The Role of Financial Markets in Mitigating Credit Market Bubbles

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We investigate how long an insolvent debtor can avoid default when survival is beneficial to creditors collectively, but individual creditors gain by forcing early repayment. Theory predicts that the debt is not rolled over and default is immediate. With 23 experimental sessions, default is never immediate, with or without secondary debt markets. With markets, prices do not reveal survival length but correlate with payoffs. Creditors are better off with markets, but markets exacerbate wealth inequality. Survival length is reduced upon repetition with the same cohort. When new creditors are introduced, survival length remains constant, even with access to default history.

Keywords: Asset Pricing, Experimental Finance, Social Rationality, Market bubbles

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I. Introduction

The Global Financial Crisis (GFC) around 2007 is thought to have been triggered by a crash in a particular credit market, namely, the market for collateralized debt obligations (CDO). The puzzling aspect was not so much the crash but the duration of the bubble before the crash. Indeed, many key participants knew that the underlying pool of debt (consisting of subprime mortgages) was insufficient ever to pay back the obligations. Yet this knowledge did not transpire through market prices, as would be expected in an informationally efficient market.

Periods of exuberantly increasing asset prices followed by sharp price declines (crashes) have been part of competitive financial markets ever since their inception in late 15th century Antwerp (Schumpeter (1939)). Most accounts of alleged asset price bubbles focus on the detrimental effects of the eventual crash. For example, the GFC's housing and credit market bubble burst has been claimed to have caused an almost 5% drop in U.S. GDP between the second Quarter of 2008 and 2009, and about a 25% decline in wealth, much of it invested for insurance and retirement purposes. Fewer accounts mention the beneficial effects of the bubble itself, however.

There should be no doubt that the pre-GFC economy benefited immensely from the funding available because of the build-up of the bubble. Many participants in the economy, like construction and real estate businesses, benefited handsomely. Such were the profits that it is not even clear whether the net of wealth created and destroyed over the entire period was negative or positive. A recent study, see Martin, Moral-Benito, and Schmitz (2021), using micro data from Spain quells some of the concerns that "housing expansions crowd out credit that is necessary for productive investment in other sectors." Instead, the study suggests "that this concern is likely to be temporary, because housing bubbles eventually raise credit to all sectors if they last long enough." Indeed, it is possible to argue that everyone would have been worse off if the CDOs alleged to be the cause of the crash had never been allowed in the first place (had the bubble never existed).

There are other credit market bubbles that continued for a long time before crashing spectacularly, such as the Greek sovereign debt crisis and the Puerto Rico municipal bond crash. In both cases, the signs of pending default were on the wall long beforehand. It can be argued that both Greece and Puerto Rico benefited from the economic activity enjoyed during the long bubble period.

What makes these cases puzzling is the presence of a well-operating financial market where debt could freely be traded. Market participants may have had plans to hang on to the bonds for a while to reap the relatively high interest payments. But prices should have been informative about these plans, and hence, shrewd investors could have timed positions and thereby precipitated bankruptcy.

Here, we ask whether bubbles can indeed survive in a situation where everyone could be better off with the bubbles, even if it eventually leads to a crash, and despite the presence of a well-operating financial market. We focus on the theory, which we test with numerous experimental sessions. We leave it to empirical researchers, as in Martin et al. (2021), to decide whether our characterization of the events leading to the GFC, the Greek sovereign crisis, or the Puerto Rico crisis is accurate.

We consider the following abstract situation. The economy starts with a growing pool of money with multi-round claims issued against it. For consistency with the experimental design, we refer to the claims as "tickets." Each round, tickets can be cashed in against a growing face value. Crucially, the tickets are rolled over ("re-financed") regularly, at which point the creditors can decide to liquidate, partly or even fully, the pool (i.e., to "cash-in"). Essentially, the situation is one where a long-lived (but finite-horizon), growing pool of cash is being funded through debt with a shorter maturity.

We are concerned with the scenario where the pool has been eroded unexpectedly, so there will never be enough cash to repay future obligations fully. This means that there is never enough money in the pool if everyone were to request cash-in of all remaining tickets. If too many tickets are submitted for cash-in, then the pool is liquidated. We consider two liquidation rules, one where only the cash-in requests are paid ("strong-form"), and another one where all tickets will be paid ("weak-form").[1]

In the absence of financial markets, our set-up could be considered an extended version (more options, more players) of the well-studied centipede game (Rosenthal, 1981). Indeed, we set the parameters so that it is individually rational for each creditor to cash-in immediately all tickets, when she expects that the pool will be liquidated in the subsequent refinancing round, just like players in the centipede game will take when they expect the opponent to do so in the next round.

Unlike in the centipede game, players have imperfect information: at the end of each round, they only know whether the pool has (not) been liquidated; they do not know the actions of the others. Nevertheless, as in the centipede game, tension arises. When kept alive, the pool of cash grows (at a rate substantially above the going interest, which we set equal to zero, for convenience). So, if creditors roll over the debt, they will collectively be better off because more cash is available to them than when they re-invest the proceeds from the immediate liquidation elsewhere. That is, there exist Pareto improvements from keeping the pool alive. Therefore, while it is individually rational to cash-in, it is *socially rational* to wait.[2]

[1] At first, the strong-form liquidation rule may seem contrived, but various real-world situations share the same incentive structure, namely, short-term debt needs to be rolled over regularly, and is backed by a pool of assets, the value of which increases more if not liquidated. This structure is typical of many shadow banking institutions, such as Asset Backed Commercial Paper (ABCP) (Adrian, Ashcraft, et al. (2012)). Positions of some Real Estate Investment Trusts (REITS) are analogous too, when they invest in long-term tax-advantaged real-estate assets while funding through short-term loans.

[2] The tension between individual and social rationality also emerged in the recent Eurozone sovereign debt crisis: government debt holders benefited from proposed debt relief efforts, but investors were still better off

We do not take a stance on whether social rationality is an expression of other-regarding preferences (Fehr, Naef, and Schmidt, 2006). This means that in the analysis we do not account for agents caring about others' welfare.[3] In principle, moral sentiments are not needed to explain choices in our setting (Smith and Wilson, 2017; Cox, List, Price, Sadiraj, and Samek, 2019): agents realize that if everyone refrains from greed (individual rationality), all are better off. Then again, the rules induced by a formal version of the theory of moral sentiments could shed light on the dynamics of the pool, when it survives, and when it defaults.[4] Dutta, Levine, and Modica (2021) conjecture that pro-social behavior (their terminology) can be acquired, not only through incentives, but also through a process of "internalization," because pro-social behavior is something that a group could benefit from, and hence, would be willing to invest in. Through internalization, members of the group thus acquire "social rationality" (our terminology). This manifests itself in a game such as ours in the form of "mistakes." Mistakes beget mistakes when they cause Pareto improvements, as we shall point out later. Social rationality is reinforced through best-response to mistakes that reflect internalization of pro-social behavior.

Social rationality can explain behavior in a large number of games where Nash equilibrium (individual rationality) has been rejected, such as the prisoner's dilemma, the trust game, public good provision games, and the aforementioned centipede game (for a summary, see Camerer (2003)). In a Nash equilibrium, players best-respond to each other with strategies that are individually rational (selfish; greedy). What these games have in common is that there exist mutually beneficial strategies (make everyone better off; lead to Pareto superior

to "hold out," which was not to agree to the terms of the debt restructuring, and hence, to be compensated immediately (Zettelmeyer, Trebesch, and Gulati (2013))

[3]Other-regarding preferences have been used to explain the results in the centipede game but such preferences are less likely to be expressed in aggregate market behavior, Camerer (2003).

[4]The promise of the moral sentiments approach, as advocated in Smith and Wilson (2017, 2019), is in its algorithmic view of human behavior, as opposed to traditional theory, which focuses on optimization. Optimization is generally infeasible in economic problems because of their computational complexity (Murawski and Bossaerts, 2016), and recent behavioral and neuroscience research has supported the hypothesis that human choice is "as if" rule-based (Bossaerts, Yadav, and Murawski, 2018; Bossaerts, 2018; Friston, 2010).

outcomes), and evidently humans resort to playing those even if there is no reputation to gain since players meet only once.

The core issue in this paper is: how the existence of financial markets affects the cash-in game, and thus social welfare. In those markets creditors trade tickets amongst themselves and are allowed to take negative positions (short-selling). Will this destroy the ability for social rationality to be expressed? In other words, will this destroy the ability for players to let the pool survive, with the danger that selfish players use market information to front-run, collecting the much higher face value of the tickets, and hence, earn more?

Indeed, trading may reveal creditors' plans (as to when to cash-in), allowing others to predict pool liquidation, and hence, through their cash-ins, precipitate liquidation. On the other hand, the market may attenuate the tension between individual and social rationality: creditors who believe that the end of the pool is near could sell their tickets in the marketplace to players who expect the pool to survive, instead of submitting them for cash-in. This then becomes a self-fulfilling prophesy: the reduced number of cash-ins extends the life of the pool. So, markets become beneficial to the community of players since the pool survives, and since survival leads to pool growth, eventually more money in total is distributed to players.

The first possibility is not without empirical merit. In many contexts, the introduction of financial markets produces results closer to predictions from individually rational valuation and choice theory. This has been observed when (i) information would otherwise fail to aggregate (Forsythe, Palfrey, and Plott, 1984), (ii) cognitive limitations would otherwise affect choices (Laibson and Yariv, 2007; Asparouhova, Bossaerts, Eguia, and Zame, 2015; Oliven and Rietz, 2004), or (iii) traders would otherwise be confused (Porter and Smith, 1995). However, there exists empirical evidence to the contrary. Specifically, Asparouhova (2006) studied prices and allocations in markets for bank loans under asymmetric information. Markets failed to settle on the competitive equilibrium when the equilibrium was not Pareto-optimal. Pareto-improving loan contracts continued to be offered despite the constant threat of cherry-picking by competitors (who offered new contracts with the sole purpose of attracting only the high-quality borrowers).

We first analyze our game theoretically without markets. Under the strong liquidation rule, only those who have cashed in share the remaining pool on a pro-rata basis. We show that cashing in all tickets immediately is a dominant strategy. Therefore, liquidation in round 1 is the unique equilibrium outcome. Under the weak liquidation rule, participants have less incentive to cash in early since if one does not cash in and if the pool is liquidated, one still shares in the proceeds. With stricter parameters so that postponing cash-in is costly, we show that liquidation in round 1 is still the unique Nash equilibrium outcome.

The introduction of financial markets complicates the theoretical analysis since traditional asset pricing theory relies on the competitive assumption (prices are taken as given), which means that strategic considerations are ignored. In contrast, game theory explicitly considers strategy since everyone realizes that they influence the community as a whole. To resolve this discrepancy, we limit our analysis to proving that a financial market equilibrium exists that is consistent with the Nash Equilibrium of the game without markets. Consistency means: agents are always indifferent between selling in the marketplace and playing the equilibrium in the game without markets.

We then turn to experiments. We first study the case without markets. In view of the results of behavioral game theory (Camerer, 2003), it may not be surprising that the pool was never liquidated in round 1. Collectively our participants are better off letting the pool grow, and hence, a sufficient number of participants do not cash in, even if they would have gained more individually by cashing in. There is attrition: upon repetition with the same cohort, the pool is liquidated faster.

It could, of course, be that some participants do not pay sufficient attention or are confused, and hence mistakenly fail to cash in when they should, causing the pool to survive. Mistakes have become a popular way to model behavior in games such as the centipede game, sometimes formally, through the quantal response equilibrium (McKelvey and Palfrey, 1992).

To shed light on the plausibility of arbitrary mistakes to explain the results, we run two different tests. First, we change the parameters so that there are no gains from waiting (the pool is stationary) or even strong gains from cashing in early (the pool shrinks). We observe that the number of rounds it takes until the pool is liquidated is reduced significantly, to the extent that there is hardly any cost to social welfare (total payments to players) when the pool shrinks over time. In the second test, unlike in the remaining sessions, the control session was solvent in the first round. If mistakes drive pool duration, the number of cash-ins in the control and treatment sessions in the first round should be the same. However, we found that there were significantly fewer submitted tickets in the control session; hence, participants paid attention to pool solvency.

The argument that this casts doubt on arbitrary mistakes as the driving force is not water-tight, however. Even if the same number of mistakes are expected in the first round, incentives to lower one's cash-in requests are lessened as we move from a growing to a constant and decreasing pool. Likewise, when the pool is initially solvent, mistakes do not beget more mistakes, while if the pool is initially insolvent, mistakes beget more mistakes. Thus, merely expecting mistakes already explains the results without having to appeal to social rationality. Such is the power of the quantal response equilibrium: in principle, it can explain any behavior in a game (Haile, Hortaçsu, and Kosenok, 2008).

We then study the consequences of introducing a market (organized as a continuous, anonymous open-book system, or continuous double auction (CDA)) where participants could trade tickets amongst each other each round, before or while deciding on cash-ins.

Regardless of the liquidation rule, the pool survived for several rounds. Availability of a financial market did not shorten the life of the pool at all. It did, however, increase substantially social welfare (measured as the average payout realized on cash-ins per claim). The cost of this increased social welfare was the risk of ending with a claim that was worth little or nothing because everyone else cashed in, and no money was left in the pool. Indeed, the disparity of final wealth across participants was large. The wealth inequalities were not mitigated by switching to a weak-form liquidation rule (where everyone shares in the liquidation, regardless of whether cash-in was requested). Short sales further affected wealth inequalities. Participants who took short positions generated significantly higher total income. This does not mean that long-only strategies were universally doomed: we show that a simple long-only strategy generates zero profit. It is related to evidence that prices appear to be a martingale: price changes across rounds are unpredictable. Further exploring the information content of prices, we find that prices across replications did not predict pool duration, providing further support for the Efficient Markets Hypothesis (EMH). They did, however, correlate significantly with the effective (or realized average) values of the tickets. The effective value is defined to be the ex-post average per-ticket payment.

We observed unambiguous decay in pool duration upon replication within each experimental session with a fixed cohort of players. We attribute this to the strict stationarity of the replications: they were consecutive in time, and involved the same participants and the same parameters. When the experimental replications were spread over multiple weeks, and new participants were allowed to replace a portion of the current participants, the decay disappears, even if the newcomers had access to the entire history of prior replications.

Our results raise an interesting issue: could market intervention in the form of ticket purchases (one can think of them as bond purchases) temporarily release the tension between individual and social rationality, thereby enhancing social welfare, without the intervenor (a government or central bank) incurring high costs? We document that a targeted long-only strategy does not produce losses. This means that there might be scope for such intervention. The following are two examples of successful government/central bank interventions consistent with the above: purchases by the German government of shares in the airline Lufthansa during the COVID-19 pandemic, later sold at a substantial profit[5] as well as the corporate bond buying by the European Central Bank (ECB) that resulted in sizeable profits.[6] Of course, that is not to say that intervention always results in profit: while the ECB profited from buying Glencore bonds[7], it sold its holdings of Steinhoff at a loss.[8] The ECB recently announced a "Transmission Protection Instrument"[9] that could also be interpreted as attempting to reduce tension between individual rationality (to profit from selling short bonds of countries expecting to default if no-one is willing to refinance their debt) and social rationality (benefiting from further growth of the said countries if they are not forced into default). It is doubtful, however, that commitment to a pre-announced intervention rule would work. Our experimental setting could provide a unique environment to test various versions of intervention policy.[10].

The remainder of the paper is organized as follows. In the next section we contrast our setting to that in similar experimental studies. A formal analysis of our setting is provided in Section III. Details of the experimental design are presented in Section IV. The experimental results are reported in Section V. Section VII concludes.

[5] www.ft.com/content/b5cc2607-6120-4e7d-9810-889d03835ad9.

[6] www.bloomberg.com/news/articles/2019-02-21/ecb-profit-rose-in-2018-as-bond-buying-bolstered-interest-income.

[7] www.bloomberg.com/news/videos/2017-01-04/why-glencore-were-the-corporate-bonds-to-buy-for-ecb.

[8] www.ft.com/content/a842c826-eb45-3d44-b58b-e7c3a83dcb6a.

[9] www.ft.com/content/fa4a9798-016d-495e-8e59-1b691a2e1aa3.

[10] We should note that the above examples differ from our experimental setting in one important dimension: they concern situations in which the "pool" (in the form of a stream of cash flows from a corporation or from a country) is not finite-lived. With infinite-lived pools, folk theorems would predict that pool survival can be sustained in a correlated equilibrium. In effect, this means that there are multiple equilibria, some of which are detrimental because they the pool to be liquidated from the beginning. Since these bad equilibria remain a possibility, the analysis presented here remains valuable, even in cases where the pool does not expire.

II. Commonalities and Differences with Related Work

The setting we envisage has much in common with an incomplete-information version of a multi-person centipede game. The big difference, however, is that we have a market that could either promote social (as opposed to individual) rationality, or, through the signaling property of prices, destroy social rationality. In addition, the traditional centipede game is one of perfect information: players are always aware of the choices of others; in our setting, players only know whether the pool is still alive; they are not told how many tickets have already been paid out.

Experimentation with the original, perfect-information centipede game without markets has generated robust deviations from the Nash equilibrium predictions, and hence, from individually rational behavior; see McKelvey and Palfrey (1992) for a 2-person, and Murphy, Rapoport, and Parco (2004); Schotter and Yorulmazer (2009) for a multi-person version of the game.[11] Importantly, the allocations that result from non-Nash behavior Pareto-improve over the Nash Equilibrium allocation. As such, players are evidently willing to forgo individual rationality with the risk of being defected upon, as long as there is a good chance that everyone will be better off – precisely what we have been calling social rationality.

While one cannot exclude that choices partly reflect (random) mistakes, economists have generally interpreted the deviations from individually rational behavior as evidence of other-regarding preferences and social norms and have incorporated those in their models under the genre of "behavioral game theory." Explicit estimation of the relative contributions of social concerns vs. decision errors exist, not for the centipede game, but for another game in the same category, namely, the public goods provision game (Goeree, Holt, and Laury,

[11]Schotter and Yorulmazer (2009) implements the game in a bank run setup; they find that "while the theory predicts no late withdrawals, we see that 50%, 67%, and 58% of the participants withdraw in late periods in the Simultaneous, Low and High-Information Sequential treatments, respectively." The late withdrawals are robust to the of the game. This behavior is consistent with that observed in the centipede game; see Palfrey and McKelvey (1992).

2002; Anderson, Goeree, and Holt, 1998). Because these contributions are unknown for the centipede game, we set out ourselves to demonstrate that Pareto improvements do provide significant explanatory power in our setting: mistakes are not the only explanation for pool survival because eliminating Pareto improvements upon delay dramatically increases early cash-in requests. Such requests become extreme when penalizing liquidation delay, and the pool often fails to survive beyond round 1.

There is a relationship between our credit market setting and bank runs (Diamond and Dybvig, 1983; Schotter and Yorulmazer, 2009). In bank runs, however, the only way to redeem ("cash in") a claim is to clear it with the issuing institution. In our setting, agents also have the option to sell their claims in a secondary market. The bank run setting is closer to our treatment without a market. Also, bank runs are an equilibrium phenomenon, while survival of the pool beyond round 1 in our setup is not. Finally, bank runs emerge when the claim (deposit) holders unexpectedly need money, before the bank's investment successfully pays off. In contrast, in our credit market setting, neither the opportunity cost for claim holders (from investing elsewhere) nor the solvency of the (bankrupt) pool change over time.

Our financial market is different from prediction markets or derivative markets to a given game (Berg, Forsythe, Nelson, and Rietz, 2008; Wolfers and Zitzewitz, 2004), because the market is integrated into the game: the traders in the market and participants in the game are the same. Our approach is closer to Crawford and Broseta (1998); Kogan, Kwasnica, and Weber (2011), but there, the focus is on the effect of markets on equilibrium selection, while we investigate the effect of markets on non-equilibrium behavior and the feedback from non-equilibrium behavior (in the underlying game) on asset pricing.

Endogeneity of prices also distinguishes our setting from that of clock games (Abreu and Brunnermeier (2003); Moinas and Pouget (2013); He and Xiong (2012); Bosch-Rosa (2018)).

There, the emphasis has been on individual incentives to ride a bubble, or, translated to our setting, to delay cash-in. In clock games, these incentives exist in equilibrium. In our setting, the incentive to delay cash-in is inconsistent with equilibrium.

