Incentivizing Effort and Informing Investment: The Dual Role of Stock Prices

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Abstract

Stock prices reflect managerial performance and aggregate investor information about investment opportunities. These dual roles are often in tension: when prices are more informative about future opportunities, they may be less effective at incentivizing managerial effort. As a result, firm value can *decrease* with revelatory price efficiency, but *increase* with ex-post inefficient investment rules and lower transparency. The interaction of these dual roles has novel implications not only for the performance sensitivity of managerial compensation but also its duration and the optimal allocation of control rights. Finally, we demonstrate that standard empirical measures of price informativeness are incomplete absent information about the firm's investment opportunities.

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1 Introduction

Stock prices play an important role in *incentive provision* via compensation contracts because they reflect information about firm performance and managerial decisions. Standard arguments suggest that contracting efficiency and, therefore, firm value increase when prices more precisely capture managerial actions, i.e., when the "noise" in the contracting signal is reduced.[1] Yet, in practice, stock prices are inherently multi-dimensional. For instance, stock prices also aggregate investor information about future opportunities and affect real decisions via *feedback effects* (e.g., Bond, Edmans, and Goldstein (2012)). As such, what appears to be "noise" from a contracting perspective may be valuable information from the standpoint of real investment.

Given the ubiquity of firm equity in incentive provision as well as the documented importance of feedback effects, we study how these dual roles interact in equilibrium. In our model, the principal of a firm must compensate her risk-averse manager for costly effort and choose whether to pursue a new investment opportunity. In equilibrium, the price provides a contractable signal of managerial effort *and* aggregates investors' information about the value of the potential investment.

As a result, more informative prices can decrease firm value in equilibrium, even when they lead to more informationally-efficient investment decisions. Intuitively, when the price is more responsive to information about investment opportunities, it can become a noisier measure of managerial effort, which makes it more difficult for the principal to incentivize the manager. This implies that the incentive provision and feedback roles are often in tension: in equilibrium, managerial effort decreases with the quality of investors' aggregate information about future opportunities.

By formally modeling the conflict between these dual roles, we uncover novel insights for optimal firm policy. We show that the principal may strictly prefer to delegate the investment decision even though the manager has no additional information. While the manager's investment policy is ex-post inefficient, under-investment in risky, positive-NPV projects allows the principal to offer a contract which elicits more managerial effort, improving firm value. We also demonstrate how increased transparency can *decrease* firm value even when the incremental information such transparency generates improves investment efficiency.

Understanding the interaction between these dual roles is crucial for empirical analysis. Specifically, we show that it is essential to account for the contribution of new investment opportunities to the firm's cash-flows *relative* to the contribution of managerial effort — we

[1] This is the basis for the large literature following Holmström and Tirole (1993), which studies the role of stock price liquidity on compensation contracts.

denote this by δ. For example, firm value increases with price informativeness only when feedback effects are sufficiently important (i.e., δ is high). On the other hand, existing empirical measures of real efficiency (e.g., revelatory price efficiency) may be misleading because they do not account for the impact of managerial effort and contracting on firm value. In fact, when δ is sufficiently low, *higher* revelatory price efficiency corresponds to *lower* firm value and overall real efficiency.

Finally, our analysis generates novel, testable predictions for how managerial compensation contracts depend on future investment opportunities. First, pay-for-performance sensitivity (PFP) of managerial compensation should be higher for firms in which feedback effects are relatively less important (i.e., firms with lower δ). Second, for a given firm, PFP should decrease as the price becomes more informative about future investment opportunities, in line with the empirical evidence in Lin, Liu, and Sun (2019). Third, there is a positive relation between liquidity and PFP (consistent with the evidence in Fang, Noe, and Tice (2009) and Jayaraman and Milbourn (2012)), but a negative relation between liquidity and price-investment sensitivity. Finally, we show that the optimal duration of executive compensation is increasing in δ even in the absence of managerial myopia.

Model Overview and Main Trade-off. Section 3 presents the model. There is a single firm with a risk-neutral principal (she) and a risk-averse manager (he). The manager can exert costly effort to increase the terminal cash flows of the firm. This effort is incentivized by an endogenous contract set by the principal to maximize her expectation of the terminal cash flows, net of managerial compensation. The principal also chooses whether or not to invest in a new project, given the information available to her. The firm's equity is a claim to the terminal cash flows, net of compensation, and is traded by a group of investors, some of who are privately informed about the profitability of the new project.

In Section 4, we solve for the financial market equilibrium and the contract offered by the principal, which depend on each other in equilibrium. Our benchmark analysis focuses on compensation contracts that are linear in the price for analytical tractability.[2] In equilibrium, the stock price contains information about managerial effort and aggregates investors' private

[2]While a general characterization of the optimal (possibly non-linear) contract is not analytically tractable in our setting, we can view our results as characterizing the first-order approximation to this contract. Importantly, we show that our main results are not a consequence of either the assumption of linear contracts or the benchmark contract's dependence on the short-term stock price. Specifically, in Section 5.5, we allow the contract to depend on both the short-term price and the (long-term) terminal value, while in Appendix B.5, we explore how our results extend to non-linear contracts written on the short-term price. In both cases, we show our main result obtains: when prices are more informative about future opportunities, they can lead to lower managerial effort and lower firm value. More generally, our analysis is based on the premise that, while there may be other signals of managerial effort, the stock price provides an unbiased and natural instrument for compensation contracts.

information about the new project. As a result, the principal compensates the manager using a fraction of the firm's equity — this is the **incentive provision** role — while also conditioning on the price when making her investment decision, which captures the **feedback effect** from prices to real investment.

The key mechanism underlying our results is that these dual roles often operate in opposite directions. On the one hand, when the price is more informative about the new project, the principal makes a more informed investment decision, increasing firm value. On the other hand, when prices reflect more firm-specific private information, they are also more volatile, *conditional* on a given level of effort. Since the manager is risk-averse, this makes compensating him for effort more costly and so the principal optimally reduces the price-sensitivity of the manager's compensation in equilibrium. This, however, reduces his effort and, consequently, firm value.

This trade-off arises in settings where the price is more volatile when investors are better informed about investment opportunities.[3] This is consistent with the large empirical literature on price non-synchronicity, which establishes that price informativeness and firm-specific volatility are positively related (e.g., Morck, Yeung, and Yu (2000), Wurgler (2000), Durnev, Morck, Yeung, and Zarowin (2003), Durnev, Morck, and Yeung (2004)).[4] Furthermore, Chen, Goldstein, and Jiang (2007) show that price volatility (non-synchronicity) is strongly correlated with the sensitivity of investment to price, which is a measure of the feedback effect. Together, this evidence implies that the trade-off is likely to be of first-order importance in practice.

Implications. Section 5 characterizes the novel implications of the trade-off generated by the dual role of stock prices. In Section 5.1, we characterize how the relation between price informativeness and expected firm value depends on the relative importance of feedback effects, or δ. When δ is high (low), the relative impact of learning about future investment opportunities is higher (lower) than the impact of incentivizing managerial effort. We show that expected firm value unambiguously increases (decreases) with price informativeness when δ is sufficiently high (low) but is U-shaped for intermediate levels of δ. As such, the impact of price informativeness on firm value can not only vary across firms but also within firms when the quality of investors' information varies.

Our analysis implies that standard measures of price efficiency provide an incomplete

[3]As we discuss further in Section 2 and Appendix B.6, the positive relation between price informativeness and volatility obtains generally, even in the absence of feedback effects.

[4]Dávila and Parlatore (2020) argue that the relation between volatility and price informativeness is mixed, but this is based on structural estimation of a model without feedback effects. As such, it is difficult to interpret their results in the context of our analysis.

picture of firm value, as we discuss in Section 5.2. Following Bond et al. (2012), we distinguish between *forecasting price efficiency* (FPE), which captures the extent to which information in prices forecast future firm cash flows, and *revelatory price efficiency* (RPE), which measure the extent to which prices reveal information relevant for future investment decisions. The literature on feedback effects has clarified the key difference in these measures, and generally argues that when learning from prices plays an important role for investment decisions, RPE is the relevant measure of real efficiency.[5] However, this may no longer be the case when the price is also used to incentivize managerial effort. We use the term *contracting price efficiency* (CPE) to capture the extent to which the price reflects the manager's actions and, consequently, is useful for contracting purposes. When investors are better informed about future investment opportunities, this increases RPE but decreases CPE, and only increases FPE when the relative impact of the investment opportunity, δ, is sufficiently high. More importantly, we show that when δ is sufficiently low, firm value can decrease even though RPE increases. This suggests that one must be cautious in interpreting changes in existing measures of RPE (e.g., Bai et al. (2016)) as proxies for changes in firm value or real efficiency.

In Section 5.3, we endogenize the information content of prices by allowing investors to choose whether or not to acquire costly information about the new project (as in Grossman and Stiglitz (1980)). This allows us to characterize the impact of an increase in transparency (i.e., a decrease in the cost of information) on firm value as well as a measure of social surplus. Intuitively, more transparency leads to increased investment efficiency but can lower firm value when contracting is relatively important (i.e., if δ is sufficiently low). If firms can affect transparency by changing the clarity of their disclosures, our results suggest that firms should be more opaque when the incentive provision role of prices is more important, even if the information in the price is utilized for investment decisions.[6] More broadly, our analysis suggests that the impact of technological advances which increase transparency and price informativeness need not be associated with increases in firm value or social surplus when stock prices play a role in both contracting and information provision.

We characterize how our results change if the principal *delegates* the decision to the manager in Section 5.4. The manager maximizes his expected payoff, and as a result, his investment rule maximizes the short-term price. Relative to the principal's investment policy, the manager over-invests in negative ex-ante NPV projects, but under-invests in positive ex-ante NPV projects. While the manager's policy decreases investment efficiency, we show

[5]For example, Bai, Philippon, and Savov (2016) estimate a related measure of RPE which captures the response of both R&D and CAPX to prices and show that, over time, prices can explain more of the variation in firms' investment decisions.

[6]This is consistent with the evidence in Bebchuk and Fried (2004), who find that hard-to-value and opaque firms tend to use more aggressive stock-based pay for managers.

that delegation can increase overall firm value when δ is sufficiently low. In particular, when the manager under-invests in high profitability projects, the price is less volatile and, consequently, less costly to use for incentive provision. As a result, the principal to offer a higher-powered contract to the manager, which induces more effort. This analysis highlights the importance of accounting for the dual role of prices in understanding the allocation of control rights.

There is an extensive literature focused on understanding how the duration of management compensation is related to firm characteristics and managerial myopia (e.g., Gopalan, Milbourn, Song, and Thakor (2014)). In Section 5.5, we show that even in the absence of explicit preferences or incentives for short-termism, the optimal linear contract puts weight on both the price (short-term) and terminal value (long-term). While overall PFP decreases with δ, as in the benchmark analysis, we further find that the relative weight on long-term performance increases with it. Due to the novel interaction of the incentive provision and feedback effect channels, the optimal mix of short-term and long-term compensation depends upon the ex-ante profitability of new opportunities, and can even be non-monotonic in price informativeness.

Section 6 summarizes the empirical predictions of the model. A broad takeaway is that it is important to condition on a measure of δ, the relative importance of feedback effects, when trying to understand the relation among price informativeness, managerial compensation and firm value. While there is no direct measure of δ that is widely agreed upon (to the best of our knowledge), we build on the existing empirical literature to suggest some approaches to capturing this variation. This includes using indirect measures (e.g., insider trading, index inclusion, managerial overconfidence) that have previously been shown to be related the intensity of the feedback effect, as well as direct, survey evidence (e.g., Goldstein, Liu, and Yang (2021b)).

The rest of the paper is as follows. The next section briefly discusses the related literature and the paper's contribution. Section 3 presents the model and discusses the key assumptions while Section 4 characterizes the equilibrium. Section 5 characterizes the theoretical implications of the dual role of prices. Section 6 summarizes the model's empirical predictions and Section 7 concludes. All proofs and supplemental analysis are in the Appendix.

2 Related Literature

Our paper contributes to the literature on how asset prices affect real decisions (see Bond et al. (2012) for a survey). The literature broadly identifies two roles for prices through which secondary markets may affect real decisions. The early part of this literature largely analyzed how prices provide information about managerial effort and, consequently, how stock-based compensation affects the value of the firm through incentive provision (e.g., see Scholes (1991), Paul (1992), Holmström and Tirole (1993), Calcagno and Heider (2021)). More recently, the literature has focused on "feedback effects," whereby stock prices aggregate investors' dispersed information from which managers can learn in order to guide their real decisions (see Goldstein and Yang (2017) for a recent survey). However, the literature has largely considered each role in isolation, while abstracting from the other.[7] Our analysis implies that the interaction between these dual roles is crucial for understanding the impact of investors' information on firm value.

The work of Lin, Liu, and Sun (2019) and Dow and Gorton (1997) are closest to our own analysis, but differ along an important dimension. In both, the manager can utilize either their own private information to make an investment decision or, alternatively, rely on the price. Price information is a *substitute* for managerial information and so when investors are better-informed, the manager's optimal contract is lower-powered. Effectively, higher price informativeness makes contracting with the manager *easier*. Our benchmark model studies a setting in which the manager's effort does not affect the investment decision and so, in contrast, contracting becomes *harder* when investors are better-informed.[8] Given the multi-faceted nature of executives' daily responsibilities, the separation we model captures an important and natural feature of managerial actions in practice.

Our results highlight that investors' information can affect managerial actions even when the two dimensions independently affect firm value. In this sense, our paper is related to the theoretical literature which considers the implications when firm value is multi-dimensional. For instance, Goldstein and Yang (2015), Goldstein and Yang (2019) and Goldstein, Kopytov, Shen, and Xiang (2021a) consider settings where investors are heterogeneously informed about two components of fundamentals. Similarly, in our model, we can view managerial effort and information about future investment as different components of fundamentals. One key difference, however, is that in our setting the first component depends endogenously on the equilibrium contract offered to the manager.

Gjesdal (1981) shows that, in a generalized information environment, the information system which is preferable for compensating an agent may not be preferable for decision-making. Our paper demonstrates that this tension naturally arises in settings where the signal of interest is the traded stock price. We build on the findings of Paul (1992), Bushman and Indjejikian (1993), and Kim and Suh (1993) who emphasize that the weight investors

[7]For instance, while Bresnahan, Milgrom, Paul, et al. (1992) analyze both roles for prices, they shut down one dimension when analyzing the impact of change in investors' information on the other dimension.

[8]In Appendix B.4, we extend our benchmark model so that managerial effort directly impacts the investment payoff. We show that the key trade-off we highlight arises in this setting as well.

place on their information reflects the contribution of the project to firm value instead of the contribution of the manager to the project (which would be preferable for contracting purposes). Relative to these papers, our analysis highlights how investor information affects both investment efficiency and contracting efficiency, and how the interaction between these affects firm value and decisions. This interaction is also absent from Chaigneau, Edmans, and Gottlieb (2018), who study an alternative channel through which more precise information reduces effort.[9]

Our analysis focuses on a setting in which the price is more volatile when investors are better informed about (future) investment opportunities. While some recent theoretical papers feature models in which a non-monotonic relationship arises (e.g., Brunnermeier, Sockin, and Xiong (2020)) and Dávila and Parlatore (2020)), we expect the positive relation in our model to arise more generally given the environment we analyze. In particular, since the fundamental risk (and information asymmetry) in our model is firm-specific, we expect the marginal trader to be effectively risk-neutral with respect to this risk (as in our model) since it will be largely diversified away. As we show in Appendix B.6, this implies the relation between volatility and price-informativeness is generally positive.[10]

Our model also relates to the literature focused on how changes in transparency (e.g., due to public disclosures or a reduction in the cost of information acquisition) can have unintended consequences for price informativeness and real efficiency (see Goldstein and Sapra (2014), and Goldstein and Yang (2017) for recent surveys). We show how greater transparency can have a negative impact on contracting efficiency and, thus, reduce firm value. This is in contrast to, but complements, existing analyses that have focused on how increases in disclosure or transparency can lead to "crowding out" of information that is relevant for investment decisions.[11]

Finally, our paper is related to the literature which argues that commitment to ex-post inefficient rules can be optimal. For instance, in Bond and Goldstein (2015) the benefit

[9] In their setting, an improvement in the principal's information about effort leads to lower volatility in output. Since the agent is risk-neutral, however, and the optimal contract is convex, this reduction in volatility can reduce the incentive of the agent to exert effort. In our setting, when investors have more information about new investment opportunities, the price becomes a noisier signal about effort and so is less useful in incentive provision.

[10] Moreover, as discussed in the introduction, the empirical literature on price non-synchronicity, informativeness and feedback suggest that, in practice, the relation between idiosyncratic volatility and informativeness is positive.

[11] For instance, Diamond (1985), Gao and Liang (2013), and Colombo, Femminis, and Pavan (2014) discuss how public disclosure can crowd out private learning by investors. Goldstein and Yang (2019) show how disclosure about one component of fundamentals affects price informativeness about other components. Banerjee, Davis, and Gondhi (2018) show that an increase in transparency can encourage learning about fundamentals, but also about the behavior of other investors, and thereby make prices less informationally efficient.

of this ex-ante commitment to rely less on prices is that, in equilibrium, the stock price is more informative. In a setting without feedback effects, Strobl (2014) shows that by committing to over-invest in negative NPV projects, a manager can induce investors to acquire more information, and thereby improve contracting efficiency. In contrast, in our setting it can be optimal to delegate to the manager (thereby committing to an ex-post inefficient investment rule) because the resultant under-investment reduces the volatility of the price which increases contracting efficiency.[12]

3 Model

Payoffs. There are three dates $t \in \{1, 2, 3\}$, and two equally-likely states of the world, denoted by $\omega \in \{H, L\}$. The state is realized at date one, but is unobservable. The firm pays terminal cash flows V at date three, which consist of two components: (i) assets-in-place that generate x_ω and (ii) a zero-cost investment opportunity that generates δy_ω.[13] We assume that $x_H > x_L > 0$, $y_H > 0 > y_L$.[14] The parameter $\delta \geq 0$ captures the importance of the new investment opportunity relative to the firm's existing assets. There are two traded securities. The risk-free security is normalized to the numeraire. The risky security is a claim to the net cash flows of the firm, and trades at a price P at date two.

The firm. The firm consists of a principal (she) and a manager (he). The principal, indexed by p, is risk neutral and offers a take it or leave it contract to the manager at date one to maximize expected firm value, net of payments. We restrict the principal to offering the manager a *linear* compensation contract of the form:

$$W(P) = \alpha + \beta P, \tag{1}$$

[12]There are two notable distinctions between our setting and Strobl (2014). First, in Strobl (2014), the manager is better informed about the investment opportunity than the investors, and so there is no feedback from prices. In contrast, feedback plays a central role in our analysis. Second, the manager's effort choice affects the likelihood of success of the new project in Strobl (2014), while the two are not directly related in our setting.

[13]The assumption of zero cost is isomorphic to a setting in which the required investment is non-zero but can be made using the firm's existing cash (contained in the assets-in-place), i.e., it does not require equity holders to contribute additional capital.

[14]The assumption that assets-in-place and the investment opportunity are positively correlated does not qualitatively affect our results but eases the exposition. It is sufficient to assume that cash flows in the high state exhibit first-order stochastic dominance, with and without investment, under which the project can be either independent of or negatively correlated with assets-in-place. See Davis and Gondhi (2021) for a setting which constructs the analogous equilibrium under this more general assumption.

where α denotes the manager's fixed wage and β denotes the sensitivity of her compensation to the market price at date two.[15]

The manager, indexed by m, has mean-variance preferences over his payoff W, with risk aversion coefficient γ, and an outside option u_0, which we normalize to zero. At date one, the manager can exert costly effort e to increase the cash flows from assets-in-place from x_ω to $x_\omega + e$. We assume that the effort is observable but not verifiable, and requires the manager to incur a private cost, $c(e)$, where $c' \geq 0, c'' > 0, c(0) = c'(0) = 0$. The manager chooses his effort level to maximize his utility from compensation net of effort costs i.e.,

$$u_m(e; \alpha, \beta) \equiv \mathbb{E}[W(P)] - \frac{\gamma}{2}\mathbb{V}[W(P)] - c(e), \tag{2}$$

subject to $u_m(e; \alpha, \beta) \geq u_0 = 0$.

At date two, the principal chooses whether or not to invest in the new opportunity in order to maximize the expected value of the terminal cash flows, given the security price. We denote this investment decision by $I(P) \in \{0, 1\}$, where $I(P) = 1$ indicates that the investment is made. Together, this implies that the terminal cash flows of the firm are given by

$$V(\omega, e, I) \equiv x_\omega + e + \delta y_\omega \times I. \tag{3}$$

The principal's optimal contract, (α^*, β^*), maximizes the terminal value of the firm net of compensation, subject to the manager's incentive compatibility constraint, the principal's investment rule, and the manager's participation constraint, i.e., she solves

$$\max_{\alpha,\beta} \mathbb{E}[V(\omega, e^*, I^*) - W(P)], \text{ subject to} \tag{4}$$

$$e^* = \arg\max_e u_m(e; \alpha, \beta), \tag{5}$$

$$I^* = \arg\max_I \mathbb{E}[V(\omega, e^*, I) | P], \text{ and} \tag{6}$$

$$u_m(e^*; \alpha, \beta) \geq u_0. \tag{7}$$

Investors. At date two, a unit-measure continuum of risk-neutral investors indexed by $i \in [0, 1]$ trade the claim to the firm's net cash flows, i.e., $V - W$.[16] While all investors

[15]As we discuss below, a complete characterization of the optimal (possibly, non-linear) contract is not analytically tractable in our setting. However, in Appendix B.5, we show that our main result obtains when we allow the principal to offer a contract that is linear-quadratic in P. As such, our results do not appear to be a consequence of the linearity assumption.

