Too Levered for Pigou? A Model of Environmental and Financial Regulation

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January 2022

Abstract

We analyze jointly optimal emission taxes and financial regulation in the presence of environmental externalities and financial frictions. Our model highlights that climate-related transition and physical risks have opposite implications for how emission taxes interact with financial constraints. Absent physical risk the socially optimal emission tax is below the Pigouvian benchmark (equal to the social cost of emissions) because emission taxes and abatement costs amplify borrowers' financial constraints. This implies that emission taxes alone cannot implement a constrained efficient allocation, as welfare can be improved by introducing capital regulation. With physical risk the effect of emission taxes on financial constraints may revert because lower emissions reduce physical risks and thereby loosen borrowers' financial constraints. This collateral externality may motivate emission taxes above the Pigouvian benchmark and underlines the need to coordinate environmental and financial regulation.

Keywords: Pigouvian tax, financial constraints, financial regulation, emission taxes, physical risk, transition risk

JEL classifications: D62, G21, G28, G32

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1 Introduction

Tackling climate change requires large-scale emission reductions and investments in clean technologies. Absent other frictions such investments can be incentivized through emission taxes equal to the social cost of emissions, also known as Pigouvian taxes in reference to the pioneering work by Pigou (1932). However, during the transition to a low-carbon economy firms and financial institutions may suffer significant losses due to stranded assets that become technologically obsolete. At the same time, physical damages caused by more frequent extreme weather events may hit asset values. Such losses can aggravate financing frictions and limit the ability of firms to make the necessary investments in green technologies.

The risks posed by climate change have moved up the agenda of investors and policy makers with a mandate for price- and financial stability. For example, the European Central Bank and the Bank of England now include climate risks in their stress tests and institutional investors view climate change as an important source of risk that they seek to mitigate.[1] While recent contributions analyze how financing constraints affect Pigouvian taxation and firm investments in green technology (see Hoffmann et al., 2017; Oehmke and Opp, 2020; Heider and Inderst, 2022), an analytical evaluation of jointly optimal environmental and financial regulations in the presence of transition- and physical risk is still missing from the literature. The goal of this paper is to fill this gap.

We develop a tractable model in which polluters face emission taxes and financing constraints. The model economy lasts for three dates and is populated by two types of agents: borrowers and deep-pocketed, risk-neutral lenders. Borrowers enter the game with an initial endowment and an investment project of a fixed scale. At the initial date, they finance the project with a mix of inside equity and debt. Equity financing is costly because borrowers have a quasi-linear utility function and a limited initial endowment.

The borrower's project generates a pecuniary return as well as carbon emissions at the final date. The emissions of the project can be reduced through costly abatement

[1]See Alogoskoufis et al. (2021) for the ECB's stress testing methodology and results, Brunnermeier and Landau (2022) for a discussion of climate-related challenges for monetary and financial policy, and the survey responses by institutional investors in Krueger et al. (2020).

activities undertaken by borrowers. At the same time, borrowers need to roll-over debt raised in the initial period. This new debt issuance is limited by a financial constraint because the project's returns are not fully pledgeable to outside investors. Borrowers can also decide to liquidate part of the initial investment at the interim date. Liquidations can help cash-constrained borrowers to generate resources and at the same time reduce emissions, yet liquidations are inefficient due to liquidation losses.

In the model borrowers are exposed to two different types of climate-related costs. First, we consider a planner imposing emission taxes to incentivize abatement activities, which together with the costs of abatement represent the cost of transitioning to a low-emissions economy (often referred to as "transition risk" in the literature).[2] Second, we assume that the return of the project may decrease in the level of aggregate emissions to capture a borrower's exposure to losses due to environmental damages caused by a warming climate (often termed as "physical risk").[3] This allows us to explore the differences in how these two types of climate-related costs interact with financial frictions and affect optimal environmental and financial policies.

We show that an emission tax equal to the social cost of emissions (i.e., a Pigouvian tax) implements the first best allocation if financial constraints are slack. In contrast, in the presence of financial constraints Pigouvian taxes cannot implement the first best and optimal emission taxes generally differ from the Pigouvian benchmark. With binding financial constraints borrowers need to liquidate some of the project at the interim date, and the socially optimal emission tax trades off the benefit of lower emissions against the costs of higher liquidations. This implies an optimal emission tax below the Pigouvian benchmark because polluters are "too levered for Pigou".[4]

[2]Consistent with transition risks being priced in financial markets, recent evidence documents that firm-level carbon emissions are priced in corporate bonds (see Seltzer et al., 2020) and stocks (see Bolton and Kacperczyk, 2021), and that the risk of stranded fossil fuel assets is priced in bank loans (see Delis et al., 2019).

[3]Several contributions document the relevance of physical risk for asset prices and firm financing. For example, Giglio et al. (2021) find that the value of real estate in flood zones responds more to changes in climate attention, and Issler et al. (2020) document an increase in delinquencies and foreclosures after wildfires in California. Evidence in Ginglinger and Moreau (2019) indicates that physical climate risks affect a firm's capital structure. For a review discussing climate risks, see Giglio et al. (2021).

[4]The mechanism behind this result is consistent with recent evidence documenting that financial constraints affect firm abatement activities and emissions, see Xu and Kim (2022) and Bartram et al. (2021).

A key insight from our analysis is that physical climate risk can reverse the relationship between emission taxes and financial constraints. When borrowers are exposed to physical risk, they may benefit from an increase in pledgeable income when the aggregate level of emissions is brought down by a higher emission tax. This additional benefit of lower emissions is not internalized by borrowers, and therefore optimal emission taxes may be *above* the Pigouvian benchmark rate in the presence of physical risk. Generally, the optimal emission tax is above or below the Pigouvian benchmark depending on whether transition costs or physical risks dominate.

We next consider the problem of a social planner who jointly sets an emission tax and financial regulation in the form of a capital mandate that allows the planner to fix the leverage of borrowers at a given level. Importantly, we establish that the presence of financial constraints alone does not motivate financial regulation in the model. This implies that, if there is a rationale for leverage regulation in our model, it results from the interaction of environmental externalities and financial constraints. Therefore our model contributes to the debate on whether financial regulation should have a climate mandate above and beyond the motivation behind current regulatory frameworks, such as moral hazard issues associated with government guarantees (for example, see Dewatripont and Tirole, 1994; Hellmann et al., 2000; Martinez-Miera and Repullo, 2010; Bahaj and Malherbe, 2020), or pecuniary externalities (for example, see Lorenzoni, 2008).

In the absence of physical risk the optimal emission tax is below the Pigouvian benchmark rate. This wedge between the social and the private cost of emissions implies that borrowers make socially inefficient leverage choices, and that there is a role for capital regulation to improve welfare.

Whether the optimal capital mandate is above or below a benchmark without financial regulation depends on the direction of the total effect of capital on emissions. On one hand, laxer financial constraints can result in a higher level of abatement, which lowers overall emissions. On the other hand, by loosening financial constraints higher capital allows for fewer liquidations, which increases emissions.[5] If the effect of better capitalized borrowers on abatement dominates the effect on liquidations, then the optimal capital mandate is above the equity level privately chosen by borrowers. Such a policy could be implemented by a leverage ratio requirement as in the Basel III regulatory framework. Interestingly, we find that the optimal capital mandate can also be below the equity level chosen by borrowers when a change in capital affects emissions mostly through its impact on liquidations (in which case higher capital is associated with higher emissions). Which of the channels dominates generally depends on the sensitivity of emissions and abatement costs to liquidations. We show that the optimal capital mandate is above the equity level privately chosen by borrowers if functional forms imply that abatement is sufficiently more efficient at a higher investment scale.

Physical risk introduces an additional motive for capital regulation. In the presence of physical risk emissions lower the asset returns and consequently the pledgeable income of borrowers. This gives rise to a collateral externality, wherein individuals do not internalize how their choices affect the tightness of the financial constraints of other borrowers. This effect is similar to collateral externalities in models with pecuniary externalities, where borrowers do not internalize the effect of their choices on the financial constraints of other agents through prices (for a detailed discussion see Dávila and Korinek, 2018). In our setting the collateral externality operates through the physical costs of environmental damages caused by higher emissions, which reduces a borrowers' pledgeable income. As a result, in the presence of physical risk the private choice of equity is socially sub-optimal even if the emissions tax is equal to the Pigouvian benchmark. This mechanism underscores the need to correctly account for the two sources of climate risk in both environmental and financial regulation.

In additional analyses we consider alternative policy tools. While the pledgeable income of borrowers cannot be lifted by rebating emission taxes (because we assume borrowers can abscond with any cash at the final date, including rebated taxes), replacing

[5]This effect is consistent with evidence that carbon emissions drop during economic crises, as exemplified by 10% drop in carbon emissions in 2020 relative to 2019 in the EU, see the European Environmental Agency).

the emission tax by a cap-and-trade system such as the EU Emission Trading System can be beneficial if pollution permits can be allocated to borrowers for free. Freely allocating pollution permits eliminates the direct effect of emission taxes on financial constraints. However, we find that this policy cannot undo the interaction between financial constraints and environmental policy because borrowers still have to incur the costs of abatement investments. Therefore, the optimal permit price is generally different from the Pigouvian benchmark and a cap-and-trade system alone cannot implement a constrained efficient allocation – even when permits can be allocated for free.

Perhaps trivially, effective policy tools create financial slack by transferring resources from unconstrained investors to constrained borrowers. Such transfers could either be implemented directly or indirectly by subsidizing abatement rather than taxing pollution. While combining emission taxes with transfers is generally effective, policies that imply generous transfers to polluters may not be politically feasible. A problem with subsidizing abatement rather than taxing pollution is that it requires regulators to decide which technologies to subsidize, which may be difficult in the presence of information asymmetries.

We relate to several recent contributions that analyze environmental externalities under financing frictions. Oehmke and Opp (2020) study how investors with a non-pecuniary preference for sustainable investments can ease financing constraints and impact firm behavior. Heider and Inderst (2022) show how financial constraints can imply optimal carbon prices different from a Pigouvian benchmark and derive optimal industry-level carbon prices in a model with heterogeneity in emissions within and across industries. Similarly, Hoffmann et al. (2017) find that, in the presence of agency problems and heterogeneity in abatement costs, optimal emission taxes are below the Pigouvian benchmark and optimally non-linear.[6] None of these papers analyze the interaction of financial reg-

[6]Previous literature shows that a Pigouvian solution may be sub-optimal also in the presence of heterogeneous exposure to the externalities or interactions between several externality generating activities, if a targeted policy tool is unavailable (Diamond, 1973; Rothschild and Scheuer, 2014). Moreover, a wedge between the optimal tax rate and the marginal social cost emerges when the planner seeks to regulate an externality in the presence of other distortionary taxes (Sandmo, 1975; Lee and Misiolek, 1986; Bovenberg and Goulder, 1997; Bovenberg and De Mooij, 1997) or when consumers have self-control problems Haavio and Kotakorpi (2011). In these cases, as in our setting, the indirect effects of the policy motivate the deviation from the Pigouvian solution.

ulation and emission taxes, which is the main focus of this paper. Another related strand of literature uses DSGE models with financial frictions to simulate the effect and optimal design of macroprudential and monetary policies in the presence of environmental externalities (Carattini et al., 2021; Dafermos et al., 2018; Diluiso et al., 2020; Ferrari and Landi, 2021). We contribute by providing analytical results that allow to pinpoint the friction motivating financial regulation in this context. Moreover, to the best of our knowledge our paper is the first to jointly analyze how physical and transition risks interact with financial frictions, which allows us to derive novel insights on how these two climate-related costs differ in their impact on environmental and financial policies.

Section 2 describes the model setup and derives the first best benchmark. Section 3 solves the competitive equilibrium, and Section 4 analyzes optimal financial and environmental regulation. Section 5 concludes.

2 Model Setup

There are three dates, $t = 0, 1, 2$, a unit mass of investors and a unit mass of borrowers.

Preferences and Endowments Investors are risk-neutral and deep-pocketed in that they have a large endowment A_t^i at $t = 0$ and $t = 1$. Borrowers have a limited endowment A_0^b only at $t = 0$ and quasi-linear utility over consumption. There is no discounting and all agents suffer disutility from aggregate carbon emissions E^a at $t = 2$:

$$U^i = c_0 + c_1 + c_2 - \gamma_u E^a,$$

$$U^b = u(c_0) + c_1 + c_2 - \gamma_u E^a,$$

where γ_u is a parameter governing the cost of emissions in agent's utility.

We assume a quasi-linear utility function to introduce a meaningful trade-off for borrowers in how much own funds they contribute. To ensure an interior solution we assume that $u(c_0)$ satisfies the Inada conditions, i.e., that $u(c_0)$ is strictly increasing and strictly convex, and that in the limit $u'(0) = \infty$ and $u'(\infty) = 0$.

Technology Borrowers have access to an investment project with fixed scale I_0 at $t = 0$. At $t = 1$ borrowers can liquidate some of the initial investment, so that the investment scale is adjusted to $I_1 \leq I_0$. The project generates a return of $R(I_1, E^a) = \rho I_1 - \gamma_p E^a$ at $t = 2$, and liquidations generate a payoff $\mu(I_0 - I_1)$ at $t = 1$, with $\mu \in [0, 1)$. The parameter γ_p captures the project's exposure to physical risk from environmental damages. Thus, the aggregate social cost of carbon emissions is $\gamma = \gamma_u + \gamma_p$ and consists of a direct utility cost as well as losses in asset values from environmental damages.

The project emits carbon emissions $E(X, I_1)$ at $t = 2$ that may be subject to emission taxes τ. X are abatement investments that reduce emissions at a cost $c_x(X, I_1)$ paid at $t = 1$. We define the (private) net benefit of emission reductions as

$$NBE(X, I_1, \tau) = -\tau E(X, I_1) - c_x(X, I_1) \tag{1}$$

and make the following functional form assumptions.