III. Formal Theoretical Setup

A. Preliminary Comments

We first study equilibria when there are no markets. We apply concepts of non-cooperative game theory. Since agents will know neither the cash-in requests of other agents nor how much money is left in the pool until liquidation, ours is a game of incomplete information. We study survival of the pool in equilibria of the games under strong and under weak liquidation rules.

Under the *strong-form* rule, only those who requested cash-in will share in the liquidation of the pool when it cannot pay the face values. Under the *weak-form* rule, everyone shares, even those who did not request cash-in. One would expect the strong-form rule to be easier to deal with, since under the alternative (weak-form) rule, if the pool is going to be liquidated, a player is indifferent between requesting cash-in or doing nothing. That is, it will matter less under the weak-form rule that the agent is not paying attention. Hence, one should expect a higher chance of mistake under the weak-form rule. We show that the distinction does not matter so that the equilibrium prediction is the same under both rules: the pool will be liquidated in round 1.

We subsequently introduce a market for tickets. To predict prices and trades, we appeal to standard competitive asset pricing theory. The approach is fundamentally different from that of game theory: players no longer have to posit beliefs about what others could potentially do, and what would happen if they make mistakes. Instead, they merely react to prices. As is standard in intertemporal asset pricing, we will impose "perfect foresight," which means that players predict the right prices in all future contingencies (Radner, 1982).

It is difficult to merge game theory and competitive analysis. Therefore, we limit our analysis to answering a simple question: does there exist a market equilibrium that supports the Nash Equilibrium of the cash-in game without markets? Our approach is analogous to that of Duffie, Malamud, and Manso (2014), where game theory is used to study pairwise bidding in a decentralized financial market, and rational expectations (perfect foresight) is used to set expectations of the bidders about future trading opportunities. This massively simplified the analysis (both for us, the theorists, and for the players).

B. No Markets

We study a T-period economy with N risk-neutral creditors. In period 1, each creditor has an endowment of D securities that are claims to a pool of money (the pool can be thought of as the wealth of a fictitious debtor) with a face value of F_1 each. For consistency with the experimental design, we refer to the claims as "tickets" and the period as "round." The face value of a ticket grows deterministically at rate R each round, i.e., in round t the face value is $F_t = F_1(1+R)^{t-1}$ $(t = 1, ..., T)$. Agents choose when to cash-in their ticket against the money pool. In round 1, the pool holds P_1 dollars, so the intrinsic value of a single claim equals $IV_1 = P_1/(ND)$. After each round, the amount left in the money pool grows at a rate of r. The decision to "not cash in" is equivalent to "rolling over" the tickets if those tickets were one-round-lived and could be rolled over to the next round instead of being paid back.

Let s_t^n denote the number of tickets player n plans to cash-in in round t $(t = 1, ..., T)$. Obviously,

$$\sum_{\tau<t} s_\tau^n \leq D,$$

for all t. Let s_t denote the vector whose elements consist of the number of tickets submitted for cash-in by each player in round t: $s_t = (s_t^1, s_t^2, ..., s_t^N)$. Abusing notation, we will also denote $s_t = \sum_{n=1}^N s_t^n$, the total number of tickets submitted in round t. Let $S_t = (s_1, s_2, s_3, \ldots s_t)$ denote the history of cash-ins till t, inclusive. To reduce notation, we set $S = S_T$. Let P_t denote the cash remaining in the pool in round t before ticket submissions have been decided on (i.e., at the start of round t). The value of P_t can be determined recursively. For $t \geq 1$, $P_{t+1} = \max\{0, [P_t - s_t F_t](1 + r)\}$. If $P_t - s_t F_t < 0$, the pool is "in default," and hence, will be liquidated. In that case, payouts are determined in two possible ways.

Under the *weak-form liquidation rule*, all tickets that are not cashed in yet are paid an equal share of the remaining cash in the pool, whether they were submitted for cash-in in round t or not. As such, the players are paid "pro rata." Under the *strong-form liquidation rule*, only tickets submitted for cash-in share (equally) in the remaining funds.

We add one additional restriction: if one reaches a round where only one player holds all the remaining tickets, then the pool will be liquidated that round. This will ensure that a pool can only survive with at least two players. The presence of at least two players plays a role in ensuring early liquidation of the pool, since players must never believe that they are the sole "survivors" of a pool that is still "alive."

Let $\pi_t(S_t)$ denote the payoff to each submitted ticket in round t given the history S_t. Table I shows the payoff $\pi_t(S_t)$ under the weak-form and strong-form liquidation rules, for $t \in [1, T]$, and where $m_t = ND - \sum_{\tau<t} s_\tau$ denotes the total number of remaining tickets at the beginning of round t.

Table I
Payoff $\pi_t(S_t)$ under the weak-form and strong-form liquidation rules

Case	Weak-form	Strong-form
If $1 \leq s_t \leq \frac{P_t}{F_t}$ and more than 1 player holds tickets in t	F_t	F_t
If $s_t > \frac{P_t}{F_t}$ or only 1 player holds tickets in t	$\dfrac{P_t}{m_t}$	$\dfrac{P_t}{s_t}$
If $s_t = 0$	0	0

The total payoff for a player n is simply the sum of all the planned payouts across rounds:

$$\sum_{t=1}^{T} s_t^n \pi_t(S_t).$$

It is the player's goal to maximize the expected total payoff. The dependence of π_t on S_t makes it clear that this is a strategic game: the payoffs depend on the cash-in strategy of all players.

Players do not observe other players' cash-in requests. At the end of a round, players are not even told how many tickets were requested for cash-in. They are only told whether the pool is liquidated or not. Therefore, the above represents an *N-person game of incomplete information*. Beside the initial values of parameters (T, N, D, F_1, IV_1), the only information commonly known by all players is:

Condition 1. *There are Pareto improvements if everyone rolls over all tickets.*

Condition 2. *There is never enough money in the pool to pay the face value of all remaining tickets.*

Conditions 1 and 2 can be obtained by simply ensuring that: (i) the intrinsic values grow ($r > 0$); (ii) the face values grow at a higher rate ($R > r$); and, (iii) the face value in the first round is above the intrinsic value ($F_1 > IV_1$). Condition 2 may seem strange, because most collateralized securities in the real world are issued against a collateral pool that (one would presume) is sufficient to pay for the obligations in case the pool is liquidated immediately. However, we are not interested in the evolution of cash-ins and prices in a solvent pool, but instead want to study whether an insolvent pool is liquidated immediately. So, we start with a pool "in default," assuming it was solvent at inception (some time $t < 0$), but an adverse event was realized at time $t = 0$.

We already pointed out the relation between our game and the centipede game. But there are three differences: (i) we allow for more than 2 players; (ii) everyone holds multiple tickets ($D \geq 2$), so a player could "pass" on (not to cash in) one ticket but "take" (to cash in) another one; (iii) no player knows how many tickets players are holding except at the start of the game (nobody knows how many tickets others "passed").

In the strong liquidation rule, our predictions will be based on dominance arguments, as in the centipede game (McKelvey and Palfrey, 1992). Specifically, we use iterative elimination of strictly dominated strategies to show that the pool must always be liquidated in the initial round. This also implies that the same holds for *any* Nash equilibrium. We do have to impose restrictions on the parameters, as in the centipede game; here, they are a bit more involved, however.

The core of our argument is based on answering the following question. Given a cash-in profile of others, if a player predicts liquidation in round t + 1, how should she respond given that, in round t, she is holding k ($\leq D$) tickets. Properties 1, 2, 3, 4, and 5 in the Internet Appendix A are used to derive the player's inference, i.e. to estimate minimum and maximum payoffs from distributing cash-in of her tickets across the rounds leading up to the liquidation round. Subsequently, we show that cashing in all tickets at a round prior to the liquidation round is a strictly dominant strategy. Everyone will expects so, and hence, by iteration, the pool will be liquidated in round 1 (see Lemma 1).

Under the weak liquidation rule, incentives to cash in early are weaker since if one does not cash in and if the pool is liquidated, one still shares in the proceeds. Therefore, we impose stricter parameters so that postponing cash-in is costly, and we show that there is no Nash equilibrium where the pool liquidates in rounds $t > 1$. So, liquidation in round 1 is the unique Nash equilibrium outcome (see Lemma 2).

Lemma 1. *Assume* $N \geq (D+1)(1+r) - 1$, $r < 1/D$, *and* $\Xi < (D-1)/D$, *where* $\Xi = IV_1/F_1$. *Under the strong liquidation rule, if a player expects liquidation in round* t^*, *and if the player still holds* $0 < k$ $(\leq D)$ *tickets in the round prior to* t^*, *then cashing in all* k *tickets at* $t^* - 1$ *is a strictly dominant strategy. Therefore, liquidation in round 1 is the unique outcome from iterated elimination of strictly dominated strategies.*

Lemma 2. *Assume* $N \geq (D+1)(1+R) - 1$, *and:*

$$\frac{1}{D+1} - \frac{D^{T-3}}{(D-1)(D+1)^{T-3}} \leq r < \frac{1}{D},$$

$$\max\{r, \frac{1}{D+1}\} < R \leq \min\left\{r + \frac{1}{(D-1)(1+r)^{T-3}}, 1 - \frac{D-1}{D+1}(1+r), \frac{D}{(D-2)(D+1)}\right\},$$

$$\text{and } \frac{D-1}{D+1} \leq \Xi < \frac{D-1}{D}.$$

Under the weak liquidation rule, liquidation in round 1 is the unique Nash equilibrium outcome.

Proof. (Both Lemmas.) See Internet Appendix A. ∎

Liquidation in round 1 is the unique Nash equilibrium outcome in Lemma 2 if parameters N, r, R and Ξ satisfy the Lemma's conditions. In the Internet Appendix B, we show that there exist feasible parameter values. The parameter values we chose in the experiments satisfy the conditions of both lemmas.

Important remark on off-equilibrium play: What happens if someone makes a mistake and forgets to submit all tickets in round 1, or even in later rounds? This cannot happen in equilibrium, but we do have to consider this situation. We show that it must be that lots of mistakes were made and those mistakes must be correlated so the pool can survive beyond round 1. We claim that these are unreasonable assumptions. While one can expect mistakes, it is unlikely that rational – yet at times erring – agents expect to make lots of mistakes, and that if one agent makes a mistake, many others will too. As a result, our equilibrium is robust to small, uncorrelated mistakes, and therefore ours is a sequential equilibrium (Kreps and Wilson, 1982).

First, assume that, a priori, everyone believes that the chance of making one or more mistakes is very small. From Property 3 in the Internet Appendix A, it follows that the pool survives only if many made mistakes: at least $N+1$ tickets must have remained in circulation. This implies that *mistakes must be strongly correlated*: if one player makes a mistake, others are expected to make mistakes.

In fact, for survival to happen, the number of mistakes needs to be even bigger than $N+1$. To see this, let z be the number of the *remaining tickets per player* that would allow the pool to survive. Therefore, z satisfies: $N\,D\,IV_1 - N(D-z)F_1 > 0$. This implies: $z > D(1-\Xi)$. Given the restriction on Ξ ($\Xi \leq (D-1)/D$), this means: $z > 1$, or z is at least 2.[12] So, the probability of the mistake is in fact substantial.

[12] Since z is an integer.

Therefore, given a prior belief that the chance of making mistakes is very small, mistakes alone cannot explain pool survival beyond round 1. More must be going on. We attribute the presence (and acceptance) of a lot of "mistakes" to social rationality, which Dutta et al. (2021) would explain as the consequence of a society's investment in internalization of the social benefits of non-Nash play.

C. With Markets

We now allow players, before deciding on cash-ins each round, to trade tickets in an anonymous, public market. As is traditional in asset pricing theory, the specifics of the exchange are left unspecified. We merely assume that the market is competitive (meaning that prices have to be taken as given), and that players have perfect foresight in the sense of Radner (1972). The latter means that they correctly anticipate (equilibrium) prices for each contingency (each possible profile of cash-ins), even if they may be incorrect about the chances of the contingency happening. Our approach is akin to the modeling of decentralized double-sided markets in Duffie et al. (2014). There too, rational expectations (perfect foresight) is used to extend a purely game-theoretic analysis of bilateral trade to a fully fledged theory for a market where multiple bilateral trades may happen simultaneously and repeatedly.

Because of trade, cash-ins are no longer constrained by endowments, i.e., it is no longer true that $\sum_{\tau<t} s_\tau^n \leq D$, all t. Players can request cash-in for more than D tickets since they can buy more in the open market using their cash endowment C.

Shortsales will be allowed. This means that the short-seller creates a ticket, referred to as a *short-sell ticket*, and incurs the obligation to compensate the buyer exactly in the same way as a regular ticket (one that was issued against the pool). Practically, when regular tickets are paid the face value because the pool is not in default, the short seller owes the buyer the face value if the buyer submits the short-sell ticket for cash-in. Conversely, if the pool is in liquidation, and the weak-form liquidation rule holds, then all short-sellers owe their buyers whatever the per-ticket liquidation value of the pool is. Under the strong-form liquidation rule, the short-sellers owe nothing if their buyers failed to submit their short-sell tickets for cash-in. The short-seller keeps the sales price, however. The buyer of a short-sell ticket does not know whether the ticket she bought was created in a short sale or was a ticket issued against the pool. Because short-sell tickets are not taken into account to determine whether the pool can pay for the face value of the submitted tickets, the schedules of payoffs under the two liquidation rules displayed in Table I remain valid.

Each round, exchange takes place at the (competitive) price p_t ($t = 1, ..., T$). Once the pool is liquidated, at say t^*, any remaining tickets (under the weak-form liquidation rule) are worthless, and hence, the price is set to $p_t \equiv 0$, $t > t^*$.

To distinguish between short-sell tickets and tickets originally issued against the pool, continue to use the notation s_t for cash-ins of the latter (original tickets), while letting v_t denote short-sell tickets submitted for cash-in in round t. The submitting player cannot distinguish between the two types, by assumption, so can only determine total cash-in request $z_t^n = s_t^n + v_t^n$ (≥ 0).

Let D_t^n be player n's total holdings of tickets, both original and short-sell ones, at the end of round t, before cash-in requests. Let C_t^n denote player n's cash holdings at the beginning of round t, before the trade. The feasibility of trade for player n is determined by the budget constraint:

$$[D_t^n - (D_{t-1}^n - z_{t-1}^n)]p_t \leq C_t^n. \tag{1}$$

Because shortsales are allowed, D_t^n can be strictly negative, while cash-in requests require positive holdings. Hence:

$$(0 \leq)\ z_t^n \leq \max(0, D_t^n).$$

In any round until liquidation, cash will change from one round to another because of trade and cash-in payoffs. That is, for $t > 1$, and provided there is no liquidation in prior rounds

$$C_t^n - C_{t-1}^n = -[D_{t-1}^n - (D_{t-2}^n - z_{t-2}^n)]p_{t-1} + z_{t-1}^n \pi_{t-1}(S_{t-1}), \tag{2}$$

where $\pi_t(S_t)$ is as in table I. We set $D_0^n = D^n = D$ (initial holdings of tickets issued against the pool) and $z_0^n = 0$. For $t = 1$, $C_1^n = C^n = C$, initial cash holdings.

Our game is still parameterized as before, though now includes cash endowments C: $(T, N, D, C,\ F_1, IV_1, r, R)$.

Our game continues to be one of incomplete information, but now includes trade in a competitive market. The competitive nature of the ticket market implies that best-response strategies in the market are not based on conjectured trading strategies of other players, but *only* on conjectured prices. We assume that the conjectures coincide with the equilibrium prices, i.e., that perfect-foresight obtains (Radner, 1972). These equilibrium prices should reflect the payoffs players can get in the cash-in game.

In general, the issue of the nature of a Nash Equilibrium in the cash-in game alongside a Perfect-Foresight Equilibrium in the ticket market is a complicated one. While indeed other players' trading strategies need not be considered in determining the best response in the ticket market (only prices are relevant), these trading strategies lead to re-distribution of tickets among players of the cash-in game. So, in principle, all permutations of allocations that can be obtained through trading should be considered.

We restrict our attention only to the Nash Equilibrium allocations in the cash-in game and ask whether there is a Perfect-Foresight Equilibrium that supports it in the sense that it does not change the incentives in the cash-in game. We define it as follows.

Definition 1. *A Perfect Foresight Equilibrium in the cash-in-cum-trade game*

$$(T, N, D, C, F_1, IV_1, r, R)$$

is consistent with the Nash equilibrium of the cash-in-only game

$$(T, N, D, F_1, IV_1, r, R)$$

if prices in the marketplace are such that, in any round, players are indifferent between trade and no-trade, and players predict that these prices would obtain.

Indifference will obtain if prices in a round t, p_t, equal equilibrium payoffs of the cash-in only game (see Proposition 1) if started in round t. This may sound like an easy requirement to satisfy, but there is only one price, while there are N players. If these players face different equilibrium payoffs, the Perfect Foresight Equilibrium consistent with the Nash Equilibrium cannot possibly exist since there is only one price (per round). Fortunately, in the Nash Equilibria of the cash-in game, everyone faces the same (per-ticket) equilibrium payoffs, which is the intrinsic value of the ticket, i.e., the value of the pool divided by the total number of tickets. We have the following result.

Proposition 1. *There exists a Perfect Foresight Equilibrium of the ticket market consistent with Nash equilibrium of the cash-in-only game whereby prices always equal intrinsic value, i.e.,* $p_t = IV_1(1+r)^{t-1}$.

Proof. As mentioned before stating the Proposition, the Proposition is true because in the cash-in-only game, everyone faces the same (Nash Equilibrium) payoffs. □

IV. Details About The Experimental Sessions

All our experimental sessions implemented the above theoretical setting, varying between treatments that

- Allowed markets or not.
- Used the weak-form or strong-form liquidation rule.

We report on 20 experimental sessions. Four of the sessions are baseline sessions with no markets (BASE1 to BASE4). Sixteen sessions are treatment sessions with markets, with five under the strong form liquidation rule (MKT-S1 to MKT-S5) and eleven under the weak liquidation rule (MKT-W1 to MRK-W11). The treatments are not balanced because the baseline treatment draws upon rich literature and an abundance of results in the context of the centipede game. With the main focus of the paper being the effect of markets on the outcomes of such games, the vast majority of sessions have markets. Because of a change of design after the first five market sessions, most of the sessions use the weak liquidation rule (5 under the original design, and 6 under an improved one). In addition, the weak rule is one that has the highest empirical relevance. Each session consisted of as many replications of the game as the allotted time (two hours) allowed, with as little as one replication and as many as five. The experiments were conducted at the campuses of the California Institute of Technology, the University of Utah, the Ecole Polytechnique Fédérale Lausanne, and the University of Melbourne. Participants were drawn from a participant pool of undergraduate and graduate students at the respective universities. All experiments were approved by the relevant Institute Review Boards to protect human subjects in academic research.

Experimental procedures were explained in detailed instructions. A set of instructions is reproduced in the Appendix. Each session started with the experimenter reading the instructions out loud. Participants were allowed to interrupt the reading if they had any questions. Next, there was a practice round. This round replicated the first couple of rounds of a real replication, except for the size of the money pool. The practice round helped participants familiarize themselves with the software, Flex-E-Markets,[13] and with the rules of the game before their actions counted towards the take-home pay. All accounting was

[13] Ad Hoc Markets, Salt Lake City, UT, USA.

done in real (local) currency: USD (US dollar), CHF (Swiss franc), or AUD (Australian dollar). Participants were paid based on performance. The money that they made during the experimental session was theirs to keep. Including the sign-up reward, participants made approximately $35 (or the equivalent in Swiss francs or Australian dollars), and the range of payoffs was from about $10 to $55. In total, about 450 participants from three continents participated. Details of the sessions can be found in Table II.

The following parameter values remained the same across sessions: the face value in round 1 was $F_1 = 1.25$, the face value growth was $R = 0.20$, and the remaining pool growth was $r = 0.10$. All participants started out with $D = 6$ tickets. If the session had markets, they also started with $C = 6$ units of the local currency (USD/CHF/AUD), except for the MKT-W1, where everyone started with cash $C = 10$. The initial intrinsic value of a ticket (total value of pool divided by the number of tickets outstanding) was $IV_1 = 1.04$, except in MKT-W2, where it was 1 (the pool opened with less money). Because the initial intrinsic value was always below the face value, the pool was in default (could not pay all the outstanding tickets at face value) from the beginning. All chosen parameter values satisfy the conditions in Lemmas 1 and 2. This means that in all sessions, it is individually optimal to cash-in in round 1 but it is socially optimal to wait until the last round to cash in.

