Hybrid Marketplaces with Free Entry of Sellers

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Abstract

We study a hybrid marketplace such as Amazon selling its own products and setting a commission rate on revenues of sellers engaged in monopolistic competition with free entry. For a large class of microfoundations based on a representative agent, the introduction of products by the marketplace is neutral on consumer welfare for a given commission, but exerts an ambiguous impact through its changes: a "demand substitution mechanism" pushes for a higher commission, but an "extensive margin mechanism" pushes for a lower commission aimed at attracting new sellers and more purchases on the marketplace. With constant demand elasticities, a hybrid marketplace sets lower (higher) commissions and increases (decreases) consumer welfare compared to a pure marketplace if its products face a less (more) elastic demand. We extend the analysis to alternative timing, Bertrand competition between sellers, endogenous product selection by the marketplace, specific commissions and advertising for product discovery.

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1 Introduction

A hybrid marketplace is monetized through percentage commissions on third party sales and through direct sales of its own products and services. In the case of Amazon, as well as other prominent platforms (as the app stores of Apple and Google), this double role as "umpire and player" has been at the center of a lively debate under the presumption that a hybrid marketplace would systematically promote its own products or increase commissions on third party ones to favor its own sales, and this may harm consumers in the long run (as in the New Brandesian view of Khan, 2016). In this work we ask whether this presumption is consistent with the endogenous market structure emerging on a hybrid marketplace open to third party sellers. Contrary to the common presumption, we show that the introduction of own products can actually increase both consumer welfare and total welfare through a reduction of commissions on sellers which reduces all prices and expands gains from variety.

Recent works by Anderson and Bedre-Defloie (AB, 2021) and Zennyo (2020) have introduced tractable frameworks based on a Logit model of product differentiation to address these issues. In both these works, a hybrid marketplace acts as a Stackelberg leader in selecting commissions and prices, and faces endogenous entry of third party sellers engaged in monopolistic competition. Zennyo (2020), adopting a commission on units sold, shows that a hybrid marketplace should be neutral on commissions and consumer welfare, while AB (2021), adopting a percentage commission on revenues (the empirically relevant case), argue that a hybrid marketplace sets excessive commission rates to shift demand toward its own products, which reduces consumer welfare. We unveil the nature of this apparent contradiction by developing an alternative microfoundation of demand systems which nests the Logit demand system and an entire class of alternative ones, and we derive conditions under which a hybrid marketplace can either increase or decrease welfare compared to a pure marketplace.

More formally, we adopt a representative agent framework based on a quasi-linear indirect utility depending on additive aggregators of the prices of all the products sold on the marketplace. The sellers are engaged in monopolistic competition with free entry.[2] For a given commission rate, the entry and pricing strategies of the marketplace are neutral on consumer welfare, a consequence of an old result applying to aggregative games with free entry.[3] The only impact of the introduction of products by the marketplace on consumer welfare occurs through a change in the commission set on sellers: when this is increased, con-

[2]Representative agent models of monopolistic competition with indirect additivity were introduced in Bertoletti and Etro (2017). It should be emphasized that AB (2021) and Zennyo (2020) rely on discrete choice models with random utility augmented respectively with random search costs for consumers and a consideration set depending on consumers' search efforts. For empirical applications of related models on Amazon see Lee and Musolff (2021) and Gutierrez (2021).

[3]This neutrality applies in any aggregative game where symmetric profit functions depend on a sum of functions of the strategies of the rivals. For related applications see Etro (2008, 2011), Ino and Matsumura (2012), Anderson *et al.* (2020), Alfaro (2020) and Alfaro and Lander (2021).

sumers are harmed through higher prices and lower gains from variety, while a reduction of the commission reduces prices and expands the gains from variety, increasing consumer welfare as well as total welfare. In general, and this is our novel result, the introduction of products by the hybrid marketplace exerts two effects: on one side, there is an incentive for the marketplace to increase the commission and shift demand toward its own products, but, on the other side, there is an incentive to reduce the commission to attract new sellers collecting more commission revenues on the extensive margin and to expand purchases by customers on the marketplace. One can regard the former as a "*demand substitution mechanism*" which harms consumers biasing their purchases toward the marketplace's products, and the latter as an "*extensive margin mechanism*" which benefits consumers expanding purchases of all products.

Under additional restrictions on the microfoundation we can obtain more precise results on the conditions under which each effect is dominant. The case of loglinear preferences in a price aggregator *à la* Nocke and Schutz (2018) is interesting because it delivers a variety of standard demand systems including isoelastic, Logit, linear demand systems and others. In the prominent case of demands with constant elasticity (which implies constant markups and full cost pass-through), if the marketplace faces the same demand elasticity as the sellers, its products are introduced at the same price and commission rates are left unchanged. If the marketplace faces a less elastic demand than the sellers, it reduces the commission on third party sales after introducing its own products because the extensive margin mechanism is dominant. Instead, under a Logit demand system the demand substitution effect is dominant, which is consistent with findings by AB (2021). Examples based on translated power surplus functions can give raise to ambiguous results.

We extend the analysis to more general microfoundations, confirming the results above for constant demand elasticities and showing an additional tendency for a hybrid marketplace to reduce commissions when its aggregate demand is highly elastic with respect to the quality of the marketplace. All the main findings are also robust when there is no commitment by the marketplace on its prices (which actually strengthens the extensive margin mechanism), with Bertrand competition between sellers, which is relevant when few of them are active in the same product category (and increases seller's markups), and with endogenous product introduction by the marketplace (which may not even take place when it increases commissions). We also consider the case of commissions on units sold: again the two effects are present and either can dominate, but now they compensate each other in the Logit case, which is consistent with the analysis of Zennyo (2020). Finally, we discuss the role of ads for product discovery that are purchased by sellers to attract "clicks" and sales to their own products: in this case the marketplace selects the ratio between ad revenues and commission revenues taking into account the substitutability between the two sources of monetization, independently from the introduction of its own products, which has always an ambiguous impact on the total fees paid by the sellers.

Our findings suggest that the presumption that a hybrid marketplace such as Amazon tends to favour its own products through worse conditions or higher commissions for third party sellers lacks a solid foundation in this class of models. In practice, Amazon sets different commission rates across wide product categories, and these rates have been quite stable over time, also after the introduction of private labels or directly retailed products by Amazon. Public data from Amazon in the US reveal that the commission rates between 2017 and 2021 have been constant at 8% for consumer electronics, cameras, cell phone devices and video game consoles, 12% for industrial & scientific products including janitorial & sanitation, 15% for books, mattresses, kitchen, home & garden, office products, music, sports, toys and much more, and 45% for Amazon device accessories. Changes in commission rates during the last five years have been reductions (below a price threshold) from 15% to 8% for baby products, beauty and health & personal care, from 20% to 15% for sports collectibles, and (above a price threshold) from 15% to 10% for the category furniture & decor and from 20% to 5% for jewelry, with the only increase being from 15% to 17% for clothing & accessories and from 6% to 8% for personal computers.[4] Remarkably, private label products had been introduced for the categories with unchanged commissions, as well as the categories with a reduction of commissions and clothing & accessories, but not for personal computers. Such a state of affairs, broadly confirmed in other countries, does not appear consistent with the thesis that the introduction of Amazon's products has generated systematic harm to consumers through higher commissions.

Literature review This work is related to the literature on platforms with competing sellers (Hagiu, 2009; Belleflamme and Toulemonde, 2016; Belleflamme and Peitz, 2019; Teh, 2020; Zennyo, 2021; Bisceglia *et al.*, 2021; Bertoletti, 2021; Jeon and Rey, 2021) and especially to the expanding literature on online marketplaces, which includes also Hagiu *et al.* (2020), Tremblay (2020), Zennyo (2020), AB (2021), Kittaka and Sato (2021), Lam and Liu (2021), Hervas-Drane and Shelegia (2021), Masden and Vellodi (2021), Ronayne and Taylor (2021) and others. A common theme emerging in this literature, and confirmed in the present work, is that the business model of an online marketplace, based on monetization of all products on the platform including those of third party sellers through commissions, is a key factor that disciplines the incentives to introduce, price and promote its own products.[5] In a static perspective, it has been emphasized that entry by the marketplace tends to materialize in case of cost efficiencies or demand advantages that benefit also consumers (Hagiu and Wright, 2015; Etro, 2021a; Hervas-Drane and Shelegia, 2021). In

[4] There was also a momentarily increase from 15% to 18% for the commission on shoes, handbags & sunglasses in 2018, but the commission was decreased to its original level of 15% in 2020. Note that while referral fees have remained mostly constant across time, other fees, as those for Fulfilment By Amazon, have been increasing. Those fees are not central to our arguments.

[5] Recent evidence that Amazon better internalizes the interest of consumers in setting prices of its own products compared to third party sellers is in Cabral and Xu (2021), who study prices of face masks and hand sanitizers at the beginning of the pandemic phase. For an early analysis of how business models affect the incentives of digital platforms see Caffarra (2019).

a dynamic perspective, it has been emphasized that even when imitative entry by the marketplace disincentivizes investment by sellers, there is an incentive to commit to a limited copycat activity internalizing the impact on future product creation for the same marketplace, which creates benefits also for consumers (Jiang *et al.*, 2011; Etro, 2021a; Masden and Vellodi, 2021).

