Tying with Network Effects*

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Abstract

We develop a leverage theory of tying in markets with network effects. When a monopolist in one market cannot fully extract the whole surplus from consumers, tying can be a mechanism through which unexploited consumer surplus is used as a demand-side leverage to create a "quasi installed base" advantage in another market characterized by network effects. Our mechanism does not require any precommitment to tying; rather tying emerges as a best response that lowers the quality of tied-market rivals. While tying can lead to exclusion of tied-market rivals, it can also expand use of the tying product, leading to ambiguous welfare effects.

1 Introduction

The leverage theory of tying typically considers the following scenario: There is a monopolistic firm in one market (say A). This firm, however, faces competition in another market (say B). According to this theory, the monopolistic firm in market A can monopolize market B using the leverage provided by its monopoly power in market A through tying or bundling arrangements. The Chicago School, however, criticized this theory and proposed instead price discrimination as the main motivation for tying. The gist of the Chicago school criticism is based on the so-called "one monopoly theorem," which states that "[a] seller cannot get two monopoly profits from one monopoly." (Blair and Kaserman, 1985).

We demonstrate that in the presence of imperfect rent extraction in a monopolized market and network effects in a market where the monopolist faces competition, tying can be a mechanism through which unexploited consumer surplus in the monopolized market is used

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as a demand-side leverage to create a strategic "quasi installed-base" advantage in the competing market, raising the perceived quality of the monopolist's tied product and lowering the perceived quality of its tied market rivals.

In markets with network effects, consumer utility consists of stand-alone benefits and network benefits. Under independent pricing, all firms compete on a level playing field with respect to network effects. Even though markets with strong network effects are typically characterized by tipping equilibria in which all consumers choose the same product, yielding maximal network benefits, the network-augmented utility component can be competed away in equilibrium to consumers' benefit. With tying, however, the tying firm can use unexploited consumer surplus in the tying market in competition against a rival firm in a tied market. We show that this advantage allows the tying firm to lock in consumers who have a high value for the tying product, ensuring that it captures the network effect and enabling it to win in the tied market even against a more efficient rival.

More precisely, consider a situation in which there are two markets, A and B. Firm 1 is a monopolist of product A and sells its product $B1$ in market B against a rival, firm 2, that produces product $B2$. Consumers in the monopolized market A are heterogeneous and some consumers receive surplus in this market under independent pricing. In such a scenario, if firm 1 offers only a bundle that ties purchase of its product A to purchase of its product $B1$, consumers with high valuations for product A may prefer to purchase the bundle even if all other consumers purchase the rival firm's product $B2$. The existence of such consumers ensures a guaranteed market share in market B for firm 1, which is akin to firm 1 having an installed base. This advantage in terms of the quasi-installed base can in turn induce low valuation consumers to purchase the bundle instead of buying $B2$. We show that a process of iterated elimination of dominated strategies can lead to tipping toward the monopolist's bundle.

Notably, and in contrast to much of the literature on the strategic use of tying (which we discuss below), this mechanism does not rely on the ability to precommit to tying, such as through technological bundling. Rather, the incentive to leverage unexploited consumer surplus in the tying market to degrade the relative quality of rivals in the tied market makes tying a best response by the monopolist absent any precommitment.[1]

We first develop our theory in the context of independent products to illustrate how network effects in the tied good market may provide incentives to tie. Specifically, we provide a sufficient condition for the monopolist to tie in equilibrium and thereby monopolize the tied good market. In situations in which the tying good market is covered (i.e., all consumers purchase the tying product under independent pricing), we show that pure bundling is an optimal strategy for the monopolist. When the tying good market is not covered under independent pricing, firm 1 may instead find it optimal to tie using a mixed bundling strategy in which consumers can choose between buying an $A/B1$ bundle and buying product $B1$ only. This mixed bundling enables firm 1 to screen consumers with respect to their willingness to pay for the monopolized product A while maximizing the network effects for its product $B1$.

[1] Our mechanism can still be effective and attractive for the monopolist when precommitment is possible, but precommitment is not necessary (nor assumed in our analysis).

When the number of consumers buying the bundle is large enough, firm 1 is able to sell even its inferior $B1$ at a profit as a stand-alone product against product $B2$.

We then extend our analysis to the case of complementary products. With pure monopoly in the tying product market, we confirm the Chicago School critique that tying cannot be a leverage mechanism even with network effects. However, pure monopoly with absolutely no competitive products is rare. We show that in the presence of an inferior alternative to the tying good we can restore our mechanism with parallel results to the independent products case; we formally demonstrate the equivalence of the complementary products case to the independent products case, with the inferior alternative in the tying market playing the same role as does the no purchase option in the independent products case.

Our analysis can be used to develop a theory of harm for tying cases when network effects are critical in the determination of the market winner, and sheds light on some recent cases involving tying.

As we noted above, the literature on tying as an anticompetitive foreclosure mechanism has focused most on situations in which a monopolist firm commits to use of a tying strategy, as first developed in Whinston (1990).[2] If the market structure in the tied good market is oligopolistic with scale economies, tying can be an effective and profitable strategy to alter market structure by making continued operation unprofitable for tied good rivals. This effect occurs because a commitment to tying leads the monopolist to price aggressively in order to ensure sales of the valuable product A. However, in Whinston (1990), inducing the exit of the rival firm is essential for the profitability of tying arrangements. Thus, if the competitor has already paid the sunk cost of entry and there is no avoidable fixed cost, tying cannot be a profitable strategy. In contrast, our mechanism requires neither commitment power of the tying firm nor exit of the rival.[3]

While the commitment assumption makes sense when firms employ technological ties, in many tying cases the tie is a pricing choice that seems to involve little commitment.[4] For example, in the recent EU *Android* case, Google was found guilty of requiring Android OEMs to pre-install the Google search app as a condition of gaining a license to Google's app store (the Play Store). The tie was not technological, but rather purely contractual, and raises the question of why Google would not have simply paid the OEMs for Android pre-installation. While the literature on bundling provides an answer by showing that with heterogeneous valuations tying can indeed be a best response as a price discrimination mechanism (as the Chicago School claimed), in these cases tying might be viewed as "innocent" and any effects on rivals inadvertent. What differs in our theory, however, is that tying can be a best response precisely because it *lowers the perceived quality of the monopolist's tied market rivals* by reducing the network benefits they can provide.

The idea of using unexploited consumer surplus as a leverage mechanism appears in some other papers. Burstein (1960) and Greenlee et al. (2008) analyze a setting in which

[2]Fumagalli et al. (2018) provide an excellent survey of tying as an exclusionary practice along with discussions on major antitrust cases.

[3]If network effects are absent, we replicate his result that bundling is not profitable if firm 2's exit is not induced.

[4]In some cases, reputational concerns might lead to an element of commitment.

the monopolist in the tying product market sells to consumers with multiunit demands and is unable to fully extract consumer surplus with linear pricing. By tying, even to competitively-supplied tied goods, the monopolist can require buyers to purchase additional products at elevated prices. In essence, tying serves as a substitute for a fixed fee.[5] In contrast, in our model consumers have single-unit demands for the tying good and so tying cannot serve this function.

Calzolari and Denicolo's (2015) theory of exclusive dealing is also based on uncaptured consumer surplus with multiunit buyers. They consider a single-market situation in which there is a dominant firm with a competitive advantage over a competitive fringe of rivals, but buyers are able to obtain information rents due to private information even if they deal exclusively with the dominant firm. Without exclusive dealing, the dominant firm needs to compete for each marginal unit of a buyer's demand; in contrast with exclusive dealing, the dominant firm competes for the entire volume demanded by a buyer. This change enables the dominant firm to exclude rivals by leveraging on the information rents left on inframarginal units. Thus, the dominant firm is able to exclude rivals with a lower discount with the imposition of exclusive deals. Exclusive dealing serves as a more profitable pricing mechanism despite the fact that it has no effects on the prices or qualities offered by the dominant firm's rivals. In contrast, in our model with heterogeneous consumers with single-unit demands, our mechanism leverages the network effect provided by inframarginal tying good consumers who are "committed" to the bundle in order to monopolize the tied good market.

Carlton and Waldman (2002) is the paper most closely related to ours. They also consider a model in which network effects exist in the tied good market and they note that tying can be an effective strategy in such cases without commitment. They consider a dynamic two-period model in which the tying and tied goods are perfect complements. In period 1 there is a monopoly in the tying "primary" market, while in period 2 the tied ("complementary good") market rival can potentially enter the primary market. In contrast to our focus on tying as a mechanism to profitably monopolize the tied good market, the purpose of tying by the primary good monopolist in their model is to preserve its market power by preventing entry into the primary market, even though it may entail short-run losses. As well, in our model these effects operate within a single period.[6]

The rest of the paper is organized in the following way. In Section 2, we illustrate the main intuitions behind our tying mechanism through a simple example with discrete consumer types. In Section 3, we describe our model for independent products with a more general demand structure. In Section 4, we analyze this model and provide conditions under which tying (offering only a bundle and possibly product $B1$ for sale, and not offering A by itself) emerges in equilibrium and leads to monopolization of market B. In Section 5, we consider complementary products, and show that we can derive parallel results to

[5] However, tying is less efficient than a fixed fee since it causes distortions in the tied markets.