[16]In Appendix B.1, we consider a setting in which investors trade a claim to the firm's terminal cash flow *without* netting out the manager's payoff.

condition on the information in the equilibrium price, P, a fraction $\lambda < 1$ of these investors also observe a private, conditionally-independent signal, $s_i \in \{s_H, s_L\}$, where

$$\mathbb{P}\left[s_i = s_H | \omega = H\right] = \mathbb{P}\left[s_i = s_L | \omega \in L\right] = \rho > \tfrac{1}{2}. \tag{8}$$

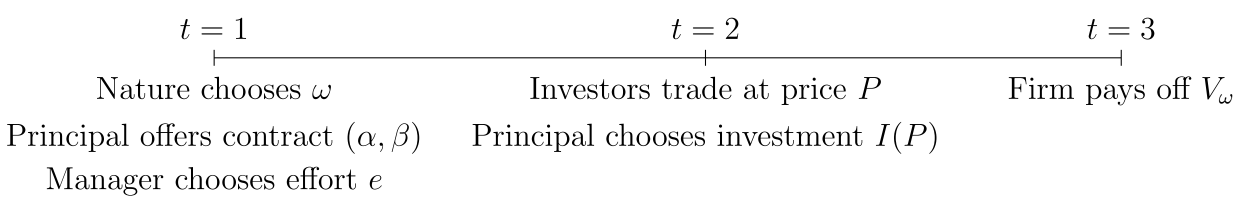
The remaining fraction, $1 - \lambda$, are uninformed, which we denote by $s_i = \emptyset$ with some abuse of notation.Investors can take both long and short positions in the risky security, but they are subject to position limits – specifically, we normalize the positions so that $d_i \in [-1, 1]$. This implies that the optimal demand is given by

$$d\left(s_i, P\right) \equiv \arg \max_{d_i \in [-1,1]} \mathbb{E}\left[d_i \times (V - W - P) \,|s_i, P\right]. \tag{9}$$

To ensure that the equilibrium price is not fully revealing, the aggregate supply of the risky security available for trade is stochastic and given by $u \sim U\left[-1, 1\right]$, where u is independent of all other random variables in the economy.[17] The equilibrium price, P, is determined by market clearing, i.e.,

$$\int_i d(s_i, P) di = u. \tag{10}$$

Figure 1: The timeline of the economy



Timing of events. The timeline of events is summarized in Figure 1. At date one, nature chooses $\omega \in \{H, L\}$. The principal offers the manager a linear contract (α, β), and the manager chooses costly effort e to maximize his expected compensation. At date two,

[17]Our analysis generalizes to a setting in which the distribution of noise trading is $U[-u_b, u_b]$ where $u_b \leq 1$. As in canonical models of financial markets, as the amount of noise trading falls (i.e., as u_b shrinks towards zero), the price signal becomes (unconditionally) more informative. Our main trade-off, however, is unchanged by this generalization: when investors' are better informed, it is more costly for the principal to incent managerial effort. We assume that $u[-1, 1]$ because it both (i) naturally corresponds to the assumed position limits for informed investors and (ii) adds tractability, so that the derived equilibrium expressions are more transparent.

investors observe their signals (if informed) and the price, and trade the risky security. The principal observes the equilibrium price, P, and chooses whether or not to invest in the new opportunity. At date three, the firm's assets pay off cash flows V_ω given by (3).

Equilibrium. An equilibrium consists of a linear contract (α, β), effort level e, investment rule $I(P)$, equilibrium demands $\{d(s_i, P)\}_i$ and equilibrium price P, such that:

(i) $d(s_i, P)$ maximizes the investor's objective in (9),

(ii) the equilibrium price P clears the risky security market (i.e., (10) holds),

(iii) the investment rule maximizes the expected firm cash flows (i.e., solves (6)),

(iv) the optimal effort level e maximizes the manager's expected utility (i.e., solves (5)),

(v) the optimal linear contract (α, β) solves the principal's problem, characterized by (4)-(7), and

(vi) posterior beliefs satisfy Bayes' rule whenever applicable.

3.1 Discussion of Assumptions

Our benchmark analysis assumes that some investors are exogenously endowed with private information about the state of the world, ω. In Section 5.3, we extend the analysis to allow for costly information acquisition. Specifically, we assume that before date one, investors choose whether or not to pay a cost, c_0, to become informed. We then derive the equilibrium fraction of informed investors. This allows us to characterize how changes in transparency (driven by changes in c_0) affect the optimal contract, the expected value of the firm, and social surplus.

The principal's investment policy maximizes the firm's terminal cash flows given the information in the price in our benchmark model. We interpret the principal as representing long-term shareholders or the board of directors. In Section 5.4, we relax this constraint and assume that the *manager* chooses whether or to invest to maximize his payoff and, consequently, the price. While this can lead to inefficient investment decisions, the main results are qualitatively similar. Moreover, we show that delegation to manager can be optimal, even though manager's investment policy is sub-optimal, due to its positive impact on managerial effort. This result suggests that it could be optimal for the principal to commit to an ex-post inefficient investment rule. We rule such commitment out to focus on the economic mechanism we wish to highlight; moreover, such a commitment may attenuate but not eliminate the impact of this mechanism.

For our benchmark model, we assume that the principal cannot contract on the firm's realized value. This is a standard assumption in the literature, commonly motivated by the observation that the manager's tenure at the firm may be shorter than the life of the project. However, in Section 5.5, we relax this assumption and allow the equilibrium contract to linearly depend on both the date two price as well as the terminal value, $V(\omega, e, I)$. We show that the principal always chooses to include the price in the equilibrium contract and, moreover, the interaction between investors' information and equilibrium managerial effort is preserved. Moreover, we show how the dual rule of prices generates a new channel through which heterogeneity in short- and long-term compensation can arise.

We assume that the principal and investors can observe the effort level chosen by the manager but cannot directly contract on this information. This allows us to introduce the trade-off between the allocative and contracting roles of prices in a transparent manner. In Appendix B.2, we consider a setting in which investors have access to a noisy signal of the manager's effort. We show that our key trade-off obtains in this setting: when the price is more informative about future investment opportunities, it is less effective from a contracting perspective.

We assume that investors are risk-neutral whereas the manager is risk-averse. This is natural. Generally, investors are able to diversify their exposure to firm-specific risk, e.g., by holding a large portfolio of securities. Moreover, as Albagli, Hellwig, and Tsyvinski (2021) emphasize, focusing on a setting with risk-neutral investors with position limits allows us to interpret the results as being about firm-specific information and risks. However, managers are generally under-diversified and, therefore, have concentrated exposure to firm-specific risk via their human capital and due to equity-based compensation.[18] We show in Appendix B.3 that our main trade-off (between price informativeness and equilibrium effort) also obtains when the manager is risk-neutral, but the optimal contract must satisfy a limited liability constraint.

We refer to the positive impact of the manager's decision as arising through "effort" but this need not be interpreted literally. For instance, the private cost borne by the manager can reflect his disutility from completing value-enhancing but monotonous tasks or making difficult but valuable decisions. Given the many dimensions through which managerial actions impact firm value, we follow the existing literature (e.g., Holmström and Tirole (1993)) and assume that effort increases the value of the firm irrespective of whether or not the investment is made. Allowing for the impact of effort to be correlated with the investment payoff introduces additional forces which obscure the economic mechanism we want to focus

[18]Moreover, top executives are often not permitted to buy put options or short their own company's stock, and hence they cannot hedge away this exposure to firm-specific risk.

on and reduces analytical tractability. Our setting allows us to highlight how contracting and feedback affect each other even when the two components do not mechanically depend on each other. In Appendix B.4, we demonstrate the robustness of our results in the narrower setting in which managerial effort increases the payoffs of the new opportunity.

Finally, we restrict attention to linear contracts because a complete analytical characterization of the optimal contract is otherwise intractable. However, in Appendix B.5, we show numerically that our main results also arise when we allow the principal to offer a contract that is non-linear in the price. As such, we interpret the contract characterized in our benchmark analysis as a first-order approximation to the more general, optimal contract (and the contract in Appendix B.5 as a second-order approximation). We expect that our key trade-off arises more generally, so long as the optimal compensation to the manager becomes more volatile when investors are more informed about future opportunities.

4 Equilibrium

We solve the model by working backwards.

4.1 Principal's investment decision

The principal can condition on the information in the equilibrium price when choosing whether or not to invest in the new project. Let $q(P) \equiv \mathbb{P}[\omega = H|P]$ denote her posterior beliefs. The principal optimally chooses to invest if and only if doing so is ex-post efficient and so, she chooses to follow a threshold strategy:

$$I(P) = \begin{cases} 1 & \text{if } q(P) \geq \frac{-y_L}{y_H - y_L} \equiv K \\ 0 & \text{if } q(P) < K. \end{cases} \tag{11}$$

The principal invests only if she is sufficiently optimistic that the investment will pay off positively, i.e., if $q(P)$ is sufficiently large. Intuitively, the investment threshold, K, increases with the size of the potential loss ($-y_L$) but decreases with the potential gain (y_H).

4.2 Financial market equilibrium

Given investor i's objective function in Eq. (9), he optimally adopts a threshold strategy:

$$d(s_i, P) = \begin{cases} 1 & \text{if } P < \mathbb{E}[V - W | s_i, P] \\ [-1, 1] & \text{if } P = \mathbb{E}[V - W | s_i, P] \\ -1 & \text{if } P > \mathbb{E}[V - W | s_i, P]. \end{cases} \tag{12}$$

Investors can observe the manager's effort choice, e, and infer the principal's investment choice, $I(P)$, since neither the manager nor the principal possess any private information and condition only on public information, i.e., P. As such, conditional on (ω, P), all investors agree on the *state-dependent* value of the firm, $V(\omega, e^*, I(P))$, defined in (3).

However, since they are privately informed, investors generally differ in their beliefs about the likelihood of each state. Let $q(s_i, P) \equiv \mathbb{P}[\omega = H | s_i, P]$ denote investor i's beliefs conditional on observing the price P and the private signal $s_i \in \{s_H, s_L, \emptyset\}$. To emphasize the role of the traded price and to simplify our notation, let us denote $V_\omega(P) \equiv V(\omega, e^*, I(P))$. Then investor i's conditional expectation of V is given by

$$\mathbb{E}[V | s_i, P] = V_L(P) + q(s_i, P) \times \left[\underbrace{V_H(P) - V_L(P)}_{\equiv \Delta V(P)} \right], \tag{13}$$

where $\Delta V(P)$ reflects the information sensitivity of cash flows. Note that $\Delta V(P) > 0$ for any P, and so investor i's valuation $\mathbb{E}[V | s_i, P]$ is increasing in $q(s_i, P)$.[19]

To understand the nature of the financial market equilibrium, it is useful to derive the beliefs of a *marginal investor*, who is indifferent between buying and selling at the equilibrium price. We conjecture and verify that, in equilibrium, there are three, distinct price levels $p_H > p_U > p_L$, each of which correspond to the market-clearing price when the marginal investor's valuation of the firm's cash flows is given by

$$\mathbb{E}[V | s_H, p_H] > \mathbb{E}[V | \emptyset, p_U] > \mathbb{E}[V | s_L, p_L], \tag{14}$$

respectively.

For instance, suppose that the supply is relatively high. Then, in order for the market to clear, it must be the case that the marginal investor is a "pessimistic" informed trader (i.e., someone who observed $s_i = s_L$). His valuation of the claim determines the market-clearing price ($P = p_L$) and all such "pessimistic" investors are indifferent between buying and selling.

[19]This is because $\Delta V(P) = x_H - x_L + \delta(y_H - y_L) \times I(P) > 0$.

However, both the uninformed and "optimistic" investors (i.e., those who observed $s_i = s_H$) optimally take long positions since

$$\mathbb{E}\left[V|s_H, p_L\right] > \mathbb{E}\left[V|\emptyset, p_L\right] > \mathbb{E}\left[V|s_L, p_L\right]. \tag{15}$$

Whether this price can be supported at a given supply level depends on ω. For instance, if $\omega = H$, market clearing implies that $P = p_L$ as long as

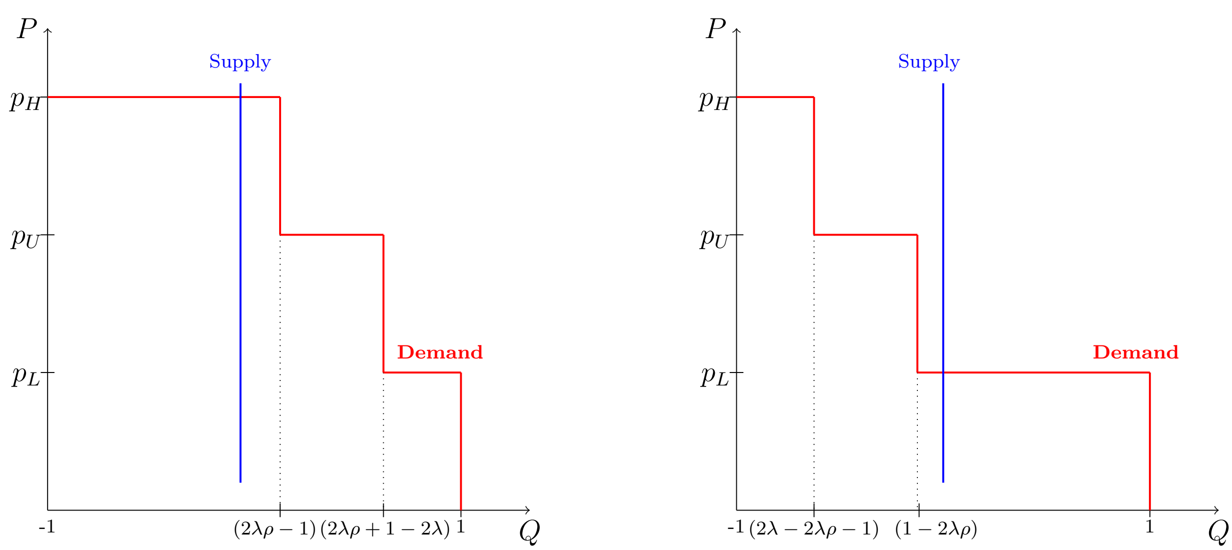
$$u > \underbrace{\lambda\rho}_{\text{observed } s_i = s_H} + \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_L} = 1 - 2\lambda(1-\rho). \tag{16}$$

If the supply of the risky asset, u, is any lower, then the market can only clear if one of the uninformed investors is indifferent between buying and selling. But for this to be the case, the price must reflect the valuation of the uninformed, i.e., $P = p_U \neq p_L$. Suppose instead that $\omega = L$. In this case, the measure of informed investors who observed $s_i = s_L$ is higher while the measure that observed $s_i = s_H$ is lower. Thus, the threshold supply such that $P = p_L$ falls to

$$u > \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_H} + \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda\rho}_{\text{observed } s_i = s_L} = 1 - 2\lambda\rho. \tag{17}$$

Following similar steps, we can determine the supply thresholds for both p_U and p_H.

Figure 2: Market clearing price for a fixed supply



(a) Equilibrium price when $\omega = H$

(b) Equilibrium price when $\omega = L$

Figure 2 illustrates this market-clearing process. While the supply of the traded claim does not depend on the realized state, ω, the aggregate demand from investors does. This is clear when comparing panel (a), where $\omega = H$ to panel (b), where $\omega = L$. While the supply is the same in both panels, there are only enough optimistic investors to support the high price when $\omega = H$, while the price falls to p_L when the true state is low.

The information contained in the price is also determined by these thresholds. For instance, given the distribution of u and the thresholds found in (16) and (17), we know that

$$\mathbb{P}\left[P = p_L | \omega = L\right] = \lambda\rho \quad \mathbb{P}\left[P = p_L | \omega = H\right] = \lambda\left(1-\rho\right). \tag{18}$$

By Bayes' rule, this implies that $\mathbb{P}\left[\omega = H | P = p_L\right] = 1-\rho$. Following similar steps yields the corresponding belief when the marginal investor is "optimistic": $\mathbb{P}\left[\omega = H | P = p_H\right] = \rho$. However, the price is not always informative since, independent of the state, the marginal investor is uninformed in equilibrium (i.e., $P = p_U$) with probability $1-\lambda$. In this latter case, conditioning on the price provides no additional information to the agents in the model.

From this, we can derive the beliefs of both the manager and the marginal investor. Note that, for the manager, who observes only the price, we have:

$$q_m\left(p_L\right) = 1-\rho, \quad q_m\left(p_U\right) = \frac{1}{2}, \text{ and } \quad q_m\left(p_H\right) = \rho. \tag{19}$$

Plugging these beliefs into (11) gives the equilibrium investment rule. In addition to accounting for the information in the price, the beliefs of the marginal investor must also account for any private information he possesses. For example, when the marginal investor is optimistic (so that $P = p_H$), his valuation must also account for his private signal, $s_i = s_H$ and so his expectation of cash flows is given by

$$\tilde{\mu}\left(p_H\right) \equiv \mathbb{E}\left[V | s_H, p_H\right] = V_L\left(p_H\right) + \tilde{\rho}\Delta V\left(p_H\right), \tag{20}$$

where

$$\tilde{\rho} \equiv \mathbb{P}\left[\omega = H | s_i = s_H, P = p_H\right] = \tfrac{\rho^2}{\rho^2+(1-\rho)^2}. \tag{21}$$

Similar arguments imply that the marginal investor is more pessimistic after conditioning on p_L, since

$$\tilde{\mu}\left(p_L\right) \equiv \mathbb{E}\left[V | s_L, p_L\right] = V_L\left(p_L\right) + \left(1-\tilde{\rho}\right)\Delta V\left(p_L\right). \tag{22}$$

In contrast, when the marginal investor is uninformed, the price provides no additional information and so his valuation is given by

$$\tilde{\mu}(p_U) \equiv \mathbb{E}[V|\emptyset, p_U] = V_L(p_U) + \frac{1}{2}\Delta V(p_U). \tag{23}$$

From these conditional expectations of the firm's cash flows, we can derive the market-clearing price since, in equilibrium, the marginal investor is indifferent between buying and selling i.e.,

$$\begin{aligned} P &= \tilde{\mu}(P) - W &(24)\\ &= \frac{1}{1+\beta}(\tilde{\mu}(P) - \alpha). &(25) \end{aligned}$$

With this, we can formally establish the existence of a financial market equilibrium with feedback to firm investment.

Proposition 1. *There exists a unique financial market equilibrium with price P and investment rule $I(P)$, where:*
(i) the equilibrium price $P(\omega, u)$ is given by

$$P(\omega, u) = \begin{cases} \frac{1}{1+\beta}(\tilde{\mu}(p_L) - \alpha) \equiv p_L & \text{if } u < u_\omega - (1-\lambda) \\ \frac{1}{1+\beta}(\tilde{\mu}(p_U) - \alpha) \equiv p_U & \text{if } u_\omega - (1-\lambda) \leq u \leq u_\omega + (1-\lambda) \\ \frac{1}{1+\beta}(\tilde{\mu}(p_H) - \alpha) \equiv p_H & \text{if } u > u_\omega + (1-\lambda) \end{cases}, \tag{26}$$

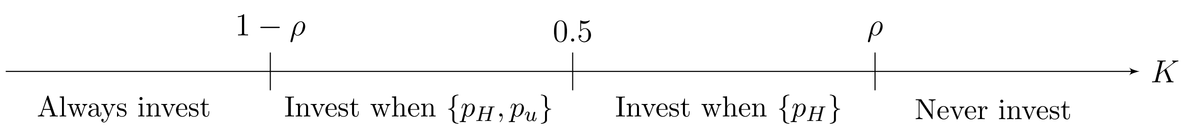
where $u_H \equiv \lambda(2\rho - 1)$, $u_L = \lambda(1 - 2\rho)$, $p_H > p_U > p_L$, and where the marginal investor's conditional expectations $\tilde{\mu}(P)$ are given by (20)–(23), and
(ii) the equilibrium investment rule is given by

$$I(P) = \begin{cases} 1 & \text{if } 1 - \rho > K \\ \begin{cases} 1 & P \in \{p_U, p_H\} \\ 0 & P = p_L \end{cases} & \text{if } \frac{1}{2} > K > 1 - \rho \\ \begin{cases} 1 & P = p_H \\ 0 & P \in \{p_U, p_L\} \end{cases} & \text{if } \rho > K > \frac{1}{2} \\ 0 & \text{if } \rho < K \end{cases}. \tag{27}$$

In equilibrium, there exists a feedback loop between firm investment and the financial market. The principal's decision to invest depends on the price through the beliefs of the marginal investor while the realized price level is determined by the investment policy through the state-dependent firm value, $V_\omega(P)$. The first part of the above proposition characterizes the financial market equilibrium, given the investment rule.

The second part of the result characterizes the investment rule, given the equilibrium price – Figure 3 provides an illustration of this. Recall that the principal only invests if she is sufficiently optimistic that the true state is high. Given the ordering of her beliefs (see equation (19)) and the investment threshold, K, this gives rise to the four distinct investment policies described above. These results are intuitive. When the investment threshold K is sufficiently high (low), the principal never invests (always invests, respectively). When the threshold is in the intermediate region, the principal conditions on the information in prices: she never invests when the price is low (i.e., $P = p_L$) but always invests when the price is high (i.e., $P = p_H$). In these regions, she only invests when the price is uninformative if the threshold K is below 1/2 which corresponds to an investment opportunity with an ex-ante positive return (i.e., $\frac{y_H + y_L}{2} > 0$).