In a more general perspective, Hagiu *et al.* (2020) have shown that hybrid marketplaces create gains for consumers through more competition on the platform, but could raise concerns related to self-preferencing and, in the absence of commitment policies, excessive imitation of sellers. While their framework is centred on search by consumers across products by sellers with market power and competitive fringes of rivals active also through a direct channel, our framework is centred on free entry of monopolistically competitive sellers providing differentiated goods only on the marketplace. A common conclusion with our work is that a hybrid marketplace such as Amazon can create benefits for consumers, and potential concerns should be addressed by antitrust policy through behavioral remedies (and not structural ones).

Our result that hybrid marketplaces can reduce commissions with benefits for customers through gains from variety resonates well with recent work by Shopova (2021) in a vertical differentiation framework. She shows that a marketplace has an incentive to introduce low quality private labels and reduce commissions on sellers of high quality rival goods, generating always an increase in consumer welfare. The intuition in her case is that the marketplace introduces an additional variety and reduces commissions because it internalizes the lower demand of the sellers and the higher pass-through on their prices.

Finally, our welfare analysis is also related to the theoretical literature on market competition with free entry (see Spence, 1976, Dixit and Stiglitz, 1977, Bertoletti and Etro, 2016, 2017) and Stackelberg leadership in aggregative games with free entry (Etro, 2008; Tesoriere, 2008; Ino and Matsumura, 2012; Anderson *et al.*, 2020; Alfaro, 2020; Alfaro and Lander, 2021) and to the empirical literature (see Berry and Waldfogel, 1999, and Dutta, 2011). In particular, Lee and Musolff (2021) have recently provided an empirical analysis of self-preferencing by Amazon in a nested Logit framework with free entry of heterogeneous sellers, and their results suggest that practices adopted by Amazon have not harmed consumers, even without accounting for endogenous commissions by the marketplace.[6] More broadly, our work provides a framework that can be used to explore how policy commitments affect sellers active in a market with free entry and what is their impact on consumer welfare, an issue emerging in various fields, including industrial and trade policy.

The work is organized as follows. Section 2 presents the structure of the benchmark model. Section 3 derives the key results on the impact of percentage commissions on sellers. Section 4 discusses extensions. Section 5 concludes.

[6] Gutierrez (2021) has provided an empirical analysis of vertical integration by Amazon (in the spirit of the work by Crawford *et al.*, 2018) in a nested Logit framework with endogenous commissons by the marketplace, and his results suggest that consumer welfare is lower in a pure marketplace compared to a hybrid one, even without accounting for endogenous entry of sellers.

2 The Model

Let us consider a hybrid marketplace offering $n > 0$ products, of which $m \in [0, n)$ directly provided by the same marketplace and the remaining ones provided by third party sellers engaged in monopolistic competition.[7] As in standard models of monopolistic competition in partial equilibrium *à la* Spence (1976) we adopt quasilinear preferences for a representative customer of the marketplace. We express preferences through an indirect utility that is a convex function of the price vector $\mathbf{p}$ of all products sold on the marketplace:

$$V = G(D(\mathbf{p})) + E \tag{1}$$

where $D(\mathbf{p})$ is a price aggregator representing the quality of the marketplace, $G(D)$ is an increasing and concave transformation and E is expenditure, assumed large enough to allow purchases of an outside *numeraire* good. The aggregator is assumed indirectly additive (IA) in the prices of the products as in:[8]

$$D(\mathbf{p}) \equiv \sum_{j=1}^{n} v_j(p_j) + H \tag{2}$$

where each product $j = 1, 2, .., m$ by the marketplace generates an incremental surplus function $v_j(\bar{p}_j)$ assumed positive, decreasing and convex in the price $\bar{p}_j$ of product j, each product by seller $j = m+1, ..., n$ generates a common surplus function $v(p_j)$ which is also positive, decreasing and convex in the price p_j, and $H \geqslant 0$ is a constant reflecting an exogenous surplus obtained from the marketplace (or from other goods purchased by consumers outside the marketplace).[9]

Applying the Roy's identity to these quasilinear IA preferences, each seller i faces the direct demand:

$$q_i(p_i) = |v_i'(p_i)|\, G'(D(\mathbf{p})) \tag{3}$$

which emphasizes that the additive aggregator crucially determines both welfare and the demand system. We can illustrate this microfoundation with two relevant cases that will be widely employed in the rest of the analysis.

2.1 Loglinear preferences

In case of a logarithmic transformation:

$$G(D) = \log D \tag{4}$$

[7] Amazon intermediates about 44% of e-commerce sales in the U.S., and the majority of these sales are by third party sellers hosted on the platform, with 36% sales by Amazon as a first party retailer and 5% sales through private labels by Amazon in 2020 (see, for instance, Gutierrez, 2021).

[8] Monopolistic competition under indirect additivity is analyzed in Bertoletti and Etro (2017), and under quasi-linearity also in Nocke and Schutz (2018).

[9] We could obtain analogous results adopting a quasilinear direct utility that is a function of an aggregator of quantities as in Spence (1976). But notice that the underlying preferences and demand systems are not overlapping (unless the monotonic transformation is linear).

we obtain the loglinear preferences employed by Nocke and Schutz (2018) to study multiproduct pricing with imperfect substitutability. They deliver the demand functions:

$$q_i(p_i) = \frac{|v_i'(p_i)|}{\sum_j v_j(p_j) + H} \tag{5}$$

which are clearly decreasing in the aggregator. In the particular specification with exponential subutilities $v_j(p) = e^{-p/\mu}$ where $\mu > 0$ determines product differentiation, the demand function becomes $q_i(p_i) = e^{-p_i/\mu}/\mu(\sum_j e^{-p_j/\mu} + H)$, and the model is isomorphic to one based on a Logit foundation (Zennyo, 2021; AB, 2021). Of course, different subutilities would deliver different demand systems, determining the perceived demand elasticities of each product.[10]

2.2 Isoelastic preferences

A slightly more general monotonic transformation of the price aggregator involves the power function:

$$G(D) = \frac{D^\kappa}{\kappa} \quad \text{with } \kappa \in (0, 1) \tag{6}$$

which provides the demand functions:

$$q_i(p_i) = \frac{|v_i'(p_i)|}{\left[\sum_j v_j(p_j) + H\right]^{1-\kappa}} \tag{7}$$

Keeping in mind that the aggregator at the denominator measures the quality of the marketplace (relative to the outside good), the parameter κ governs the elasticity of consumer utility with respect to the aggregator and also the elasticity of the demand of each product with respect to the aggregator: for $\kappa \to 0$ we approach the loglinear case $G(D) = \log D$ with demand functions that are highly elastic with respect to the aggregator and each other price, while for $\kappa \to 1$ we approach the linear preferences $G(D) = D$ with demand functions that are inelastic with respect to the aggregator and unrelated to the price of the other products. Loosely speaking, a higher κ implies that the products of the marketplace are less substitutable between themselves in generating utility, though they are jointly more substitutable with respect to the outside good.[11]

This framework is also compatible with an alternative microfoundation based on competition for heterogeneous customers (Etro, 2021a) or two-sidedness of the market (AB, 2021). Consider a unit mass of consumers with loglinear preferences, so that the individual demand conditional on purchasing from the marketplace is the same as in (5), but consumers are heterogeneous in the price

[10] On the correspondence between discrete choice models and representative agent models based on indirectly additive aggregators see Thisse and Ushchev (2016).

[11] It can be verified that the cross price elasticity between the products of the marketplace is decreasing in κ, while outside substitutability (a measure of how much expenditure on the marketplace gets lost toward the outside commodity when all its prices increase proportionally) is increasing in κ (see Bertoletti, 2018).

aggregator obtained from alternative outlets $\tilde{D}$, which is drawn from a Pareto distribution $(\tilde{D}/\bar{D})^{\kappa}$ with some upperbound $\bar{D}$. Then, the marketplace attracts a fraction of consumers $(D(\mathbf{p})/\bar{D})^{\kappa}$, generating the same aggregate demand as in (7) up to an exogenous constant. In such a case we can interpret the parameter κ as the elasticity of consumers's entry on the marketplace with respect to its quality: for $\kappa \to 0$ entry is inelastic and all consumers purchase from the marketplace independently from alternatives, and for $\kappa \to 1$ the outside option is uniformly distributed and entry of consumers is instead highly elastic. All these microfoundations converge toward interpreting an increase of κ as an increase of the elasticity of purchases on the marketplace with respect to its quality (due to substitutability with outside alternatives).

2.3 Surplus functions

In the monopolistic competition framework adopted here, sellers facing the general demand function (3) set prices taking as given the price aggregator, therefore what matters for pricing is the perceived demand elasticity, that is the elasticity of the $v'(p)$ function. The underlying subutility function $v(p)$ determines the additional surplus obtained by consumers in function of the price, therefore its shape determines both the surplus elasticity $\zeta(p) \equiv -\frac{v'(p)p}{v(p)}$ and the elasticity of demand $\varepsilon(p) \equiv -\frac{v''(p)p}{v'(p)}$, both of which are positive under our assumptions.