[6] On pp. 206-7, Carlton and Waldman suggest that tying could be profitable in the presence of network effects even "if the alternative producer cannot enter the primary good market," although without providing any analysis. Our paper can be viewed as following up on this point of theirs.

the independent products case if we assume an inferior alternative to the monopolized tying good. We discuss the application of our results to recent antitrust cases and offer concluding remarks in Section 6.

2 An Illustrative Example

To explain the main mechanism and intuition behind our model, we provide an illustrative example. There are two markets A and B. Market A is served by a monopolist, firm 1. In market B, firm 1 and firm 2 compete. These two product markets are independent. Firms' production costs are normalized to zero in all markets. There are two consumers.

2.1 Market A

The two consumers are heterogeneous in terms of their valuations for product A. One is a high (H) type consumer and the other is a low (L) type consumer. Each consumer's willingness to pay for product A is given by u_k, where $k = H, L$, with $u_H = u_L + s > u_L > 0$ so $s > 0$. We assume that $u_L > s$. This implies that the optimal monopoly price in market A is $p_A^* = u_L$ and consumer H receives a surplus of s. An important feature of market A is that firm 1 is unable to extract the whole surplus in the market despite its monopoly power.

2.2 Market B

Market B is characterized by network effects and firm 1's product $B1$ is inferior to firm 2's product $B2$. Products $B1$ and $B2$ are not compatible with each other. In this market, we assume that the two consumers have the same preferences. More specifically, firm i's product provides a stand-alone value of v_i to consumers, where $v_2 > v_1 > 0$. If the two consumers purchase the same product i, there are additional network benefits of $n > 0$, yielding a total value of $v_i + n$. In other words, given that a consumer buys product Bi, her gross surplus is v_i if she is the only consumer buying the product and is the larger amount $v_i + n$ if the other consumer buys also the same product.

2.3 Independent Pricing Equilibrium

Consider first the market equilibrium when the two products are sold independently by firm 1. In this case, the two markets can be analyzed independently. As noted above, firm 1 will charge $p_A^* = u_L$ in market A. In market B, given any prices (p_{B1}, p_{B2}), there is no continuation equilibrium in which the two consumers choose different products; in equilibrium, either both consumers purchase $B1$ or both purchase $B2$. We restrict attention to coalition-proof Nash equilibrium (CPNE) of the consumer response [Bernheim, Peleg, and Whinston (1987)]. An implication of the "coalition-proofness" is that when players have identical preferences they coordinate on their Pareto-optimal continuation equilibrium. Then, we have a unique equilibrium in which firm 2 charges $p_{B2} = \Delta$, firm 1 charges $p_{B1} = 0$

and both consumers buy product $B2$.[7] Hence, consumers capture the full network benefits for themselves, and firm 2 captures only its quality advantage as profit. In this case, firm 1 earns nothing in market B and an overall profit of

$$\Pi_1^* = 2u_L.$$

2.4 Equilibrium with Tying

Now suppose that firm 1 engages in tying: it requires that any consumer who purchases its monopolized product A also buy product $B1$ (and only $B1$), and sells the bundle at the price of P.[8]

We show that if there is sufficient unexploited surplus in market A – specifically if $s > 2n$ – then firm 1 has a profitable deviation to offering only a bundle starting from any independent pricing equilibrium in which both consumers buy $B2$ (which can occur only when $n > \Delta$).[9] By doing so it leverages the unexploited consumer surplus enjoyed by consumer H in market A to monopolize market B. The argument shows that when firm 1 ties in this fashion there is a unique continuation equilibrium in which consumers' choices to buy the bundle are pinned down by iterated dominance.

The leverage mechanism with two discrete-type consumers operates in two steps. First, tying allows firm 1 to leverage the surplus from the monopoly product A that is enjoyed by consumer H to gain purchases of $B1$. Once consumer H is secured to buy the bundle, firm 1 achieves a strategic advantage in selling to consumer L as if it had consumer H as an installed-base of its product. These network benefits allow firm 1 to induce consumer L to buy the bundle as well.

To see this point, suppose that firm 1 offers only a bundle at price $P = u_L + \varepsilon$ where $\varepsilon > 0$. Observe, first, that if both consumers buy the bundle, then firm 1's profit will strictly exceed its profit in an independent pricing equilibrium in which the consumers coordinate on $B2$ (equal to $2u_L$). Moreover, as we now show, both consumers will indeed buy the bundle for small enough ε.

First, with $P = u_L + \varepsilon$, for small enough ε it is a dominant strategy for consumer H to purchase the bundle: she prefers the bundle even under the most unfavorable condition that $B2$ is offered free and consumer L purchases $B2$ since

$$(u_H + v_1) - P > v_2 + n \Leftrightarrow s > \Delta + n + \varepsilon, \tag{1}$$

which is satisfied for small enough ε given that $s > 2n > \Delta + n$. Given that consumer H purchases the bundle, for small enough ε it is also optimal for consumer L to purchase the

[7] As usual, we break the tie in favor of a more efficient firm.

[8] Our argument requires that consumers will not buy both the bundle and product $B2$ and then use product $B2$ instead of $B1$. Firm 1 can prevent this behavior with a contract that prevents use of $B2$. Alternatively, such a contractual requirement will not be necessary when there are large enough production costs of product $B1$, which would make purchase of the bundle for the purpose of using only product A undesirable.

[9] The assumption of $s > 2n$ is a sufficient condition for tying to be profitable, which is stronger than necessary. We make this assumption for simplicity to reduce the number of cases to consider.

bundle even if $B2$ is offered free since

$$(u_L + v_1) + n - P > v_2 \Leftrightarrow n > \Delta + \varepsilon \tag{2}$$

given that $n > \Delta$. Thus, by deviating in this fashion, firm 1 is certain to monopolize market B and increases its profit.

In the next section, we will examine in a more general model the equilibrium that results when firm 1 can tie in this fashion. When, as here, firm 1 would sell product A to all consumers under independent pricing (the case of "full coverage"), the equilibrium involves firm 1 offering only a bundle and monopolizing both markets. In the example above, that equilibrium outcome involves firm 2 offering to sell product $B2$ at cost ($p^*_{B2} = 0$) and firm 1 selling its bundle at the price $P = u_L + (n - \Delta)$. Thus, by tying firm 1 is able to achieve the same profit as when under independent pricing consumers fail to coordinate on their Pareto-preferred continuation equilibrium, all buying $B1$ instead at price $p^*_{B1} = n - \Delta$.

3 The Independent-Products Model

In this section, we lay out a more general model of tying in markets with network effects. As in the illustrative example in the previous section, we study here the case in which products A and B are independent and can be used separately. (We discuss the case of complements in Section 5.)

Market A is monopolized by firm 1. In market B, there are direct network effects, firm 1 and firm 2 compete, and consumers have homogeneous valuations: their willingness to pay for each firm's product is given by $v_1 + \beta N_1 > 0$ and $v_2 + \beta N_2 > 0$, respectively, where $v_1 > 0, v_2 > 0$, $\beta > 0$, and N_i represents the number of consumers using firm i's product Bi. We normalize the total number of consumers to 1. All marginal costs are normalized to zero.

At the heart of our leverage mechanism is "unexploited consumer surplus" in the tying market which can be used in competition with a competitor in another market. If there are high valuation consumers who receive sufficiently large consumer surpluses, they may be willing to purchase the bundle (rather than product $B2$ only) even if all other consumers purchase $B2$. The existence of such high valuation consumers in market A provides a demand-side leverage for firm 1 in market B akin to having an installed base. If network effects are sufficiently strong, this strategic advantage more than makes up for any quality disadvantage of firm 1 and enables firm 1 to monopolize market B, extracting the resulting network effects as profit.