Both the baseline and the treatment sessions use the Flex-e-Markets trading platform. In the baseline session the platform was organized as a single private market. All participants were connected to the experimenter only, and had no connections to one another. Each round included private communication from each participant to the experimenter about the number of tickets to be submitted for cash-in. The communication was implemented via the private order books between the experimenter and each of the participants. If a participant i wanted to submit s_i tickets in round t, they simply submitted a sell order for s_i units to the experimenter at a random price.[14] At the end of each round the total sum of the face values of the submitted tickets was checked against the money in the pool and if the pool was sufficient, the face values would be paid out. In that case, it was publicly announced that the game transitions to the next round, which is a repetition of the previous but with a different amount of money in the pool. That amount was not announced. Also, for those who cashed in, the number of their tickets decreased with the amount of tickets submitted, and their cash increased by the face value for that round, per ticket submitted. If the pool amount in round t was insufficient to cover the face values, it was announced that the pool would be liquidated. All baseline sessions were under the weak liquidation rule. Thus, the amount in the pool was divided equally between all outstanding tickets.

For the market sessions, a second public continuous double auction (CDA) market was introduced. The two markets had a common endowment (D tickets and C cash), and a common budget constraint. For example, if one had 6 tickets in total, one could not both offer to sell 4 and submit 4 for cash-in. The platform would give a message of violation of the budget constraint. In the public market, as in any CDA market, all orders were limit buys and sells that accumulated in the book, anonymously. When an incoming order crossed a sitting order in the book (an incoming buy at a price at or higher than the best sell sitting in the book, or an incoming sell at a price at or lower than the best buy in the book) a transaction immediately executed at a price equal to the standing order price. Figure 1 presents the interface through which participants were trading and interacting with the experimenter ticket submission.

[14] Note that the platform Flex-E-Markets was used in this baseline session as the communication device, only for consistency purposes with the treatment session. The private market serves no market purpose, it is just a communication device. However, since with each sell order, participants had to also submit a price, per the platform requirements, they were instructed to submit a random price. The inclusion of the price is only for the purpose of the private sell order going through. On the experimenter end, prices were ignored and only information about total cash-ins was collected.

Shortsales were allowed in all but five sessions (MKT-W1 to MKT-W5). When selling short, traders were exposed to the risk of being chosen to pay the face value (or liquidation value) of a holder who submitted his/her ticket for cash-in. Except for those same sessions, sessions under the weak-form liquidation rule had a trading round following the announcement of the liquidation of the pool, but before the tickets paid off. There, participants had a choice to buy or sell tickets, with the knowledge that the pool would be liquidated and all outstanding tickets would pay the pro-rata value of the remaining money in the pool. The resulting prices give us an indication as to market expectations of the intrinsic value of a ticket upon liquidation. See Figure 3.

In the early sessions (MKT-W1 and MKT-W2), requests for cash-in were submitted *after* trading in the marketplace. Unfortunately, the amount of mouse clicking (needed) to submit cash-in requests) could signal actual cash-in intentions, and hence, artificially reduce the duration of the pool. For this reason, in sessions MKT-W3, MKT-W4, and MKT-W5, loud background clicking noise was used to mask individual cash-in submissions. In the remainder of the experiments, we merged trading and submissions for cash-in so that background clicking noise was no longer needed; the clicking generated by regular trading was sufficient to mask cash-in submissions.

V. Results

A. Preliminary

Figures 2, 3 and 4 illustrate the nature of the data that our experiments generated. Each figure presents one session only in each of the three treatments — baseline (BASE4), market sessions with weak-liquidation rule (MKT-W6), and market with strong-liquidation rule (MKT-W9). The full set of results will be discussed later on.

Figure 2 shows how requests for cash-ins evolve over time. Very few tickets are submitted in the first round, and increase over the rounds, suggesting that mistakes (of forgetting to cash in) alone are not the reason for pool survival, unless mistakes are far more common early on. We will return to a direct test of the conjecture that mistakes constitute the sole reason for pool survival. Upon replication, pool duration is reduced. The reduction is gradual but always incomplete: we have never observed a session where the pool is eventually liquidated in round 1.

Before explaining Figure 3 several definitions are needed.

Definition 2. *The intrinsic value of the ticket in round t is equal to* $IV_t = \frac{P_t}{m_t}$.[15]

Definition 3. *The effective value of the ticket in round t is equal to* $EV_t = \frac{\sum_{\tau=t}^{t^*} \pi_\tau s_\tau}{m_t}$.[16]

The intrinsic value is an ex-ante measure of ticket value. The effective value of a ticket, on the other hand, is an ex-post measure of ticket value at round t, conditional on the actual pool survival, it represents the average value of each ticket that is still not cashed in at round t.

Figure 3 displays the evolution of transaction prices against several value measures: (i) current face value, (ii) current intrinsic value; (iii) current effective value. Transaction prices are remarkably stable over time; the current round's price is the best predictor of the subsequent round's price, in accordance with simple formulations of the Efficient Markets Hypothesis, Fama (1998). Transaction prices, in general, are way above intrinsic value, suggesting that there are sizable price bubbles. However, effective values are also above intrinsic values, so this in itself does not imply that prices do not correctly reflect actual payments. Nevertheless, effective values are consistently below transaction prices as well, suggesting over-pricing

[15] P_t is the pool size and m_t is the number of outstanding tickets at the beginning of the round t.

[16] s_t is the number of submitted tickets in round t, π_t is the payoff per ticket in round t, and t^* is the liquidation round.

even if one accounts for the fact that the pool survives several rounds, and hence, total payments will be higher.

Formal tests will show that transaction prices correlate strongly with effective values (across replications), but that transaction prices do not drop sufficiently as effective value decreases in later rounds of the replication. Formal tests will also show that prices do not predict pool duration. This latter observation is crucial to understand why the introduction of markets fails to eliminate pool survival. Prices do not signal collective cash-in plans, and hence, pool survival. Prices do not aggregate information about cash-in plans in an unbiased way, in violations with sophisticated formulations of the Efficient Markets Hypothesis (Radner, 1979; Grossman and Stiglitz, 1980; Grossman, 1981).

Finally, Figure 4 shows how pool survival generates wealth inequality and how short-sellers tend to generate more wealth. The solid line indicates the per capital level of wealth that would have been reached under the Nash equilibrium, i.e. the number of tickets each participant initially holds times the intrinsic value in the first round plus initial cash endowment. Actual average wealth is always higher; however, about a third of the participants have a realized wealth that is lower than what they would have had if the pool has liquidated in round one (represented by the solid line). Later on, we will formally study the determinants of wealth inequality.

B. Pool Duration

Table III lists the number of rounds the pool survived across all replications and sessions (not only the ones depicted in Figures 2 to 4). The theory predicts that the pool never survives more than one round. However, when pool growth is positive, there are social welfare gains from letting the pool survive, counteracted by the increased incentives to cash in early and collecting the (much higher) face value. “Social rationality” partly trumps individual rationality.

Furthermore, the existence of the secondary market does not shorten the life of the pool. Indeed, the average and median pool durations (4.18 and 4) of the market treatment sessions are not statistically significantly different from those of the baseline treatment (3.7 and 3.5).

C. Welfare Creation

Table III documents that the per-ticket payout has effectively been higher than the payout predicted in the theory. This implies that the effective value of a ticket as of round 1 has invariably been above its intrinsic value, by an average of 22% across the three treatments. Because the pool is never liquidated in the first round, welfare generated is always positive.

Of interest to us is the impact of markets. Wealth created above the equilibrium prediction is significantly higher in the two markets treatments (average of 26% across the two treatments) than in the absence of markets (17%; the difference is significant at $p = 0.02$). We conclude that the ability to trade tickets is beneficial for social welfare.

Theory predicts that the liquidation rule (weak vs. strong) should have no impact. This prediction does not hold up. The wealth created under the weak liquidation rule, at 28%, is significantly higher than that created under the strong liquidation rule, at 22% with p value of 0.03. In addition, when comparing each market treatment against the baseline treatment, while the weak liquidation rule provides significantly higher social welfare (28% vs 17%, $p = 0.003$), the strong liquidation rule does not (22% vs 17%, p value of 0.24).

D. Prices

In theory (Lemmas 1 and 2), the pool never survives, and hence, the value of a ticket equals the intrinsic value. We define a bubble as a situation where actual (transaction) prices are above the intrinsic value. The bubble could be rationalized by the fact that some participants postpone cash-in, and hence, the pool has the opportunity to grow, which means that the effective value may be higher than the intrinsic value. Only if prices are above effective value is there irrational over-pricing, and hence, an irrational bubble.

We ran a number of regressions (general linear modeling, with fixed/random effects where appropriate, based on Akaike and Bayesian Information Criteria) to assess the informational content of prices. We study two aspects of prices: (i) their ability to predict bubble duration; (ii) their ability to predict effective value.

The motivation for (i) is that, in a rational expectations equilibrium, prices should reveal plans of participants in an unbiased way, and hence, eventual pool duration (Radner, 1979; Grossman and Stiglitz, 1980; Grossman, 1981). To the extent that prices do so, participants can infer when the pool will be liquidated, and attempt to cash in the round before. In turn, this will hasten the demise of the pool, and hence, convergence to Nash equilibrium. However, we already know that pool duration is no shorter in the presence of markets, indicating that prices did not reveal pool duration, or, if they did, participants failed to pay attention. Here, we test whether pool duration could be inferred from prices.

In order to ensure that our regressions were well specified, we ran an exhaustive model selection analysis using two information criteria (Akaike criterium AIC and Bayesian criterium BIC) to select the best one (see Bossaerts and Hillion (1999) for an early application in finance). Various potential confounding factors were considered, including the number of participants, replication number, and session-specific random coefficients (intercepts). In all cases, AIC and BIC agreed on the ranking of the models. We report only the best model, and if this model did not include the regressor of interest (in this case, average trade price in round 1), we display results for both the best model and the model including the regressor of interest, and the corresponding levels of AIC and BIC.

Table IV presents the results. Regardless of the liquidation rule, only the replication number predicts duration: on average, pool duration is reduced by one round every 4 replications ($4 \times (-0.280) \approx -1$ and $4 \times (-0.267) \approx -1$ in weak-form and strong-form rules). The average round-1 trade price is not retained as a regressor in the best model. When we nevertheless introduce it, the t statistic is a mere 0.527 (weak-form liquidation rule) or 0.915 (strong-form liquidation rule). Note that the regression results (intercepts, slope coefficients, session random effects) are very much alike regardless of liquidation rule, as predicted in the theory even if the theory does not explain pool duration. It suggests that some of the discussion regarding off-equilibrium mistakes and incentives to take advantage of them are relevant to explain observed behavior.

Table V displays the results for effective values. This time, the average first-round trade price is retained in the optimal model, in both treatments. The coefficient sizes across treatments are 0.266 for Weak-Form Liquidation Rule sessions and 0.151 for Strong-Form Liquidation Rule sessions. Ideally, the coefficient should have been 1 (and the intercept 0), yet the estimates are all much smaller, suggesting that transaction prices over-estimate effective value. This is *prima facie* evidence of irrational bubbles.

There is a negative interaction between replication number and price. This means that over-pricing (relative to effective value) is even more pronounced in later replications. The magnitudes of the interaction effects are at -0.031 and -0.041 and significant. Over-pricing increases by 13% (weak-form liquidation rule) and 37% (strong-form liquidation rule) for every additional replication. The interaction term captures how effective value decreases over time while trade prices remain relatively constant.

E. Wealth Inequality

While the presence of financial markets leads to significant increases in social welfare (see Table III), it also exacerbates wealth inequality (Figure 4). We now show this formally.

Measuring wealth inequality using the Gini coefficient G of final wealth (final cash, accumulated through trading and cash-in requests), we discovered that G was significantly higher in sessions with markets than without markets (p value of 0.04 based on a two-sample t test; the difference is significant at $p = 0.04$).

We ran regressions to identify the determinants of wealth inequality; see Table VI. The best model (lowest AIC and BIC) included pool duration as the most significant explanatory variable. The effect sizes vary between 0.024 (No Markets) and 0.051 (Weak-Form Liquidation rule).

Figure 4 illustrates how participants who short-sold tickets tend to be among the top earners. The results for all sessions confirms that short-sellers rarely end up with wealth below the median. This does not mean, however, that long-only trading strategies are loss-making. As an example, Table VII estimates returns on a simple long-only strategy whereby tickets are bought in round 1, at average round 1 prices, and sold, at average prices, in the first round after the face value increased beyond the average transaction price. In the first replication of MKT-W9 session, for instance, Figure 3 indiciates that the strategy that would buy in round 1 and sell in round 4 is profitable. Overall, in the two market treatments, the return is positive in 55% of the replications. The mean and median returns are -0.5% and 1% and both are insignificantly different from zero. The mean is insignificant, suggesting that prices are a martingale. Markets can be said to be "efficient" in the simple meaning of the term (Fama, 1998).

VI. Robustness tests

A. Control sessions

It is popular to attribute the failure of traditional (individually rational) game theory in terms of mistakes. The quantal response equilibrium (McKelvey and Palfrey, 1992) builds on this. Mistakes could indeed be an alternative explanation for why our pool survives beyond the first round. We now demonstrate that mistakes alone cannot explain pool survival beyond round 1. Specifically, we ran three additional sessions (CTR-r1, CTR-r2, and CLASS). In CTR-r1 and CTR-r2 we changed the growth rate r, while in CLASS we changed the solvency of the pool in round 1, as well as the composition of participants in each replication, by including a small portion of newcomers, as to change the socially and individually optimal dissolution of the pool respectively. Table VIII shows the parameters used in these control sessions.

In replications of sessions without market CTR-r1 and CTR-r2, we randomly switched from a growing ($r = 0.10$) to a zero-growth or a negative-growth pool (see Table IX), so that there are no gains or even losses from waiting. If mistakes are the only reason for why the pool survives, we should see equal pool durations across the three parameterizations of the growth rate. If instead, pool survival is partly attributable to social rationality, we expect longer pool duration when the pool grows. There is no social benefit to waiting in the replications with no growth or decreasing growth in these control sessions, so pool duration should be lower. In addition, if participants react to those incentives, round 1 cash-in requests should increase significantly under negative growth, while if mistakes are random, there should be no effect.

In CLASS sessions we test the sensitivity of individual cash-in decisions to pool solvency in round 1. When the pool is solvent there is no tension between social and individual rationality. In the CLASS sessions, we allowed the initial intrinsic value to be higher than the face value in round 1 and so the pool was solvent in that round. If participants paid attention to pool solvency and their behavior was not driven by mistakes, one would observe the number of cash-ins in round 1 of CLASS to be significantly less than in the other treatment sessions. Otherwise, pool duration is attributable to mistakes.

B. Mistakes

When the pool growth is *negative* (CTR-r1 and CTR-r2), there are no social welfare gains from letting the pool survive. Evidently, participants paid attention to this and cashed in many more tickets sooner in the first few early rounds, leading the pool liquidation in round 1 or 2 (see Table IX). When the pool lasted more than one round, wealth was destroyed (welfare created is negative) by 3.5%. Yet we observe that wealth changes are often zero under negative growth. This means that if the pool had been liquidated immediately, wealth destruction would have been avoided. This also implies that participants were aware of the potential for wealth destruction, and hence, cashed in early, in contrast to their behavior when pool growth was zero or positive. In addition, in CLASS sessions, we found that participants also paid attention to pool solvency, because fewer cash-ins occurred in round 1. In eight out of eleven replications, no submission happened in round 1. Furthermore, we found that across all replications, on average, 0.5% of the tickets were cashed in at the end of round 1. It is significantly lower, with p value of 0.001, compared to an average of 6.3% in the first round of other sessions with markets, where the pool is always insolvent from round 1 on.

The results in Table X confirm these findings. In particular, we estimate how pool growth affected percentage tickets submitted in round 1, pool duration, and effective value. The Table displays regression results for the best-fitting model (measured using the AIC and BIC) for two experimental sessions CTR-r1 and CTR-r2. The regressions confirm that round 1 cash-in requests increase dramatically, from 29% (of outstanding tickets) to 81% (29 + 52) when the pool experiences negative growth. Pool duration decreases marginally when the pool either remains constant or decreases.

The magnitude of effective value should be reduced significantly under zero or negative growth, and more so when participants do not cash-in early in these replications. The effective value is reduced from 1.195 dollars under pool growth to an estimated 1.031 under zero or negative growth. If we compare the latter to the round 1 intrinsic value, which equals 1.04 dollars, we conclude that participants changed their strategies sufficiently to eliminate most losses caused by negative growth. This is consistent with the limited welfare losses in sessions with zero or negative growth as well as with shorter pool durations as listed in Table IX.

Therefore, human mistakes alone cannot explain pool survival beyond round 1.

C. Decay in pool duration

To address the decay in pool duration upon replication, in the CLASS session, a portion of participants were replaced by new ones for the subsequent replication. 11 replications were organized (online), and a special website was set up to facilitate communication and logistics. All new and current participants could access the past results on this website, such as transaction prices, ticket face value, an intrinsic value, as well as an effective value in each round, and how many rounds it took in previous replications to liquidate the pool.

The CLASS session was organized as part of an introductory finance class at Caltech. All students in the class were free to sign up for the replications and received fixed class credit for participation. In addition, they were paid for performance exactly as before, without any impact on their class grade. Students in the class came from various backgrounds (graduate, undergraduate, covering diverse majors, from physics to biology and business economics and management). In total, 125 students participated, with 80 repeated participants across replications. The average and the median number of participants per replication were 11 and 12.

In these sessions, the decay disappears, even if the newcomers had access to the entire history of prior replications. Table IX shows no decline in duration after 11 replications. We perform a formal test similar to that in Table IV to predict pool duration, and the replication number does not have predictive power anymore.

VII. Concluding Remarks

We document how social rationality invalidates predictions from traditional game theory, even in the presence of markets that, in principle, could potentially signal players' plans, and hence, expose socially rational players to exploitation by greedy players. Financial markets actually enhance social rationality, by allowing assets to be moved from individually rational to more socially conscious players, at the cost of higher wealth inequality. Pricing appears to be in line with the Efficient Markets Hypothesis, except that the level is wrong: overall, irrational bubbles emerge, since prices are generally above effective value, even if they correlate significantly with it. Mistakes alone cannot explain our findings. Finally, we observe the usual decay in social rationality. Decay disappears, however, once players are not identical across replications, even if newcomers have access to the entire past history of outcomes.

Because irrational overpricing re-emerges robustly upon replication, we consider our design ideal to study asset price bubbles experimentally. Our setting has the advantage that it is more intuitive than the traditional design with which irrational bubbles are analyzed experimentally, namely the Smith-Suchanek-Williams setup (Smith, Suchanek, and Williams, 1988). We would also claim that our setting captures the essence of real-life bubbles (Martin et al. (2021)). First, its emergence is beneficial for the group (the total payout to the participants is higher than in the absence of bubbles). Second, the bubble will eventually burst. Third, bubbles robustly re-emerge upon replication as long as new players are allowed into the game.

