer (2003). In contrast, in the two-sided bargaining game we study, a given negotiation may end with one side of the bounds unobserved (e.g., because no agent quits). We handle this complication by relying on *probabilities* of events, rather than on empirical CDFs of prices/bids alone.

Importantly, A1 imposes only a weak rationality condition, not requiring that agents behave according to any equilibrium concept, although the conditions are weak enough to be satisfied by standard concepts, such as Bayes Nash or Perfect Bayes.

Important implications of this assumption are that $X_Q^S \leq S \leq X_{AC}^S$ and $X_{AC}^B \leq B \leq X_Q^B$. These inequalities imply what we call our *unconditional* bounds on F_S and F_B:

$$P(X_{AC}^S \leq x) \leq F_S(x) \leq P(X_Q^S \leq x) \tag{3}$$

$$P(X_Q^B \leq x) \leq F_B(x) \leq P(X_{AC}^B \leq x) \tag{4}$$

Theorem 1. *(3) gives a sharp lower bound for F_S under A1.i and a sharp upper bound for F_S under A1.ii. (4) gives a sharp lower bound for F_B under A1.iv and a sharp upper bound for F_B under A1.iii.*

All proofs are in the appendix. The proof follows immediately from $X_Q^S \leq S \leq X_{AC}^S$ and $X_{AC}^B \leq B \leq X_Q^B$. For sharpness of Theorem 1, given that we place no restrictions on behavior other than A1, nothing in the data or assumptions rules out the possibility that the play of the game is such that $X_{AC}^S = S$, and similarly for the other bounds. These bounds can be relatively tight in some cases and loose in others. Appendix F offers Monte Carlo simulations illustrating this point.

The bounds are nonparametric. They are weakly increasing and lie in $[0,1]$, and thus can correspond themselves to a CDF. The bounds will be valid even if the game has multiple equilibria, and, in particular, even if the data is not all generated by the same equilibrium or by any standard notion of equilibrium play. Furthermore, if the true DGP does not in fact entail sellers all drawing from the same distribution F_S — that is, if sellers (or, analogously, buyers) are asymmetric — the bounds will remain valid for the mixture distribution of values in the data.

These bounds place no restrictions on the dependence between B and S. For example, the bounds allow for the possibility that B and S are correlated through game-level heterogeneity that is either unobservable or observable to the econometrician.[18] One form of heterogeneity is $S = W + \tilde{S}$ and $B = W + \tilde{B}$, where $\tilde{S}$, $\tilde{B}$, and W are independent, and where

[18]In the case of used cell phones that we examine in Section 5, unobserved heterogeneity can include

W is known to both agents but not the econometrician. In this scenario, S and B are independent conditional on W, but, from the analyst's perspective, are correlated *across instances of the game* through W. A related possibility is multiplicative separability, where $S = W\tilde{S}$ and $B = W\tilde{B}$.[19] We do not impose either structure but highlight these below as special cases allowed for by our moderate assumptions and ruled out by our strongest assumptions (and by some existing theoretical models).

3.5. What Assumptions Are Suggested by Bargaining Theory? We now explore whether theoretical bargaining models of the environment we study suggest assumptions to tighten the unconditional bounds. There are very few extensive-form equilibria studied in the literature from bargaining games close in generality to the one we study — a bargaining game with two-sided incomplete information and a continuous value distribution where both parties can make offers.[20] The two models closest to our setting are those of Perry (1986) and Cramton (1992). Appendix G describes these in detail. Our bounds allow for a much wider range of possible outcomes than the equilibria in these two papers; indeed, these equilibria are among infinitely many that our bounds accommodate. We focus on these examples only because they are the two existing examples general enough to relate to our framework. Table 2 summarizes the assumptions and theoretical models we consider.

3.5.1. Monotonicity. In both Perry (1986) and Cramton (1992), offers satisfy a property we refer to as *monotonicity*, describing how an agent's first offer relates to her own value:

Assumption A2 (Monotonicity)**.** *(i) S is weakly increasing in P_1^S, and (ii) B is weakly increasing in P_2^B conditional on P_1^S.*

A2.i describes own-offer weak monotonicity for sellers: for $y < y'$, a seller with $P_1^S = y$ has a weakly lower value than a second seller who has chooses $P_1^S = y'$, and therefore

aspects of the seller's reputation or of the cell phone that both the buyer and seller observe but not the econometrician (e.g., a cracked screen in a listing photo or a protective covering included with the phone).

[19] Multiplicative or additive separability are two structures for unobserved heterogeneity commonly assumed in empirical auction work (e.g., Krasnokutskaya 2011; Freyberger and Larsen 2022).

[20] See Table A9 of Larsen (2021) for a breakdown of the theoretical literature modeling extensive-form, incomplete-information bargaining games. This literature largely focuses on models where only one side has a private value, only one side is allowed to make offers, or agents have only two possible values.

Table 2: Assumptions' Relationships to Models of Two-Sided Incomplete Information

Model/Environment	Uncond. A1	Monoton. A2	Indep. A3	Stoch. Monoton. A4	Positive Corr. A5
Cramton (1992)	✓	✓	✓	✓	✓
Cramton (1992) + Unobs. Het.	✓			✓	✓
Perry (1986)	✓	✓	✓	✓	✓
Perry (1986) + Unobs. Het.	✓			✓	✓
Assumption Type	Behavioral	Behavioral	Envir.	Behavioral	Envir.

Notes: Table shows which assumptions are satisfied in Cramton (1992) and Perry (1986) models, as well as in modifications of those models we derive with unobserved heterogeneity (Appendix G). Final row distinguishes between assumptions about behavior vs. those about information environment.

the lowest price at which the second seller counters or accepts is an upper bound on the value of the first seller. This is precisely what is represented by $X_{AC}^{S*}(y)$, defined in Section 3.1. Similarly, A2.i implies that the highest quit price among sellers with first offers less than y provides a lower bound on the value of the seller who has $P_1^S = y$. Part (ii) of A2, monotonicity for the buyer, is weaker, as it is *conditional* on the seller's first offer. These arguments yield

$$\int \mathbf{1}(X_{AC}^{S*}(y) \leq x) dF_{P_1^S}(y) \leq F_S(x) \leq \int \mathbf{1}(X_Q^{S*}(y) \leq x) dF_{P_1^S}(y) \tag{5}$$

$$\int \mathbf{1}(X_Q^{B*}(y,z) \leq x) dF_{P_1^S,P_2^B}(y,z) \leq F_B(x) \leq \int \mathbf{1}(X_{AC}^{B*}(y,z) \leq x) dF_{P_1^S,P_2^B}(y,z) \tag{6}$$

Theorem 2. *(5) gives a sharp lower bound for F_S under A1.i and A2.i and a sharp upper bound for F_S under A1.ii and A2.i. (6) gives a sharp lower bound for F_B under A1.iv and A2.ii and a sharp upper bound for F_B under A1.iii and A2.ii*

These *monotonicity* bounds are derived as follows: Under A2 and conditional on $P_1^S = y$, we have $X_Q^{S*}(y) \leq S \leq X_{AC}^{S*}(y)$, and the objects $X_{AC}^{S*}(y)$ and $X_Q^{S*}(y)$ are non-random. We plug these objects into (1) to obtain (5), where $\mathbf{1}(\cdot)$ is the indicator function. The buyer bounds follow similarly. The bounds improve upon the unconditional ones by comparing the accept/counter or quit actions of agents across instances of the game. Appendix F shows simulations where the bounds do vs. do not improve upon the unconditional ones. Appendix G shows that Perry (1986) and Cramton (1992) satisfy monotonicity.

3.5.2. Independence. This assumption relates an agent's *value* to the opponent's *offer*:

Assumption A3 (Independence). *(i) S is independent of P_2^B conditional on P_1^S, and (ii) B is independent of P_1^S.*

As the seller makes the first move, a natural assumption is that the seller's first offer depends on S. A3.ii takes this one step further and assumes that the seller's first offer does not depend on B; A3.i describes a similar condition for the seller's value, but conditional on the seller's first offer. Through this relationship between an agent's value and an opponent's offer, A3 captures a notion of independence between values. This assumption is satisfied in the Perry (1986) and Cramton (1992) environments.

A3 implies the following *independence* bounds:

$$\int \max_z m_{AC}^S(x,y,z)dF_{P_1^S}(y) \leq F_S(x) \leq \int \min_z m_Q^S(x,y,z)dF_{P_1^S}(y) \tag{7}$$

$$\max_{y'} P(X_Q^B \leq x \mid P_1^S = y') \leq F_B(x) \leq \min_{y'} P(X_{AC}^B \leq x \mid P_1^S = y') \tag{8}$$

where $m_{AC}^S(x,y,z) = P(X_{AC}^S \leq x \mid P_1^S = y, P_2^B = z)$, $m_Q^S(x,y,z) = P(X_Q^S \leq x \mid P_1^S = y, P_2^B = z)$.

Theorem 3. *(7) gives a sharp lower bound for F_S under A1.i and A3.i and a sharp upper bound for F_S under A1.ii and A3.i. (8) gives a sharp lower bound for F_B under A1.iv and A3.ii and a sharp upper bound for F_B under A1.iii and A3.ii*

These bounds are obtained by applying (1) and (2) and then using A1 and A3. The bounds can be narrow or wide in practice; Monte Carlo simulations in Appendix F illustrate both cases and discusses data features affecting the bounds' width.[21]

While the models of Cramton (1992) and Perry (1986) satisfy both monotonicity and independence, it is not hard to modify their environments to violate these assumptions. Appendix G shows that additively separable unobserved heterogeneity would violate both assumptions in both models, as listed in Table 2.

[21] The independence case shows how our bounds would become unwieldy, as alluded to in Section 3.2, if we were to attempt to exploit data from a single agent negotiating with multiple opponents over time, even if these negotiations are non-overlapping. Consider a seller negotiating with multiple buyers over time, and assume S and P_1^S are constant across negotiations. The generalization of the independence assumption would be that, conditional on P_1^S, S is independent of *each* P_2^B the seller faces, and our bounds would then condition on the (potentially large) set of such offers — a set that may differ in size across instances of the game. As highlighted in Section 3.2, this complication is one reason why we limit our data to only one buyer per seller and vice versa. The independence case is only one such example; other bounds become unwieldy as well without this restriction.

Table 2 also distinguishes between assumptions about behavior (e.g. monotonicity) vs. the environment (e.g. independence). Either type of assumption may be violated in a given dataset if they fail to describe behavior or an environment well, or because of a data weakness (e.g. unobserved heterogeneity). This raises an important point for empirical work: theoretical equilibrium models of bargaining may be unhelpful for empirics if their results do not hold in the presence of unobserved heterogeneity across instances of the game. It is precisely empirical challenges such as this that motivate our incomplete modeling approach, which can help bridge the gap between restrictive, extensive-form (and complete) models and analysis of bargaining in actual negotiation data.

3.6. Weakening Monotonicity and Independence. Our next assumption generalizes monotonicity:

Assumption A4 (Stochastic Monotonicity)**.** *(i) $P(S \leq x \mid P_1^S = y)$ weakly decreases in y $\forall$ x, and (ii) $P(B \leq x \mid P_1^S = y, P_2^B = z)$ weakly decreases in z $\forall$ y, x.*

This assumption is implied by monotonicity. Like monotonicity, stochastic monotonicity describes how an agent's offer is related to her value and allows us to exploit comparisons across instances of the game. A4 means that an agent's value is more likely high when her first offer is high.[22] Combined with (1) and (2), we obtain the following *stochastic monotonicity* bounds:

$$\int \max_{y' \geq y} P(X_{AC}^S \leq x \mid P_1^S = y') dF_{P_1^S}(y) \leq F_S(x) \leq \int \min_{y' \leq y} P(X_Q^S \leq x \mid P_1^S = y') dF_{P_1^S}(y) \tag{9}$$

$$\int \max_{z' \geq z} m_Q^B(x, y, z') dF_{P_1^S, P_2^B}(y, z) \leq F_B(x) \leq \int \min_{z' \leq z} m_{AC}^B(x, y, z') dF_{P_1^S, P_2^B}(y, z) \tag{10}$$

where $m_Q^B(x,y,z) = P(X_Q^B \leq x \mid P_1^S = y, P_2^B = z)$, $m_{AC}^B(x,y,z) = P(X_{AC}^B \leq x \mid P_1^S = y, P_2^B = z)$.

Theorem 4. *(9) gives a sharp lower bound for F_S under A1.i and A4.i and a sharp upper bound for F_S under A1.ii and A4.i. (10) gives a sharp lower bound for F_B under A1.iv and A4.ii and a sharp upper bound for F_B under A1.iii and A4.ii.*

[22] In another recent application of a similar assumption, Frandsen and Lefgren (2021) exploit stochastic monotonicity to bound treatment effects of charter school attendance.

Our next assumption weakens independence:

Assumption A5 (Positive correlation)**.** *(i) $P(S \leq x \mid P_1^S = y, P_2^B = z)$ is weakly decreasing in z for all y and x, and (ii) $P(B \leq x \mid P_1^S = y)$ is weakly decreasing in y for all x.*

A5 states that one agent's value is stochastically increasing in the other agent's first offer, capturing a notion of correlation between buyer and seller values.[23] A5 is implied by A3. Under A5 we obtain

$$\int \max_{z' \geq z} m_{AC}^S(x,y,z')dF_{P_1^S,P_2^B}(y,z) \leq F_S(x) \leq \int \min_{z' \leq z} m_Q^S(x,y,z')dF_{P_1^S,P_2^B}(y,z) \tag{11}$$

$$\int \max_{y' \geq y} P(X_Q^B \leq x \mid P_1^S = y')dF_{P_1^S}(y) \leq F_B(x) \leq \int \min_{y' \leq y} P(X_{AC}^B \leq x \mid P_1^S = y')dF_{P_1^S}(y) \tag{12}$$

Theorem 5. *(11) gives a sharp lower bound for F_S under A1.i and A5.i and a sharp upper bound for F_S under A1.ii and A5.i. (12) gives a sharp lower bound for F_B under A1.iv and A5.ii and a sharp upper bound for F_B under A1.iii and A5.ii*

Appendix G demonstrates that, when modifying Cramton (1992) and Perry (1986) to include unobserved heterogeneity, independence and monotonicity can be violated even while these weaker conditions hold. Table 2 summarizes these results.

3.7. Combining Assumptions on Marginal Distributions. A2–A5 can be combined to obtain tighter bounds. For example, we can combine A2 and A3 — monotonicity and independence — our two strongest assumptions. Or we can combine A4 and A5 — stochastic monotonicity and positive correlation — two weaker assumptions. Appendix C derives these combined bounds and proves sharpness.

The choice of which assumptions to adopt should be based on which seem reasonable in a given setting. For example, seller monotonicity and buyer independence may be inappropriate in settings with suspected unobserved heterogeneity, where stochastic monotonicity and positive correlation would be more appropriate. Other assumptions that may be less

[23]While these bounds exhibit a form of opponent-offer stochastic monotonicity, we refer to them as the *positive correlation* bounds to distinguish them from A4, (own-offer) stochastic monotonicity.

sensitive to unobserved heterogeneity include buyer monotonicity, which, through conditioning on the seller's first offer, may purge some unobserved heterogeneity. In Section 5, we estimate each of the bounds and test for crossings.

4 Estimation

We now describe estimators for the bounds. For some of the bounds, the natural plug-in estimators are unbiased or have an outward bias. In other cases, however, the plug-in estimator are inward biased and might be artificially tight (and unbiased estimators do not exist; Hirano and Porter 2012). We therefore modify the plug-in estimator to be either outward biased or half-median-unbiased in the spirit of Chernozhukov et al. 2013. We describe the basics of our estimators here and relegate additional details, along with discussions of inference and testing, to Appendix E. An observation $i = \{1, \ldots, n\}$ is the bargaining sequence for a given buyer-seller pair negotiating over a given product. We do all estimation separately by product and thus omit any notation denoting products.

4.1. Preliminary Ingredients for Estimation. For each i, the variables $X^S_{AC,i}$, $X^S_{Q,i}$, $X^B_{AC,i}$, and $X^B_{Q,i}$ are observed. We estimate the conditional probability $P(X^B_Q \leq x \mid P^S_1 = y') = E\left[\mathbf{1}(X^B_Q \leq x) \mid P^S_1 = y'\right]$ for each value of x using the Nadaraya-Watson kernel estimator with an Epanechnikov kernel and bandwidth $n^{-1/4}$. This bandwidth choice implies undersmoothing, which facilitates inference. Let $\widehat{P}(X^B_Q \leq x \mid P^S_1 = y')$ denote the estimator. We proceed analogously for $P(X^B_{AC} \geq x \mid P^S_1 = y')$, $P(X^S_{AC} \leq x \mid P^S_1 = y')$, and $P(X^S_Q \geq x \mid P^S_1 = y')$.

Similarly, we estimate the function $m^B_Q(x,y,z)$ using the Nadaraya-Watson kernel estimator with an Epanechnikov kernel and bandwidth $n^{-1/5}$. Due to the higher dimension of $m^B_Q(\cdot)$, the bandwidth converges at a slower rate. Denote the estimator $\hat{m}^B_Q(x,y,z)$. We estimate $\hat{m}^B_{AC}(x,y,z)$, $\hat{m}^S_{AC}(x,y,z)$, and $\hat{m}^S_Q(x,y,z)$ analogously.

$X^{S*}_{AC}(y)$ and $X^{S*}_Q(y)$ can be estimated with sample analogs, $\hat{X}^{S*}_{AC}(y) = \min_{i:P^S_{1,i} \geq y} X^S_{AC,i}$ and $\hat{X}^{S*}_Q(y) = \max_{i:P^S_{1,i} \leq y} X^S_{Q,i}$. Notice that $\hat{X}^{S*}_{AC}(y) \geq X^{S*}_{AC}(y)$ and $\hat{X}^{S*}_Q(y) \leq X^{S*}_Q(y)$, which implies that the estimated seller monotonicity bounds are outward biased (i.e., they are on average too wide) and thus are conservative when it comes to bounding the true CDF.

Estimating $X_{AC}^{B*}(y,z)$ and $X_{Q}^{B*}(y,z)$ is more complicated, as these condition on a specific value of the continuous variable P_1^S. Let $N(y) = \{z \in \mathbb{R} : |z-y| \leq h_n(y)\}$ be a neighborhood of y of a size $h_n(y)$ dependent on n and decreasing to 0 as $n \to \infty$. Define $\hat{X}_{AC}^{B*}(y,z) = \max_{i: P_{2,i}^B \leq z, P_{1,i}^S \in N(y)} X_{AC,i}^B$ and $\hat{X}_{Q}^{B*}(y,z) = \min_{i: P_{2,i}^B \geq z, P_{1,i}^S \in N(y)} X_{Q,i}^B$. As opposed to $\hat{X}_{AC}^{S*}(y)$, the bias of $\hat{X}_{AC}^{B*}(y,z)$ cannot be signed. To obtain a conservative estimator, we assume that $X_{AC}^{B*}(y,z)$ is Lipschitz continuous. That is, there exists $C \geq 0$ such that $|X_{AC}^{B*}(y,z) - X_{AC}^{B*}(y',z)| \leq C|y-y'|$ for all y, y', z. Then $\hat{X}_{AC}^{B*}(y,z) \leq \overline{\text{supp}}(X_{AC}^B : P_2^B \leq z, P_1^S \in N(y)) \leq X_{AC}^{B*}(y,z) + Ch_n(y)$. To choose $h_n(y)$, we use a matching approach. Let K_n be the number of neighbors and let $h_n(y)$ be such that $\sum_{i=1}^n \mathbf{1}(|P_{1,i}^S - y| \leq h_n(y)) = K_n$. We choose $K_n = n^{1/4}$. If the density of $P_1^S(y)$ is bounded and bounded away from 0 in a neighborhood of y, then $h_n(y)$ is proportional to $n^{-3/4}$ and therefore goes to 0 as $n \to \infty$. Finally, we let $\tilde{X}_{AC}^{B*}(y,z) = \hat{X}_{AC}^{B*}(y,z) - \eta_n$ where $\eta_n = n^{-1/2}$, which ensures that $\tilde{X}_{AC}^{B*}(y,z) \leq X_{AC}^{B*}(y,z)$ with probability approaching 1. Similarly, we use $\tilde{X}_{Q}^{B*}(y,z) = \hat{X}_{Q}^{B*}(y,z) + \eta_n$.