A classic specification is based on power functions:

$$v(p) = p^{1-\varepsilon} \tag{8}$$

with $\varepsilon > 1$, and delivers an isoelastic demand function, with a demand elasticity $\varepsilon(p) = \varepsilon$ and a surplus elasticity $\zeta(p) = \varepsilon - 1$. In case of exponential functions:

$$v(p) = e^{-\frac{p}{\mu}} \tag{9}$$

with $\mu > 0$ parametrizing product differentiation, the demand function becomes loglinear, with the two elasticities equal to $\varepsilon(p) = \zeta(p) = \frac{p}{\mu}$ and increasing in the price. Another useful case is based on the translated power surplus:

$$v(p) = \frac{(a-p)^{1+\gamma}}{1+\gamma} \tag{10}$$

where $a > 0$ and $\gamma > 0$ parameterize willingness to pay and shape of demand (nesting linear, perfectly rigid and elastic demand), with elasticities $\varepsilon(p)/\zeta(p) = \gamma/(1+\gamma)$. We will repeatedly use these specifications for illustrative purposes.

It is easy to compute that the surplus elasticity changes with the price according to $\zeta'(p) = \frac{\zeta(p)}{p}\left[1 + \zeta(p) - \varepsilon(p)\right]$, and therefore it can be either constant in the price (under power functions) or variable (as in the other examples) depending on its relation with the demand elasticity, which can also be either constant or variable in the prices.

2.4 Technology and timing

Each seller bears a fixed cost of entry $f > 0$. The marketplace provides a good j at marginal cost $\bar{c}_j \geqslant 0$ setting the price $\bar{p}_j$, while any third party seller i provides its good at a common marginal cost $c \geqslant 0$ setting the price p_i under monopolistic competition. The revenues of the sellers are subject to a uniform percentage commission at rate $\tau \in [0, 1]$ paid to the marketplace. In Section 4 we will also extend the model to the case of Bertrand competition between sellers, to the case of specific rather than percentage commissions, and to an endogenous advertising choice by the sellers.

The timing of the benchmark game is the following: 1) the marketplace sets the uniform commission rate on third party sellers; 2) the marketplace sets the prices of its own products; 3) entry of sellers takes place and 4) the sellers set their prices under monopolistic competition. This reflects the rather stable commitment of Amazon to its commission rates per product category, and its ability to introduce own products affecting entry of sellers. In Section 4 we will also extend the model with a different timing (with price decisions of the marketplace taking place in the last stage) and with a preliminary stage of product selection by the marketplace (when also the latter bears fixed costs of product introduction).

3 Equilibrium analysis

In this section we solve for the subgame perfect equilibrium of the benchmark model by backward induction. Our final aim is to compare the choices made by a hybrid marketplace offering multiple own products and a pure marketplace offering only products by third party sellers.

3.1 Pricing and entry of the sellers

Given the strategies of the marketplace and the number of sellers, each seller i sets the price p_i to maximize profits:

$$\pi(p_i) = [(1 - \tau)p_i - c] \left|v'(p_i)\right| G'(D(\mathbf{p})) - f \tag{11}$$

taking as given the price aggregator (2) under monopolistic competition. This provides a common price rule $p = p(\tau)$ for each product satisfying:

$$p = \frac{\varepsilon(p)c}{(\varepsilon(p) - 1)(1 - \tau)} \tag{12}$$

where the demand elasticity $\varepsilon(p)$ is now assumed larger than unity in equilibrium with a positive marginal cost (but approaching unity for zero marginal cost). The independence of pricing from the prices of the other products relies on the IA property of the price aggregator. The positive impact of the commission on the price depends on the shape of the demand function, and can be computed as $p'(\tau) = \eta(p(\tau))p(\tau)/(1 - \tau)$, where the pass-through elasticity of the price with respect to the marginal cost $\eta(p) \equiv \partial \ln p / \partial \ln c$ can be easily shown to be less (more) than unitary if $\varepsilon'(p)$ is positive (negative) as long as the marginal cost is positive (but approximately null for zero marginal cost).

For instance, under a power surplus function (8), we obtain the price rule:

$$p(\tau) = \frac{\varepsilon c}{(\varepsilon - 1)(1 - \tau)}$$

and the cost pass-through is full with $\eta(p(\tau)) = 1$. Instead, with the exponential subutility function (9), the price of the sellers is:

$$p(\tau) = \mu + \frac{c}{1 - \tau}$$

as in common Logit models, and the pass-through is incomplete with elasticity $\eta(p(\tau)) = 1/[1 + (1 - \tau)\frac{\mu}{c}]$. Finally, in case of translated power functions the price is:

$$p(\tau) = \frac{a + \frac{\gamma c}{1-\tau}}{1 + \gamma}$$

with incomplete pass-through and $\eta(p(\tau)) = 1/[1 + (1 - \tau)\frac{a}{\gamma c}]$.

Using the price rule, we can express the profits of each seller as a decreasing function of the commission rate $\pi(p(\tau))$, and this expression decreases also in the value of the price aggregator due to the concavity of $G(D)$. Given the commission and the prices of the products of the marketplace, entry of new third party sellers increases n and, therefore, the value of the price aggregator:

$$D(\mathbf{p}) = \sum_{j=1}^{m} v_j(\bar{p}_j) + (n - m)v(p(\tau)) + H \tag{13}$$

reducing the gross profits of each seller until they match the fixed cost f (focusing, of course, on cases where entry takes place). Accordingly, free entry pins down the equilibrium value of the aggregator as a function of the commission rate $D = D(\tau)$ such that:

$$[(1 - \tau)p(\tau) - c] \left|v'\left(p(\tau)\right)\right| G'(D) = f \tag{14}$$

The equilibrium aggregator is decreasing in the commission with derivative:

$$D'(\tau) = \frac{-\varepsilon(p(\tau))D(\tau)}{(1 - \tau)\sigma(D(\tau))} < 0 \tag{15}$$

where we introduced an index of curvature for the monotonic transformation $\sigma(D) \equiv \frac{-G''(D)D}{G'(D)} \geqslant 0$, which is actually constant under loglinear preferences ($\sigma(D) = 1$) and isoelastic preferences ($\sigma(D) = 1 - \kappa$).

An increase of the commission exerts a direct negative impact on the profits of the sellers (while the impact through prices is null by the envelope theorem), which reduces the value of the aggregator independently from the provision and the pricing of products by the marketplace. This implies that consumer welfare (which here is also "user welfare" due to zero profits of the sellers) amounts to $V = G(D(\tau)) + E$, independent from products and prices of the marketplace for a given commission. Given the generality of this result, we formalize it as follows:

Proposition 1. *Under monopolistic competition with free entry of sellers on a marketplace serving customers with quasilinear IA preferences, the introduction of products by the marketplace is neutral on consumer welfare for a given commission.*

To the extent that the marketplace is not changing commissions while introducing new products, there are no consequences on the prices of the sellers due to monopolistic competition and on the IA price aggregator due to free entry. Therefore, this framework, as already those of Zennyo (2020) and AB (2021), implies that the hybrid marketplace is neutral on consumer welfare, independently from the prices and the qualities of the products of the marketplace. The only impact of the introduction of new products is to crowd out some sellers and affect their number, which can be derived as follows:

$$n(\tau) - m = \frac{D(\tau) - \sum_{j=1}^{m} v_j(\bar{p}_j) - H}{v(p(\tau))} \tag{16}$$

and is assumed positive in equilibrium to focus on interesting cases.[12] The introduction of a product generating lower (higher) surplus than a seller expands (reduces) the total number of products because it opens more space for entry of third party sellers, but with no ultimate impact on the aggregator. The neutrality on the aggregator and consumer welfare relies on a well known property of this class of aggregative games with a Stackelberg leader and endogenous entry of followers - for related statements of the neutrality property see Etro (2008, 2011) and Anderson *et al.* (2020).[13]

Accordingly, in the rest of the work we will examine the indirect impact that a hybrid marketplace has on welfare through changes of the commission. Before doing that, however, we need to understand how the marketplace is going to price its own products.

3.2 Pricing by the marketplace

Taking into account that third party products generate commission revenues and own products generate direct profits, we can rearrange the gross profits of

[12] Notice that a higher surplus from goods purchased outside the marketplace H reduces the total number of products sold on the marketplace because it reduces the effective demand of each product. Accordingly, we are implicitly assuming either that H or τ are not too high. As usual, we ignore the integer constraint on the number of sellers.

[13] The neutrality of the aggregator applies with any demand system based on a single symmetric aggregator (see Bertoletti and Etro, 2021, 2022), and also under Bertrand competition between sellers discussed in Section 4.2. Related applications are, for instance, in Etro (2011), Ino and Matsumura (2012), Cato and Oki (2012) and elsewhere. For an extension to strategies of the leader that affect the profitability of followers and therefore free entry see Alfaro (2020) and Alfaro and Lander (2021).

the marketplace using (16) as follows:

$$\begin{aligned}\Pi &= \left[\tau(n(\tau)-m)p(\tau)\left|v'\left(p(\tau)\right)\right| + \sum_{j=1}^{m}(\bar{p}_j-\bar{c}_j)\left|v_j'(\bar{p}_j)\right|\right]G'(D(\tau)) \quad (17)\\ &= \tau\zeta(p(\tau))[D(\tau)-H]G'(D(\tau)) + \Delta(\mathbf{p},\tau)G'(D(\tau))\end{aligned}$$

where we remind the reader that $\zeta(p)$ is the elasticity of the surplus function, which determines also the ratio of marginal revenue and surplus from third party products. We isolated in the first term the commission revenues of a pure marketplace and in the second term the profits generated by the products of the marketplace net of the lost commission revenues, and we defined:

$$\Delta(\mathbf{p},\tau) \equiv \sum_{j=1}^{m}\left[(\bar{p}_j-\bar{c}_j)\left|v_j'(\bar{p}_j)\right| - \tau\zeta(p(\tau))v_j(\bar{p}_j)\right] \quad (18)$$

as an index of differential profits between own and third party products. Such an index is corrected to internalize the impact of the products supplied by the marketplace on the (reduced) entry of third party sellers and therefore on the (lost) commission revenues. In particular, setting a higher price generates lower surplus from the product of the marketplace, which attracts a larger number of sellers and more commission revenues.