Figure 3: Different investment policies



Finally, we note that investors' information impacts the investment decision through both ρ and λ. While the precision of their information, ρ, determines the price level(s) at which she invests, as discussed above, it is the measure of informed investors, λ, which determines the likelihood of observing each price since

$$\mathbb{P}[P = p_H] = \mathbb{P}[P = p_L] = \tfrac{\lambda}{2}, \mathbb{P}[P = p_U] = 1 - \lambda. \tag{28}$$

In Section 5.3, we allow λ to be endogenous and analyze its determinants.

4.3 Managerial effort

Given the terms of the principal's offered contract (α, β), we can characterize the manager's optimal effort choice at date one. Specifically, the manager maximizes his expected utility over compensation $W = \alpha + \beta P$, net of effort costs i.e.,

$$\max_e \mathbb{E}[\alpha + \beta P] - \frac{\gamma}{2}\mathbb{V}[\alpha + \beta P] - c(e), \tag{29}$$

where P is the equilibrium price of the traded claim, characterized in Proposition 1.

Note that the cash-flows $V_\omega(P)$ shift uniformly upward in each state with an increase in the manager's effort, e, and consequently, so does the price P. This implies that the variance of the price, and hence, the variance of the manager's payoff does not depend upon his effort level, given β. As a result, and by utilizing (25), the first order condition for the manager's problem in (29) is given by

$$\frac{\beta}{1+\beta} = c'(e). \tag{30}$$

Given the convexity of the cost function, the second order condition always holds, and so the optimal choice of effort is characterized by (30).

As expected, the manager's optimal level of effort increases with the sensitivity of his compensation to the market price. In contrast to contracting models with exogenous signals, the increase in effort e (as β increases) is attenuated due to the nature of the endogenous signal on which the contract is written. In particular, the risky security price accounts for the manager's payoff and so is scaled by $\frac{1}{1+\beta}$. In Appendix B.1 we show, however, that this does not affect managerial effort in equilibrium. Specifically, we consider a variant of the model in which the traded security is a claim to V and show that the principal chooses to provide *lowered*-powered incentives (i.e., lower β) relative to the benchmark model. Intuitively, the principal is able to recover the same signal of managerial effort, $\tilde{\mu}(P)$, in either setting and so equilibrium effort is unaffected.

4.4 Optimal Linear Contract

Given the date two equilibrium characterized in Proposition 1, and the manager's optimal effort choice given by (30), the principal chooses the linear contract (α, β) that maximizes the (unconditional) expected firm value, net of the manager's payoff. Specifically, the principal's problem can be re-written as

$$\max_{\alpha,\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I - (\alpha + \beta P)\right], \qquad \text{subject to :} \tag{31}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{32}$$

$$\mathbb{E}\left[\alpha + \beta P\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta P\right] - c(e) \geq 0. \tag{33}$$

Since α does not affect the manager's effort choice, the principal can adjust it, given β, so that the manager's participation constraint (33) binds. Moreover, note that the variance of the price can be written in terms of the variance of the marginal investor's valuation, $\tilde{\mu}(P)$:

$$\mathbb{V}[P] = \frac{1}{(1+\beta)^2}\mathbb{V}\left[\tilde{\mu}(P)\right]. \tag{34}$$

With these observations, the principal's objective simplifies to

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}\left(P\right)\right] - c\left(e\right) \text{ subject to } \frac{\beta}{1+\beta} = c'\left(e\right). \tag{35}$$

The following proposition provides a characterization of the optimal linear contract.

Proposition 2. *Suppose the financial equilibrium, (P, I), is given by Proposition 1 and the manager's optimal effort choice, e, is given by (30). Then the principal's optimal linear contract is given by (α, β), where*

$$\alpha = c\left(e\right) + \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}\left(P\right)\right] - \beta\mathbb{E}\left[P\right], \text{ and} \tag{36}$$

$$\beta = \frac{1}{c''\left(e\right)\gamma\mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}. \tag{37}$$

The principal's payoff in equation (35) has three components. The first component, $\mathbb{E}\left[x_\omega + e + \delta y_\omega I\right]$, is the expected value of the firm, which increases with the effort choice of the manager. The second term, $\frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]$, captures the reduction in the manager's expected utility because his payoff is tied to the price of the risky security. The third term, $c\left(e\right)$, is the manager's private cost of providing effort.

The principal must compensate the manager for both of these costs in order for the contract to be accepted. As a result, she faces a trade-off: while an increase in β increases the effort exerted by the manager and, hence, firm value, it also increases the compensation he demands. Intuitively, when the manager is more risk-averse (higher γ), the price is more variable (higher $\mathbb{V}\left[\tilde{\mu}\left(P\right)\right]$), or the cost of effort increases quickly ($c''\left(e\right)$), the optimal β is lower. This is seen clearly in the expression for the optimal β i.e., equation (37).

5 Implications

In this section, we characterize the implications of our benchmark model. The first subsection highlights the key trade-off in our setting: more informative prices lead to more efficient investment decisions, but also increase the cost of incentivizing managerial effort. This implies that expected firm value is non-monotonic in the quality of investors' aggregated information. The second subsection explores the implications of this observation for several distinct measures of price efficiency. The third subsection endogenizes information acquisition by investors and characterizes the effect of changes in transparency on firm value and social surplus. The fourth subsection shows how the trade-off between the incentive provision and feedback effects can naturally lead to the optimality of delegating the investment decisions to managers in some states. The final subsection illustrates how the duration of the manager's compensation varies when the principal also contracts on the terminal payoff.

5.1 Price informativeness and expected value

We begin by characterizing the impact of investment and managerial effort on the expected value, EV, where

$$\begin{aligned} EV &= \mathbb{E}\left[x_\omega + e + \delta y_\omega \times I\right] \\ &= \frac{x_H + x_L}{2} + e + \delta \times \begin{cases} 0 & \text{if } \rho < K, \\ \frac{\lambda}{2}\left(y_L + \rho\left(y_H - y_L\right)\right) & \text{if } \rho > K > \frac{1}{2}, \\ \frac{y_H + y_L}{2} - \frac{\lambda}{2}\left(y_L + (1-\rho)\left(y_H - y_L\right)\right) & \text{if } \frac{1}{2} > K > 1-\rho \\ \frac{y_H + y_L}{2} & \text{if } 1-\rho > K \end{cases} \end{aligned}$$

The above expression implies that, holding fixed managerial effort, the expected value increases in λ and ρ as long as the principal's investment decision depends on the price, that is, when $\rho > K > 1-\rho$.[20] Similarly, holding fixed investors' information, the expected value increases in managerial effort, e.

However, both the offered contract and, hence, equilibrium managerial effort, depend upon the informativeness of the price in equilibrium. Intuitively, since ρ measures the precision of informed investors' information and λ measures the fraction of informed traders, an increase in either makes the price more informative about the new investment opportunity, y_ω. This improves the efficiency of the investment decision since the principal is more likely to invest when the payoff is positive. However, it also makes the price more volatile, which increases the principal's cost of incentivizing effort. In equilibrium, the principal responds to an increase in either ρ or λ by reducing the sensitivity of the manager's compensation to the price (i.e., she reduces β), which in turn reduces the effort, e, exerted by the manager. We show this formally in the proof of the following proposition.

Proposition 3. *The optimal choice of β and e decrease with δ, λ and ρ.*

The proposition highlights the key trade-off in our model. While the expected value increases in both the manager's effort choice and the efficiency of the investment decision,

[20] While this is straightforward to see when $\rho > K > \frac{1}{2}$, it is also the case when $\frac{1}{2} > K > 1-\rho$, since

$$\begin{aligned} y_L + (1-\rho)\left(y_H - y_L\right) < 0 &\iff \qquad (38) \\ 1-\rho < \frac{-y_L}{y_H - y_L} = K. & \qquad (39) \end{aligned}$$

an increase in λ or ρ leads to a decrease in managerial effort in equilibrium. Intuitively, as the price becomes more informative about investment opportunities, it increases the volatility of the price for a given level of effort. Together with equation (37), this implies that β decreases with price informativeness (i.e., λ and ρ).

Thus, the impact of more informed investors depends upon the relative strength of these two countervailing effects. In the following proposition, we characterize how the impact of changes in λ depend on both (i) the quality of investors' information and (ii) the relative importance of the investment project, δ.

Proposition 4. *If $\rho < K$ or $1-\rho > K$, then firm value is decreasing in ρ and λ. If $\rho > K > 1-\rho$ and if the cost of effort satisfies Condition (65) in the Appendix, then*

(i) EV is increasing in λ if $\delta > \bar{\delta}$,

(ii) EV is U-shaped in λ if $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$,

(iii) EV is decreasing in λ if $\delta \leq \underline{\delta}$.

Condition (65) is satisfied if the cost of effort is quadratic.

Note that the impact of an increase in the fraction of informed investors on expected value can be expressed as:

$$\frac{\partial EV}{\partial \lambda} = \underbrace{\frac{\partial e}{\partial \lambda}}_{<0} + \delta \times \begin{cases} 0 & \text{if } \rho < K, \\ \frac{y_H - y_L}{2}\left(\rho - K\right) & \text{if } \rho > K > \frac{1}{2}, \\ \frac{y_H - y_L}{2}\left(K - (1-\rho)\right) & \text{if } \frac{1}{2} > K > 1-\rho \\ 0 & \text{if } 1-\rho > K. \end{cases} \tag{40}$$

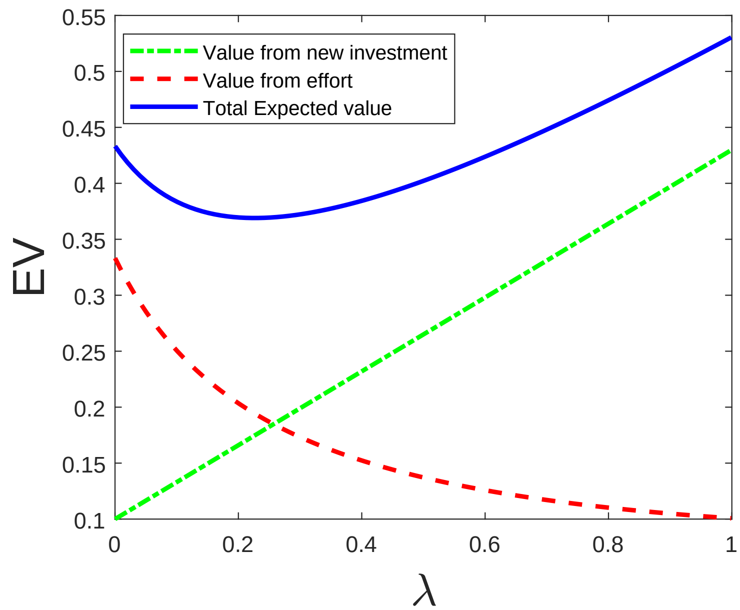
When the principal never invests (i.e., if $\rho < K$) or always invests (i.e., if $1-\rho > K$), increasing λ has no impact on investment efficiency. As a result, more informative prices necessarily decrease the expected value because they lead to a decrease in managerial effort.[21]

When $\rho > K > 1-\rho$, the impact depends upon δ, which parameterizes the relative importance of the investment project. Intuitively, when the investment project is sufficiently important (i.e., δ is sufficiently high), expected value increases with λ. On the other hand, if the investment project is sufficiently small relative to assets-in-place, the effect of λ on effort choice dominates and expected value decreases with λ. Condition (65) in the Appendix provides a sufficient condition to ensure this is the case. Specifically, the condition implies that the manager's effort choice is convex and decreasing in λ. Note that the impact of λ on investment efficiency is constant. When δ is sufficiently high, the decrease in managerial effort which follows from the increase in λ is always smaller than the increase in investment

[21] Changes in ρ give rise to the same effect in these regions.

Figure 4: Expected value versus fraction of informed investors

The figure plots the expected value of the firm as a function of λ. The cost of effort is $\frac{c_e}{2}e^2$. The other parameters of the model are set to: $x_H = 1$, $x_L = 0.5$, $c_e = 2$, $\rho = 0.7$ and $y_H = 1, y_L = -0.9$.



efficiency. As a result, firm value strictly increases. When δ is sufficiently low, this is reversed: the increased investment efficiency is always outweighed by the decline in effort, even as λ approaches one, and so the expected value strictly declines.

For $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$, however, the effect is no longer monotonic. The decline in effort for "low" λ causes the expected value to decrease initially, then increase. The plots in Figure 4 provide a numerical illustration for this case. The figure plots expected value (solid blue line) and its components: the manager's equilibrium effort level (red dashed line) and the expected value of the investment opportunity (green dotted line). Since the manager's effort is decreasing and convex in λ, while the value from the new investment opportunity increases linearly, the expected value is U-shaped in λ when $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$.

To summarize, Proposition 4 establishes that expected value need not increase with price informativeness. In fact, the relation between the two is either decreasing or non-monotonic unless feedback effects are sufficiently important. The result is particularly important for empirical tests of the relation between the quality of investors' aggregated information and expected value, since the nature of this relation can vary across firms (with different δ's) and even within firms (e.g., for different levels of λ) as a consequence of the trade-off between incentive provision and feedback effects.

5.2 Efficiency measures

Next, we characterize how alternate measures of price efficiency proposed in the literature depend on the fraction of informed investors and the precision of their signals in our setting.

1. **Revelatory price efficiency (*RPE*):** In our model, the price guides the principal's real investment decision. Revelatory price efficiency measures the extent to which prices reveal information which is relevant for investment decisions. In our model, this can be measured as
$$RPE = \frac{\mathbb{V}[y_\omega] - \mathbb{V}[y_\omega|P]}{\mathbb{V}[y_\omega]}$$

2. **Contracting price efficiency (*CPE*):** Importantly, the price not only conveys information about the investment opportunity, but also affects the manager's incentive to exert effort. Intuitively, given the manager's risk-aversion, the price is more useful for contracting if it tracks the manager's effort more precisely. This is captured by the noise-to-signal ratio of the price about the manager's action, which is a common measure of contracting efficiency in the literature (e.g., Holmström and Tirole (1993)). Since the signal which the principal can extract is $\tilde{\mu}(P)$, which moves one-for-one with managerial effort, this measure is simply the inverse of the noise in price signal:
$$CPE = [\mathbb{V}(\tilde{\mu}(P)|e)]^{-1}.$$

3. **Forecasting price efficiency (*FPE*):** Forecasting price efficiency captures the extent to which the information in the price correctly forecast future firm cash flows. A common measure of forecasting price efficiency is the inverse of the variance of cash flows conditional on observing the price. In our model, this measure is given by
$$FPE = [\mathbb{V}(x_\omega + e + \delta y_\omega I(P)|P)]^{-1}.$$

With these definitions in mind, we formally characterize how each type of efficiency varies when investors, in the aggregate, have more information.

Proposition 5. *In the equilibrium characterized by Propositions 1 and 2, we have:*

(i) RPE increases in λ and ρ,

(ii) CPE decreases in λ and ρ,

(iii) FPE decreases in λ iff $\rho > K > \frac{1}{2}$ and δ is sufficiently high i.e., $\left(1 + \delta \frac{y_H - y_L}{x_H - x_L}\right)^2 > \left(\frac{\rho^2 + (1-\rho)^2}{2\rho(1-\rho)}\right)$. FPE decreases in ρ iff $K > 0.5$ at $\rho = K$.

Figure 5: Efficiency metrics versus fraction of informed

The figure plots the RPE, CPE and FPE as a function of λ. The other parameters of the model are set to: $x_H = 1$, $x_L = 0.5$, $c_e = 2$, $\rho = 0.7$ and $y_H = 1, y_L = -1.1$.

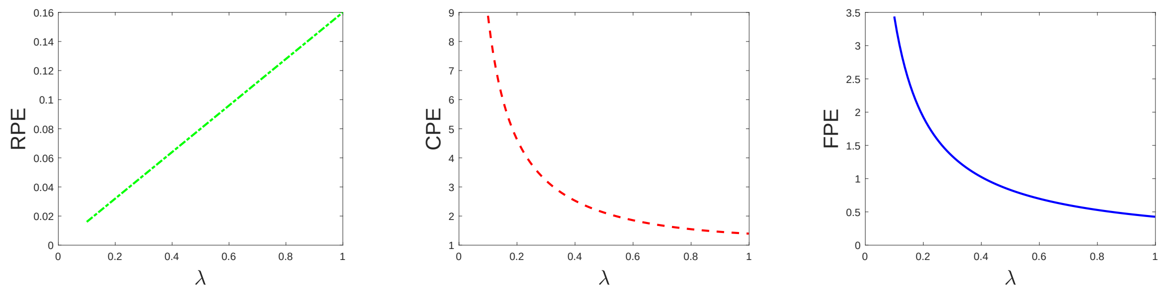


Figure 5 provides an illustration of this result by plotting RPE, CPE and FPE as a function of λ. As ρ increases, the signal which the principal can extract from the price is more precise. As λ increases, it is more likely that the price provides an informative signal to the principal. This implies that revelatory price efficiency increases in both ρ and λ (e.g., Panel (a) of Figure 5). On the other hand, as the price becomes more informative about investors' information, it becomes relatively less informative about the manager's action, that is, the signal to noise ratio of the price signal about e decreases. This leads to lower contracting price efficiency, as illustrated by Panel (b) of Figure 5.

The link between investors' information and forecasting price efficiency is more nuanced because the variance of the firm's cash flows is endogenous. Holding fixed the likelihood of investment, an increase in either λ or ρ increases the precision of the marginal investor's forecast, on average, which increases FPE. However, λ and ρ also affect the likelihood of investment, which can increase the variance in the firm's cash flows and push FPE downward. For example, when $\rho < K$, the manager never invests; once ρ crosses the threshold K, there is a discontinuous drop in FPE due to the likelihood of investment. Once in this region (i.e., when $\rho > K > \frac{1}{2}$), an increase in λ makes investment more likely. When the relative importance of the investment is sufficiently large (i.e., δ is sufficiently high) relative to the precision of investors' information, ρ, such increases in λ can lead to a decrease in FPE, as we see in Figure 5 panel (c).[22]

The recent literature on feedback effects has emphasized the tension between forecasting price efficiency (FPE) and revelatory price efficiency (RPE) (e.g., see Bond et al. (2012)). In these models, what matters for real efficiency is whether the price reveals information necessary for decision makers to take value-maximizing actions; as a result, this literature

[22]Note that the right-hand side of the inequality found in the proposition is increasing in ρ.

has focused on RPE as the relevant measure of incremental real efficiency introduced through the information found in security prices. Our analysis highlights that RPE is an incomplete measure of incremental real efficiency when stock prices are used to provide managerial incentives, because it does not account for this information's impact on contracting efficiency. In particular, while increases in either RPE or CPE lead to increases in firm value (holding the impact of the other channel fixed), in equilibrium, they move in opposite directions (see Proposition 5). Thus, using either of these measures in isolation as a proxy for incremental real efficiency can be misleading because it fails to account for the attenuation (or even reversal) in real efficiency captured by the other measure.

5.3 Costly Information Acquisition and Transparency

Our benchmark analysis shows that while an increase in λ increases the extent to which prices reveal information about y_ω (i.e., increases RPE), it can decrease firm value by making it costlier for the principal to incentivize the manager to exert effort. Given this trade-off, we explore the incentives of investors to acquire costly information by endogenizing λ.

Specifically, suppose that before date one, each investor chooses whether or not to become informed by paying a cost c_0 to obtain an informative private signal $s_i \in \{s_H, s_L\}$. Each investor takes as given the manager's optimal effort level (i.e., the solution to equation (30)) as well as the information acquisition decision of all other investors. Acquiring a private signal is only valuable if the incremental increase in his expected profits exceeds the fixed cost, c_0, given the nature of the financial market equilibrium. This implies that, in any interior equilibrium (i.e., when $\lambda \in (0,1)$) where investors are indifferent between being informed and uninformed, it must be the case that

$$\mathbb{E}\left[d\left(s_i, P\right)\left(V-P\right)\right] - \mathbb{E}\left[d\left(\emptyset, P\right)\left(V-P\right)\right] = c_0. \tag{41}$$

This yields the following equilibrium characterization.

Proposition 6. *There exists a $\hat{c}$, defined in the appendix, such that:*

(i) If $c_0 \geq \hat{c}$, then $\lambda = 0$.

(ii) If $c_0 < \hat{c}$, the measure of informed investors in equilibrium is given by

$$\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)\Delta V\left(p_U\right)} < 1,$$

and is decreasing in the cost of information c_0, increasing in the precision of the signal ρ, and the information sensitivity of the risky security when the price is uninformative ($\Delta V\left(p_U\right)$).

When information acquisition costs are sufficiently high (i.e., $c_0 > \hat{c}$), the net benefit from acquiring information is too low, even when all other investors are uninformed. As a result, all investors optimally choose not to acquire any information. On the other hand, when costs are sufficiently low, a fraction of investors choose to acquire information. This fraction of informed investors increases with the precision of the signal ρ and decreases with the fixed cost c_0.

Intuitively, the measure of informed investors depends upon the information-sensitivity of the risky security. However, in our setting, it only depends on the sensitivity when the price is uninformative. As we show in the proof, the expected trading gains when the price is high or low (i.e., $P = p_H$ and $P = p_L$, respectively) are the same for both informed and uninformed investors. This is a consequence of our assumption that investors are risk-neutral and face position limits. As such, the incremental benefit from acquiring private information arises when the price is uninformative (i.e., $P = p_U$). Since investors differ only in their beliefs about the payoff across the two states, the benefit of trading at the uninformed price is $\Delta V(p_U) = V_H(p_U) - V_L(p_U)$ and is scaled by their information advantage relative to the uninformed, $\rho - \frac{1}{2}$. This is also the reason why there are always uninformed investors in equilibrium: if $\lambda = 1$, the price is never uninformative and thus, there is no benefit to being informed.