More specifically, we assume that consumers' valuations for product A, denoted u, are distributed on $[\alpha, \alpha + \overline{u}]$, where α represents the lower bound for the consumers' valuations.[10] It will be convenient to define a consumer's "type" as $x = u - \alpha$, which we assume is

[10]For example, consider a product that has a basic functionality plus some additional features. We can imagine a situation in which the basic functionality provides the same utility of α to all consumers, but additional features may generate different levels of extra utility to consumers, which is distributed on $[0, \overline{u}]$.

distributed on $[0, \overline{u}]$ according to a c.d.f. G with a strictly positive density g.[11]

Remark 1 *Our model can also be applied to two-sided markets where in market A firm 1 is a two-sided platform that receives advertising revenue whenever it is chosen by a consumer. If we assume that there is an associated advertising revenue of $\alpha > 0$ for each consumer in market A and consumers' valuations for product A are distributed on $[0, \overline{u}]$, then our one-sided market model is isomorphic to a two-sided model with additional advertising revenue per consumer.*

Let p_A be the price of product A. With a change of variables of $\widehat{p}_A = p_A - \alpha$, we have a demand function $D(\widehat{p}_A) = 1 - G(\widehat{p}_A)$ in market A. We assume that $G(\cdot)$ satisfies the monotone hazard rate condition, that is, $\frac{g(\cdot)}{1-G(\cdot)}$ is strictly increasing. In market A, with independent pricing firm 1 chooses $\widehat{p}_A$ to maximize

$$\max_{\widehat{p}_A} (\widehat{p}_A + \alpha) [1 - G(\widehat{p}_A)] . \tag{3}$$

Remark 2 *In the case of a two-sided tying market with advertising revenues α earned by firm 1, the price $\widehat{p}_A = p_A - \alpha$ is the price paid by consumers while α is a negative marginal cost of the firm, as reflected in problem (3).*

In market B, we make the following assumption:

Assumption 1: $\Delta \equiv v_2 - v_1 > 0$ and $\Delta < \beta < \frac{1}{2g(x)}$ for all $x \in [0, \overline{u}]$.

As in Section 2, the condition that $\Delta > 0$ means that firm 2's product $B2$ has higher quality than firm 1's product $B1$. The assumption that $\beta > \Delta$ means that network effects are sufficiently important relative to the quality differential Δ: if all consumers buy product $B1$ from firm 1, then its (network-augmented) quality $v_1 + \beta$ becomes higher than that of product $B2$, v_2. Last, the assumption that $\beta < 1/[2g(x)]$ for all x is a stability condition for interior equilibrium in the tying regime[12] and guarantees a unique cut-off type when firm 1 offers a pure bundle (see the proof of Lemma 2).

We will consider two simultaneous pricing games and compare them, in one firm 1 can offer only single-product prices p_A and p_{B1} ("independent-pricing" game), while in the other it is free to also offer a bundle at price P. In the latter case, no pre-commitment is involved; rather, firm 1 decides on whether to bundle purely as a best response to firm 2's price p_{B2}.

4 Analysis of the Independent-Products Model

In this section, we examine the impact of allowing tying in the independent-products model. In the rest of the paper, we restrict attention to coalition-proof Nash equilibrium (CPNE) of

[11] We admit the possibility that $\overline{u} = \infty$.

[12] If $G(\cdot)$ is uniform, $\beta < \frac{1}{2g(x)} = \frac{\overline{u}}{2}$ is the necessary and sufficient condition for an interior equilibrium to be stable. For general distributions, it is a sufficient, but not necessary, condition for the stability of an interior equilibrium because the violation of the condition implies only local instability.

the consumer response, a stronger notion of self-enforceability that accounts for coalitional deviations [Bernheim, Peleg, and Whinston (1987)]. We use this refinement of Nash equilibrium to simplify exposition and help avoid the issue of multiplicity of consumer responses given offered prices: an implication of the "coalition-proofness" in consumer response is that when players have identical preferences they coordinate on their Pareto-optimal continuation equilibrium. We also assume that a firm's removal of a purchase option that no consumer is choosing does not change consumer responses; this eliminates the possibility that a set of prices is sustained by the threat that consumers might resolve their indifference in a disadvantageous way for the firm once the option is removed. We continue to restrict attention to equilibria in undominated price offers.

4.1 Independent-Pricing Game

In the absence of tying, as in Section 2 the two markets can be analyzed independently as we assume independent products.

In market B, all consumers have the same preference. Since we restrict our attention to CPNE of the consumer response, all consumers purchase product $B2$ in the unique equilibrium.[13] In the equilibrium, firm 1 charges zero price ($p_{B1}^* = 0$) and firm 2 charges $p_{B2}^* = \Delta$. Thus, when consumers coordinate their purchase responses with neither firm having an advantage in network effects, all consumers end up purchasing product $B2$ at a price at which the consumers capture the network benefits.

In market A, independent pricing may or may not result in all consumers purchasing product A. We denote the solution to the monopoly pricing problem (3) by p_A^*. Under the monotone hazard assumption on $G(\cdot)$, $p_A^* = \alpha$ (or, equivalently, $\widehat{p}_A^* = 0$) so that all consumers buy A (there is "full coverage") when

$$\alpha \geq \frac{1 - G(0)}{g(0)} = \frac{1}{g(0)} \tag{4}$$

and $p_A^* > \alpha$ (or, equivalently, $\widehat{p}_A^* > 0$) otherwise. In the latter case, firm 1 sets a price of $p_A^* = \alpha + \widehat{p}_A^*$, where $\widehat{p}_A^*$ satisfies the following condition.

$$\widehat{p}_A^* = \frac{1 - G(\widehat{p}_A^*)}{g(\widehat{p}_A^*)} - \alpha \ (> 0). \tag{5}$$

In both cases, the mass of consumers buying product A without tying is given by $1 - G(\widehat{p}_A^*)$.

Hence, without tying, firm 1 receives a profit of

$$\Pi_1^* = \begin{cases} \alpha & \text{if } \widehat{p}_A^* = 0 \\ (\alpha + \widehat{p}_A^*)(1 - G(\widehat{p}_A^*)) = \frac{[1-G(\widehat{p}_A^*)]^2}{g(\widehat{p}_A^*)} & \text{if } \widehat{p}_A^* > 0 \end{cases}$$

[13] As is standard in the literature, we make the tie-breaking assumption in favor of the firm that can offer the highest consumer surplus to avoid the open set problem.

Firm 2's profit is the same as in Section 2:

$$\Pi_2^* = \Delta.$$

4.2 Tying Equilibrium

We now derive the equilibrium when firm 1 is allowed to tie, requiring purchase (and use) of product $B1$ in order to acquire product A.[14] We will show that, under certain conditions, in this equilibrium firm 1 offers for sale only either a bundle (at price P) and possibly also product $B1$ (at price p_{B1}), and by doing so monopolizes market B. Moreover, in cases of "full coverage" in which all consumers would buy product A at its monopoly price p_A^*, firm 1 offers only the bundle.

To begin, we first show that in response to any price offered by firm 2, $p_{B2} \geq 0$, firm 1 optimally either adopts independent-good pricing or refuses to sell product A separately by offering for sale only a bundle and (possibly) $B1$:

Lemma 1 *For any price $p_{B2} \geq 0$ offered by firm 2, firm 1 optimally either adopts independent pricing or refuses to sell product A separately, instead tying sale of A to purchase and use of $B1$; i.e., it always has a best response in which it either sells products A and $B1$ separately at prices (p_A, p_{B1}) or it sells only a bundle and product $B1$ at prices (P, p_{B1}).*

Proof. If firm 1 sets prices such that $P \geq p_A + p_{B1}$, then consumer purchases are the same as if it had instead simply offered independent prices (p_A, p_{B1}) as consumers all weakly prefer separate purchases of products A and $B1$ to buying the bundle. So suppose instead that firm 1 offers a bundle discount so that $P < p_A + p_{B1}$. Figure 1 depicts the purchase decisions of three sets of consumers in this case, who are distinguished by their value u for product A:

Consumer Set I ($u < P - p_{B1}$): These consumers buy either $B1$ or $B2$ and do not buy A. This choice is independent of the level of u for these consumers, who all make the same choice (given the CPNE refinement).

Consumer Set II ($u \in [P - p_{B1}, p_A)$): These consumers buy either $B2$ or the bundle. This choice depends on the value of u, with a consumer in this set more likely to buy the bundle for higher values of u.

Consumer Set III ($u \geq p_A$): These consumers buy either A and $B2$ or they buy the bundle. This choice is independent of the level of u for these consumers, who all make the same choice (given the CPNE refinement).

Observe, first, that firm 1 sells the separately offered product A at most to consumers in Set III. If firm 1 is not selling any of product A to consumers in this set, then dropping the separate sale of product A and only offering to sell the bundle and product $B1$ has no

[14] As we noted earlier, the requirement that the consumer not also purchase and use $B2$ is not necessary if the cost of producing $B1$ and $B2$ (which we have set here to zero) is sufficiently high; if it is, no consumer who has purchased an $A/B1$-bundle will also find it worthwhile to pay a price above cost for product $B2$.