The platform selects the prices of its own products to maximize profits (17) taking as given the price aggregator $D(\tau)$, since this is expected to be constant under free entry for a given commission, but taking into account the opportunity cost of losing commission revenues on sellers's products, which, as mentioned, tends to increase the prices. Since only $\Delta(\tau)$ is affected by the prices of the marketplace, its maximization for any $j = 1, 2, ..., m$ provides rules $\bar{p}_j = \bar{p}_j(\tau)$ that, assuming an interior solution, satisfy:

$$\bar{p}_j = \frac{\varepsilon_j(\bar{p}_j)\bar{c}_j}{\varepsilon_j(\bar{p}_j) - 1 - \tau\zeta(p(\tau))} \quad (19)$$

where we defined $\varepsilon_j(p) \equiv -\frac{v_j''(p)p}{v_j'(p)}$ as the demand elasticity for the product j of the marketplace. Assuming that $\tau\zeta(p(\tau))$ increases in the commission, also the prices of the platforms's products increase in the commission, taking into account its impact on both commission revenues per seller and number of sellers.

An interesting case to focus on is the one of a good produced at the same cost and generating the same surplus whether it is supplied by the marketplace or by the seller. For such a good, marketplace and seller set the same price when the commission is null, but the comparison is ambiguous for a positive commission. In particular, the price of the marketplace is lower than the price of the seller if:

$$\bar{p} = \frac{\varepsilon(\bar{p})c}{\varepsilon(\bar{p}) - 1 - \tau\zeta(p)} \leqslant \frac{\varepsilon(p)c}{(\varepsilon(p)-1)(1-\tau)} = p$$

which, using the shape of the surplus function, is equivalent to:

$$\varepsilon(p) - \varepsilon(\bar{p}) \leqslant \tau \left[(\varepsilon(\bar{p}) - \varepsilon(p))(\varepsilon(p) - 1) - \varepsilon(p)\zeta'(p)p/\zeta(p)\right]$$

Given any positive commission, this condition is always satisfied as an inequality when $\varepsilon'(p), \zeta'(p) < 0$, is satisfied as an equality when $\varepsilon'(p) = \zeta'(p) = 0$, and is never satisfied when $\varepsilon'(p), \zeta'(p) > 0$. Indeed, in case of a power surplus function with a constand demand elasticity marketplace and seller set always the same markup, in case of exponential and translated power functions with increasing elasticities the marketplace sets a higher markups and the opposite outcome can emerge in other cases.[14] We summarize the essential finding as follows:

Proposition 2. *Under monopolistic competition with free entry of sellers on a marketplace serving customers with quasilinear IA preferences, a product with given cost and surplus function is sold at the same price for any commission rate by marketplace and sellers when they face the same constant demand elasticity, otherwise either the marketplace or the sellers can set a higher price.*

We can also consider asymmetric situations where the marketplace faces different demand and cost functions than the sellers building on earlier specifications. We will often consider marketplace's products with a surplus function $v_j(p) = z\bar{v}(p)$ and marginal cost $\bar{c}$, where the demand shift parameter $z > 0$ measures the intensity of demand for the product of the marketplace, but is neutral on pricing. First, let us consider a power surplus function:

$$\bar{v}(p) = p^{1-\bar{\varepsilon}} \tag{20}$$

where $\bar{\varepsilon} > 1$ represents the constant demand elasticity faced by the marketplace. This provides a price $\bar{p}(\tau) = \frac{\bar{\varepsilon}\bar{c}}{(\bar{\varepsilon}-1)(1-\tau)}$. If sellers face a power function (8), the markup is the same in case of a common elasticity ($\varepsilon = \bar{\varepsilon}$), otherwise the platform sets higher markups when facing a more rigid demand and lower markups when facing a more elastic demand than the sellers. Second, let us consider the Logit framework with the following exponential surplus function for the marketplace:

$$\bar{v}(p) = e^{-\frac{p}{\bar{\mu}}} \tag{21}$$

where the parameter $\bar{\mu} > 0$ refers to the marketplace's products. The prices of the marketplace can be computed as $\bar{p}(\tau) = \bar{\mu} + \bar{c} + \tau p(\tau)$, where the marginal cost is augmented by the opportunity cost of giving up to commission revenues, as in AB (2021).[15] Finally, translated power surplus functions for the marketplace as:

$$\bar{v}(p) = \frac{(a-p)^{1+\bar{\gamma}}}{1+\bar{\gamma}} \tag{22}$$

[14] For instance the surplus function $v(p) = (p+h)^{1-\varepsilon}$ implies $\varepsilon'(p), \zeta'(p) \lesseqgtr 0$ and $\bar{p}(\tau) \lesseqgtr p(\tau)$ if $h \lesseqgtr 0$.

[15] When the sellers face the function (9) and the same marginal cost, the marketplace sets a higher price than the sellers if $\bar{\mu} \geqslant \mu$, otherwise it sets a lower price for any $\tau < 1 - \bar{\mu}/\mu$ and a higher price for $\tau > 1 - \bar{\mu}/\mu$.

where $\bar{\gamma} > 0$ provide the price $\bar{p}(\tau) = \frac{a+\bar{\gamma}\bar{c}+a\tau\zeta}{1+\bar{\gamma}+\tau\zeta}$.

Given equilibrium pricing, we can now rewrite the index of differential profits (18) as a function of the commission only:

$$\Delta(\tau) = \sum_{j=1}^{m} v_j(\bar{p}_j(\tau)) \left[\frac{\zeta_j(\bar{p}_j(\tau)) \left[1 + \tau\zeta(p(\tau))\right]}{\varepsilon_j(\bar{p}_j(\tau))} - \tau\zeta(p(\tau)) \right] \tag{23}$$

where we defined the surplus elasticity for marketplace's products $\zeta_j(p) \equiv -\frac{v_j'(p)p}{v_j(p)}$, and we focus on cases where (23) is positive (otherwise there would be no reason to provide a hybrid marketplace). It is also useful to define the additional surplus from the marketplace's products as $\Psi(\tau) \equiv \sum_{j=1}^{m} v_j(\bar{p}_j(\tau))$. Notice that, by the envelop theorem, the impact of the commission on the index of differential profits can be computed from (18) as:

$$\begin{aligned} \Delta'(\tau) &= -\Psi(\tau)\left[\zeta(p(\tau)) + \tau\zeta'(p(\tau))p'(\tau)\right] = \\ &= -\Psi(\tau)\zeta(p(\tau))\left[1 + \frac{\tau}{1-\tau}\eta(p(\tau))\left[1 + \zeta(p(\tau)) - \varepsilon(p(\tau))\right]\right] \end{aligned} \tag{24}$$

where we used the slope of the surplus elasticity and the pass-through. We focus on cases where this expression is negative (as it is always for a low enough commission): since the products's prices are set to maximize the index of differential profits, the only impact of the commission is the direct impact on the lost revenues. The analysis simplifies further when we adopt a common surplus function for all the marketplace's products $z\bar{v}(p)$ and marginal cost $\bar{c}$, which still allows for differences from the sellers: we denote this as the case of homogeneous products of the marketplace: since all these products are now sold at the same price, we simplify $\Psi \equiv mz\bar{v}(\bar{p})$ and $\Delta = \Psi[\bar{\zeta}(\bar{p})\left[1 + \tau\zeta(p)\right]/\bar{\varepsilon}(\bar{p}) - \tau\zeta(p)]$, where upperbars identify the elasticities of the marketplace.

Finally, given the index (23) we express the profits of the marketplace as:

$$\Pi(\tau) = \left[\tau\zeta(p(\tau))\left(D(\tau) - H\right) + \Delta(\tau)\right] G'(D(\tau)) \tag{25}$$

which is a function of the commission rate only, and will be assumed concave in what follows.

3.3 Commissions by the marketplace

We now move to study of the commission set by the marketplace with the purpose of comparing the choices made by pure and hybrid marketplaces. Our previous analysis has shown that the commission rates are sufficient statistics for consumer welfare, therefore this comparison allows us to answer the question whether a hybrid platform harms consumers by setting worse conditions for the sellers or not. To build intuitions, we start by considering simpler classes of preferences, the logarithmic preferences of Nocke and Schutz (2018) based on (4) and then the case of isoelastic preferences based on (6). We conclude by presenting the general case.