Given the impact of λ on the expected value, this result suggests a natural role for policy changes that increase transparency by reducing the cost of information acquisition, c_0. We characterize the impact of transparency on firm value FV, which captures the expected value of the firm net of compensation to the manager, i.e.,

$$FV = \text{Expected Value } - \text{Managerial Compensation.} \tag{42}$$

Since the optimal linear contract sets the manager's compensation to ensure that he is at his reservation utility, this implies that firm value can be expressed as

$$FV = \mathbb{E}\left[x_\omega + e + \delta y_\omega \times I\right] - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c(e). \tag{43}$$

The following result characterizes how an increase in transparency, or equivalently a decrease in c_0, affects firm value.

Proposition 7. *Firm Value increases with transparency (decreases with c_0) if $1-\rho < K < \rho$ and δ is sufficiently large. Firm Value decreases with transparency (increases with c_0) if either (i) $1 - \rho > K$, or (ii) $\rho < K$, or (iii) $1 - \rho < K < \rho$ and δ is sufficiently small.*

The proposition provides sufficient conditions for when an increase in transparency increases firm value, and when it decreases firm value. To gain some intuition, note that a change in information acquisition costs affects firm value via two channels:

$$\frac{d}{dc_0}FV = \frac{\partial}{\partial\lambda}FV \times \frac{d\lambda}{dc_0} \tag{44}$$

$$= \delta \times \underbrace{\frac{\partial}{\partial\lambda}\mathbb{E}\left[y_\omega \times I\right]\frac{d\lambda}{dc_0}}_{\text{investment channel}} + \underbrace{\frac{\partial}{\partial\lambda}\left(e - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right)\right)\frac{d\lambda}{dc_0}}_{\text{incentive channel}} \tag{45}$$

The first channel, which we call the **investment channel**, captures the impact of transparency on the efficiency of the investment decision. Since an increase in λ increases the expected value of the investment, an increase in c_0 reduces firm value (FV) through this term. This channel only arises when there is feedback from prices (i.e., when $1-\rho < K < \rho$) and is increasing in the scale of the investment, δ.

The second channel, which we refer to as the **incentive channel**, captures the impact of transparency on equilibrium effort provision net of costs. As the proof of Proposition 7 establishes,

$$\frac{\partial}{\partial\lambda}\left(e - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right)\right) = -\frac{\gamma}{2}\left(\frac{1}{1 + c''\left(e\right)\gamma\mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}\right)^2\frac{\partial}{\partial\lambda}\mathbb{V}\left(\tilde{\mu}\left(P\right)\right) < 0, \tag{46}$$

which reflects the fact that when the price becomes more informative about the investment project, it increases the variance of the price. This increases the potential risk borne by the manager due to the compensation contract and, in equilibrium, decreases the effort the manager exerts. As such, an increase in c_0, which reduces λ, decreases firm value (FV) through this channel.

In general, the impact of transparency on firm value depends on the relative magnitude of these two channels, and can be ambiguous. Proposition 7 provides sufficient conditions for when the direction of the effect is unambiguous. Specifically, the investment channel dominates when the feedback effect is operational (i.e., $1-\rho < K < \rho$) and the relative importance of the new investment opportunity is sufficiently large (i.e., δ is sufficiently large). In this case, an increase in transparency unambiguously leads to an increase in firm value. On the other hand, when the feedback effect is not operational (i.e., if $1-\rho > K$ or $\rho < K$) or the relative importance of the new investment is sufficiently low, the incentive channel dominates and firm value decreases with transparency.

It has been suggested that one potential solution to the trade-off faced by the principal would be to sell the investment opportunity since the resultant decline in the volatility of the price would lead to higher equilibrium effort. First, we note that in many settings such a sale may not be feasible. The payoff of the investment may derive from the firm's existing assets, including its human capital or intellectual property. Second, we note that when λ is endogenous, the value of the investment is increasing in the information-sensitivity which depends not only on the investment payoff but the correlation of the payoff to the firm's assets-in-place (as proxied by $x_H - x_L$). As such, if a potential buyer anticipated having access to less information, the resultant reduction in its willingness-to-pay may outweigh the benefit of higher managerial effort if the investment is sold.

In Appendix B.7, we characterize the aggregate impact of transparency on a measure of social surplus that accounts for not only the expected value of the firm net of compensation (i.e., FV), but also the manager's expected utility net of effort costs, and the aggregate trading gains and losses net of information acquisition costs. We find that social value increases with transparency when the feedback effect is relatively more important in determining firm value, but can decrease with transparency otherwise. This result cautions against an "one-size-fits-all" approach to regulatory policy that affects transparency in financial markets.[23] In particular, the dual role of stock prices highlighted by our analysis implies that higher transparency may not be socially desirable, even when it lowers investors' information costs and improves investment outcomes, because of its negative impact on contracting.

5.4 Price maximization versus value maximization

In the benchmark model, we assumed that the principal's investment rule, $I(P)$, maximized her expectation of the firm's terminal cash flows, given the price. In this section, we examine the implications of relaxing this assumption and allow the *manager* to choose the investment policy. Specifically, suppose the manager invests (i.e., chooses $I_m(P) \in \{0, 1\}$) in order to maximize his payoff $(\alpha + \beta P)$ which, given his contract, is equivalent to maximizing the price.

The date-one price is maximized when the manager invests (i.e., $I_m(P) = 1$), as long as

$$q_m(P) > \frac{-y_L}{y_H - y_L} \equiv K,$$

where $q_m(P)$ denotes the beliefs of the marginal investor, not of the principal. This change in the investment policy does not alter the financial market equilibrium given in Proposition 1 but the investment rule is modified to reflect the manager's use of $q_m(P)$:

- If $1 - \tilde{\rho} > K$, the manager always invests.

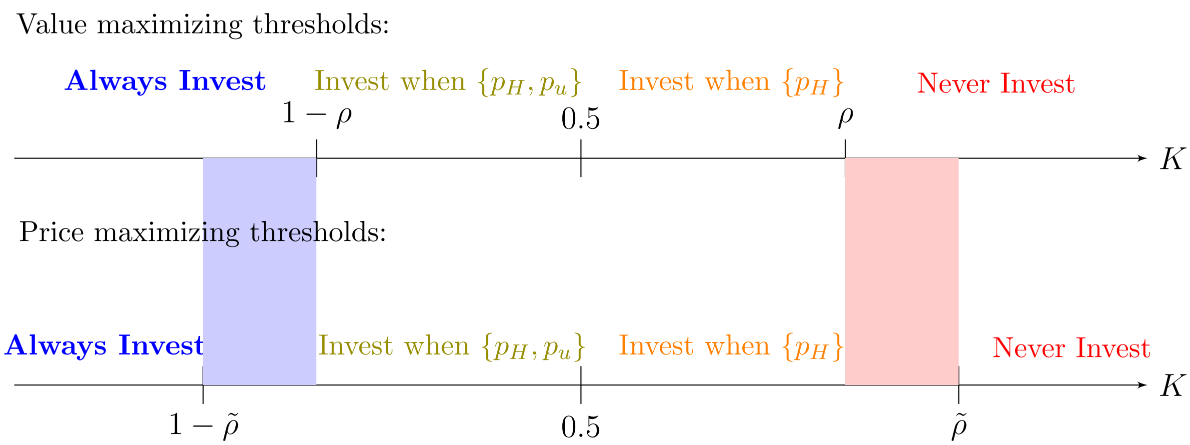
[23] Notably our results are not driven by the Hirshleifer (1971) effect, whereby more public information reduces opportunities for risk sharing, and are also distinct from the "crowding out" channel highlighted by other papers in the feedback literature (e.g, Goldstein and Yang (2019)).

- If $\frac{1}{2} > K > 1 - \tilde{\rho}$, the manager invests if $P \in \{p_u, p_H\}$.
- If $\tilde{\rho} > K > \frac{1}{2}$, the manager invests if $P = p_H$.
- If $\tilde{\rho} < K$, the manager never invests.

In Figure 6, we compare the new investment thresholds to those found in our baseline specification. In the two shaded regions, the manager and principal follow different investment rules. If $K \in (\rho, \tilde{\rho})$ the principal never invests while the manager invests only if $P = p_H$. If $K \in (1 - \tilde{\rho}, 1 - \rho)$, the principal always invests, but the manager refrains from investing when $P = p_L$. This change in policy reflects the difference in the principal's and the marginal investors' beliefs about the likelihood of each state.

Figure 6: Thresholds

The figure characterizes the investment rule when the principal maximizes terminal value (top) to the one when the manager maximizes the price (bottom), as a function of $K = \frac{-y_L}{y_H - y_L}$.



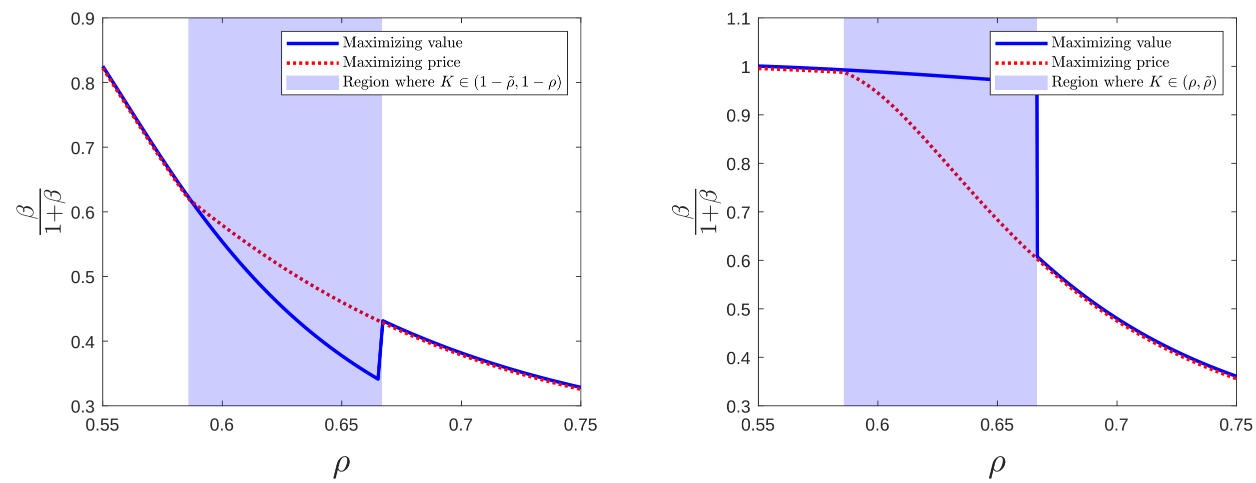
These thresholds suggests that contracting on date-one price has a downside when the manager also chooses the firm's investment policy: it can encourage the manager to focus too much on maximizing the firm's short-term value at the expense of its long-term cash flows. As a result, an increase in revelatory price efficiency (i.e., an increase in either λ or ρ) can lead to a decrease in firm value, even when managerial effort is held constant.

Proposition 8. *Holding e fixed, when the manager invests to maximize price, firm value decreases in λ if and only if $\rho < K < \tilde{\rho}$ or $1 - \tilde{\rho} < K < 1 - \rho$.*

In the baseline model, an increase in either the quality of investors' information (ρ) or the fraction of informed investors (λ) increases investment efficiency. More information increases investment efficiency because the principal's investment rule is also efficient. However,

Figure 7: Optimal pay-for-performance versus ρ

The figure plots the optimal linear contract chosen by the principal as a function of ρ. Other parameters are: $\gamma = 1$, $\delta = 2$, $x_H = 1, x_L = 0.5$, $y_H = 1$.



(a) $K = 0.33$ (b) $K = 0.67$

Proposition 8 highlights this is no longer the case when the manager maximizes expected price. This reflects the fact that, as in other rational expectations models with a continuum of investors, the marginal investors' equilibrium expectation of cash flows (reflected in the price) need not coincide with expected cash flows conditional on the information in the price. As a result, an investment policy that maximizes the expected price need not maximize expected value. This wedge between price maximization and value maximization arises more generally (e.g., Albagli et al. (2021), Banerjee, Breon-Drish, and Smith (2021a)).

Despite this source of inefficiency, however, the price maximization investment rule may be desirable in our setting because of its impact on managerial effort. Note that the manager's effort and the offered contract are the same as in the baseline model. However, the variance in prices $\mathbb{V}[\tilde{\mu}(P)]$, which affects β and e in equilibrium, depends upon the investment policy. Figure 7 plots the optimal pay-for-performance component when the investment policy maximizes the terminal value (blue, solid line) and when it maximizes the price (red, dashed line). In the shaded region of panel (a), $K \in (1 - \tilde{\rho}, 1 - \rho)$, which implies that the manager always invests if he maximizes the firm's terminal value, but refrains from investing when $P = p_L$ when he maximizes the price. This reduction in investment lowers the unconditional variance of the price which allows the principal to choose a higher β in equilibrium. As a result, the effort chosen by the manager will also be higher in this region when he maximizes the price. In the shaded region of panel (b), $K \in (\rho, \tilde{\rho})$, which implies that the manager never invests if he maximizes the terminal value, but does invest when $P = p_H$

when he maximizes the price. This decision to invest increases the unconditional variance of the price which leads the principal to choose a lower β, which lowers managerial effort.

These results suggest that allowing the manager to determine the firm's investment policy when the project's unconditional net present value is high (i.e., when K is low) can be optimal even if the manager's preferred policy is ex-ante inefficient. For instance, suppose that the conditional value of the investment when the price is low ($P = p_L$) is close to zero, i.e., the difference between $1-\rho$ and K is arbitrarily small. If the manager maximizes the price, he will not invest in this project (since $1-\tilde{\rho} < K$). This has an arbitrarily small effect on the value of the investment project but, as panel (a) makes clear, can lead to a non-trivial increase in β and, hence, managerial effort. When the latter outweighs the former, it is optimal for the principal to delegate the investment decision to the manager. The following proposition formalizes this intuition.

Proposition 9. *If $K < 0.5$, there exists a $\bar{\rho} < 1-K$ such that the principal strictly prefers to delegate investment to the manager for all $\rho \in (\bar{\rho}, 1-K)$.*

5.5 Contracting on Price and Value

In this section, we allow the principal to offer a linear contract that depends on both the short-term (date-one) price and long-term value. Suppose the manager's compensation is

$$W(P,V) = \alpha + \beta\left(\pi P + (1-\pi) Z\right) \tag{47}$$

where $Z \equiv V - W$ denotes the firm's net cash flows and α, β, and π are chosen optimally by the principal. Given the terms of the principal's offered contract (α,β,π), the manager maximizes his expected utility over his compensation, $W(P,V)$, net of the cost of effort, that is, he solves

$$\max_e \mathbb{E}\left[\alpha + \beta\left(\pi P + (1-\pi) Z\right)\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta\left(\pi P + (1-\pi) Z\right)\right] - c(e), \tag{48}$$

As before, the manager optimally chooses his effort so that

$$\frac{\beta}{1+\beta} = c'(e),$$

since both the price and the firm's terminal value increase one-for-one with effort. The principal chooses the contract (α,β,π) that maximizes the (unconditional) expected firm value, net of the manager's compensation. Specifically, she now solves

$$\max_{\alpha,\beta,\pi} \mathbb{E}\left[x_\omega + e + \delta y_\omega I - \left(\alpha + \beta\left(\pi P + (1-\pi) Z\right)\right)\right], \qquad \text{subject to :} \tag{49}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{50}$$

$$\mathbb{E}\left[\alpha + \beta\left(\pi P + (1-\pi) Z\right)\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta\left(\pi P + (1-\pi) Z\right)\right] - c(e) \geq 0. \tag{51}$$

The following proposition provides a characterization of the optimal linear contract.

Proposition 10. *Suppose the financial equilibrium, (P, I), is given by Proposition 1 and the manager's optimal effort choice, e, is given by (50). Then the principal's optimal linear contract is given by (α, β, π), where*

$$\pi^* = 1 + \frac{cov\left(V - \tilde{\mu}(P), \tilde{\mu}(P)\right)}{(1+\beta^*)\mathbb{V}\left(V-\tilde{\mu}\right) + \beta^* cov\left(V-\tilde{\mu}, \tilde{\mu}\right)} < 1, \text{ and} \tag{52}$$

$$\beta = \frac{1}{c''(e)\gamma\left(\mathbb{V}\left[\tilde{\mu}(P)\right] - \frac{cov(V-\tilde{\mu}(P), \tilde{\mu}(P))^2}{\mathbb{V}(V-\tilde{\mu}(P))}\right)}. \tag{53}$$

Since the manager's participation constraint is always binding, the principal's objective simplifies to

$$\max_{\beta,\pi} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \left(\frac{\beta^2\gamma}{2}\mathbb{V}\left[\pi P + (1-\pi) Z\right] + c(e)\right). \tag{54}$$

Moreover, as before, the manager's choice of effort depends upon β, only. As a result, the optimal π is chosen to minimize

$$\mathbb{V}\left[\pi P + (1-\pi) Z\right] \tag{55}$$

which, it is straightforward to show, yields equation (52). Given this choice of π, the optimal level of β is then given by equation (53). In this setting, the manager's contract is more sensitive to her effort relative to the baseline model (i.e, the optimal β is always higher, see equation (37)). This increase in β, and the resultant increase in managerial effort and firm value, arises because $\mathbb{V}[P] \geq \mathbb{V}\left[\pi P + (1-\pi) Z\right]$ at the optimal π. Intuitively, this is because $\text{cov}\left(V - \tilde{\mu}(P), \tilde{\mu}(P)\right) < 0$, and this implies $\pi < 1$: the manager always receives a positive loading on both price and value. This arises in our model because prices exhibit reversal, a feature which arises across a wide range of rational expectations equilibria models (e.g., Hellwig (1980)) and which is empirically relevant over the longer horizons over which investment payoffs are realized (e.g., De Bondt and Thaler (1985)). As a result, putting positive weights on the price and the terminal cash flow provides diversification, lowering the total risk borne by the manager. In contrast, if the price were set by a risk-neutral market maker (as in Vives (1995)), π would be one in our setting. That is, the principal would only contract on the price since $\text{cov}\left(V - \tilde{\mu}\left(P\right), \tilde{\mu}\left(P\right)\right) = 0$. This is because the price is a less noisy signal of the manager's effort relative to the terminal value.[24]

In Figure 8, we plot the optimal π and a scaled measure of performance-sensitivity, $\frac{\beta}{1+\beta}$, as a function of the fraction of informed investors, λ, and the relative importance of investment opportunities, δ. When the measure of informed investors increases, the volatility of both the price and the realized cash flow increases. As panels (a) and (b) illustrate, this leads to lower-powered incentives, i.e., β falls with λ, as in our benchmark model.

The impact on the weight the principal places on the price, however, may be non-monotonic. Recall that when the investment threshold, K, is lower than 1/2 (panel (a)), the principal invests in the project as long as the price is not $P = p_L$. In this case, the relative weight on the price, π, decreases with λ. On the other hand, when the investment threshold is higher than 1/2 (panel (b)), the relative weight is U-shaped in λ: the principal optimally puts relatively more weight on short-term signals (the price) relative to long term signals (V) when the mass of informed investors is sufficiently low or sufficiently high. These results suggest a novel channel through which the manager's optimal short-term compensation varies as a function of investors' information.[25]

In panels (c) and (d), we plot the optimal π and β as a function of δ. As in the benchmark model, β decreases with δ since price and realized cash-flows become more volatile as the scale of the investment payoff grows. However, as δ increases, principal places relatively more weight on the long-term value, i.e., the duration of managerial compensation increases with the size of the investment opportunity. When δ is zero, the manager is paid utilizing only the short-term price which is less volatile. Intuitively, as δ grows, there is an increased benefit from hedging which leads the principal to utilize more long-term compensation. This is consistent with the results of Gopalan et al. (2014) who document that increased pay duration is associated with more growth opportunities and R&D intensity.

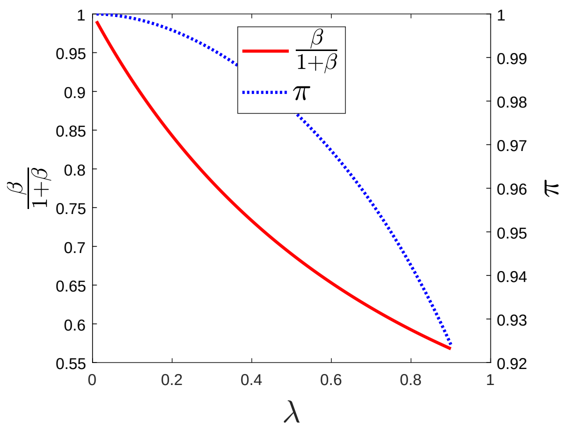
To the extent that λ is negatively related to liquidity, these results also speak to the literature on the relation between liquidity and stock-based compensation. Our results imply that the relative weight on short-term price compensation (i.e., π) increases with market liquidity (decreases with λ) when investment opportunities are ex-ante positive NPV (i.e., K is low), such compensation may decrease with market liquidity when investment opportunities

[24] If $\text{cov}\left(V - P, P\right) \geq 0$, conditioning on the terminal value adds volatility (since investors only observe a noisy signal of the true state, ω) and provides no additional incentive to exert effort.

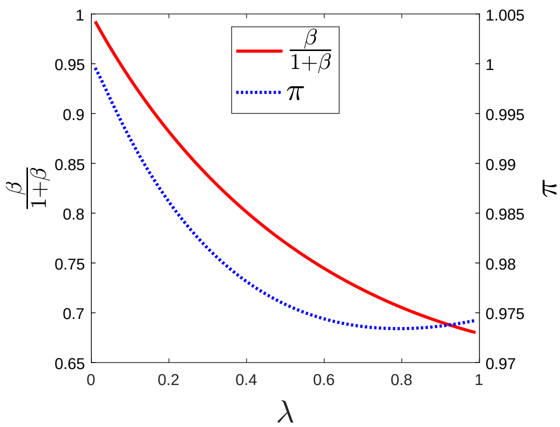
[25] For instance, in Peng and Röell (2014), changes in short-term compensation reflect changes in the manager's ability to manipulate the firm's value.