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Figure 1. The Trading Interface

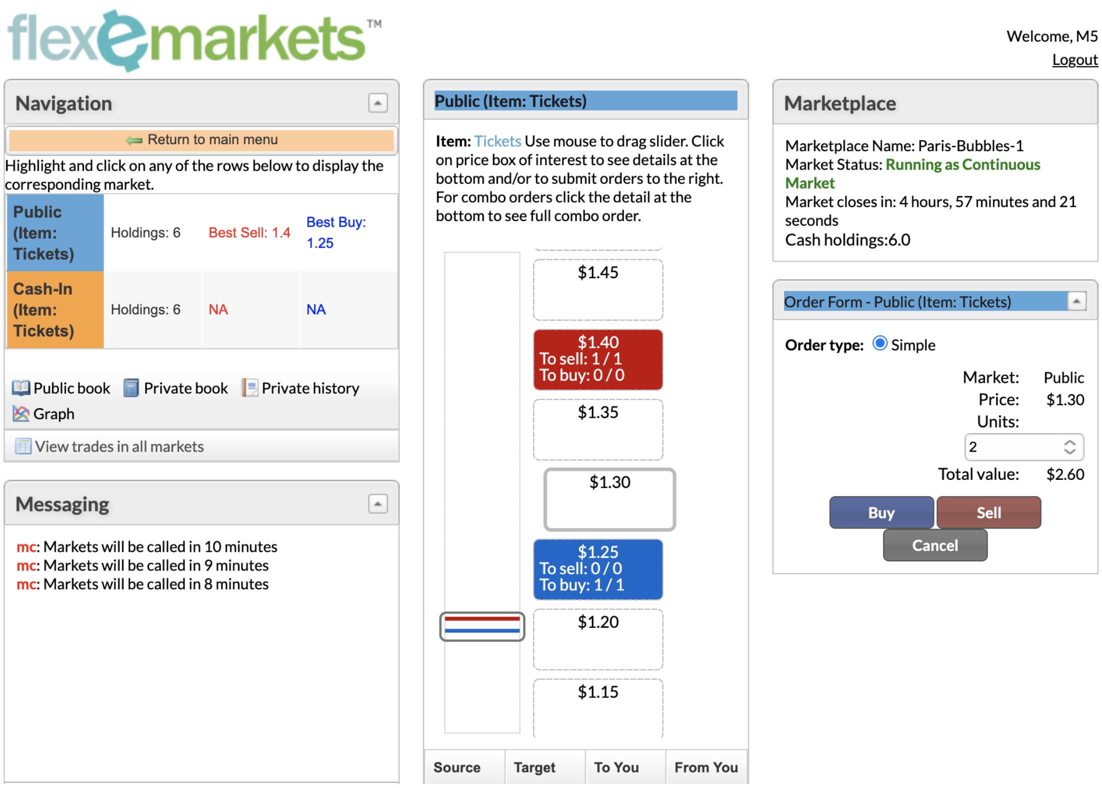


Figure 2. Ticket Cash-ins

Figures show the evolution of the percentage of tickets cashed in at the end of each round in sessions with no secondary ticket market, BASE4, and with secondary ones, MKT-W9 and MKT-S3.

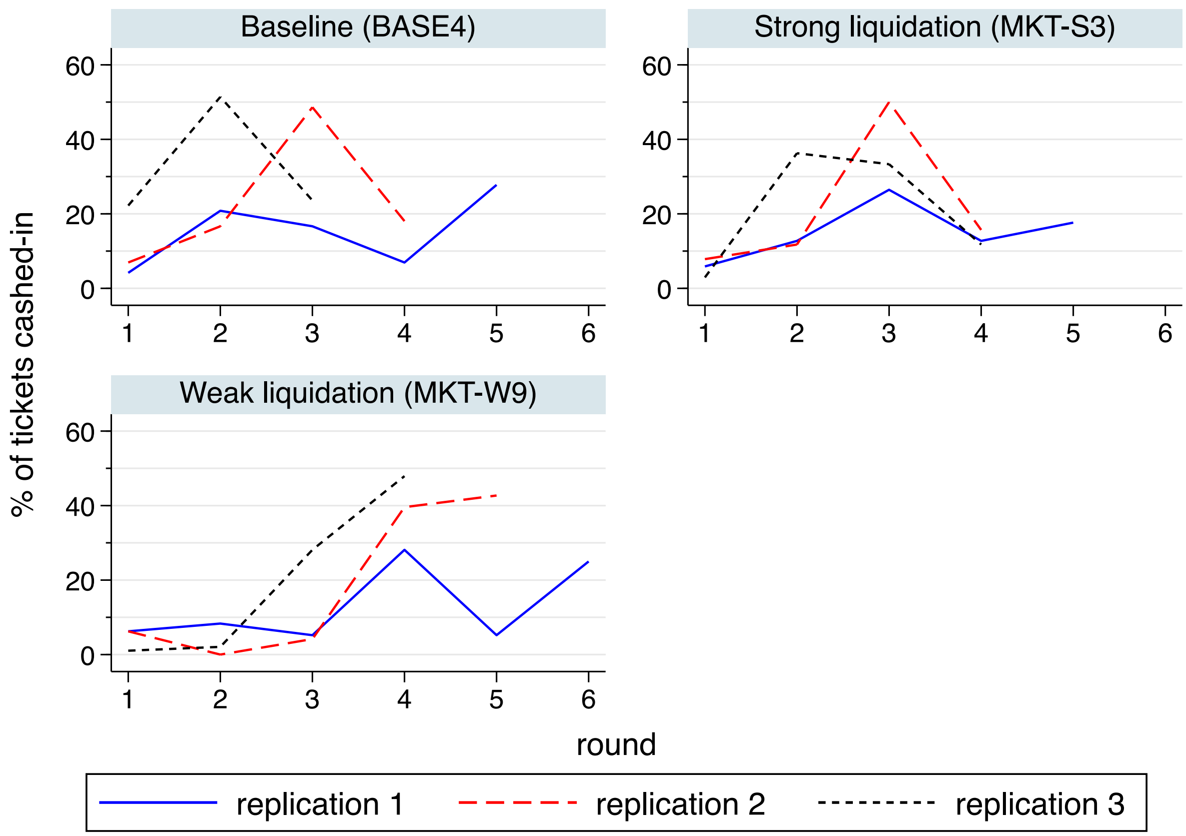


Figure 3. Evolution of Prices and Values

Figures illustrate the transaction price, effective value, face value, and intrinsic value in each replication. In the weak liquidation session (MKT-W9), there was an extra trading round after the pool was declared in default.

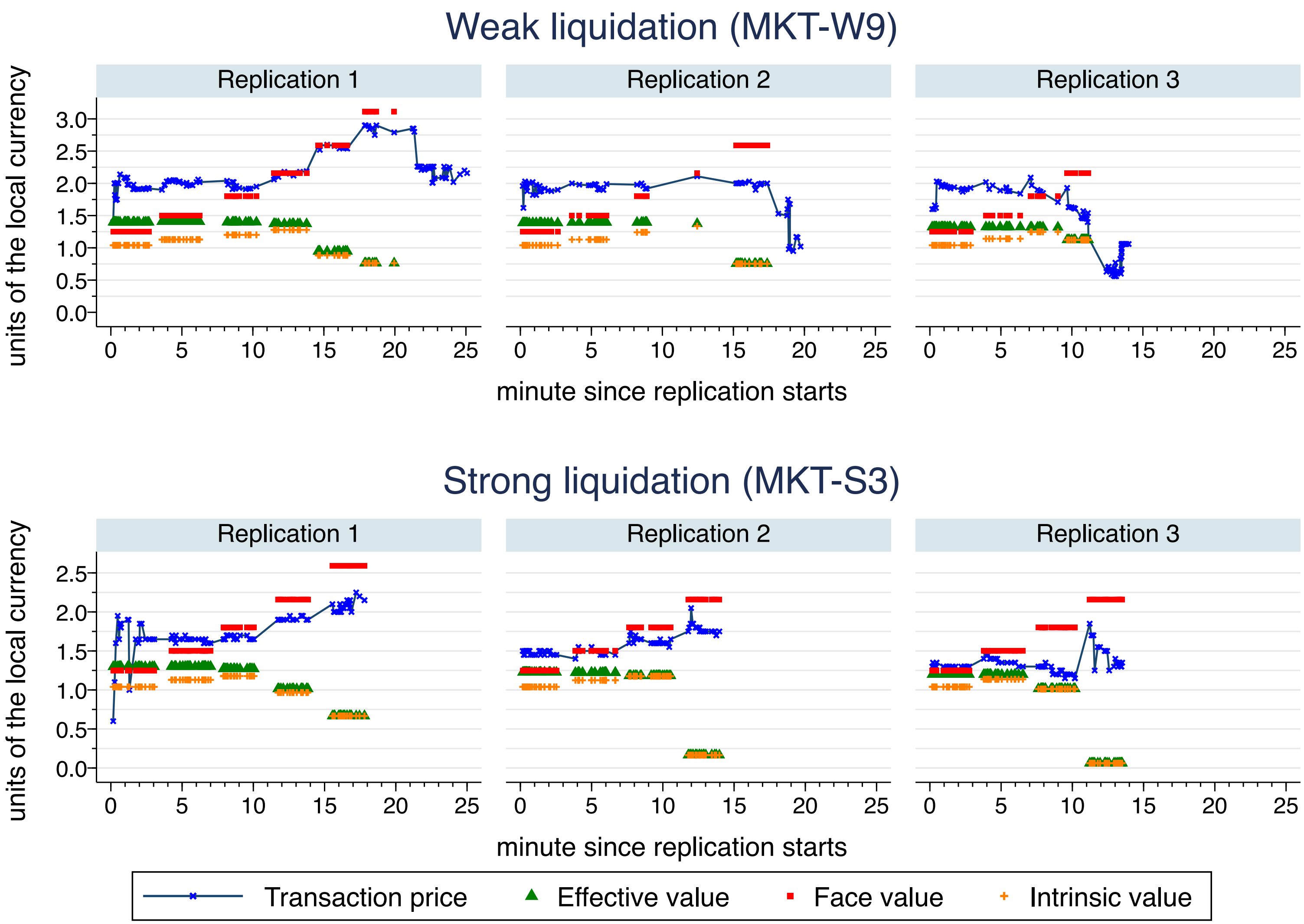


Figure 4. Final Wealth Distribution and Short-sales

Figures present the final wealth and the number of short-sold tickets per participant. The *Wealth benchmark* is the level of wealth that can be achieved if theory held.

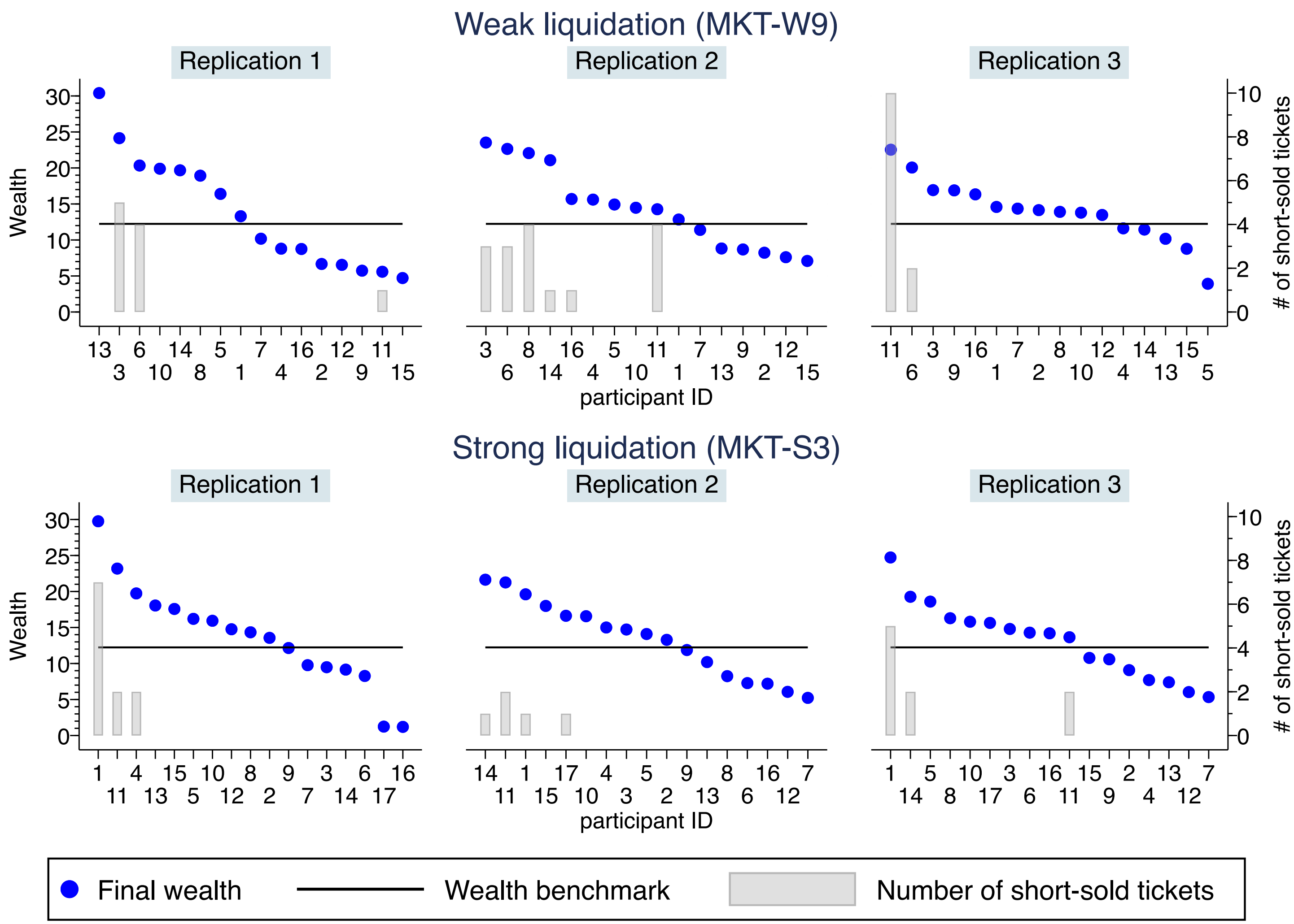


Table II
Experimental Sessions

Listed are: session name (column 1); location (column 2); whether participants could trade tickets in a public market (column 3); the liquidation rule used (S: Strong-form; W: Weak-form; column 4); whether cash-in requests followed trade or were simultaneous with trade (column 5); whether short sales were allowed (column 6); the number of participants (column 7); the number of replications (column 8). Session MKT-W10 had only 13 participants in replications 2 and 3. The following parameter values remained the same across sessions: $IV_1 = 1.04$, $F_1 = 1.25$, $R = 0.20$, $r = 0.10$, $D = 6$, and $C = 6$ units of the local currency. A few sessions have different parameter values that are indicated in column 9.

(1) Session	(2) Location	(3) Markets?	(4) Liquid. Rule?	(5) Cash-in After?	(6) Short Sales?	(7) # of Subjects	(8) # of Replic.	(9) Parameter Values
BASE1	Caltech	No	W	NA	NA	8	3	$C = 0$
BASE2	Caltech	No	W	NA	NA	12	3	$C = 0$
BASE3	Caltech	No	W	NA	NA	12	5	$C = 0$
BASE4	Caltech	No	W	NA	NA	12	3	$C = 0$
MKT-W1	EPFL	Y	W	Y	N	16	1	$C = 10$
MKT-W2	Caltech	Y	W	Y	N	22	2	$IV_1 = 1$
MKT-W3	Caltech	Y	W	Y	N	14	2	
MKT-W4	Caltech	Y	W	Y	N	18	3	
MKT-W5	Caltech	Y	W	Y	N	13	3	
MKT-W6	Caltech	Y	W	N	Y	10	3	
MKT-W7	Caltech	Y	W	N	Y	14	4	
MKT-W8	Caltech	Y	W	N	Y	15	3	
MKT-W9	U Utah	Y	W	N	Y	16	3	
MKT-W10	U Utah	Y	W	N	Y	19/13	3	
MKT-W11	U Utah	Y	W	N	Y	10	3	
MKT-S1	Melbourne	Y	S	N	Y	17	5	
MKT-S2	Melbourne	Y	S	N	Y	15	3	
MKT-S3	Melbourne	Y	S	N	Y	17	3	
MKT-S4	Melbourne	Y	S	N	Y	14	5	
MKT-S5	Melbourne	Y	S	N	Y	19	4	

Table III
Pool Duration and Wealth Created

Table reports: [1] *Pool durations*, which is the number of rounds before pool is liquidated and [2] *Wealth created* or per-ticket value created, which is the effective value divided by round-1 intrinsic value minus 1.

	Replication Number					Replication Number				
Session	1	2	3	4	5	1	2	3	4	5
	Pool durations					*Wealth created*				
BASE1	6	4	3			0.27	0.20	0.11		
BASE2	5	3	2			0.26	0.17	0.06		
BASE3	6	4	3	2	2	0.40	0.25	0.10	0.03	0.01
BASE4	5	4	3			0.27	0.17	0.08		
MKT-W1	7					0.53				
MKT-W2	6	5				0.28	0.33			
MKT-W3	5	3				0.24	0.26			
MKT-W4	5	4	3			0.25	0.25	0.17		
MKT-W5	6	4	3			0.36	0.31	0.21		
MKT-W6	5	3	3			0.38	0.24	0.15		
MKT-W7	5	4	3	2		0.41	0.28	0.20	0.12	
MKT-W8	5	4	4			0.38	0.30	0.21		
MKT-W9	6	5	4			0.40	0.39	0.32		
MKT-W10	6	4	3			0.44	0.33	0.19		
MKT-W11	4	3	3			0.24	0.18	0.15		
MKT-S1	5	4	3	2	2	0.31	0.24	0.16	0.10	0.02
MKT-S2	8	6	5			0.45	0.38	0.25		
MKT-S3	5	4	4			0.25	0.17	0.15		
MKT-S4	5	4	3	2	2	0.26	0.16	0.17	0.10	0.07
MKT-S5	6	5	4	3		0.34	0.34	0.26	0.16	

Table IV
Prices as Predictors of Pool Duration

Table reports the regressions of pool duration on average first-round prices, and regressors capturing potential confounding factors. Those factors are: session random effects, replication number, dummies for short-sale restrictions, cash-in submissions simultaneous with trading, and interaction terms. Only the best model according to information criteria (AIC; BIC) is shown, except if the criteria exclude the price as a predictor, in which case a second regression, with the price as a regressor, is displayed. Modeling is based on a Poisson distribution for pool duration, where the mean is governed by a log-linear relation with explanatory variables. Session random effects are allowed for (standard deviation of effects is shown), and only if retained under the best AIC/BIC. t statistics in parentheses.

Sessions	Intercept	Average Price	Replication Number	Session Random Effects	AIC/BIC	Number of Observations
Weak-Form Liquidation	1.965 (9.367)		-0.280 (-2.598)	NO	19.9/ 22.66	30
	1.362 (1.052)	0.309 (0.527)	-0.249 (-2.061)	NO	21.5/ 25.7	30
Strong-Form Liquidation	2.049 (8.746)		-0.267 (-2.834)	NO	16.8/ 18.8	20
	0.935 (0.754)	0.663 (0.915)	-0.241 (-2.426)	NO	18.1/ 21.1	20

Table V
Prices as Predictors of Effective Value

Table reports the regressions of effective value on average first-round prices, and regressors capturing potential confounding factors. The effective value is defined as the total payout from the pool during the replication, divided by the number of tickets issued. The confounding factors are: session random effects, replication number, dummies for short-sale restrictions, cash-in submissions simultaneous with trading, and interaction terms. Only the best model according to information criteria (AIC; BIC) is shown; this always included the average first-round price as a regressor. Modeling is based on a gaussian distribution for effective value, where the mean is governed by a linear relation with explanatory variables. Session random effects are allowed for (standard deviation of effects is shown), and only if retained under the best AIC/BIC. t statistics in parentheses.

Sessions	Intercept	Average Price	Replication Number Interaction	Session Random Effects	AIC/BIC	Number of Observations
Weak-Form Liquidation	0.927 (10.100)	0.266 (5.796)	-0.031 (-5.300)	0.044	-87.1/ -80.1	30
Strong-Form Liquidation	1.197 (14.774)	0.151 (3.308)	-0.041 (-11.026)	0.059	-61.6/ -56.7	20

Table VI
Wealth Inequality With and Without Markets

Table reports the regressions of Gini coefficients of wealth on potential explanatory variables: pool duration, average first-round prices (if there were markets), and the number of participants. Only the best model according to information criteria (AIC; BIC) is displayed and compared to the full model. Modeling is based on a gaussian distribution for the Gini coefficient, where the mean is governed by a linear relation with explanatory variables. Session random effects (standard deviation of effects is shown) are allowed for, and only if retained under the best AIC/BIC. t statistics in parentheses.

Sessions	Intercept	Average Price	Participant Number	Pool Duration	Session Random Effects	AIC/ BIC	# of Obs.
Weak-Form Liquidation	0.108	-0.097		0.051	0.033	-70.7/	30
	(1.457)	(-1.893)		(5.993)		-65.0	
	0.058	-0.097	0.004	0.050		-69.0/	30
	(0.557)	(-1.924)	(0.719)	(5.191)		-62.2	
Strong-Form Liquidation	0.036			0.034	NO	-59.1/	20
	(1.190)			(4.950)		-56.2	
	0.019	-0.092	0.009	0.039	NO	-58.7/	20
	(1.600)	(-1.440)	(1.570)	(5.720)		-53.8	
No Markets	0.044			0.024	NO	-37.3/	14
	(1.083)			(2.174)		-35.4	

Table VII
Return on a Sample Long-Only Strategy

Table shows returns from buying one ticket at average trade prices in round 1 and selling at average trade prices in the first round when the face value increases above the average trade price.