3.3.1 Loglinear preferences

Under loglinear preferences (4), consumer welfare can be expressed as $V = \log D(\tau) + E$, where the equilibrium value of the aggregator can be computed from (14) as:

$$D(\tau) = \frac{[(1-\tau)p(\tau) - c]\, |v'(p(\tau))|}{f}$$

and is decreasing and convex in the commission on sellers. The profits of the marketplace (25) are simplified as follows:

$$\Pi(\tau) = \tau\zeta(p(\tau)) \left(1 - \frac{H}{D(\tau)}\right) + \frac{\Delta(\tau)}{D(\tau)} \tag{26}$$

A pure marketplace facing $\Delta(\tau) = 0$ sets the profit-maximizing commission rate τ^p that satisfies the first order condition:

$$\zeta(p(\tau^p)) + \tau^p \zeta'(p(\tau^p))p'(\tau^p) = \frac{|D'(\tau^p)|\, H\tau^p\zeta(p(\tau^p))}{D(\tau^p)\,[D(\tau^p) - H]} \tag{27}$$

assuming that both sides are positive and the second order condition for the interior maximum is satified. The left hand side of (27) represents the marginal revenue from the commission on an active seller and the right hand side the marginal costs of reducing the value of the price aggregator and therefore reducing the number of sellers active on the platform with the associated commission revenues.

Using the shape of the surplus elasticity and the pass-through as already in (24) and the impact of the commission on the price aggregator (15) with $\sigma(D) = 1$, we can also rearrange the implicit expression for the commission rate as follows:

$$\tau^p = \frac{1}{1 - \eta(p(\tau^p))[1 + \zeta(p(\tau^p)) - \varepsilon(p(\tau^p))] + \varepsilon(p(\tau^p))\frac{H}{D(\tau^p)-H}} \tag{28}$$

which depends on the various elasticities (all evaluated at the same commission rate) and on the relevance of the exogenous surplus obtained from other purchases H compared to the surplus obtained from the marketplace represented by the equilibrium aggregator $D(\tau^p)$. In particular, the commission decreases when the sellers face a more elastic demand because the platform internalizes the negative impact on their sales, and it decreases also when the buyers expect a higher surplus from goods purchased outside the platform compared to what they obtain on the platform.[16]

Under power surplus functions (8),where $\varepsilon = \zeta + 1$ is a constant, the formula boils down to:

$$\tau^p = \frac{D(\tau^p) - H}{D(\tau^p) - H + \varepsilon H} \tag{29}$$

[16] In case of zero marginal costs, which is relevant for sales of software apps (for instance on app stores), we simply have $\tau^p = 1 - H/D(\tau^p)$.

and, since the price aggregator is mononically decreasing (so that the right hand side is a decreasing function in the unit interval), there is always an interior solution for the commission rate. The commission for the case of exponential subutility (9) and translated power subutility (10) can be expressed through the pass-through elasticities and demand elasticities derived above.

Let us now move to a hybrid marketplace with $\Delta(\tau) > 0$. This sets its commission rate to maximize (26) taking into account not only the effects on the price aggregator, but also the opportunity cost of losing commission revenues on its own products. Using (24), the profit-maximizing commission τ^h satisfies a first order condition that can be rearranged as follows:

$$\zeta(p(\tau^h)) + \tau^h \zeta'(p(\tau^h))p'(\tau^h) = \frac{\left|D'(\tau^h)\right| \left[H\tau^h \zeta(p(\tau^h)) - \Delta(\tau^h)\right]}{D(\tau^h)\left[D(\tau^h) - \Psi(\tau^h) - H\right]} \tag{30}$$

again assuming an interior solution.

The comparison between commissions set by a pure marketplace in (27) and a hybrid marketplace in (30) is ambiguous in general. Heuristically, the introduction of own products generates an incentive to shift demand toward them with a higher commission - the effect at the numerator of (30) depending on the differential profit index $\Delta(\tau^h)$, but it also reduces the number of sellers and increases the incentives to expand sales of products and generate commission revenues on new sellers - the effect at the denominator of (30) depending on the incremental surplus generated by own products $\Psi(\tau^h)$. The first effect is the demand substitution effect emphasized by AB (2021) aimed at diverting demand where it is more profitable for the marketplace, and the second effect is an extensive margin effect aimed at expanding demand for all products.

We now focus on the case of homogeneous products by the marketplace. Repeating the steps above, we can rearrange the formula for the commission as:

$$\tau^h = \frac{D - H - \Psi\left[1 - \frac{\varepsilon(p)\bar{\zeta}(\bar{p})}{\zeta(p)\bar{\varepsilon}(\bar{p})}\right]}{\{1 - \eta(p)[1 + \zeta(p) - \varepsilon(p)]\}(D - H - \Psi) + \varepsilon(p)\left[H + \Psi\left(1 - \frac{\bar{\zeta}(\bar{p})}{\bar{\varepsilon}(\bar{p})}\right)\right]} \tag{31}$$

where upperbars refer to the products of the marketplace and we dropped the arguments related to the commission on the right hand side. We still cannot determine whether a positive or increasing value of the surplus from marketplace's products Ψ increases or reduces the commission. Nevertheless we can show that already within our examples both cases can emerge.

Under common power functions (8) for sellers with demand elasticity ε and (20) for the marketplace with demand elasticity $\bar{\varepsilon}$, the last formula boils down to:

$$\tau^h = \frac{D - H - \Psi\frac{\varepsilon - \bar{\varepsilon}}{(\varepsilon - 1)\bar{\varepsilon}}}{D - H + \varepsilon H + \Psi\frac{\varepsilon - \bar{\varepsilon}}{\bar{\varepsilon}}} \lesseqgtr \tau^p \quad \text{if} \quad \varepsilon \gtreqless \bar{\varepsilon} \tag{32}$$

where the comparison with (29) is straightforward.

In the specification where not only the products of the sellers are identical but also the products of the marketplace provide power surplus functions with the same elasticity (namely $\varepsilon = \bar{\varepsilon}$), we have a constant commission given by (29): a hybrid marketplace sets the same markups as the sellers and does not change its commission compared to a pure marketplace. The hybrid marketplace is therefore completely neutral on consumer welfare, in spite of potential differences in both costs and demand (scale) parameters between the products of sellers and those of the marketplace.

Of course, if the marketplace faces a different demand elasticity than the sellers, the commission can change. In particular, when the marketplace faces a less elastic demand than the sellers for its products ($\bar{\varepsilon} < \varepsilon$), it reduces the commissions while introducing the products, and when it faces a more elastic demand ($\bar{\varepsilon} > \varepsilon$) it increases the commissions. Intuitively, when third party sellers face a relatively more elastic demand, the marketplace sets higher markups on its own products, but recovers entry of sellers and expenditure of buyers on the marketplace by reducing the commission. This may well be the case for Amazon if customers have indeed a more rigid demand for its products compared to those of third party sellers hosted on the platform, for instance due to Amazon reputation for more reliable shipping and post-sale services.

One can verify that in case of exponential subutilities, a hybrid marketplace sets always a higher commission than a pure one, as already shown by AB (2021).[17] Instead, in case of translated power subutilities a hybrid marketplace sets a higher commission when $\bar{\gamma} = \gamma$ but not necessarily when $\bar{\gamma} > \gamma$. The broad message is that the introduction of products by a marketplace increases its profits also through adjustments of the commissions, but exerts an ambiguous impact on consumer welfare, whose quantitative measure is unlikely to be relevant in practice.

3.3.2 Isoelastic preferences

We now consider the extended model based on the isoelastic transformation (6) where $\kappa \in (0,1)$ governs the elasticity of consumers's benefits and purchases on the marketplace with respect to its quality. In this case welfare is $V = D(t)^\kappa/\kappa + E$, where the aggregator can be expressed from (14) as follows:

$$D(\tau) = \left[\frac{[(1-\tau)p(\tau) - c]\,|v'(p(\tau))|}{f} \right]^{\frac{1}{1-\kappa}}$$

and the impact of the commission depends crucially on κ. For the purpose of this section we adopt the simplifying assumption $H = 0$, so that the expected profits of a hybrid marketplace are:

$$\Pi(\tau) = D(\tau)^\kappa \left[\tau\zeta(p(\tau)) + \frac{\Delta(\tau)}{D(\tau)} \right] \tag{33}$$

[17] In particular, since in the Logit framework we have $\zeta(p)/\varepsilon(p) = \bar{\zeta}(\bar{p})/\bar{\varepsilon}(\bar{p}) = 1$ for any prices, the equilibrium commission satisfies:

$$\tau^h = \frac{D - H}{[1 - \eta(p)](D - H - \Psi) + \varepsilon(p)H}$$

which is necessarily higher when Ψ is positive compared to when it is null.

A pure marketplace (with $\Delta(\tau)$) sets a commission τ^p that satisfies:

$$\zeta(p(\tau^p)) + \tau^p \zeta'(p(\tau^p))p'(\tau^p) = \frac{|D'(\tau^p)|\,\kappa \tau^p \zeta(p(\tau^p))}{D(\tau^p)} \tag{34}$$

Using the shape of the surplus elasticity, the pass-through and the equilibrium aggregator above, we can also express the equilibrium commission rate as:

$$\tau^p = \frac{1}{1 - \eta(p(\tau^p))[1 + \zeta(p(\tau^p)) - \varepsilon(p(\tau^p))] + \frac{\varepsilon(p(\tau^p))\kappa}{1-\kappa}} \tag{35}$$

which is assumed interior to the unit interval. Notice that the commission is decreasing in κ: when demand and profits of sellers are more elastic with respect to the aggregator and therefore to the same commission, the demand shifting effect is weak and it is convenient to reduce commissions to attract more sellers and expenditure on the marketplace.[18] In the case of isoelastic demand, the formula boils down to the following explicit expression:

$$\tau^p = \frac{1}{1 + \frac{\varepsilon\kappa}{1-\kappa}} \tag{36}$$

with associated prices $p(\tau^p) = \frac{c(\varepsilon + 1/\kappa - 1)}{\varepsilon - 1}$ decreasing in κ. When the demand of products is highly elastic with respect to the overall quality of the marketplace, the commissions are set at lower levels reducing the prices.[19]

When the marketplace has its own products on sale, it sets the commission rate τ^h to maximize (33) according to:

$$\zeta(p(\tau^h)) + \tau^h \zeta'(p(\tau^h))p'(\tau^h) = \frac{\left|D'(\tau^h)\right| \left\{\kappa \tau^h \zeta(p(\tau^h)) - (1-\kappa)\frac{\Delta(\tau^h)}{D(\tau^h)}\right\}}{D(\tau^h) - \Psi(\tau^h)} \tag{37}$$

The comparison between commissions set by hybrid and pure marketplaces is still ambiguous because the introduction of own products generates an incentive to shift demand toward them with a higher commission, but it also increases the incentives to recover entry of sellers and expenditure by buyers on the platform through a lower commission.