Assumption 1. *$E(X, I_1)$ and $c_x(X, I_1)$ satisfy*

(i) $\frac{\partial E(X,I_1)}{\partial X} \leq 0, \; \frac{\partial E(X,I_1)}{\partial I_1} \geq 0, \; \frac{\partial c_x(X,I_1)}{\partial X} \geq 0, \; \frac{\partial c_x(X,I_1)}{\partial I_1} \geq 0,$

(ii) $E(X \to \infty, I_1) = E(X, I_1 = 0) = 0, \;\; E(X = 0, I_1 = I_0) = \bar{E},$
$c_x(X = 0, I_1) = c_x(X, I_1 = 0) = 0,$

(iii) $\frac{\partial^2 NBE(X,I_1,\tau)}{\partial X^2} < 0.$

Part (i) of Assumption 1 ensures that higher abatement investment is costly but reduces emissions, and that a higher investment scale is associated with higher emissions and abatement costs; (ii) defines boundaries such that costs and emissions are non-negative and there is an upper bound $\bar{E}$ on emissions. Part (iii) implies that the net benefit of emission reductions (NBE) is strictly concave in X to ensure that the problem is well-behaved and NBE has a unique maximum in X.

Agents are sufficiently dispersed, so that they do not internalize the effect of their decisions on aggregate carbon emissions E^a.

Financing Borrowers need to finance the investment I_0 at $t = 0$ and abatement X at $t = 1$. At $t = 0$ they can contribute their own funds as equity financing $e \leq A_0^b$. Additionally, borrowers can raise debt financing d_t from investors at $t = 1, 2$,[7] but borrowing is limited by a moral hazard problem. We assume that borrowers can abscond with a fraction $\theta \in [0, 1]$ of asset returns and any other resources (cash) at $t = 2$ (as in Rampini and Viswanathan, 2013). Thus, the pledgeable income of borrowers is given by $\tilde{R}(I_1, E^a) = (1 - \theta)R(I_1, E^a)$ and there is a wedge between the project's return and pledgeable income. At $t = 1$ the liquidation proceeds $\mu(I_0 - I_1)$ can be seized by investors who provided $t = 0$ financing (that is, liquidation proceeds are pledgeable).[8] Investors can demand liquidation if they choose not to roll over their debt and are not repaid in full at $t = 1$. We assume that $A_0^b \geq (1 - \mu)I_0$ to ensure that borrowers have sufficient funds to cover liquidation losses.

Financial and Environmental Regulation Throughout we assume that an environmental regulator imposes an emission tax τ per unit of emissions. Emission taxes are rebated lump-sum to investors (T^i) and borrowers (T^b) such that $T^i + T^b = \tau E^a$. Since borrowers can abscond with any cash at $t = 2$, these lump-sum transfers are not pledgeable. This implies that, even when fully rebating the tax to borrowers, the tax reduces pledgeable income. In Section 4.4.1 we show that replacing emission taxes by an emission trading market in which pollution permits are allocated for free can help alleviating the negative effect of emission taxes on financial constraints but does does not fully overcome the problem because polluters still have to pay for abatement costs.[9]

In Section 3 we derive the competitive equilibrium for a given emission tax τ. In Section 4 we also introduce financial regulation in the form of a capital mandate $\bar{e}$ that requires borrowers to contribute $e = \bar{e}$, which allows us to analyze jointly optimal emission

[7]Since there is no risk in our model, there is no difference between external debt and external equity financing. For simplicity we refer to d_t as debt.

[8]One interpretation of this assumption is that a fraction μ of assets are tangible assets that can be liquidated at $t = 1$ and pledged to outside investors, while a fraction $1 - \mu$ are intangible assets that cannot be pledged and have no liquidation value at $t = 1$.

[9]Emission taxes are equivalent to setting up a market for tradeable pollution permits (such as the EU Emission Trading System), in which polluters are forced to purchase permits and the regulator issues an aggregate number of permits that results in a permit price τ (see Montgomery, 1972). In Section 4.4.1 we analyze an emission trading market in which pollution permits are allocated for free.

taxes and financial regulation.

Risk Strictly speaking there is no climate risk in the model because we assume that the social cost of emissions γ is known at $t = 0$. Our model nevertheless captures the concepts of transition risk and physical risk discussed in the literature. One interpretation of the fixed investment scale I_0 is that legacy assets from previous investments determine the investment scale, and that transition risk has materialized as agents have just learnt the level γ and associated emission taxes.[10] Moreover, the parameter γ_p can be interpreted as resembling any effects of environmental damages on asset prices, including through physical risk that has not yet materialized.

Variable Definitions For the further analysis it will be useful to introduce the following variable definitions:

Definition 1. *The project's private net marginal return $r(\tau, E^a, X, I_1)$ and pledgeable net marginal return $\tilde{r}(\tau, E^a, X, I_1)$ are respectively defined as*

$$r(\tau, E^a, X, I_1) = \rho - \left[\mu + \frac{\partial c_x(X, I_1)}{\partial I_1} + \tau \frac{\partial E(X, I_1)}{\partial I_1}\right],$$
$$\tilde{r}(\tau, E^a, X, I_1) = (1 - \theta)\rho - \left[\mu + \frac{\partial c_x(X, I_1)}{\partial I_1} + \tau \frac{\partial E(X, I_1)}{\partial I_1}\right].$$

The following conditions ensure that continuing the investment project has positive NPV at $t = 1$ even when the emission tax is equal to the Pigouvian level $\tau = \gamma$ and emissions are at the highest possible level $E^a = \bar{E}$. This implies that liquidations are inefficient. The second condition ensures that, while inefficient, liquidations do relax financial constraints:

Assumption 2. *ρ and θ are sufficiently large such that*

(i) $r(\gamma, \bar{E}, X, I_1) > 0, \ \forall X, I_1,$

(ii) $\tilde{r}(0, 0, X, I_1) < 0, \ \forall X, I_1.$

[10] This can be modelled by introducing a pre-stage $t = -1$ in which borrowers decide on the investment scale, while the level of γ is only revealed at $t = 0$.

2.1 First Best Benchmark

Before analyzing the competitive equilibrium, the following proposition derives the first best benchmark:

Proposition 1. *In the first best allocation $I_1 = I_0$ and optimal $t = 0$ consumption by borrowers c_0^b and optimal abatement X are defined by the following conditions:*

$$u'(c_0^b) = 1,$$
$$\gamma \frac{\partial E(X, I_1)}{\partial I_1} + \frac{\partial c_x(X, I_1)}{\partial I_1} = 0.$$

Proof. See Appendix A.1.1 □

In the first best allocation there are no liquidations because liquidations are inefficient by Assumption 2. The borrower's consumption is such that the marginal utility is equalized across agents and time. The optimal abatement is at a level at which the marginal gain from lower emissions is equal to the marginal cost of abatement.

3 Competitive Equilibrium

In this section we solve the problem of borrowers and define a competitive equilibrium given an emission tax τ but without financial regulation. We compare this allocation to an equilibrium with financial regulation chosen by a social planner in the next section.

3.1 Borrower Problem

The borrower's utility is given by

$$U^b = u(c_0) + c_1 + c_2$$

We can eliminate c_0, c_1 and c_2 and formulate the objective as choosing e, d_1, X and I_1 so as to maximize U^b, given an emission tax τ and aggregate emissions E^a. The problem is subject to non-negativity constraints and a financial constraint:[11]

$$c_0 = A_0^b - e \geq 0, \tag{2}$$

$$c_1 = (I_0 - I_1)\mu + d_1 - (I_0 - e) - c_x(X, I_1) \geq 0, \tag{3}$$

$$c_2 = R(I_1, E^a) - \tau E(X, I_1) - d_1 + T^b \geq 0, \tag{4}$$

$$d_1 \leq \tilde{R}(I_1, E^a) - \tau E(X, I_1), \tag{5}$$

$$e \geq 0, \;\; I_1 \in [0, I_0]. \tag{6}$$

Equations (2), (3) and (4) are non-negativity constraints on consumption at $t = 0, 1,$ and 2, respectively. Equation (5) is a financial constraint that ensures borrowing does not exceed the borrower's pledgeable income. The non-negativity constraint for c_0 is always satisfied since we assume that $u'(0) = \infty$. Moreover, due to the financial constraint (5) c_2 is always positive, so that (4) never binds. The non-negativity constraint on e remains slack too because the following lemma shows that for the project to continue borrowers need to contribute a minimum amount of equity:

Lemma 1. *Borrowers need to contribute at least $e \geq (1 - \mu)I_0$, else the project is not financed at $t = 0$.*

Proof. See Appendix A.1.2. □

Put differently, borrowers cannot borrow more than $d_0 \leq \mu I_0$. When borrowing more than μI_0 borrowers cannot repay d_0 even when liquidating the entire investment at $t = 1$. Liquidating less results in an even lower pledgeable income because by Assumption 2 liquidations increase financial slack (since $\tilde{r}(\tau) < 0$). This implies that an initial debt $d_0 > \mu I_0$ cannot be repaid in full. Anticipating this outcome lenders are not willing to extend a loan $d_0 > \mu I_0$.

Thus, at $t = 0$ a borrower can follow one of two paths: either put up a minimum level of equity $e \geq I_0(1 - \mu)$ to start the project; or consume the full endowment $c_0 = A_0^b$ and do not fund the project. In what follows we focus on the interesting case in which the

[11]Note that we eliminate $d_0 = I_0 - e$ from Eq. (3).

project is not abandoned at $t = 0$ and discuss the condition for this to be the case in Appendix A.1.2.[12]

Using these insights the borrower's problem can be formulated as the following Lagrangian:

$$
\begin{aligned}
\max_{X,I_1,d_1,e} \mathcal{L} = {} & u(A_0^b - e) - I_0(1-\mu) + e + R(I_1, E^a) - \mu I_1 - c_x(X, I_1) - \tau E(X, I_1) + T^b \\
& + \lambda \left[\tilde{R}(I_1, E^a) - \tau E(X, I_1) - d_1\right] + \underline{\kappa}_I I_1 + \overline{\kappa}_I [I_0 - I_1] \\
& + \kappa_{c_1} \left[d_1 - I_0(1-\mu) - I_1\mu + e - c_x(X, I_1)\right] + \kappa_e[e - (1-\mu)I_0],
\end{aligned} \tag{7}
$$

where λ is the Lagrange multiplier for the financial constraint and κ's are the multipliers for lower and upper bounds on variables. The first order condition w.r.t. d_1 implies that the multiplier on the non-negativity constraint for c_1 is equal to λ,

$$
-\lambda + \kappa_{c_1} = 0. \tag{8}
$$

Intuitively, if the financial constraint binds, borrowers are at a corner solution and do not consume at $t = 1$, so that $c_1 = 0$ and $\lambda = \kappa_{c_1} > 0$.

3.2 Borrower Decisions at $t = 1$

The first order conditions with respect to X and I_1 are given by, respectively,

$$
(1+\lambda)\left(\tau \frac{\partial E(X, I_1)}{\partial X} + \frac{\partial c_x(X, I_1)}{\partial X}\right) = 0, \tag{9}
$$

$$
\rho(1+\lambda(1-\theta)) - (1+\lambda)\left[\mu + \frac{\partial c_x(X, I_1)}{\partial I_1} + \tau \frac{\partial E(X, I_1)}{\partial I_1}\right] - \overline{\kappa}_I + \underline{\kappa}_I = 0. \tag{10}
$$

[12] One way to ensure that the project is never abandoned at $t = 0$ is to assume that borrowers receive a private benefit B from starting the project that is sufficiently large such that $u'(A_0^b - (1-\mu)e) + B > u'(A_0^b)$, which implies that borrowers prefer starting the project even when they have to fully liquidate it at $t = 1$.

Together with the following condition that combines the complementary slackness conditions of the financial constraint (5) and the non-negativity constraint (3),

$$\lambda \left[(1-\theta)\rho I_1 - \tau E(X, I_1) - I_0(1-\mu) + e - c_x(X, I_1) - \mu I_1\right] = 0, \tag{11}$$

these equations define the optimal $t = 1$ allocations I_1, X, and λ for a given e (the optimality condition for equity is derived below). In Eq. (9) borrowers choose abatement trading off the tax bill associated with carbon emissions against the cost of abatement.[13] Eq. (10) is the first order condition with respect to I_1. Using the definitions for private net return and net pledgeable return, $r(\tau, E^a, X, I_1)$ and $\tilde{r}(\tau, E^a, X, I_1)$ from Definition 1 we can rewrite this first order condition as

$$\lambda = -\frac{r(\tau, E^a, X, I_1) - \overline{\kappa}_I + \underline{\kappa}_I}{\tilde{r}(\tau, E^a, X, I_1)} \tag{10'}$$

If the financial constraint is slack, then $\lambda = 0$ and the pledgeable return $\tilde{r}(\tau, E^a)$ does not affect the first order condition (10). In this case the optimal liquidation depends only on the investment's net marginal return $r(\tau, E^a)$. The first condition in Assumption 2 implies that the net marginal return is positive and therefore it is optimal to continue the project without any liquidations, i.e., there is a corner solution with $I_1 = I_0$ and $\overline{\kappa}_I = r(\tau) > 0$:

Lemma 2. *If $\tau \leq \gamma$, borrowers do not liquidate any investment if the financial constraint (5) is slack. That is, if $\lambda = 0$, then $I_1 = I_0$.*

Proof. Equation (10) evaluated at $\lambda = 0$ is $r(\tau, E^a, X, I_1) - \overline{\kappa}_I + \underline{\kappa}_I = 0$. By Assumption 2 $r(\tau, E^a, X, I_1) > 0$, which implies that the solution requires $\overline{\kappa}_I > 0$ (i.e., $I_0 = I_1$). □

The second condition in Assumption 2 implies that the pledgeable net marginal return

[13] Note that dividing by $(1 + \lambda)$ removes any direct dependence of the borrower's abatement decision on the tightness of the financial constraint. This is because emission taxes have the same seniority as debt and therefore reduce pledgeable income as much as total (pledgeable plus non-pledgeable) income, in contrast to the wedge $R - \tilde{R}$ between total asset returns and pledgeable returns. This implies that the financial constraint affects abatement only indirectly through I_1, which is important to understand our results on optimal financial regulation in Section 4. We show in Appendix B.1 that λ would enter the borrower's abatement first order condition directly if we instead assumed taxes were junior to debt.

from investment is negative. Put differently, the liquidation proceeds μ exceed the loss in pledgeable income and therefore liquidating investments eases financial constraints. In case of a binding financial constraint ($\lambda > 0$), the borrower may liquidate some investment to generate financial slack.