4.2. Estimation of Bounds. With the ingredients from above, we estimate the bounds. For brevity, we describe here the estimation of each lower bound; estimators for upper bounds are analogous. The unconditional lower bound estimators are simply the empirical analogs of (3) and (4): $\frac{1}{n}\sum_{i=1}^n \mathbf{1}(X_{AC,i}^S \leq x)$ and $\frac{1}{n}\sum_{i=1}^n \mathbf{1}(X_{Q,i}^B \leq x)$, both of which are unbiased.

For the monotonicity bounds, note that $\int \mathbf{1}(X_{AC}^{S*}(y) \leq x) dF_{P_1^S}(y) = E_{P_1^S}[\mathbf{1}(X_{AC}^{S*}(P_1^S) \leq x)]$, where $E_{P_1^S}[\cdot]$ is an expectation over P_1^S. We estimate the lower bounds from (5) and (6) by $\frac{1}{n}\sum_{i=1}^n \mathbf{1}(\hat{X}_{AC}^{S*}(P_{1,i}^S) \leq x)$ and $\frac{1}{n}\sum_{i=1}^n \mathbf{1}(\tilde{X}_{Q}^{B*}(P_{1,i}^S, P_{2,i}^B) \leq x)$, which are both conservative.

We estimate the stochastic monotonicity lower bounds in (9) and (10) by $\frac{1}{n}\sum_{i=1}^n \max_{y' \in \omega_1(P_{1,i}^S)} \widehat{P}(X_{AC}^S \leq x \mid P_1^S = y')$ and $\frac{1}{n}\sum_{i=1}^n \max_{z' \in \omega_2(P_{2,i}^B)} \hat{m}_Q^B(x, P_{1,i}^S, z')$, where we define $\omega_1(P_{1,i}^S) = \{y : y \geq P_{1,i}^S, y \in Q_{0.05}(P_{1,i}^S) \cup \{P_{1,i}^S\}\}$, and for any random variable Y, $Q_\alpha(Y)$ is the interval between the α and $1-\alpha$ quantiles of Y; $\omega_2(P_{2,i}^B)$ is defined analogously. Notice that the sample analog estimator of the seller's stochastic monotonicity lower bound is $\frac{1}{n}\sum_{i=1}^n \max_{y' \geq P_{1,i}^S} \widehat{P}(X_{AC}^S \leq x \mid P_1^S = y')$, but we use the additional constraint that $y \in Q_{0.05}(P_{1,i}^S)$ since $P(X_{AC}^S \leq x \mid P_1^S = y')$ can be poorly estimated at the support boundary. Moreover, to ensure that we maximize over a nonempty set, we always include $P^S_{1,i}$. Applying this tail truncation yields conservative estimates; the same is true for all estimators in the paper using this truncation. These estimators might still be inward biased due to the maxima. To address this, we modify our estimators to be half-median-unbiased (Appendix E). Simulations in Appendix F compare bias-corrected and uncorrected estimators.

We estimate the independence lower bounds in (7) and (8) by $\frac{1}{n}\sum_{i=1}^{n}\max_{z\in Q_{0.05}(P^B_{2,i})}\hat{m}^S_{AC}(x,P^S_{1,i},z)$ and $\max_{y'\in Q_{0.05}(P^S_{1,i})}\widehat{P}(X^B_Q\leq x\mid P^S_1=y')$, and the positive correlation lower bounds in (11) and (12) by $\frac{1}{n}\sum_{i=1}^{n}\max_{z'\in\omega_2(P^B_{2,i})}\hat{m}^S_{AC}(x,P^S_{1,i},z')$ and $\frac{1}{n}\sum_{i=1}^{n}\max_{y'\in\omega_1(P^S_{1,i})}\widehat{P}(X^B_Q\leq x\mid P^S_1=y')$. Appendix E discusses corresponding half-median-unbiased estimators. Appendices C and E address estimators combining assumptions.[24]

5 Bounding Values in eBay Bargaining

5.1. Bounds on Buyer and Seller Values for Cell Phones. We apply these estimators to bound buyer and seller value distributions separately for all 44 products. We normalize prices by the product's reference price to aid interpretation. To illustrate, we first focus on one product: the most popular product from the cell phone category from Table 1.

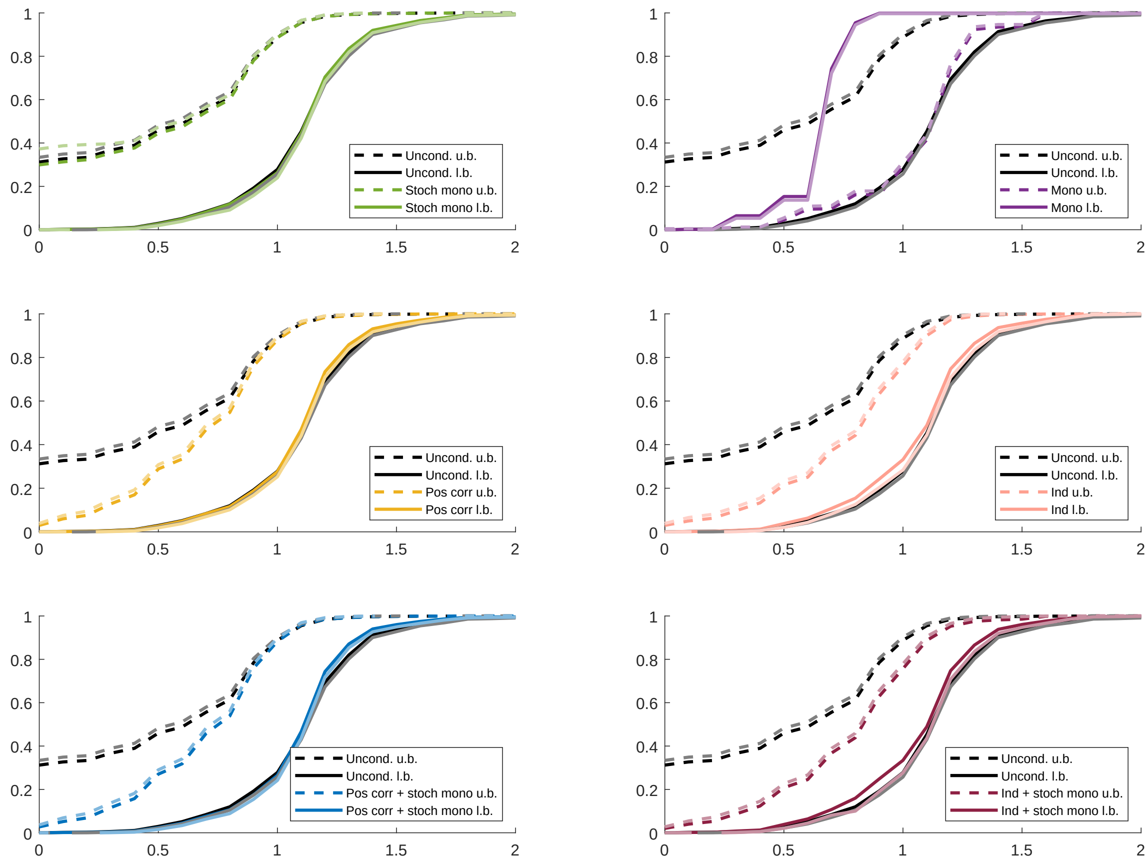
5.1.1. Bounds on Seller Values. Figure 2 shows bounds on F_S for this product under different assumptions. Every panel also shows the unconditional bounds for comparison. Dashed lines are upper and solid lines are lower bounds. Next to point estimates, we also show 95% one-sided pointwise confidence bands.

The unconditional bounds — which rely on our weakest assumptions — can be wide, the upper bound in particular. This is because it is constructed using prices at which a seller quits (walking away from bargaining), which are unobserved if a sequence either ends in agreement or if the buyer quits (rather than the seller).[25] Depending on the product, some

[24]While we do address this in this paper, one could condition bounds arguments on covariates and apply this conditioning in the estimated conditional probability functions. In this case, a parametric approximation, such as a probit, may be preferred to the nonparametric estimators we propose here. The matching approach described above could then be used to estimate support bounds for the monotonicity assumptions.

[25]In such cases, we cannot rule out the seller having a value of zero, and hence the upper bound CDF

Figure 2: Bounds on Seller Distribution for Cell Phone



Notes: Bounds on F_S for most popular cell phone product. Top two panels show stochastic monotonicity bounds (left) and monotonicity bounds (right). Middle panels show positive correlation bounds (left) and independence bounds (right). Bottom panels show combined positive correlation + stochastic monotonicity bounds (left) and combined independence + stochastic monotonicity bounds (right). Every panel shows unconditional bounds for comparison. Upper bounds are dashed lines and lower bounds are solid lines. Faded lines represent 95% confidence bands (constructed via subsampling; see Appendix E) for the bounds represented in the corresponding color. All prices are scaled by product's reference price, and thus units on horizontal axis are fraction of the reference price.

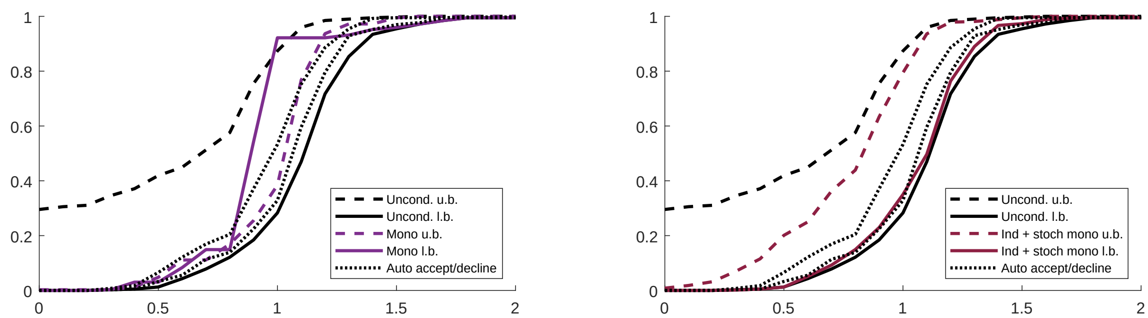
assumptions do little to improve the unconditional bounds. For example, for this product, the stochastic monotonicity bounds (top left panel), are nearly as wide as the unconditional bounds. Stochastic monotonicity implies that a seller with a high first offer is likely to have a high value. This assumption will lead to a tightening of the lower bound if, for example,

is (graphically) high. The lower bound, on the other hand, relies on prices at which the seller accepts or counters, and at least one of these prices is *always* available in the data (P_1^S). The lower bound will thus, by construction, be surjective (i.e., mapping to each value in [0,1]).

in some instances of the game in which sellers have relatively high first offers the seller eventually ends up accepting a relatively *low* offer.[26] If instead sellers with high first offers always end up accepting higher prices, the stochastic monotonicity assumption (even if true) will not tighten the bounds.

The monotonicity bounds (top right) illustrate the potential for crossings; here the lower bound lies entirely above the upper bound. As demonstrated in Section 3.5 and Appendix G, monotonicity can be violated by unobserved game-level heterogeneity, which can generate non-monotonicities in the relationship between S and P_1^S. This finding highlights the importance of our weaker assumption (stochastic monotonicity), which is not rejected by the data. In the bottom right panel of Figure 2, we display the tightest bounds for this product that do not cross (independence with stochastic monotonicity). Monte Carlo simulations in Appendix F demonstrate that any of these bounds — even the unconditional bounds — can be quite narrow or wide, depending on the DGP.

Figure 3: Bounds on Seller Distribution and Auto Accept/Decline Prices



Notes: Bounds on F_S using only negotiations in which seller reported a non-zero auto-accept and auto-decline price. Left panel shows monotonicity bounds and right panel shows combined independence and stochastic monotonicity bounds, with upper bounds as dashed lines and lower bounds as solid lines. Every panel also shows the unconditional bounds for comparison. Empirical CDFs of auto-accept and auto-decline prices are shown in gray dotted lines. All prices are scaled by the reference price for the product, and thus units on the horizontal axis are fraction of the reference price.

For seller values, secret auto-accept and auto-decline prices serve as a novel validation

[26] This can happen due to the randomness in the buyer to which the seller is matched or other DGP features for later offers, which we make no assumptions about. A similar argument applies to the positive correlation bounds (left middle panel). These bounds rely on assuming the seller's value is stochastically increasing in the buyer's first offer conditional on the seller's first offer, and will only tighten the lower bound, say, if, conditional on cases where $P_1^S = y$, sellers who face high P_2^B sometimes accept relatively low offers. If this is not the case, positive correlation, while not rejected by the data, will do little to improve the bounds.

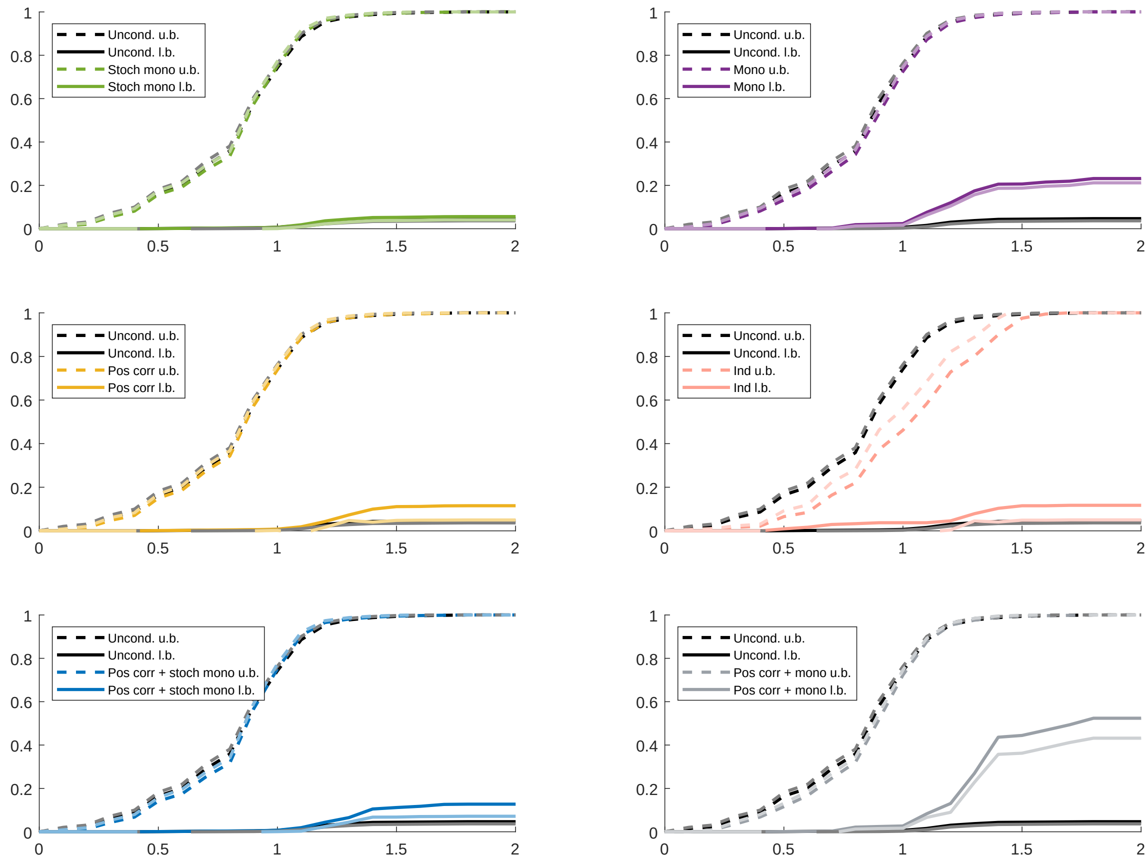
check — private information of the seller that offers an immediate upper and lower bound on the true F_S.[27] Here we re-estimate F_S bounds limiting to the 396 negotiations of this product in which the seller reported these secret thresholds. Figure 3 shows the results, along with empirical CDFs of auto-accept and auto-decline prices (gray dotted lines). The left panel shows the monotonicity bounds, which still cross. The right panel (independence with stochastic monotonicity) is our primary interest, as these are the tightest bounds we obtain without the (overly strong) assumption of seller monotonicity. Here we observe that our assumptions are consistent with the tight bounds implied by the secret prices: the lower bound implied by the auto-accept price CDF lies below our upper bound and the auto-decline price CDF lies above our lower bound. We repeat this exercise for the two other products in our sample with at least 200 negotiations with auto-accept/decline prices and find similar results, suggesting the bounds are consistent with these secret thresholds.[28]

5.1.2. Bounds on Buyer Values. Figure 4 displays bounds on F_B for this same product. We find wider bounds for F_B than for F_S. The lower bound is not onto on $[0,1]$ because it depends on quit prices, similar to the F_S upper bound. The combined positive correlation and monotonicity bounds (bottom right) especially help to tighten the buyer bounds for this product. Unlike for F_S, the monotonicity bounds for F_B do not cross. The monotonicity assumption for B is weaker, requiring only that, *conditional* on the seller's first offer, a buyer's value be higher at higher buyer offers (whereas the seller monotonicity assumption is particularly strong). We reiterate that all bounds are sharp, implying they are the best possible under their corresponding assumptions. Any tightening of the bounds necessarily requires stronger assumptions.

[27] A given seller's auto-accept price is a weak upper bound on her value and the auto-decline price a weak lower bound. If, rather than alternating offers, the protocol involved only a take-it-or-leave-it offer by the buyer, the seller's optimal choice would be to set both the auto-accept and auto-decline prices equal to her value. The auto-accept/decline prices are not used anywhere in identifying or estimating our bounds; we intentionally withhold that information to use it in this validation exercise.

[28] Cases where the auto-accept/decline prices are inconsistent with our bounds are when the F_S lower bound lies above the auto-decline price CDF or the F_S upper bound lies below the auto-accept price CDF. We always find such crossings for seller monotonicity bounds (as Table 3 below shows, seller monotonicity bounds cross for all products). For one product, we also find a crossing of the auto-decline price under the seller independence assumption, but the size of the crossing is small, corresponding to 0.0002 of the mass of seller types (estimated via the integrated violation error, described in Section 5.2.).