The assumption of homogeneous products by the marketplace allows to obtain further results. If all products face a constant demand elasticity ε the introduction of products by the marketplace is neutral on the commission (36). However, in the case of power functions (8) for sellers with demand elasticity ε and (20) for the marketplace with demand elasticity $\bar{\varepsilon}$, we obtain:

$$\tau^h = \frac{1 - \frac{\Psi}{D}\frac{\varepsilon - \bar{\varepsilon}}{(\varepsilon - 1)\bar{\varepsilon}}}{1 + \frac{\varepsilon\kappa}{1-\kappa} + \frac{\Psi}{D}\frac{\varepsilon - \bar{\varepsilon}}{\bar{\varepsilon}}} \lesseqgtr \tau^p \quad \text{if} \quad \varepsilon \gtreqless \bar{\varepsilon} \tag{38}$$

[18] For instance, in case of zero marginal costs, we have $\tau^p = 1 - \kappa$.

[19] As a back of the envelope computation, $\varepsilon = 4$ and $\kappa = 0.5$ provide a commission of 20% and net markups of 25% for the sellers, which are not far from realistic values, even without invoking investment in customer building (see Gutierrez, 2021).

which confirms qualitatively our earlier comparison for any value of κ.

More generally, the hybrid marketplace sets a lower commission if the sign of the derivative of its profits with respect to the commission is negative when evaluated at τ^p - or the right hand side of (34) is lower than the one of (37) in τ^p. Assuming homogeneous products of the marketplace and using the expression for τ^p in (35), after some manipulation we can express the condition for a lower commission by the hybrid marketplace as follows:

$$[1-\eta(p)][1+\zeta(p)-\varepsilon(p)](1-\kappa) < \frac{\bar{\varepsilon}(\bar{p})\zeta(p)-\varepsilon(p)\bar{\zeta}(\bar{p})}{\bar{\zeta}(\bar{p})}$$

where upperbars refer to the elasticities of marketplace's products. In the special case of power functions for sellers (8) with constant elasticity ε, the left hand term is null and we only need the right hand term to be positive. This requires $\bar{\varepsilon}(\bar{p})(\varepsilon-1) > \varepsilon\bar{\zeta}(\bar{p})$ or, using the slope of the surplus function, $\varepsilon > \frac{\bar{\varepsilon}(\bar{p})}{1-\bar{\zeta}'(\bar{p})\bar{p}/\bar{\zeta}(\bar{p})}$. As just noticed, if also the marketplace's products generate power surplus functions (20) with constant elasticity $\bar{\varepsilon}$, all what is needed is that this demand elasticity is lower than the one faced by the sellers, but the condition can be satisfied also with different surplus functions for the marketplace.

When we allow for general surplus functions of the sellers, the role of κ becomes crucial. Under the assumption of increasing elasticities $\varepsilon'(p), \zeta'(p) > 0$, which implies incomplete pass-through $\eta(p) < 1$ and $1+\zeta(p)-\varepsilon(p) > 0$, the condition above can be simplified as:

$$\kappa > 1 - \frac{\bar{\varepsilon}(\bar{p})\zeta(p)-\varepsilon(p)\bar{\zeta}(\bar{p})}{[1-\eta(p)][1+\zeta(p)-\varepsilon(p)]\bar{\zeta}(\bar{p})}$$

While the right hand side is still an implicit expression, the condition is always satisfied for a high enough κ as long as $\frac{\zeta(p)}{\varepsilon(p)} > \frac{\bar{\zeta}(\bar{p})}{\bar{\varepsilon}(\bar{p})}$, which depends on the relative ratios between surplus and demand elasticities. Such inequality never holds under exponential subutilities (in line with AB, 2021), but it can hold for instance under translated power subutilities as long as $\bar{\gamma} > \gamma$.[20] Recalling our interpretation of κ in this model, when the utility of consumers and their demand of products on the marketplace are highly elastic with respect to the quality of the same marketplace, it is more tempting to reduce commissions to attract new sellers and more spending on the marketplace.

3.3.3 General preferences

In case of an increasing and concave transformation $G(D)$, the analysis is slightly more cumbersome. The equilibrium aggregator is defined by (14) for a given commission. The expression for the profits of a hybrid marketplace is given by (25). Assuming an interior solution, the formula for the profit-maximizing

[20] For instance, when $c \to 0$, the hybrid marketplace reduces the commission if $\kappa > 1 - \frac{\bar{\gamma}-\gamma}{(1+\bar{\gamma})(1+\gamma)}$. I am grateful to Paolo Bertoletti for useful comments on this section.

commission rate can be expressed as follows:

$$\zeta(p(\tau))+\tau\zeta'(p(\tau))p'(\tau)=\frac{|D'(\tau)|\left\{[1-\sigma(D(\tau))+\frac{\sigma(D(\tau))}{D(\tau)/H}]\tau\zeta(p(\tau))-\frac{\sigma(D(\tau))}{D(\tau)/\Delta(\tau)}\right\}}{D(\tau)-\Psi(\tau)-H}$$

where we used the index of curvature of the concave transformation $\sigma(D)$, and kept assuming the existence of an interior solution. The left hand side is always the marginal revenue from the commission on a seller and the right hand side is the marginal cost of reducing entry of sellers and welfare, affected in opposite directions by the differential profits and incremental surplus on own products.

Under a power surplus function for sellers with demand elasticity ε the commission selected by a pure marketplace can be derived through usual computations as:

$$\tau^p=\frac{1}{1+\varepsilon\left[\frac{D(\tau^p)}{\sigma(D(\tau^p))(D(\tau^p)-H)}-1\right]}$$

If the marketplace introduces products facing the constant demand elasticity $\bar{\varepsilon}$, we obtain:

$$\tau^h=\frac{1-\frac{\Psi(\tau^h)}{D(\tau^h)-H}\frac{\varepsilon-\bar{\varepsilon}}{(\varepsilon-1)\bar{\varepsilon}}}{1+\varepsilon\left[\frac{D(\tau^h)}{\sigma(D(\tau^h))(D(\tau^h)-H)}-1\right]+\frac{\Psi(\tau^h)}{D(\tau^h)-H}\frac{\varepsilon-\bar{\varepsilon}}{\bar{\varepsilon}}}\lesseqgtr\tau^p\quad\text{if}\quad\varepsilon\gtreqless\bar{\varepsilon}\qquad(39)$$

which confirms the neutrality under a common elasticity and the incentive to reduce the commission for a marketplace if and only if this faces less elastic demands. We summarize our main general findings as follows:

Proposition 3. *Under monopolistic competition with free entry of sellers on a marketplace serving customers with quasilinear IA preferences, the introduction of homogeneous products by the marketplace is neutral on consumer welfare if the sellers face the same constant demand elasticity as the marketplace, and otherwise can either increase or decrease consumer welfare.*

Our main policy implication differs from the one of AB (2021), because banning the dual mode inducing a hybrid marketplace to turn into a pure marketplace may actually harm rather than benefit consumers. However, other results obtained by AB (2021) extend naturally to our framework. In particular, a hybrid marketplace has an interest in promoting higher perceived quality or lower (production and shipping) costs for both its own products and those of third party sellers, and there is no obvious incentive to self-preferencing. Moreover, the introduction of a tax on third party revenues tends to increase the commission set by the marketplace, while a tax on the revenues of products directly sold by the marketplace tends to reduce the commission, with opposite effects on consumer welfare.

4 Extensions

In this section we extend the model in a few directions. First, we change the timing of the baseline model by considering a marketplace that cannot commit to price choices before the sellers. Next, we introduce strategic price competition between sellers, which is relevant when few of them are active in the same product category. Subsequently, we analyze the choice of product selection by the marketplace, essentially endogenizing which products are actually introduced and extending the analysis of Etro (2021a). Then, we consider the alternative case of specific commissions to relate our results with those of Zennyo (2020). Finally, we consider advertising as an additional source of monetization which is becoming always more important for online marketplaces.

Another extension of some interest is the one to competing subcription-funded marketplaces, which is the direction taken by Amazon with its Prime membership fee (and by device-funded platforms as the one of Apple). This is explored elsewhere (Etro, 2021b): the demand-substitution and extensive margin effects are present also in that context, but the platforms internalize also the direct impact of their strategies on consumer welfare, because this affects monetization through the access fees, and competition leads the platforms to shift revenues to consumers through lower membership fees, which amplifies benefits for consumers.