3.3 Borrower Decisions at $t = 0$

At $t = 0$ borrowers decide on their capital structure by choosing the optimal inside equity e (debt financing follows as the residual $d_0 = I_0 - e$). The first order condition of the Lagrangian (7) w.r.t. e is

$$u'(A_0^b - e) - \kappa_e = 1 + \lambda \tag{12}$$

This condition shows that borrowers contribute equity trading off the utility cost of lower $t = 0$ consumption on the left-hand side against loosening a potentially binding financial constraint as indicated by the multiplier λ on the right-hand side.

The first order conditions and complementary slackness condition together define the competitive equilibrium:

Definition 2. *Given a carbon tax τ, the competitive equilibrium is the set of allocations $I_1^{ce}(\tau), X^{ce}(\tau), \lambda^{ce}(\tau), e^{ce}(\tau)$ defined by Equations (9), (10), (11), and (12), and aggregate emissions are given by $E^a = E(X^{ce}, I_1^{ce})$. The allocations $c_0^{ce}(\tau), c_1^{ce}(\tau), c_2^{ce}(\tau)$, and $d_0^{ce}(\tau)$ follow as residuals from Eqs. (2), (3), (4), and $d_0 = I_0 - e$.*

Using this equilibrium definition, the following lemma characterizes the conditions under which the financial constraint binds in the competitive equilibrium:

Lemma 3. *If $u'(A_0^b - (1-\mu)I_0) > 1$, then the financial constraint binds for any $\tau \geq 0$ in the competitive equilibrium.*
If $u'(A_0^b - (1-\mu)I_0) < 1$, then there exists a threshold $\hat{e}(\tau)$ such that, for $e \leq \hat{e}(\tau)$, the financial constraint binds ($\lambda \geq 0$) in the competitive equilibrium. If $u'(A_0^b - \hat{e}(\tau)) \leq 1$, then the optimal level of equity is $e^{ce} \geq \hat{e}(\tau)$ and the financial constraint is slack in the competitive equilibrium. The threshold $\hat{e}(\tau)$ is weakly increasing in τ.

Proof. See Appendix A.1.3. □

Lemma 3 shows that, if the borrower's endowment is sufficiently small such that at the minimum level $e = (1-\mu)I_0$ the marginal utility from consumption exceeds 1 (i.e., $u'(A_0^b-(1-\mu)I_0) > 1$), then the financial constraint binds in the competitive equilibrium. Intuitively, if inside equity is relatively scarce, then the level of debt borrowers want to take on to smooth consumption is so high that the financial constraint binds.

In contrast, if the borrower's endowment is relatively large, such that $u'(A_0^b - (1-\mu)I_0) \leq 1$, then the financial constraint may be either slack or binding depending on the size of a borrower's endowment relative to the threshold $\hat{e}(\tau)$ (defined in Appendix A.1.3). If the endowment is sufficiently large that even $u'(A_0^b - \hat{e}(\tau)) \leq 1$, then the constraint is slack at the optimum. Importantly $\hat{e}(\tau)$ increases in τ. This shows that higher emission taxes lower financial slack. This interaction between emission taxes and financial constraints may motivate welfare-improving financial regulation, as we show in Section 4.

3.4 Pigouvian Tax

If the second case in Lemma 3 applies and the financial constraint is slack, then by Lemma 2 borrowers can avoid inefficient liquidations for any emission tax $\tau \leq \gamma$. Consequently, the optimal Pigouvian emission tax can implement the first best allocation, as shown in the following proposition:

Proposition 2. *If $\lambda^{ce}(\gamma) = 0$, then the competitive equilibrium with $\tau = \gamma$ is equivalent to the first best allocation.*

Proof. With $\lambda = 0$, it follows from Lemma 2 that $I_1 = I_0$ under Assumption 2. In this case the FOCs of borrowers w.r.t. X and e in Eqs. (9) and (12) are equivalent to those in the first best given in Proposition 1. □

Proposition 2 established an important benchmark and implies that, with a slack financial constraint, there is no case for additional regulation because an emission tax equal to the social cost of emissions γ can implement the first best. Accordingly, throughout we refer to a tax $\tau = \gamma$ as the Pigouvian benchmark. In the next section we analyze how optimal emission taxes change when the financial constraint binds and ask whether there is a rationale to combine emission taxes with financial regulation in this case.

4 Optimal Emission Taxes and Financial Regulation

We consider the problem of a utalitarian social planner who maximizes welfare by setting the optimal emission tax τ^*. Given that emission taxes may interact with financial constraints we also analyze whether it is welfare-improving to combine emission taxes with financial regulation in the form of a capital mandate that requires borrowers to contribute equity $e = \bar{e}$.

To be able to compare the competitive equilibrium to an allocation with a capital mandate, we define a *financial regulation equilibrium* as an equilibrium in which borrowers choose abatement and liquidations given an emission tax τ, but e is mandated by a regulator rather than chosen by borrowers according to their first order condition (12):

Definition 3. *Given an emission tax τ and a capital mandate $\bar{e}$ that requires the borrower to contribute $e = \bar{e}$, a financial regulation equilibrium is a set of allocations $I_1^r(\tau, \bar{e}), X^r(\tau, \bar{e}), \lambda^r(\tau, \bar{e})$ defined by Equations (9), (10), and (11), and aggregate emissions are given by $E^a = E(X^r, I_1^r)$. The allocations $c_0^r(\tau), c_1^r(\tau), c_2^r(\tau)$, and $d_0^r(\tau)$ follow as residuals from Eqs. (2), (3), (4), and $d_0 = I_0 - e$. The optimal financial regulation equilibrium is defined as the financial regulation equilibrium with $\tau = \tau^*$ and $\bar{e} = \bar{e}^*$ that maximize social welfare $W(\tau, e) = U^i(\tau, e) + U^b(\tau, e)$.*

To derive τ^* and $\bar{e}^*$ we solve the problem of a planner choosing the optimal tax and capital mandate. This problem can be written as the following Lagrangian with κ_e the multiplier lower bound constraint $\bar{e} \geq (1 - \mu)I_0$ (see Lemma 1) and κ_τ multiplier on the non-negativity constraint on τ:

$$\max_{\tau, \bar{e}} \; W = u(A_0^b - \bar{e}) + R(I_1, E^a) - \mu I_1^r - \gamma E(X^r, I_1^r) - (1 - \mu)I_0 + \bar{e} - c_x(X^r, I_1^r),$$
$$+ \kappa_e(\bar{e} - (1 - \mu)I_0) + \kappa_\tau \tau.$$

In analyzing this problem we proceed in steps and initially abstract from physical risk, i.e., we focus on the case $\gamma_p = 0$. The case $\gamma_p > 0$ is covered in Section 4.3

As we show in Appendix A.1.5 and A.1.6, the first order conditions with respect to τ and $\bar{e}$ can be written as, respectively,

$$r(\gamma, \bar{e})\frac{\partial I_1^r}{\partial \tau} - (\gamma - \tau)\frac{\partial E(X^r, I_1^r)}{\partial X}\frac{\partial X^r}{\partial \tau} + \kappa_\tau = 0 \tag{13}$$

$$u'(A_0^b - \bar{e}) - \kappa_e = 1 + r(\tau, \bar{e})\frac{\partial I_1^r}{\partial \bar{e}} - (\gamma - \tau)\left(\frac{\partial E(X^r, I_1^r)}{\partial I_1^r}\frac{\partial I_1^r}{\partial \bar{e}} + \frac{\partial E(X^r, I_1^r)}{\partial X^r}\frac{\partial X^r}{\partial \bar{e}}\right) \tag{14}$$

where we denote the net marginal income evaluated at the optimal abatement and investment scale in the regulation equilibrium by $r(\tau, \bar{e}) = r(\tau, E^a(\tau, \bar{e}), X^r(\tau, e), I_1^r(\tau, \bar{e}))$ (and accordingly $\tilde{r}(\tau, \bar{e})$ can be used for the net pledgeable marginal income).

Note that the investment scale I_1^r and abatement X^r are optimal choices by private agents in the financial regulation equilibrium and functions of τ and $\bar{e}$. The following lemma clarifies how the equilibrium allocations I_1^r and X^r respond to the policy instruments τ and $\bar{e}$.

Lemma 4. *Consider the case when $\gamma_p = 0$. If $\lambda^r(\tau, \bar{e}) = 0$, then $\frac{dI^r}{d\tau} = \frac{dI^r}{d\bar{e}} = \frac{dX^r}{d\bar{e}} = 0$. If $\lambda^r(\tau, \bar{e}) > 0$, then*

$$\frac{\partial I_1^r}{\partial \tau} = \frac{E(X^r, I_1^r)}{\tilde{r}(\tau)} < 0, \qquad \frac{\partial I_1^r}{\partial \bar{e}} = \frac{-1}{\tilde{r}(\tau)} > 0.$$

The derivatives of X^r with respect to $\bar{e}$ and τ are given by, respectively,

$$\frac{\partial X^r}{\partial \tau} = \frac{\frac{\partial E(X, I_1)}{\partial X^r} - \frac{\partial^2 NBE(X^r, I_1^r)}{\partial X^r \partial I_1^r}\frac{\partial I_1^r}{\partial \tau}}{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial (X^r)^2}}, \qquad \frac{\partial X^r}{\partial \bar{e}} = -\frac{\frac{\partial^2 NBE(X^r, I_1^r, \tau)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r, I_1^r, \tau)}{\partial (X^r)^2}}\frac{\partial I_1^r}{\partial \bar{e}},$$

where $NBE(X^r, I_1^r, \tau)$ is the net benefit of emission reductions as defined in Eq. (1).

Proof. See Appendix A.1.4 □

If borrowers are financially unconstrained ($\lambda^r(\tau, \bar{e}) = 0$), liquidations are not affected by emission taxes, and neither liquidations nor abatement are affected by the capital mandate $\bar{e}$. As shown in Proposition 2, in this case the competitive equilibrium with a Pigouvian tax $\tau = \gamma$ is already equivalent to the first best allocation.

By contrast, if the financial constraint binds, borrowers need to liquidate investments to be able to roll-over their debt. An increase in emission taxes tightens financial constraints and induces borrowers to liquidate more (i.e., $\frac{\partial I_1^r}{\partial \tau} < 0$). An increase in equity has the opposite effect. By relaxing the financial constraint it enables borrowers to liquidate less (i.e., $\frac{\partial I_1^r}{\partial \bar{e}} > 0$).

The effect on abatement is more complex. The direct effect of higher emission taxes on abatement is positive because emission taxes increase the cost of polluting, captured by the term $\frac{\partial E(X,I_1)}{\partial X^r}$ in the numerator of the derivative $\frac{\partial X^r}{\partial \tau}$. But there is also an indirect effect through the impact on the tightness of the financial constraint and thus the level of liquidations, captured by the term $-\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r}\frac{\partial I_1^r}{\partial \tau}$. This indirect effect can be positive or negative, depending on the functional forms of $E(X, I_1)$ and $c_x(X, I_1)$.[14] The direction of the overall effect of taxes on abatement depends on the direction and relative magnitude of the two effects. In the plausible case in which the direct effect of emission taxes on abatement dominates higher taxes result in lower emissions, so that $\frac{\partial X^r}{\partial \tau} > 0$. But theoretically it is also possible that the indirect effect of emission taxes on liquidations dominates. In this case, emission taxes can have a perverse effect and decrease abatement due to tightening financial constraints.

Similarly, capital affects abatement indirectly by reducing liquidations. From the derivative $\frac{\partial X^r}{\partial \bar{e}}$ in Lemma 4 it can be seen that, because $\frac{\partial I_1^r}{\partial \bar{e}} > 0$, abatement increases in the capital mandate if the functional forms of $E(X, I_1)$ and $c_x(X, I_1)$ are such that the net marginal benefit of emission reductions is higher at a greater investment scale, i.e., if $\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r} > 0$.[15]

With the effect of emission taxes and capital mandates on abatement and liquidations understood, we are ready to analyze how τ and $\bar{e}$ are set optimally in the following subsections.

[14] Recall from the Eq, (1) that $NBE(X^r, I_1^r, \tau) = -\tau E(X^r, I_1^r) - c_x(X^r, I_1^r)$ is the net benefit of emission reductions. By Assumption 1 $\frac{\partial^2 NBE(X^r,I_1^r,\tau)}{\partial (X^r)^2} < 0$. This implies that the direct effect of emission taxes on abatement is positive. The direction of the indirect effect depends on the sign of the cross-derivative of the net benefit of emission $\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r}$. If it is positive, then abatement is more efficient at a larger investment scale I_1 and the indirect effect of emission taxes on abatement is negative.

[15] Since $\frac{\partial I_1^r}{\partial \bar{e}} > 0$ and $\frac{\partial^2 NBE(X^r,I_1^r,\tau)}{\partial (X^r)^2} < 0$, the derivative $\frac{\partial X^r}{\partial \bar{e}}$ is positive if and only if $\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r} > 0$.