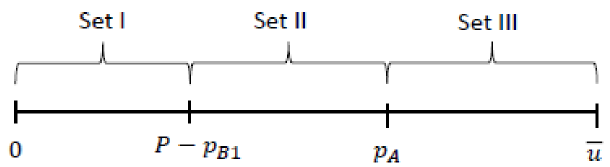

Figure 1: Consumer purchase decisions as a function of their type x

effect on firm 1's profit. Suppose, instead, that consumers in Set III all purchase product A and product $B2$. If so, then the consumer type with valuation $u = p_A$ must weakly prefer purchasing product $B2$ over buying the bundle. In that case, all other consumers in Set II, who have lower values u for product A, must strictly prefer purchasing product $B2$ over buying the bundle. Finally, if that is so, then the consumer with type $u = P - p_{B1}$ must strictly prefer buying $B2$ over buying only $B1$, and hence so must all consumers in Set I. In sum, if any consumers are buying the separately sold product A, then these are the *only* sales that firm 1 is making. If so, then firm 1 would do at least as well by offering instead its optimal independent-pricing best response (which includes pricing product A at p_A^*) and thereby earning at least as much profit from sales of product A and possibly also profits from sales of product $B1$. ■

We next state what we will show is a sufficient condition for firm 1 to monopolize market B when tying is permitted:

Assumption 2: $\frac{1}{g(0)} - G^{-1}\left(\frac{\beta-\Delta}{2\beta}\right) < \alpha$

Assumption 2 will guarantee that if all consumers of product A under independent pricing purchase the bundle, then product $B1$ offers a higher utility than product $B2$ even if all remaining consumers purchase $B2$: i.e., $\beta(1 - G(\widehat{p}_A^*)) > \Delta + \beta G(\widehat{p}_A^*)$ holds.[15] Assumption 2, which ensures that enough consumers buy A under independent pricing, is more likely to be satisfied if α is large and $\frac{\Delta}{\beta}$ is small.[16] Note that if the full coverage condition (4) holds, then Assumption 2 necessarily holds since then

$$\frac{1}{g(0)} - G^{-1}\left(\frac{\beta - \Delta}{2\beta}\right) < \frac{1}{g(0)} < \alpha.$$

It is useful to first consider how consumers would react to firm 1 offering only a pure bundle. The following lemma shows that the consumer response is pinned down by iterated

[15]The proof of Lemma 3 shows that the assumption implies the inequality.

[16]If x is uniformly distributed with $g = 1/\overline{u}$ over $[0, \overline{u}]$, the condition in Assumption 2 can be written as $\frac{(\beta+\Delta)}{2\beta}\overline{u} < \alpha$.

dominance.

Lemma 2 *When firm 1 offers only a bundle for sale, given prices of P for the bundle and p_{B2} for product $B2$, and defining $\widehat{P} \equiv P - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*[17]
(i) If $(\widehat{P} - p_{B2}) \in (\beta - \Delta, \overline{u} - \beta - \Delta)$, consumers whose valuation for A is higher than $\widetilde{x} \in (0, \overline{u})$ purchase the bundle while consumers whose valuation is lower than $\widetilde{x}$ purchase $B2$, where $\widetilde{x}$ satisfies

$$\widetilde{x} + v_1 + \beta(1 - G(\widetilde{x})) - \widehat{P} = v_2 + \beta G(\widetilde{x}) - p_{B2}. \tag{6}$$

(ii) If $(\widehat{P} - p_{B2}) \leq \beta - \Delta$, all consumers purchase the bundle (i.e., $\widetilde{x} = 0$).
(iii) If $(\widehat{P} - p_{B2}) \geq \overline{u} - \beta - \Delta$, all consumers purchase $B2$ only (i.e., $\widetilde{x} = \overline{u}$) .

Remark 3 *Note that $\beta - \Delta < \overline{u} - \beta - \Delta$ since $1 = \int_0^{\overline{u}} g(x)dx \leq [\max_x g(x)]\overline{u}$ implies that $\overline{u} \geq 1/[\max_x g(x)] > 2\beta$, where the last inequality follows from Assumption 1.*

Proof. Let $\psi(t, x)$ be the payoff gain from purchasing the bundle over purchasing $B2$ for a type t consumer (i.e., whose willingness to pay for A is $\alpha + t$) if all other players whose types are higher than x choose the bundle.

$$\psi(t, x) = t + \beta[1 - 2G(x)] - \Delta - (\widehat{P} - p_{B2}).$$

Note that $\psi(t, x)$ is continuous in t and x, increasing in t, and decreasing in x. Define as well the function $\Psi(x)$ as follows:

$$\Psi(x) \equiv \psi(x, x) = x + \beta[1 - 2G(x)] - \Delta - (\widehat{P} - p_{B2}).$$

Note that under Assumption 1, $\Psi(x)$ is increasing in x because by Assumption 1

$$\Psi'(x) = 1 - 2\beta g(x) > 0$$

and when $\beta - \Delta < (\widehat{P} - p_{B2}) < \overline{u} - \beta - \Delta$, we have $\Psi(0) < 0 < \Psi(\overline{u})$. Therefore, in any equilibrium response by consumers the "cut-off" type $\widetilde{x}$ who is indifferent between the bundle and product $B1$ (if interior) must be the unique solution to $\Psi(\widetilde{x}) = 0$. As in the analysis of global games, we use an induction argument to show that the choices of types above and below $\widetilde{x}$ are in fact pinned down by iterated dominance.[18] Observe that:

(i) If $(\widehat{P} - p_{B2}) < \overline{u} - \beta - \Delta$, then even when all other consumers are expected to choose $B2$ (i.e., the cut-off type is $\overline{x}^0 = \overline{u}$) , it is optimal to choose the bundle for any consumer whose type is higher than $\overline{x}^1 = \beta + \Delta + (\widehat{P} - p_{B2}) < \overline{u} = \overline{x}^0$. Given that at least a measure of $1 - G(\overline{x}^1)$ consumers choose the bundle, we can derive another cut-off value $\overline{x}^2 < \overline{x}^1$. Note that $\overline{x}^n$ is

[17] When there is a unique consumer response that survives iterated elimination of dominant strategies, it is also the unique Nash equilibrium of consumer responses, and hence the only possible CPNE.
[18] For an excellent survey of global games, see Morris and Shin (2010).

a decreasing sequence. Similarly, if $\beta - \Delta < (\widehat{P} - p_{B2})$, then even when all other consumers are expected to choose the bundle (i.e., the cut-off type $\underline{x}^0 = 0$), it is optimal to choose $B2$ for any consumer whose type is lower than $\underline{x}^1 = -\beta + \Delta + (\widehat{P} - p_{B2}) > 0 = \underline{x}^0$. Given that at least a measure of $G(\underline{x}^1)$ consumers choose $B2$, we can derive another cut-off value $\underline{x}^2 > \underline{x}^1$. Note that $\underline{x}^n$ is an increasing sequence. Thus, when $\beta - \Delta < (\widehat{P} - p_{B2}) < \overline{u} - \beta - \Delta$, the continuity of $\psi(t, x)$ and the way the two sequences $\overline{x}^n$ and $\underline{x}^n$ are constructed imply that $\psi(\underline{x}, \underline{x}) = \psi(\overline{x}, \overline{x}) = 0$, where $\overline{x} = \lim_{n\to\infty} \overline{x}^n$ and $\underline{x} = \lim_{n\to\infty} \underline{x}^n$. Given that there is a unique $\widetilde{x}$ such that $\Psi(\widetilde{x}) = 0$, it must be that $\underline{x} = \overline{x} = \widetilde{x}$.

(ii) If $(\widehat{P} - p_{B2}) \leq \beta - \Delta$, the process of iterated deletion of dominated strategies leads to $\widetilde{x} = 0$ because $\Psi(0) > 0$.

(iii) Similarly, if $(\widehat{P} - p_{B2}) \geq \overline{u} - \beta - \Delta$, the process of iterated deletion of dominated strategies leads to $\widetilde{x} = \overline{u}$ because $\Psi(\overline{u}) < 0$. ■

A consequence of Lemma 2 is that even if $p_{B2} = 0$, by choosing $P = \alpha + \beta - \Delta$, firm 1 can induce all consumers to buy the bundle and realize a profit of $P = \alpha + \beta - \Delta$, which is strictly larger than the profit under independent pricing when full coverage is optimal. More generally, Lemma 2 implies that if tying is allowed and Assumptions 1 and 2 hold, then no independent-pricing equilibrium exists: firm 1 has a profitable deviation in which it offers only a bundle:

Lemma 3 *If tying is allowed and Assumptions 1 and 2 hold, then no independent-pricing equilibrium exists.*

Proof. Suppose that firm 2 is setting price $p_{B2} \geq 0$ in an independent pricing equilibrium. Using (6), define the bundle price that implements critical type $\widetilde{x}$ as

$$\widehat{P}(\widetilde{x}|p_{B2}) \equiv p_{B2} + \widetilde{x} + \beta[1 - 2G(\widetilde{x})] - \Delta. \tag{7}$$