Figure 4: Bounds on Buyer Distribution for Cell Phone



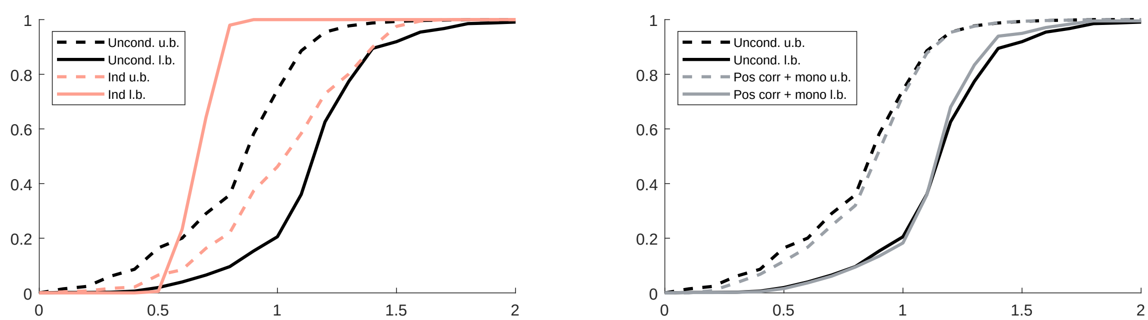
Notes: Bounds on F_B for most popular cell phone product. Top two panels show stochastic monotonicity bounds (left) and monotonicity bounds (right). Middle panels show positive correlation bounds (left) and independence bounds (right). Bottom panels show combined positive correlation + stochastic monotonicity bounds (left) and combined positive correlation + monotonicity bounds (right). Every panel shows unconditional bounds for comparison. Upper bounds are dashed lines and lower bounds are solid lines. Faded lines represent 95% confidence bands (constructed via subsampling; see Appendix E) for the bounds represented in the corresponding color. All prices are scaled by product's reference price, and thus units on horizontal axis are fraction of the reference price.

We can examine how the bounds in Figure 4 change — and whether they cross — when we impose a stronger assumption that raises the lower bound: the *list-price-recall condition* described in Section 3.2.[29] For this condition, recall that we incorporate into X_Q^B the concept that a buyer can always purchase at the list price, and therefore, under this condition, $B \leq P_1^S$. Under this condition, we should observe the buyer independence bounds crossing,

[29]The list-price-recall condition only affects the F_B lower bound, the only bound that uses X_Q^B.

because $B \leq P_1^S$ implies B and P_1^S are not independent. Figure 5 shows bounds incorporating this condition. We observe clear evidence of the independence bounds being violated: the lower bound lies nearly entirely above the upper. The unconditional bounds (black lines) are also much tighter under list-price-recall, and do not cross. Exploiting the list-price-recall condition also offers a check on the validity of the tightest bounds we obtained in Figure 4, those relying on monotonicity and positive correlation. Even applying the list-price-recall condition, these bounds do not cross. The fact that the positive correlation bounds do not cross, while the independence bounds do, highlights the benefit of the weaker assumption underlying these bounds.

Figure 5: Bounds on Buyer Distribution with List Price Recall



Notes: Bounds on F_B for most popular cell phone product using the list-price-recall condition. Upper bounds are equivalent to those in Figure 4; only lower bounds change. Left panel shows independence bounds and right panel shows combined positive correlation and monotonicity bounds. Every panel shows unconditional bounds for comparison. Upper bounds are dashed lines and lower bounds are solid lines. All prices are scaled by product's reference price, and thus units on horizontal axis are fraction of the reference price.

5.2. Exploring All Products. We now summarize bounds for all 44 products. In Table 3, we show, under each assumption, the fraction of products for which bounds cross.[30] Crossings indicate a violation of the underlying assumption(s). Some crossings may be due to finite sample estimation error, and we account for this by testing whether any crossing is statistically significant (Frac. Reject); Appendix E describes details. We also compute the *integrated violation error* (IVE), which ranges from 0 to 1 and measures the average

[30] In practice we choose a grid on which to evaluate the bounds and check for crossings (upper bound lying below lower bound) at each grid point. We use 0 to 2.5 in increments of 0.1. To allow for machine rounding errors, we only consider violations exceeding 2e-10.

difference between the upper and lower bound in cases where they cross.[31]

Table 3: Bound Crossings Under Different Assumptions

	Seller Bounds			Buyer Bounds		
	Frac. Cross	Frac. Reject	IVE	Frac. Cross	Frac. Reject	IVE
Unconditional (A1)	0	0	0	0	0	0
Monotonicity (A2)	1.00	1.00	0.22	0	0	0
Independence (A3)	0.05	0	0	0.41	0.14	0.01
Stochastic Monotonicity (A4)	0	0	0	0	0	0
Positive Correlation (A5)	0	0	0	0.0227	0	0
Mon. + Indep. (A2 + A3)	1.00	1.00	0.22	0.64	0.50	0.06
Mon. + Pos. Corr (A2 +A5)	1.00	1.00	0.22	0.02	0	0
Stoch. Mon. + Indep. (A4 + A3)	0.14	0.02	0.00	0.34	0.16	0.01
Stoch. Mon. + Pos. Corr (A4 + A5)	0	0	0	0	0	0

Notes: Across all products in the estimation sample, table shows the fraction of products for which seller lower bound crosses the upper bound, fraction of products for which crossings are statistically significant, and the IVE. Table shows similar quantities for the buyer bounds.

Table 3 shows that the results from Figures 2 and 4 are representative of products in our data. In particular, we find that seller monotonicity bounds cross for 100% of products, with the upper monotonicity bound violating the lower monotonicity bound by an average of 22% (the IVE).[32] For buyer values, where the monotonicity assumption is weaker, the bounds never cross. The opposite is true for the independence bounds, which only cross for 5% of products for sellers (and no crossings are statistically significant) but for 41% of products when bounding F_B, 14% of which are statistically significant. In general, we observe that, even for bounds with some statistically significant crossings, the size of the crossings are small, as indicated by the low IVE.[33]

[31] For a generic upper and lower bound by F^U and F^L, the IVE is given by $\int \max\{F^L(x) - F^U(x), 0\} dG(x)$, where the distribution function G is equal the unconditional lower bound in the case of sellers and the unconditional buyer bound in the case of buyers. We choose these distributions as they are surjective on $[0,1]$ (mapping to every point in $[0,1]$), as the seller lower bound depends on P_1^S and the buyer upper bound depends on P_2^B, which are both always observed in our sample.

[32] Appendix C proves that the F_S monotonicity bounds are unchanged when combined with independence or positive correlation.

[33] Imposing the list-price-recall condition leads to additional products having statistically significant crossings only for F_B bounds involving independence (consistent with our findings in Section 5.1.2) and to 1 out of the 44 products having a statistically significant crossing for the F_B monotonicity + positive correlation bounds.

Table 4: Statistics Across Products on Width of Bounds

	Seller Bounds			Buyer Bounds		
	Min	Mean	Max	Min	Mean	Max
Unconditional (A1)	0.330	0.411	0.512	0.404	0.430	0.459
Monotonicity (A2)	–	–	–	0.294	0.380	0.445
Independence (A3)	0.172	0.273	0.419	0.095	0.238	0.339
Stochastic Monotonicity (A4)	0.312	0.408	0.499	0.391	0.420	0.452
Positive Correlation (A5)	0.241	0.365	0.498	0.366	0.415	0.441
Mon. + Indep. (A2 + A3)	–	–	–	0.113	0.223	0.316
Mon. + Pos. Corr (A2 +A5)	–	–	–	0.265	0.361	0.423
Stoch. Mon. + Indep. (A4 + A3)	0.164	0.258	0.382	0.124	0.256	0.356
Stoch. Mon. + Pos. Corr (A4 + A5)	0.239	0.366	0.483	0.368	0.410	0.442

Notes: Table shows the minimum, mean, and max (across products) of the average width of the bounds, where the average width for a given product is computed by the upper minus lower bound integrated against the density of the unconditional lower bound for sellers or the unconditional upper bound for buyers. These statistics are computed for a given set of bounds only for products for which the bounds do not cross (thus, the width of bounds and any bounds relying on seller monotonicity is omitted, as these always cross).

In Table 4, for a given set of bounds, we compute the average width of the bounds for a given product and take the mean, minimum, and maximum of these average widths across products.[34] The unconditional seller bounds can be relatively tight for some products, with an average probability gap of 0.330 for the minimum-width product, and quite wide for others, with an average gap of 0.512 for the maximum product. The average width for some bounds — such as the stochastic monotonicity bounds — is quite similar to that of the unconditional bounds. Given that all of the bounds are sharp under their corresponding assumptions, these widths could only be reduced under stronger assumptions. We find that stochastic monotonicity and positive correlation assumptions for the seller improve the bounds for some products, decreasing the minimum average width to 0.239. For buyer bounds, monotonicity and positive correlation improve tightness relative to the unconditional bounds, with a minimum average width of 0.265.

[34]For a given type of bounds for a given product, we integrate the upper bound minus the lower bound against the density of the unconditional lower bound in the case of seller values and the unconditional upper bound in the case of buyer values, as with the IVE. This average width metric is similar to the IVE, ranging from 0 to 1, with a lower number meaning the bounds are tighter.

6 Quantifying Inefficient Impasse and Uncertainty

6.1. Motivation. We now consider bounds on the counterfactual first-best trade probability. In a first-best world, a buyer with value B and seller with value S will trade whenever $B \geq S$. The Myerson and Satterthwaite (1983) Theorem demonstrated that the surplus offered by such a mechanism is unattainable when buyers and sellers have independent private values with overlapping supports. Cramton (1992) demonstrated that, while the first-best *surplus* is infeasible, it is possible for negotiators to achieve the first-best *quantity* of trade after costly delay. The first-best quantity of trade will be weakly higher than the realized volume of trade in the data; the question is, how much higher? A lower bound of $P(B \geq S)$ can be compared to the sale probability in the data to bound the degree of inefficient impasse occurring in reality.[35] As important as the object $P(B \geq S)$ is for quantifying inefficiency, existing empirical tools and theoretical models are insufficient for identifying it, and data from bargaining settings where agents have private information will not typically contain data on those private values themselves (S and B) except in lab experiments.

As explained in Section 3.2, we allow for the possibility that buyers and sellers may have a continuation value after failed trades (and may, for example, re-enter the marketplace). Buyer/seller values are then interpretable as willingness to pay/sell. This interpretation is not problematic for studying inefficient impasse because we are quantifying, for a fixed set of buyer-seller pairs, how well real-world bilateral bargaining performs relative to the first-best. We do not model or study the process by which those pairs came to be matched, which would be necessary to study the efficiency of the searching and matching process. Both types of efficiencies — the efficiency of the search and matching process and the efficiency of the incomplete-information bilateral bargaining conditional on matching — are important components of the efficiency of the market as a whole. Given the complexities

[35]Note that the trade probability is intimately related to the conditional surplus (the gains from trade conditional on trade occurring); the latter is weakly increasing in the former. The trade probability is a more useful object for our empirical purposes because the real-world counterpart is observable in the data, whereas the conditional surplus is not. This is because, whenever trade occurs, trade must be the efficient outcome, but the size of those gains is not necessarily identified. Whenever trade fails even though gains from trade exist, the outcome is necessarily inefficient, but the size of the loss is not necessarily identified.

of incomplete-information bargaining (a continuum of equilibria with no full characterization; Ausubel et al. 2002), this paper focuses only on the latter: the inefficiency of the bargaining *conditional* on the buyer-seller pairs matched to one another in the data.

It also bears emphasis that the goal of our exercise is not to construct a policy counterfactual — indeed, the counterfactual we examine, the first-best, is not feasible — but rather to quantify how well real-world bargaining performs relative to this benchmark. We hope this quantification can inform several audiences. For the platform, it may offer insights into how much value is left on the table in the current sales mechanism and whether it might be worth investigating alternatives, such as encouraging the use of the auto accept/decline feature or encouraging more communication between agents (which has been shown in lab experiments to reduce inefficient impasse in incomplete-information bargaining; Valley et al. 2002). For empirical bargaining researchers, who typically model negotiated prices as arising from a protocol with no inefficient impasse (some form of Nash bargaining; e.g., Crawford and Yurukoglu 2012), this quantification exercise may offer some insights into how reasonable such an abstraction may be, albeit only within the eBay platform. For bargaining theory, we hope the exercise can motivate future models capturing elements of inefficient impasse and unobserved heterogeneity.

6.2. Bounds on Surplus. In this section we construct bounds on the distribution of $B-S$ (the surplus) and use these to bound $P(B \geq S)$. Two of these bounds follow immediately from steps in Section 3. The other two are new, building on ideas from the marginal distribution bounds in Section 3.

Our weakest bounds for $P(B-S \geq x)$ rely only on revealed preferences (A1), which implies $P(B-S \geq x) \geq P(X^B_{AC} - X^S_{AC} \geq x)$. Buyers will generally not accept or counter at a price that is strictly higher than the seller (as this would be strange behavior indeed). Therefore, $P(X^B_{AC} - X^S_{AC} \geq x)$ typically corresponds to $P(X^B_{AC} - X^S_{AC} = x)$, which, at $x = 0$, is equal to $P(sale)$ (the sale probability in the data), as this represents cases where one agent accepts a price the other proposes. The upper bound is similar: A1 implies $P(B-S \geq x) \leq P(X^B_Q - X^S_Q \geq x)$. At $x = 0$, this latter probability is always equal to 1, because only one party (the buyer or seller) can quit in a given negotiation. When the seller quits, $X_Q^B = \infty$, and when the buyer quits, $X_Q^S = 0$. Thus, revealed preferences alone yield uninformative bounds; the most we learn is that $P(B \geq S) \in [P(sale), 1]$.

We next consider bounds that rely on buyer monotonicity (A2.ii) as well as the following weak assumption on $B - S$:

Assumption A6. *(Surplus stochastic monotonicity). $P(B - S \geq x \mid P_1^S = y, P_2^B = z)$ is increasing in z for all y.*

To interpret this assumption, consider two negotiations, both of which involve $P_1^S =$ \$300. Suppose that in the first negotiation $P_2^B =$ \$200 and the parties trade, while in the second negotiation $P_2^B =$ \$290 and trade fails. A6 implies that gains from trade likely exist in the second negotiation, even though trade fails.[36] A6 is akin to our stochastic monotonicity assumption on *values* applied instead to the *difference in values*.[37] Combined with buyer monotonicity, it yields the following:

$$P(B - S \geq x) \geq \int \max_{z' \leq z} P(X_{AC}^{B*}(y,z) - X_{AC}^S \geq x \mid P_1^S = y, P_2^B = z')dF_{P_1^S, P_2^B}(y,z) \qquad (13)$$

$$P(B - S \geq x) \leq \int \min_{z' \geq z} P(X_Q^{B*}(y,z) - X_Q^S \geq x \mid P_1^S = y, P_2^B = z')dF_{P_1^S, P_2^B}(y,z) \qquad (14)$$

Theorem 6. *(13) gives a sharp lower bound for $P(B - S \geq x)$ under A1.i, A1.iii, A2.ii, and A6. (14) gives a sharp upper bound for $P(B - S \geq x)$ under A1.ii, A1.iv, A2.ii, and A6.*

A stronger variant of the last assumption is the following:

Assumption A7. *(Surplus weak monotonicity). $B - S$ is weakly increasing in P_2^B conditional on P_1^S.*

Revisiting the example discussed after A6, A7 implies that gains from trade definitely (rather than only likely) existed in the second negotiation. While stronger than surplus

[36] Sufficient (but not necessary) conditions for A6 are a strict version of A2.ii (buyer monotonicity) and A3.i (seller independence), two assumptions for which we do not find large crossings in Section 5. To see that these assumptions are sufficient, suppose we can write $P_2^B = f(B, P_1^S)$ where $f(\cdot, P_1^S)$ is increasing for all P_1^S with inverse function $g(\cdot, P_1^S)$. Then the conditional probability statement in A6 can be written $P(B - S \geq x \mid P_1^S = y, B = g(z,y)) = P((z,y) - S \geq x \mid P_1^S = y, B = g(z,y))$. This latter statement is equivalent to $P(g(z,y) - S \geq x \mid P_1^S = y)$, which is increasing in z for all y.

[37] Note that we do not rely on assumptions about the correlation structure between S and B, as these are unhelpful when examining the *difference* $B - S$.

stochastic monotonicity (Assumption A6), this assumption is weaker than assuming monotonicity for *both* the buyer and seller.[38] As A6 is akin to a stochastic monotonicity assumption applied to $B-S$, A7 is akin to weak monotonicity. Combined with buyer monotonicity, A7 yields the following:

$$P(B-S \geq x) \geq \int \mathbf{1}(X_Q^{B*-S}(y,z) \geq x) dF_{P_1^S,P_2^B}(y,z) \tag{15}$$

$$P(B-S \geq x) \leq \int \mathbf{1}(X_{AC}^{B*-S}(y,z) \geq x) dF_{P_1^S,P_2^B}(y,z) \tag{16}$$

where $X_{AC}^{B*-S}(y,z) = \overline{\text{supp}}(X_{AC}^{B*}(y,z) - X_{AC}^S : P_2^B \geq z, P_1^S = y)$ and $X_Q^{B*-S}(y,z) = \underline{\text{supp}}(X_Q^{B*}(y,z) - X_Q^S : P_2^B \leq z, P_1^S = y)$.

Theorem 7. *(15) gives a sharp lower bound for $P(B-S \geq x)$ under A1.i, A1.iii, A2.ii, and A7. (16) gives a sharp upper bound for $P(B-S \geq x)$ under A1.ii, A1.iv, A2.ii, and A7.*

Finally, the strongest assumptions we consider for bounding $P(B-S \geq x)$ rely on monotonicity (A2) for both the buyer *and* seller, implying that $P(X_{AC}^{B*} - X_{AC}^{S*} \geq x) \leq P(B-S \geq x) \leq P(X_Q^{B*} - X_Q^{S*} \geq x)$. Section 5 shows that seller (but not buyer) monotonicity is rejected by the data. We nonetheless estimate these bounds to illustrate that they are indeed too strong and can cross. Estimators for all of our bounds on $P(B-S \geq x)$ are similar to those for F_S and F_B and are discussed in Appendix E.

6.3. Inefficient Impasse Results. We evaluate bounds on $P(B-S \geq x)$ at $x = 0$, or, equivalently, $P(B \geq S)$, the first-best trade probability. We are primarily interested in the lower bound on $P(B \geq S)$ but we evaluate the upper bound as well to look for crossings.

We display estimates of lower bounds on $P(B \geq S)$, along with confidence intervals on these lower bounds, under these assumptions in Table 5. For the electronics product, $P(sale)$ is 0.432 and the lower bound on the counterfactual $P(B \geq S)$ under surplus stochastic monotonicity and buyer monotonicity (column 2) is 0.463, suggesting that the real-world bargaining misses some efficient trades. However, the confidence interval con-

[38] Strict versions of A2.i and A2.ii are sufficient for A7 to hold, but not necessary. Specifically, suppose we can write $P_1^S = f_1(S)$ and $P_2^B = f_2(B, P_1^S)$ where $f_1(\cdot)$ and $f_2(\cdot, P_1^S)$ are increasing with inverse functions $g_1(\cdot)$ and $g_2(\cdot, P_1^S)$, respectively. Then $B - S = g_2(P_2^B, P_1^S) - g_1(P_1^S)$ and, conditional on P_1^S, this latter difference is an increasing function of P_2^B.

tains $P(sale)$, and thus the evidence of inefficient impasse under this set of assumptions is relatively weak. Column 3 shows bounds relying on surplus weak monotonicity and buyer monotonicity. Here we find confidence intervals that lie above $P(sale)$ for most products, suggesting that, maintaining these assumptions, bargaining is indeed inefficient. The column 4 lower bounds are higher still, but recall that these invoke seller monotonicity and hence are rejected by the data.