4.1 Socially Optimal Emission Tax

When emission taxes interact with financial constraints a social planner needs to trade off the desired effect of taxes on emissions against the undesired side-effect of forcing borrowers to inefficiently liquidate investments (since $\frac{\partial I^r}{\partial \tau} < 0$, see Lemma 4).

Proposition 3. *Consider the case $\gamma_p = 0$ and $\gamma_u > 0$. The optimal emission tax for a given capital mandate $\bar{e}$ is:*

- $\tau^* = \gamma$ *if* $\lambda^r(\tau^*, \bar{e}) = 0$*,*
- $\tau^* < \gamma$ *if* $\lambda^r(\tau^*, \bar{e}) > 0$ *and* (13) *evaluated at* $\kappa_\tau = 0$ *has a unique solution.*

Proof. See Appendix A.1.5 □

If the capital mandate $\bar{e}$ is sufficiently high so that the financial constraint is slack, then the negative effect of emission taxes on liquidations can be avoided altogether and it is optimal to set the Pigouvian emission tax $\tau^* = \gamma$. This result mirrors Proposition 2.

However, if the borrower's endowment is small it may be too costly to set such a high capital mandate. In this case optimal emission taxes are strictly below the Pigouvian level γ because higher emission taxes would result in too much liquidations given the high leverage of borrowers. Put differently, borrowers are "too levered for Pigou".[16]

These results clarify how optimal emission taxes are set for a given $\bar{e}$. The following subsection complements these results by deriving implications for the optimal capital mandate.

4.2 Socially Optimal Capital Mandate

The trade-off faced by the social planner when choosing the optimal capital mandate depends on the effect of equity on emissions. As a first step, Lemma 5 establishes how higher borrower equity affects emissions through its impact on the final investment scale I_1 and abatement X. The overall effect of equity on emissions can be either positive or

[16] In the proposition we focus on the case in which Eq. (13) has a unique solution when evaluated at $\kappa_\tau = 0$ because, if the equation has multiple solutions, we cannot determine analytically whether the maximum is global.

negative depending on whether equity has a greater effect on abatement or liquidations, which in turn depends on the functional forms of $E(X, I_1)$ and $c_x(X, I_1)$. This intermediate result is important to understand why the optimal capital mandate can be above or below the level of equity chosen by borrowers in the competitive equilibrium, as we establish below in Proposition 4.

Lemma 5. *In the financial regulation equilibrium with $\gamma_p = 0$ the total effect of the capital mandate on emissions is given by $Z(\tau, \bar{e})\frac{\partial I_1^r}{\partial \bar{e}}$, where*

$$Z(\tau, \bar{e})\frac{\partial I_1^r}{\partial \bar{e}} = \frac{dE(X^r, I_1^r)}{dI^r}\frac{\partial I_1^r}{\partial \bar{e}} = \underbrace{\frac{\partial E(X^r, I_1^r)}{\partial I_1^r}\frac{\partial I_1^r}{\partial \bar{e}}}_{\text{direct effect of } \bar{e} \text{ on } E} - \underbrace{\frac{\partial E(X^r, I_1^r)}{\partial X^r}\frac{\frac{\partial^2 NBE(X^r, I_1^r, \tau)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r, I_1^r, \tau)}{\partial (X^r)^2}}\frac{\partial I_1^r}{\partial \bar{e}}}_{\text{indirect effect of } \bar{e} \text{ on } E}$$

is the total effect of the final investment scale I_1^r on emissions $E(X^r, I_1^r)$.

Proof. Follows from totally differentiating $E(X^r, I_1^r)$ with respect to $\bar{e}$ and using Lemma 4. $\square$

Equity relaxes the financial constraint and thus affects emissions through two channels. First, the direct effect of a laxer financial constraint is an increase in the investment scale, which leads to an increase in emissions. The second effect operates through the change in the optimal abatement by borrowers. This effect is indirect since abatement depends on equity only through the effect that it has on the investment scale. As discussed in Lemma 4, a higher investment scale results in more abatement if and only if the marginal net benefit of emission reductions is higher at a higher investment scale, i.e., if $\frac{\partial^2 NBE(X^r, I_1^r, \tau)}{\partial X^r \partial I_1^r} > 0$. Overall, these effects are collected in the term $Z(\tau, \bar{e})$ in Lemma 5, which captures the total effect of increasing the investment scale on emissions.

Since $\frac{\partial I_1^r}{\partial \bar{e}} > 0$, higher equity leads to lower emissions whenever $Z(\tau, \bar{e}) < 0$. This is the case when the effect of a higher investment scale on the optimal abatement is sufficiently large. By contrast, if $Z(\tau, \bar{e}) > 0$ higher equity leads to higher emissions. In this case a stronger capitalization of borrowers mostly leads them to avoid liquidating polluting assets, while the effect on abatement is limited.

Using the insights from Lemma 5, Proposition 4 below shows that, if $\lambda^{ce} > 0$, then the socially optimal capital mandate can be either above or below the level of equity chosen by borrowers in the competitive equilibrium, depending on the total impact of the final investment scale on emissions.

Proposition 4. *Consider the case $\gamma_p = 0$. If $\lambda^r(\tau^*, \bar{e}^*) > 0$ and $Z(\tau^*, \bar{e}^*) \neq 0$, then the competitive equilibrium is not constrained efficient for any τ. The optimal capital mandate is*

- $\bar{e}^* > e^{ce}$ *if* $Z(\tau^*, \bar{e}^*) < 0$,
- $\bar{e}^* < e^{ce}$ *if* $Z(\tau^*, \bar{e}^*) > 0$.

Proof. See Appendix A.1.6 □

Proposition 4 shows that, if the financial constraint binds at the optimal carbon tax and leverage mandate ($\lambda^r(\tau^*, \bar{e}^*) > 0$), then the competitive equilibrium is generally not constrained efficient because a planner can improve welfare by imposing a capital mandate $\bar{e} \neq e^{ce}$. The optimal capital mandate can be above or below e^{ce} depending on the effect of a higher investment scale on emissions, $Z(\tau, \bar{e})$.

To understand this result recall that the optimal carbon tax is below the Pigouvian level γ if the financial constraint binds (see Proposition 3). When choosing leverage borrowers internalize the effect of higher equity on easing financing constraints and thereby avoiding liquidations (higher I_1). But borrowers value the marginal effect of emissions according to the tax rate $\tau E(X, I_1)$, yet from a welfare perspective the cost of emissions is $\gamma E(X, I_1)$. This drives a wedge between the leverage choice by private agents and the socially optimal leverage because $\tau < \gamma$.

This wedge can be seen when comparing the borrower's first order condition w.r.t. $\bar{e}$ in Eq. (12) to the planner's first order condition w.r.t. $\bar{e}$ in Eq. (14), which are re-stated for convenience below (for the case of an interior solution of I_1):

$$u'(A_0^b - e) - \kappa_e = 1 - \frac{r(\tau, e)}{\tilde{r}(\tau, e)}, \tag{12}$$

$$u'(A_0^b - \bar{e}) - \kappa_e = 1 - \frac{r(\tau, \bar{e})}{\tilde{r}(\tau, \bar{e})} + (\gamma - \tau)\frac{Z(\tau, \bar{e})}{\tilde{r}(\tau, \bar{e})}, \tag{14}$$

where we use shorthand notation $r(\tau, e) = r(\tau, E^a(\tau, e), X^{ce}(\tau, e), I_1^{ce}(\tau, e))$ (and accordingly for $\tilde{r}(\tau, e)$).

With $\tau < \gamma$, the wedge pulls the optimal leverage above or below the level in the competitive equilibrium depending on whether the total effect of capital on emissions is positive or negative, i.e., on the sign of $Z(\tau, \bar{e})\frac{\partial I_1^r}{\partial \bar{e}}$.

If $Z(\tau, \bar{e}) < 0$, then higher capital results in lower emissions because the impact of equity on emissions through abatement dominates the effect through liquidations. The marginal benefit of equity from the perspective of the borrower, $\tau\frac{dE(X^r, I_1^r)}{d\tau}$, is lower than the social benefit $\gamma\frac{dE(X^r, I_1^r)}{d\tau}$. Thus, the planner opts for a capital mandate that is above the privately optimal level of equity, $\bar{e}^* > e^{ce}$.

By contrast, if $Z(\tau, \bar{e}) > 0$, then higher capital implies higher emissions. In this case the wedge between the private and social cost of emissions $\tau - \gamma$ implies the marginal cost of equity is lower from the perspective of the borrower than it is for the planner. As a result, the planner sets a capital mandate below a borrower's optimal choice of equity in the competitive equilibrium, $\bar{e}^* < e^{ce}$

A motive to include climate risks in financial regulation. The finding in Proposition 4 that capital regulation can improve welfare may not seem surprising given the large body of literature that shows how financial constraints can motivate financial regulation (for an overview, see Dewatripont and Tirole, 1994). Yet the following proposition shows that the financial constraint in itself does not motivate a capital mandate in our model:

Proposition 5. *If $\gamma = 0$, then $\bar{e}^* = e^{ce}$ regardless of whether $\lambda^{ce} = 0$ or not.*

Proof. In Appendix A.1.6 □

Proposition 5 implies that, in the absence of environmental externalities, the competitive equilibrium coincides with the optimal financial regulation equilibrium – irrespective of whether the financial constraint binds or not. This is important because it implies that financial constraints alone are not enough to motivate capital regulation in our model. Instead, the motive for implementing a capital mandate $\bar{e}$ comes from the interaction between environmental externalities and financial constraints because binding financial constraints imply that the optimal emission tax is below the Pigouvian benchmark. Therefore, the model provides a rationale for including environmental externalities in the mandate of financial regulators and more broadly that there is a case to coordinate environmental and financial regulations.

A related insight that follows from Propositions 2 and 3 is that it is never optimal for the planner to set a capital mandate to achieve a slack financial constraint if it binds in the competitive equilibrium:

Corollary 1. *Consider the case $\gamma_p = 0$. If $\lambda^{ce}(\tau^*, e^{ce}) > 0$ then also $\lambda^r(\tau^*, \bar{e}^*) > 0$.*

Proof. Suppose that $\lambda^{ce}(\tau^*, \bar{e}^{ce}) > 0$ in the competitive equilibrium and the planner sets a leverage mandate $\bar{e}' > \bar{e}^{ce}$ sufficiently strict for the financial constraint to turn slack, $\lambda^r(\tau^*, \bar{e}') = 0$. Then, by Proposition 3 the optimal tax is equal to the social cost of carbon, $\tau^* = \gamma$. But if $\tau^* = \gamma$, then by Proposition 2 the allocation is already at the first best and there is no reason to impose capital regulation. This implies $\bar{e}'$ cannot be optimal, which contradicts $\lambda^r(\tau^*, \bar{e}*) = 0$. □

Corollary 1 highlights that the goal of capital regulation is not to ensure that borrowers never become financially constrained. This again underlines the insight from Proposition 5 that the motive for capital regulation is a result of the interaction between environmental externalities and financial constraints that drive a wedge between the private and social cost of emissions. Without this wedge there is no case for financial regulation in the model in the absence of physical risk.

4.3 Physical Risk

This section analyzes the case in which $\gamma_p > 0$, so that emissions affect asset values through physical risk. Throughout this section we focus on the interesting case of a binding financial constraint. We also introduce additional parameter assumptions in order to streamline the discussion of the equilibrium in this case.

Assumption 3. *We assume that the parameters satisfy:*

(i) $Z(\tau, \bar{e}) < 0, \ \forall \tau, \bar{e}$

(ii) $\tilde{r}(\tau, \bar{e}) < \gamma_u Z(\tau, \bar{e}), \forall \tau, \bar{e}$

By Assumption 3 (i) the effect of equity on abatement dominates the effect on liquidations. In this case $\bar{e}^* > e^{ce}$ in the absence of physical risk (the first case in Proposition 4). We also replace part (ii) of Assumption 2 by Assumption 3 (ii), which ensures liquidations can increase financial slack also when the indirect effect of liquidations on pledgeable returns through physical risk is internalized. Without this assumption borrowers would not face a meaningful trade-off between inefficient liquidations and easing financial constraints in the presence of physical risk.

The presence of physical risk can change how emission taxes interact with financial constraints. To see this, consider a hypothetical borrower who is exposed only to physical risk and has no polluting assets. Such a borrower would unambiguously benefit from higher emission taxes that result in lower emissions and increase the borrower's (pledgeable) income. For borrowers that are exposed to both physical and transition risk the net effect of emission taxes on financial constraints depends on whether physical risk is sufficiently large, as shown in the following lemma:

Lemma 6. *Let $S(I_1, e, \tau)$ be a measure of a borrower's financial slack for a given I_1, e and τ, defined as*

$$S(I_1, e, \tau) = \tilde{R}(I_1, E^a) - \mu I_1 + e - I_1(1 - \mu) - \tau E(X^{ce}, I_1) - c_x(X^{ce}, I_1).$$

There exists a threshold $\hat{\gamma}_p(I_1, \tau)$ such that, if $\gamma_p > \hat{\gamma}_p(I_1, \tau)$, an increase in emission taxes τ increases financial slack, i.e., $\frac{\partial S(I_1,e,\tau)}{\partial \tau}\big|_{I_1,e} > 0$ if and only if $\gamma_p > \hat{\gamma}_p(I_1, \tau)$.

Proof. See Appendix A.2.1. □

When emission taxes increase rather than decrease financial slack the trade-offs faced by a social planner change fundamentally, as highlighted in the following proposition:

Proposition 6. *If the financial constraint binds ($\lambda^r > 0$) and (13) evaluated at $\kappa_\tau = 0$ has a unique solution, the optimal emission tax in the financial regulation equilibrium is*

- $\tau^* > \gamma$, *if* $\gamma_p > \hat{\gamma}_p(I_1^r(\tau^*, e), \tau^*)$,
- $\tau^* < \gamma$, *if* $\gamma_p < \hat{\gamma}_p(I_1^r(\tau^*, e), \tau^*)$,

where $\hat{\gamma}_p$ is defined in Lemma 6.