	Replication Number				
Session	1	2	3	4	5
MKT-W1	0.17				
MKT-W2	0.14	0.09			
MKT-W3	-0.02	-0.09			
MKT-W4	0.00	-0.01	0.02		
MKT-W5	0.05	0.05	-0.25		
MKT-W6	-0.04	-0.35	0.04		
MKT-W7	0.05	-0.09	-0.07	-0.09	
MKT-W8	0.01	0.05	-0.12		
MKT-W9	0.01	0.10	-0.19		
MKT-W10	0.03	-0.19	-0.11		
MKT-W11	0.05	0.05	0.18		
MKT-S1	0.01	-0.01	-0.11	-0.12	N/A
MKT-S2	0.20	0.06	-0.09		
MKT-S3	0.02	0.10	0.09		
MKT-S4	-0.08	0.06	0.06	-0.03	-0.04
MKT-S5	0.22	0.08	0.03	-0.16	

Table VIII
Control Sessions

Listed are control sessions with weak-form liquidation rule: session name (column 1); location (column 2); whether participants could trade tickets in a public market (column 3); whether cash-in requests followed trade or were simultaneous with trade (column 4); whether short sales were allowed (column 5); the number of participants (column 6); the number of replications (column 7). In total, 125 participants participated in Session CLASS, and 80 repeat-participants were allowed; average/median number of participants per session: 11/12.). The following parameter values remained the same across sessions: $F_1 = 1.25$ and $R = 0.20$. Other parameter values are in column 8.

(1) Session	(2) Location	(3) Markets?	(4) Cash-in After?	(5) Short Sales?	(6) # of Subjects	(7) # of Replic.	(8) Parameter Values
CTR-r1	U Utah	No	NA	NA	17	6	IV_1=1.04, $r \leq 0$, D=6, C=0
CTR-r2	U Utah	No	NA	NA	18	6	IV_1=1.04, $r \leq 0$, D=6, C=0
CLASS	Caltech	Y	N	Y	80/125/11	11	IV_1=1.30, r=0.1, D=5, C=6

Table IX
Control sessions: Pool Duration and Wealth Created

Panel A shows pool durations. Pool growth: everywhere increasing at $r = 10\%$ except where indicated in parentheses (0: constant pool; $-$: pool decreases at $r = -10\%$ per round). Panel B reports welfare created or per-ticket value created, which is the effective value divided by round 1 intrinsic value minus 1.

	Replication Number										
Session	1	2	3	4	5	6	7	8	9	10	11
Panel A: Pool durations											
CTR-r1	5	3	3	2	2	1					
		(0)		(–)	(0)	(–)					
CTR-r2	4	2	4	3	1	2					
	(0)	(–)			(–)	(0)					
CLASS	5	5	4	5	8	8	6	6	8	6	7
Panel B: Welfare created											
CTR-r1	0.23	0	0.13	-0.03	0	-0.00					
CTR-r2	0	-0.04	0.15	0.09	-0.00	0					
CLASS	0.65	0.62	0.49	0.70	0.86	0.77	0.85	0.86	0.88	0.84	0.75

Table X
The Effect of Mistakes

Table reports the estimates of the regressions of (i) percentage tickets submitted for cash-in in round 1, (ii) pool duration, and (iii) effective value, on a pool growth dummy (pool constant or decreasing, or only decreasing). Percentage tickets submitted and effective value are modeled with a gaussian distribution and a linear link function; duration is modeled using a Poisson distribution and log-linear link function. Only the best-fitting model is displayed; fit determined by information criteria (AIC/BIC). No session fixed/random effects were needed. t statistics in parentheses.

Dependent Variable	Intercept	Pool Constant/ Decreases	Pool Decreases	AIC/ BIC	Number Observations
Round-1 Percent Cash-in Requests	28.813 (4.302)		52.437 (4.521)	110.7/ 112.1	12
Pool Duration	1.322 (5.119)	-0.568 (-1.603)		18.7/ 19.7	12
Effective Value	1.195 (70.003)	-0.164 (-7.832)		-41.0/ -39.5	12

Appendix: Instructions for Participants (Weak-Form Liquidation)

1. Summary

This experiment concerns a market in tickets (claims) issued against a pool of money. Across a number of rounds, you will be able to trade the tickets with others at prices determined in the marketplace. Each round, you will also be able to submit the tickets to us, the experimenters, to be cashed in for a known amount called the face value. This face value grows over time (across rounds). The pool of money against which the tickets are issued is used to pay those who want to cash-in. This pool also grows over time, but at a slower rate than the face value. If there is insufficient money in the pool to honor the requested cash-ins at face value, the pool is put in liquidation, and all outstanding tickets will be paid pro rata.

The experiment will last at most 9 rounds. Your earnings for the experiment depend entirely on the cash you hold at the end of the last round. Changes in cash are determined by your trading choices (you want to buy low and sell high) and cash-in decisions (you want to maximize income from ticket submissions).

2. Details Of The Setup

In round 1, you and the other participants hold a total of N tickets (N will be announced at the beginning of the experiment). Each ticket has an initial face value of $1.25. In Round 2 the face value goes up by 20% and becomes $1.50. The face value continues to grow at the rate of 20% (subject to rounding to the nearest cent) in each subsequent round. The face values in all rounds are listed in red in the second column of the table below.

Round	Ticket Face Value (USD)	Value (USD) If Liquidation*
1	1.25	1.04
2	1.50	1.15
3	1.80	1.26
4	2.16	1.39
5	2.59	1.53
6	3.11	1.68
7	3.73	1.85
8	4.48	2.03
9	5.37	2.23

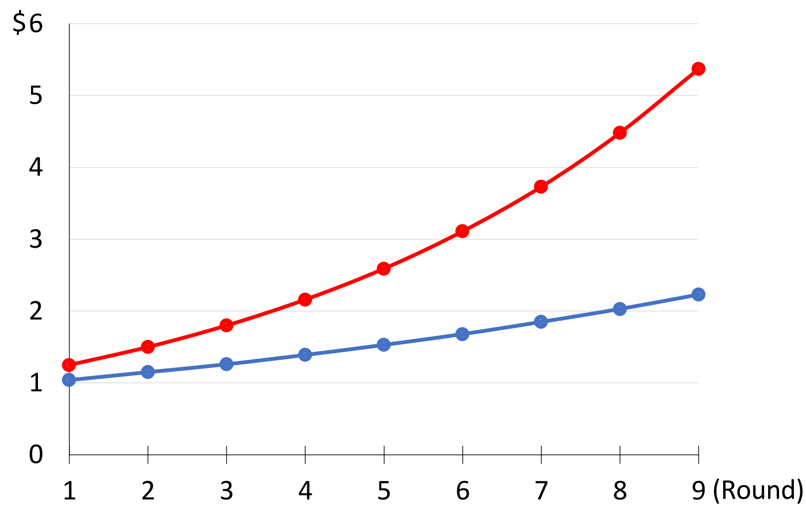
*Assuming no tickets were submitted for cash-in earlier

Each ticket is a claim to a pool of money. The pool of money starts with \$X in Round 1 of the experiment (X will be announced at the beginning of the experiment). Given the total number of tickets (N), there is insufficient money in the pool to pay everyone the face value even if *all* tickets are submitted for cash-in. For instance, at the end of round 1, there is \$X in the pool, so the amount of money per tickets, X/N, is insufficient to cover the face value, \$1.25. But the pool of money grows over time, so that there is more money available per ticket if everyone waits to submit tickets. The pool grows *after* paying tickets submitted for cash-in.

When not enough money is available to honor the requested cash-ins, the pool is put into liquidation, and the remaining money is divided among all outstanding tickets. If, say, \$45 is left in the pool when it runs out of money, and, say, 30 tickets remain outstanding, each ticket fetches \$45/30=\$1.50. This liquidation payment will necessarily be less than the face value even if nobody cashed in earlier. The third column of the Table above shows, in blue, how much each ticket would be paid if no tickets were submitted for cash-in earlier and the pool is liquidated after rounds 1,2,3,...,9, respectively.

Importantly, you will *not* know how many tickets were submitted in any given round (except your own, of course). As such, you will never be sure whether the pool will be put into liquidation until we announce so.

The figure below is a graphical depiction of the Table above; the red line indicates how the face value increases per round, and the blue line depicts how the liquidation value climbs.



In addition to tickets, you will begin the experiment with some cash. Use this cash to purchase more tickets if you wish. You can increase your cash either by selling tickets in the open market or by submitting tickets for cash-in. At the end of the experiment, all the cash you have is yours to keep.

3. Trading And Requesting Cash-Ins

In each round, you will be able to trade the tickets among yourselves in a market called **Public**. You can buy tickets and thus increase the number of tickets you held in the beginning of the round, or sell tickets, and thus decrease your ticket allotment. Public is a public market: everyone can see all orders that have been submitted, and trade always takes place at the best possible price.

In **Public**, you are allowed to short sell tickets. When you short sell, you keep the purchase price; however, if the buyer decides to submit your ticket for cash-in, we will make YOU pay for the face value or liquidation payment (whichever applies to tickets that were not acquired in a short sale). That is, we will take the payment out of YOUR cash, not the money pool. If you do not have enough cash, we will take away all your cash as penalty and approach another short-seller to cover the liability. Notice that this means that you (as short-seller) may end up paying for someone else's shortsales!

In addition, you will be able to submit for cash-in *some*, *all*, or *none* of your tickets by submitting sell orders in a market called **Cash-In**. This is the market you should use to request cash-in of tickets against the pool of money. Unlike Public, in this market you need to identify the person you want to sell to. Thus, you should only submit sell orders to us, the experimenters; we will be logged in as the participant with name "50." The Cash-In market is a private market, which means that nobody else will be able to see your order. Your request would be "valid" as long as you submit a sell order at a price equal to or below the face value.

Do not submit any other orders in Cash-In. If you submit an order to buy, or attempt to submit an order to sell to someone else besides us ("50"), you may forfeit your earning for the session.

While markets are open for trading, nothing happens to your sell orders in the Cash-In market (which also means that you may cancel them at any time before markets are paused). When markets are paused at the end of the round, the following will happen to your valid sell orders in the market Cash-In.

1. If the money pool is sufficient to cover the face values for all submitted sell orders, we will buy each submitted ticket for the face value for that round (as listed in the Table above).
2. If there is not enough money in the pool to honor all sell orders at the face value, the pool will be put in liquidation. This means that all remaining tickets (not only those submitted through sell orders in Cash-In) will receive the liquidation payment.

Liquidation proceeds as follows. First, we announce publicly that there is not enough money left in the pool. We then allow for one more trading round before we actually liquidate the pool, so that you still have the opportunity to trade tickets among each other before we pay the tickets. While Cash-In will remain open during that extra trading round, you do not need to submit orders in that market. At the end of the extra round, we pay the liquidation payment to all outstanding tickets.

The maximum number of rounds is 9. If at the end of any round before round 9, only one participant is still holding tickets, the experiment terminates automatically, and the remaining tickets are paid either the face value or the liquidation value, whichever is smaller.

Markets are open for 3 minutes in rounds 1 to 3, and for 2:30 minutes in all remaining rounds. There will be three sessions. Including instructions, practice, inevitable pauses between sessions and rounds, the experiment lasts about two hours.

Good luck!

Internet Appendix

A. Proofs of Lemmas 1 and 2

We put minimal restrictions on our parameters in order to be able to solve our game. The pool growth r has to be less than the growth of the face value (R). The round 1 intrinsic value of a ticket IV_1 should be less than the face value F_1. We shall let Ξ denote the ratio of the two, so $\Xi = IV_1/F_1 < 1$.

We put a restriction on the speed at which the pool grows (if not liquidated of course), namely r. The restriction is: $r < 1/D$. If the growth is too fast, it may be worth not cashing in early if nobody else cashes in yet the pool is close to liquidation. This restriction is analogous to the one in the centipede game, where the pile of money one receives in the next round upon passing must not be higher than the pile one gives up in the current round (because of passing). Since $D > 1$, $r < 1$, i.e., the pool is not allowed to double across rounds.

We need more restrictions, but at this point it is instructive to derive a few properties that hold without these additional restrictions, to understand how much we have gained. Indeed, we can already come up with good estimates of the critical number of tickets it takes for the pool to become liquidated if it has not liquidated yet.

First, define s_t^* as the maximum number of tickets that can be offered in round t without forcing the pool into liquidation. That is, s_t^* is defined as:

$$s_t^* F_t \leq P_t < (s_t^* + 1) F_t, \tag{A1}$$

where P_t denotes total pool size in round t. We refer s_t^* as the "sub-critical number." Let s_t the actual number of tickets submitted for cash-in in round t. Let $j_t = s_t^* - s_t$. We will refer to j_t as the *margin*, referring to the extra number of tickets that could have been submitted without forcing the pool into liquidation. Given a cash-in profile $\{s_1, s_2, ..., s_T\}$, let t^* be the round in which the pool is liquidated ($t^* \leq T$).

Property 1. *For* $1 < t \leq t^*$,

$$(0 \leq)\ \ j_{t-1}\frac{1+r}{1+R} \leq \frac{P_t}{F_t} < (1+j_{t-1})\frac{1+r}{1+R}.$$

In words, Property 1 states that the margin (the number of tickets that could be added to the actual number of cash-in requests without defaulting the pool) determines the ratio of the pool size and the face value in the *subsequent* round. The margin will therefore also determine the sub-critical number of tickets in the subsequent round (s_t^*), something we deal with in Property 2.

Proof. From the definition of s_t^*:

$$s_{t-1}^* F_{t-1} \quad \leq \quad P_{t-1} \quad < (s_{t-1}^* + 1)F_{t-1}.$$

Transform this into an expression involving the margin $j_{t-1} = s_{t-1}^* - s_{t-1}$:

$$j_{t-1}F_{t-1} \leq P_{t-1} - s_{t-1}F_{t-1} < (1 + j_{t-1}F_{t-1}).$$

Rearranging, and using $P_t = (P_{t-1} - s_{t-1}F_{t-1})(1+r)$ while $F_t = F_{t-1}(1+R)$,

$$j_{t-1}\frac{1+r}{1+R} \leq \frac{P_t}{F_t} < (1+j_{t-1})\frac{1+r}{1+R}.$$

$\square$

Property 2. *For* $1 < t \leq t^*$,

$$(0 \leq)\ \ s_t^* \leq j_{t-1} < (1+s_t^*)\frac{1+R}{1+r}.$$

Proof. We first prove the left-hand-side inequality, namely, that the margin number in the *prior* round provides an upper bound to the sub-critical number. Suppose not, i.e., suppose (because both numbers are integers) $s_t^* \geq j_{t-1}+1$. Then, starting from the definition of s_t^*,

$$\begin{aligned} P_t &\geq s_t^* F_t \\ &\geq (j_{t-1}+1)F_t \\ &= (j_{t-1}+1)F_{t-1}(1+R) \\ &> (j_{t-1}+1)F_{t-1}(1+r). \end{aligned} \tag{A2}$$

Since $P_t = (P_{t-1} - s_{t-1}F_{t-1})(1+r)$, from (A2) we have

$$\begin{aligned} P_{t-1} &> (s_{t-1}+j_{t-1}+1)F_{t-1} \\ &= (1+s_{t-1}^*)F_{t-1}. \end{aligned}$$

This contradicts the definition of s_t^*. So, since we are dealing with integers, $s_t^* \leq j_{t-1}$.

Next we prove the right-hand-side inequality. Again using the definition of s_t^*, and applying Property 1, we therefore have:

$$\begin{aligned} & P_t < (1+s_t^*)F_t \\ \Leftrightarrow\quad & \frac{P_t}{F_t} < 1+s_t^* \\ \Rightarrow\quad & j_{t-1}\frac{1+r}{1+R} < 1+s_t^* \quad \text{(using property 1)} \\ \Leftrightarrow\quad & j_{t-1} < (1+s_t^*)\frac{1+R}{1+r}. \end{aligned}$$

□

The above properties allow us to obtain an estimate of how many tickets are still around in excess of the sub-critical number, $m_t - s_t^*$. These tickets are so-to-say not "covered" by the pool. We refer to this number as the *excess*.

We need a lower bound on the excess, so that, first, players can assess how many tickets are still out there if the pool has not been liquidated yet (Property 3), and second, we ensure that the face value of a round is always larger than the liquidated value in the subsequent round (Property 4). The latter incentivizes players to cash in early, offsetting the effect of pool growth. We need a restriction on the relation between the initial number of tickets per capita D, the initial intrinsic value (IV_1) and the initial face value (F_1). Specifically, we want the initial face value to be high enough relative to the initial intrinsic value: Ξ ($= IV_1/F_1$) $< (D-1)/D$. If the pool starts barely "out of the money" (IV_1 is below but close to F_1), then D has to be high. Conversely, if D is low, then the pool has to start more "out of the money."

In the experiment we chose: $\Xi = 1.04/1.25 < 5/6$. We could afford to choose parameters close to the boundary because our restrictions are only *sufficient* for the existence of dominant strategies that result in liquidation of the pool in round 1.

Since we have been assuming that $r < 1/D$, the restriction $\Xi < (D-1)/D$ implies that $\Xi < 1 - r$. In our experiment, $r = 0.10$, so that the restriction does not bind: $\Xi = 5/6 < 1 - 0.10 = 0.9$.

Here is the implication of the extra restriction we have just introduced.

Property 3. *For $1 < t \leq t^*$, and provided $\Xi < (D-1)/D$,*

$$m_t - s_t^* \geq m_{t-1} - s_{t-1}^* \geq ... \geq m_1 - s_1^* \geq N + 1.$$

In words, as long as the pool has not been liquidated yet, the excess increases (weakly). This means that, in later rounds, at least as many tickets are not covered by the pool, or conversely, it takes an equal or lower number of tickets to force the pool into liquidation. The excess is larger than the number of players, a property we use for players to assess how many tickets are still out there if the pool has not been liquidated yet.

Proof. First note that, by definition (for $1 \leq t \leq t^*$): $m_{t-1} - s_{t-1} = m_{t-1} - s^*_{t-1} + j_{t-1}$. So,

$$m_t - s^*_t = m_{t-1} - s_{t-1} - s^*_t = m_{t-1} - s^*_{t-1} + j_{t-1} - s^*_t.$$

By Property 2,

$$m_{t-1} - s^*_{t-1} + j_{t-1} - s^*_t \geq m_{t-1} - s^*_{t-1} + s^*_t - s^*_t = m_{t-1} - s^*_{t-1}.$$

So, $m_t - s^*_t \geq m_{t-1} - s^*_{t-1}$. The smallest in this series obtains when $t = 1$. By the definition of s^*_t:

$$m_1 - s^*_1 \geq m_1 - \frac{P_1}{F_1} = ND(1 - \Xi).$$

Since $\Xi < (D-1)/D$, $D(1-\Xi) > 1$. Hence:

$$m_1 - s^*_1 \geq N + 1,$$

since m_1 and s^*_1 are integers. *A fortiori*, $m_t - s^*_t \geq N + 1$. □

Property 4. *For* $1 \leq t < t^*$*, and provided* $\Xi < (D-1)/D$ *and* $r < 1/D$*,*

$$\frac{(P_t - s_t F_t)(1+r)}{m_t - s_t} < F_t.$$

In words, the face value in round t is always higher than the liquidated value in round $t+1$.

Proof. By Definition 2, we have $P_t = m_t IV_t$. Since $IV_1/F_1 < 1$ and $r < R$, $IV_t/F_t < 1$ for all t. Hence, $P_t = m_t IV_t < m_t F_t$. We have

$$\frac{d}{ds_t}\frac{P_t - s_t F_t}{m_t - s_t} = \frac{P_t - m_t F_t}{(m_t - s_t)^2} < 0$$

Therefore, $(P_t - s_t F_t)/(m_t - s_t)$ decreases in s_t. Since, $s_t \geq 0$, we have

$$\begin{aligned}
\frac{(P_t - s_t F_t)(1+r)}{m_t - s_t} &\leq \frac{P_t(1+r)}{m_t} \\
&< \frac{F_t(1+r)(s_t^* + 1)}{m_t} \\
&< F_t \frac{D+1}{D}\frac{s_t^*+1}{m_t} \quad \text{(because } r < 1/D\text{)} \\
&\leq F_t \frac{D+1}{D}(1 - \frac{N}{m_t}) \quad \text{(because Property 3)} \\
&\leq F_t \frac{D+1}{D}(1 - \frac{1}{D}) \quad \text{(because } m_t \leq DN\text{)} \\
&< F_t \frac{D^2 - 1}{D^2} \\
&< F_t.
\end{aligned}$$

□

Property 3 has an immediate consequence.