Table 5: Lower Bounds on First-Best Trade Probability for Most Popular Products

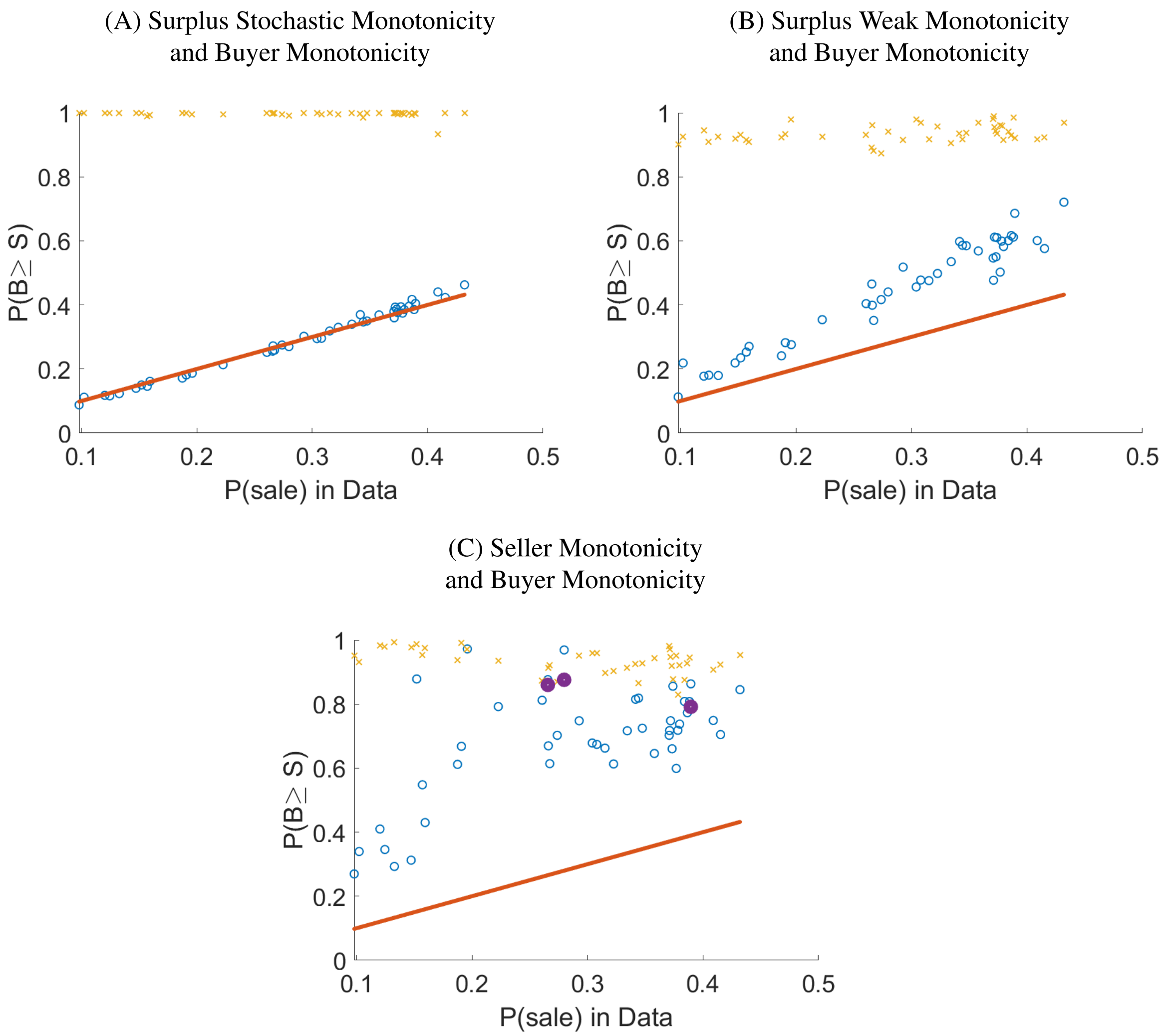
	n	(1) Uncond. $P(sale)$	(2) Surplus Stoch Mon, Buyer Mon	(3) Surplus Weak Mon, Buyer Mon	(4) Seller Mon, Buyer Mon
Consumer Electronics	419	0.432	0.463	0.721	0.845
		[0.385,0.480]	[0.400,0.527]	[0.629,0.766]	[0.779,0.892]
Video Games/Consoles	342	0.415	0.423	0.576	0.705
		[0.363,0.468]	[0.385,0.514]	[0.467,0.631]	[0.624,0.759]
Cell Phones	1227	0.266	0.256	0.465	0.876
		[0.241,0.290]	[0.229,0.291]	[0.409,0.496]	[0.845,0.904]
Computers/Tablets	321	0.125	0.116	0.181	0.346
		[0.088,0.161]	[0.078,0.157]	[0.083,0.229]	[0.269,0.432]

Notes: For the most popular products with each category, table displays lower bounds on $P(B \geq S)$ under different assumptions. 95% confidence intervals (obtained via subsampling) are shown in square braces below each estimate.

We can construct a measure of *inefficient impasse*, which we define as the fraction of cases where gains from trade exist and yet trade fails, or $1 - P(sale)/P(B \geq S)$. A lower bound on this quantity is obtained by plugging in a lower bound on $P(B \geq S)$. The bounds in Table 5 suggest that the real-world bargaining for these products exhibits inefficient impasse, but not as much as implied by the (overly strong) assumption of seller monotonicity. For example, for the popular cell phone product, the sale probability in the data is 0.266, but in column 3 we find that it would be as high as 0.465 in a first-best world, suggesting that, when the buyer values the phone more than the seller, the pair still fails to reach an agreement 43% of the time (i.e., $1 - 0.266/0.465$).

Figure 6 extends this analysis to all 44 products. In each panel, we order products on the horizontal axis by their sale probability in the data. On vertical axes, an "$\times$" represents

Figure 6: Bounds on $P(B \geq S)$, All Products



Notes: Figure display upper bounds (marked with "×" and lower bounds (marked with hollow circles) for the counterfactual first-best trade probability ($P(B \geq S)$) under different assumptions, for each product in the estimation sample. Horizontal axes rank products by sale probability in the data. The solid line represents the 45-degree line. Solid purple dots in panel C highlight the lower bound for products where bounds cross.

the estimated upper bound and a hollow circle the lower bound.[39] These hollow circles are made solid purple for products where the estimated bounds cross (i.e., where the lower

[39]To facilitate uncovering possible violations, we tighten the upper bound by invoking the list-price-recall condition. This condition only affects the upper bound on $P(B \geq S)$ because only the upper bound is related to X_Q^B. The gain from invoking this condition is small here, lowering the upper bound only by 3.2 percentage points for the average product in panel B and by 0.3 percentage points in panels A and C.

bound lies above the upper bound). Panel A displays bounds relying on surplus stochastic monotonicity and buyer monotonicity. As in Table 5, these assumptions are too weak to yield informative bounds, as they are close to $P(sale)$ and 1. In panel C, we find that assuming monotonicity for both agents yields bounds that are tighter, but violated for some products (3 out of 44).[40] These crossings are expected, as seller monotonicity bounds cross for all products (Table 3). The Goldilocks-like assumptions are in panel B (surplus weak monotonicity and buyer monotonicity), where we observe informative bounds that do not cross. The lower bounds suggest that the real-world bargaining indeed exhibits inefficient impasse, ranging from 13.0% to 53.2% across products, with a median of 35.4%.

As discussed in Section 3.7, the choice of which bounds to favor should be guided by the details of a particular setting, where possible. We favor the bounds in Figure 6.B because both buyer monotonicity and surplus weak monotonicity allow us to condition on the seller's first offer, allowing us to potentially condition on some unobserved heterogeneity, which seems wise in the eBay setting. We have additional confidence that these assumptions may be capturing accurate general properties of eBay bargaining in that, like the buyer monotonicity bounds alone (Table 3), they consistently do not cross for any product.

6.4. When is Bargaining More Efficient? We now evaluate inefficient impasse ($1 - P(sale)/P(B \geq S)$) separately in various cuts of the data in which agents, items, or other aspects of the negotiation satisfy particular conditions (such as whether or not the agents exchange any messages). We rely on our lower bounds based on surplus weak monotonicity and buyer monotonicity (Theorem 7). For each condition we consider, we limit the analysis to products for which we observe at least 100 negotiations where the condition is satisfied and 100 where it is not. We then evaluate the difference in the inefficient impasse lower bound between those two subsets. We also compare $P(sale)$ between the two subsets, as well as the $P(B \geq S)$ lower bound. This exercise is exploratory and speculative in nature. We have no model for *why* bargaining would be more efficient under certain conditions. Our goal here is to provide a number of findings that may motivate future investigation.

[40]Figure 6 only shows bounds on $P(B-S \geq x)$ at $x = 0$; the bounds may cross at other x as well.

Table 6: Heterogeneity in Inefficient Impasse Lower Bound

A. Diff. in Inefficient Impasse Lower Bound Between Obs. Where Condition Met vs. Not							
	$P(sale)$		$P(B \geq S)$		Ineff. Impasse		
Condition	Diff.	S.E.	Diff.	S.E.	Diff.	S.E.	# Prod
Has $\geq$ One Message	0.123	(0.0329)	0.159	(0.0304)	-0.039	(0.0411)	2
Seller is an eBay Store	0.001	(0.0170)	0.003	(0.0173)	-0.002	(0.0232)	9
Buyer is from U.S.	0.369	(0.0083)	0.503	(0.0087)	-0.082	(0.0299)	25
Has Auto Accept/Decline	0.004	(0.0143)	-0.064	(0.0155)	-0.100	(0.0253)	10
High Num. of Photos	-0.022	(0.0095)	-0.041	(0.0100)	-0.032	(0.0165)	29
High Seller Average Rating	0.006	(0.0084)	-0.001	(0.0089)	-0.007	(0.0143)	39
High Seller Num. of Reviews	-0.015	(0.0081)	0.000	(0.0085)	0.033	(0.0140)	43
High Seller Experience	-0.036	(0.0079)	-0.038	(0.0084)	0.022	(0.0143)	44
High Buyer Experience	-0.038	(0.0080)	-0.033	(0.0085)	0.041	(0.0143)	43
B. Diff. in Inefficient Impasse Lower Bound Between Obs. Where Condition Met vs. Obs. with Low Seller & Low Buyer Experience							
High Seller/High Buyer Exp	-0.062	(0.0217)	0.023	(0.0003)	0.031	(5.0000)	5
High Seller/Low Buyer Exp	-0.018	(0.0222)	0.023	(0.0037)	0.030	(5.0000)	5
Low Seller/High Buyer Exp	-0.041	(0.0220)	0.023	(0.0218)	0.031	(5.0000)	5
C. Diff. in Inefficient Impasse Lower Bound Between Prod. Where Condition Met vs. Obs.							
New Product (vs. Used)	-0.018	(0.0169)	0.019	(0.0647)	0.046	(4.0000)	4
High Reference Price Prod	-0.156	(0.0085)	0.009	-(0.0138)	0.011	(44.0000)	44

Notes: Analysis using various subsamples. For each row in panels A and B, we limit to products with at least 100 negotiations where the condition is satisfied and 100 where it is not, resulting in fewer than 44 products for many conditions. This is true even for conditions relying on within-product medians ("High" rows), because some products have multiple observations at the median, resulting in fewer than 100 observations weakly greater than (or fewer than 100 strictly less than) the median. We compute $P(sale)$, $P(B \geq S)$, and the inefficient impasse $(1 - P(sale)/P(B \geq S))$ within the subsample where the condition is satisfied vs. not, compute the difference between these values in the two subsamples for a given product, and then average across products. These average differences are reported in the "Diff." columns, with standard errors on these differences shown in the "S.E." columns, computed via the delta method. The final column shows the number of products available for a given condition. Panel B measures differences relative to negotiations with low seller and low buyer experience. Panel C instead computes differences between products that are used vs. new (using the 2 product identifiers that appear in our sample both as used and new products) in the first row. Panel C second row compares differences between products with a reference price above the the median (across products) vs. below the median.

The results are shown in Table 6. We illustrate this exercise first by comparing observations in which agents exchanged some text communication during the negotiation.[41] The first row of Table 6 shows that, on average, the sale probability is 12.3 percentage point

[41] Our data indicates whether some agent sends a message, not who sends it. In the samples used in Table 6.A, the average product has the following percentages of observations satisfying the following conditions: has a message, 11.6%; eBay store, 27.9%; U.S. buyer, 55.1%; has auto accept/decline price, 32.4%.

higher with communication, and the lower bound on the first-best trade probability is 15.9 percentage points higher. Together, the inefficient impasse lower bound is 3.9 percentage points lower with communication, but this difference is not statistically significant. Because we only have lower bounds on inefficient impasse, the type of inference statement we could make, if this result were more precisely estimated, would be that there is a degree of efficiency that *may* be reached in negotiations satisfying this condition that *cannot* be reached in other negotiations. For simplicity, we will say, for example, that a negotiation is "more efficient" or, equivalently, "has less inefficient impasse" when the condition is met. This particular finding about communication, though insignificant, is consistent with laboratory experiments in Valley et al. (2002) and with descriptive evidence from a natural experiment on eBay's Germany site (Backus et al. 2023) in which the sale probability increased when the site began allowing communication. Importantly, here we find only a correlation, not a causal statement, as agents choose whether to convey such messages.

Table 6 shows only a small and insignificant difference in the inefficient impasse bound in negotiations with the seller being an eBay store vs. not.[42] However, bargaining does appear to be more efficient with U.S. buyers (a decrease in the inefficient impasse bound of 8.2 percentage points).[43] We also find that bargaining is more efficient (an increase in the bound of 10 percentage points) when the seller reports auto-accept/decline prices. This feature is meant to eliminate the need for the seller to consider very low or very high offers, saving the seller time. Our results suggest that this form of automation — allowing the seller to commit *a priori* to an acceptable range of offers — may also reduces inefficient impasse.

The remaining conditions in panel A of Table 6 refer to "high" vs "low" characteristics, e.g., a high number of photos in the listing. We define a listing to have a high number of photos if it has weakly more photos than the median listing for that product. Other "high"

[42] A *store* is a status larger sellers may pay for, giving them access to special marketing tools.

[43] Previous work has documented mixed results comparing negotiations in the U.S. and elsewhere. Roth et al. 1991, studying lab ultimatum games, found higher offers and lower acceptance rates in the U.S. than in Japan or Israel. Keniston et al. 2021 showed that agents have preferences for splitting the difference between previously proposed bargaining offers in the U.S., Spain, and India, and in cross-country tariff negotiations.

vs. "low" conditions are defined analogously.[44] Listings with a high number of photos have an inefficient impasses lower bound that is 3.2 percentage points lower, consistent with more information about product quality improving efficiency, and this difference is relatively precisely estimated.[45]

The remaining variables in Table 6 refer to the seller's average rating, the seller's number of reviews, and the buyer and seller experience. These experience variables are the total number of eBay negotiations the agent has engaged in prior to the current one.[46] We find that a higher quantity of reviews and higher buyer and seller experience are all associated with a *higher* inefficient impasse lower bound. These differences are precisely measured (and we find no difference based on the level of the seller's ratings).

A possible explanation for greater inefficiency accompanying more reviews/more experience lies in an interesting feature of incomplete-information bargaining settings. In such settings, there is a clear trade off between efficiency and rent extraction (Myerson and Satterthwaite 1983; Loertscher and Marx 2022).[47] If additional experience (or reviews) gives an agent more power to extract rents from her opponent, and agents exploit this power, we would expect additional experience to harm efficiency, consistent with Table 6. In panel B of Table 6, we peel back these experience results further, reporting the difference between the inefficient impasse lower bound in observations with different combinations of high and low experience. In each case, the difference is relative to negotiations between low-experience sellers and low-experience buyers. We find the largest difference when the buyer is experienced and the seller is not, but this difference is not precisely measured.

[44]The number of photos is a choice of the seller; eBay requires at least one. Th median number of photos (across observations within a product) is 3.2 photos for the average product.

[45]Backus et al. (2020) documented that the first buyer offer on a listing arrives more quickly to listings with more photos, arguing that the additional photos may reduce asymmetric information, a problem that reduces efficiency in some models (Deneckere and Liang 2006). Lewis (2011) similarly associates an increase in the number of photos on eBay listings with a reduction in asymmetric information.

[46]The median cutoff conditions defining "High" realizations of these variables are, for the average product, a 99.8 rating (out of 100), 763.3 reviews, 300.8 previous negotiations for the seller, and 16.9 previous negotiations for the buyer.

[47]Under incomplete information, giving one agent more bargaining power increases her payoff but also increases deadweight loss, reducing the size of the total pie. In a complete-information settings (Nash bargaining, say), an increase in the bargaining power of one agent does not imply any change to total surplus.

Finally, panel C of Table 6 compares inefficiency *across* products (rather than within) based on certain characteristics. We find that the inefficient impasse lower bound is 6.5 percentage points higher for negotiations over new products than used products, although this difference is not statistically significant.[48] We find a small and insignificant difference between higher reference price products relative to lower reference price products.

7 Discussion and Conclusion

This study provides bounds on the private-value distributions of buyers and sellers and on the first-best trade probability from sequential-offer bargaining data on eBay. The bounds are sharp and nonparametric. We rely on revealed preferences arguments and other assumptions on behavior or information without specifying a full model of equilibrium play. Bounds relying on our strongest assumptions (monotonicity of sellers' first offers and independence between buyer's values and seller's first offers) can cross. While these strong assumptions are satisfied in equilibria of two-sided incomplete-information bargaining games analyzed in the theoretical literature, they appear too strong for empirical work. This underscores the importance of our more moderate assumptions, which allow for empirical features such as unobserved game-level heterogeneity.

Our approach circumvents a major theoretical problem arising in bargaining games of incomplete information: signaling. Each action taking by a player signals information to the opposing player, yielding a multiplicity (even a continuum) of equilibria that are qualitatively very different depending on how off-equilibrium beliefs are specified (see Ausubel et al. 2002). The bounds we propose do not rely on any specification of beliefs, equilibrium refinements, or equilibrium selection, allowing us to study how well bargaining performs in this real-world market without strongly constraining the answer a priori.

Given that the bounds rely on assumptions that, in many cases, are quite weak, they

[48] This comparison uses the two bar codes that appear in our sample both as new and used item; that is, we observe at least 200 negotiations for used versions and at least 200 for new versions of these bar codes. Because we define a *product* as a bar-code-condition-type pair (where condition type means used vs. new), as described in Section 2, there are four *products* in the analysis of the first row of Table 6

can naturally be wide. We show that, in spite of this, the bounds can highlight which behavioral properties are consistent with real-world bargaining and allow us to quantify inefficient impasse — a question that is not possible to address under complete-information frameworks such as Nash bargaining. Under our preferred assumptions, we find evidence of inefficient impasse: for the median product, at least 35% of failed trades are cases where positive trade gains exist. Thus, viewing consumer negotiations in this market through the lens of a complete-information model would be incorrect. We also find that it would be misleading to impose too strong of an assumption on behavior. Though satisfied by existing theoretical models, the strongest assumptions we explore would suggest that inefficient breakdown is far more prevalent. We are able to falsify these stronger assumptions.