Proof. See Appendix A.2.2. □

If physical risk is sufficiently high such that $\gamma_p > \hat{\gamma}_p(I_1^r(\tau^*, e^*), \tau^*)$, then higher emission taxes ease financial constraints. This changes the trade-offs faced by a social planner and implies optimal emission taxes above the social cost of emissions γ. Intuitively, if the emission reductions achieved through taxes have the additional benefit of easing financial constraints, then this benefit is taken into account in setting the optimal tax rate. Such a case may apply to economies that are heavily exposed to the risk of environmental damages from floodings and other weather disasters that have a negative effect on economic output and asset values and become increasingly likely as the planet warms up.

By contrast, if physical risk is relatively small such that $\gamma_p < \hat{\gamma}_p(I_1^r(\tau^*, e^*), \tau^*)$, then the direct effect of emission taxes on polluters' financial constraints dominates and optimal taxes are below the social cost of emissions. This case applies where transition risks dominate physical risks, for example in economies with large polluting industries.

The following proposition complements these results and shows how the optimal capital mandate is set depending on the optimal emission tax.

Proposition 7. *Consider the case $\gamma_p > 0$ (physical risk) and $\lambda^r > 0$ (binding financial constraint).*

- *If $\tau^* = \gamma$ then the socially optimal capital mandate is higher than the privately optimal level of equity, i.e., $\bar{e}^* > e^{ce}$.*
- *There exists a threshold $\hat{\tau} > \gamma$ such that the socially optimal capital mandate is higher than the privately optimal level of equity, $\bar{e}^* > e^{ce}$, whenever $\tau^* < \hat{\tau}$.*

Proof. See Appendix A.2.3 □

According to Proposition 7 the optimal capital mandate $\bar{e}^*$ exceeds the level of equity chosen in the competitive equilibrium e^{ce} even if the emission tax is equal to the Pigouvian benchmark $\tau^* = \gamma$. This is in stark contrast to the benchmark model without physical risk, in which there is no motive for capital regulation if emission taxes are equal to the Pigouvian benchmark (see Proposition 2).

To understand this result note that borrowers do not internalize the effect of emissions on the tightness of other borrower's financial constraints. In the presence of physical risk lower emissions increase asset values and therefore ease financing constraints. Higher equity is associated with more abatement and consequently lower emissions. As a result, even when $\tau^* = \gamma$ borrowers' choice of equity is socially inefficient. The optimal capital mandate is $\bar{e}^* > e^{cb}$ even for $\tau^* > \gamma$, as long as τ^* does not exceed the threshold $\hat{\tau}$ defined in Proposition 7. The threshold $\hat{\tau}$ coincides with the tax rate at which borrowers correctly internalize the effect of more equity on lower emissions and other borrower's financial constraints.

A collateral externality The mechanism at play here can be referred to as a *collateral externality* because borrowers do not internalize the negative effect of emissions on other borrowers' pledgeable income and financial constraints. Collateral externalities can also emerge in models with pecuniary externalities, where borrowers do not internalize how their choices affect the financial constraint of other agents through their impact on prices (for a detailed discussion, see Dávila and Korinek, 2018). As in these settings, here borrowers choose a socially sub-optimal leverage because they do not internalize their impact on financial constraints. Unlike in the pecuniary externality literature, in our setting the collateral externality operates through physical costs from environmental damages, which reduce other borrowers' pledgeable income. Via this collateral externality physical risk provides a rationale for capital regulation even if the tax is set at the Pigouvian benchmark (see Proposition 7). The collateral externality is also the mechanism that explains why the optimal emission tax may be above the Pigouvian benchmark if physical risk is sufficiently high (the case $\gamma_p > \hat{\gamma}_p$ in Proposition 6).

4.4 Other Policies

4.4.1 Pollution Permits

An alternative policy available to the planner is to issue a limited quantity Q of tradeable pollution permits (such as the EU Emission Trading Scheme). For each unit of emissions the borrower needs to surrender a permit to the planner. Remaining permits can be sold at the market price p.

Absent other frictions such pollution permit markets are equivalent to emission taxes (see Montgomery, 1972). The emission tax policy we study in the baseline model is equivalent to a pollution permit scheme in which borrowers need to purchase permits from the planner at the market price p at $t = 2$. In this case it is optimal for the planner to set the quantity of permits such that the equilibrium price equals the optimal emissions tax rate $p = \tau^*$.

Due to the negative effect of emission taxes on financial constraints a more desirable policy may be to distribute permits for free. Yet, focusing on the baseline case of no physical risk we show below that even when using such a policy the presence of financial constraints still implies the planner may prefer to implement emission permit prices below the Pigouvian benchmark and that there is still a motive for capital regulation.[17]

With a free endowment of tradeable permits the problem of the borrower can be restated by reformulating the financial constraint and the non-negativity constraints on

[17] Introducing physical risk would have the same directional effects on the optimal emission tax and capital mandate as discussed in Section 4.3. The key insight here that even with a pollution permit market with freely allocated permits financial frictions inhibit a Pigouvian tax from implementing the first best is therefore not affected by the presence of physical risk and we focus on the case without physical risk for simplicity.

consumption at $t = 1$ and $t = 2$ as:

$$c_1 = I_0(1-\mu) - I_1\mu + d_1 + e - c_x(X, I_1) + p(Q - E(X, I_1)) \geq 0 \tag{3'}$$

$$c_2 = R(I_1, E^a) - d_1 \geq 0 \tag{4'}$$

$$d_1 \leq \tilde{R}(I_1, E^a) \tag{5'}$$

The first order conditions of the borrower remain the same as in the benchmark (with p taking the place of τ) but the complementary slackness condition (11) is now given by

$$\lambda[\tilde{R}(I_1, E^a) - \mu I_1 + e - c_x(X, I_1) + p(Q - E(X, I_1)]. \tag{11'}$$

Pollution permit market clearing requires that $Q = E(X^r(p, \bar{e}), I_1^r(p, \bar{e}))$, which implies that the planner can choose the quantity of permits so that to implement the preferred market price p. The planner's first order conditions are:

$$r(\gamma, \bar{e})\frac{\partial I_1^r}{\partial p} - (\gamma - p)\frac{\partial E(X^r, I_1^r)}{\partial X^r}\frac{\partial X^r}{\partial p} + \kappa_p = 0 \tag{13'}$$

$$u'(A_0^b - \bar{e}) - \kappa_e = 1 + r(\gamma, \bar{e})\frac{\partial I_1^r}{\partial \bar{e}} - (\gamma - p)\left(\frac{\partial E(X^r, I_1^r)}{\partial I_1^r}\frac{\partial I_1^r}{\partial p} + \frac{\partial E(X^r, I_1^r)}{\partial X^r}\frac{\partial X^r}{\partial p}\right) \tag{14'}$$

Market clearing in permits also implies that the tightness of a borrower's financial constraint is not affected by the extra cost associated with buying the permits (equivalent to the cost of tax payment in the benchmark model). However, since the shadow cost of permits still induces the borrower to engage in abatement the financial constraint is still affected by the costs of abatement. As a consequence, also in this case the planner may prefer to set a quantity that implements a market price that is lower than the Pigouvian benchmark $p < \gamma$.

Proposition 8. *Let $\gamma_p = 0$. If the net pledgeable marginal return is sufficiently low such that $\tilde{r}(p, \bar{e}) < -\tau Z(p, \bar{e})$ for all $p < \gamma$ and $\bar{e} \in (I_0(1-\mu), A_0^b)$, then the optimal price of pollution permits is*

- *$p^* = \gamma$ if $\lambda^r(p^*, \bar{e}^*) = 0$,*

- *$p^* < \gamma$ if $\lambda^r(p^*, \bar{e}^*) > 0$ and (13') has a unique solution at $\kappa_p = 0$.*

The optimal capital mandate generally differs from the equity choice in the competitive equilibrium, $\bar{e}^* \neq e^{ce}$, whenever $Z(p, \bar{e})\,(\theta\rho p + \tilde{r}(\gamma, \bar{e})) \neq 0$.

If the net pledgeable marginal return is sufficiently low liquidations generate financial slack also in the case of freely allocated permits. In this case, the optimal price of the permit set by the planner trades off the benefits of lower emissions against the cost of triggering more inefficient liquidations. The latter effect is driven by the impact of rising costs of abatement on the tightness of the financial constraint.

In this case the planner again has motive for regulating capital. As in the baseline model with emission taxes the planner and the borrower differ in their assessment of the marginal value of equity on liquidations. The borrower's choice of abatement trades off the marginal cost of abatement against the shadow price of the permit, so that she disregards the effect that equity has through these channels due to the Envelope Theorem. At the same time, the planner internalizes the fact that the permit endowment and surrender have a net zero effect on the financial constraint. Thus, the social benefit of equity accounts only for the marginal cost of abatement and disregards any impact through changes in emissions. The result is that the private choice of equity generally does not coincide with the optimal capital mandate.

Beyond the scope of our model, a challenge to such a policy may be that freely allocating pollution permits may be difficult in the presence of heterogeneity. For example, it would be difficult for regulators to correctly allocate permits if polluters were privately informed about heterogeneous abatement costs. This may imply that financial constraints are tightened for some polluters and slackened for others, with potentially undesirable distributional consequences.

4.4.2 Transfers

We next consider policies that transfer resources from investors to borrowers. Such policies may be helpful because they can ease financial constraints of borrowers.

Transfers Consider a transfer $\mathcal{T}$ to borrowers financed by raising lump-sum taxes from investors. Note that this transfer needs to be paid at $t = 1$ because at $t = 2$ borrowers can abscond with cash. With this transfer the complementary slackness condition (11) becomes

$$\lambda \left[\tilde{R}(I_1, E^a) - \tau E(X, I_1) + \mathcal{T} - I_0(1-\mu) + e - c_x(X, I_1) - \mu I_1 \right] = 0.$$

Clearly if $\mathcal{T}$ is sufficiently large, then the financial constraint becomes slack. As shown in Proposition 2, this implies that an emission tax equal to the Pigouvian benchmark can implement the first best. Complementing Pigouvian emission taxes with transfers to polluters can therefore (almost trivially) overcome the problems caused by financial constraint. Whether generous transfers to polluters are politically feasible in the real world is less clear.

Abatement Subsidy A politically less contentious policy option could be to subsidize abatement. To analyze such a policy in the context of our model suppose that $\tau = 0$ and consider instead a subsidy s on abatement financed by lump-sum taxes such that $-sX = T^b + T^i$. Again, it is better to pay the subsidy at $t = 1$ because at $t = 2$ borrowers can abscond with cash. The first order condition with respect to X in Eq. (9) becomes

$$(1+\lambda)\left(s - \frac{\partial c_x(X, I_1)}{\partial X}\right) = 0.$$

This equation is equivalent to the original first order condition (9) when setting $s = \tau \frac{\partial E(X,I_1)}{\partial X}$. Whether the subsidy can implement the first best allocation or needs to be complemented with capital regulation therefore depends on whether the financial constraint binds. To see if this is the case note the complementary slackness condition (11) now becomes

$$\lambda \left[\tilde{R}(I_1, E^a) + sX - T^b - I_0(1-\mu) + e - c_x(X, I_1) - \mu I_1 \right] = 0,$$

If the subsidy is (partially) financed by investors, then $T^b < -sX$ and the subsidy constitutes a net transfer of resources from investors to borrowers and can therefore ease financial constraints. As shown in Proposition 2, with a slack financial constraint a subsidy that mirrors an emission tax equal to the Pigouvian benchmark can implement the first best.

While subsidies for abatement investments are likely politically less controversial, in practice subsidies may be more difficult to implement than emission taxes for other reasons. A key challenge to subsidies is that abatement investments need to be verifiable and may be subject to other asymmetric information problems. Moreover, this solution requires the government to be able to correctly identify which technologies to support. Arguably, a technology-neutral policy is more flexible because it allows the market to pick the most efficient emission-reduction technology.

5 Conclusion

Climate change is one of the biggest challenges in recent history and has moved up the agenda of policy makers and investors. This paper provides an analytical framework to understand how to best design and coordinate environmental regulation in the form of emission taxes and financial regulation in the form of capital mandates.

The model shows that in the presence of financial constraints emission taxes alone cannot implement a constrained efficient allocation and welfare can be improved when complementing emission taxes with leverage regulation. This result provides a rationale for including climate risks in financial regulatory frameworks and motivates coordinating environmental and financial regulation.

Another important insight is that the way in which financial constraints interact with emission taxes critically depends on whether transition risks or physical risks dominate. Higher emission taxes tighten the financial constraints of borrowers exposed to transition risk because polluters bear the cost of emission taxes and abatement. By contrast, higher emission taxes can ease the financial constraints of borrowers with assets that are exposed to physical risk because lower emissions have a positive effect on their pledgeable income and financial constraints. This collateral externality provides an additional motive for capital regulation even if emission taxes are equal to the Pigouvian benchmark. The optimal emission tax may be above or below a Pigouvian benchmark, depending on whether the effect of transition risk or physical risk dominates.

Other policies that help slacken constraints may be desirable. For example, replacing emission taxes by a pollution permit market with ex-ante freely allocated permits can alleviate the direct effect of taxes on financial constraints. However, because the shadow cost of permits induces borrowers to engage in abatement financial constraints are still affected by the costs of abatement and inhibit the permit scheme alone to implement a constrained efficient allocation. Subsidies to abatement may be superior to emission taxes from a welfare perspective if the subsidy constitutes a net transfers to polluters and thereby slackens financial constraints. A challenge is that, in contrast to emission taxes, subsidies require abatement investments to be verifiable and regulators to know which emission-reducing technologies are the most efficient.