Figure 8: Optimal pay-for-performance versus λ and δ

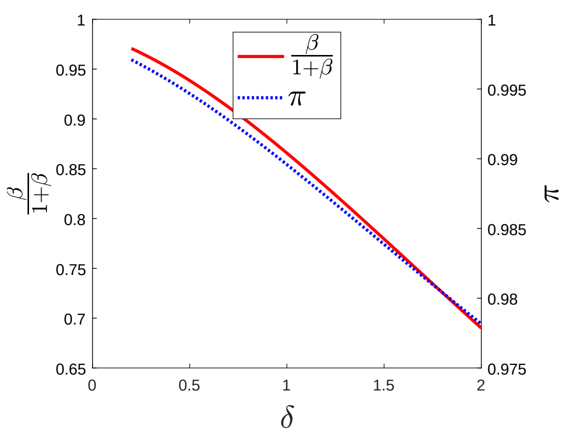
The figure plots the optimal linear contract chosen (β, π) by the principal as a function of λ and δ. The cost of effort is $\frac{e^2}{2}$. Other parameters are: $\gamma = 1$, $x_H = 1, x_L = 0.5$, $y_H = 1$, $\rho = 0.7$.



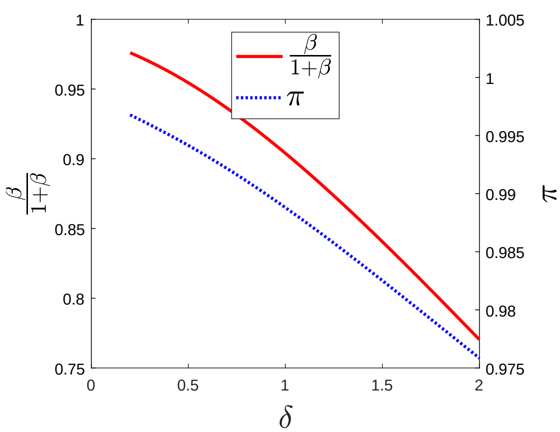
(a) $K = 0.33$, $\delta = 2$



(b) $K = 0.67$, $\delta = 2$



(c) $K = 0.33$, $\lambda = 0.5$



(d) $K = 0.67$, $\lambda = 0.5$

are ex-ante negative NPV. This suggests that the impact of market liquidity on stock-based compensation and market liquidity depends in part on the nature of investment opportunities available to firms and, as a result, should vary across industries and over market conditions.

6 Empirical Predictions

In this section, we summarize a number of novel, testable predictions based on our analysis above. The main predictions highlight the interaction between the key variables of the model: incremental price informativeness about investment opportunities, managerial compensation and firm value.

Testing the model's predictions requires a proxy for the relative importance of feedback effects, or δ, for a given firm. While we are not aware of any agreed-upon proxies for this measure, we build on the existing literature to propose some potential approaches. For example, broadly speaking, the feedback effect is likely to be more important for firms for which price sensitivity to investment is higher (e.g., Chen et al. (2007)). This recommends the use of the investment-price sensitivity as a potential proxy for δ.[26] Edmans, Jayaraman, and Schneemeier (2017) argue that the feedback effect is weaker for firms with greater incidence of insider trading, suggesting that all else equal, insider trading intensity and δ are negatively related. As recently demonstrated by Goldstein et al. (2021b), a particularly promising approach may be to use surveys to directly solicit managers' views of how important the feedback effect is for their firm. Finally, market-to-book ratio, R&D investment, and intangible assets, are likely to be higher for firms in which future investment opportunities play an important role (i.e., δ is high) — however, one must be careful since each of these is also affected by other firm characteristics.

Given these potential proxies for δ, our model predicts that an increase in investors' private information (e.g., as measured by price non-synchronicity, PIN, or other measures) is positively related to firm value for those firms where the feedback effect is most important (Proposition 4). On the other hand, when the feedback effect is absent or dampened (e.g., overconfident managers or high incidence of insider trading), more informative prices should be associated with lower firm value. One potential proxy for the quality of investor information for this feedback channel is index inclusion – as Billett, Diep-Nguyen, and Garfinkel (2020) argue, index inclusion leads to a decline in investment price sensitivity. Moreover, Banerjee, Huang, Nanda, and Xiao (2021b) show that over-confident managers are more

[26] We acknowledge, however, that since the investment-price sensitivity depends not only on the relative importance of feedback effects, but also on the informativeness of prices, liquidity and other (deeper) parameters, it provides a noisy measure.

likely to ignore information reflected in stock prices. Our model predicts that, in contrast to rational managers, more investor information should always lead to a decrease in firm value for such managers.[27]

Our model predicts that firms for which the feedback effect is relatively less important should have higher pay-for-performance sensitivity for compensation (Proposition 3). This suggests that, all else equal, firms with lower investment-price sensitivity should be associated with higher powered executive compensation contracts. Similarly, an increase in investors' information should lead to a lower pay-for-performance sensitivity for executives, all else equal (Proposition 3). Lin et al. (2019) provide evidence for this prediction using a natural experiment, namely, the introduction of the Regulation SHO Pilot Program.[28] They show that pay-for-performance decreases and price sensitivity of investment increases for such firms, consistent with the prediction of our model.[29]

A large literature has focused on the impact of stock market liquidity on firm performance through its effect on price informativeness. In our model, stock price liquidity is inversely related to price informativeness. As such, our model predicts that higher liquidity is associated with higher pay-for-performance sensitivity, consistent with both Fang et al. (2009) and Jayaraman and Milbourn (2012). On the other hand, our model predicts that higher liquidity should be associated with lower revelatory price efficiency. In this vein, Fang, Tian, and Tice (2014) show that an increase in liquidity is associated with less innovation.

The results of Section 5.2 emphasize one must be careful in interpreting existing empirical measures of real efficiency. For instance, in the absence of contracting considerations, Bai et al. (2016) interpret any increase in revelatory price efficiency as necessarily associated with an increase in aggregate efficiency (expected firm value). However, Proposition 5 implies that when contracting is sufficiently important (δ is low), aggregate real efficiency is *negatively* related to revelatory price efficiency in equilibrium. Similarly, while Fang et al. (2009) demonstrate how an increase in contract efficiency can increase firm value, our results caution against a universal application of this result. Specifically, if the cause of the increase in contracting efficiency comes at the expense of revelatory price efficiency, firm value may fall instead.

To the extent that firms influence transparency, e.g., by changing the clarity of their financial reporting, our results in Section 5.3 predict that the level of transparency should

[27] This is because the benefit of more informative prices does not accrue when the manager ignores the price: instead, the only effect is the decline in contracting price efficiency and, hence, firm value.

[28] The Reg SHO program removed short-sale restrictions for randomly selected (pilot) firms from May 2005 to August 2007, which lowered trading frictions and improved price informativeness.

[29] One could use additional shocks to price informativeness, such as a decline in analyst coverage due to brokerage mergers or closures (e.g., Hong and Kacperczyk (2010), Kelly and Ljungqvist (2012)).

predictably vary with both firm and investor characteristics (see Li (2008); You and Zhang (2009); Miller (2010)). In particular, firms should be more opaque when price-based managerial compensation plays an important role, even when feedback from market prices can improve the firm's investment decisions. This is consistent with empirical evidence that documents hard-to-value and opaque firms tend to use more aggressive stock-based pay for their managers (Bebchuk and Fried (2004)).

We examine the impact of allowing the manager to make the investment decision in Section 5.4. Our model predicts that, relative to the setting in which the principal (e.g., the board of directors) is responsible for the investment decision, there should be higher (lower) pay-for-performance when the firm has access to better (worse) investment opportunities. Moreover, we expect to see voluntary delegation of investment by the principal when the investment is ex-ante more valuable.

Finally, in Section 5.5, we show how price informativeness and the strength of the feedback effect change the optimal mix of short- and long-term compensation. Our model implies that the duration of executive compensation increases with δ. Gopalan et al. (2014) analyze the cross-sectional determinants of pay duration and find that pay duration is longer in firms with high market to-book ratios and in more R&D intensive firms (high δ firms), consistent with our model. Finally, our model also implies that the relationship between pay duration and price informativeness depends upon the ex-ante value of the investment project and can even be non-monotonic when the project is ex-ante less likely to be undertaken.

7 Conclusions

In this paper, we developed a model which highlights the dual role of a firm's stock price. First, it provides the principal a contractable signal about managerial effort, and so is used for managerial compensation. Second, it aggregates investor information about a new project and so affects real investment via feedback effects. In our model, these roles are at odds: when the price is more informative about future investment opportunities, it is more volatile and, therefore, less effective for incentive provision. Our analysis highlights why it is critical for researchers to account for the relative contribution of new investments and managerial effort when determining the impact of investor information on both firm value and its policies.

We show that, as a result, firm value and social surplus can decrease with price informativeness and existing measures of price efficiency (including forecasting price efficiency and revelatory price efficiency) are incomplete without accounting for the impact of this information on incentive provision. Moreover, we show how the optimal level of transparency and firm investment policy depend critically upon the relative importance of each role. This analysis also leads to novel implications for how the composition and price-sensitivity of managerial compensation depends on price informativeness.

Our model is stylized for tractability and clarity of exposition, but naturally lends itself to further study. For instance, the dual role of stock prices suggests a natural role for the joint choice of project (as in Davis and Gondhi (2021)) and the types of projects on which the manager exerts effort. Our analysis also suggests that the dual role of the stock price should influence the manager's incentive to acquire, and publicly disclose, information about assets in place and future investment opportunities. We leave this analysis for future research.

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A Proofs

A.1 Proof of Proposition 1

First, we establish the market-clearing price as a function of u and ω. In equations (16) and (17), we establish the thresholds for $P = p_L$. We now establish analogous thresholds for when the price is p_u and p_H.

If $\omega = H$, then $P = p_H$ as long as

$$u < \underbrace{\lambda\rho}_{\text{observed } s_i = s_H} - \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_L} = 2\lambda\rho - 1, \tag{56}$$

while if $\omega = L$, then $P = p_H$ as long as

$$u < \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_H} - \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(\rho)}_{\text{observed } s_i = s_L} = 2\lambda(1-\rho) - 1. \tag{57}$$

As a result, if $\omega = H$, then $P = p_u$ as long as

$$2\lambda\rho - 1 \leq u \leq 1 - 2\lambda(1-\rho), \tag{58}$$

while if $\omega = L$, then $P = p_u$ as long as

$$2\lambda(1-\rho) - 1 \leq u \leq 1 - 2\lambda\rho. \tag{59}$$

Note, then, that if we define $u_H \equiv \lambda(2\rho - 1)$, $u_L = \lambda(1 - 2\rho)$, the thresholds above correspond to those found in the proposition.

With these market-clearing conditions, we confirm the signal that both investors and the principal can extract from the price and, in so doing, confirm the beliefs of the marginal investor at each price level. Specifically,

$$\begin{aligned}
\mathbb{P}[\omega = H|p_h] &= \frac{\mathbb{P}[\omega = H \cap P = p_H]}{\mathbb{P}[P = p_h]} = \frac{\lambda\rho}{\lambda\rho + \lambda(1-\rho)} = \rho \\
\mathbb{P}[\omega = H|p_l] &= \frac{\mathbb{P}[\omega = H \cap P = p_l]}{\mathbb{P}[P = p_l]} = \frac{\lambda(1-\rho)}{\lambda\rho + \lambda(1-\rho)} = 1 - \rho \\
\mathbb{P}[\omega = H|p_u] &= \frac{\mathbb{P}[\omega = H \cap P = p_u]}{\mathbb{P}[P = p_u]} = \frac{1-\lambda}{1-\lambda+1-\lambda} = \tfrac{1}{2}.
\end{aligned}$$

Note that, given these beliefs, the optimal investment rule, (6), yields the investment rule specified in the proposition.

Finally, in order for the conjectured equilibrium to exist, the equilibrium price levels must be distinct. To do so, we show that it is always the case that $p_H > p_u > p_L$. If the principal's investment decision is the same across any two prices, then this ordering holds trivially: V_H and V_L are the same across the two prices (given the investment policy) while $\tilde{\rho} > \frac{1}{2} > 1 - \tilde{\rho}$. Suppose instead that the principal's investment decision differs across two adjacent prices. There are two cases to consider.

(1) Suppose that the principal only invests if she observes $P = p_H$. Then, given the logic above, $p_u > p_L$ and it remains to be shown that $p_H > p_u$. Note that

$$\begin{aligned}
p_H &= \frac{1}{1+\beta}\left[(1-\tilde{\rho})\, V_L(p_H) + \tilde{\rho} V_H(p_H) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[(1-\rho)\, V_L(p_H) + \rho V_H(p_H) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[(1-\rho)\, V_L(p_u) + \rho V_H(p_u) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[\tfrac{1}{2} V_L(p_u) + \tfrac{1}{2} V_H(p_u) - \alpha\right] \\
&= p_u,
\end{aligned}$$

where the first and third inequalities follows from $\tilde{\rho} > \rho > \frac{1}{2}$, while the second inequality follows from the fact that if the principal invests, it increases the value of the firm's expected cash flows.

(2) Suppose that the principal only invests if she observes $P \in \{p_H, p_u\}$. Then, given the logic above, $p_H > p_u$ and it remains to be shown that $p_u > p_L$. Note that

$$\begin{aligned}
p_u &= \frac{1}{1+\beta}\left[\tfrac{1}{2} V_L(p_u) + \tfrac{1}{2} V_H(p_u) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[\tfrac{1}{2} V_L(p_L) + \tfrac{1}{2} V_H(p_L) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[\tilde{\rho} V_L(p_L) + (1-\tilde{\rho})\, V_H(p_L) - \alpha\right] \\
&= p_L,
\end{aligned}$$

where the second inequality follows from $\frac{1}{2} > 1-\tilde{\rho}$, while the first inequality follows from the fact that if the principal invests, it increases the value of the firm's expected cash flows. □

A.2 Proof of Proposition 2

Principal's objective is

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - c(e) \text{ subject to } \frac{\beta}{1+\beta} = c'(e)\,. \quad (60)$$

The FOC is

$$\frac{\partial e}{\partial \beta}\left(1 - c'(e)\right) - \gamma \frac{\beta}{(1+\beta)^3} \mathbb{V}\left[\tilde{\mu}(P)\right] = 0$$

which implies

$$\frac{1}{(1+\beta)^2}\left(\frac{1-c'(e)}{c''(e)}\right) - \gamma \frac{\beta}{(1+\beta)^3} \mathbb{V}\left[\tilde{\mu}(P)\right] = 0$$

which in turn implies

$$\beta = \frac{1}{\gamma c''(e)\,\mathbb{V}\left[\tilde{\mu}(P)\right]}.$$

Let $B = \frac{\beta}{1+\beta}$. The second order condition is

$$\frac{\partial^2 e}{\partial B^2}\left(1 - c'(e)\right) - \frac{\partial e}{\partial B}\frac{\partial e}{\partial B}c''(e) - \gamma\mathbb{V}\left[\tilde{\mu}(P)\right]$$

Note that

$$B = c'(e) \implies 1 = c''(e)\frac{\partial e}{\partial B} \implies 0 = c'''(e)\frac{\partial e}{\partial B}\frac{\partial e}{\partial B} + c''(e)\frac{\partial^2 e}{\partial B^2} = 0.$$

Substituting these into SOC, we get

$$-\frac{c'''(e)\frac{\partial e}{\partial B}\frac{\partial e}{\partial B}}{c''(e)}\left(1 - c'(e)\right) - \frac{1}{c''(e)} - \gamma\mathbb{V}\left[\tilde{\mu}(P)\right]$$

which simplifies to

$$-\frac{c'''(e)}{\left[c''(e)\right]^2}c'(e)\left(1 - c'(e)\right) - 1.$$

For the SOC to hold, we need

$$\frac{c'''(e)}{\left[c''(e)\right]^2}c'(e)\left(1 - c'(e)\right) + 1 > 0$$

□

A.3 Proof of Proposition 3

Note that

$$\beta = \frac{1}{\gamma c''(e)\,\mathbb{V}\left[\tilde{\mu}(P)\right]}$$

which implies that

$$\frac{\partial\beta}{\partial\lambda} = -\frac{1}{\gamma c''(e)\left(\mathbb{V}\left[\tilde{\mu}(P)\right]\right)^2}\frac{\partial\mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial\lambda}$$
$$\frac{\partial\beta}{\partial\rho} = -\frac{1}{\gamma c''(e)\left(\mathbb{V}\left[\tilde{\mu}(P)\right]\right)^2}\frac{\partial\mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial\rho}.$$
$$\frac{\partial\beta}{\partial\delta} = -\frac{1}{\gamma c''(e)\left(\mathbb{V}\left[\tilde{\mu}(P)\right]\right)^2}\frac{\partial\mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial\delta}.$$

$$\mathbb{V}\left[\tilde{\mu}(P)\right] = \frac{\lambda}{2}\left(\tilde{\mu}(p_H)^2 + \tilde{\mu}(p_L)^2\right) + (1-\lambda)\,\tilde{\mu}(p_u)^2 - \left[\frac{\lambda}{2}\left(\tilde{\mu}(p_H) + \tilde{\mu}(p_L)\right) + (1-\lambda)\,\tilde{\mu}(p_u)\right]^2 \tag{61}$$

which implies

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \lambda} = \left(\tilde{\mu}\left(p_H\right) - \tilde{\mu}\left(p_u\right)\right)\left(\tilde{\mu}\left(p_u\right) - \tilde{\mu}\left(p_L\right)\right) + \frac{1-\lambda}{2}\left(\tilde{\mu}\left(p_H\right) + \tilde{\mu}\left(p_L\right) - 2\tilde{\mu}\left(p_u\right)\right)^2 > 0.$$

Similarly, since $\frac{\partial \tilde{\mu}(p_L)}{\partial \rho} < 0$ and $\tilde{\mu}\left(p_L\right) < \mathbb{E}\left[\tilde{\mu}\left(P\right)\right]$, then

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \rho} = \lambda\left[\frac{\partial \tilde{\mu}\left(p_H\right)}{\partial \rho}\left(\tilde{\mu}\left(p_H\right) - \mathbb{E}\left[\tilde{\mu}\left(P\right)\right]\right) + \frac{\partial \tilde{\mu}\left(p_L\right)}{\partial \rho}\left(\tilde{\mu}\left(p_L\right) - \mathbb{E}\left[\tilde{\mu}\left(P\right)\right]\right)\right] > 0 \tag{62}$$

Moreover,

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \delta} > 0 \tag{63}$$

This implies that

$$\frac{\partial \beta}{\partial \lambda} < 0 \qquad \frac{\partial \beta}{\partial \rho} < 0 \qquad \frac{\partial \beta}{\partial \delta} < 0$$

Note that effort solves

$$\frac{\beta}{1+\beta} = c'\left(e\right)$$

which implies

$$\frac{1}{\left(1+\beta\right)^2}\frac{\partial \beta}{\partial \delta} = c''\left(e\right)\frac{\partial e}{\partial \delta}$$
$$\frac{1}{\left(1+\beta\right)^2}\frac{\partial \beta}{\partial \lambda} = c''\left(e\right)\frac{\partial e}{\partial \lambda}$$
$$\frac{1}{\left(1+\beta\right)^2}\frac{\partial \beta}{\partial \rho} = c''\left(e\right)\frac{\partial e}{\partial \rho}$$

which implies that effort decreases with δ, λ and ρ. □

A.4 Proof of Proposition 4

Let $B = \frac{\beta}{1+\beta}$. Note that B and e solves

$$B = c'\left(e\right) \qquad B = \frac{1}{1 + \gamma c''\left(e\right)\mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}$$

Differentiating with respect to λ, we get

$$c''\left(e\right)\frac{\partial e}{\partial \lambda} = \frac{\partial B}{\partial \lambda} \qquad \frac{\partial B}{\partial \lambda} = -\gamma B^2\left[c''\left(e\right)\frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda} + \mathbb{V}\left[\tilde{\mu}\right]c'''\left(e\right)\frac{\partial e}{\partial \lambda}\right]$$

This implies that

$$\frac{\partial e}{\partial \lambda} = -\gamma B^2\left[\frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda} + \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''\left(e\right)}{c''\left(e\right)}\frac{\partial e}{\partial \lambda}\right]$$

$$\frac{\partial e}{\partial \lambda}\left(\underbrace{1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''\left(e\right)}{c''\left(e\right)}}_{>0}\right) = -\gamma B^2 \frac{\partial \mathbb{V}\left[P\right]}{\partial \lambda}$$

The term in the underlying brace in the above equation is

$$1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''\left(e\right)}{c''\left(e\right)} = 1+c'\left(e\right)\left(1-c'\left(e\right)\right)\frac{c'''\left(e\right)}{\left[c''\left(e\right)\right]^2} > 0$$

because of the second order condition. This implies

$$\frac{\partial e}{\partial \lambda} = -\frac{\gamma B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}}{1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)}} < 0. \tag{64}$$

Note that firm value is given by

$$\frac{\partial FV}{\partial \lambda} = \frac{\partial e}{\partial \lambda} + \delta \frac{\partial E\left[\mathbb{I}_{d_m = I} y_\omega\right]}{\partial \lambda}$$

If $\rho < K$ or $1-\rho > K$, then firm value is decreasing in λ. For the rest of the proof, assume that $\rho > K > 1-\rho$. Differentiating equation 64 , we get

$$\frac{\partial^2 e}{\partial \lambda^2} = -\frac{\gamma B^2 \left(1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)}\right)\frac{\partial^2 \mathbb{V}[\tilde{\mu}]}{\partial \lambda^2} + 2\gamma B \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}\frac{\partial B}{\partial \lambda} - \gamma^2 B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}\left(B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}\frac{c'''(e)}{c''(e)}\right)}{\left(1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)}\right)^2}$$

Note that $\frac{\partial^2 \mathbb{V}[\tilde{\mu}]}{\partial \lambda^2} = -\frac{1}{2}\left(p_H + p_L - 2p_u\right)^2 \leq 0$. This implies

$$\frac{\partial^2 e}{\partial \lambda^2} = \gamma B^2 \left(1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''\left(e\right)}{c''\left(e\right)}\right)\frac{-\partial^2 \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda^2} + 2\gamma B c''\left(e\right)\frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}\frac{-\partial e}{\partial \lambda} + \gamma^2 B^4 \frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}\left(\mathbb{V}\left[\tilde{\mu}\right]\frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + \frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}\frac{c'''\left(e\right)}{c''\left(e\right)}\right)$$

The first two terms are positive. So, the function is convex iff the third term is also positive

$$\mathbb{V}\left[\tilde{\mu}\right]\frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + \frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}\frac{c'''\left(e\right)}{c''\left(e\right)} > 0. \tag{65}$$

If the above condition is true, effort is convex in λ, which implies that firm value is also convex in λ. This implies that there are only three possible shapes for FV: increasing, decreasing, U shaped. There exists $\bar{\delta} > \underline{\delta} > 0$ such that, for $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$, FV is U shaped in λ. If $\delta \leq \underline{\delta}$, FV is decreasing in λ. If $\delta > \bar{\delta}$, FV is increasing in λ. □

A.5 Proof of Proposition 5

We begin by characterizing how the efficiency measures depend on the underlying parameters, given the financial market equilibrium and the optimal contract.