It is possible that the assumptions we use — even those that are not the strongest — are still too strong. To assess this, there are relatively few empirical analyses of bargaining under incomplete information to which ours can be directly compared — and none, to our knowledge, from real-world negotiations involving consumers. However, several studies offer useful comparisons. First, Valley et al. (2002) studied laboratory participants in a two-sided incomplete-information bargaining game, and found that participants fail to trade in 46% of cases where trade gains exist. They found that this impasse is reduced substantially (to 15%) when negotiators are allowed to communicate. Bochet et al. (2023) and Huang et al. (2023) also studied two-sided incomplete-information experimentally, finding a corresponding level of inefficient impasse of 30% and 17%, respectively. In a structural model, Larsen (2021), studying professionals negotiating over used-car inventory, found that at least 21% of first-best trades fail, and Larsen et al. (2022) found that skilled mediators substantially reduce this inefficient breakdown.[49] Relative to these numbers, our estimates of inefficient impasse for the median product suggest that the performance of bargaining in real-world consumer settings is in the ballpark of (but perhaps more inefficient than) those involving laboratory participants or business-to-business negotiations.

[49]These numbers are found in (or can be constructed from) Table 1 of Valley et al. (2002), Figure 3 of Bochet et al. (2023), Table 2 of Huang et al. (2023), and Table 3 of Larsen (2021). Note that in Huang et al. (2023), gains from trade always exist, which is not the case for the other experimental studies.

We see our findings as useful benchmarks to which future studies of bargaining in various contexts may be compared.

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A Auxiliary Lemmas

We prove two auxiliary lemmas. The first is used to prove that an lower or upper bound is in the identified set and the second to show that any function above the lower bound (or below the upper bound) is also.

Lemma 1. *Let X be a random variable, Y a random vector, and denote the conditional CDF by $F_{X|Y}(x \mid y)$. Let $g(x,y) : \mathbb{R}^{1+\dim(Y)} \to [0,1]$ be a CDF for all y.*

1. *If $g(x,y) \geq F_{X|Y}(x \mid y)$ for all x and y, then there exists a random variable W such that $W \leq X$ and $F_{W|Y}(x \mid y) = g(x,y)$.*
2. *If $g(x,y) \leq F_{X|Y}(x \mid y)$ for all x and y, then there exists a random variable W such that $W \geq X$ and $F_{W|Y}(x \mid y) = g(x,y)$.*

Proof. We only prove part 1; part 2 follows from analogous arguments. Define $g^{-1}(z,y) = \inf\{x \in \mathbb{R} : g(x,y) \geq z\}$. Let $U \sim U[0,1]$ be independent of (X,Y) and define $\tilde{F}_{X,U|Y}(x,u,y) = P(X < x \mid Y = y) + P(X = x \mid Y = y)u$. Finally, let $W = g^{-1}\left(\tilde{F}_{X,U|Y}(X,U,Y), Y\right)$. We show that $W \leq X$ and $F_{W|Y}(x \mid y) = g(x,y)$.

For the second part, it is sufficient to prove that $\tilde{F}_{X,U|Y}(X,U,Y) \mid Y = y \sim U[0,1]$ for all y. To do so, let $\bar{x}_1(y), \bar{x}_2(y), \ldots, \bar{x}_{M(y)}(y)$ be the mass points of X conditional on y. Then for all $m = 1,2,\ldots,M(y)$

$$\tilde{F}_{X,U|Y}(X,U,Y) \sim U[P(X < \bar{x}_m(y) \mid Y = y), P(X = \bar{x}_m(y) \mid Y = y)]$$

conditional on $Y = y$ and $X = \bar{x}_m(y)$ and for all $m = 1,\ldots,M(y)-1$

$$\tilde{F}_{X,U|Y}(X,U,Y) \sim U[P(X \leq \bar{x}_m(y) \mid Y = y), P(X < \bar{x}_{m+1}(y) \mid Y = y)]$$

conditional on $Y = y$ and $X \in (\bar{x}_m(y), \bar{x}_{m+1}(y))$. Finally if $P(X < \bar{x}_1(y) \mid Y = y) > 0$, then

$\tilde{F}_{X,U|Y}(X,U,Y) \mid Y=y, X<\bar{x}_1(y) \sim U[0, P(X<\bar{x}_1(y) \mid Y=y)]$ and if $P(X>\bar{x}_{M(y)}(y) \mid Y=y)>0$, then $\tilde{F}_{X,U|Y}(X,U,Y) \mid Y=y, X>\bar{x}_{M(y)}(y) \sim U[P(X \leq \bar{x}_{M(y)}(y) \mid Y=y), 1]$. Since the supports of the intervals of these uniforms only overlap at the boundaries, the union of the supports is $[0,1]$, and the difference of the upper and lower bound is equal to the probability that X is in the respective set (that is either $X=\bar{x}_m(y)$ or $X \in (\bar{x}_m(y), \bar{x}_{m+1}(y))$), it follows that $\tilde{F}_{X,U|Y}(X,U,Y) \mid Y=y \sim U[0,1]$ for all y.

To show that $W \leq X$, notice that, since $g(x,y) \geq F_{X|Y}(x \mid y)$ for all x and y, it holds that

$$g^{-1}(z,y)=\inf\{x \in \mathbb{R}: g(x,y) \geq z\} \leq \inf\{x \in \mathbb{R}: F_{X|Y}(x \mid y) \geq z\} = F_{X|Y}^{-1}(z \mid y)$$

Next, notice that, if $Y=y$ and $X=\bar{x}_m(y)$, then $\tilde{F}_{X,U|Y}(x,u,y) \leq P(X=\bar{x}_m(y) \mid Y=y)$ for all $u \in [0,1]$ and

$$W=g^{-1}\left(\tilde{F}_{X,U|Y}(\bar{x}_m(y),U,y),y\right) \leq F_{X|Y}^{-1}(P(X=\bar{x}_m(y) \mid Y=y) \mid y)=\bar{x}_m(y)$$

Finally, for all $Y=y$ and $X=x \notin \{\bar{x}_1(y), \bar{x}_2(y), \ldots, \bar{x}_{M(y)}(y)\}$,
$W=g^{-1}\left(\tilde{F}_{X,U|Y}(x,U,y),y\right) \leq F_{X|Y}^{-1}(P(X \leq x \mid Y=y) \mid y) \leq x.$ □

Lemma 2. *Let X be a random variable, Y a random vector, and $g(x): \mathbb{R} \to [0,1]$ a CDF.*

1. *If $g(x) \geq F_X(x)$ for all x, then there exists a random variable W such that $W \leq X$ and $F_W(x)=g(x)$.*
2. *If $g(x) \leq F_X(x)$ for all x, then there exists a random variable W such that $W \geq X$ and $F_W(x)=g(x)$.*

Moreover, in both cases, if $F_{X|Y}(x \mid y)$ is either weakly increasing, weakly decreasing, or constant in an element of y for all x, then $F_{W|Y}(x \mid y)$ shares this property.

Proof. Define $g^{-1}(z)=\inf\{x \in \mathbb{R}: g(x) \geq z\}$. Let $U \sim U[0,1]$ be independent of (X,Y) and define $\tilde{F}_{X,U}(x,u)=P(X<x)+P(X=x)u$. Finally, let $W=g^{-1}\left(\tilde{F}_{X,U}(X,U)\right)$. The first two parts of the lemma follow immediately from the proof of Lemma 1. To show that

$F_{X|Y}(x \mid y)$ and $F_{W|Y}(x \mid y)$ share the same monotonicity properties, define $p_X(x) = P(X = x)$ and notice that

$$
\begin{aligned}
P(W \leq w \mid Y = y) &= P\left(g^{-1}\left(\tilde{F}_{X,U}(X,U)\right) \leq w \mid Y = y\right) \\
&= \int_0^1 P\left(g^{-1}\left(F_X(X) + p_X(X)(u-1)\right) \leq w \mid Y = y\right) du
\end{aligned}
$$

The function $h(x,u) = g^{-1}\left(F_X(x) + p_X(x)(u-1)\right)$ is weakly increasing in x for all u. It follows that there exists $x(u)$ such that

$$
P(h(X,u) \leq w \mid Y = y) = P(X \leq x(u) \mid Y = y)
$$

or

$$
P(h(X,u) \leq w \mid Y = y) = P(X < x(u) \mid Y = y)
$$

If the right hand side is weakly increasing/decreasing/constant in y for all u, it follows that $P(W \leq w \mid Y = y)$ shares the same monotonicity property. $\square$

B Proofs of Main Theorems

*Proof of **Theorem 1** (Unconditional Bounds).* $X_Q^S \leq S \leq X_{AC}^S \Rightarrow P(X_{AC}^S \leq x) \leq P(S \leq x) \leq P(X_Q^S \leq x)$. Similarly, $X_{AC}^B \leq B \leq X_Q^B \Rightarrow P(X_Q^B \leq x) \leq P(B \leq x) \leq P(X_{AC}^B \leq x)$.

If $S = X_{AC}^S$, then $P(X_{AC}^S \leq x) = P(S \leq x)$. Hence, the lower bound can be attained. Moreover, it follows from Lemma 2 (with $X = X_{AC}^S$) that any CDF $F(x)$ with $F(x) \geq P(X_{AC}^S \leq x)$ for all x is also in the identified set. Similar arguments imply sharpness of the upper bound and the buyer bounds. $\square$

*Proof of **Theorem 2** (Monotonicity).* Conditional on $P_1^S = y$, $P_2^B = z$, $X_{AC}^{B*}(y,z) \leq B \leq X_Q^{B*}(y,z)$ and $X_{AC}^{B*}(y,z)$ and $X_Q^{B*}(y,z)$ are non-random. Then (2) and $X_{AC}^B \leq B \leq X_Q^B \Rightarrow P(B \leq x) \geq \int \mathbf{1}(X_Q^{B*}(y,z) \leq x) dF_{P_1^S,P_2^B}(y,z)$. The buyer upper bound is analogous. For the seller, conditional on y, we have $X_Q^{S*}(y) \leq S \leq X_{AC}^{S*}(y)$ and $X_Q^{S*}(y)$ and $X_{AC}^{S*}(y)$ are non-random. Combining this with (1) implies $\int \mathbf{1}(X^{S*}_{AC}(y) \le x) dF_{P^S_1}(y) \le P(S \le x) \le \int \mathbf{1}(X^{S*}_{Q}(y) \le x) dF_{P^S_1}(y)$.

The seller lower bound is attained with $S = X^{S*}_{AC}(P^S_1)$, in which case S is increasing in P^S_1. We can apply a modification of Lemma 2 (with $X = X^{S*}_{AC}(P^S_1)$) to show that any CDF $F(x)$ with $F(x) \ge P(X^{S*}_{AC}(P^S_1) \le x)$ for all x is also in the identified set. To do so, let $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_L$ be the mass points of $X^{S*}_{AC}(P^S_1)$. Let $\tilde{U} \sim U[0,1]$ be independent of $X^{S*}_{AC}(P^S_1)$ and

$$U = \begin{cases} F_{P^S_1 | X^{S*}_{AC}(P^S_1) = \bar{x}_l}(P^S_1) & \text{if } X^{S*}_{AC}(P^S_1) = \bar{x}_l \qquad \forall\, l = 1,2,\ldots,L \\ \tilde{U} & \text{if } X^{S*}_{AC}(P^S_1) \in (\bar{x}_l, \bar{x}_u) \end{cases}$$

and define $\tilde{F}_{X^{S*}_{AC}(P^S_1),U}(x,u) = P\left(X^{S*}_{AC}(P^S_1) < x\right) + P(X^{S*}_{AC}(P^S_1) = x)u$ Finally, let $F^{-1}(z) = \inf\{x \in \mathbb{R} : F(x) \ge z\}$ and $W = F^{-1}\left(\tilde{F}_{X,U}(X^{S*}_{AC}(P^S_1), U)\right)$.

Since $U \mid X^{S*}_{AC}(P^S_1) \sim U[0,1]$, the arguments from the proof of Lemma 2 imply $P(W \le x) = F(x)$. The construction ensures that W is increasing in P^S_1 and hence, the CDF is attained when $S = W$. Sharpness of the upper bound follows analogously.

Similarly, the lower bound for the buyer is attained when $B = X^{B*}_Q(P^S_1, P^B_2) \le X^B_Q$ and, just like above, a modification of Lemma 2 implies that any CDF $F(x)$ with $F(x) \ge P(X^{B*}_Q(P^S_1, P^B_2) \le x)$ for all x is also in the identified set. □

Proof of ***Theorem 3*** *(Independence).* First, note $P(S \le x) = \int P(S \le x \mid P^S_1 = y) dF_{P^S_1}(y)$, which, by A3.i, is $\int \max_z P(S \le x \mid P^S_1 = y, P^B_2 = z) dF_{P^S_1}(y)$. The lower bound follows from $S \le X^S_{AC}$. The seller upper bound is analogous. For the buyer, $P(B \le x) = \max_y P(B \le x \mid P^S_1 = y)$ and $P(B \le x) = \min_y P(B \le x \mid P^S_1 = y)$. The bounds follow from $X^B_{AC} \le B \le X^B_Q$.

Attainment of the seller lower bounds follows from Lemma 1 with $X = X^S_{AC}$ and $Y = (P^S_1, P^B_2)$, $g(x,y,z) = \max_{z'} P(X^S_{AC} \le x \mid P^S_1 = y, P^B_2 = z')$, and $W = S$. Since g does not depend on z, the implied distribution of S is independent of P^B_2 conditional on P^S_1. Any function above the CDF being in the identified set follows from Lemma 2. Sharpness of the upper bound and the buyer bounds follows from analogous arguments. □

Proof of ***Theorem 4*** *(Stochastic Monotonicity).* By (2), $P(B \le x) = \int P(B \le x \mid P^S_1 = y, P^B_2 = z) dF_{P^S_1, P^B_2}(y,z)$. By A4 this can then be written $\int \max_{z' \ge z} P(B \le x \mid, P^S_1 = y, P^B_2 = z') dF_{P^S_1, P^B_2}(y,z)$

or $\int \min_{z' \leq z} P(B \leq x \mid P_1^S = y, P_2^B = z') dF_{P_1^S, P_2^B}(y,z)$. The bounds follow from $X_{AC}^B \leq B \leq X_Q^B$. For the seller, we have, by (1) and A4, $P(S \leq x) = \int \max_{y' \geq y} P(S \leq x \mid P_1^S = y') dF_{P_1^S}(y)$ and $P(S \leq x) = \int \min_{y' \leq y} P(S \leq x \mid P_1^S = y') dF_{P_1^S}(y)$. The bounds follow from $X_Q^S \leq S \leq X_{AC}^S$.

Attainment of the seller lower bounds follows from Lemma 1 with $X = X_{AC}^S$, $Y = P_1^S$, $g(x,y) = \max_{y' \geq y} P(X_{AC}^S \leq x \mid P_1^S = y')$, and $W = S$. Since g is weakly increasing y for all x, the implied conditional distribution of S satisfies A4.i. Any function above the CDF being in the identified set follows from Lemma 2. Sharpness of the upper bound and the buyer bounds follows from analogous arguments. □

Proof of ***Theorem 5*** *(Positive Correlation).* First, note $P(S \leq x) = \int P(S \leq x \mid P_1^S = y, P_2^B = z) dF_{P_1^S, P_2^B}(y,z)$, which, by A5, is $\int \max_{z' \geq z} P(S \leq x \mid P_1^S = y, P_2^B = z') dF_{P_1^S, P_2^B}(y,z)$. The lower bound follows from $S \leq X_{AC}^S$. The seller upper bound is analogous. For the buyer, we have $P(B \leq x) = \int \max_{y' \geq y} P(B \leq x \mid P_1^S = y') dF_{P_1^S}(y)$ and $P(B \leq x) = \int \min_{y' \leq y} P(B \leq x \mid P_1^S = y') dF_{P_1^S}(y)$. The bounds follow from $X_{AC}^B \leq B \leq X_Q^B$. The sharpness arguments follow from similar arguments as those in the proof of Theorem 4. □

Proof of ***Theorem 6*** *(Surplus Stochastic Monotonicity and Buyer Monotonicity).* Note $P(B - S \geq x) = \int P(B - S \geq x \mid P_1^S = y, P_2^B = z) dF_{P_1^S, P_2^B}(y,z)$. By A6, this can be written $\int \max_{z' \leq z} P(B - S \geq x \mid P_1^S = y, P_2^B = z') dF_{P_1^S, P_2^B}(y,z)$. The lower bound follows from A1.i and A2.ii, implying $X_{AC}^{B^*}(P_1^S, P_2^B) - X_{AC}^S \leq B - S$. The upper bound is analogous.

Theorem 2 applied with $X = X_{AC}^S - X_{AC}^{B^*}(P_1^S, P_2^B)$, $Y = (P_1^S, P_2^B)$, $g(x,y,z) = \max_{z' \leq z} \Pr(X_{AC}^S - X_{AC}^{B^*}(y,z) \leq x \mid P_1^S = y, P_2^B = z')$, and $W = S - B$ implies that there exists a random variable W such that $W \geq X_{AC}^{B^*}(P_1^S, P_2^B) - X_{AC}^S$ and

$$P(W \geq x \mid P_1^S = y, P_2^B = z) = \int \max_{z' \leq z} P(X_{AC}^{B^*}(y,z) - X_{AC}^S \geq x \mid, P_1^S = y, P_2^B = z') dF_{P_1^S, P_2^B}(y,z)$$

Letting $B = X_{AC}^{B^*}(P_1^S, P_2^B)$ and $S = W + B$ implies that all assumptions are satisfied and the lower bound is attained. Any function above the CDF being in the identified set follows modifying Lemma 2 as in the proof of Theorem 8. □

Proof of ***Theorem 7*** *(Surplus Weak Monotonicity and Buyer Monotonicity).* By A1 and A2,

$S \leq X_{AC}^{S}$ and $B \geq X_{AC}^{B*}(y,z)$ conditional on $P_1^S = y$ and $P_2^B = z$. Define $X_{AC}^{B*-S}(y,z) = \overline{\text{supp}}(X_{AC}^{B*}(y,z) - X_{AC}^{S} : P_2^B \leq z, P_1^S = y)$. The assumptions imply that $B - S \geq X_{AC}^{B*-S}(y,z)$ conditional on $P_1^S = y$ and $P_2^B = z$ and thus, $P(B - S \geq x) \geq \int \mathbf{1}(X_{AC}^{B*-S}(y,z) \geq x) dF_{P_1^S, P_2^B}(y,z)$. The upper bound is analogous.

The lower bound is attained when $B = X_{AC}^{B*}(P_1^S, P_2^B)$ and $S = X_{AC}^{B*}(P_1^S, P_2^B) - X_{AC}^{B*-S}(P_1^S, P_2^B)$ in which case both monotonicity assumptions hold. That any function above the CDF is also in the identified set follows from modifying Lemma 2, as in the Theorem 2 proof. $\square$

C Bounds Based on Combined Assumptions

B.1. Derivation of Bounds Combining Assumptions.

Theorem 8. *(Independence + Monotonicity.) (5) gives a sharp lower bound for F_S under A1.i, A2.i, and A3.i and a sharp upper bound for F_S under A1.ii, A2.i, and A3.i. The following inequalities give a sharp lower bound for F_B under A1.iv, A2.ii, and A3.ii and a sharp upper bound for F_B under A1.iii, A2.ii, and A3.ii:*

$\max_y \int \mathbf{1}(X_{Q}^{B*}(y,z) \leq x) dF_{P_2^B|P_1^S}(z|y) \leq F_B(x) \leq \min_y \int \mathbf{1}(X_{AC}^{B*}(y,z) \leq x) dF_{P_2^B|P_1^S}(z|y)$

Proof. Sharpness of the seller lower bound follows from the proof of Theorem 2 and the observation that $X_{AC}^{S*}(P_1^S)$ is independent of P_2^B conditional of $P_1^S = y$ (because $X_{AC}^{S*}(y)$ is deterministic).