A Proofs

A.1 Benchmark Economy

A.1.1 First Best (Proposition 1)

Proof. The first best allocation can be found by the abatement, investment and consumption levels that maximize social welfare defined by the sum of agent's utilities

$$\max_{I_1 \leq I_0, X, c_t^b \geq 0, c_t^i \geq 0} W = u(c_0^b) + c_0^i + c_1^b + c_1^i + c_2^b + c_2^i - \gamma E(X, I_1),$$

subject to the aggregate resource constraints

$$\begin{aligned} c_0^b + c_0^i &= A_0^b + A_0^i - I_0, \\ c_1^b + c_1^i + c_x(X, I_1) &= A_1^i + \mu(I_0 - I_1), \\ c_2^b + c_2^i &= \rho I_1 - \psi\gamma E(X, I_1). \end{aligned}$$

Eliminating $c_0^i, c_1^b, c_1^i, c_2^b + c_2^i$ the problem can be formulated as follows:

$$\begin{aligned} \max_{I_1, X, c_0^b} W = & \, u(c_0^b) + A_0^b + A_0^i - I_0 - c_0^b + A_1^i + \mu(I_0 - I_1) - c_x(X, I_1) \\ & + A_2^i + \rho I_1 - (1+\psi)\gamma E(X, I_1) + \bar{\kappa}_{I_1}(I_0 - I_1), \end{aligned}$$

with $\bar{\kappa}_{I_1}$ the Lagrange multiplier on the constraint that $I_1 \leq I_0$. The first order conditions w.r.t. c_0^b, I_1 and X are given by, respectively,

$$\begin{aligned} u'(c_0^b) &= 1, \\ \rho - \mu - (1+\psi)\gamma \frac{\partial E(X, I_1)}{\partial I_1} - \frac{\partial c_x(X, I_1)}{\partial I_1} - \bar{\kappa}_{I_1} &= 0, \\ (1+\psi)\gamma \frac{\partial E(X, I_1)}{\partial I_1} + \frac{\partial c_x(X, I_1)}{\partial I_1} &= 0. \end{aligned}$$

By Assumption 2 liquidations are inefficient, which implies $\bar{\kappa}_{I_1} > 0$ and $I_1 = I_0$. $\square$

A.1.2 Proof of Lemma 1

Proof. The maximum $t = 1$ consumption that the borrower can achieve for a given e is $e - I_0(1 - \mu)$. For the borrower to obtain external financing the consumption at $t = 1$ must be non-negative as required by (3). Thus, the borrower needs to contribute at least $e = I_0(1 - \mu)$ of equity. By Assumption 2 liquidating less cannot generate more financial slack. Therefore, a higher level of equity $e > I_0(1 - \mu)$ is needed for the borrower to continue the project without full liquidation. □

Discussion: Note that if the project is fully liquidated at $t = 1$, the borrower is better off not continuing the project and instead consuming A_0^b at $t = 0$. Since $r(\tau, E^a, X, I_1) > 0$, the payoff from continuation increases in the final investment scale I_1. When the financial constraint binds, the optimal investment scale, abatement and equity levels are pinned down jointly by Equations (12), (9) and (11). This yields $I_1^{ce}(\tau)$, $X^{ce}(\tau)$ and $e^{ce}(\tau)$. The borrower finds it optimal to continue the project if and only if

$$u(A_0^b) < u(A_0^b - e^{ce}(\tau)) + e + R(I_1^{ce}, E^a) - \mu I_1^{ce}(\tau) - c_x(X^{ce}(\tau), I_1^{ce}(\tau)) - \tau E(X^{ce}(\tau), I_1^{ce}(\tau)) + T^b$$

A.1.3 Proof of Lemma 3

Proof. Represent the complementary slackness condition (11) as:

$$\lambda S(I_1, e, \tau) = \lambda[\tilde{R}(I_1, E^a) - \tau E(X^{ce}(I_1, \tau), I_1) - I_0(1 - \mu) - I_1\mu + e - c_x(X^{ce}(I_1, \tau), I_1)]$$

where $X^{ce}(I_1, \tau)$ follows from (9). Note that $S(I_1, e; \tau) = c_1$ is a measure of $t = 1$ financial slack for a given I_1 and e. Higher carbon taxes lower financial slack and equity increases slack:

$$
\begin{aligned}
\left.\frac{\partial S(I_1,e,\tau)}{\partial \tau}\right|_{I_1,e} &= -E(X^{ce},I_0) - \underbrace{\left(\tau\frac{\partial E(X^{ce},I_0)}{\partial X^{ce}} + \frac{\partial c_x(X^{ce},I_0)}{\partial X^{ce}}\right)}_{=0 \text{ by Eq. (9)}}\frac{\partial X^{ce}}{\partial \tau} \\
&= -E(X^{ce},I_0) \leq 0, \\
\left.\frac{\partial S(I_0,e,\tau)}{\partial e}\right|_{I_1} &= 1 > 0.
\end{aligned}
$$

(i) With $e < (1-\mu)I_0$ it is not possible to finance the project by (i), implying $\kappa_e = 0$. $u'(A_0^b - (1-\mu)I_0) > 1$ implies $u'(A_0^b - e) > 1$ for $e > (1-\mu)I_0$ because $u''(.) > 0$. It follows from Eq. (12) that it must be that $\lambda > 0$ for $e \geq (1-\mu)I_0$, which in turn implies that the financial constraint binds irrespective of the level of τ.

(ii) The financial constraint is slack ($\lambda = 0$) if equity and tax are such that $S(I_0,e,\tau) > 0$. In this case, there are no liquidations $I_1^{ce} = I_0$ ($\overline{\kappa}_I > 0$).

Consider the threshold $\hat{e}(\tau)$ such that $S(I_0,\hat{e}(\tau),\tau) = 0$. If the initial endowment is high enough so that $u'(A_0^b - \hat{e}(\tau)) \leq 1$, then the borrower chooses equity that is at least equal to the threshold $e \geq \hat{e}(\tau)$, so the financial constraint is slack.

Totally differentiation of $S(I_0,\hat{e}(\tau),\tau) = 0$ with respect to τ yields:

$$
\frac{d\hat{e}}{d\tau} = E(X^{ce}(I_1,\tau),I_1) > 0
$$

□

A.1.4 Proof of Lemma 4

Recall that if the financial constraint is binding, $\lambda^r(\tau,\bar{e}) > 0$, then $X^r(\tau,\bar{e})$ and the interior solution of $I_1^r(\tau,\bar{e})$ are pinned down by:

$$
\tau\frac{\partial E(X^r,I_1^r)}{\partial X^r} = -\frac{\partial c_x(X^r,I_1^r)}{\partial X^r}, \tag{9}
$$

$$
\tilde{R}(I_1^r,E^a) - I_0(1-\mu) + \bar{e} - c_x(X^r,I_1^r) - \mu I_1^r - \tau E(X^r,I_1^r) = 0 \tag{11}
$$

Totally differentiating (9) with respect to τ and $\bar{e}$ allows us to find $\frac{dX^r}{d\tau} = \frac{\partial X^r}{\partial \tau}$ and $\frac{dX^r}{d\bar{e}} = \frac{\partial X^r}{\partial \bar{e}}$. The respective total derivatives are:

$$\left[\tau\frac{\partial^2 E(X^r, I_1^r)}{\partial (X^r)^2} + \frac{\partial^2 c_x(X^r, I_1^r)}{\partial (X^r)^2}\right]\frac{dX^r}{d\tau} + \left[\frac{\tau\partial^2 E(X^r, I_1^r)}{\partial X^r \partial I_1^r} + \frac{\partial^2 c_x(X^r, I_1^r)}{\partial X^r \partial I_1^r}\right]\frac{dI_1^r}{d\tau} + \frac{\partial E(X^r, I_1^r)}{\partial X^r} = 0$$
$$\left[\tau\frac{\partial^2 E(X^r, I_1^r)}{\partial (X^r)^2} + \frac{\partial^2 c_x(X^r, I_1^r)}{\partial (X^r)^2}\right]\frac{dX^r}{d\bar{e}} + \left[\tau\frac{\partial^2 E(X^r, I_1^r)}{\partial X^r \partial I_1^r} + \frac{\partial^2 c_x(X^r, I_1^r)}{\partial X^r \partial I_1^r}\right]\frac{dI_1^r}{d\bar{e}} = 0$$

Which can be re-arranged to yield:

$$\frac{dX^r}{d\tau} = \frac{\frac{\partial E(X^r, I_1^r)}{\partial X^r} - \frac{\partial^2 NBE(X^r, I_1^r)}{\partial X^r \partial I_1^r}\frac{dI_1^r}{d\tau}}{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial (X^r)^2}} \tag{15}$$

$$\frac{dX^r}{d\bar{e}} = -\frac{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial (X^r)^2}}\frac{dI_1^r}{d\bar{e}} \tag{16}$$

Likewise, totally differentiating (11) with respect to τ and $\bar{e}$ allows us to find $\frac{dI_1^r}{d\tau} = \frac{\partial I_1^r}{\partial \tau}$ and $\frac{dI_1^r}{d\bar{e}} = \frac{\partial I_1^r}{\partial \bar{e}}$. The respective total derivatives are:

$$-\frac{\partial c_x(X^r, I_1^r)}{\partial X^r}\frac{dX^r}{d\tau} - \frac{\partial c_x(X^r, I_1^r)}{\partial I_1^r}\frac{dI_1^r}{d\tau} - (\mu - \rho(1-\theta))\frac{dI_1^r}{d\tau}$$
$$-\tau\frac{\partial E(X^r, I_1^r)}{\partial X^r}\frac{dX^r}{d\tau} - \tau\frac{\partial E(X^r, I_1^r)}{\partial I_1^r}\frac{dI_1^r}{d\bar{e}} - E(X^r, I_1^r) = 0$$
$$-\frac{\partial c_x(X^r, I_1^r)}{\partial X}\frac{dX^r}{d\bar{e}} - \frac{\partial c_x(X^r, I_1^r)}{\partial I_1^r}\frac{dI_1^r}{d\bar{e}} - (\mu - \rho(1-\theta))\frac{dI_1^r}{d\bar{e}} - \tau\frac{\partial E(X^r, I_1^r)}{\partial X^r}\frac{dX^r}{d\bar{e}} - \tau\frac{\partial E(X^r, I_1^r)}{\partial I_1^r}\frac{dI_1^r}{d\bar{e}} + 1$$

Which can be re-arranged to yield:

$$\frac{dI_1^r}{d\tau} = \frac{E(X^r, I_1^r)}{\tilde{r}(\tau)} \tag{17}$$

$$\frac{dI_1^r}{d\bar{e}} = \frac{-1}{\tilde{r}(\tau)} \tag{18}$$

Where $\frac{dI_1}{de} = \frac{\partial I_1^*}{\partial e} > 0$ and $\frac{dI_1}{de} = \frac{\partial I_1^*}{\partial \tau} < 0$, since $\tilde{r}(\tau) < 0$ and $E(X, I_1) > 0 \; \forall X < \infty$.

A.1.5 Proof of Proposition 3

The first order condition of the social planner with respect to τ is given by:

$$\left(\rho - \mu - \gamma\frac{\partial E}{\partial I_1} - \frac{\partial c_x}{\partial I_1}\right)\frac{\partial I_1^r}{\partial \tau} - \left(\gamma\frac{\partial E}{\partial X} + \frac{\partial c_x}{\partial X}\right)\frac{\partial X^r}{\partial \tau} + \kappa_\tau = 0 \tag{19}$$

Using (9) and definitions of $\frac{\partial X^r}{\partial \tau}$ and $\frac{\partial I_1^r}{\partial \tau}$ in (15) and (17) we can rewrite (19) as:

$$r(\gamma, \bar{e})\frac{\partial I_1^r}{\partial \tau} - (\gamma - \tau)\frac{\partial E(X^r, I_1^r)}{\partial X}\frac{\partial X^r}{\partial \tau} + \kappa_\tau = 0 \tag{13}$$

If $\lambda^r(\gamma, \bar{e}) > 0$ the LHS of (13) at $\tau = \gamma$ is negative, since $\frac{\partial I_1^r}{\partial \tau} < 0$ and $r(\gamma, \bar{e}) > 0$. The first term of (13) is negative for all τ such that $\lambda^r(\tau, \bar{e})$, since $r(\gamma, \bar{e})\frac{\partial I_1^r}{\partial \tau} < 0$.

If $\frac{\partial X^r}{\partial \tau} > 0$, then $-(\gamma - \tau)\frac{\partial E(X^r, I_1^r)}{\partial X}\frac{\partial X^r}{\partial \tau}$ is positive if and only if $\tau < \gamma$, since $\frac{\partial E(X^r, I_1^r)}{\partial X} < 0$. Thus, if an interior solution of (13) exists it is $\tau^* < \gamma$.

If $\frac{\partial X^r}{\partial \tau} < 0$, then $-(\gamma - \tau)\frac{\partial E(X^r, I_1^r)}{\partial X}\frac{\partial X^r}{\partial \tau}$ is positive if and only if $\tau > \gamma$. In this case, if (13) evaluated at $\kappa_\tau = 0$ has a unique interior solution, it is $\tau^* > \gamma$. However, since LHS of (13) is negative at $\tau = \gamma$, we must have that LHS of (13) is crossing zero from below at the solution $\tau^* > \gamma$. This implies that $\tau^* > \gamma$ represent a minimum of the social welfare function and is thus not the optimum. Therefore, if $\frac{\partial X^*}{\partial \tau} < 0$, then $\kappa_\tau > 0$ and the planner sets $\tau = 0$.