Property 5. *For $1 < t \leq t^*$ and provided $\Xi < (D-1)/D$, there always exists at least one player who holds at least two tickets at the beginning of round t.*

Proof. Suppose all players hold at most 1 ticket each at t, hence, the highest the number of tickets can be is N. However, Property 3 says that the number of remaining tickets is always larger than the total number of players, since $m_t \geq s_t^* + N + 1 \geq N + 1$. This contradicts the assumption. Therefore, there always exists at least one player who holds at least two tickets in any round t. □

A1. Strong liquidation rule

We now study responses of a player to possible cash-in profiles that the other players could choose. More specifically, given a cash-in profile of others, the player predicts liquidation in round $t+1$ and wonders how to respond given that, in round t, she is holding k ($\leq D$) tickets. Properties 1, 2, 3, 4, and 5 guide the player's inference. They allow the player to estimate minimum and maximum payoffs from distributing cash-in of her tickets across rounds up to expected liquidation.

We will need the number of players N to be larger than the number of tickets per player D. Specifically, $N \geq (D + 1)(1 + r) - 1$. Effectively, we require that all players be "small" – in terms of ticket holdings – relative to the volume of tickets available. Notice that, when we introduce markets so that players can accumulate tickets, there is the danger that this condition is violated, to the extent that it may no longer be optimal to cash in early. But bidding may reveal accumulation plans, and hence, it is not obvious that accumulation can occur in a way that is profitable. We exploit this revelation by assuming, when there are markets, that prices are set in a fully revealing perfect-foresight equilibrium. Here, there are no markets, and hence, D is fixed throughout the game.

Given a profile of cash-in strategies (of other players), t^* will now denote the round in which our player predicts that the pool will be liquidated *ignoring her own choices* ($0 < t^* \leq T$). For the strong liquidation rule (which we are dealing with now), we restrict our attention to cases where other players submit *all* their tickets (that remain in their possession as of t^*).

Lemma 1. *Assume $N \geq (D+1)(1+r)-1$, $r < 1/D$, and $\Xi < (D-1)/D$. Under the strong liquidation rule, if a player still holds $0 < k$ $(\leq D)$ tickets in the round prior to t^*, and the player expects liquidation in t^*, then cashing-in all k tickets at $t^* - 1$ is a strictly dominant move. Therefore, liquidation in round 1 is the unique outcome by iterated elimination of dominated strategies (IEDS).*

Proof. To simplify notation, let us refer to round t^* as "round $t+1$." As of the previous round, t, our player still holds k $(0 < k \leq D)$ tickets. Let s_t^{-i} denote the number of tickets all other players together plan to submit for cash-in in round t.

We consider three (3) situations that round t could be in, given the strategy profiles of all other traders: (i) $s_t^* = 0$, or equivalently, $F_t > P_t$ (in which case, necessarily $s_t^{-i} = 0$ for otherwise the pool would be liquidated at t instead of $t+1$), (ii) $s_t^* = s_t^{-i} > 0$, and (iii) $s_t^* > s_t^{-i} \geq 0$. In situations (i) and (ii), our player is pivotal (cashing in one of the k tickets forces the pool into liquidation). In situation (iii), she has slack: she can cash in at least one ticket in round t without forcing the pool into liquidation.

i. $s_t^* = 0$. Since $s_t^{-i} = 0$ (no other player submits tickets in round t), our player can submit any or all k tickets and receive whatever is in the pool, P_t $(< F_t)$. Alternatively, she can wait till $t+1$, and share the pool with others. She submits k tickets, and others submit s_{t+1}^{-i}. Since the pool is expected to be liquidated even without her submission, $s_{t+1}^{-i} \geq 1$. In the best case, she shares with only one (1) ticket from another player. So, at best her payoff equals $P_t(1+r)k/(k+1)$. Since $k \leq D$, this payoff is at most $P_t(1+r)D/(D+1)$, and since $r < 1/D$, $(1+r) < (D+1)/D$ and hence, the payoff is strictly less than $P_t[(D+1)/D][D/(D+1)]$ $= P_t$. So our player is better off cashing in at t rather than waiting. Hence, cashing-in all k tickets is a weakly dominant strategy.

ii. $s_t^* = s_t^{-i} > 0$, and our player is pivotal just like in the first case. She can submit any $q \in [1, k]$ tickets, pool will be liquidated. Her proceed is $qP_t/(s_t^* + q)$. This payoff monotonically increases in the number of submitted tickets q. In other words, cashing in 1 and k tickets generates the lowest and highest payoffs of $P_t/(s_t^*+1)$ and $kP_t/(s_t^*+k)$. If our player postpones cash-in of all k tickets, she will receive

$$
\begin{aligned}
k\frac{(P_t - s_t^* F_t)(1+r)}{m_t - s_t^*} &< k\frac{(P_t - s_t^* \frac{P_t}{s_t^*+1}))(1+r)}{m_t - s_t^*} \quad \text{(since, } F_t > \frac{P_t}{s_t^*+1}\text{)} \\
&= k\frac{P_t}{s_t^*+1}\frac{1+r}{m_t - s_t^*} \\
&\leq D\frac{P_t}{s_t^*+1}\frac{1+r}{N+1} \quad \text{(since, } k \leq D \text{ and using Property 3)} \\
&\leq D\frac{P_t}{s_t^*+1}\frac{1}{D+1} \quad \text{(since, } N \geq (D+1)(1+r)-1\text{)} \\
&< \frac{P_t}{s_t^*+1}.
\end{aligned}
$$

Therefore, cashing in all k tickets is a strictly dominant strategy.

iii. $s_t^* > s_t^{-i} \geq 0$. Our player is not pivotal. That is, cashing in all k tickets may or may not force the pool into liquidation, i.e. [a] $k > s_t^* - s_t^{-i}$ or [b] $k \leq s_t^* - s_t^{-i}$. We will show that cashing in all k tickets is a dominant strategy in both cases.

a. $k \leq s_t^* - s_t^{-i}$: When all tickets are cashed in early, our player receives kF_t. Postponing cash-in of the k tickets leads to:

$$
k\frac{(P_t - s_t^{-i}F_t)(1+r)}{m_t - s_t^{-i}} < kF_t \quad \text{(Property 4).}
$$

In addition, cashing in any $q \in [1, k)$ tickets and postponing $k - q$ tickets leads to:

$$
\begin{aligned}
&qF_t + (k-q)\frac{[P_t - (s_t^{-i}+q)F_t](1+r)}{m_t - (s_t^{-i}+q)} \\
&< qF_t + (k-q)F_t \quad \text{(Property 4)} \\
&= kF_t.
\end{aligned}
$$

Therefore, cashing in all k tickets is a strictly dominant strategy.

b. $k > k^* = s_t^* - s_t^{-i}$: Cashing in $q_1 \in [k^* + 1, k]$ tickets will liquidate the pool and receive the payoff $q_1 P_t/(s_t^{-i} + q_1)$. When $s_t^{-i} > 0$, this payoff monotonically increases in the number of submitted tickets q_1. In other words, cashing in k tickets generates the highest payoffs, and cashing in $k^* + 1$ generates the lowest payoff of

$$\frac{(k^* + 1)P_t}{s_t^* + 1} = \frac{(k^* + 1)(s_t^* F_t + e_t)}{s_t^* + 1} \tag{A3}$$

where $e_t \equiv P_t - s_t^* F_t < F_t$. When $s_t^{-i} = 0$, the player will receive whatever is in the pool, P_t, which is greater than $\frac{(k^*+1)P_t}{s_t^*+1}$.

Cashing in $q_2 \in [1, k^*]$ tickets and postponing $k - q_2$ tickets will generate the payoff

$$\begin{aligned} & q_2 F_t + (k - q_2) \frac{[P_t - (s_t^{-i} + q_2)F_t](1 + r)}{m_t - (s_t^{-i} + q_2)} \\ & = q_2 F_t + (k - q_2) \frac{[(k^* - q_2)F_t + e_t](1 + r)}{m_t - s_t^* + k^* - q_2} = f(q_2). \end{aligned} \tag{A4}$$

If all k tickets are cashed in at $t + 1$, then the pool will have $k^* F_t(1 + r) + e_t(1 + r)$ upon liquidation, which is shared among $m_{t+1} = m_t - s_t = m_t - s_t^{-i}$ tickets. So, our player receives:

$$\begin{aligned} k \frac{k^* F_t(1 + r) + e_t(1 + r)}{m_t - s_t^{-i}} &= k \frac{e_t(1 + r)}{m_t - s_t^{-i}} + k^* F_t \frac{k(1 + r)}{m_t - s_t^{-i}} \\ &< k \frac{e_t(1 + r)}{m_t - s_t^*} + k^* F_t \frac{k(1 + r)}{m_t - s_t^*} \quad \text{(since } s_t^* > s_t^{-i}\text{)}. \end{aligned} \tag{A5}$$

To prove that cashing in all k tickets at t is a dominant strategy, first, we need to prove that if player decides to cash in at least 1 ticket at t, cashing all k tickets is a dominant strategy, i.e. the payoff (A14) is greater than payoff (A13) for all q_2. Second, postponing cashing-in all k tickets is dominated by submitting k^* tickets at t, i.e. $f(k^*)$ is greater than payoff (A5).

First, (A14) > (A13) for all q_2 is equivalent with

$$\begin{aligned}
\frac{(k^*+1)(s_t^*F_t+e_t)}{s_t^*+1} &> q_2F_t+(k-q_2)\frac{[(k^*-q_2)F_t+e_t](1+r)}{m_t-s_t^*+k^*-q_2} \\
\Leftrightarrow \frac{(k^*+1)(s_t^*F_t+e_t)}{s_t^*+1}-q_2F_t &> (k-q_2)\frac{[(k^*-q_2)F_t+e_t](1+r)}{m_t-s_t^*+k^*-q_2} \\
\Leftrightarrow \frac{(k^*+1)s_t^*F_t-q_2(s_t^*+1)F_t}{s_t^*+1}+\frac{k^*+1}{s_t^*+1}e_t &> (k-q_2)\frac{[(k^*-q_2)F_t+e_t](1+r)}{m_t-s_t^*+k^*-q_2} \\
\Leftrightarrow (k^*-q_2)F_t+\frac{s_t^*-k^*}{s_t^*+1}F_t+\frac{k^*+1}{s_t^*+1}e_t &> (k-q_2)\Bigg[\frac{(k^*-q_2)F_t(1+r)}{m_t-s_t^*+k^*-q_2} \\
&\qquad +\frac{e_t(1+r)}{m_t-s_t^*+k^*-q_2}\Bigg]. \qquad \text{(A6)}
\end{aligned}$$

To show the inequality (A15) is true is equivalent to showing that:

$$(k^*-q_2)F_t > (k-q_2)\frac{(k^*-q_2)F_t(1+r)}{m_t-s_t^*+k^*-q_2} \qquad \text{(A7)}$$

$$\text{and} \quad \frac{s_t^*-k^*}{s_t^*+1}F_t+\frac{k^*+1}{s_t^*+1}e_t > (k-q_2)\frac{e_t(1+r)}{m_t-s_t^*+k^*-q_2} \qquad \text{(A8)}$$

are true. We have $k^*-q_2 \geq 0$. In addition, since $k \leq D$ and $q_2 \geq 1$, $k-q_2 \leq D-1$. From Property 3, $m_t-s_t^* \geq N+1$ and $N \geq (D+1)(1+r)-1$, we have

$$(k-q_2)\frac{(1+r)}{m_t-s_t^*+k^*-q_2} \leq \frac{D-1}{D+1} < 1.$$

Therefore, (A16) is true. Furthermore, since $F_t > e_t$, $\frac{s_t^*-k^*}{s_t^*+1}F_t+\frac{k^*+1}{s_t^*+1}e_t > \frac{s_t^*-k^*}{s_t^*+1}e_t+\frac{k^*+1}{s_t^*+1}e_t = e_t$, (A17) is true. Hence, (A14) > (A13).

Second, we have

$$f(k^*) = k^* F_t + (k - k^*)\frac{e_t(1+r)}{m_t - s_t^*}$$
$$= k\frac{e_t(1+r)}{m_t - s_t^*} + k^*(F_t - e_t\frac{1+r}{m_t - s_t^*})$$

Since $e < F_t$ and using Property 3, we have $F_t - e_t\frac{1+r}{m_t-s_t^*} > F_t\frac{N+1-(1+r)}{m_t-s_t^*} \geq F_t\frac{D(1+r)}{m_t-s_t^*} \geq F_t\frac{k(1+r)}{m_t-s_t^*}$. Hence, $f(k^*) >$ (A5). Consequently, cashing in all k tickets is a strictly dominant strategy.

Therefore, regardless what information set (size of pool, history of cash-ins) a player is at in round t, immediate cash-in of all k tickets is a dominant strategy. Everyone will conclude this, and hence, the will be liquidated in round t instead of $t+1$. We now reset players' conjectures of pool liquidation at t and continue with the recursion until $t = 2$. We end with the result that the pool will be liquidated in round 1. □

A2. Weak liquidation rule

For the weak liquidation rule, we need to impose tighter bounds on parameters so that delaying cash-in is costly for players despite the fact that they automatically share in the proceeds if the pool is liquidated earlier. Given these additional restrictions, we show that liquidation in round 1 is the unique Nash equilibrium outcome (Lemma 2). We use the following Propositions A1-A3 to prove this.

Proposition A1. *Assume $N \geq (D+1)(1+R) - 1$, $r < 1/D$, and $\Xi < (D-1)/D$. If $s_t^* - s_t \geq 1$ and pool is liquidated in round $t+1$, every player who survives until round $t+1$ would regret for not cashing-in at least one more ticket in round t.*

Proof. The proposition states that if there is a room for at least one more ticket in round t without liquidating the pool, any player who still holds k_{t+1} tickets at $t+1$ would want to cash in at least one more ticket in round t. We consider two cases: [a] $k_{t+1} \leq s_t^* - s_t$ and [b] $k_{t+1} > s_t^* - s_t$.

a. $k_{t+1} \leq s_t^* - s_t$: When all tickets are cashed in early, our player receives $k_{t+1}F_t$. Postponing cash-in of the k_{t+1} tickets leads to:

$$k_{t+1}\frac{(P_t - s_tF_t)(1+r)}{m_t - s_t} < k_{t+1}F_t \quad \text{(using Property 4).}$$

Therefore, the player is better off by cashing-in additional k_{t+1} in round t.

b. $k_{t+1} > k^* = s_t^* - s_t$: If all k_{t+1} tickets are cashed in at $t+1$, then the pool will have $k^*F_t(1+r) + e_t(1+r)$ upon liquidation, where $e_t \equiv P_t - s_t^*F_t$. This cash is shared among $m_{t+1} = m_t - s_t$ tickets. So, our player receives:

$$\begin{aligned} k_{t+1}\frac{k^*F_t(1+r) + e_t(1+r)}{m_t - s_t} &= k_{t+1}\frac{e_t(1+r)}{m_t - s_t} + k^*F_t\frac{k_{t+1}(1+r)}{m_t - s_t} \\ &< k_{t+1}\frac{e_t(1+r)}{m_t - s_t^*} + k^*F_t\frac{k_{t+1}(1+r)}{m_t - s_t^*} \quad \text{(since } s_t^* > s_t\text{).} \end{aligned}$$

If the player cashes in early k^* out of k_{t+1} tickets in round t and postponing $k_{t+1} - k^*$ tickets, it will generate the payoff

$$\begin{aligned} & k^*F_t + (k_{t+1} - k^*)\frac{[P_t - (s_t + k^*)F_t](1+r)}{m_t - (s_t + k^*)} \\ & = k^*F_t + (k_{t+1} - k^*)\frac{e_t(1+r)}{m_t - s_t^*} \quad \text{(using } e_t \equiv P_t - s_t^*F_t \text{ and } k^* = s_t^* - s_t\text{)} \\ & = k_{t+1}\frac{e_t(1+r)}{m_t - s_t^*} + k^*(F_t - e_t\frac{1+r}{m_t - s_t^*}). \end{aligned}$$

Since $e_t < F_t$ and using Property 3 and the proposition's condition of N, we have $F_t - e_t\frac{1+r}{m_t - s_t^*} > F_t\frac{N+1-(1+r)}{m_t - s_t^*} > F_t\frac{D(1+r)}{m_t - s_t^*} \geq F_t\frac{k_{t+1}(1+r)}{m_t - s_t^*}$. Therefore, cashing-in early k^*

tickets in round t would generate higher payoff.

$\square$

Proposition A2. *Assume $N \geq (D+1)(1+R)-1$, $\Xi < (D-1)/D$, and:*

$$\frac{1}{D+1} - \frac{D^{T-3}}{(D-1)(D+1)^{T-3}} \leq \; r < \frac{1}{D},$$

$$\max\left\{r, \frac{1}{D+1}\right\} < \; R \leq r + \frac{1}{(D-1)(1+r)^{T-3}}.$$

Let $e_t \equiv P_t - s_t^* F_t < F_t$. For $1 \leq t \leq t^* - 2$, if $s_t^* = s_t \geq 1$ and $e_t \geq F_t \frac{R-r}{1+r}$, every player who submits ticket(s) in round t would be better off delaying to round $t+1$ cashing in one ticket.

Proof. The proposition states that t is the last round when the pool has enough cash to fulfill at least one submitted ticket and that there is at least one more round between round t and the liquidation round t^* where no player requests for cashing-in. If the residual cash e_t is large enough, every player who cashed in ticket(s) at round t would have higher payoff if she delayed submission of one ticket to round $t+1$.

Since $s_t^* = s_t$, $s_{t+1}^* = 0$, which implies that one submitted ticketed in round $t+1$ would liquidate the pool. However, we first show that when the player postpones submitting one ticket in round t, the pool in round $t+1$ *can* fulfill the one submitted ticket without liquidating the pool. Subsequently, we show that by delaying cashing-in this one ticket, she would have higher payoff.

a. Since t is the last round when the pool has enough cash to fulfill at least one submitted ticket, $s_{t+1}^* = 0$ or $P_{t+1} < F_{t+1}$. However, since $s_t^* = s_t$, when reducing one submitted ticket at round t, the pool size at round $t+1$ will become $P_{t+1}^{new} = (F_t + e_t)(1+r)$. If $e_t \geq F_t \frac{R-r}{1+r}$, $P_{t+1}^{new} = (F_t + e_t)(1+r) \geq F_t(1+R) = F_{t+1}$. Hence the pool has enough cash to pay for one submitted ticket at $t+1$. If a player submits one ticket at $t+1$ instead of at t, the residual cash in round $t+1$ will decline by $F_t(R-r)$. This is because the postponement raises the pool's cash by $F_t(1+r)$ going into round $t+1$, while the pool pays $F_{t+1} = F_t(1+r)$ for the cashed-in ticket. Net, the pool total changes by $F_t(1+r) - F_t(1+R) = F_t(r-R)$ (it loses $F_t(R-r)$).

b. Next, we show that every player who cashes in ticket(s) at round t would have higher payoff if she delayed submitting one ticket to round $t+1$. There are two types: [1] players who submit all of their remaining tickets at t; and [2] players who did not submit all of their tickets at t, originally planning to wait for part of them till t^*. Type-1 players obviously win from postponing cash-in of one ticket till $t+1$ since it will earn F_{t+1} instead of F_t ($F_{t+1} > F_t$). Type-2 players planned to postpone cash-n of $k > 0$ tickets until t^*. Those players increase the payoff on the postponed ticket by $F_{t+1} - F_t = RF_t$. However, the cash in the pool declines by $F_t(R-r)$ in round $t+1$ (see [a]), and hence, by $F_t(R-r)(1+r)^{t^*-(t+1)}$ by round t^*. Therefore, at t^*, they lose at most $F_t(R-r)\frac{(1+r)^{t^*-(t+1)}}{m_t - s_t^*}$ at t^* for their k remaining tickets. However, this reduction payoff of k tickets is less than the additional cash of RF_t they earn on the one ticket they postponed from t to $t+1$, since $k \leq D-1$ and

$$\begin{aligned}
kF_t(R-r)\frac{(1+r)^{t^*-(t+1)}}{m_t - s_t^*} &\leq (D-1)F_t(R-r)\frac{(1+r)^{t^*-(t+1)}}{m_t - s_t^*} \\
&\leq (D-1)F_t(R-r)\frac{(1+r)^{T-2}}{m_t - s_t^*} \quad \text{(since } t \geq 1 \text{ and } t^* \leq T\text{)} \\
&\leq (D-1)F_t(R-r)\frac{(1+r)^{T-2}}{N+1} \quad \text{(Property 3)} \\
&\leq F_t\frac{1+r}{N+1} \quad \left(R \leq r + \tfrac{1}{(D-1)(1+r)^{T-3}}\right) \\
&< F_t\frac{1}{D+1} \quad (N \geq (D+1)(1+R) - 1 \text{ and } R > r) \\
&< F_t R \quad (R > 1/(D+1)).
\end{aligned}$$

□

Proposition A3. *Assume $N \geq (D+1)(1+R)-1$ and:*

$$\frac{1}{D+1} - \frac{D^{T-3}}{(D-1)(D+1)^{T-3}} \leq r < \frac{1}{D},$$
$$\max\left\{r, \frac{1}{D+1}\right\} < R \leq r + \frac{1}{(D-1)(1+r)^{T-3}},$$
$$\text{and } \frac{D-1}{D+1} \leq \Xi < \frac{D-1}{D}.$$

Under the weak liquidation rule, there does not exist a Nash equilibrium where $s_1^* = s_1$ and the pool is liquidated in round $t^* \in [2, T]$.