4.1 No price commitments

Our baseline analysis has analyzed a marketplace acting as a Stackelberg leader able to pre-commit on the commission on third party sellers and also on the prices of its own products. In practice, it is not clear that a marketplace as Amazon has any first mover advantage in setting prices compared to other sellers: most of the price changes occur in real time time for both Amazon and the sellers hosted on its platform. While a pre-commitment allows the marketplace to increase its own prices in function of the pre-determined commission rate and to better monetize the demand of its own products, it is not crucial to obtain lower commissions and benefit consumers compared to a pure marketplace. To verify this, we now change the timing of the baseline model, assuming that both the marketplace and the sellers set their prices in the last stage under monopolistic competition, though analogous results would apply if either the marketplace or the sellers were acting strategically setting (higher) prices.

While sellers set prices according to the usual rule $p(\tau)$ in (12), the marketplace does not internalize the impact on entry and sets lower prices $\bar{p}_j = \bar{p}_j(0)$ that satisfy:

$$\bar{p}_j = \frac{\varepsilon_j(\bar{p}_j)\bar{c}_j}{\varepsilon_j(\bar{p}_j) - 1} \tag{40}$$

because it is not subject to the payment of a commission. Free entry, however, determines the same aggregator $D(\tau)$ as before, which is determined by (14). The expressions for the equilibrium commissions are the same as in the baseline model, with the only difference that the index of differential profits takes into account the new prices set by the marketplace as in:

$$\Delta(\tau) = \sum_{j=1}^{m} v_j(\bar{p}_j(\tau)) \left[\frac{\zeta_j(\bar{p}_j)}{\varepsilon_j(\bar{p}_j)} - \tau\zeta(p(\tau)) \right] \tag{41}$$

This expression is smaller than (23) for a given commission because the marketplace cannot precommit to higher prices exploiting the demand diversion generated by the commission. Which pushes for lower commissions set by a hybrid marketplace.

To illustrate, let us consider the case of loglinear preferences with homogeneous products on the marketplace. Replacing (41) in (30) we can rearrange the commission for the hybrid marketplace as:

$$\tau^h = \frac{D - H - \Psi \left[1 - \frac{\varepsilon(p)\bar{\zeta}(\bar{p})}{\zeta(p)\bar{\varepsilon}(\bar{p})} \right]}{\{1 - \eta(p)[1 + \zeta(p) - \varepsilon(p)]\}(D - H - \Psi) + \varepsilon(p)\,(H + \Psi)} \tag{42}$$

Under a common and constant demand elasticity for all products or under Logit demand systems the parenthesis at the numerator is null and the terms at the denominator push alone for a lower commission rate compared to a pure marketplace. Intuitively, when the marketplace cannot commit to optimally monetize the demand diversion generated by the commissions through appropriate pricing of its products, the demand substitution mechanism tends to be dominated by the extensive margin mechanism.

4.2 Strategic interactions

Our main framework assumed monopolistic competition between sellers, which appears the relevant scenario for marketplaces as Amazon that host a huge number of products. However, when product categories subject to the same commission are narrowly defined, sellers may take strategic interactions into account at the pricing stage. Here we verify how Bertrand competition between sellers affects our results.

At the pricing stage, each seller maximizes (11) taking into account the impact of its price choice on the true demand function, and therefore also the aggregator. Exploiting symmetry across sellers, this delivers the price rule $p(\tau)$ such that:

$$p = \frac{\epsilon(p, D)c}{(1-\tau)(\epsilon(p, D) - 1)} \quad \text{with } \epsilon(p, D) \equiv \varepsilon(p) - \zeta(p)v(p)\frac{\sigma(D)}{D} \tag{43}$$

where $\sigma(D)$ is the curvature index and the lower demand elasticity implies a higher markup: strategic sellers set higher prices compared to monopolistically competitive sellers. Free entry implies always the zero profit condition (14), so that the system of two equations determines jointly (p, D) as functions of the commission τ.[21] This preserves the neutrality of the aggregator and welfare with respect to the provision of products by the marketplace, which should not be surprising since this neutrality in free entry models was orginally observed in the presence of strategic interactions (Etro, 2008, 2011).

Once we know how the commission affects the prices of sellers $p(\tau)$ and the aggregator $D(\tau)$, nothing changes qualitatively in the derivation of prices of the marketplace and of its commission. In practice, competition is softened between a small number of sellers increasing also the prices of the marketplace and reducing welfare compared to the case of monopolistic competition, but the ambiguous impact of hybrid platforms on the commission remains.

4.3 Product selection

Our next investigation is about the conditions under which the marketplace enters and with which products in the baseline model. The problem was explored by Hagiu and Wright (2015) and Etro (2021a) for a given set of product varieties and sellers under the assumptions of independent demands and (specific) commissions optimally set on each product. In our framework with free entry of sellers, interdependent demands and a uniform (percentage) commission set on all products, the issue is complicated because the marketplace must take in consideration not only the relative profitability of direct and third party sales, but also the impact on demand allocation across products.[22]

Given the gross profits of the marketplace (17), let us consider for simplicity the case where the marketplace bears the same fixed costs as the sellers for each product. Then the introduction of a new product is profitable if it augments the net profits:

$$\Pi(\tau^h) - mf = [\tau^h \zeta(p(\tau^h)) \left(D(\tau^h) - H\right) + \Delta(\tau^h)] G'(D(\tau^h)) - mf \qquad (44)$$

by increasing enough the index of differential profits to cover the fixed cost. Taking as given the commission (which applies to all products), using the index (23) and omitting arguments, the condition for the introduction of a product with surplus function $z\bar{v}(\bar{p})$ to be profitable is:

$$z\bar{v}(\bar{p}) \left[\frac{\bar{\zeta}(\bar{p}) \left[1 + \tau^h \zeta(p)\right]}{\bar{\varepsilon}(\bar{p})} - \tau^h \zeta(p) \right] - \frac{f}{G'(D)} > 0$$

Employing the zero profit condition for the sellers (14) to replace the fixed cost and rearranging, we can obtain the exact condition under which the platform profits from providing the given good:

$$\frac{z\bar{v}(\bar{p})\bar{\zeta}(\bar{p})}{\bar{\varepsilon}(\bar{p})} - \frac{v(p)\zeta(p)}{\varepsilon(p)} + \tau^h \zeta(p) \left[\frac{v(p)}{\varepsilon(p)} - z\bar{v}(\bar{p}) \frac{\bar{\varepsilon}(\bar{p}) - \bar{\zeta}(\bar{p})}{\bar{\varepsilon}(\bar{p})} \right] > 0 \qquad (45)$$

[21] For instance, under loglinear preferences and power surplus functions one can derive $p = \frac{\varepsilon c}{(\varepsilon - 1)(1-\tau) - f}$ for a small enough fixed cost per customer.

[22] The related problem of product selection in a market with monopolistic competition is addressed in Spence (1976) and more recently in Bertoletti and Etro (2022).

The first two terms represent the difference in gross profits between marketplace and sellers, and under a zero commission this must be positive for entry by the marketplace to cover the fixed cost and create positive net profits. The last term can be either positive or negative, and accounts for the role of the commission: on one side, a positive commission reduces the incentives of the marketplace to enter because profitability must compensate for the lost commission revenues, but, on the other side, it increases the incentives because it shifts demand toward the products of the marketplace whose markup can be increased.

It is immediate to verify that in our example with power functions (8) for sellers with demand elasticity ε and (20) for the marketplace with demand elasticity $\bar{\varepsilon}$, the condition becomes:

$$z\bar{p}^{1-\bar{\varepsilon}}\left[1-\frac{1+\tau^h(\varepsilon-1)}{\bar{\varepsilon}}\right] > p^{1-\varepsilon}\left[1-\frac{1+\tau^h(\varepsilon-1)}{\varepsilon}\right]$$

If marketplace and sellers face the same surplus functions and costs, the expressions on each side are identical and the marketplace is indifferent between introducing or not its product. Otherwise the condition is satisfied if the marketplace has large enough advantages either in demand or costs. In any case, when the marketplace finds it profitable to introduce the product, this does not affect consumer welfare (because it does not change the commission).

Similarly, our examples of loglinear demands provide the simplified condition $z\exp(p/\mu-\bar{p}/\bar{\mu}) > 1-\tau^h$, and our translated power surplus functions with $\gamma=\bar{\gamma}$ imply $z\left(\frac{a-\bar{p}}{a-p}\right)^{\gamma} > \frac{1-\tau^h}{1+\tau^h\varepsilon(p)/\gamma}$. These conditions are satisfied only if the marketplace has large enough advantages either in demand or costs compared to the sellers. The consequence is that in this case where the introduction of a product by the marketplace reduces consumer welfare (by raising the commission), the product may not be introduced to start with.[23]

4.4 Specific commissions

The benchmark analysis assumed *ad valorem* commissions on the sellers. We now consider the case where the only commission available is a commission t on the quantity sold rather than the revenues, as in Zennyo (2020). We keep the rest of the notation as in the baseline model and follow its development.

Given the specific commission on sales, each seller i sets the price to maximize gross profits:

$$\pi(p_i) = (p_i - t - c)\,|v'(p_i)|\,G'(D(\mathbf{p})) \tag{46}$$

ignoring the impact on the price aggregator. This provides price rules $p = p(t)$ that satisfy:

$$p = \frac{\varepsilon(p)(c+t)}{\varepsilon(p)-1} \tag{47}$$

[23] This may be relevant since in practice online marketplaces often introduce products of lower quality than the sellers (see Shopova, 2021).