Thus, whenever (13) has a unique solution when evaluated at $\kappa_\tau = 0$ and $\lambda^r(\tau^*, \bar{e}) > 0$ the planner sets a tax below the Pigouvian rate $\tau^* < \gamma$.

If $\lambda^r(\tau, \bar{e}) = 0$, then $\frac{\partial I_1^r}{\partial \tau} = 0$. Thus (13) simplifies to:

$$-(\gamma - \tau)\frac{\partial E}{\partial X}\frac{\frac{\partial E(X^r, I_1^r)}{\partial X^r}}{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial (X^r)^2}} + \kappa_\tau = 0 \tag{20}$$

which is solved at $\tau = \gamma$ and $\kappa_\tau = 0$. This represents the maximum of the social welfare function because $\frac{\partial^2 NBE(X^r, I_1^r)}{\partial (X^r)^2} < 0$. For this to be the optimal tax rate we need that $\lambda^r(\gamma, \bar{e}) = 0$

So far we focused on the case of interior solution for $I^r(\tau, \bar{e})$. If liquidations are at the corner solution $I_1^r(\tau,\bar{e}) = 0$ with $\underline{\kappa}_I > 0$, we have that $\frac{dI_1^r}{d\tau} = 0$. Thus the optimal tax solves:

$$(\gamma-\tau)\frac{-\left(\frac{\partial E}{\partial X}\right)^2}{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^2}} + \kappa_\tau = 0$$

It implies that when liquidations are in the corner solution, planner sets $\tau^* = \gamma$.

A.1.6 Proof or Proposition 4 and Proposition 5

The first order conditions of the social planner with respect to $\bar{e}$ is:

$$\left(\rho - \mu - \gamma\frac{\partial E}{\partial I_1} - \frac{\partial c_x}{\partial I_1}\right)\frac{\partial I_1^r}{\partial \bar{e}} - \left(\gamma\frac{\partial E}{\partial X} + \frac{\partial c_x}{\partial X}\right)\frac{\partial X^r}{\partial \bar{e}} - u'(A_0^b - \bar{e}) + 1 + \kappa_e = 0 \tag{21}$$

Using definition of $r(\tau, E^a)$ we and the fact that $r(\gamma,\bar{e}) = r(\tau,\bar{e}) + \tau\frac{\partial E(X^r,I_1^r)}{\partial I_1^r} - \gamma\frac{\partial E(X^r,I_1^r)}{\partial I_1^r}$, we can rewrite (21) as:

$$u'(A_0^b - \bar{e}) - \kappa_e = 1 + r(\tau,\bar{e})\frac{\partial I_1^r}{\partial \bar{e}} - (\gamma - \tau)\left(\frac{\partial E(X^r,I_1^r)}{\partial I_1^r}\frac{\partial I_1^r}{\partial \bar{e}} + \frac{\partial E(X^r,I_1^r)}{\partial X^r}\frac{\partial X^r}{\partial \bar{e}}\right) \tag{14}$$

Using (11) and definitions of $\frac{\partial X^r}{\partial e}$ and $\frac{\partial I_1^r}{\partial e}$ in (16) and (18) we can further rewrite (14) as:

$$u'(A_0^b - \bar{e}) - \kappa_e = 1 - \frac{r(\tau,\bar{e})}{\tilde{r}(\tau,\bar{e})} - \frac{\tau-\gamma}{\tilde{r}(\tau,\bar{e})}\left(\frac{\partial E(X^r,I_1^r)}{\partial I_1^r} - \frac{\partial E(X^r,I_1^r)}{\partial X^r}\frac{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial (X^r)^2}}\right)$$

Which can be readily compared to FOC wrt. e of the private problem when $I_1^{ce}(\tau, e)$ is at the interior solution, which is given by:

$$u'(A_0^b - e) - \kappa_e = 1 - \frac{r(\tau,\bar{e})}{\tilde{r}(\tau,\bar{e})} \tag{12'}$$

If $\gamma = \tau$ (which is optimal when $\gamma = 0$ or $\lambda^r = 0$) the planner's FOC (14') is identical to the borrower's FOC (12'), so the planner does not regulate leverage.

If $\gamma > \tau$ the RHS of planner's FOC (14') is higher than the RHS of borrower's FOC (14') if and only if:

$$\frac{\partial E(X^r, I_1^r)}{\partial I_1^r} - \frac{\partial E(X^r, I_1^r)}{\partial X^r} \frac{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial X \partial I_1^r}}{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial (X^r)^2}} < 0 \tag{22}$$

If the RHS of planner's FOC (14') is higher than the RHS of borrower's FOC (14') then the planner prefers a higher level of equity than the borrower. In this case, social planner implements binding leverage regulation.

A.2 Physical Risk

A.2.1 Proof of Lemma 6

Recall that the complementary slackness condition (11) is given by:

$$\lambda S(I_1, e, \tau) = \lambda[\tilde{R}(I_1, E^a) - \tau E(X^{ce}(I_1, \tau), I_1) - I_0(1-\mu) - I_1\mu + e - c_x(X^{ce}(I_1, \tau), I_1)]$$

where $X^{ce}(I_1, \tau)$ follows from (9). In the presence of physical risk $\gamma_p > 0$, $\tilde{R}(I_1, E^a) = (1-\theta)(\rho I_1 - \gamma_p E^a(X^a, I_1^a))$. In this case, the derivative of $S(I_1, e; \tau)$ with respect to τ when holding I_1 and e fixed is:

$$\begin{aligned}
&\left.\frac{\partial S(I_1, e, \tau)}{\partial \tau}\right|_{I_1, e} \\
&= -\gamma_p \frac{\partial E^a(X^a, I_1^a)}{\partial X^a} \left.\frac{\partial X^a}{\partial \tau}\right|_{I_1, e} - E(X^{ce}, I_1) - \underbrace{\left(\tau \frac{\partial E(X^{ce}, I_0)}{\partial X^{ce}} + \frac{\partial c_x(X^{ce}, I_0)}{\partial X^{ce}}\right)}_{= 0 \text{ by Eq. (9)}} \frac{\partial X^{ce}}{\partial \tau} \\
&= -\gamma_p \frac{\partial E^a(X^a, I_1^a)}{\partial X^a} \left.\frac{\partial X^a}{\partial \tau}\right|_{I_1, e} - E(X^{ce}, I_1)
\end{aligned}$$

Equation (9) pins down $X^{ce}(I_1, \tau)$. Taking the total derivative of (9) when holding I_1 and e fixed yields:

$$\left.\frac{\partial X^{ce}}{\partial \tau}\right|_{I_1, e} = \frac{\frac{\partial E(X^{ce}, I_1)}{\partial X^{ce}}}{\frac{\partial^2 NBE(X^{ce}, I_1)}{\partial X^2}}$$

Thus, for a given I_1 an increase in tax generates financial slack $\frac{\partial S(I_1,e,\tau)}{\partial \tau}\Big|_{I_1,e} > 0$ if and only if

$$\gamma_p > -\frac{E(X^{ce}, I_1)\frac{\partial^2 NBE(X^{ce},I_1)}{\partial X^2}}{\left(\frac{\partial E(X^{ce},I_1)}{\partial X^{ce}}\right)^2} \equiv \hat{\gamma}_p(I_1, \tau) \tag{23}$$

Thus, there exists a threshold $\hat{\gamma}_p(I_1, \tau)$ such that if $\gamma_p > \hat{\gamma}_p(I_1, \tau)$ higher tax results in more slack in the financial constraint.

A.2.2 Proof of Proposition 6

If $\gamma_p > 0$, the first order condition of the planner's problem with respect to τ is given by:

$$\begin{aligned} &\left(\rho - \mu - (\gamma_u + \gamma_p)\frac{\partial E(X^r, I_1^r)}{\partial I_1^r} - \frac{\partial c_x(X^r, I_1^r)}{\partial I_1^r}\right)\frac{\partial I_1^r}{\partial \tau} \\ -&\left((\gamma_u + \gamma_p)\frac{\partial E(X^r, I_1^r)}{\partial X^r} + \frac{\partial c_x(X^r, I_1^r)}{\partial X^r}\right)\frac{\partial X^r}{\partial \tau} + \kappa_\tau = 0 \end{aligned} \tag{24}$$

Using the above and (9) we can represent planner's FOC as:

$$\hat{r}(\gamma, \bar{e})\frac{\partial I_1^r}{\partial \tau} - (\gamma_u + \gamma_p - \tau)\frac{\partial E(X^r, I_1^r)}{\partial X^r}\frac{\partial X^r}{\partial \tau} + \kappa_\tau = 0 \tag{24'}$$

As in the benchmark model total differentiation of (9) yields:

$$\frac{\partial X^r}{\partial \tau} = \frac{\frac{\partial E(X^r,I_1^r)}{\partial X^r} - \frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r}\frac{dI_1^r}{d\tau}}{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial (X^r)^2}} \tag{15}$$

If the financial constraint is slack, then $\frac{\partial I_1^r}{\partial \tau} = 0$, in this case the FOC of the planner simplifies to:

$$-(\gamma_u + \gamma_p - \tau)\frac{\partial E(X^r, I_1^r)}{\partial X^r}\frac{\frac{\partial E(X^r,I_1^r)}{\partial X^r}}{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial (X^r)^2}} + \kappa_\tau = 0 \tag{25}$$

Thus the optimal tax in the case of a slack financial constraint is Pigouvian, $\tau = \gamma$.

If the financial constraint binds, then we can find $\frac{\partial I_1^r}{\partial \tau}$ by totally differentiating (11), which yields:

$$\left(\rho(1-\theta)-\mu-(\tau+\gamma_p)\frac{\partial E(X^r,I_1^r)}{\partial I_1^r}-\frac{\partial c_x(X^r,I_1^r)}{\partial I_1^r}\right)\frac{\partial I_1^r}{\partial \tau}-E(X^r,I_1^r)-\gamma_p\frac{\partial E(X^r,I_1^r)}{\partial X^r}\frac{\partial X^r}{\partial \tau}=0$$

where we use the fact that $\tau\frac{\partial E(X^{ce},I_0)}{\partial X^{ce}}+\frac{\partial c_x(X^{ce},I_0)}{\partial X^{ce}}=0$ by (9). Using the definition of $\frac{\partial X^r}{\partial \tau}$ this can be represented as:

$$(\tilde{r}(\tau,\bar{e})-\gamma_p Z(\tau,\bar{e}))\frac{\partial I_1^r}{\partial \tau}=E(X^r,I_1^r)+\gamma_p\frac{\left(\frac{\partial E(X^r,I_1^r)}{\partial X^r}\right)^2}{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial (X^r)^2}} \tag{26}$$

If the pledgeable net income is sufficiently low, so that:

$$\tilde{r}(\tau,\bar{e})<\gamma_p Z(\tau,\bar{e}) \tag{27}$$

then the LHS of (26) is negative. The RHS of (26) is negative if and only if (23) is satisfied. Thus, if conditions (23) and (27) are satisified $\frac{\partial I_1^r}{\partial \tau}>0$.

If $\frac{\partial I_1^r}{\partial \tau}>0$, then the first term of RHS of (24') is positive. Throughout our discussion of the physical risk we assume that $Z(\tau,\bar{e})<0$ for all τ and $\bar{e}$. This requires that $\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r}>0$, which implies that $\frac{\partial X^r}{\partial \tau}>0$ whenever $\frac{\partial I_1^r}{\partial \tau}>0$. Thus the second term of RHS of (24') is negative if and only if $\tau>\gamma_u+\gamma_p$.

Thus if (24') evaluated at $\kappa_\tau=0$ has a unique solution it is $\tau>\gamma_u+\gamma_p$

If $\frac{\partial I_1^r}{\partial \tau}<0$, then the LHS of (24') is negative. In this case, if (24') evaluated at $\kappa_\tau=0$ has a unique solution it is $\tau<\gamma_u+\gamma_p$. The proof is the same as in Proposition 3 (see Appendix A.1.5).

A.2.3 Proof of Proposition 7

If $\gamma_p>0$, the first order condition of the planner's problem with respect to $\bar{e}$ is given by:

$$\begin{aligned}-u'(A_0^b-\bar{e})+\left(\rho-\mu-(\gamma_u+\gamma_p)\frac{\partial E(X^r,I_1^r)}{\partial I_1^r}-\frac{\partial c_x(X^r,I_1^r)}{\partial X^r}\right)\frac{\partial I_1^r}{\partial \bar{e}}\\ -(\gamma_u+\gamma_p-\tau)\frac{\partial E(X^r,I_1^r)}{\partial X^r}\frac{\partial X^r}{\partial \bar{e}}+1+\kappa_r=0\end{aligned} \tag{28}$$

where we use the fact that $\tau \frac{\partial E(X^{ce}, I_0)}{\partial X^{ce}} + \frac{\partial c_x(X^{ce}, I_0)}{\partial X^{ce}} = 0$ by (9).

As in the benchmark model total differentiation of (9) yields:

$$\frac{\partial X^r}{\partial \tau} = -\frac{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial (X^r)^2}} \frac{dI_1^r}{d\tau} \tag{16}$$

If the financial constraint is slack $\lambda^r(\tau, \bar{e}) = 0$, then $\frac{\partial I_1^r}{\partial \bar{e}} = 0$, in this case the FOC of the planner simplifies to:

$$u'(A_0^b - \bar{e}) - \kappa_r = 1 \tag{29}$$

In this case, the leverage mandate corresponds to the privately optimal equity level, so the economy is constrained efficient.