Proof. We show that when $s_1^* = s_1$ and the pool is liquidated at round $t^* \in [2, T]$, there always exists a player who survives to round t^* and regrets not having submitted all D tickets at round 1, of course, this then liquidates the pool in round 1, or there exists a player who wants to delay cashing-in a ticket in round 1.

From Property 5, there always exists a player who holds at least two tickets at t^*. Hence, she cannot have submitted more than $D-2$ tickets in round 1. Let k be her number of tickets submitted in round 1, $k \leq D-2$. Since $s_1^* = s_1$, she will receive at most $e_1(1+r)^{t^*-1}/(m_1-s_1^*)$ for each of the remaining $D-k$ tickets in round t^*, where $e_1 < F_1$. Her total payoff is at most

$$kF_1 + (D-k)e_1\frac{(1+r)^{t^*-1}}{m_1 - s_1^*}. \tag{A9}$$

a. When $t^* = 2$, using Property 3, the condition $N \geq (D+1)(1+R)-1$, and $r < R$, we have $\frac{(1+r)}{m_1-s_1^*} \leq \frac{(1+r)}{N+1} < \frac{1}{D+1}$. Hence her total payoff is at most

$$kF_1 + (D-k)e_1\frac{(1+r)}{m_1 - s_1^*} < kF_1 + (D-k)e_1\frac{1}{D+1}$$
$$< kF_1 + (D-k)F_1\frac{1}{D+1} \quad \text{(since } e_1 < F_1\text{)}.$$

b. When $t^* > 2$, there will be at least a round between round 1 and the liquidation round t^*. If $e_1 \geq F_1(R-r)/(1+r)$, by Proposition A2, every player who submitted ticket(s) at round 1 would regret not delaying one of their submitted tickets to the next round. This implies that there will be defection and $s_1 < s_1^*$, so in this case, $s_1^* = s_1$ cannot be a Nash equilibrium. In the alternative scenario, $e_1 < F_1(R-r)/(1+r)$, the player who still holds at least two tickets in round t^* would regret not submitting all D tickets in round 1. To see this, using $t^* \leq T$, Property 3, $e_1 < F_1(R-r)/(1+r)$, the conditions of $N \geq (D+1)(1+R)-1$, $r < R$, and $R \leq r + \frac{1}{(D-1)(1+r)^{T-3}}$, we derive that

$$e_1 \frac{(1+r)^{t^*-1}}{m_1 - s_1^*} \leq e_1 \frac{(1+r)^{T-1}}{N+1} < F_1(R-r)\frac{(1+r)^{T-2}}{N+1} < F_1(R-r)\frac{(1+r)^{T-3}}{D+1} < F_1 \frac{1}{D+1}. \tag{A10}$$

Hence, applying (A10) to (A9), the total payoff for this player from cashing in k tickets at $t = 1$ and the remainder at t^* is, at best:

$$kF_1 + (D-k)e_1 \frac{(1+r)^{t^*-1}}{m_1 - s_1^*} < kF_1 + (D-k)F_1 \frac{1}{D+1}.$$

Therefore, when $t^* \geq 2$, the player's total payoff in (A9) is at most:

$$\begin{aligned} kF_1 + (D-k)F_1 \frac{1}{D+1} &= F_1 \frac{D(k+1)}{D+1} \\ &\leq F_1 \frac{D(D-1)}{D+1} \quad \text{(since } k \leq D-2\text{)} \\ &\leq IV_1 D \ \ \text{(since } \Xi = IV_1/F_1 \geq (D-1)/(D+1)\text{)}. \end{aligned}$$

But $IV_1 D$ is the payoff she would have gotten if she had submitted all tickets in round 1. The player would have been better off even though the pool is liquidated immediately. Therefore, the player is better off submitting all tickets in round 1. $\square$

Lemma 2. *Assume* $N \geq (D+1)(1+R)-1$, *and:*

$$\frac{1}{D+1} - \frac{D^{T-3}}{(D-1)(D+1)^{T-3}} \leq \ r < \frac{1}{D},$$

$$\max\{r, \frac{1}{D+1}\} < \ R \leq \min\left\{r + \frac{1}{(D-1)(1+r)^{T-3}}, 1 - \frac{D-1}{D+1}(1+r),\ \frac{D}{(D-2)(D+1)}\right\},$$

$$\textit{and}\ \frac{D-1}{D+1} \leq \ \Xi < \frac{D-1}{D}.$$

Under the weak liquidation rule, liquidation in round 1 is the unique Nash equilibrium outcome.

Proof. We first show that there exists a Nash equilibrium whereby everyone decides to cash in all tickets in round 1, and hence, the pool is liquidated in round 1. Would any player want to deviate and delay cash-in? Delaying cash-in of one ticket does not avoid liquidation in round 1, so it is pointless. Indeed, even delaying cash-in of all one's tickets does not avoid liquidation in round 1, by Property 3: the minimum number of tickets that need to remain outstanding to avoid liquidation is $m_1 - s_1^*$ which is greater than or equal to (see Lemma conditions) $N+1 \geq (D+1)(1+R) > D$. That is, the minimum number of tickets that need to remain outstanding to avoid liquidation is bigger than the number of tickets a player holds.

Next, we prove, by contradiction, that no other Nash equilibrium exists whereby the pool is liquidated in round $t^* > 1$. Assume that the equilibrium exists. We show that there is always a player who planned to wait till t^*, yet is better off moving cash-in of one or more tickets to some prior round $t^* - l$ ($l \geq 1$), contradicting that the player is "best-responding," as required for Nash equilibrium.

There can be two situations: [1] there is a room for at least one ticket in round $t^* - 1$ without liquidating the pool, or [2] a single additional ticket would liquidate the pool at $t^* - 1$. Similar to Lemma 1, let us refer to round t^* as "round $t+1$," hence $t \leq T-1$.

Consider situation [1]: the Proposition A1 shows that *any* player who still holds tickets at $t+1$ would want to cash in at least one more ticket in round t as long as the pool is not liquidated in round t. Iterating this argument will eventually lead to situation [2]: cashing in one more ticket earlier causes liquidation at t.

Let us therefore move to situation [2]. There, $s_t^* = s_t$. Proposition A3 shows that, if $t = 1$, all Nash equilibria imply immediate liquidation, so the posited equilibrium cannot be true.

We now consider $t \in [2, T-1]$. By the rules of the game, at least two players "survive" until (= still hold tickets by) round $t+1$. We consider three situations the previous round (t) could be in: (i) at least one player cashes in at least one ticket and at least one of the remaining players does not cash in any; (ii) none of the players submits a ticket, in which case $s_t^* = s_t = 0$; and (iii) all players submit at least one ticket in round t.

Since $t > 1$, there always exists at least one round prior round t. Define $\hat{t}$ as follows: for cases (i) and (iii), $\hat{t} = t$; for case (ii), $\hat{t} = \tau$, where τ is the last round when the pool can fulfil at least one submitted ticket. Define k_{t+1} (≥ 1) as the number of tickets a surviving player holds in the liquidation round $t+1$; k_a (≥ 0) as the number of tickets she submits throughout rounds 1 to $\hat{t}-1$ ($\hat{t} \geq 2$), and k_b (≥ 0) as the number of tickets she submits throughout $\hat{t}$ to t. We now show that the player prefers to submit some of the $k_b + k_{t+1}$ tickets in round $\hat{t}-1$ rather than later. Refer to the number that she would prefer to submit in $\hat{t}-1$ as $K_{\hat{t}-1}$ ($\leq k_b + k_{t+1}$). We choose $K_{\hat{t}-1}$ so that: [1] the early submission of an additional $K_{\hat{t}-1}$ tickets does not liquidate the pool in round $\hat{t}-1$, which means: $j_{\hat{t}-1} \geq K_{\hat{t}-1}$; and [2] the payoff of this change in strategy is higher than that of cashing in $k_b + k_{t+1}$ later. The first part also ensures that additional early submission of $K_{\hat{t}-1}$ tickets does not change the player's payoff of the remaining k_a (they continue to generate the face value).

Situation (i): At least one player cashes in at least one ticket and at least one other player does not cash in any at t. Therefore, $s_t^* = s_t \geq 1$. From Property 2, we have $j_{t-1} \geq s_t^*$; hence, $j_{t-1} \geq 1$. Since $j_{t-1} \geq 1$, the player who holds $k_{t+1} \geq 1$ tickets at $t+1$ but did not submit a single ticket at t, could submit $K_{t-1} = 1$ extra ticket at $t-1$ without liquidating the pool at $t-1$. The payoff on the early cash-in is F_{t-1}. Compare this to the payoff on *all* k_{t+1} tickets if cashed in at $t+1$:

$$k_{t+1}e_t\frac{1+r}{m_t - s_t^*} < k_{t+1}F_t\frac{1+r}{m_t - s_t^*} = k_{t+1}F_{t-1}\frac{(1+r)(1+R)}{m_t - s_t^*}.$$

Applying Property 3, $r < 1/D$ (so $(1+r) < (D+1)/D$), and $k_{t+1} \leq D$,

$$k_{t+1}e_t\frac{1+r}{m_t - s_t^*} < F_{t-1}\frac{(D+1)(1+R)}{N+1}.$$

Since $N \geq (D+1)(1+R) - 1$, this payoff is less than F_{t-1}, which is a lower bound on the payoff the player can get from cashing in early one of the k_{t+1} tickets since it disregards the payoff on the remaining $k_{t+1} - 1$ tickets. Hence, situation (i) cannot be an equilibrium situation.

Situation (ii): None of the players submits a ticket at t, i.e. $s_t^* = s_t = 0$. Since the pool starts in the money, i.e. pool has enough money to fulfil at least 1 submitted ticket in round 1, let denote τ ($\tau \leq t-1$) be the last round when pool is in the money. We will show that there exist the planned moves (None of the players submits a ticket at t) can be improved upon, and hence, that the situation cannot occur in equilibrium. In fact, we show more: any strategy whereby $s_\tau^* = s_\tau \geq 1$, $s_l^* = s_l = 0$ for all $l \in [\tau+1, t]$, $\tau \geq 2$, and $t \in [3, T-1]$, can be improved upon. In words: it is impossible to have strategies that cause there to be at least a round prior to round τ and at least a round with no submitted tickets between round τ and the liquidation round $t+1$.

If the residual cash in the pool of round τ, e_τ, is at least $F_\tau(R-r)/(1+r)$, by Proposition A2, every player who submitted ticket(s) at round τ would regret not delaying one of their submitted tickets to the next round. Hence, the posited equilibrium is inconsistent with $e_\tau \geq F_\tau(R-r)/(1+r)$.

What if $e_\tau < F_\tau(R-r)/(1+r)$? We consider two scenarios for round τ: [ii.a] there is at least one player who does not submit a ticket in round τ, and [ii.b] all players submit at least one ticket in round τ.

ii.a At least one player does not submit any ticket in τ. Such a player has $k_{t+1} \leq D$ tickets in round $t+1$ and her payoff for these tickets at $t+1$ is $k_{t+1}e_\tau(1+r)^{t+1-\tau}/(m_\tau - s_\tau^*)$. From Property 2, we have $j_{\tau-1} \geq s_\tau^*$. Since $s_\tau^* \geq 1$, we have $j_{\tau-1} \geq 1$. Therefore, this player could have submitted 1 extra ticket at $\tau-1$, with payoff $F_{\tau-1}$. Compare this to the payoff in case *all* k_{t+1} tickets are cashed in at $t+1$:

$$
\begin{aligned}
k_{t+1}e_\tau\frac{(1+r)^{t+1-\tau}}{m_\tau - s_\tau^*} &< k_{t+1}F_\tau(R-r)\frac{(1+r)^{t-\tau}}{m_\tau - s_\tau^*} \quad (\text{since } e_\tau < F_\tau(R-r)/(1+r)) \\
&< k_{t+1}F_\tau(R-r)\frac{(1+r)^{T-3}}{m_\tau - s_\tau^*} \quad (\, t-\tau \leq T-3, \text{ since } \tau \geq 2 \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{and } t \leq T-1) \\
&< DF_{\tau-1}(1+R)(R-r)\frac{(1+r)^{T-3}}{N+1} \quad (\text{Property 3, and } k_{t+1} \leq D) \\
&\leq F_{\tau-1}\frac{D}{D+1}(R-r)(1+r)^{T-3} \quad (\text{Lemma's restriction on } N) \\
&\leq F_{\tau-1}\frac{D}{D+1}\frac{1}{D-1} \quad (\text{using } R \leq r + \tfrac{1}{(D-1)(1+r)^{T-3}}) \\
&< F_{\tau-1}.
\end{aligned}
$$

Hence, the posited scenario is inconsistent with equilibrium.

ii.b All remaining players submit at least one ticket in round τ. From Property 5, there always exists a player who holds at least two tickets in round $t+1$. Hence, this player can submit at most $D-2$ tickets in round τ. Use k^* ($\leq D-2$) to denote this number, and k_{t+1} for the remaining tickets she holds by round $t+1$. She receives $e_\tau(1+r)^{t+1-\tau}/(m_\tau - s^*_\tau)$ for each of the remaining k_{t+1} tickets in round $t+1$. Her total payoff is

$$
\begin{aligned}
k^*F_\tau + k_{t+1}e_\tau\frac{(1+r)^{t+1-\tau}}{m_\tau - s^*_\tau} &< k^*F_\tau + k_{t+1}F_\tau(R-r)\frac{(1+r)^{t-\tau}}{m_\tau - s^*_\tau} \quad (e_\tau < F_\tau(R-r)/(1+r)) \\
&\leq k^*F_\tau + k_{t+1}F_\tau(R-r)\frac{(1+r)^{T-3}}{m_\tau - s^*_\tau} \quad (\text{since } t-\tau \leq T-3) \\
&\leq k^*F_\tau + (D-1)F_\tau(R-r)\frac{(1+r)^{T-3}}{N+1} \; (k_{t+1} \leq D-1, \\
&\qquad\qquad\qquad\qquad\qquad\qquad \text{and Property 3}) \\
&\leq k^*F_\tau + F_\tau\frac{(D-1)(R-r)(1+r)^{T-3}}{(D+1)(1+R)} \; (\text{restriction on } N) \\
&\leq k^*F_\tau + F_\tau\frac{D-1}{(D+1)(1+R)}\frac{1}{D-1} \quad (R \leq r + \tfrac{1}{(D-1)(1+r)^{T-3}}) \\
&\leq k^*F_\tau + F_\tau\frac{1}{(D+1)(1+R)}.
\end{aligned}
$$

Because there are at least two players in round τ, $s^*_\tau \geq k^*+1$. From Property 2, we infer: $j_{\tau-1} \geq s^*_\tau \geq k^*+1$. Therefore, our player could actually submit $K_{\tau-1} = k^*+1$ tickets in round $\tau-1$ without liquidating the pool. If she does so, she would receive $(k^*+1)F_{\tau-1} = (k^*+1)F_\tau/(1+R)$ on the earlier cash-ins. We now show that this payoff is larger than the above upper bound on the player's original strategy, i.e., that:

$$(k^*+1)F_\tau\frac{1}{1+R} \geq k^*F_\tau + F_\tau\frac{1}{(D+1)(1+R)}$$

$$
\begin{aligned}
\Leftrightarrow (k^*+1)\frac{1}{1+R} - k^* &\geq \frac{1}{(D+1)(1+R)}\\
\Leftrightarrow \frac{1-Rk^*}{1+R} &\geq \frac{1}{(D+1)(1+R)}\\
\Leftrightarrow 1-Rk^* &\geq \frac{1}{D+1}\\
\Leftrightarrow R &\leq \frac{D}{k^*(D+1)}.
\end{aligned}
$$

The above inequality is true since $R \leq \frac{D}{(D-2)(D+1)}$, and $k^* \leq D-2$. Our player thus generates a higher payoff for submitting k^*+1 tickets in round $\tau-1$. Therefore, the posited scenario is inconsistent with equilibrium.

Situation (iii): All players submit at least one ticket at t. We consider two scenarios: [iii.a] all players submit just one ticket in round t, and [iii.b] at least one player submits two or more tickets in round t.

iii.a All players submit exactly one ticket at round t. From Property 2, we have $j_{t-1} \geq s_t^*$; and since there are at least 2 players in round t, $s_t^* \geq 2$; hence $j_{t-1} \geq 2$. So, any player could have submitted $K_{t-1} = 2$ tickets in round $t-1$ without liquidating the pool and earn $2F_{t-1}$. They would prefer this since the original payoff is the sum of F_t in round t and $e_t(1+r)/(m_t - s_t^*)$ for each $k_{t+1} \leq D-1$ tickets in round $t+1$, or

$$
\begin{aligned}
F_t + k_{t+1}e_t\frac{1+r}{m_t - s_t^*} &\leq F_t + (D-1)e_t\frac{1+r}{m_t-s_t^*}\\
&< F_t\Big[1+(D-1)\frac{1+r}{N+1}\Big] \text{ (using Property 3 and } e_t < F_t).\\
&\leq F_t\Big[1+\frac{D-1}{D+1}\frac{1+r}{1+R}\Big] \text{ (using Lemma's restriction on } N)\\
&= \frac{F_t}{1+R}\Big[1+R+\frac{D-1}{D+1}(1+r)\Big]\\
&\leq 2\frac{F_t}{1+R} \text{ (using } R \leq 1-\tfrac{D-1}{D+1}(1+r))\\
&\leq 2F_{t-1}.
\end{aligned}
$$

So, even disregarding payoffs on the remaining tickets, the payoff from cashing in two tickets early is strictly larger than the payoff on the original strategy. Hence, this case cannot occur in equilibrium.

iii.b At least one player submits at least two tickets in round t. Let $A^{2,1}$ denote the set consisting of players who submitted at least two tickets in t, $k_t^{A^{2,1}} \geq 2$. In round $t+1$, from Property 5, there always exists a player who holds at least two tickets. Let $A^{1,2}$ denote the set consisting of these players. We consider two cases: [b1] $A^{2,1} \neq A^{1,2}$, and [b2] $A^{2,1} = A^{1,2}$, or players submit at least 2 tickets in both t and $t+1$; we use $A^{2,2}$ to denote this set of players.

To simplify the notation, let denote $k_t^A \geq 1$ be the number of submitted tickets in round t of players in sets $A^{1,2}$ and $A^{2,2}$. We will show that, in both [b1] and [b2], players in sets $A^{1,2}$ and $A^{2,2}$ prefer cashing in $k_t^A + 2$ tickets in round $t-1$ instead of cashing in k_t^A ticket(s) in t and the remaining ones in $t+1$. In both cases, the pool has enough cash in round $t-1$ to pay for the extra $k_t^A + 2$ tickets without liquidating.