Suppose that firm 1 deviates from independent pricing and instead offers only a bundle at price $\widehat{P}(\widehat{p}_A^*|p_{B2})$ where $\widehat{p}_A^* = p_A^* - \alpha$ is the lowest type buying product A in the independent pricing equilibrium. Firm 1 then makes the same number of sales as under independent pricing (since we will have $\widetilde{x} = \widehat{p}_A^*$), but at a higher price since

$$\begin{aligned} \widehat{P}(\widehat{p}_A^*|p_{B2}) &\geq \widehat{p}_A^* + (\beta - \Delta) - 2\beta G(\widehat{p}_A^*) \\ &= \widehat{p}_A^* + 2\beta \left[\frac{\beta - \Delta}{2\beta} - G(\widehat{p}_A^*) \right] \\ &> \widehat{p}_A^* \end{aligned}$$

where the first inequality follows from (7) and the fact that $p_{B2} \geq 0$, while the final inequality follows because

$$G^{-1}\left(\frac{\beta - \Delta}{2\beta}\right) > \frac{1}{g(0)} - \alpha \geq \frac{1 - G(\widehat{p}_A^*)}{g(\widehat{p}_A^*)} - \alpha = \widehat{p}_A^*,$$

where the first inequality follows from Assumption 2, the second from the monotone hazard rate property, and the third from (5). Hence, this deviation increases firm 1's profit. ■

Remark 4 *Because for any $p_{B2} \in [0, \Delta]$ firm 1's independent-pricing best response results in the same profit for firm 1 as in the independent pricing equilibrium (firm 1 sells only product A at price p_A^*), the proof of Lemma 3 also shows that independent-pricing is not a best response to any such price offer by firm 2.*

We now establish our main result:

Proposition 1 *If tying is allowed and Assumptions 1 and 2 hold, the unique equilibrium involves firm 2 tying (offering for sale only the bundle and possibly product $B1$) and fully monopolizing market B. Specifically,*

(i) If $\alpha + \beta \geq \frac{1}{g(0)}$: firm 1 sells only the bundle at price $P^* = \alpha + (\beta - \Delta)$ while firm 2 sets $p_{B2}^* = 0$.

(ii) If $\alpha + \beta < \frac{1}{g(0)}$: firm 1 sells the bundle to consumers with valuations above the critical type $\widetilde{x}^* < \widehat{p}_A^*$ satisfying

$$\widetilde{x}^* = \left[\frac{(1 - G(\widetilde{x}^*))}{g(\widetilde{x}^*)} - \alpha\right] - \beta \tag{8}$$

and sells $B1$ to the remaining consumers, with equilibrium prices given by

$$\begin{aligned} P^* &= \alpha + \widetilde{x}^* + \beta[1 - G(\widetilde{x}^*)] - \Delta \\ p_{B1}^* &= \beta[1 - G(\widetilde{x}^*)] - \Delta \\ p_{B2}^* &= 0. \end{aligned}$$

Remark 5 *Comparing (4) with the condition for case (i), we can immediately see that case (i) of the Proposition applies whenever there is full coverage of market A under independent pricing.*

Proof. By Lemma 3 any equilibrium must involve sales of the bundle to some consumers. (If not, then considering the discussion of Figure 1 we see that firm 1 must be making no sales of $B1$. It must therefore be charging p_A^* to not have a profitable deviation to independent pricing, and must also (by undominated pricing) be setting $p_{B1}^* = 0$ and $\widehat{P}^* = p_A^*$. Hence, the equilibrium would be equivalent to an independent pricing equilibrium, in contradiction to Lemma 3.) By our previous logic, these must be the consumers with the highest value for product A. Moreover, all lower-type consumers not purchasing the bundle must (by the CPNE refinement) be making the same choice between product $B1$ and product $B2$.

We first show that they cannot be buying product $B2$. Suppose they were. In that case, we can show that firm 1 would have a profitable deviation to undercut firm 2's sales of $B2$, switching firm 2's customers to product $B1$. First, we show that the critical type $\widetilde{x}^*$ who is indifferent between the bundle and product $B2$ must have $\widetilde{x}^* \leq \widehat{p}_A^*$. If $\widetilde{x}^* = 0$, this is necessarily the case, so suppose that $\widetilde{x}^* > \widehat{p}_A^* \geq 0$. By the definition of $\widetilde{x}^*$, we have

$$\widetilde{x}^* = \arg\max_{\widetilde{x}} \Pi_1(\widetilde{x}|p_{B2}) \equiv [\widehat{P}(\widetilde{x}|p_{B2}) + \alpha] \cdot (1 - G(\widetilde{x}).$$

That is,

$$\Pi_1(\widetilde{x}^*|p_{B2}) \equiv [\widehat{P}(\widetilde{x}^*|p_{B2}) + \alpha] \cdot (1 - G(\widetilde{x}^*) \geq [\widehat{P}(\widetilde{x}|p_{B2}) + \alpha] \cdot (1 - G(\widetilde{x}) \text{ for any } \widetilde{x}$$

However,

$$\begin{aligned}
\Pi_1(\widehat{p}_A^*|p_{B2}) &= [p_{B2} + \widehat{p}_A^* + \beta(1 - 2G(\widehat{p}_A^*)) - \Delta + \alpha)] (1 - G(\widehat{p}_A^*)) \\
&= (\widehat{p}_A^* + \alpha)(1 - G(\widehat{p}_A^*)) + [p_{B2} + \beta(1 - 2G(\widehat{p}_A^*)) - \Delta)] (1 - G(\widehat{p}_A^*)) \\
&> (\widetilde{x}^* + \alpha)(1 - G(\widetilde{x}^*)) + [p_{B2} + \beta(1 - 2G(\widetilde{x}^*)) - \Delta)] (1 - G(\widetilde{x}^*)) \\
&= [p_{B2} + \widetilde{x}^* + \beta(1 - 2G(\widetilde{x}^*)) - \Delta + \alpha)] (1 - G(\widetilde{x}^*)) = \Pi_1(\widetilde{x}^*|p_{B2}),
\end{aligned}$$

where the inequality is coming from (i) $(\widehat{p}_A^* + \alpha)(1 - G(\widehat{p}_A^*)) \geq (\widetilde{x}^* + \alpha)(1 - G(\widetilde{x}^*))$ by the definition of $\widehat{p}_A^*$ as the optimal price in market A under independent pricing and (ii) $\beta(1 - 2G(\widehat{p}_A^*)) - \Delta > 0$ under Assumption 2 (from the same argument as in the proof of Lemma 3) and $-G(\widehat{p}_A^*) > -G(\widetilde{x}^*)$ by our supposition of $\widetilde{x}^* > \widehat{p}_A^*$. Thus, we have a contradiction and we can conclude that $\widetilde{x}^* \leq \widehat{p}_A^*$. Given this fact, observe that firm 1 can undercut firm 2, even if firm 2 is charging $p_{B2} = 0$, and switch the consumers buying $B2$ to buying $B1$ by offering price $p_{B1} = \beta[1 - 2G(\widetilde{x})] - \Delta - \varepsilon$ for sufficiently small $\varepsilon > 0$: this follows using iterated dominance because, given the fact that all consumers with $x > \widetilde{x}^*$ purchase the bundle (since $\widehat{P}(\widetilde{x}^*|p_{B2}) < \widehat{P}(\overline{u}|p_{B2}) = \overline{u} - \beta - \Delta$ these consumers' choices are themselves pinned down by iterated dominance, as shown by Lemma 2), all consumers with $x \leq \widetilde{x}^*$ prefer to purchase product $B1$ rather than $B2$ even if all other consumers with $x \leq \widetilde{x}^*$ choose $B2$ since

$$v_1 + \beta(1 - G(\widetilde{x}^*)) - p_{B1} > v_2 + \beta G(\widetilde{x}^*)$$

Given that $\widetilde{x}^* \leq \widehat{p}_A^*$ this deviation would be profitable because then

$$\begin{aligned}
p_{B1} &= \beta[1 - 2G(\widetilde{x}^*)] - \Delta - \varepsilon \\
&\geq \beta - \Delta - 2\beta G(\widehat{p}_A^*) - \varepsilon \\
&> 0
\end{aligned}$$

for sufficiently small $\varepsilon > 0$, which contradicts firm 2 making sales of $B2$.