If the financial constraint is binding $\lambda^r(\tau, \bar{e}) > 0$, then liquidations are pinned down by (11). By total differentiation of this equation we get:

$$\left(\rho(1-\theta) - \mu - (\tau + \gamma_p)\frac{\partial E(X^r, I_1^r)}{\partial I_1^r} - \frac{\partial c_x(X^r, I_1^r)}{\partial I_1^r}\right) \frac{\partial I_1^r}{\partial \bar{e}} + 1 - \gamma_p \frac{\partial E(X^r, I_1^r)}{\partial X^r} \frac{\partial X^r}{\partial \bar{e}} = 0 \tag{30}$$

where we use the fact that $\tau \frac{\partial E(X^{ce}, I_0)}{\partial X^{ce}} + \frac{\partial c_x(X^{ce}, I_0)}{\partial X^{ce}} = 0$ by (9).

Using the definition of $\frac{\partial X^r}{\partial \bar{e}}$ we find:

$$\frac{\partial I_1^r}{\partial \bar{e}} = \frac{-1}{\tilde{r}(\tau, \bar{e}) - \gamma_p \left(\frac{\partial E(X^r, I_1^r)}{\partial I_1^r} - \frac{\partial E(X^r, I_1^r)}{\partial X^r} \frac{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r, I_1^r)}{\partial (X^r)^2}} \right)} \tag{31}$$

If condition (27) is satisfied, then $\frac{\partial I_1^r}{\partial \bar{e}} > 0$.

Using this in the FOC of the social planner with respect to $\bar{e}$, gives:

$$u'(A_0^b - \bar{e}) - \kappa_e = 1 + \lambda^{SP} \tag{32}$$

where λ^{SP} is defined as:

$$\lambda^{SP} = -\frac{r(\tau,\bar{e}) + (\tau-\gamma_u-\gamma_p)\left(\frac{\partial E(X^r,I_1^r)}{\partial I_1^r} - \frac{\partial E(X^r,I_1^r)}{\partial X^r}\frac{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial (X^r)^2}}\right)}{\tilde{r}(\tau,\bar{e}) - \gamma_p\left(\frac{\partial E(X^r,I_1^r)}{\partial I_1^r} - \frac{\partial E(X^r,I_1^r)}{\partial X^r}\frac{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial (X^r)^2}}\right)} \tag{33}$$

Recall, that private FOC with respect to e is:

$$u'(A_0^b - \bar{e}) - \kappa_e = 1 + \lambda^{ce}$$

To compare the privately optimal equity level to the optimal leverage mandate set by the planner, we define the wedge between λ^{SP} and that the λ^{ce} of the private problem (in the interior solution).

$$L(\tau,\bar{e}) = \lambda^{SP} - \lambda^{ce} = \frac{-r(\tau,\bar{e}) - Z(X^r,I_1^r)(\tau-\gamma_u-\gamma_p)}{\tilde{r}(\tau,\bar{e}) - \gamma_p Z(X^r,I_1^r)} + \frac{r(\tau,\bar{e})}{\tilde{r}(\tau,\bar{e})} \tag{34}$$

Where we defined $Z(X^r,I_1^r) = \left(\frac{\partial E(X^r,I_1^r)}{\partial I_1^r} - \frac{\partial E(X^r,I_1^r)}{\partial X^r}\frac{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial X^r \partial I_1^r}}{\frac{\partial^2 NBE(X^r,I_1^r)}{\partial (X^r)^2}}\right)$ to simplify the exposition. Rearranging (34) yields:

$$\frac{-Z(X^r,I_1^r)(\tau-\gamma_u-\gamma_p)\tilde{r}(\tau,\bar{e}) - r(\tau,\bar{e})\gamma_p Z(X^r,I_1^r)}{\left(\tilde{r}(\tau,\bar{e}) - \gamma_p Z(X^r,I_1^r)\right)\tilde{r}(\tau,\bar{e})} \tag{35}$$

Notice that the denominator of (35) is positive if condition (27) is satisfied. The numerator evaluated at $\tau = \gamma_u + \gamma_p$, is positive if $Z(X^r,I_1^r) < 0$. This implies that when $\tau = \gamma_u + \gamma_p$ planner's λ^{SP} is larger than that of the borrower, λ^{ce},. Thus, the planner sets a leverage mandate above the borrower's choice of equity: $\bar{e}^* > e^{ce}$.

Notice that the first term of the numerator $-Z(X^r,I_1^r)(\tau-\gamma_u-\gamma_p)\tilde{r}(\tau,\bar{e})$ is negative for $\tau > \gamma_u + \gamma_p$ and positive for $\tau < \gamma_u + \gamma_p$. Moreover, since $\frac{\partial r(\tau,\bar{e})}{\partial \tau} < 0$, there exists a threshold τ' defined in $r(\tau', e^{ce}) = 0$, such that if $\tau < \tau'$ then $r(\tau, e^{ce}) > 0$ and if $\tau > \tau'$ then $r(\tau, e^{ce}) > 0$. Thus, $r(\tau, e^{ce})\gamma_p Z(X^r,I_1^r) > 0$ if and only if $\tau < \tau'$. Therefore, there exists a level of emissions tax: $\tau' > \tau > \gamma_u + \gamma_p$ such that $\lambda^{SP} = \lambda^{ce}$ and $\bar{e} = e^{ce}$.

Thus there exists a threshold $\tau' > \hat{\tau} > \gamma_u + \gamma_p$ such that if $\tau < \hat{\tau}$ then the optimal leverage mandate is above the borrower's choice of equity: $\bar{e}^* > e^{ce}$.

B Extensions and Additional Results

B.1 Carbon Taxes Junior to Debt

In this appendix we derive the borrower's first order condition with respect to abatement X for the case in which carbon taxes are junior to debt. In this case $\tau E(X, I_1)$ does not enter the financial constraint (5) and the constraint becomes

$$d_1 \leq \tilde{R}(I_1, E^a).$$

The borrower's problem (7) is given by

$$\begin{aligned}\max_{X,I_1,d_1,e} \mathcal{L} = &\, u(A_0^b - e) - I_0(1-\mu) + e + R(I_1, E^a) - \mu I_1 - c_x(X, I_1) - \tau E(X, I_1) + T^b \\ &+ \lambda\left[\tilde{R}(I_1, E^a) - d_1\right] + \underline{\kappa}_I I_1 + \overline{\kappa}_I [I_0 - I_1] \\ &+ \kappa_{c_1}\left[d_1 - I_0(1-\mu) - I_1\mu + e - c_x(X, I_1)\right] + \kappa_e[e - (1-\mu)I_0].\end{aligned}$$

The first order conditions with respect to X, (9) is now

$$\tau \frac{\partial E(X, I_1)}{\partial X} + (1+\lambda)\frac{\partial c_x(X, I_1)}{\partial X} = 0.$$

Importantly, λ does not drop out from the equation in this case, which implies that in this case the tightness of the financial constraint has a direct effect on abatement.

B.2 Cap-and-trade with Free Permits

B.2.1 Optimal Price of Permits

The first order conditions of the social planner are given by Equations (13) and (14). To find the effect of τ on liquidations we totally differentiate the complementary slackness constraint which in the case of cap-and-trade framework reads as:

$$\lambda[(\rho(1-\theta)-\mu)I_1 - I_0(1-\mu) - c_x(X,I_1) + e] = 0 \tag{36}$$

By total differentiation we get:

$$\left(\rho(1-\theta)-\mu-\frac{\partial c_x(X,I_1)}{\partial I_1}\right)\frac{\partial I_1}{\partial \tau} - \frac{\partial c_x(X,I_1)}{\partial X}\frac{\partial X}{\partial p} = 0 \tag{37}$$

Using the expression for $\frac{\partial X}{\partial \tau}$ from Lemma 4 this simplifies to

$$\left(\rho(1-\theta)-\mu-\frac{\partial c_x(X,I_1)}{\partial I_1}\right)\frac{\partial I_1}{\partial p} - \frac{\partial c_x(X,I_1)}{\partial X}\frac{\frac{\partial E(X,I_1)}{\partial X} - \frac{\partial^2 NBE(X,I_1,p)}{\partial X \partial I_1}\frac{\partial I_1}{\partial p}}{\frac{\partial^2 NBE(X,I_1,p)}{\partial X^2}} = 0$$

$$\frac{\partial I_1}{\partial p} = \frac{\frac{\partial E(X,I_1)}{\partial X}\frac{\partial c_x(X,I_1)}{\partial X}}{\tilde{R}-\mu-\frac{\partial c_x(X,I_1)}{\partial I_1}+\frac{\partial c_x(X,I_1)}{\partial X}\frac{\frac{\partial^2 NBE(X,I_1,p)}{\partial X \partial I_1}}{\frac{\partial^2 NBE(X,I_1,p)}{\partial X^2}}}\frac{1}{\frac{\partial^2 NBE(X,I_1,p)}{\partial X^2}}$$

Using the FOC of the borrower with respect to X, this can be represented as:

$$\frac{\partial I_1}{\partial p} = \frac{\frac{\partial E(X,I_1)}{\partial X}\frac{\partial c_x(X,I_1)}{\partial X}}{\tilde{r}(p,\bar{e})+p\left(\frac{\partial E(X,I_1)}{\partial I_1}-\frac{\partial E(X,I_1)}{\partial X}\frac{\frac{\partial^2 NBE(X,I_1,p)}{\partial X \partial I_1}}{\frac{\partial^2 NBE(X,I_1,p)}{\partial X^2}}\right)}\frac{1}{\frac{\partial^2 NBE(X,I_1,p)}{\partial X^2}} = \frac{\frac{\partial E(X,I_1)}{\partial X}\frac{\partial c_x(X,I_1)}{\partial X}}{\tilde{r}(p)+pZ(p,e)}\frac{1}{\frac{\partial^2 NBE(X,I_1,p)}{\partial X^2}}$$

The problem of the planner is equivalent to that discussed in the benchmark case if $\frac{\partial I_1}{\partial p} < 0$, which holds whenever $\tilde{r}(p,\bar{e})+pZ(p,\bar{e}) < 0$. In this case the planner sets $p^* < \gamma$, by the same argument as the one used to prove that planner sets $\tau^* < \gamma$ in the benchmark model.

B.2.2 Optimal Capital Mandate

In a setting with free emission permits, the impact of equity on liquidations can be found by total differentiation of the adjusted complementary slackness constraint in Equation

36, which yields:

$$\left(\rho(1-\theta)-\mu-\frac{\partial c_x(X,I_1)}{\partial I_1}\right)\frac{\partial I_1}{\partial e}-\frac{\partial c_x(X,I_1)}{\partial X}\frac{\partial X}{\partial e}+1=0 \tag{38}$$

$$\tag{39}$$

The sensitivity of abatement to equity is the same as in the benchmark setting and thus corresponds to the one given in Lemma 4. Using this the above can be rewritten as:

$$\left(\rho(1-\theta)-\mu-\frac{\partial c_x(X,I_1)}{\partial I_1}+\frac{\partial c_x(X,I_1)}{\partial X}\frac{\frac{\partial^2 NBE(X,I_1,p)}{\partial X\partial I_1}}{\frac{\partial^2 NBE(X,I_1,p)}{\partial X^2}}\right)\frac{\partial I_1}{\partial e}+1=0$$

Using the FOC of the borrower with respect to X, this can be represented as:

$$\frac{\partial I_1}{\partial e}=\frac{-1}{\tilde{r}(p)+p\left(\frac{\partial E(X,I_1)}{\partial I_1}-\frac{\partial E(X,I_1)}{\partial X}\frac{\frac{\partial^2 NBE(X,I_1,p)}{\partial X\partial I_1}}{\frac{\partial^2 NBE(X,I_1,p)}{\partial X^2}}\right)}=\frac{-1}{\tilde{r}(p)+pZ(p,e)} \tag{40}$$

Thus, the first order condition of the planner with respect to e can be rewritten as:

$$u'(A-\bar{e})-\kappa_e=1\underbrace{-\frac{r(p,\bar{e})}{\tilde{r}(p,\bar{e})+pZ(p,\bar{e})}+\frac{(\gamma-p)}{\tilde{r}(p,\bar{e})+pZ(p,\bar{e})}Z(p,\bar{e})}_{=\lambda^{SP}} \tag{41}$$

To see whether the planner prefers a higher or lower equity than the private agent, we compare λ^{SP} with λ^b:

$$\begin{aligned}\lambda^{SP}-\lambda^b&=\frac{-r(p,\bar{e})+(\gamma-p)Z(p,\bar{e})}{\tilde{r}(p,\bar{e})+pZ(p,\bar{e})}+\frac{r(p,\bar{e})}{\tilde{r}(p,\bar{e})}\\&=\frac{-r(p,\bar{e})\tilde{r}(p,\bar{e})+(\gamma-p)Z(p,\bar{e})\tilde{r}(p,\bar{e})+r(p,\bar{e})r(p.\bar{e})+r(p,\bar{e})pZ(p,\bar{e})}{(\tilde{r}(p,\bar{e})+pZ(p,\bar{e}))r(p,\bar{e})}\\&=Z(p,\bar{e})\frac{r(p,\bar{e})p+(\gamma-p)\tilde{r}(p,\bar{e})}{(\tilde{r}(p,\bar{e})+pZ(p,\bar{e}))r(p,\bar{e})}\end{aligned}$$

Generally, the private agent prefers a different level of equity than the socially optimal level.

If $\tilde{r}(p,\bar{e})+\tau Z(p,\bar{e})<0$, then the denominator of the fraction is positive. It also implies that in equilibrium $p^*<\gamma$. If $Z(p^*,\bar{e})\left(r(p^*,\bar{e})p^*+(\gamma-p^*)\tilde{r}(p^*,\bar{e})\right)>0$ then

$\lambda^{SP} - \lambda^b > 0$ and the planner chooses equity above the privately optimal level $\bar{e}^* > e^{ce}$. If $Z(p^*, \bar{e})\left(r(p^*, \bar{e})p^* + (\gamma - p^*)\tilde{r}(p^*)\right) < 0$ then $\lambda^{SP} - \lambda^b < 0$ and the planner chooses equity above the privately optimal level $\bar{e}^* < e^{ce}$.

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