- First, we show that the pool has enough cash in round $t-1$ to pay for the extra $k_t^A + 2$ tickets at face value. From Property 2, we have $j_{t-1} \geq s_t^*$. In case [b1], the maximum number of tickets that can be submitted without liquidating the pool at t, s_t^*, must at least be $k_t^A + k_t^{A^{2,1}} \geq k_t^A + 2$; hence, $j_{t-1} \geq k_t^A + 2$.

  In case [b2], if set $A^{2,2}$ has at least two players, s_t^* must at least be $k_t^A + k_t^A \geq k_t^A + 2$; hence, $j_{t-1} \geq k_t^A + 2$. If set $A^{2,2}$ has only one player, the remaining players in the pool submit exactly 1 ticket in both t and $t+1$; we use $A^{1,1}$ to denote this set of players. From Property 3, the number of tickets at the beginning of round $t+1$ is

$$m_{t+1} \geq s_{t+1}^* + N + 1 \geq N + 1,$$

$s^*_{t+1} = 0$ since $s^*_t = s_t$. The player in set $A^{2,2}$ submits at least 2 tickets at t, hence she has at most $D-2$ tickets at $t+1$. Therefore, the pool must have at least four players in set $A^{1,1}$, since if we subtract the number of tickets that player in set $A^{2,2}$ holds from m_{t+1}, the total number of tickets held by players in set $A^{1,1}$ is at least

$$m_{t+1} - (D-2) \geq N - D + 3 > 3,$$

since $N > D$ which is from the Lemma's condition of $N \geq (1+D)(1+R) - 1$. Therefore, s^*_t must be at least $k^A_t + 4$; hence, $j_{t-1} > k^A_t + 2$. Therefore, the pool at $t-1$ always has enough money to fulfil an extra $K_{t-1} = k^A_t + 2$ tickets for players in sets $A^{1,2}$ and $A^{1,2}$ without liquidating the pool.

- Second, we show that players prefer cashing in $k^A_t + 2$ tickets in round $t-1$. These players' current payoff is the sum of $k^A_t F_t$ in round t and $e_t(1+r)/(m_t - s^*_t)$ for each of k_{t+1} ($\leq D-1$) tickets submitted in round $t+1$. This sum is bounded above as follows:

$$\begin{aligned}
k^A_t F_t + k_{t+1} e_t \frac{1+r}{m_t - s^*_t} &\leq k^A_t F_t + (D-1) e_t \frac{1+r}{m_t - s^*_t} \\
&< F_t \Big[k^A_t + (D-1) \frac{1+r}{N+1} \Big] \text{ (Property 3 and } e_t < F_t) \\
&\leq F_t \Big[k^A_t + \frac{D-1}{D+1} \frac{1+r}{1+R} \Big] \ \ (N \geq (D+1)(1+R) - 1) \\
&\leq F_t \Big[k^A_t + \frac{D-1}{D} \frac{1}{1+R} \Big] \ \ (r < 1/D).
\end{aligned}$$

If instead submitting $k^A_t + 2$ tickets in round $t-1$, they would earn at least $(k^A_t + 2) F_{t-1} = (k^A_t + 2) F_t / (1+R)$. They would prefer so if

$$(k^A_t + 2) \frac{F_t}{1+R} \geq F_t \Big[k^A_t + \frac{D-1}{D} \frac{1}{1+R} \Big]$$

$$\Leftrightarrow \frac{k_t^A+2}{1+R} \geq k_t^A + \frac{D-1}{D}\frac{1}{1+R}$$
$$\Leftrightarrow \frac{k_t^A+2}{1+R} - k_t^A \geq \frac{D-1}{D}\frac{1}{1+R}$$
$$\Leftrightarrow 2 - k_t^A R \geq \frac{D-1}{D}. \tag{A11}$$

Since $R \leq \frac{D}{(D-2)(D+1)}$ and $\frac{D}{(D-2)(D+1)} < \frac{D+1}{D(D-2)}$ for all D, we have $R < \frac{D+1}{D(D-2)}$. In addition, $k_t^A \leq D-2$ (since they already submit at least 2 tickets at $t+1$) we have

$$2 - k_t^A R \geq 2 - (D-2)\frac{D+1}{D(D-2)} = \frac{D-1}{D}.$$

Hence, inequality (A11) is true.

Therefore, scenario [iii.b] is inconsistent with equilibrium.

□

B. Proof of parameter feasibility for Lemma 2

We demonstrate that given D and T, there always exist parameters r, R, and Ξ such that:

$$\frac{1}{D+1} - \frac{D^{T-3}}{(D-1)(D+1)^{T-3}} \leq r < \frac{1}{D},$$
$$\max\{r, \frac{1}{D+1}\} < R \leq \min\left\{r + \frac{1}{(D-1)(1+r)^{T-3}}, 1 - \frac{D-1}{D+1}(1+r),\ \frac{D}{(D-2)(D+1)}\right\},$$
$$\text{and } \frac{D-1}{D+1} \leq \Xi < \frac{D-1}{D}.$$

Given D and T, there always exists r such that $\frac{1}{D+1} - \frac{D^{T-3}}{(D-1)(D+1)^{T-3}} \leq r < \frac{1}{D}$, since $\frac{1}{D+1} < \frac{1}{D}$ for all $D > 1$. Then given r, there always exits R such that $R > r$ and $\max\{r, \frac{1}{D+1}\} <$

$R \leq \min\left\{r + \frac{1}{(D-1)(1+r)^{T-3}}, 1 - \frac{D-1}{D+1}(1+r),\ \frac{D}{(D-2)(D+1)}\right\}$. To prove this, we need to show that each of the upper bounds is greater than both lower bounds r and $\frac{1}{D+1}$.

For the upper bound $r + \frac{1}{(D-1)(1+r)^{T-3}}$, it is larger than r since $r + \frac{1}{(D-1)(1+r)^{T-3}} > r$ for all $D > 1$, and

$$\begin{aligned} r + \frac{1}{(D-1)(1+r)^{T-3}} &> r + \frac{1}{(D-1)(\frac{D+1}{D})^{T-3}} \quad \text{(since } r < 1/D\text{)} \\ &\geq \frac{1}{D+1} \quad \text{(since } r \geq \tfrac{1}{D+1} - \tfrac{D^{T-3}}{(D-1)(D+1)^{T-3}}\text{)}. \end{aligned}$$

For the upper bound $1 - \frac{D-1}{D+1}(1+r)$, since $r < 1/D$, we have

$$1 - \frac{D-1}{D+1}(1+r) > 1 - \frac{D-1}{D+1}(1+\frac{1}{D}) = \frac{1}{D}.$$

For the upper bound $\frac{D}{(D-2)(D+1)}$, we have

$$\frac{D}{(D-2)(D+1)} - \frac{1}{D} = \frac{D^2 - (D-2)(D+1)}{D(D-2)(D+1)} = \frac{D+2}{D(D-2)(D+1)} > 0.$$

Therefore, both upper bounds are larger than $\frac{1}{D}$. Since $\frac{1}{D}$ is larger than both r and $\frac{1}{D+1}$, the two upper bounds are larger. For $\frac{D-1}{D+1} \leq \Xi < \frac{D-1}{D}$, there exists Ξ, since $\frac{D-1}{D+1} < \frac{D-1}{D}$, for all $D > 1$. $\square$

Modified Lemma 1: Anh

Lemma 1 new. *Assume $N \geq (D+1)(1+r) - 1$, $r < 1/D$, and $\Xi < (D-1)/D$. Under the strong liquidation rule, liquidation in round 1 is the unique Nash equilibrium outcome.* ~~*if a player still holds $0 < k$ ($\leq D$) tickets in the round prior to t^*, and the player expects all remaining tickets of other players to be cashed in at t^*, then cashing in all k tickets at $t^* - 1$ is a dominant strategy. Therefore, liquidation in round 1 is the unique equilibrium outcome.*~~

Proof. We first show that there exists a Nash equilibrium whereby everyone decides to cash in all tickets in round 1, and hence, the pool is liquidated in round 1. Would any player want to deviate and delay cash-in? Delaying cash-in of one ticket does not avoid liquidation in round 1, so it is pointless. Indeed, even delaying cash-in of all one's tickets does not avoid liquidation in round 1, by Property 3: the minimum number of tickets that need to remain outstanding to avoid liquidation is $m_1 - s_1^*$ which is greater than or equal to (see Lemma conditions) $N + 1 \geq (D + 1)(1 + r) > D$. That is, the minimum number of tickets that need to remain outstanding to avoid liquidation is bigger than the number of tickets a player holds.

Next, we prove, by contradiction, that no other Nash equilibrium exists whereby the pool is liquidated in round $t^* > 1$. Assume that the equilibrium exists. There are two (2) situations that round t^* could be in: [1] at least one player does not cash in all of her tickets, or [2] every player submits all tickets for cash-in. To simplify notation, let us refer to round t^* as "round $t + 1$." Let s_t^{-i} denote the number of tickets all other players together submit for cash-in in round t.

Consider situation [1]: let denote $k_s \geq 0$ and $k_l \geq 1$ be the number of cash-in tickets and number of un-submitted ones in round $t + 1$ of the player A who does not cash in all of her tickets in this round. There are two (2) scenarios that round $t + 1$ could be in: [a] $s_{t+1}^{-A} = 0$, or [b] $s_{t+1}^{-A} \geq 1$. In scenario [a], beside player A, all other players do not submit any tickets for cash-in, and they receive nothing in this round by the strong-form liquidation rule. These players must be better off to submit at least one ticket to share the pool's payoff with player A, contradicting that the players are "best-responding," as required for Nash equilibrium. In scenario [b], we will show that player A is better off cashing in all of her tickets, contradicting that the player is "best-responding". Her payoff in this round is $k_s P_{t+1}/(s_{t+1}^{-A} + k_s)$. If she submitted all of her tickets, she would have higher pay off of $(k_s + k_l)P_{t+1}/(s_{t+1}^{-A} + k_s + k_l)$, since

$$(k_s + k_l)\frac{P_{t+1}}{s_{t+1}^{-A} + k_s + k_l} > k_s \frac{P_{t+1}}{s_{t+1}^{-A} + k_s}$$
$$\Leftrightarrow k_l s_{t+1}^{-A} > 0. \tag{A12}$$

Since $k_l \geq 1$ and $s_{t+1}^{-A} \geq 1$, inequality (A12) is true. Therefore, the posited situation is inconsistent with equilibrium.

Consider situation [2]: we now show that all players who planned to wait till $t+1$ are better off moving cash-in of all of their tickets to round t, contradicting that the players are "best-responding," as required for Nash equilibrium. Let k ($1 \leq k \leq D$) denote the number of tickets that our representative player still holds at the beginning of round t, and k_t ($0 \leq k_t < k$) and k_{t+1} ($k_{t+1} = k - k_t$ and $1 \leq k_{t+1} \leq D$) denote the number of tickets that our player submits for cash-in in rounds t and $t+1$. We consider three (3) situations that round t could be in: (i) $s_t^* = s_t^{-i} = 0$, or equivalently, $F_t > P_t$, (ii) $s_t^* = s_t^{-i} > 0$, and (iii) $s_t^* > s_t^{-i} \geq 0$. In situations (i) and (ii), our player does not submit any ticket in round t, or $k_t = 0$ and $k_{t+1} = k$, otherwise the pool would be liquidated at t instead of $t+1$. ~~our player is pivotal (cashing in one of the k tickets forces the pool into liquidation). In situation (iii), she has slack: she can cash in at least one ticket in round t without forcing the pool into liquidation.~~

i. $s_t^* = s_t^{-i} = 0$. Her payoffs in rounds t and $t+1$ are 0 and $P_t(1+r)k/(k + s_{t+1}^{-i})$. Since all other players submit all of their tickets in round $t+1$, $s_{t+1}^{-i} \geq 1$. In the best case, she shares with only one (1) ticket from another player. So, at best her payoff equals $P_t(1+r)k/(k+1)$. Since $k \leq D$, this payoff is at most $P_t(1+r)D/(D+1)$, and since $r < 1/D$, $(1+r) < (D+1)/D$ and hence, the payoff is strictly less than $P_t[(D+1)/D][D/(D+1)]$ $= P_t$. Therefore, her total payoff in both rounds t and $t+1$ is strictly less than P_t. So our player is better off cashing in at t and receive whatever is in the pool, P_t, rather than waiting. Hence, situation (i) cannot be an equilibrium situation. ~~Hence, cashing in all k tickets is a weakly dominant strategy.~~

ii. $s_t^* = s_t^{-i} > 0$. Her payoffs in rounds t and $t+1$ are 0 and

$$\begin{aligned}
k\frac{(P_t - s_t^* F_t)(1+r)}{m_t - s_t^*} &< k\frac{(P_t - s_t^* \frac{P_t}{s_t^*+1}))(1+r)}{m_t - s_t^*} \quad (\text{since, } F_t > \frac{P_t}{s_t^*+1}) \\
&= k\frac{P_t}{s_t^*+1}\frac{1+r}{m_t - s_t^*} \\
&\leq D\frac{P_t}{s_t^*+1}\frac{1+r}{N+1} \quad (\text{since, } k \leq D \text{ and using Property 3}) \\
&\leq D\frac{P_t}{s_t^*+1}\frac{1}{D+1} \quad (\text{since, } N \geq (D+1)(1+r)-1) \\
&< \frac{P_t}{s_t^*+1}.
\end{aligned}$$

If the player cashed in early all k tickets, she would receive $kP_t/(s_t^*+k)$. This payoff monotonically increases in the number of submitted tickets k. Since $k \geq 1$, $kP_t/(s_t^*+k) \geq P_t/(s_t^*+1)$. Therefore, cashing in all k tickets in round t provides better payoff for the player. Hence, situation (ii) cannot be an equilibrium situation. ~~In other words, cashing in 1 and k tickets generates the lowest and highest payoffs of $P_t/(s_t^*+1)$ and $kP_t/(s_t^*+k)$. If our player postpones cash-in of all k tickets, she will receive. Therefore, cashing in all k tickets is a strictly dominant strategy.~~.

iii. $s_t^* > s_t^{-i} \geq 0$. ~~Our player is not pivotal.~~ Cashing in all k tickets may or may not force the pool into liquidation, i.e. [a] $k > s_t^* - s_t^{-i}$ or [b] $k \leq s_t^* - s_t^{-i}$. ~~We will show that cashing in all k tickets is a dominant strategy in both cases.~~ We will show that cashing in all k tickets in round t instead of k_t and $k - k_t$ in rounds t and $t+1$ provides better payoff for the player in both cases.

a. $k \leq s_t^* - s_t^{-i}$: Cashing in k_t and $k - k_t$ tickets in rounds t and $t+1$ generates the total payoff of

$$k_t F_t + (k - k_t)\frac{[P_t - (s_t^{-i} + k_t)F_t](1+r)}{m_t - (s_t^{-i} + k_t)}$$

If the player cashed in early all tickets, our player would receive kF_t in round t. This payoff is strictly greater than what she received, since

$$\begin{aligned}
&k_t F_t + (k - k_t)\frac{[P_t - (s_t^{-i} + k_t)F_t](1+r)}{m_t - (s_t^{-i} + k_t)} \\
&< k_t F_t + (k - k_t)F_t \quad \text{(Property 4)} \\
&= kF_t.
\end{aligned}$$

Hence, situation [a] cannot be an equilibrium situation. ~~Therefore, cashing in all k tickets is a strictly dominant strategy.~~

b. $k > s_t^* - s_t^{-i}$: Let $k^* \equiv s_t^* - s_t^{-i}$. Cashing in k_t and $k - k_t$ tickets in rounds t and $t+1$ generates the total payoff of

$$\begin{aligned}
&k_t F_t + (k - k_t)\frac{[P_t - (s_t^{-i} + k_t)F_t](1+r)}{m_t - (s_t^{-i} + k_t)} \\
&= k_t F_t + (k - k_t)\frac{[(k^* - k_t)F_t + e_t](1+r)}{m_t - s_t^* + k^* - k_t}.
\end{aligned} \tag{A13}$$

where $e_t \equiv P_t - s_t^* F_t < F_t$. If the player cashed in early all k tickets, the pool would be liquidated and our player would receive the payoff $kP_t/(s_t^{-i} + k)$. When $s_t^{-i} > 0$, this payoff monotonically increases in the number of submitted tickets k. In other words, cashing in k tickets generates the highest payoffs, and cashing in $k^* + 1$ generates the lowest payoff of

$$\frac{(k^*+1)P_t}{s_t^* + 1} = \frac{(k^*+1)(s_t^* F_t + e_t)}{s_t^* + 1}. \tag{A14}$$

When $s_t^{-i} = 0$, the player will receive whatever is in the pool, P_t, which is greater than $\frac{(k^*+1)P_t}{s_t^*+1}$. Therefore, regardless of s_t^{-i}, cashing all k tickets in round t would generate a payoff that is greater than the payoff A14.

To show that the player is better off by cashing all tickets in round t is equivalent with showing that the payoff (A14) is greater than payoff (A13) for all k_t, or

$$\begin{aligned}
\frac{(k^*+1)(s_t^*F_t+e_t)}{s_t^*+1} &> k_tF_t + (k-k_t)\frac{[(k^*-k_t)F_t+e_t](1+r)}{m_t-s_t^*+k^*-k_t} \\
\Leftrightarrow \frac{(k^*+1)(s_t^*F_t+e_t)}{s_t^*+1} - k_tF_t &> (k-k_t)\frac{[(k^*-k_t)F_t+e_t](1+r)}{m_t-s_t^*+k^*-k_t} \\
\Leftrightarrow \frac{(k^*+1)s_t^*F_t - k_t(s_t^*+1)F_t}{s_t^*+1} + \frac{k^*+1}{s_t^*+1}e_t &> (k-k_t)\frac{[(k^*-k_t)F_t+e_t](1+r)}{m_t-s_t^*+k^*-k_t} \\
\Leftrightarrow (k^*-k_t)F_t + \frac{s_t^*-k^*}{s_t^*+1}F_t + \frac{k^*+1}{s_t^*+1}e_t &> (k-k_t)\left[\frac{(k^*-k_t)F_t(1+r)}{m_t-s_t^*+k^*-k_t}\right. \\
&\qquad \left. + \frac{e_t(1+r)}{m_t-s_t^*+k^*-k_t}\right].
\end{aligned} \tag{A15}$$

To show the inequality (A15) is true is equivalent to showing that:

$$(k^*-k_t)F_t > (k-k_t)\frac{(k^*-k_t)F_t(1+r)}{m_t-s_t^*+k^*-k_t} \tag{A16}$$

$$\text{and} \quad \frac{s_t^*-k^*}{s_t^*+1}F_t + \frac{k^*+1}{s_t^*+1}e_t > (k-k_t)\frac{e_t(1+r)}{m_t-s_t^*+k^*-k_t} \tag{A17}$$

are true. Since, the pool is not liquidated in round t when the player submits k_t tickets, $k^*-k_t \geq 0$. In addition, from Property 3, $m_t - s_t^* \geq N+1$, and we also have $N \geq (D+1)(1+r)-1$ and $k-k_t \leq D$. Therefore

$$(k-k_t)\frac{(1+r)}{m_t-s_t^*+k^*-k_t} \leq \frac{D}{D+1} < 1.$$

Therefore, (A16) is true. Furthermore, since $F_t > e_t$, $\frac{s_t^*-k^*}{s_t^*+1}F_t + \frac{k^*+1}{s_t^*+1}e_t > \frac{s_t^*-k^*}{s_t^*+1}e_t + \frac{k^*+1}{s_t^*+1}e_t = e_t$, (A17) is true. Hence, (A14) > (A13). Hence, situation [b] cannot be an equilibrium situation.

Therefore, situation (iii) is inconsistent with equilibrium. ~~regardless what information set the players are at, cashing in all~~ $\cancel{k}$ ~~tickets is a dominant strategy at~~ $\cancel{t}$ ~~for all players. Hence, pool is liquidated in round~~ $\cancel{t}$ ~~instead of~~ $\cancel{t+1}$~~. We now reset players' conjectures of pool liquidation at~~ $\cancel{t}$ ~~and continue with the recursion until~~ $\cancel{t=2}$~~. We end with the result that the pool will be liquidated in round 1.~~ □