Lemma 1. *Consider the unique financial market equilibrium described in proposition 1 and the optimal contract described in proposition 2. Then,*

(i) Revelatory price efficiency is

$$RPE = \lambda \left(2\rho - 1\right)^2$$

(ii) Forecasting price efficieny is

$$FPE^{-1} = \begin{cases} \left(x_H - x_L + \delta y_H - \delta y_L\right)^2 \left[\frac{1-\lambda(2\rho-1)^2}{4}\right] & \text{if } 1-\rho > K \\ \left(x_H - x_L + \delta y_H - \delta y_L\right)^2 \left(\frac{\rho(1-\rho)\lambda}{2} + \frac{1-\lambda}{4}\right) + \left(x_H - x_L\right)^2 \frac{\rho(1-\rho)\lambda}{2} & \text{if } \frac{1}{2} > K > 1-\rho \\ \left(x_H - x_L + \delta y_H - \delta y_L\right)^2 \frac{\rho(1-\rho)\lambda}{2} + \left(x_H - x_L\right)^2 \left(\frac{\rho(1-\rho)\lambda}{2} + \frac{1-\lambda}{4}\right) & \text{if } \rho > K > \frac{1}{2}, \\ \left(x_H - x_L\right)^2 \left[\frac{1-\lambda(2\rho-1)^2}{4}\right] & \text{if } \rho < K \end{cases} \tag{66}$$

(iii) Contracting price efficiency is

$$CPE = \left[\frac{\lambda}{2}\left(\tilde{\mu}\left(p_H\right)^2 + \tilde{\mu}\left(p_L\right)^2\right) + (1-\lambda)\,\tilde{\mu}\left(p_U\right)^2 - \left(\frac{\lambda}{2}\left(\tilde{\mu}\left(p_H\right) + \tilde{\mu}\left(p_L\right)\right) + (1-\lambda)\,\tilde{\mu}\left(p_U\right)\right)^2\right]^{-1}$$

(iv) Firm value is

$$FV = \frac{x_H + x_L}{2} + e + \begin{cases} \delta \frac{y_H + y_L}{2} & \text{if } 1-\rho > K \\ (1-\lambda)\,\delta \frac{y_H + y_L}{2} + \frac{\lambda}{2}\delta\left(\rho y_H + (1-\rho)\,y_L\right) & \text{if } \frac{1}{2} > K > 1-\rho \\ \frac{\lambda}{2}\delta\left(\rho y_H + (1-\rho)\,y_L\right) & \text{if } \rho > K > \frac{1}{2}, \\ 0 & \text{if } \rho < K \end{cases}$$

where $\tilde{\mu}(p_H)$,$\tilde{\mu}(p_U)$ *and* $\tilde{\mu}(p_L)$ *are defined in proposition (1).*

Note that

$$\begin{aligned} RPE &= \frac{V\left[y_\omega\right] - V\left[y_\omega | p\right]}{V\left[y_\omega\right]} \\ &= \frac{\frac{(y_H - y_L)^2}{4} - \left[\frac{(y_H - y_L)^2}{4}\left(1-\lambda\right) + \lambda\rho\left(1-\rho\right)\left(y_H - y_L\right)^2\right]}{\frac{(y_H - y_L)^2}{4}} \\ &= \lambda\left(2\rho - 1\right)^2 \end{aligned} \tag{67}$$

Moreover, by definition, $FPE^{-1} = \mathbb{V}\left(x_\omega + y_\omega \mathbb{I}_{d_m = I} | p\right)$ and this simplifies to 66. Contracting price efficiency is captured by

$$\begin{aligned} CPE &= \left[\mathbb{V}\left[\tilde{\mu}\left(p\right)|e\right]\right]^{-1} \\ &= \left[\frac{\lambda}{2}\left(\tilde{\mu}\left(p_H\right)^2 + \tilde{\mu}\left(p_L\right)^2\right) + (1-\lambda)\,\tilde{\mu}\left(p_U\right)^2 - \left(\frac{\lambda}{2}\left(\tilde{\mu}\left(p_H\right) + \tilde{\mu}\left(p_L\right)\right) + (1-\lambda)\,\tilde{\mu}\left(p_U\right)\right)^2\right]^{-1} \end{aligned}$$

□

A.6 Proof of Proposition 6

Recall that investors differ only in their belief about the relative likelihood of each state. This implies that

$$\mathbb{E}\left[d\left(\{s_i,P\}\right)\left(x_\omega+e+y_\omega I(P)-P\right)\right]=\left[\tfrac{\rho\lambda(1-\rho)}{2}+\tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i=s_H,p_H\right)-q\left(s_i=s_L,p_H\right)\right)\Delta V\left(p_H\right) \tag{68}$$

$$+\left[\tfrac{\rho\lambda(1-\rho)}{2}+\tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i=s_H,p_L\right)-q\left(s_i=s_L,p_L\right)\right)\Delta V\left(p_L\right) \tag{69}$$

$$+\left[\tfrac{(1-\lambda)\rho}{2}+\tfrac{(1-\lambda)(1-\rho)}{2}\right]\left(q\left(s_i=s_H,p_U\right)-q\left(p_U\right)\right)\Delta V\left(p_U\right) \tag{70}$$

$$+\left[\tfrac{(1-\lambda)\rho}{2}+\tfrac{(1-\lambda)(1-\rho)}{2}\right]\left(q\left(p_U\right)-q\left(s_i=s_L,p_U\right)\right)\Delta V\left(p_U\right) \tag{71}$$

while

$$\mathbb{E}\left[d\left(\{P\}\right)\left(x_\omega+e+y_\omega I(P)-P\right)\right]=\left[\tfrac{\rho\lambda}{2}+\tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(s_i=s_H,p_H\right)-q\left(p_H\right)\right)\Delta V\left(p_H\right) \tag{72}$$

$$+\left[\tfrac{\rho\lambda}{2}+\tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(p_L\right)-q\left(s_i=s_L,p_L\right)\right)\Delta V\left(p_L\right) \tag{73}$$

We can simplify by noting that

$$\left[\tfrac{\rho\lambda(1-\rho)}{2}+\tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i=s_H,p_H\right)-q\left(s_i=s_L,p_H\right)\right)=\rho\lambda\left(1-\rho\right)\left(\tilde{\rho}-\tfrac{1}{2}\right) \tag{74}$$

$$\left[\tfrac{\rho\lambda}{2}+\tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(s_i=s_H,p_H\right)-q\left(p_H\right)\right)=\tfrac{\lambda}{2}\left(\tilde{\rho}-\rho\right) \tag{75}$$

$$\left[\tfrac{\rho\lambda(1-\rho)}{2}+\tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i=s_H,p_L\right)-q\left(s_i=s_L,p_L\right)\right)=\rho\lambda\left(1-\rho\right)\left(\tfrac{1}{2}-\left(1-\tilde{\rho}\right)\right) \tag{76}$$

$$\left[\tfrac{\rho\lambda}{2}+\tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(p_L\right)-q\left(s_i=s_L,p_L\right)\right)=\tfrac{\lambda}{2}\left(\left(1-\rho\right)-\left(1-\tilde{\rho}\right)\right) \tag{77}$$

Substituting in the expression for $\tilde{\rho}$ reveals that all four expressions are equal to

$$\tfrac{\rho\lambda(1-\rho)}{2}\left(\tfrac{2\rho-1}{\rho^2+(1-\rho)^2}\right). \tag{78}$$

As a result, the indifference condition reduces to

$$\tfrac{1-\lambda}{2}\left(\rho-\tfrac{1}{2}\right)\Delta V\left(p_U\right)+\tfrac{1-\lambda}{2}\left(\tfrac{1}{2}-\left(1-\rho\right)\right)\Delta V\left(p_U\right)=c \tag{79}$$

and so in an interior equilibrium, the measure of informed investors is

$$\lambda=1-\tfrac{c}{\left(\rho-\frac{1}{2}\right)\Delta V(p_U)}. \tag{80}$$

$$=1-\tfrac{c}{\left(\rho-\frac{1}{2}\right)(x_H-x_L+I(p_U)(y_H-y_L))} \tag{81}$$

More generally,

$$\lambda=\max\left\{0,1-\tfrac{c}{\left(\rho-\frac{1}{2}\right)\Delta V(p_U)}\right\}. \tag{82}$$

Thus, in any setting, the measure of informed investors is increasing in ρ but does *not* depend upon e. $\square$

A.7 Proof of Proposition 7

Let $B = \frac{\beta}{1+\beta}$ and note that $c'(e) = B$ and $\beta = \frac{1}{c''(e)\gamma\mathbb{V}(\tilde{\mu}(P))}$, which implies $c''(e)\frac{\partial e}{\partial\lambda} = \frac{\partial B}{\partial\lambda}$ and

$$B = \frac{\beta}{1+\beta} = \frac{1}{1 + c''(e)\gamma\mathbb{V}(\tilde{\mu}(P))} \tag{83}$$

$$\Rightarrow \frac{\partial B}{\partial\lambda} = -B^2\gamma\left(c''(e)\frac{\partial}{\partial\lambda}\mathbb{V}(\tilde{\mu}(P)) + \mathbb{V}(\tilde{\mu}(P))\,c'''(e)\frac{\partial e}{\partial\lambda}\right). \tag{84}$$

This implies

$$\frac{\partial}{\partial\lambda}\left(e - c(e) - \frac{\gamma}{2}B^2\mathbb{V}(\tilde{\mu}(P))\right) = (1 - c'(e))\frac{\partial e}{\partial\lambda} - \gamma\mathbb{V}(\tilde{\mu}(P))\,B\frac{\partial B}{\partial\lambda} - \frac{\gamma}{2}B^2\frac{\partial}{\partial\lambda}\mathbb{V}(\tilde{\mu}(P)) \tag{85}$$

$$= (1 - B - \gamma B\mathbb{V}(\tilde{\mu}(P))\,c''(e))\frac{\partial e}{\partial\lambda} - \frac{\gamma}{2}B^2\frac{\partial}{\partial\lambda}\mathbb{V}(\tilde{\mu}(P)) \tag{86}$$

$$= (1 - B - \gamma\mathbb{B}\mathbb{V}(\tilde{\mu}(P))\,c''(e))\frac{\partial e}{\partial\lambda} + \frac{1}{2}\left(1 + \gamma B^2\mathbb{V}(\tilde{\mu}(P))\frac{c'''(e)}{c''(e)}\right)\frac{\partial e}{\partial\lambda} \tag{87}$$

$$= \left(\frac{1}{2} + \frac{\gamma}{2}B^2\mathbb{V}(\tilde{\mu}(P))\frac{c'''(e)}{c''(e)}\right)\frac{\partial e}{\partial\lambda} \tag{88}$$

$$= -\frac{\gamma}{2}B^2\frac{\partial}{\partial\lambda}\mathbb{V}(\tilde{\mu}(P)) < 0 \tag{89}$$

and so we can express the change in firm value as

$$\frac{\partial}{\partial c_0}FV = \frac{\partial}{\partial\lambda}\mathbb{E}[\delta y_\omega \times I]\frac{d\lambda}{dc_0} + \frac{\partial}{\partial\lambda}\left(e - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2\mathbb{V}(\tilde{\mu}(P)) - c(e)\right)\frac{d\lambda}{dc_0} \tag{90}$$

$$= \left(\delta\frac{\partial}{\partial\lambda}\mathbb{E}[y_\omega \times I] - \frac{\gamma}{2}B^2\frac{\partial}{\partial\lambda}\mathbb{V}(\tilde{\mu}(P))\right)\frac{d\lambda}{dc_0}, \tag{91}$$

Next, note that

$$\frac{\partial\mathbb{E}[\delta y_\omega \times I]}{\partial\lambda} = \begin{cases} 0 & \text{if } 1-\rho > K \\ \frac{\delta(y_H - y_L)}{2}(\rho - 1 + K) & \text{if } \frac{1}{2} > K > 1-\rho \\ \frac{\delta(y_H - y_L)}{2}(\rho - K) & \text{if } \rho > K > \frac{1}{2}, \\ 0 & \text{if } \rho < K \end{cases}$$

and

$$\frac{\partial\mathbb{V}[\tilde{\mu}(P)]}{\partial\lambda} = \frac{1}{2}\left(\mu_H^2 + \mu_L^2 - 2\mu_U^2\right) - \mu_U(\mu_H + \mu_L) - \frac{1}{2}\lambda(\mu_H + \mu_L - 2\mu_U)^2 \tag{92}$$

$$= \frac{1}{2}\left((\mu_H - \mu_U)^2 + (\mu_L - \mu_U)^2\right) - \frac{1}{2}\lambda(\mu_H + \mu_L - 2\mu_U)^2 \tag{93}$$

$$\geq (\mu_H - \mu_U)(\mu_U - \mu_L) > 0 \tag{94}$$

So that

$$\frac{\partial}{\partial c_0}FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial\lambda}\mathbb{E}\left[\delta y_\omega \times I\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} (\mu_H-\mu_U)^2+(\mu_L-\mu_U)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{95}$$

Next, note that if $1-\rho > K$ or $\frac{1}{2} > K > 1-\rho$:

$$\lim_{\delta\to 0}\lambda = 1 - \frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(x_H-x_L\right)}, \tag{96}$$

$$\lim_{\delta\to\infty}\lambda = 1 \tag{97}$$

$$\lim_{\delta\to\infty}\left(1-\lambda\right)\delta = \lim_{\delta\to\infty}\frac{c_0\delta}{\left(\rho-\frac{1}{2}\right)\left(x_H-x_L+\delta\left(y_H-y_L\right)\right)} \tag{98}$$

$$= \frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(y_H-y_L\right)} \tag{99}$$

and if $\rho > K > \frac{1}{2}$ or $\rho < K$, then

$$\lambda = 1 - \frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(x_H-x_L\right)}. \tag{100}$$

We have to consider four cases:

(1) If $1-\rho > K$, then

$$\frac{\partial}{\partial c_0}FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c} (\mu_H-\mu_U)^2+(\mu_L-\mu_U)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right)\right), \tag{101}$$

which is positive.

(2) If $\frac{1}{2} > K > 1-\rho$, then

$$\frac{\partial}{\partial c_0}FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial\lambda}\mathbb{E}\left[\delta y_\omega \times I\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} (\mu_H-\mu_U)^2+(\mu_L-\mu_U)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{102}$$

$$\Rightarrow \lim_{\delta\to\infty}\frac{\partial}{\partial c_0}FV = \lim_{\delta\to\infty} -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial\lambda}\mathbb{E}\left[\delta y_\omega \times I\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} (\mu_H-\mu_U)^2+(\mu_L-\mu_U)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{103}$$

$$= -\frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(y_H-y_L\right)}\mathbb{E}\left[y_\omega \times I\right] - 1 < 0 \tag{104}$$

On the other hand, as delta shrinks, the following is positive:

$$\lim_{\delta\to 0}\frac{\partial}{\partial c_0}FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c} (\mu_H-\mu_U)^2+(\mu_L-\mu_U)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right)\right). \tag{105}$$

(3) If $\rho > K > \frac{1}{2}$, then $\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)}$, and

$$\frac{\partial}{\partial c_0} FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial \lambda}\mathbb{E}\left[\delta y_\omega \times I\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2 \\ -\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{106}$$

$$\Rightarrow \lim_{\delta \to 0} \frac{\partial}{\partial c_0} FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2 \\ -\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \end{array}\right)\right), \tag{107}$$

which is positive.

(4) If $\rho < K$, then $\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)}$ and following analogous steps to Case 1 shows that FV increases in c_0: the only impact of increased transparency is the reduction in managerial effort since the firm never invests in this case. □

A.8 Proof of Proposition 8

Note that firm value (ignoring assets in place) is given by

$$\begin{aligned} FV &= E\left[e + \delta y_\omega \times I\right] \\ &= \begin{cases} e + \delta \frac{y_H + y_L}{2} & \text{if } 1 - \tilde{\rho} > K \\ e + (1-\lambda)\delta \frac{y_H + y_L}{2} + \frac{\lambda}{2}\delta\left(\rho y_H + (1-\rho)y_L\right) & \text{if } \frac{1}{2} > K > 1 - \tilde{\rho} \\ e + \frac{\lambda}{2}\delta\left(\rho y_H + (1-\rho)y_L\right) & \text{if } \tilde{\rho} > K > \frac{1}{2}, \\ e & \text{if } \tilde{\rho} < K \end{cases} \end{aligned}$$

Note that,

$$\frac{\partial FV}{\partial \lambda} = \begin{cases} \frac{\partial e}{\partial \lambda} & \text{if } 1 - \tilde{\rho} > K \\ \frac{\partial e}{\partial \lambda} + \delta \frac{(y_H - y_L)}{2}(\rho - 1 + K) & \text{if } \frac{1}{2} > K > 1 - \tilde{\rho} \\ \frac{\partial e}{\partial \lambda} + \delta \frac{(y_H - y_L)}{2}(\rho - K) & \text{if } \tilde{\rho} > K > \frac{1}{2}, \\ \frac{\partial e}{\partial \lambda} & \text{if } \tilde{\rho} < K \end{cases}$$

In the first and fourth cases, firm value decreases with λ since effort decreases with λ. Next consider the second case. As λ increases, the second term is positive only when $K > 1 - \rho$. In the region where $1 - \hat{\rho} < K < 1 - \rho$, the second term is negative. High λ can be bad for firm value in this region. Finally, consider the third case. As λ increases, the second term is positive only when $\rho > K$. In the region where $\rho < K < \hat{\rho}$, the second term is negative. Again, high λ can be bad for firm value in this region. □

A.9 Proof of Proposition 9

Let the principal's utility given the optimal linear contract be denoted by

$$U_p = \underbrace{e - \frac{\gamma}{2}B^2\mathbb{V}\left[\tilde{\mu}(P)\right] - c\left(e\right)}_{\equiv U_{p,e}} + \delta\mathbb{E}\left[y_\omega \times I\right]. \tag{108}$$

U_p does not depend upon delegation unless the manager's investment decision differs from her own; this occurs when $\rho < K < \hat{\rho}$ and $1-\hat{\rho} < K < 1-\rho$.

First, note that in either case, the manager's investment decision is inefficient (ex-ante and as a result, $\delta\mathbb{E}[y_\omega \times I]$ is lower under delegation. However, by the envelope theorem, $\frac{\partial U_{p,e}}{\partial \mathbb{V}[\tilde{\mu}(P)]} = -\frac{\gamma}{2}B^2 < 0$: if delegation reduces the variance of $\tilde{\mu}(P)$, then the principal can be better off letting the manager invest.

If $\rho < K < \hat{\rho}$, the manager invests when the principal would not (when $P = p_H$) which increases $\tilde{\mu}(p_H)$. Since

$$\frac{\partial \mathbb{V}[\tilde{\mu}(P)]}{\partial \tilde{\mu}(p_H)} = \lambda\left(\tilde{\mu}(p_H) - \mathbb{E}[\tilde{\mu}(P)]\right) > 0, \tag{109}$$

this implies that the principal is necessarily worse off in this region. On the other hand, if $1-\hat{\rho} < K < 1-\rho$, the manager does not invest when $P = p_L$ which increases $\tilde{\mu}(p_L)$. But note that

$$\frac{\partial \mathbb{V}[\tilde{\mu}(P)]}{\partial \tilde{\mu}(p_L)} = \lambda\left(\tilde{\mu}(p_L) - \mathbb{E}[\tilde{\mu}(P)]\right) < 0, \tag{110}$$

which implies that principal can improve his net utility in this region.