So, consider instead the equilibrium cut-off type when firm 1 makes sales of the bundle and also possibly of product $B1$, while firm 2 makes no sales and sets $p_{B2}^* = 0$. Since the consumer types below the equilibrium cut-off $\widetilde{x}$ all have the same preference for $B1$ versus $B2$ and coordinate their purchase decisions (given the CPNE refinement) the most firm 1 can charge for $B1$ and secure these consumers' business is $p_{B1} = \beta(1 - G(\widetilde{x})) - \Delta$.[19] The equilibrium cut-off type $\widetilde{x}$ who is indifferent between purchasing the bundle and purchasing $B1$ then satisfies $\widetilde{x} = \widehat{P} - p_{B1}$. If firm 1 charges $p_{B1} = \beta(1 - G(\widetilde{x})) - \Delta$ and $\widehat{P} = \widetilde{x} + \beta(1 - G(\widetilde{x})) - \Delta$,

[19] Purchasing $B1$ gives these consumers $v_1 + \beta - p_{B1}$ while deviating to purchase $B2$ would instead give them $v_2 + \beta G(\widetilde{x})$, so the maximum firm 1 can charge for $B1$ without leading this set of consumers to deviate collectively (and in a self-enforcing way) is $p_{B1} = \beta(1 - G(\widetilde{x})) - \Delta$.

the cut-off that maximizes its profit would solve

$$\underset{\widetilde{x}}{Max}\ [\alpha + \widetilde{x} + \beta(1 - G(\widetilde{x})) - \Delta](1 - G(\widetilde{x})) + [\beta(1 - G(\widetilde{x})) - \Delta]G(\widetilde{x})$$

or equivalently

$$\underset{\widetilde{x}}{Max}\ (\widetilde{x} + \alpha + \beta)(1 - G(\widetilde{x})) - \Delta.$$

The first-order condition of this (relaxed) problem is (8) and the conclusion that $\widetilde{x}^* < \widehat{p}_A^*$ follows from a comparison of (8) and (5).[20]

Finally, we argue that at the proposed equilibrium prices no set of consumer types would wish to collectively deviate to purchasing $B2$. If any set of types would do so, it would be a set of types $[0, \overline{x}]$.[21] We have already argued that set $[0, \widetilde{x}^*]$ will not wish to deviate when $p_{B1}^* = \beta[1 - G(\widetilde{x}^*)] - \Delta$. It is immediate that no set $[0, x']$ with $x' < \widetilde{x}^*$ would want to deviate as all such consumers have the same preferences and the network benefits of the deviation are largest for $x' = \widetilde{x}^*$. So, consider sets $[0, x'']$ where $x'' > \widetilde{x}^*$. For consumers with $x \in (\widetilde{x}^*, x'')$, who all buy the bundle in the proposed equilibrium, to each be better off with this deviation it must be that

$$x + v_1 + \beta - \widehat{P}^* < v_2 + \beta G(x'') \text{ for all } x \in (\widetilde{x}^*, x'')$$

and in particular,

$$\widehat{P}^* > x'' + \beta[1 - G(x'')] - \Delta.$$

However,

$$\widehat{P}^* = \widetilde{x}^* + \beta[1 - G(\widetilde{x}^*)] - \Delta < x'' + \beta[1 - G(x'')] - \Delta,$$

where the last inequality follows from Assumption 1 which implies that $x + \beta[1 - G(x)]$ is a strictly increasing function.

■

It is worth discussing the mechanism and intuition behind Proposition 1. In our model, tying through mixed bundling – where firm 1 offers both a bundle and $B1$ for sale – enables firm 1 to screen consumers with more price instruments while still maintaining the ability to leverage surplus of inframarginal consumers for its monopoly product A to the competitive market B: as in the case of pure bundling, it is as if firm 1 already had an installed base advantage in competition for product B, which ensures firm 1's market dominance and enables it to expropriate the resulting network benefits of consumers in market B. By contrast,

[20] The problem is a relaxed problem because when $\widetilde{x}$ is the cut-off type that is indifferent between the bundle and $B1$ firm 1 may not be able to charge as high a price as $p_{B1} = \beta[1 - G(\widetilde{x})] - \Delta$ without encouraging a collective deviation to purchase $B2$ by some set of consumers (we have only argued that it would prevent the consumers with types below $\widetilde{x}$ from collectively deviating). We show next that, in fact, with this price for $B1$ no set of types will deviate and so the solution to the relaxed problem is in fact the solution to firm 1's pricing problem.

[21] This cutoff property is due to the monotonicity in incentives to deviate; if a type $\overline{x}$ has the incentive to deviate, any types below $\overline{x}$ would have an incentive as well since the lower types suffer less from not consuming product A. In addition, they can enjoy greater network benefits from $B2$ with their own deviation.

under independent pricing, the equilibrium market prices are independent of network effects and all benefits from network effects are competed away. We can contrast this result with the results in most models of strategic leverage theory including Whinston (1990). In these models, mixed bundling replicates the outcome under independent pricing by undoing the strategic effects of pure bundling. This is because pure bundling is optimal only ex ante when the tying firm is able to commit to pure bundling and its competition-intensifying effects lead tied-market rivals to exit (or not enter). Without such commitment, mixed bundling has no bite as a strategic instrument. In the presence of network effects, in contrast, mixed bundling retains strategic value as a leverage mechanism even without any commitment assumption as it is ex post optimal due to its ability to lower the value of rivals' products.

4.3 Welfare Effects of Tying

We now investigate welfare implications of tying in our model. In the case in which market A is covered under independent pricing (i.e., $\widehat{p}_A^* = 0$), bundling is profitable, but always welfare-reducing, lowering aggregate surplus by the amount Δ, as it results in substitution of the inferior product $B1$ for the superior product $B2$. Regarding consumer surplus, as tying reduces aggregate surplus by Δ while changing total industry profit by $(\alpha + (\beta - \Delta)) - (\alpha + \Delta) = \beta - 2\Delta$, in this case it reduces consumer surplus by $(\beta - \Delta)$.

On the other hand, when market A is instead not covered under independent pricing (i.e., $\widehat{p}_A^* > 0$), there is an opposing welfare effect of tying: it expands the use of product A since $\widetilde{x}^* < \widehat{p}_A^*$. Its welfare impacts thus can be ambiguous. More precisely, the loss in welfare in market B, which equals Δ, must be compared to the increase in welfare in market A:[22]

$$\widetilde{AS} - AS = \underbrace{\int_{\widetilde{x}^*}^{\widehat{p}_A^*} (x+\alpha)g(x)dx}_{\text{Market Expansion Effect in } A} - \underbrace{\Delta}_{\text{Efficiency Loss in } B}$$

To explore further how the market expansion effect depends on key parameters of the model, consider first case (ii) of Proposition 1 in which market A is not fully covered even with tying (i.e., $\widehat{p}_A^* > 0$ with $\alpha + \beta < \frac{1}{g(0)}$). In this case, note from condition (8) that $\widetilde{x}^*$ is decreasing in β, whereas $\widehat{p}_A^*$ is independent of β. This implies that the market expansion effect is positively related to β. In contrast, in case (i) of Proposition 1 in which market A is fully covered (i.e., $\widetilde{x}^* = 0$), $\widetilde{x}^*$ is invariant in both α and β as long as $\alpha + \beta \geq \frac{1}{g(0)}$, whereas $\widehat{p}_A^*$ is decreasing in α. As a result, the extent of expansion in purchases of product A, $\widehat{p}_A^* - \widetilde{x}^*$, decreases in α in this case. We illustrate these effects with a uniform distribution of x in the following Example:

Example 1 *In the uniform distribution case with $\overline{u} = 1$, we can derive closed-form expressions for welfare analysis. More specifically, Assumptions 1 and 2 can be written as $\Delta < \beta < \frac{1}{2}$ and $\frac{(\beta+\Delta)}{2\beta} < \alpha$, respectively and we assume as well that $\alpha < 1$ so that market A*

[22]We denote by AS the aggregate surplus under independent pricing and by $\widetilde{AS}$ aggregate surplus when tying is allowed; similarly for consumer surplus, denoted in these two situations by CS and $\widetilde{CS}$.

is not covered under independent pricing where we have $\widehat{p}_A^* = (1-\alpha)/2$. Aggregate surplus under independent pricing is then

$$AS = \underbrace{\frac{3}{8}(\alpha+1)^2}_{AS_A} + \underbrace{v_2 + \beta}_{AS_B}$$

With bundling, we consider two cases, corresponding to the two cases of Proposition 1.

(i) If $\alpha+\beta \geq 1$, all consumers buy the bundle and market A is fully covered (i.e., $\widetilde{x}^* = 0$). We then have

$$\widetilde{AS} = \underbrace{\alpha + \frac{1}{2}}_{\widetilde{AS}_A} + \underbrace{v_1 + \beta}_{\widetilde{AS}_B}$$

(ii) If $\alpha + \beta < 1$, consumers with $x \geq \widetilde{x}^* = \frac{1-\alpha-\beta}{2} (< \widehat{p}_A^* = \frac{1-\alpha}{2})$ buy the bundle and those with $x < \widetilde{x}^*$ buy B1. We then have

$$\widetilde{AS} = \underbrace{\frac{(3+3\alpha-\beta)(1+\alpha+\beta)}{8}}_{\widetilde{AS}_A} + \underbrace{v_1 + \beta}_{\widetilde{AS}_B}$$

Taken together,

$$\widetilde{AS} - AS = \underbrace{\left\{ \begin{array}{ll} \frac{(1+3\alpha)(1-\alpha)}{8} & \textit{if } \alpha+\beta \geq 1 \\ \frac{\beta(2+2\alpha-\beta)}{8} & \textit{if } \alpha+\beta < 1 \end{array} \right\}}_{\textit{Market Expansion Effect in A}} - \underbrace{\Delta}_{\textit{Efficiency Loss in B}}$$

The expansion in sales of product A decreases in α in case (i) and increases in β in case (ii).