Note $P(B \leq x) = \max_y P(B \leq x \mid P_1^S = y)$. The lower bound follows by (2) and $B \leq X_{Q}^{B*}(y,z)$. The upper bound is analogous. Attainment of the lower bound follows from modifying Lemma 2. In particular, denote the lower and upper mass point of $X_{Q}^{B*}(P_1^S, P_2^B)$ conditional on $P_1^S = y$ by $\tilde{x}_1(y), \ldots, \tilde{x}_{L(y)}(y)$ and let $U = F_{P_2^B|P_1^S = y, X_Q^{B*}(y, P_2^B) = \bar{x}_u(y)}(P_2^B, P_1^S)$ if $P_1^S = y$ and $X_{Q}^{B*}(P_1, P_2^B) = \bar{x}_l(y)$ and $U \sim U[0,1]$ independent of all other random variables if $P_1^S = y$ and $X_{Q}^{B*}(P_1, P_2^B) \neq \bar{x}_l(y)$ for all $l = 1, 2, \ldots L(y)$. Then $U \mid P_1^S = y, X_{Q}^{B*}(P_1, P_2^B) = x \sim U[0,1]$ for x and y. Using this random variable U, Lemma 1 applied with $X = X_{Q}^{B*}(P_1^S, P_2^B)$, $Y = P_1^S$, $g(x,y) = \max_{y'} P(X_{Q}^{B*}(P_1^S, P_2^B) \leq x \mid P_1^S = y')$, and $W = B$ implies that the lower bound can be attained. The construction also ensures that W is weakly increasing in P_2^B

conditional on P_S^1. Any function above the CDF being in the identified set follows from a similar modification of Lemma 2. □

Theorem 9. *(Positive Correlation + Monotonicity.) (5) gives a sharp lower bound for F_S under A1.i, A2.i, and A5.i and a sharp upper bound for F_S under A1.ii, A2.i, and A5.i. The following inequalities give a sharp lower bound for F_B under A1.iv, A2.ii, and A5.ii and a sharp upper bound for F_B under A1.iii, A2.ii, and A5.ii:*

$$\int \max_{y' \geq y} \int \mathbf{1}(X_Q^{B*}(y',z) \leq x) dF_{P_2^B|P_1^S}(z|y) \leq F_B(x) \leq \int \min_{y' \leq y} \int \mathbf{1}(X_{AC}^{B*}(y',z) \leq x) dF_{P_2^B|P_1^S}(z|y)$$

Proof. Sharpness of the seller lower bound follows from the proof of Theorem 2 and the observation that $X_{AC}^{S*}(P_1^S)$ is independent of P_2^B conditional of $P_1^S = y$ (because $X_{AC}^{S*}(y)$ is deterministic).

Note $P(B \leq x) = \int \max_{y' \geq y} P(B \leq x \mid P_1^S = y') dF_{P_1^S}(y)$. The lower bound follows by (2) and $B \leq X_Q^{B*}(y,z)$. The upper bound is analogous. Sharpness follows from the same arguments as those in the proof of Theorem 8. □

Theorem 10. *(Independence + Stochastic Monotonicity.) The following inequalities give a sharp lower bound for F_S under A1.i, A4.i, and A3.i; a sharp upper bound for F_S under A1.ii, A4.i, and A3.i; a sharp lower bound for F_B under A1.iv, A4.ii, and A3.ii; and a sharp upper bound for F_B under A1.iii, A4.ii, and A3.ii:*

$$\int \max_{y' \geq y} \max_z m_{AC}^S(x,y',z) dF_{P_1^S}(y) \leq F_S(x) \leq \int \min_{y' \leq y} \min_z m_Q^S(x,y',z) dF_{P_1^S}(y)$$
$$\max_y \int \max_{z' \geq z} m_Q^B(x,y,z') dF_{P_2^B|P_1^S}(z|y) \leq F_B(x) \leq \min_y \int \max_{z' \geq z} m_{AC}^B(x,y,z') dF_{P_2^B|P_1^S}(z|y)$$

Proof. Note $P(S \leq x) = \int \max_{y' \geq y} P(S \leq x \mid P_1^S = y') dF_{P_1^S}(y)$, and $P(S \leq x \mid P_1^S = y') = \max_z P(S \leq x \mid P_1^S = y', P_2^B = z)$. The lower bound follows from $S \leq X_{AC}^S$. For the buyer, $P(B \leq x) = \max_y P(B \leq x \mid P_1^S = y)$. Applying (2) and A4 yields $P(B \leq x \mid P_1^S = y) = \int \max_{z' \geq z} P(B \leq x \mid P_1^S = y, P_2^B = z) dF_{P_2^B|P_1^S}(z|y)$. The lower bound follows from $B \leq X_Q^B$. Analogous arguments yield the upper bounds. Sharpness follows from the same arguments as those with the corresponding single assumption case. □

Theorem 11. *(Positive Correlation + Stochastic Monotonicity.) The following inequalities give a sharp lower bound for F_S under A1.i, A4.i, and A5.i; a sharp upper bound for F_S under A1.ii, A4.i, and A5.i; a sharp lower bound for F_B under A1.iv, A4.ii, and A5.ii; and a sharp upper bound for F_B under A1.iii, A4.ii, and A5.ii:*

$$
\begin{aligned}
F_S(x) &\geq \int \max_{y' \geq y} \int \max_{z' \geq z} m^S_{AC}(x, y', z') dF_{P_2^B | P_1^S}(z|y') dF_{P_1^S}(y) \\
F_S(x) &\leq \int \min_{y' \leq y} \int \min_{z' \geq z} m^S_{Q}(x, y', z') dF_{P_2^B | P_1^S}(z|y') dF_{P_1^S}(y) dy \\
F_B(x) &\geq \int \max_{y' \geq y} \int \max_{z' \geq z} m^B_{Q}(x, y', z') dF_{P_2^B | P_1^S}(z|y') dF_{P_1^S}(y) \\
F_B(x) &\leq \int \min_{y' \leq y} \int \min_{z' \geq z} m^B_{AC}(x, y', z') dF_{P_2^B | P_1^S}(z|y') dF_{P_1^S}(y)
\end{aligned}
$$

Proof. Note $P(S \leq x) = \int \max_{y' \geq y} P(S \leq x \mid P_1^S = y') dF_{P_1^S}(y)$ and

$$
\begin{aligned}
P(S \leq x \mid P_1^S = y') &= \int P(S \leq x \mid P_1^S = y', P_2^B = z) dF_{P_2^B | P_1^S}(z|y') \\
&= \int \max_{z' \geq z} P(S \leq x \mid P_1^S = y', P_2^B = z) dF_{P_2^B | P_1^S}(z|y')
\end{aligned}
$$

The lower bound follows from $S \leq X^S_{AC}$. For the buyer, $P(B \leq x) = \int \max_{y' \geq y} P(B \leq x \mid P_1^S = y') dF_{P_1^S}(y)$ and

$$
\begin{aligned}
P(B \leq x \mid P_1^S = y') &= \int P(B \leq x \mid P_1^S = y', P_2^B = z) dF_{P_2^B | P_1^S}(z|y') \\
&= \int \max_{z' \geq z} P(B \leq x \mid P_1^S = y', P_2^B = z) dF_{P_2^B | P_1^S}(z|y')
\end{aligned}
$$

The lower bound follows from $B \leq X^B_Q$. Upper bounds follow analogously. Sharpness follows from arguments as those with the corresponding single assumption case. □

D Additional Discussion of Data and Assumptions

D.1. Sample Restrictions. As discussed in Section 2, what we refer to as our *original data* consists of all eBay Best-Offer-enabled listings from June 2012 through May 2013 satisfying the following: a buyer makes an offer, the item has a product identifier, and the product's reference price is computed based on at least ten non-Best-Offer posted price sales. We impose several sample restrictions on this dataset to obtain our estimation sample. These restrictions are described in the notes to Table 7, which shows the number of observations from the original sample that are dropped due to each restriction.[50] Several restrictions are data cleaning steps that drop only a small fraction of observations. Our major restrictions are the following. First, we remove negotiations in which an agent is involved in other negotiations simultaneously, dropping 48.55% of observations. Second, after imposing this restriction, we then keep only the first seller with whom a given buyer interacts for a given product, and the first buyer with whom a given seller negotiates, dropping 16%. Third, we limit to products for which we have at least 200 observations, dropping 29%.

Table 7: Data Cleaning

1) Frac. incomplete sequences	0.0226
2) Frac. overlapping sequences	0.4855
3) Frac. additional incomplete sequences	0.0004
4) Frac. extreme outlier offers/prices	0.0340
5) Frac. dropped by keeping only first seller/buyer	0.1604
6) Frac. with fewer than 200 negotiations per product	0.2905

Notes: Table shows the order in which our additional sample restrictions are enforced on the original data, and the fraction of observations dropped at each step. First row shows a small fraction are dropped due to incomplete or nonsensical bargaining data, including observations where (i) an offer or the final price is higher than the Buy-It-Now price, (ii) more than one offer arrives at the same time from the same buyer, (iii), additional actions take place after one party accepts, (iv) one or both parties make more than three offers, (v) the data indicates a counteroffer takes place but the offer itself is not recorded, or (vi) agents make non-monotonic offers (e.g. a buyer offers more than the seller has asked for or a seller asks for less than the buyer has offered). Second row shows the fraction of observations dropped due to overlapping negotiations. Third row shows an additional small fraction of incomplete/nonsensical observations more easily identified after overlapping sequences are dropped in step 2. Fourth row shows observations dropped because of offers or auto accept/decline prices being greater than 2.5 times the reference price. Fifth row shows fraction dropped when we keep only the first seller a buyer negotiates with and vice versa. Final row shows fraction of the data dropped because the product had fewer than 200 negotiations.

We now consider how our results change when we modify step 5 of these sample restrictions. Table 8 replicates Table 3 from the body of the paper but where, instead of step 5, we keep the *last* (in panel A) or a *random* (in panel B) seller among those with whom a given buyer interacts, and similarly for sellers interacting with multiple buyers. The results are

[50]The precise fraction of observations dropped due to each restriction depends on the order in which the restrictions are imposed. The order we followed is the order in which they are listed in Table 7.

similar to those in Table 3. Let the *last version* of the data denote the sample used in panel A and the *random version* denote the sample used in panel B. Recall that the inefficient impasse lower bound for the median product is 0.354 in our main sample. This number decreases to 0.298 when we use the last version of the data, and decreases to 0.328 when we use the random version. Thus, the implications for inefficient impasse are similar in these samples as in our main sample, albeit slightly lower.

Table 8: Bounds Crossing with Different Bilateral Bargaining Pairs

	Seller Bounds			Buyer Bounds		
	Frac. Cross	Frac. Reject	IVE	Frac. Cross	Frac. Reject	IVE
A. Restricting to last seller a buyer negotiates with, and vice versa						
Unconditional (A1)	0	0	0	0	0	0
Monotonicity (A2)	1.00	1.00	0.23	0	0	0
Independence (A3)	0.07	0	0	0.40	0.14	0.01
Stochastic Monotonicity (A4)	0	0	0	0	0	0
Positive Correlation (A5)	0	0	0	0	0	0
Mon. + Indep. (A2 + A3)	1.00	1.00	0.23	0.60	0.47	0.06
Mon. + Pos. Corr (A2 +A5)	1.00	1.00	0.23	0	0	0
Stoch. Mon. + Indep. (A4 + A3)	0.19	0	0.00	0.37	0.14	0.01
Stoch. Mon. + Pos. Corr (A4 + A5)	0	0	0	0	0	0
B. Restricting to random seller a buyer negotiates with, and vice versa						
Unconditional (A1)	0	0	0	0	0	0
Monotonicity (A2)	1.00	1.00	0.23	0	0	0
Independence (A3)	0.05	0	0	0.27	0.12	0.01
Stochastic Monotonicity (A4)	0	0	0	0	0	0
Positive Correlation (A5)	0	0	0	0	0	0
Mon. + Indep. (A2 + A3)	1.00	1.00	0.23	0.61	0.39	0.05
Mon. + Pos. Corr (A2 +A5)	1.00	1.00	0.23	0	0	0
Stoch. Mon. + Indep. (A4 + A3)	0.15	0	0	0.27	0.10	0.00
Stoch. Mon. + Pos. Corr (A4 + A5)	0	0	0	0	0	0

Notes: Table replicates Table 3 using different samples. The main sample in the paper restricts to the first seller a given buyer negotiates with and the first buyer a given seller negotiates with. Panel A above uses the *last* seller and *last* buyer. Panel B selects a *random* seller among those a given buyer negotiates with, and a *random* buyer among those that a given seller negotiates with.

D.2. Bargaining Costs. We do not explicitly model bargaining costs in this paper. Such costs could take many forms; we highlight only a few here and show how they would fit into our framework. Suppose the buyer faces an additive disutility, call it χ, when making or accepting an offer, and the buyer's gross utility and outside option are V and μ (as in

Section 3.2). The buyer's value (willingness to pay) for this trade would be $B = V - \mu - \chi$. The object B is the buyer's value net of bargaining costs, just as it is net of any outside option. An alternative cost form is the fee τ that a seller pays eBay when a sale occurs; with this fee, the seller's willingness to sell is $S \equiv \breve{S} + \tau$, where $\breve{S}$ is the least the seller would be willing to accept absent eBay fees.[51] Costs could also include a shipping cost ψ a buyer pays, resulting in buyer value of $B = V - \mu - \psi$. Finally, costs might take the form of discounting, such that, for a discount factor δ, the buyer receives $\delta(V - P)$ by accepting an offer P and μ otherwise. The buyer's willingness to pay that could be bounded by observing accept or quit decisions would then be $B = V - \mu/\delta$.[52]

E Estimation and Inference

E.1. Inference and Median Unbiased Estimation with Single Assumptions

Here we focus only on lower bound estimators. Upper bound estimators are analogous. Let $z_{1-\alpha}$ be the $1-\alpha$ quantile from a standard normal distribution. The unconditional lower bound estimators are empirical distribution functions and pointwise, one-sided $1-\alpha$ confidence bands are therefore $\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(X_{AC,i}^{S} \leq x) - z_{1-\alpha}\sqrt{\frac{\left(\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(X_{AC,i}^{S} \leq x)\right)\left(1-\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(X_{AC,i}^{S} \leq x)\right)}{n}}$ and $\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(X_{Q,i}^{B} \leq x) - z_{1-\alpha}\sqrt{\frac{\left(\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(X_{Q,i}^{B} \leq x)\right)\left(1-\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(X_{Q,i}^{B} \leq x)\right)}{n}}$. Similarly, we calculate confidence bands under monotonicity by replacing $X_{AC,i}^{S}$ with $\hat{X}_{AC}^{S*}(P_{1,i}^{S})$ and $X_{Q,i}^{B}$ with $\tilde{X}_{Q}^{B*}(P_{1,i}^{S}, P_{2,i}^{B})$

The large-sample distributions of the stochastic monotonicity bounds are nonstandard because the bounds are only directionally differentiable functions of conditional mean functions. Similar inference problems arise in Chernozhukov et al. (2013) and Fang and Santos (2018). However, neither paper applies in our setting because Chernozhukov et al. (2013) focus on maxima and minima of conditional mean functions and Fang and Santos (2018) is

[51] In practice, this fee is a percentage commission (typically 10%) but in this discussion we consider it to be additive for simplicity.

[52] If bargaining costs were heterogeneous across agents — for example, if each faced distinct additive disutilities or discount rates — other assumptions we work with, such as monotonicity, could be violated.

not applicable with nonparametric estimators. Developing a (nonstandard) bootstrap procedure, as in Fang and Santos (2018), while allowing for nonparametric estimators, is beyond the scope of our paper. We therefore use subsampling, which is known to be consistent under weak assumptions (Politis and Romano 1994). We use a subsample size of $b_n = n^{-3/4}$ and bandwidths of $b_n^{-1/3}$ and $b_n^{-1/4}$ for one- and two-dimensional functions, respectively. We undersmooth relatively more in the subsamples than in our main estimation sample to ensure that, due to the smaller sample size, the finite sample biases of the nonparametric estimators in the subsamples do not dominate those in the original sample. Confidence bands for the independence and positive correlation bounds are also based subsampling.

The estimated stochastic monotonicity, independence, and positive correlation bounds are generally inward biased due to the maxima and minima. As explained in Chernozhukov et al. (2013) , a *half-median-unbiased estimator* is given by simply constructing a 50% one-sided confidence interval, which we calculate using our subsampling procedure.

E.2. Estimation and Inference of Bounds Combining Assumptions. Here we focus only on lower bound estimators. Upper bound estimators are analogous. We first consider the bounds in Theorem 8. For the buyer, we write the lower bound as

$$\begin{aligned}\max_y \left(\int \mathbf{1}(X_Q^{B*}(y,z) \leq x) dF_{P_2^B|P_1^S}(z|y) \right) &= \max_y \left(P(X_Q^{B*}(y,P_2^B) \leq x \mid P_1^S = y) \right) \\ &= \max_y P(X_Q^{B*}(P_1^S,P_2^B) \leq x \mid P_1^S = y)\end{aligned}$$

We estimate $P(X_Q^{B*}(P_1^S,P_2^B) \leq x \mid P_1^S = y)$ using $\tilde{X}_Q^{B*}(P_1^S,P_2^B)$ instead of $X_Q^{B*}(P_1^S,P_2^B)$ and the Nadaraya-Watson kernel estimator with an Epanechnikov kernel and bandwidth.

We write the buyer lower bound from Theorem 9 as

$$\int \max_{y' \geq y} P(X_Q^{B*}(y',P_2^B) \leq x \mid P_1^S = y') dF_{P_1^S}(y) = \int \max_{y' \geq y} P(X_Q^{B*}(P_1^S,P_2^B) \leq x \mid P_1^S = y') dF_{P_1^S}(y)$$

Using the estimator $\widehat{P}(\tilde{X}_Q^{B*}(P_1^S,P_2^B) \leq x \mid P_1^S = y')$ from above, we estimate the lower bound by $\frac{1}{n}\sum_{i=1}^n \max_{y' \in \omega_1(P_{1,i}^S)} \widehat{P}(\tilde{X}_Q^{B*}(P_1^S,P_2^B) \leq x \mid P_1^S = y')$.

We estimate the seller lower bound in Theorem 10 using the sample analog $\frac{1}{n}\sum_{i=1}^{n}\max_{y'\in\omega_1(P_{1,i}^S)}\max_z \hat{m}_{AC}^S(x,y',z)$. For the buyer, define $g_Q^B(x,y,z)=\max_{z'\geq z} m_Q^B(x,y,z')$. Then we can write the lower bound as $\max_y\left(E[g_Q^B(x,P_1^S,P_2^B)\mid P_1^S=y]\right)$. For each x, we estimate $E[g_Q^B(x,P_1^S,P_2^B)\mid P_1^S=y]$ using the Nadaraya-Watson kernel estimator with an Epanechnikov kernel function and bandwidth $n^{-1/4}$. Let $\hat{E}[g_Q^B(x,P_1^S,P_2^B)\mid P_1^S=y]$ denote the estimator. Our estimator is $\max_{y\in Q_{0.05}(P_{1,i}^S)}\left(\hat{E}[g_Q^B(x,P_1^S,P_2^B)\mid P_1^S=y]\right)$, where $g_Q^B(x,y,P_2^B)=\max_{z'\in\omega_2(P_{2,i}^B)} m_Q^B(x,y,z')$.