Second, note that when $1-\rho = K$, $\mathbb{E}[y_\omega \times I_p] = \mathbb{E}[y_\omega \times I_m]$, where I_p (I_m) denotes the investment rule under the principal (manager). While the investment return under either investment rule is increasing in ρ, the difference between them is strictly decreasing in ρ when $1-\hat{\rho} < K < 1-\rho$ since

$$\mathbb{E}[y_\omega \times I_p] - \mathbb{E}[y_\omega \times I_m] = \frac{\lambda}{2}\left((1-\rho)y_H + \rho y_L\right) \tag{111}$$

which decreases in ρ. On the other hand, $U_{p,e}(I_m) - U_{p,e}(I_p)$ is strictly bounded above zero for any ρ in this interval. Since $U_{p,e}(I_m) - U_{p,e}(I_p)$ and $\mathbb{E}[y_\omega \times I_p] - \mathbb{E}[y_\omega \times I_m]$ are continuous in ρ, there exists a $\bar{\rho}$ such that for all ρ such that $1-K > \rho > \bar{\rho}$ the principal strictly prefers delegation. Finally, note that in order for $\rho < 1-K$ it must be the case that $K < 0.5$. This completes the proof of the statement. □

A.9.1 Proof of Proposition 10

Note that

$$W = \alpha + \beta\left(\pi P + (1-\pi)(V-W)\right) \implies W = \frac{\alpha}{(1+\beta(1-\pi))} + \frac{\beta}{(1+\beta(1-\pi))}\left(\pi P + (1-\pi)V\right)$$

Moreover, the market clearing condition implies

$$\begin{aligned} P &= \mathbb{E}[V-W] \\ &= \mathbb{E}\left[V - \frac{\alpha}{(1+\beta(1-\pi))} - \frac{\beta}{(1+\beta(1-\pi))}\left(\pi P + (1-\pi)V\right)\right] \end{aligned}$$

which implies that

$$P\left(1+\frac{\beta\pi}{\left(1+\beta\left(1-\pi\right)\right)}\right)=\mathbb{E}\left[V\right]\left(1-\frac{\beta\left(1-\pi\right)}{\left(1+\beta\left(1-\pi\right)\right)}\right)-\frac{\alpha}{\left(1+\beta\left(1-\pi\right)\right)}$$

$$P=\frac{\mathbb{E}\left[V\right]-\alpha}{1+\beta}\equiv\frac{\tilde{\mu}\left(P\right)-\alpha}{1+\beta}$$

Manager's effort problem is

$$\max_{e}\mathbb{E}\left[\frac{\alpha}{\left(1+\beta\left(1-\pi\right)\right)}+\frac{\beta\left(\pi P+\left(1-\pi\right)V\right)}{\left(1+\beta\left(1-\pi\right)\right)}\right]-\frac{\gamma}{2}\mathbb{V}\left[\alpha+\beta\left(\pi P+\left(1-\pi\right)Z\right)\right]-c\left(e\right),$$

The FOC is

$$\frac{\beta}{1+\beta}=c'\left(e\right).$$

The optimal contract solves

$$\max_{\alpha,\beta,\pi}\mathbb{E}\left[x_{\omega}+e+\delta y_{\omega}I-\left(\alpha+\beta\left(\pi P+\left(1-\pi\right)Z\right)\right)\right],\qquad\text{subject to :}\tag{112}$$

$$\frac{\beta}{1+\beta}=c'\left(e\right),\tag{113}$$

$$\mathbb{E}\left[\alpha+\beta\left(\pi P+\left(1-\pi\right)Z\right)\right]-\frac{\gamma}{2}\mathbb{V}\left[\alpha+\beta\left(\pi P+\left(1-\pi\right)Z\right)\right]-c\left(e\right)\geq0.\tag{114}$$

This simplifies to

$$\max_{\beta,\pi}\mathbb{E}\left[x_{\omega}+e+\delta y_{\omega}I\right]-\frac{\gamma}{2}\beta^{2}\mathbb{V}\left[\left(\pi P+\left(1-\pi\right)Z\right)\right]-c\left(e\right)\quad\text{subject to }\frac{\beta}{1+\beta}=c'\left(e\right).$$

Moreover

$$\begin{aligned}\beta^{2}\mathbb{V}\left[\left(\pi P+\left(1-\pi\right)Z\right)\right]&=\frac{\beta^{2}\mathbb{V}\left[\left(\pi P+\left(1-\pi\right)V\right)\right]}{\left(1+\beta\left(1-\pi\right)\right)^{2}}\\&=\frac{\beta^{2}\mathbb{V}\left[\left(\pi\frac{\tilde{\mu}}{1+\beta}+\left(1-\pi\right)V\right)\right]}{\left(1+\beta\left(1-\pi\right)\right)^{2}}\\&=\frac{\beta^{2}}{\left(1+\beta\right)^{2}}\mathbb{V}\left[\left(\tilde{\mu}+\frac{\left(1-\pi\right)\left(1+\beta\right)}{\left(1+\beta\left(1-\pi\right)\right)}\left(V-\tilde{\mu}\right)\right)\right]\end{aligned}$$

Note that $\tilde{\mu}$ is independent of π and the optimal π is characterized by minimizing

$$\min_{e}\ \mathbb{V}\left[\left(\tilde{\mu}+\frac{\left(1-\pi\right)\left(1+\beta\right)}{\left(1+\beta\left(1-\pi\right)\right)}\left(V-\tilde{\mu}\right)\right)\right]$$

The FOC is

$$\pi=1+\frac{\operatorname{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)}{\left(1+\beta\right)\mathbb{V}\left(V-\tilde{\mu}\right)+\beta\operatorname{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)}$$

Substituting this into the principal's objective, we get

$$\begin{aligned}\mathbb{V}[W] &= \frac{\beta^2}{(1+\beta)^2}\left[\left(\mathbb{V}[\tilde{\mu}] + \left(\frac{(1-\pi)(1+\beta)}{(1+\beta(1-\pi))}\right)^2 \mathbb{V}(V-\tilde{\mu}) + 2\frac{(1-\pi)(1+\beta)}{(1+\beta(1-\pi))}\text{cov}(V-\tilde{\mu},\tilde{\mu})\right)\right] \\ &= \frac{\beta^2}{(1+\beta)^2}\left[\left(\mathbb{V}[\tilde{\mu}] - \frac{[\text{cov}(V-\tilde{\mu},\tilde{\mu})]^2}{\mathbb{V}(V-\tilde{\mu})}\right)\right]\end{aligned}$$

The objective is

$$\max_{\beta} \mathbb{E}[x_\omega + e + \delta y_\omega I_m] - \frac{\gamma}{2}\frac{\beta^2}{(1+\beta)^2}\left[\left(\mathbb{V}[\tilde{\mu}] - \frac{[\text{cov}(V-\tilde{\mu},\tilde{\mu})]^2}{\mathbb{V}(V-\tilde{\mu})}\right)\right] - c(e) \quad \text{subject to } \frac{\beta}{1+\beta} = c'(e).$$

The FOC is

$$\begin{aligned}(1-c'(e))\frac{\partial e}{\partial \beta} &= \gamma\frac{\beta}{(1+\beta)^3}\left(\mathbb{V}[\tilde{\mu}] - \frac{[\text{cov}(V-\tilde{\mu},\tilde{\mu})]^2}{\mathbb{V}(V-\tilde{\mu})}\right) \\ \beta &= \frac{1}{\gamma c''(e)\left[\left(\mathbb{V}[\tilde{\mu}] - \frac{[\text{cov}(V-\tilde{\mu},\tilde{\mu})]^2}{\mathbb{V}(V-\tilde{\mu})}\right)\right]}.\end{aligned}$$

□

B Robustness and Additional Analysis

B.1 Traded Cash Flows

In what follows, we assume that investors trade a claim to the firm's terminal cash flows *without* netting out the compensation to the manager. This could correspond to a setting in which the principal, who owns the firm, pays the manager directly instead of utilizing the firm's cash flows. In practice, this arrangement is akin to a private equity or venture capital firm paying one of its employee to improve and oversee a firm in its portfolio. We denote the price of this claim as $\hat{P}$. Note that

$$\hat{P} = \tilde{\mu}(P) \tag{115}$$

$$= (1+\beta) P + \alpha \tag{116}$$

The proof of the existence of the financial market equilibrium is unchanged since α and β are scalars and do not play a role in the proof of Proposition 1.

As a result of this change, the manager's optimal level of effort now solves

$$\beta = c'(e), \tag{117}$$

which becomes the principal's new IC constraint. Specifically, we can rewrite the principal's objective as

$$\max_{\beta} \mathbb{E}[x_\omega + e + \delta y_\omega I] - \frac{\gamma}{2}\beta^2 \mathbb{V}[\tilde{\mu}(P)] - c(e) \text{ subject to } \beta = c'(e). \tag{118}$$

Then the principal's optimal contract is given by $\left(\hat{\alpha}, \hat{\beta}\right)$, where

$$\hat{\alpha} = c(e) + \frac{\gamma}{2}\hat{\beta}^2 \mathbb{V}[\tilde{\mu}(P)] - \hat{\beta}\mathbb{E}[P], \text{ and} \tag{119}$$

$$\hat{\beta} = \frac{1}{1 + c''(e)\gamma \mathbb{V}[\tilde{\mu}(P)]}. \tag{120}$$

Thus, the contract is "lower-powered", i.e., less sensitive to the stock price. Note that, relative to the baseline model, this has no impact on the manager's equilibrium level of effort since $\hat{\beta} = \frac{\beta}{1+\beta}$, i.e., in equilibrium the manager's first-order condition is unchanged.

B.2 Noisy signal about effort

In the baseline model, we assumed that investors can perfectly observe the manager's equilibrium effort. In this section, we relax this restriction and assume that investors can only condition on a noisy signal of his chosen effort level.

Specifically, suppose that the terminal value of the firm's cash flows, V, is given by

$$V(\omega, e, I, \theta) = x_\omega + (e + \theta) + \delta y_\omega \times I, \tag{121}$$

where $\theta \sim N(0, \sigma_\theta^2)$ is independent of ω. Further suppose that in addition to observing private signals s_i about ω, all investors observe a *non-contractible* signal $s_\theta = e + \theta + \eta$, where $\eta \sim N\left(0, \sigma_\eta^2\right)$ is independent of all other random variables.

Let $V_\omega\left(P; s_\theta\right)$ denote the investors' expected cash-flow in state ω given the price P and signal s_θ. Then, we can express

$$V_\omega\left(P, s_\theta\right) = \mathbb{E}\left[V\left(\omega, e, I, \theta\right)|\omega, P, s_\theta\right] = x_\omega + \hat{e} + \kappa\left(s_\theta - \hat{e}\right) + \delta y_\omega \times I\left(P\right), \tag{122}$$

where $\hat{e}$ denotes the investors' inference of the manager's choice of effort, and $\kappa \equiv \frac{\mathbb{C}(s_\theta,\theta)}{\mathbb{V}(s_\theta)} = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2} \in [0,1]$ is a measure of the informativeness of s_θ. As in Holmström and Tirole (1993), investors correctly infer the manager's effort level in equilibrium. The public signal, however, provides information about θ and so investors utilize it when forming beliefs about the firm's value. Since the public signal is observed by all investors, its inclusion does not change the nature of the financial market equilibrium but it does alter the equilibrium price. Specifically, using steps analogous to those in the benchmark analysis, one can show that the equilibrium price in this setting is given by

$$P\left(\omega, u, s_\theta\right) = \begin{cases} \frac{1}{1+\beta}\left(\tilde{\mu}\left(p_L; s_\theta\right) - \alpha\right) \equiv p_L\left(s_\theta\right) & \text{if } u < u_\omega - (1-\lambda) \\ \frac{1}{1+\beta}\left(\tilde{\mu}\left(p_U; s_\theta\right) - \alpha\right) \equiv p_U\left(s_\theta\right) & \text{if } u_\omega - (1-\lambda) \leq u \leq u_\omega + (1-\lambda) \\ \frac{1}{1+\beta}\left(\tilde{\mu}\left(p_H; s_\theta\right) - \alpha\right) \equiv p_H\left(s_\theta\right) & \text{if } u > u_\omega + (1-\lambda) \end{cases}, \tag{123}$$

where the marginal investor's beliefs are given by

$$\tilde{\mu}\left(p_H; s_\theta\right) \equiv \mathbb{E}\left[V|s_H, p_H, s_\theta\right] = V_L\left(p_H, s_\theta\right) + \tilde{\rho}\Delta V\left(p_H, s_\theta\right), \tag{124}$$

$$\tilde{\mu}\left(p_L; s_\theta\right) \equiv \mathbb{E}\left[V|s_H, p_H, s_\theta\right] = V_L\left(p_L, s_\theta\right) + \left(1-\tilde{\rho}\right)\Delta V\left(p_L, s_\theta\right), \text{ and} \tag{125}$$

$$\tilde{\mu}\left(p_U; s_\theta\right) \equiv \mathbb{E}\left[V|s_U, p_U, s_\theta\right] = V_L\left(p_U, s_\theta\right) + \frac{1}{2}\Delta V\left(p_U, s_\theta\right) \tag{126}$$

and the equilibrium investment rule is equation (27).

Given this characterization of the financial market equilibrium, the manager's optimal choice of effort is given by the first order condition

$$\frac{\beta}{1+\beta}\kappa = c'\left(e\right). \tag{127}$$

While investors' inference of the manager's effort level, $\hat{e}$, is correct in equilibrium, changes in the manager's effort affect the price only through the public signal. Since this public signal of managerial effort is noisy (i.e., $\kappa \leq 1$), the equilibrium level of effort is lower (holding fixed β) than in the benchmark (see equation (30)). Holding fixed the contract, as the investors' signal becomes more precise, κ increases which leads to an increase in the manager's effort as well.

The following proposition characterizes the optimal linear contract in this setting.

Proposition 11. *Suppose the financial market equilibrium is characterized by the price in* (123) *and beliefs* (124)-(126)*, and the manager's optimal choice of effort, e, is given by*

(127). *Then, the principal's optimal linear contract is given by* (α, β) *where*

$$\alpha = c(e) + \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \left(\mathbb{V}\left(\tilde{\mu}(P; s_\theta)\,|s_\theta\right) + \kappa\sigma_\theta^2\right) - \beta\mathbb{E}[P], \text{ and} \tag{128}$$

$$\beta = \frac{\kappa}{c''(e)\,\gamma\left(\mathbb{V}\left(\tilde{\mu}(P; s_\theta)\,|s_\theta\right) + \kappa\sigma_\theta^2\right) - \kappa(1-\kappa)}. \tag{129}$$

Moreover, the optimal choice of β*:*
(i) decreases with both λ *and* ρ*, and*
(ii) increases with κ *if and only if* $\gamma c''(e)\,\mathbb{V}\left(\tilde{\mu}(P; s_\theta)\,|s_\theta\right) > \kappa^2$*.*
Finally, the equilibrium effort decreases with both λ *and* ρ*, and increases with* κ*.*

The optimal linear contract now depends on the two sources of investor information reflected in the price. To gain some intuition, note that the variance in the manager's compensation can be expressed as

$$\mathbb{V}(\alpha + \beta P) = \beta^2\mathbb{V}(P) = \left(\frac{\beta}{1+\beta}\right)^2 \left(\mathbb{V}\left(\tilde{\mu}(P; s_\theta)\,|s_\theta\right) + \kappa\sigma_\theta^2\right). \tag{130}$$

As in the benchmark model, for a fixed β, when the price is more informative about the true state of the world, ω, (i.e., if either λ or ρ are higher), the volatility of the price too, (captured by the $\mathbb{V}\left(\tilde{\mu}(P; s_\theta)\,|s_\theta\right)$ term). This makes the manager's compensation more risky, and hence it becomes costlier for the principal to induce the manager to exert effort. As in our benchmark analysis, the principal optimally responds to such an increase by reducing the optimal β.

When the price is more informative about effort (i.e., as κ increases), there are two opposing effects on the contract. First, as (127) highlights, when the signal is more informative increasing β induces more effort, on the margin. However, this increase in κ also increases the volatility of the price (as captured by the $\kappa\sigma_\theta^2$) term, which in turn, makes it more costly to incentivize effort provision. When κ is sufficiently low, Proposition 11 shows that β is increasing in κ — in this case, the positive impact on the equilibrium effort choice dominates the negative impact of more costly risk compensation. As a result, the principal optimally chooses to increase the price-sensitivity of the contract. On the other hand, when κ is sufficiently high, the former effect dominates, and the principal optimally chooses to decrease β when κ increases. Despite this non-monotonicity in the offered contract, the optimal effort chosen by the manager always increases with κ as in, e.g., Fishman and Hagerty (1989) and Holmström and Tirole (1993).

In short, our key trade-off obtains even when investors cannot observe the manager's effort perfectly: all else equal, when the price becomes more informative about investment opportunities, the price-sensitivity of the offered contract and managerial effort decrease. In contrast, when the price becomes more informative about managerial effort, the price-sensitivity of the contract can increase or decrease, depending upon on the relative informativeness of the two signals.[30]

[30] This result differs superficially from the findings of Holmström and Tirole (1993) due to the signals they utilize in their optimal contract. Specifically, the manager's compensation depends upon a transformation

B.3 Risk-neutral Managers

The key trade-off in our analysis is that when the price is more informative about future investment opportunities, it becomes more volatile signal about effort and, consequently, it is more costly for the principal to incentivize managerial effort. While the assumption of managerial risk aversion makes this trade-off particularly transparent in our benchmark analysis, the analysis in this section establishes that it is not necessary. Specifically, we show that the same trade-off can obtain when the manager is risk neutral, but the optimal contract satisfies a limited liability constraint, defined as follows.

Definition 1. *The optimal contract satisfies limited liability if for all $P \in \{p_H, p_U, p_L\}$, the contract satisfies $\alpha + \beta P \geq 0$.*

Note that the manager's effort choice is still given by (30). However, the objective of the principal is now given by:

$$\max_{\alpha,\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I - (\alpha + \beta P)\right], \qquad \text{subject to :} \tag{131}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{132}$$

$$\mathbb{E}\left[\alpha + \beta P\right] - c(e) \geq 0, \tag{133}$$

$$\alpha + \beta p_L \geq 0. \tag{134}$$

Given that $p_H > p_u > p_L$, condition (134) ensures that the limited liability constraint holds for all price levels. The above problem implies that the principal can always decrease α to improve her payoff till the manager's participation constraint (133) or (134) binds. This implies that the principal's objective can be rewritten as

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \max\left\{\beta E\left[P\right] - \beta p_L, c(e)\right\} \text{ subject to } \frac{\beta}{1+\beta} = c'(e). \tag{135}$$

The following lemma provides a characterization of how more information in prices affects the contract offered in equilibrium.

Lemma 2. *$E[P]$ increases in λ. Moreover, suppose at the optimal contract, the limited liability constraint (134) binds. Then, an increase in λ leads to a decrease in β, and consequently, the equilibrium level of effort e.*

Intuitively, the above result implies that the cost of incentivizing the manager increases with price informativeness when the limited liability condition binds. As such, our main trade-off can also obtain when the manager is risk-neutral.

B.4 Effort Impacts Investment Payoff

In our benchmark model, managerial effort affects the payoff of assets-in-place. In some settings, managerial effort may improve the value of a potential investment, if undertaken.

of the price so that the only impact of an increase in κ is on the informativeness of this signal. As a result, more information about managerial effort always leads to a more price-sensitive contract in equilibrium.

For example, this may reflect ex-ante work which increases potential synergies between the firm's existing assets and the new investment. To examine the effect of such an interaction, in this section we consider a setting in which managerial effort does not affect the value of assets-in-place but instead increases the payoff of the investment from y_ω to $y_\omega + e$. For tractability, we also assume that investors trade the firms' terminal cash flow, i.e., they do not net out the manager's compensation. All other features of the model are unchanged.

In this case, the date-one expected value of the firm is maximized when the principal invests as long as

$$q(P) > \frac{-y_L - e}{y_H - y_L} \equiv \hat{K}(e). \tag{136}$$

The construction of the financial market equilibrium is unchanged. Investors share common beliefs about the state-dependent value of the firm and their valuation of the traded claim can be ranked by their respective beliefs about the likelihood that $\omega = H$.