We now investigate the effects of tying on consumer welfare. Consumer welfare under independent pricing can be written as the sum of consumer surplus in market A and market B.

$$CS = \underbrace{\int_{\widehat{p}_A^*}^{\overline{u}} [1 - G(x)]dx}_{CS_A} + \underbrace{v_1 + \beta}_{CS_B = (v_2+\beta)-\Delta}$$

Consider again the two cases:

(i) If $\alpha + \beta \geq \frac{1}{g(0)}$, market A is covered when tying is allowed and all consumers purchase the bundle at the price of $P^* = \alpha + \beta - \Delta$. In this case, it is useful to think that consumers pay a (fictitious) prices of $\widetilde{p}_A^* = \alpha$ for product A and $\widetilde{p}_{B1}^* = \beta - \Delta$ for product $B1$, with $P^* = \widetilde{p}_A^* + \widetilde{p}_{B1}^*$. Then,

$$\widetilde{CS} = \underbrace{\int_0^{\overline{u}} [1 - G(x)]dx}_{\widetilde{CS}_A} + \underbrace{v_1 + \Delta}_{\widetilde{CS}_B = (v_1+\beta)-(\beta-\Delta)}$$

We have

$$\widetilde{CS} - CS = \underbrace{\int_0^{\widehat{p}_A^*} [1 - G(x)]dx}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{(\Delta - \beta)}_{\widetilde{CS}_B - CS_B < 0}$$

Tying increases consumer surplus in market A by expanding use of product A with a lower (fictitious) price, but decreases consumer surplus in market B. The overall effect depends on the relative magnitude of these two opposing effects. Consumers are more likely to suffer from tying in case (i) if α is higher because it will reduce the positive market expansion effect in market A. A higher β also makes tying less favorable for consumers. With independent pricing, the network-augmented utility term (represented by β) is competed away and passed on to consumers, but it is expropriated by the firm 1 when it ties. In contrast, an increase in Δ directly increases consumer surplus under tying. The reason is that Δ (the quality advantage of $B2$ over $B1$) is captured as a profit by firm 2 under independent pricing. However, under tying, as firm 2 charges zero price, Δ is fully compensated by the tying firm to induce consumers to purchase the inferior product $B1$ as part of the bundle. Thus, the effects of Δ on social welfare and consumer surplus are opposite.

(ii) If $\alpha + \beta < \frac{1}{g(0)}$, only a measure $1 - G(\widetilde{x}^*)$ of consumers buy the bundle (at the price of $P^* = \widetilde{x}^* + \beta(1 - G(\widetilde{x}^*)) - \Delta$), whereas the remaining consumers buy product $B1$ (at the price of $p_{B1}^* = \beta(1 - G(\widetilde{x}^*)) - \Delta$). In this case, we can define a fictitious price of A by firm 1 as $\widetilde{p}_A^* = P^* - p_{B1}^*$. In other words, we treat consumers who purchase the bundle at the price of P as if they pay an effective price of $\widetilde{p}_A^*$ and p_{B1}^*, respectively, for products A and $B1$. With a change of variables, $\widehat{\widetilde{p}}_A^* = \widetilde{p}_A^* - \alpha$, we have $\widehat{\widetilde{p}}_A^* = P^* - p_{B1}^* = \widetilde{x}^*$. Then, we can decompose the total consumer surplus into (fictitious) consumer surplus in market A and consumer surplus in market B.

$$\widetilde{CS} = \underbrace{\left[\int_{\widetilde{x}^*}^{\overline{u}} [1 - G(x)]dx\right]}_{\widetilde{CS}_A} + \underbrace{[v_1 + \beta - p_{B1}^*]}_{\widetilde{CS}_B}$$

Thus, we have

$$\widetilde{CS} - CS = \underbrace{\int_{\widetilde{x}^*}^{\widehat{p}_A^*} [1 - G(x)]dx}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{[\Delta - \beta(1 - G(\widetilde{x}^*)]}_{\widetilde{CS}_B - CS_B < 0}$$

Once again, the effects of tying on total consumer surplus depend on the relative magnitudes of two opposite effects in markets A and B.

Example 2 *For the uniform distribution case with $\overline{u} = 1$,*

$$CS = \underbrace{\frac{(\alpha + 1)^2}{8}}_{CS_A} + \underbrace{v_1 + \beta}_{CS_B}$$

(i) If $\alpha + \beta \geq 1$,

$$\widetilde{CS} = \frac{1}{2} + v_1 + \Delta$$

In this case, we have

$$\widetilde{CS} - CS = \underbrace{\left[\frac{1}{2} - \frac{(\alpha+1)^2}{8}\right]}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{[\Delta - \beta]}_{\widetilde{CS}_B - CS_B < 0}$$

(ii) If $\alpha + \beta < 1$,

$$\widetilde{CS} = \underbrace{\frac{(1+\alpha+\beta)^2}{8}}_{\widetilde{CS}_A} + \underbrace{\left[v_1 + \frac{\beta(1-\alpha-\beta)}{2} + \Delta\right]}_{\widetilde{CS}_B}$$

We thus have

$$\begin{aligned} \widetilde{CS} - CS &= \underbrace{\frac{\beta(2+2\alpha+\beta)}{8}}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{\left[\Delta - \frac{\beta(1+\alpha+\beta)}{2}\right]}_{\widetilde{CS}_B - CS_B < 0} \\ &= -\frac{\beta(2+2\alpha+3\beta)}{8} + \Delta, \end{aligned}$$

In both cases, $(\widetilde{CS} - CS)$ *is decreasing in both* α *and* β *with the uniform distribution, but increasing in* Δ.

5 Complementary Products

In this section, we consider complementary products. In line with the Chicago school logic, we first show that tying is not a profitable strategy as a leverage mechanism to suppress competition in complementary product markets. However, as in Whinston (1990), we show that if there is an inferior competitively-supplied alternative to the tying product then results that parallel those for the independent products case re-emerge. One major difference from Whinston (1990) is, once again, we do not rely on the commitment assumption and subsequent exit of the rival firm in the tied product market.

5.1 The Basic Model: The Chicago argument

We consider a setting that parallels the baseline model, where firm 1 is a monopolist in market A, except that products A and B are now complementary. For the purpose of exposition, consider product A as the primary product whereas B is an add-on product, that is, for the use of product B, product A is necessary; without A, product B is of no use.[23] For instance, product A can be considered as an operating system whereas B is application software.

When products are sold independently, consumers can use one of the two system products, $(A, B1)$ and $(A, B2)$, depending on which firm's product B is used. To simplify the analysis, let us assume that consumers' valuations for the combined products $A - B1$ and $A - B2$

[23]Similar points can be made if products A and B are instead perfect complements that must be used in fixed proportions.

are respectively given by $u + (v_1 + \beta N_1)$ and $u + (v_2 + \beta N_2)$, where $u = (\alpha + x)$ with x distributed on $[0, \overline{u}]$ according to a c.d.f. $G(\cdot)$ and a strictly positive density $g(\cdot)$, which satisfy the monotone hazard rate condition, and $\Delta \equiv v_2 - v_1 > 0$ as in the independent products case although, for simplicity, we also assume that condition (4) holds.

We first show that for the complementary products case, firm 1 has no incentive to tie as it can benefit from the presence of product $B2$. We maintain the same parametric assumptions (i.e., Assumptions 1 and 2) made in the independent products model.

Observe that if firm 1 ties then firm 2 is unable to make any sales. In that case, firm 1 would optimally sell a bundle at price $P = \alpha + v_1 + \beta$.[24] Firm 1's profit would then be $\alpha + v_1 + \beta$. However, firm 1 can actually earn more by adopting independent pricing that both induces consumers to use product $B2$ instead of product $B1$, but extracts the resulting increase in aggregate surplus through a higher price of product A. In fact, there is a continuum of Nash equilibria due to firm 1's ability to "price squeeze" and extract a portion of the surplus Δ (Choi and Stefanadis, 2001) which are parameterized by $\lambda \in [0, 1]$, the degree of price squeeze exercised by firm 1:

$$p_A = \alpha + v_1 + \beta + \lambda\Delta,\ p_{B1} = 0,\ p_{B2} = (1 - \lambda)\Delta$$

Firm 1's profit is then given by $\Pi_1 = \alpha + v_1 + \beta + \lambda\Delta$ so that all equilibria under independent pricing yield a higher profit than that under tying unless $\lambda = 0$ (in which case the profits are the same), establishing the Chicago school argument.

5.2 An Inferior Alternative Product in the Tying Market

We now suppose that there is an inferior alternative product that is competitively supplied at the marginal cost of zero.[25] We now call firm 1's product in the tying market $A1$ while the alternative product is called $A2$. To maintain mathematical isomorphism between the complementary and independent product cases, we normalize consumers' valuations for the combined products that include this alternative $A2 - B1$ and $A2 - B2$ to $(v_1 + \beta N_1)$ and $(v_2 + \beta N_2)$, respectively.[26] Thus, $\alpha + x$ represents the added value that product $A1$ brings to a system over use of product $A2$ for a consumer of type x.