For the bounds from Theorem 11, we estimate the seller lower bound by $\frac{1}{n}\sum_{i=1}^{n}\max_{y'\in\omega_1(P_{1,i}^S)}\left(\hat{E}[g_{AC}^S(x,P_1^S,P_2^B)\mid P_1^S=y']\right)$ as well as the buyer lower bound by $\frac{1}{n}\sum_{i=1}^{n}\max_{y'\in\omega_1(P_{1,i}^S)}\left(\hat{E}[g_Q^B(x,P_1^S,P_2^B)\mid P_1^S=y']\right)$.

Inference is based on subsampling in all of these cases, as explained in Section E.1. Using subsampling, we then also obtain median unbiased estimators as described in the previous subsection.

E.3. Estimation and Inference of Bounds on First-Best Trade Probability. For the lower bound in Theorem 6, we first estimate $P(X_{AC}^{B*}-X_{AC}^S\geq x\mid P_1^S=y,P_2^B=z)$ by replacing $X_{AC}^{B*}-X_{AC}^S$ with $\hat{X}_{AC}^{B*}(y,z)-\hat{X}_{AC}^S(y)$ and using the Nadaraya-Watson kernel estimator with an Epanechnikov kernel function and bandwidth $n^{-1/5}$. Denote the estimator by $\hat{m}^{B-S}(y,z)$. The estimated lower bound is then $\frac{1}{n}\sum_{i=1}^{n}\max_{z'\in\omega_2(P_{2,i}^B)}\hat{m}^{B-S}(P_{1,i}^S,z')$. For the lower bound in Theorem 7, define $\hat{X}_{AC}^{B*-S}(y,z)\equiv\min_{i:P_{2,i}^B\leq z,P_{1,i}^S\in N(y)}(X_{AC,i}^B-X_{AC,i}^S)$, where the neighborhood $N(y)$ is as in Section 4. The estimated lower bound is then $\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(\hat{X}_{AC}^{B*-S}(P_{1,i}^S,P_{2,i}^B)\geq x)$. The estimator that is based on both buyer and seller monotonicity is $\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(\hat{X}_{AC}^{B*}(P_{1,i}^S)-\hat{X}_{AC}^{S*}(P_{1,i}^S)\geq x)$.

For the confidence bands of the marginal distributions, we explain in Section E.1 that we use conservative estimators and one-sided confidence bands. To construct the two-sided confidence intervals for the lower bound in Theorem 6 (shown in Table 5), we first use the previously described estimators and subsampling to approximate the quantiles of the centered distribution. We then use these quantiles as well as conservative point estimators to construct confidence intervals. In particular, for the lower endpoint of the confidence interval, we use $\hat{X}^{B*}_{AC}(y,z)+\eta_n-\hat{X}^{S}_{AC}(y)$ instead of $\hat{X}^{B*}_{AC}(y,z)-\hat{X}^{S}_{AC}(y)$, and for the upper endpoint we use $\hat{X}^{B*}_{AC}(y,z)-\eta_n-\hat{X}^{S}_{AC}(y)$, where $\eta_n=n^{-1/2}$ as in Section 4.1. The confidence interval for the sale probability is simply based on the large sample distribution of the empirical distribution function. The confidence interval for the lower bound in Theorem 7 is also based on the empirical distribution function but using conservative estimates of X^{B*-S}_{AC}, as this estimator suffers from the same potential inward bias as the buyer monotonicity bounds. In particular, for the lower and upper endpoints of the confidence interval, we use $\tilde{X}^{B*-S}_{AC}(y,z)+\eta_n$ and $\tilde{X}^{B*-S}_{AC}(y,z)-\eta_n$, respectively, where again $\eta_n=n^{-1/2}$.

E.4. Testing. Let $g_l(x)$ and $g_u(x)$ denote a lower and upper bound derived under some assumptions. To test the null hypothesis that the imposed assumptions are true, we use a scaled estimated version of $\frac{1}{J}\sum_{j=1}^{J}\min\{g_u(x_j)-g_l(x_j),0\}$, which is equal to 0 under the null hypothesis and negative if the bounds cross (at one of the J grid points). We use an equally spaced grid on $[0,2.5]$ with $J=25$. Note that bounds might not cross even if the assumptions do not hold.

The test statistic is based on the estimators discussed in Sections 4 and E.2. For the unconditional and the monotonicity bounds, we use the large sample distribution of the empirical distribution functions to approximate the distribution function of the test statistic. For all other assumptions, we use subsampling, as described in sections E.1 and E.2, to approximate the distribution of the test statistic. We reject the null if the test statistic is smaller than the α quantile of that distribution (essentially using a one-sided test).

F Monte Carlo Simulations

We present a Monte Carlo study of the buyer and seller distribution bounds. There is naturally a great deal of flexibility in how to simulate two-sided bargaining; here we simply simulate outcome data consistent with our assumptions. We do not simulate actual equilibrium play of a two-sided bargaining game, as the equilibria focused on in previous work (Perry 1986; Grossman and Perry 1986; Cramton 1992) do not result in multiple offers by a given party that vary with the party's value.

F.1. Algorithm for Simulating Bargaining Data. The primary parameters we vary in this exercise are α_b and α_s, which we refer to as *shade factors*; the probability a buyer and seller accept/decline; and the means of buyer and seller value distributions. Shade factors allow us to vary how aggressive agents' offers are: a buyer with value b and shade factor α_b makes the same offers as a buyer with value $b+\alpha_b$ and shade factor 0, and in this sense shade factors set a minimum level of offer shading. The probability a buyer or seller accepts or quits (instead of making a counteroffer) allows us to investigate how countering frequency affects bound tightness. Varying mean values allows us to adjust the potential surplus. The DGP is described in Table 9.

Table 9: Alogorithm for Simulating Bargaining Data

0. **Initialize:** Draw $B \sim F_B$ and $S \sim F_S$. Set shade factors α_B and α_S, and set cap $T_{\max}$ on number of rounds. Set functions $p_{BQ}(k)$, $p_{BA}(k)$, $p_{SQ}(k)$, and $p_{SA}(k)$ specifying probabilities, in round k, of buyer quitting, buyer accepting, seller quitting, or seller accepting
1. **Round 1:** Seller offers $P_1^S = g_1(S, \alpha_s, U_1)$ where $U_1 \sim U[0,1]$ and g_1 is a function weakly increasing in all arguments (we vary g_1 in our illustrations)
2. **Round 2:** Buyer offers $P_2^B = U_2(B-\alpha_b)$ if $P_1^S > B$ and $P_2^B = U_2 \min\{P_1^S, B-\alpha_b\}$ if $P_1^S \leq B$, where $U_2 \in (0,1)$ is random or fixed depending on specific setup
3. **Round 3$\geq k < T_{\max}$, k odd:** Seller responds to buyer's last offer
   - **Case 1.** $P_{k-1}^B < S$: Seller quits with probability $p_{SQ}(k)$, or else makes counteroffer $P_k^S = U_3 P_{k-2}^S + (1-U_3)(S+\alpha_S)$, where $U_3 \in (0,1)$ and its distribution depends on specific setup
   - **Case 2.** $P_{k-1}^B \geq S$: Seller accepts with fixed probability $p_{SA}(k)$, or else makes counteroffer $P_k^S = U_3 P_{k-2}^S + (1-U_3)\max(P_{k-1}^B, S+\alpha_S)$
4. **Round 4$\geq k < T_{\max}$, k even:** Buyer responds to seller's last offer P_{k-1}^S
   - **Case 1.** $P_{k-1}^S > B$: Buyer quits with probability $p_{BQ}(k)$, or else makes counteroffer $P_k^B = U_4 P_{k-2}^B + (1-\lambda)(B-\alpha_B)$, where $U_4 \in (0,1)$ and its distribution depends on specific setup
   - **Case 2.** $P_{k-1}^S \leq B$: Buyer accepts with probability $p_{BA}(k)$, or else makes counteroffer $P_k^B = U_4 P_{k-2}^B + (1-U_4)\min(P_{k-1}^S, b-\alpha_B)$
5. **Round $T_{\max}$:** Terminate with no trade occurring

F.2. Results of Monte Carlo Exercise. Figure 7 illustrates several of our bounds estimated using this simulated data. For each panel, we simulate 100 replications of the DGP and then report the true distribution, the true bounds as well as the estimated bounds along with 95% one-sided, pointwise confidence bands averaged across these replications. In each example we set $n = 200$ and $T_{max} = 8$. We draw the values from a Beta distribution, which has support on $[0,1]$. The Beta distribution has two parameters, α and β. We set $\alpha = 2$ and set β depending on which mean value we want to achieve. We then add the maximum shading factor to ensure that bids are always non-negative. We vary the parameters of the DGP in each panel in order to illustrate what features will lead to bounds that are loose (panels on the left) or tight (panels on the right). We focus only on three sets of bounds — the seller unconditional bounds, seller monotonicity bounds, and buyer independence bounds — to conserve space and because the intuition gained by these three cases extends to the other bounds in the paper. In each panel, lower bounds are shown with solid lines and upper bounds with dashed lines. The true CDF as well as the true bounds are shown with a dot-dash line. We obtain the true bounds by estimating the bounds on a very large sample (1 million observations). Comparing the estimated bounds on small samples to these true bounds allows us to evaluate bias in the estimated bounds.

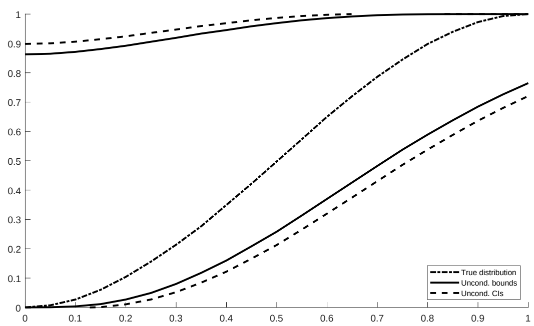
We show the seller unconditional bounds in panels A and B. For the seller unconditional bounds to be relatively tight, it must be the case that sellers quit at prices close to their values and also counter at prices close to their values. As an example, consider a setting where buyer and seller values are highly correlated and have a similar mean. Suppose the typical play of the game is that the seller offers a price a little above her value, the buyer counters at a price a little below the seller's value (and also below the buyer's value, naturally), and the seller then quits. This sequence of play is consistent with the weak revealed preference assumptions that the unconditional bounds are built on (A1) and it yields the tight bounds on seller values illustrated in panel B.[53] We can also easily generate wide unconditional bounds. For example, consider a case where the seller typically makes offers far above her value and rarely quits. Such bounds are illustrated in panel A.[54] Here,

[53]The specification for this case is as follows: both the buyer and the seller have mean values of 0.5 and their correlation is 0.999. We set $p_{BQ}(k) = p_{BA}(k) = p_{SA}(k) = 0$, $p_{SQ}(k) = 0.95$, $g_1(S, \alpha_s, U_1) = 1.1(S + \alpha_s)$, and $U_2 = 0.9$, $U_3 \sim U[0, 0.5]$, $U_4 \sim U[0, 0.5]$.

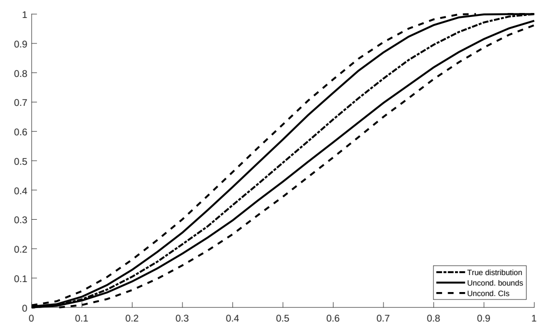
[54]The specification for this case is as follows: both the buyer and the seller have mean values of 0.5 and

Figure 7: Simulation Results

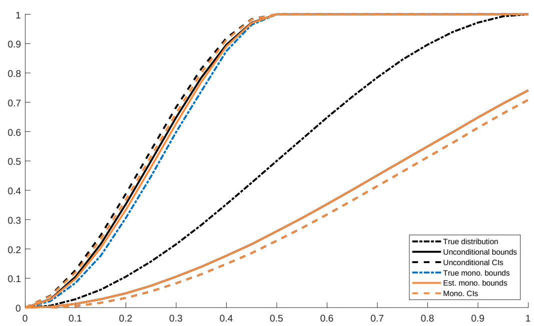
(A) Unconditional Bounds, Seller (Wide)



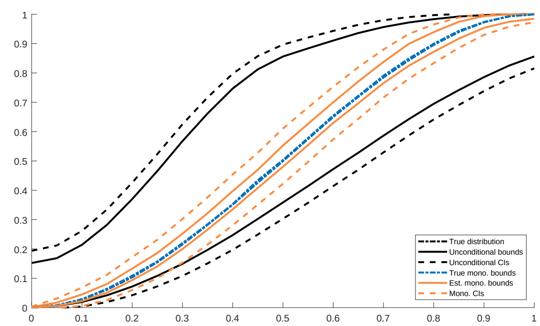
(B) Unconditional Bounds, Seller (Tight)



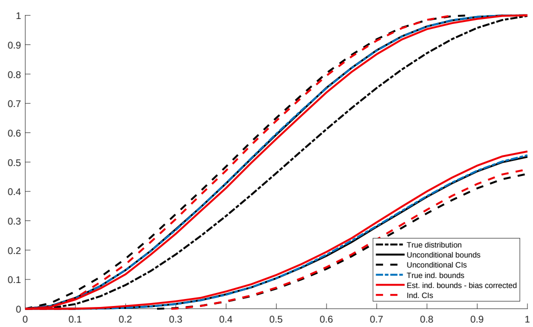
(C) Monotonicity Bounds, Seller (Wide)



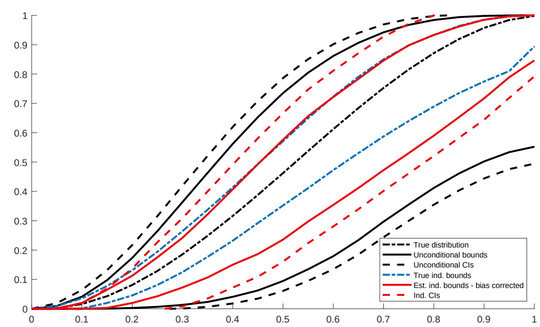
(D) Monotonicity Bounds, Seller (Tight)



(E) Independence Bounds, Buyer (Wide)



(F) Independence Bounds, Buyer (Tight)



Notes: The figure shows bounds estimated from simulated data under cases where bounds are wide (on left) vs. narrow (on right). Panels A and B show unconditional seller bounds. Panels C and D show seller monotonicity bounds. Panels E and F show buyer independence bounds. Estimated bounds are shown with solid lines, confidence bands with dashed lines, and true CDF and true bounds with a dot-dash line. Panels C–F also show unconditional bounds for comparison.

the correlation structure between buyer and seller values plays no role.

We illustrate the seller monotonicity bounds in panels C and D. The monotonicity bounds will improve upon the unconditional bounds when there is some probability that sellers who start with relatively low first offers end the game at relatively high final accept/counter or quit prices. This can occur due to randomness in the value of the buyer to whom the seller is matched and due to features of bargaining at later rounds of the game. We illustrate such a case in panel D.[55] If, however, the final accept/counter and quit prices of a seller are, like P_1^S, deterministically mononotonic in the seller's value, the monotonicity assumption will do nothing to improve upon the unconditional bounds (because $X_{AC}^{S*} = X_{AC}^S$ and $X_Q^{S*} = X_Q^S$ in that case, and the unconditional bounds will equal the monotonicity bounds). We illustrate this situation in panel C, where the monotonicity bounds are equally as wide as the unconditional bounds.[56]

Finally, we illustrate the buyer independence bounds in panels E and F. Recall that these bounds are obtained by combining $P(B \leq x|P_1^S = y) = P(B \leq x)$ (buyer independence) with weak rationality on the part of the buyer ($X_{AC}^B \leq B$ for the buyer upper bound). The buyer independence assumption will therefore yield no improvement over the buyer unconditional upper bounds if X_{AC}^B and X_Q^B are, like B, independent of P_1^S. This case is illustrated in panel E.[57] It is easy to generate a case in which the maximum accept/counter price of the buyer *does* depend on P_1^S, and this yields a much tighter upper bound. To do so, we generate data such that B is independent of P_1^S, but X_{AC}^B and X_Q^B are not because bids in later rounds directly depend on P_1^S.[58]

their correlation is 0. We set $p_{BQ}(k) = p_{BA}(k) = 1$, $p_{SA}(k) = 0$, $p_{SQ}(k) = 0.25$, $g_1(S, \alpha_s, U_1) = 1.5(S + \alpha_s) + 0.5$, and $U_2 = 0.9$, $U_3 \sim U[0, 0.5]$, $U_4 \sim U[0, 0.5]$.

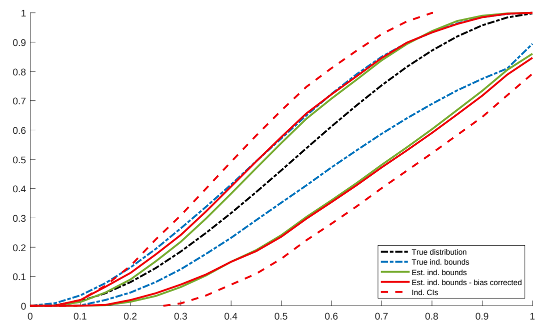
[55] The specification for this case is as follows: both the buyer and the seller have mean values of 0.5 and their correlation is 0.999. We set $p_{BQ}(k) = p_{BA}(k) = p_{SA}(k) = 0$, $p_{SQ}(k) = 0.5$, $g_1(S, \alpha_s, U_1) = 1.5(S + \alpha_s)$, and $U_2 = 0.5$, $U_2 \sim U[0, 0.5]$, $U_3 \sim U[0, 0.5]$.

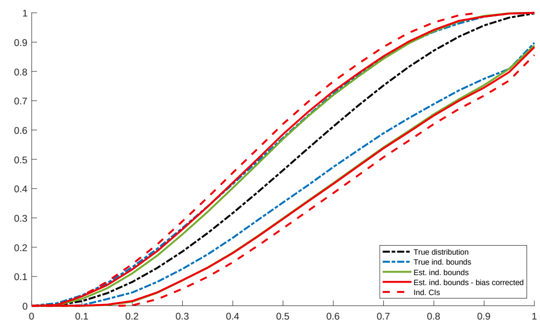
[56] The specification for this case is as follows: both the buyer and the seller have mean values of 0.5 and their correlation is 0.999. We set $p_{BQ}(k) = p_{SQ}(k) = 1$, $p_{BA}(k) = p_{SA}(k) = 0$, $g_1(S, \alpha_s, U_1) = 1.5(S + \alpha_s)$, and $U_2 = 0.5$, $U_3 \sim U[0, 0.5]$, $U_4 \sim U[0, 0.5]$.

[57] The specification for this case is as follows: both the buyer and the seller have mean values of 0.5 and their correlation is 0. We set $p_{BQ}(k) = 0.95$, $p_{SQ}(k) = p_{BA}(k) = p_{SA}(k) = 0$, $g_1(S, \alpha_s, U_1) = U_1$ with $U_1 \sim [1, 1.5]$, and $U_2 \sim U[0.75, 1]$, and $U_3 = U_4 = U_1$.

[58] The specification for this case is as follows: both the buyer and the seller have mean values of 0.5 and

Figure 8: Bias Comparisons





Notes: The figure shows buyer independence bounds estimated from simulated data and two sample sizes under cases where bounds are narrow.