The novel feature which arises in this setting is that the value of the manager's effort is only realized if the principal chooses to invest. As a result, it is now the case that $V_L(P) = x_L + 1_{I(P)=1}(y_L + e)$. If the manager always invests ($1 - \rho > \hat{K}(e)$), the analysis is as in our benchmark model since the manager's effort always increases the value of the firm's terminal cash flow. In contrast, when the principal never invests ($\rho < \hat{K}(e)$), the manager's effort has no impact on the price level. In the other two cases, managerial effort only impacts the price with probability less than one and, as a result, the volatility of the price is also impacted:

- If $\frac{1}{2} > \hat{K}(e) > 1 - \rho$, the manager only invests if $P \in \{p_u, p_H\}$ and the marginal investors' expectation of the firms terminal cash flows are

$$\tilde{\mu}(P) = \begin{cases} \tilde{\mu}(p_H) = x_L + y_L + e + \tilde{\rho}[x_H - x_L + y_H - y_L] & \text{if } u < u_\omega - (1-\lambda) \\ \tilde{\mu}(p_u) = x_L + y_L + e + \frac{1}{2}[x_H - x_L + y_H - y_L] & \text{if } u_\omega - (1-\lambda) \leq u \leq u_\omega + (1-\lambda) \\ \tilde{\mu}(p_L) = x_L + (1-\tilde{\rho})[x_H - x_L] & \text{if } u > u_\omega + (1-\lambda) \end{cases} \tag{137}$$

- If $\rho > \hat{K}(e) > \frac{1}{2}$, the manager only invests if $P = p_H$ and the marginal investors' expectation of the firms terminal cash flows are

$$\tilde{\mu}(P) = \begin{cases} \tilde{\mu}(p_H) = x_L + y_L + e + \tilde{\rho}[x_H - x_L + y_H - y_L] & \text{if } u < u_\omega - (1-\lambda) \\ \tilde{\mu}(p_u) = x_L + \frac{1}{2}[x_H - x_L + y_H - y_L] & \text{if } u_\omega - (1-\lambda) \leq u \leq u_\omega + (1-\lambda) \\ \tilde{\mu}(p_L) = x_L + (1-\tilde{\rho})[x_H - x_L] & \text{if } u > u_\omega + (1-\lambda) \end{cases} \tag{138}$$

Given this, the manager's first-order condition now implies that his optimal effort is the solution to

$$\beta \frac{\partial \mathbb{E}[P]}{\partial e} - \frac{\gamma \beta^2}{2} \frac{\partial \mathbb{V}[P]}{\partial e} = c'(e). \tag{139}$$

The principal's objective reflects this new incentive-compatibility constraint:

$$\max_{\alpha,\beta} \mathbb{E}\left[x_\omega + (y_\omega + e)\,\mathbb{I}_{d_m=I} - \beta P - \alpha\right] \tag{140}$$

$$\text{subject to} \tag{141}$$

$$\beta\frac{\partial \mathbb{E}[P]}{\partial e} - \tfrac{\gamma\beta^2}{2}\frac{\partial \mathbb{V}[P]}{\partial e} = c'(e) \tag{142}$$

$$\mathbb{E}[\beta P + \alpha] - \tfrac{\gamma}{2}\mathbb{V}[\beta P + \alpha] - c(e) \geq 0 \tag{143}$$

Since the manager's participation constraint always binds, this can be expressed as

$$\max_{\beta} \begin{cases} e - \frac{\gamma}{2}\beta^2\mathbb{V}[P] - c(e) & \text{if } 1-\rho > \hat{K}(e) \\ e\left(1-\frac{\lambda}{2}\right) - \frac{\gamma}{2}\beta^2\mathbb{V}[P] - c(e) & \text{if } \frac{1}{2} > \hat{K}(e) > 1-\rho \\ e\frac{\lambda}{2} - \frac{\gamma}{2}\beta^2\mathbb{V}[P] - c(e) & \text{if } \rho > \hat{K}(e) > \frac{1}{2} \\ -\frac{\gamma}{2}\beta^2\mathbb{V}[P] - c(e) & \text{if } \rho < \hat{K}(e) \end{cases} \tag{144}$$

subject to

$$c'(e) = \begin{cases} \beta & \text{if } 1-\rho > \hat{K}(e) \\ \beta\left(1-\frac{\lambda}{2}\right) - \gamma\beta^2\frac{\lambda}{2}\left[\frac{\lambda}{2}p_H + (1-\lambda)\,p_u - \left(1-\frac{\lambda}{2}\right)p_L\right] & \text{if } \frac{1}{2} > \hat{K}(e) > 1-\rho \\ \beta\frac{\lambda}{2} - \gamma\beta^2\frac{\lambda}{2}\left[p_H\left(1-\frac{\lambda}{2}\right) - \frac{\lambda}{2}p_L - (1-\lambda)\,p_u\right] & \text{if } \rho > \hat{K}(e) > \frac{1}{2} \\ 0 & \text{if } \rho < \hat{K}(e) \end{cases} \tag{145}$$

Thus, when $1-\rho > \hat{K}(e)$, the optimal β is the same as in Appendix B.1 and equilibrium effort is the same as in our benchmark model. When $\rho < \hat{K}(e)$, the manager never exerts effort and the optimal β is zero. In the other two cases, an increase in the measure of informed investors, λ, affects the likelihood of investment which alters both (i) the probability that the investment is chosen and (ii) the volatility of the price. In both cases, the volatility of the price is increasing in λ, as in our benchmark analysis, which reduces the optimal β and equilibrium managerial effort.

However, a more informative price also impacts the likelihood of investment when $\rho > \hat{K}(e) > 1-\rho$. If the manager does not invest when the price is low (i.e., if $\frac{1}{2} > \hat{K}(e) > 1-\rho$), then an increase in λ reduces the value of managerial investment further since it is less likely that the manager's effort is realized. In contrast, if the manger only invests when the price is high (i.e., if $\rho > \hat{K}(e) > \frac{1}{2}$, the manager's effort is more likely to be pay off: when λ increases, it is more likely that $P = p_H$ and, holding fixed the impact of increased volatility, this channel leads to an increase in managerial effort.

B.5 Non-linear Contract

For a general contract, $W(P)$, the manager's first-order condition can be written as[31]

$$\frac{\partial \mathbb{E}\left[W\left(P\right)\right]}{\partial e}-\frac{\gamma}{2}\frac{\partial \mathbb{V}\left[W\left(P\right)\right]}{\partial e}=c'\left(e\right). \tag{146}$$

Since the manager's participation constraint is always binding, this implies that, for a general contract, $W(P)$, the principal's problem is

$$\max_{W(P)}\mathbb{E}\left[e+\delta y_{\omega}I\left(P\right)\right]-\tfrac{\gamma}{2}\mathbb{V}\left[W\left(P\right)\right]-c\left(e\right) \tag{147}$$

subject to the incentive-compatibility constraint, equation (146).

To impose some structure, we will consider a quadratic approximation, i.e., let $W\left(P\right)=\alpha+\beta P+\theta P^{2}$. In practice, such a contract could be approximately implemented utilizing a combination of equity and options. Given the potential for non-linearity in the manager's contract, we will focus on the setting in which investors trade the terminal cash flow (as in Section B.1) for tractability. It is straightforward to see that

$$\frac{\partial \mathbb{E}\left[W\left(P\right)\right]}{\partial e}=\beta+2\theta\mathbb{E}\left[P\right], \tag{148}$$

and, after noting that

$$\mathbb{V}\left[W\left(P\right)\right]=\theta^{2}\mathbb{V}\left[P^{2}\right]+2\beta\theta Cov\left(P,P^{2}\right)+\beta^{2}\mathbb{V}\left(P\right), \tag{149}$$

it can be shown that

$$\frac{\partial \mathbb{V}\left[W\left(P\right)\right]}{\partial e}=4\left\{\theta^{2}Cov\left(P,P^{2}\right)+\beta\theta\mathbb{V}\left(P\right)\right\}. \tag{150}$$

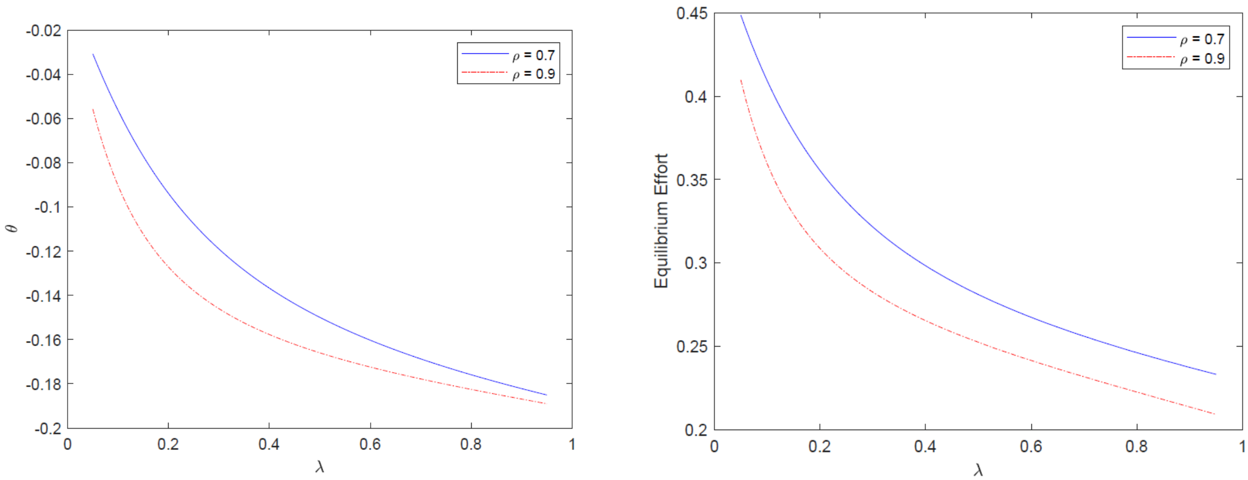
We solve for the optimal contract numerically. In contrast to our baseline model, when the price becomes more volatile (due to an increase in either ρ or λ), the principal can shift either β or θ. Figure 9 provides an illustration of how changes in investors' aggregate information affects equilibrium outcomes. While we find that the optimal β in this case is always one, the optimal θ is always negative: introducing concavity into his payoff reduces the variance of the manager's compensation. Moreover, as more investors become informed (an increase in λ) or if their signal becomes more precise (e.g., shifting from $\rho=0.7$ to $\rho=0.9$), theta becomes more negative to reduce the disutility experienced by the manager (panel (a)). This reduction in θ, however, comes at a cost: as the manager's first-order condition makes clear, as θ becomes more negative, equilibrium effort decreases as well (panel (b)).

This numerical illustration suggests that our key result — that managerial effort can decrease as investors become better informed about fundamentals — is not a consequence of linearity in the manager's contract.

[31]Note that this reduces to (30) when the contract is linear in the price since, in that setting, effort has no impact on the variance of the manager's payoff.

Figure 9: Optimal Theta and Equilibrium Effort

The figure plots the optimal θ chosen by the principal and the manager's equilibrium effort as a function of λ. The cost of effort is quadratic, $c(e) = e^2$. Other parameters are: $\gamma = 1$, $\delta = 2$, $x_H = 1, x_L = 0.5$, $y_H = 1$, $y_L = -0.5$.



(a) Optimal θ (b) Managerial Effort

B.6 Price Informativeness and Volatility

Recent work (e.g., Dávila and Parlatore (2020) and Brunnermeier et al. (2020)) has highlighted the potential for non-monotonicity in the relation between price volatility and informativeness. The specific nature of this relation, however, depends upon the primitives of the model including the marginal investor's risk preferences. This is of particular relevance to our analysis given that we expect investors to be well-diversified with respect to the firm-specific risks we analyze.

In what follows, we demonstrate in a canonical setting that when the marginal investor's risk-aversion tends to zero, this relation is always positive, as in our benchmark analysis. Suppose the terminal payoff (fundamental value) of a risky security is $F \sim \mathcal{N}\left(0, \frac{1}{\tau}\right)$. We denote the market-determined price of the risky security by P, and the aggregate supply of the risky asset by $z \sim \mathcal{N}\left(0, \frac{1}{\tau_z}\right)$.

There is a continuum of investors, indexed by $i \in [0, 1]$ who observe a private signal s_i, where

$$s_i = F + \varepsilon_i \qquad \varepsilon_i \sim N\left(0, \frac{1}{\tau_e}\right) \tag{151}$$

and ε_i is independent and identically distributed across investors so that $\int \varepsilon_i di = 0$. Investors submit limit orders which condition not only their private signals but alsothe price, P. Each investor i exhibits CARA utility with coefficient of absolute risk aversion γ over terminal wealth W_i: $W_i = W_0 + x_i (F - P)$, where x_i denotes her demand for the risky security.

The price is set by a competitive, risk neutral market maker (as in e.g., Kyle (1985), Vives (1995)) who conditions on all observable public information, including investors' aggregated demand. Following the usual steps, it is straightforward to show that there always exists a unique, linear, financial market equilibrium in which

$$P = \Lambda s_p, \text{ where } \Lambda = \frac{\tau_p}{\tau + \tau_p}, \tag{152}$$

where

$$s_p = F + \beta z, \quad \tau_p = \tau_z/\beta^2, \text{ and } \beta = -\frac{\gamma}{\tau_e}. \tag{153}$$

In this setting, price informativeness is denoted by τ_p; we denote price volatility by $\mathcal{V} \equiv \mathbb{V}(P)$. Then, given the equilibrium price,

$$\mathcal{V} = \Lambda^2 \left(\frac{1}{\tau} + \frac{1}{\tau_p}\right)$$

We can decompose the equilibrium elasticity of price volatility to price informativeness in a manner analogous to the approach taken by Dávila and Parlatore (2020):

$$\frac{d\log\mathcal{V}}{d\log\tau_p} = \underbrace{2\frac{d\log\Lambda}{d\log\tau_p}}_{\text{Equilibrium Learning}} - \underbrace{\frac{\tau}{\tau+\tau_p}}_{\text{Noise reduction}}$$

Since the marginal investor (i.e., the market maker in this setting) is risk-neutral, the equilibrium learning channel always dominates the noise reduction channel:

$$\frac{d\log\mathcal{V}}{d\log\tau_p} = \underbrace{2\frac{\tau}{\tau+\tau_p}}_{\text{Equilibrium Learning}} - \underbrace{\frac{\tau}{\tau+\tau_p}}_{\text{Noise reduction}} = \frac{\tau}{\tau+\tau_p} > 0.$$

This implies that price volatility and informativeness are positively related which, in turn, suggests that our key result — that managerial effort will decrease as investors become better informed about firm-specific risk — is a robust observation.

B.7 Social Value

In what follows, we characterize how an increase in transparency, or equivalently a decrease in c_0, affects social value (SV), where , i.e.,

$$SV = \begin{array}{c} \text{Expected Value } - \text{Managerial Compensation} \\ +\text{Managerial Utility} - \text{Manager's Cost of Effort} \\ +\text{Net Trading Gains / Losses} \\ -\text{Investor Cost of Information} \end{array} \tag{154}$$

Note that since (i) aggregate trading gains / losses sum to zero and (ii) the manager is exactly indifferent given his compensation (i.e., Manager utility - Manager's Cost of Effort matches his reservation utility, $u_0 = 0$), this simplifies to

$$SV = \mathbb{E}\left[x_\omega + e + \delta y_\omega \times I\right] - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right) - \lambda c_0 \;=\; FV - \lambda c_0. \tag{155}$$

This leads directly to the next proposition.

Proposition 12. *Social Value increases with transparency (decreases with c_0) if $1-\rho < K < \rho$ and δ is sufficiently large. Social Value decreases with transparency (increases with c_0) if $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)(x_H - x_L)$ and if either (i) $1-\rho > K$, or (ii) $\rho < K$, or (iii) $1-\rho < K < \rho$ and δ is sufficiently small.*

The above result parallels Proposition 7 in that it provides sufficient conditions when an increase in transparency has an unambiguous effect on social value. Given the expression for social value in (155), one can express the impact of a change in c_0 on social value as:

$$\frac{d}{dc_0}SV = \frac{d}{dc_0}FV - \left(\lambda + c_0\frac{d\lambda}{dc_0}\right). \tag{156}$$

In addition to the investment and incentive channels, transparency has a direct effect on the SV through the aggregate cost of information acquisition. Whether this increases or decreases the social value depends on the equilibrium level of λ, since

$$\lambda + c_0\frac{d\lambda}{dc_0} = 2\lambda - 1. \tag{157}$$

When information acquisition costs c_0 are sufficiently high (i.e., $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)(x_H - x_L)$), the equilibrium λ is lower than $\frac{1}{2}$. As a result, any further increase in the cost of learning improves social value by decreasing the fraction of investors who become informed, thereby reducing aggregate expenditure on private information acquisition. On the other hand, when information acquisition costs are low (so that $\lambda > \frac{1}{2}$), the resultant decrease in λ is not sufficiently large to offset the increase in c_0 paid by those investors who still choose to become informed. In this case, overall expenditure on information acquisition is higher and, consequently, social value is lower. We note that this is more likely to be the case when investors have access to more precise information (i.e., if ρ increases) and if the information sensitivity of assets in place is high (i.e., if $x_H - x_L$ increases). The overall impact of transparency then follows from how this channel interacts with the impact on firm value.

B.8 Proofs of Extensions

B.8.1 Proof of Proposition 11

Let $B \equiv \frac{\beta}{1+\beta}$, and note that choosing β optimally is equivalent to choosing B optimally. First, note that α can be chosen to ensure that the manager's participation constraint binds. Then, one can rewrite the principal's objective as

$$\max_{\beta}\mathbb{E}\left[x_\omega + (e+\theta) + \delta y_\omega \times I\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta P\right] - c\left(e\right) \tag{158}$$

subject to $B\kappa = c'(e)$, which implies:

$$\kappa = c''(e)\, e_B, \quad \text{and } 0 = c'''(e)\, e_B^2 + c''(e)\, e_{BB}. \tag{159}$$

The FOC w.r.t. B for the principal is given by

$$e_B (1 - c'(e)) - \gamma B \left(\mathbb{V}(\tilde{\mu}(P; s_\theta) | s_\theta) + \kappa \sigma_\theta^2\right) = 0 \tag{160}$$

$$\Leftrightarrow \frac{\kappa}{c''(e)} (1 - \kappa B) - \gamma B \left(\mathbb{V}(\tilde{\mu}(P; s_\theta) | s_\theta) + \kappa \sigma_\theta^2\right) = 0 \tag{161}$$

which follows from plugging in $e_B = \frac{\kappa}{c''(e)}$. This implies that the optimal choice of B is given by:

$$B = \frac{\kappa}{\gamma c''(e) (\mathbb{V}(\tilde{\mu}(P; s_\theta) | s_\theta) + \kappa \sigma_\theta^2) + \kappa^2} \tag{162}$$

or equivalently, the optimal choice of β is given by:

$$\beta = \frac{\kappa}{\gamma c''(e) (\mathbb{V}(\tilde{\mu}(P; s_\theta) | s_\theta) + \kappa \sigma_\theta^2) - \kappa (1 - \kappa)}. \tag{163}$$

The SOC is given by $\mathcal{S} < 0$, where

$$\mathcal{S} = e_{BB} (1 - c'(e)) - e_B^2 c''(e) - \gamma \left(\mathbb{V}(\tilde{\mu}(P; s_\theta) | s_\theta) + \kappa \sigma_\theta^2\right) \tag{164}$$

Since $e_{BB} = -\frac{c'''(e)}{c''(e)} e_B^2$ and

$$\gamma \left(\mathbb{V}(\tilde{\mu}(P; s_\theta) | s_\theta) + \kappa \sigma_\theta^2\right) = \frac{e_B (1 - c'(e))}{B} \tag{165}$$

$$= \frac{\kappa e_B (1 - c'(e))}{c'(e)} \tag{166}$$

$$= \frac{e_B^2 c''(e) (1 - c'(e))}{c'(e)} \tag{167}$$

we have

$$\mathcal{S} = -\frac{c'''(e)}{c''(e)} e_B^2 (1 - c'(e)) - e_B^2 c''(e) - \gamma \left(\mathbb{V}(\tilde{\mu}(P; s_\theta) | s_\theta) + \kappa \sigma_\theta^2\right) \tag{168}$$

$$= -e_B^2 \left(\frac{c'''(e)}{c''(e)} (1 - c'(e)) + c''(e) + \frac{c''(e) (1 - c'(e))}{c'(e)}\right) \tag{169}$$

$$= -e_B^2 \left(\frac{c''(e)}{c'(e)} + \frac{c'''(e) (1 - c'(e))}{c''(e)}\right) \tag{170}$$

So we need

$$\frac{c''(e)}{c'(e)} + \frac{c'''(e) (1 - c'(e))}{c''(e)} > 0. \tag{171}$$

Finally, the optimal effort satisfies

$$c'(e) = B\kappa = \frac{\kappa^2}{\gamma c''(e)\left(\mathbb{V}\left(\tilde{\mu}\left(P; s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) + \kappa^2} \tag{172}$$

which implies that effort increases in κ. □

B.8.2 Proof of Lemma 2

Note that price is given by (26) and the expected price is given by

$$E\left[P\right] = \frac{\lambda}{2}\left(p_L + p_H\right) + (1-\lambda)\,p_U$$

and hence

$$\begin{aligned}\frac{\partial E\left[P\right]}{\partial \lambda} &= \frac{1}{2}\left(p_L + p_H - 2p_U\right)\\ &= \frac{1}{2\left(1+\beta\right)}\left(\tilde{\mu}\left(p_L\right) + \tilde{\mu}\left(p_H\right) - 2\tilde{\mu}\left(p_U\right)\right)\end{aligned}$$

If the manager never invests or always invests, the price is symmetric and so $\frac{\partial E[P]}{\partial \lambda} = 0$.

If $\frac{1}{2} > K > 1-\rho$, the manager invests if $P \in \{p_u, p_H\}$ and

$$\begin{aligned}\frac{\partial E\left[P\right]}{\partial \lambda} &= \frac{1}{2\left(1+\beta\right)}\left(-y_L - \left(1-\tilde{\rho}\right)\left(y_H - y_L\right)\right)\\ &= \frac{\left(y_H - y_L\right)}{2\left(1+\beta\right)}\left(K - \left(1-\tilde{\rho}\right)\right) > 0\end{aligned}$$

where the last inequality holds, because $K > 1-\rho > 1-\tilde{\rho}$ in this case.

If $\rho > K > \frac{1}{2}$, the manager invests if $P = \{p_H\}$ and

$$\begin{aligned}\frac{\partial E\left[P\right]}{\partial \lambda} &= \frac{1}{2\left(1+\beta\right)}\left(\tilde{\rho}\left(y_H - y_L\right) + y_L\right)\\ &= \frac{\left(y_H - y_L\right)}{2\left(1+\beta\right)}\left(\tilde{\rho} - K\right) > 0\end{aligned}$$

where the last inequality holds, because $\tilde{\rho} > \rho > K$ in this case. □

B.8.3 Proof of Proposition 12

The proof of this follows from the observation that

$$\frac{\partial}{\partial c_0}SV = \frac{\partial}{\partial c_0}FV - \left(\lambda + c_0\frac{d\lambda}{dc_0}\right) \tag{173}$$

$$= \frac{\partial}{\partial c_0}FV - \left(2\lambda - 1\right) \tag{174}$$

since

$$\lambda = \begin{cases} 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L + \delta(y_H - y_L))} & \text{if } 1-\rho > K \text{ or } \frac{1}{2} > K > 1-\rho \\ 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)} & \text{if } \rho > K > \frac{1}{2} \text{ or } \rho < K \end{cases}, \tag{175}$$

and so

$$\lim_{\delta \to 0} \lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)}. \tag{176}$$

This implies that $2\lambda - 1 < 0$ if and only if $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)(x_H - x_L)$. □