5.2.1 Independent-pricing game

In the presence of product $A2$, a consumer of type x chooses $A1$ over $A2$ if and only if

$$\alpha + x - p_{A1} \geq 0.$$

[24] For this result, we actually need a less stringent assumption than (4), namely $\alpha + v_1 + \beta > 1/g(0)$. Note as well that firm 1 could instead acheieve the same result with independent pricing by setting any independent prices such that $p_A + p_{B1} = \alpha + v_1 + \beta$ and $p_A > \alpha$.

[25] The assumption of competitvely supplied alternative is for simplicity. It can be supplied by a firm with market power.

[26] We can allow a more general utility specificaton by assuming that consumers' valuations for the combined products $A2 - Bj$ is given by $u' + (v_j + \beta N_j)$ for $j = 1, 2$ with $u' < u$. For instance, we can assume that $u' = \alpha' + (1 - \theta)x$ with $\alpha' < \alpha$ and $1 > \theta \geq 0$ without qualitatively changing any results, where $(\alpha - \alpha')$ and θ represent the degree of quality inferiority for the alternative product.

Thus, firm 1's sales of product A are positive only if $p_{A1} < \overline{u}$ and, as in the independent products case, equal $1 - G(\widehat{p}_{A1})$ where $\widehat{p}_{A1} \equiv p_{A1} - \alpha$. We then get the following result:

Proposition 2 *Suppose Assumptions 1 and 2 as well as condition (4) hold. When tying is prohibited with complementary products and there is an inferior competitively-supplied alternative in market A, the equilibrium is identical to the one for the independent products case. Specifically:*

(i) In market A, firm 1 charges $p^*_{A1} = \alpha$ *and sells* $A1$ *to all consumers*

(ii) In market B, firm 1 charges $p^*_{B1} = 0$ *and firm 2 charges* $p^*_{B2} = \Delta$ *and all consumers buy product* $B2$*.*

Firm 1 earns $\Pi^*_1 = \alpha$ *and firm 2 earns* $\Pi^*_2 = \Delta$*.*

5.2.2 Tying

In the presence of tying, let P be the price of the $A1/B1$ bundle of firm 1 and let p_{B2} be the price of firm 2's product $B2$. As above, product $A2$ is provided competitively at the price of zero. In the presence of product $A2$ in the tying market, the case of complementary products is isomorphic to the case of independent products and there is a unique equilibrium, which involves all consumers purchasing the $A1/B1$ bundle, as described in the following proposition that parallels case (i) of Proposition 1 for the independent products case:

Proposition 3 *Suppose Assumptions 1 and 2 as well as condition (4) hold. When tying is allowed with complementary products and there is an inferior competitively-supplied alternative in market A, there is a unique equilibrium in which all consumer purchase the* $A1/B1$ *bundle and the equilibrium prices are given by*

$$P^* = \alpha + \widehat{P}^* = \alpha + (\beta - \Delta),\ p^*_{B2} = 0.$$

Moreover, firm 1's profit, equal to $\alpha + (\beta - \Delta)$ *exceeds that under independent pricing. Both consumer surplus and social welfare decrease:*

$$\begin{aligned} \widetilde{CS} &= CS^* - (\beta - \Delta) < CS^* \\ \widetilde{AS} &= AS^* - \Delta < AS^*. \end{aligned}$$

6 Applications and Conclusion

In this paper, we have developed a leverage theory of tying in markets with network effects. We first analyze incentives to tie for independent products. When a monopolist in one market cannot fully extract the whole surplus from consumers, tying can be a mechanism through which unexploited consumer surpluses in one market are used as a demand-side leverage to create a strategic "quasi installed-base" advantage in another market characterized by network effects. Our mechanism does not require any pre-commitment to tying, and hence can apply to cases in which a firm employs a purely contractual tie. Tying can lead to the exclusion of more efficient rival firms in the tied market, but can also in some cases expand purchase of the tying good if the tied market is not fully covered with independent pricing. We also extend our analysis to the complementary products case. By allowing the existence of inferior alternatives as in Whinston (1990), we show that the setup of complementary products is mathematically identical to that of independent products. We also discuss welfare implications of tying.

Our analysis can provide a theory of harm for instances of tying where network effects are critical in the determination of the market winner. For instance, our model can shed light on the recent antitrust investigation concerning Google's practices in its MADA (Mobile Application Distribution Agreement) contracts. In particular, the EC decision has concluded that Google has engaged in illegal tying by requiring Android OEM "manufacturers to pre-install the Google search app ..., as a condition for licensing Google's app store (the Play Store)" a contractual tie.[27] Google's Play Store can be considered as the tying product as a "must-have" app, with other third-party app stores being inferior alternatives. In the search market, Google faces competition from other search engines and there is evidence of network effects, in particular, stemming from the fact that search results quality increases with the scale of queries received by a search engine.[28] One might wonder why Google's tie would be any different from Google simply paying for preinstallation of Google's search app. Our model suggests that by leveraging what would otherwise be unexploited surplus from OEMs' use of the app store, this tying may be an optimal way for Google to lock in part of the search market, reducing the quality of rivals.[29]

The model also has relevance for the *Microsoft* case in Europe (IP/04/382) in 2004. The European Commission held Microsoft guilty of an abuse of dominant position by "tying its Windows Media Player (WMP), a product where it faced competition, with its ubiquitous Windows operating system."[30] Microsoft had a near monopoly position in the PC operating system market with over 90 percent of market share. We can consider Linux as an inferior alternative to Microsoft's Windows OS in the tying market. The media player market can be considered as the tied market in which Microsoft faced competition (from firms such as RealPlayer) and network effects are critical. More precisely, the media player market can be considered a two-sided market with indirect network effects. If more content is provided in the format of a particular company, then more consumers will use the company's Media

[27] See the European Commission Press Release "Antitrust: Commission fines Google €4.34 billion for illegal practices regarding Android mobile devices to strengthen dominance of Google's search engine," released on July 18, 2018. Available at http://europa.eu/rapid/press-release_IP-18-4581_en.htm.

[28] See, for example, He et al. (2017), Schäfer and Sapi (2020), and Klein et al. (2023) for empirical evidence of network effects in Internet search. The complaint in *US v. Google* also alleges the presence of such effects.

[29] One difference from our model, however, is that the "buyers" are distributors (phone OEMs/carriers) not consumers. Thus, while the search "purchase" can be interpreted as determining which search engine gets to be the pre-installed default on the distributor's device, distributors are not consumers with single-unit demands and more complicated pricing than linear pricing may be possible.

[30] Microsoft's tie of its media player to Windows may have involved a technological tie, but as we noted above the effects we highlight would apply in that case as well. The case also involved Microsoft's conduct of "deliberately restricing interoperability between Windows PCs and non-Microsoft work group servers." https://ec.europa.eu/commission/presscorner/detail/en/IP_04_382

Player to access such content. Moreover, if more consumers select a particular company's Media Player, then content providers will obviously have an incentive to make their content available in the format of the company.[31,32] Our model assumes direct network effects in the tied market, but can be considered capturing such feedback effects of two-sided markets in a reduced form.[33]

We developed our model in the context of one-sided markets. However, as shown in the Appendix, our model is mathematically equivalent to the one with two-sided tying market with advertising revenues (with a reinterpretation of α as per-consumer advertising revenue). This may have important implications for recent antitrust debates on two-sided digital platforms. We showed that welfare impacts of tying depend on the relative magnitudes of positive market expansion effects and negative market foreclosure effects of more efficient firms. When advertising revenue is important (i.e., α is high) and services would under independent pricing be provided for free (hence, the market would be covered), as is common for many digital platforms, our model indicates that there are greater incentives to engage in tying to leverage unexploited consumer surplus. In that case, however, there are no socially beneficial market expansion effects. Therefore, the effects of tying are more likely to be anticompetitive in such a case. In addition, the negative effects on consumer surplus will be more pronounced as network effects in the tied market become more important. This implies that more scrutiny may be warranted when ad-financed digital platforms engage in tying with other products or services characterized by network effects.

[31] As described in the *Official Journal of the European Union* (6.2.2007): "The decision then explains why tying in this particular case is liable to foreclose competition....WMP's ubiquitous presence induces content providers and software developers to rely primarily on Windows Media technology. Consumers will in turn prefer to use WMP, since a wider array of complimentary software and content will be available for that product. Microsoft's tying reinforces and distorts these 'network effects' to its advantage, thereby seriously undermining the competitive process in the media player market."

[32] The Korean Fair Trade Commission also fined Microsoft 33 billion won (US$32 million) for abusing its market dominant position by bundling Windows OS with its instant messaging (IM) program as well as WMP. For the messenger case, the tied market market is characterized by direct network effects as in our model.

[33] See Choi and Jeon (2021) for an analysis of tying that explicitly accounts for indirect network effects in two-sided markets.

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