F.3. Bias Correction. Estimators of unconditional bounds are unbiased and, as explained in Section 4, estimators of the monotonicity bounds have an outward bias. While we use half-median-unbiased estimators for the independence bounds, this estimator can still be biased in finite samples, which is particularly noticeable in panel F of Figure 7, where the estimate of the lower bound lies below the true bound. The estimator (and the other estimators that involve minima and maxima) have two main sources of biases that go in opposite directions. First, there is a inward bias that arises from taking the minimum (or maximum) of an estimated function. This bias term is handled by using the half-median-unbiased estimator. Second, the Nadaraya-Watson estimator is biased and, at the maximum, the estimator of the function is downward biased (and upward biased at the minimum). This bias term is handled by using undersmoothing, but still results in an outward bias in finite samples. In panel F, the second bias term dominates.

Figure 8 shows the buyer independence bounds again, but also includes the non-bias-corrected estimator. For the upper bound, where the first bias term dominates, the half-median-unbiased estimator is closer to the true bound, but the bias-adjustment has almost no effect for the lower bound. As the sample sizes increases, the biases decrease, as can be

their correlation is 0. We set $p_{BQ}(k) = 0.95$, $p_{SQ}(k) = p_{BA}(k) = p_{SA}(k) = 0$, $g_1(S, \alpha_s, U_1) = U_1(S + \alpha_s)$ with $U_1 \sim [1, 1.5]$, and $U_2 \sim U[0.75, 1]$, and $U_3 = U_4 = \max\{1 - P_1^S, 0\}$.

seen from panel B, where the sample size is $2,000$.

G Related Extensive-Form Models

In this section we consider two extensive-form bargaining models: Cramton (1992) and Perry (1986). In each case, when discussing unobserved heterogeneity, our notation here differs slightly from the body of the paper. Here we write a seller's value with additively separable unobserved heterogeneity included as $\tilde{S} = S + W$ (in the body of the paper, we instead write $S = \tilde{S} + W$). Similarly, for the multiplicative case, we write $\tilde{S} = SW$. We apply this notation to the buyer's value and to buyer and seller offers as well. We adopt this change so that variables without $(\tilde{\cdot})$ always represent those *absent* unobserved heterogeneity.

G.1. Cramton (1992). This model studies a setting similar to ours, where a seller and buyer with independent private values engage in bargaining. One possible outcome in the Cramton (1992) equilibrium is for the seller to make the first offer, P_1^S, which completely reveals the seller's value S. The buyer then either accepts, quits, or makes a counteroffer P_2^B that completely reveals her value B.[59] These first two offers are $P_1^S = \frac{\delta S + \gamma(S)}{1+\delta}$ and $P_2^B = \frac{\delta B + S}{1+\delta}$, where δ is a discount factor and the object $\gamma(S)$ is the buyer type indifferent between accepting and rejecting the seller's offer of P_1^S given that the seller has revealed her type to be S and the buyer's value is bounded above by some $\overline{b}$. In the equilibrium studied in Cramton (1992), the function $\gamma(\cdot)$ is given by the following:

$$F_B(\overline{b}) - F_B(\gamma) - (1-\delta^2)(\gamma - s) f_B(\gamma) = \int_s^{\gamma} \delta^3 \left(\frac{b-s}{\gamma - s} \right)^{1+\delta} dF_B(b) \qquad (17)$$

This object is quite complex, depending on the CDF and density of buyer values, F_B and f_B. If buyer and seller values are uniformly distributed, $\gamma(s)$ has a closed-form solution $\gamma(s) = \alpha - (2\alpha - 1)s$, where α is defined by $1 - 2\alpha = \frac{-\delta}{2+\delta-\delta^2}$. For our arguments here, we

[59] If the buyer chooses to make a counteroffer, P^B, the buyer exploits this knowledge of the seller's type and makes an offer that corresponds to the Rubinstein (1982) equilibrium offer for the case where the buyer and seller know each others' values. Note that, in the Cramton (1992) equilibrium, the timing of these offers is also important in revealing an agent's value, but the level of the offers is sufficient for our purposes.

assume $\gamma(s)$ is differentiable with $\gamma'(s) \in (-\delta, 0)$. It is possible to show that $\gamma'(s) \in (-\delta, 0)$ is satisfied with slack in the uniform case, which we state as the following lemma:[60]

Lemma 3. *In the Cramton (1992) model, if buyer and seller values are uniformly distributed, the function $\gamma(\cdot)$ satisfies (with slack) $\gamma'(\cdot) \in (-\delta, 0)$.*

Proof. Note that $\alpha \in (\frac{1}{2}, \frac{3}{4})$ for $\delta \in (0,1]$, so $\gamma'(s) \in [-.5, 0)$ for $\delta \in (0,1]$. Therefore, $\gamma'(s) < 0$ is satisfied with slack. Now note $\gamma'(s) = 1 - 2\alpha$. Setting $\gamma'(s) \geq -\delta$ yields $\frac{-\delta}{2+\delta-\delta^2} \geq -\delta \iff \delta^2 \leq 2$, and thus $\gamma'(s) > -\delta$ is satisfied with slack. $\square$

An immediate result of this property is that the equilibrium offers, $P_1^S = \frac{\delta S + \gamma(S)}{1+\delta}$ and $P_2^B = \frac{\delta B + S}{1+\delta}$, satisfy A2 (strictly, in fact): P_1^S is strictly monotone in S because $\gamma'(s) > -\delta$, and hence P_2^B is also strictly monotone in B conditional on P_1^S.

Now consider a modified setting in which a buyer and seller play the equilibrium of Cramton (1992), but in a given realization of the game buyer and seller values are both shifted additively by a common amount, W, that is independent of B and S. Specifically, a buyer's value is given by $B + W$ and a seller's by $S + W$, where $W = w$ is known to both agents but not to the econometrician. Cramton's model assumes, without loss of generality, that buyer values are distributed on $[0,1]$. In our modification, we instead have values distributed on $[w, 1+w]$. In this environment, the equilibrium offers simply shift additively by w as well, becoming $P_1^S + w$ and $P_2^B + w$, as we demonstrate in the following lemma:

Lemma 4. *Suppose seller and buyer values in the Cramton (1992) setting are given by $S + W$ and $B + W$. If, when $W = 0$, the first two offers are given by $P_1^S = p_1^S$ and $P_2^B = p_2^B$, then, when $W = w$, these offers are given by $p_1^S + w$ and $p_2^B + w$.*

Proof. We first prove the following claim: The function $\gamma(\cdot)$ satisfies additive separability. Let $\tilde{\gamma}(s, w)$ represent the value of γ in a game in which $W = w$; thus $\gamma(s) = \tilde{\gamma}(s, 0)$. We will

[60] It is also possible to derive sufficient conditions for these properties outside of the uniform case; these would be similar to the assumption referred to as "$(F\delta)$" in Cramton (1992). These conditions are cumbersome. Like Cramton, therefore, we instead show they are satisfied with slack in the uniform case and thus they do not appear to be overly restrictive.

show that $\tilde{\gamma}(s,w) = \gamma(s) + w$. To see this, let $\tilde{\overline{b}} = \overline{b} + w$, and $\tilde{s} = s + w$, and let $\tilde{F}_{\tilde{B}}$ and $\tilde{f}_{\tilde{B}}$ be the distribution and density of $\tilde{B}$.

The condition defining $\tilde{\gamma}$ is given by modifying (17) to become $\tilde{F}_{\tilde{B}}(\tilde{\overline{b}}) - \tilde{F}_{\tilde{B}}(\tilde{\gamma}) - (1 - \delta^2)(\tilde{\gamma} - \tilde{s})\tilde{f}_{\tilde{B}}(\tilde{\gamma}) = \int_{\tilde{s}}^{\tilde{\gamma}} \delta^3 \left(\frac{x-\tilde{s}}{\tilde{\gamma}-\tilde{s}}\right)^{1+\delta} d\tilde{F}_{\tilde{B}}(x)$. Note that, for any number x, $\tilde{F}_{\tilde{B}}(\tilde{x}) = F_B(\tilde{x} - w)$ and $\tilde{f}_{\tilde{B}}(\tilde{x}) = f_B(\tilde{x} - w)$. We now apply a change of variables from x to $y = x - w$ in the integral, yielding $\int_{s}^{\tilde{\gamma}-w} \delta^3 \left(\frac{y+w-\tilde{s}}{\tilde{\gamma}-\tilde{s}}\right)^{1+\delta} dF_B(y)$. Combining these results yields

$$F_B(b) - F_B(\tilde{\gamma} - w) - (1 - \delta^2)(\tilde{\gamma} - \tilde{s}) f_B(\tilde{\gamma} - w) = \int_{s}^{\tilde{\gamma}-w} \delta^3 \left(\frac{y-s}{\tilde{\gamma}-\tilde{s}}\right)^{1+\delta} dF_B(y) \tag{18}$$

Comparing (17) to (18) demonstrates that, if γ is the solution to the former then $\gamma + w$ is the solution to the latter, proving the claim.

Now consider the equilibrium conditional on a realization of W. Offers will be given by $\tilde{p}_1^S = \frac{\delta\tilde{s}+\tilde{\gamma}(s,w)}{1+\delta} = \frac{\delta s+\gamma(s)}{1+\delta} + w$ and $\tilde{p}_2^B = \frac{\delta\tilde{b}+\tilde{s}}{1+\delta} = \frac{\delta b+s}{1+\delta} + w$, satisfying additivity. □

We now demonstrate that unobserved heterogeneity can lead to a violation of the monotonicity assumption even while stochastic monotonicity is satisfied. We show that monotonicity of the seller's first offer $\tilde{P}^S$ in the seller's value $\tilde{S}$ is violated in this setting, and we prove an analogous result for the buyer. Note that here we are considering what this setting would look like to the econometrician, who would see observations of different instances of the game and where realizations of W may vary across these observations.

Lemma 5. *The Cramton (1992) equilibrium offers in the game with additive unobserved heterogeneity can violate A2, but A4 is still satisfied.*

Proof. Suppose s increases by 1 and w decreases by $\eta < 1$ (so $\tilde{s}$ increases overall). Because $\gamma(s)' \in (-\delta, 0)$, the change in $\tilde{p}_1^S$ due to the change in s is at most an increase of $\frac{\delta}{1+\delta}$, and the change in $\tilde{p}_1^S$ due to the change in w is a decrease of η. For any $\eta \in \left(\frac{\delta}{1+\delta}, 1\right)$, $\tilde{p}_1^S$ decreases even though $\tilde{s}$ increases, violating seller monotonicity.

For buyer monotonicity, suppose s increases by η_s and w decreases by η_w such that $\tilde{p}_1^S$ does not change. It then holds that $0 < \eta_w < \eta_s$. Next suppose b increases by $\eta_b \in (\eta_w < \eta_s)$. Then $\tilde{b} = b + w$ decreases, but since $\frac{\delta(b+\eta_b)+(s+\eta_s)}{1+\delta} + w - \eta_w > \frac{\delta b+s}{1+\delta} + w$, $\tilde{p}_2^B$ increases.

To see that stochastic monotonicity is satisfied for the seller, let $g(s) \equiv \frac{s-\gamma(s)}{1+\delta}$, which is strictly increasing under our assumption that $\gamma(\cdot)$ is strictly decreasing. Then we have

$$\begin{aligned}
P(\tilde{S} \leq x | \tilde{P}_1^S = y) &= P\left(S \leq x - W | P_1^S + W = y\right) \\
&= \int P\left(S \leq x - y + f(S) | w = y - f(S), W = w\right) f_{W|w=y-f(S)}(w) dw \\
&= \int P\left(S \leq x - y + \frac{\delta S + \gamma(S)}{1+\delta} \middle| w = y - f(S), W = w\right) f_W(w) dw \\
&= \int P\left(g(S) \leq x - y | f(S) = y - w, W = w\right) f_W(w) dw \\
&= \int P\left(g(S) \leq x - y | f(S) = y - w\right) f_W(w) dw \\
&= \int P\left(g(f^{-1}(y-w)) \leq x - y\right) f_W(w) dw \\
&= \int \mathbf{1}\left(g(f^{-1}(y-w)) \leq x - y\right) f_W(w) dw
\end{aligned}$$

Since $g(f^{-1}(\cdot))$ is a strictly increasing function, $P(\tilde{S} \leq x | \tilde{P}_1^S = y)$ is strictly decreasing in y. In the third and fifth line, we use that W and S are independent.

Using similar arguments, we can also show that the stochastic monotonicity condition of the buyer holds. To do so, write

$$\begin{aligned}
&P(\tilde{B} \leq x | \tilde{P}_1^S = y, \tilde{P}_2^B = z) \\
&\quad = P\left(B \leq x - W \middle| f(S) + W = y, \frac{\delta B + S}{1+\delta} + W = z\right) \\
&\quad = \int P\left(B \leq x - w \middle| f(S) + w = y, \frac{\delta B + S}{1+\delta} + w = z\right) f_W(w) dw \\
&\quad = \int P\left(B \leq x - w \middle| S = f^{-1}(y-w), \frac{\delta B + f^{-1}(y-w)}{1+\delta} = z - w\right) f_W(w) dw \\
&\quad = \int P\left(B \leq x - w \middle| S = f^{-1}(y-w), B = \frac{1}{\delta}\left((1+\delta)(z-w) - f^{-1}(y-w)\right)\right) f_W(w) dw \\
&\quad = \int \mathbf{1}\left(\frac{1}{\delta}\left((1+\delta)(z-w) - f^{-1}(y-w)\right) \leq x - w\right) f_W(w) dw
\end{aligned}$$

which is decreasing in z. In the third line, we used that W is independent of (S, B). $\square$

The Cramton model assumes independence of B and S, and this immediately yields the result that the independence assumption for the seller is satisfied in his model: S is independent of P_2^B conditional on P_1^S because P_1^S completely reveals S to the buyer, and hence, conditional on P_1^S, there is no variation left in S. However, even maintaining the independence of the components B and S, if additive unobserved heterogeneity is introduced into the game, then $B+W$ will be correlated with P_1^S+W, violating buyer independence. The proof of Lemma 6, focusing on the uniform distribution case, demonstrates that seller independence can also be violated without violating positive correlation.

Lemma 6. *The Cramton (1992) equilibrium offers in a game with additive unobserved heterogeneity can violate A3.i for the seller and A3.ii for the buyer.*

Proof. In the Cramton model *with unobserved heterogeneity*, clearly $\tilde{B}$ is correlated with $\tilde{P}_1^S$ through W, so buyer independence (A3.ii) is violated. For seller independence (A3.i), note from Lemma 5 that $\tilde{P}_2^B$ can be written as follows, where $\tilde{P}_1^S$ is fixed at y: $\tilde{P}_2^B = \frac{\delta B + f^{-1}(y-W)}{1+\delta} + W$. Now consider a change in $\tilde{S}$. Holding $\tilde{P}_1^S$ fixed at y, this change in $\tilde{S}$ must also correspond to a change in W (or else $\tilde{P}_1^S$ could not remain constant).

The change in W will necessarily affect $\tilde{P}_2^B$ unless the terms in $\tilde{P}_2^B$ depending on W offset one another; that is, unless $\frac{d}{dw}\left(\frac{f^{-1}(y-w)}{1+\delta} + w\right) = 0$. To see that this is not the case, note $\gamma' \in (-\delta, 0)$ implies $f' \in (0, \frac{\delta}{1+\delta})$, and, by the inverse function theorem, $f^{-1'} \in (\frac{1+\delta}{\delta}, \infty)$. This implies $\frac{d}{dw}\frac{f^{-1}(y-w)}{1+\delta} + w \in (-\infty, -1/\delta + 1)$. For any $\delta < 1$, this derivative is non-zero, and thus variation in W also leads to variation in $\tilde{P}_2^B$, violating seller independence. □

G.2. Perry (1986). The Perry model has no discounting. Instead, agents face a per-offer additive cost of bargaining, c_S for the seller and c_B for the buyer. While players can alternate offers, the equilibrium that Perry focuses on has the property that only one player makes an offer and the other accepts or rejects (and never makes a counteroffer). One outcome in the Perry equilibrium is for the seller to make the first offer, P_1^S, with this offer given by $P_1^S = \frac{1-F_B(P_1^S)}{f_B(P_1^S)} + S$, where f_B is the density of buyer values. In this equilibrium, the seller's first offer, P_1^S, clearly satisfies monotonicity (A2.i), and hence also satisfies the weaker condition of stochastic monotonicity (A4.i).

In a version of this model with additively separable unobserved heterogeneity, the seller's offer will also be additively separable in the unobserved heterogeneity. Specifically, $\tilde{P}_1^S = \frac{1-F_{\tilde{B}}(\tilde{P}_1^S)}{f_{\tilde{B}}(\tilde{P}_1^S)} + \tilde{S} = \frac{1-F_B(\tilde{P}_1^S - W)}{f_B(\tilde{P}_1^S - W)} + S + W = P_1^S + W$. Thus, $\tilde{P}_1^S = P_1^S + W$.

In this modified version of the model, seller monotonicity (A2.i) can be violated. To show this, we re-write $P_1^S = \frac{1-F_B(P_1^S)}{f_B(P_1^S)} + S$ as $\phi(p_1^S) = s$, where $\phi(p_1^S) \equiv p_1^S - \frac{1-F_B(p_1^S)}{f_B(p_1^S)}$ is the buyer's *virtual value* function. Implicit differentiation of $\phi(p_1^S) = s$ with respect to s yields $\frac{dp_1^S}{ds} = \frac{1}{\phi'(p_1^S)}$. Consider now a case where s increases by 1 and w decreases by $\eta < 1$, and hence $\tilde{s}$ increases overall. The object $\tilde{p}_1^S$ will increase by $\frac{1}{\phi'(p_1^S)} - \eta$. For any distribution F_B with $\phi'(\cdot) > 1$, there exists an $\eta < 1$ such that p_1^S will increase by less than when η when s increases by 1, and, in such a case, $\tilde{p}_1^S$ will decrease overall. The uniform distribution on [0,1] is one such example, where this condition is satisfied with slack, with $\phi'(\cdot) = 2$.

Consider now a case in which agents play the equilibrium of Perry (1986), but in a given realization of the game buyer and seller values are both scaled *multiplicatively* (rather than shifted additively) by some amount W which, again, is common knowledge to both agents. Thus, $\tilde{S} = WS$ and $\tilde{B} = WB$. In this case, it can be shown that $\tilde{P}_1^S = WP_1^S = W\frac{1-F_B(\tilde{P}_1^S/W)}{f_B(\tilde{P}_1^S/W)} + WS$, where P_1^S is the offer the seller would make if the realization of W were 1. The presence of this multiplicative heterogeneity can lead to violations of weak monotonicity across instances of the game. Consider, for simplicity, the case where $B \sim U[0,1]$. In this case, the expression for $\tilde{P}_1^S$ simplifies to $2p_1^S = w + ws$. Suppose s increases by 1 and w decreases, scaling down by some factor $\eta \in (\frac{1}{2}, 1)$; overall, $\tilde{s}$ increases by a factor of $2\eta > 1$. However, the expression for $\tilde{P}_1^S$ then implies that $\tilde{p}_1^S = \eta p_1^S$, and thus $\tilde{p}_1^S$ decreases.

Independence of the buyer's value from the seller's first offer (A3.ii) is satisfied in this model absent unobserved heterogeneity. However, once unobserved heterogeneity is included, the seller's first offer $\tilde{P}_1^S$ and buyer's value $\tilde{B}$ will be correlated through W in both the additively or multiplicatively separable unobserved heterogeneity models. The Perry model cannot serve for studying monotonicity of the buyer's first offer or independence of the buyer's offer from the seller's value because only one offer occurs in equilibrium.