Free entry of sellers implies the zero profit condition:

$$(p(t)-t-c)\left|v'\left(p(t)\right)\right|G'(D(t))=f$$

which determines the equilibrium aggregator $D(t)$ as a function decreasing in the specific commission and, as before, independent from the provision of marketplace's products. The profit of the hybrid platform can be expressed as:

$$\Pi=[t\xi(p(t))\left(D(t)-H\right)+\Delta(\mathbf{p},t)]G'(D(t))$$

where we defined $\xi(p)\equiv -v'(p)/v(p)$ and the relevant index of differential profits:

$$\Delta(\mathbf{p},t)\equiv\sum_{j=1}^{m}\left[\left(\bar{p}_j-\bar{c}_j\right)\left|v_j'(\bar{p}_j)\right|-t\xi(p(t))v_j(\bar{p}_j)\right]$$

The prices of the platform's products are selected to maximize this index according to the rule:

$$\bar{p}_j(t)=\bar{c}_j+\frac{1+t\xi(p(t))}{\vartheta_j(\bar{p}_j(t))} \tag{48}$$

where we defined $\vartheta_j(p)\equiv -v_j''(p)/v_j'(p)$. This allows us to express the index of differential profits as a function of the specific commission only:

$$\Delta(t)=\sum_{j=1}^{m}v_j(\bar{p}_j(t))\left[\frac{\xi_j(\bar{p}_j(t))\left[1+t\xi(p(t))\right]}{\vartheta_j(\bar{p}_j(t))}-t\xi(p(t))\right]$$

where we defined $\xi_j(p)\equiv -v_j'(p)/v_j(p)$. Finally, the profits can be expressed as:

$$\Pi(t)=[t\xi(p(t))\left(D(t)-H\right)+\Delta(t)]G'(D(t)) \tag{49}$$

whose maximization with respect to t defines the equilibrium with implications that are qualitatively analogous to those obtained under percentage commissions.

To obtain further results it is convenient to focus on the case of loglinear preferences (4), where the aggregator can be derived as:

$$D(t)=\frac{(p(t)-t-c)\left|v'\left(p(t)\right)\right|}{f}$$

In such a case, a pure marketplace sets a commission to maximize $t\xi(p(t))(1-H/D(t))$, implying the condition:

$$\xi(p(t^p))+t^p\xi'(p(t^p))p'(t^p)=\frac{\left|D'(t^p)\right|Ht^p\xi(p(t^p))}{D(t^p)\left[D(t^p)-H\right]} \tag{50}$$

which equalizes as usual the marginal revenue from the specific commission per seller on the left hand side and the marginal costs of reducing the the number of sellers on the right hand side. A hybrid marketplace maximizes $t\xi(p(t))(1 - H/D(t)) + \Delta(t)/D(t)$ setting the commission according to the following rule:

$$\xi(p(t^h)) + t^h \xi'(p(t^h))p'(t^h) = \frac{\left|D'(t^h)\right| \left[Ht^h\xi(p(t^h)) - \Delta(t^h)\right]}{D(t^h)\left[D(t^h) - \Psi(t^h) - H\right]} \tag{51}$$

where we defined the additional surplus from the products of the marketplace as $\Psi(t) \equiv \sum_{j=1}^{m} v_j(\bar{p}_j(t))$. Once again, this is compatible with either a higher or a lower commission due to opposite effects that are analogous to what emerged under *ad valorem* commissions. For the same reasons the impact on consumer welfare can go in either direction.

To compare the results with those of Zennyo (2020), let us consider the Logit demand system where all subutilities are exponential, with (9) for the sellers and (21) for all the products of the marketplace. This implies prices of sellers and marketplace given by $p(t) = c + \mu + t$ and $\bar{p}_j(t) = \bar{c}_j + \bar{\mu} + \frac{\bar{\mu}}{\mu}t$, which would match for identical products as in Zennyo (2020). Moreover, computing $\xi(p) = |D'(t)|/D(t) = 1/\mu$ and $\vartheta_j(p) = \xi_j(p) = 1/\bar{\mu}$ and $\Delta(t) = \Psi(t)$ we can simplify the implicit expression for the commission to:

$$t^* = \frac{\mu\left(D(t^*) - H\right)}{H} \tag{52}$$

The key aspect is that the commission is independent from the presence of marketplace's products, namely $t^* = t^h = t^p$. This confirms the result by Zennyo (2020) that under a Logit microfoundation a hybrid marketplace sets the same specific commission as a pure marketplace. Clearly, this neutrality does not hold with subutilities that are not exponential and, *a fortiori*, in the case of a more general microfoundation. Accordingly the introduction of products by the marketplace may either increase or decrease consumer welfare.

4.5 Advertising for product discovery

An expanding source of monetization for online marketplaces is represented by advertising by sellers, essentially aimed at product discovery. While a platform has its own incentives to promote sales that generate commission revenues, each seller has an additional incentive to spend in ads to divert "clicks" of customers to its own products from those of the rivals. To the extent that this expands sales, it can also generate additional revenues for the marketplace. This creates an imperfect substitutability between commission revenues and ad revenues for the platform and can affect also its incentives to change conditions for third party sellers after entry with its own products. In the Appendix we argue that in general this does not need to be the case and that the structure of monetization is independent from the introduction of marketplace's products.[24]

[24] Ads can also raise other issues for platforms. Kirpalani and Philippon (2020) argue that information disclosure by consumers improves the gains from match quality but may increase too much the market power of a monopolistic marketplace toward third party sellers. Latham *et al.* (2021) explore the role of Google in the ad tech stack.

More formally, we augment the baseline model with a probability of purchase of each product depending on spending in ads by the seller. The ad fee can be regarded as exogenous if the willingness to pay of the sellers depends on ad returns on alternative platforms and is not biased by the platform, but we also consider the case in which the marketplace exploits its market power and selects the ad fee (directly or by restricting the supply of ads). In the case of Amazon, ad fees are determined through ad auctions per click, and a concern is that Amazon may exploit its dual role by manipulating quality scores to increase ad costs for rival sellers.

Given percentage commissions and advertising fees, each seller selects price and ads under monopolistic competition. Prices are increased to take into account ad costs, and the ratio of ad spending and net earnings per sale is equalized to the ratio of demand elasticities with respect to ads and price. As in the baseline framework, the free entry condition determines the price aggregator, and therefore welfare, as a decreasing function of each fee and, once again, independently from the products introduced by the marketplace. When the marketplace sets its commission rate, the monetization through ads tends to reduce the marginal commission revenues and therefore the optimal commission rate: intuitively the marketplace is aware of the impact of higher ad costs on prices and therefore sales. However, the introduction of products by the marketplace exerts the usual ambiguous impact on the commission.

When the marketplace can also set the ad fees, if their impact on revenues is small, it is optimal to provide free ads. Otherwise, the ad fees are chosen in such a way that the ratio of ad revenues and commission revenues per sale should equate the ratio of the elasticities of revenue with respect to ad fee and commission rate. Such a rule is independent from the introduction of own products, implying that pure and hybrid marketplace should decide on the structure of monetization on third party sellers independently from the source of revenues. Once again, the usual trade-off between demand-shifting and extensive margin mechanisms determines whether hybrid platforms are going to increase or reduce consumer welfare through changes of the total payment for commission and ad fee. This confirms the spirit of the results of our benchmark model also when the marketplace monetizes through ads for product discovery.

5 Conclusion

We have analyzed the role of endogenous entry of monopolistically competitive sellers on hybrid marketplaces under rather general demand systems. Our microfoundation was based on indirectly additive aggregators and allowed us to show that a hybrid marketplace can set either higher or lower commissions compared to a pure marketplace, with opposite effects on consumer (and user) welfare. For instance, under constant demand elasticities, a hybrid marketplace sets lower percentage commissions increasing consumer welfare if and only if its products face a less elastic demand.

The literature so far has advanced various arguments for which a hybrid marketplace may benefit consumers by fostering competition on the platform (Hagiu *et al.*, 2020) and introducing cheaper or more valuable products (Etro, 2021a; Shopova, 2021), but may also create consumer harm favoring its own products or undermining entry by sellers. Considering differentiation between products and free entry of third party sellers on a marketplace, we have suggested that a key channel through which the hybrid marketplace can affect welfare is the change of the commissions set on rival sales. While the direction of this change remains an empirical issue, the commission rates set by Amazon have been quite stable over time and also in product categories where Amazon introduced its own products.

Further research may explore other strategies by marketplaces as those concerning search services for consumers, investments in logistics and platform-liability design in the presence of defect-reducing investments: see Hervas-Drane and Shelegia (2021) and Zennyo (2020) for early explorations in these directions. It should be remarked that the microfoundations used here and in related works generate aggregative games where the introduction of own products by the marketplace is neutral on consumer welfare for a given commission, and any welfare benefits emerge indirectly through reductions of the commission rates on third party sellers. One could also explore more general frameworks where a hybrid marketplace can benefit consumers directly, that is by introducing new products at lower prices and by strengthening competition (see Hagiu *et al.*, 2020, and Shopova, 2021). Finally, there is space for fruitful empirical work on the welfare impact of the strategies of Amazon. Lee and Musolff (2021) and Gutierrez (2021) have analyzed empirically the trade-offs generated by vertical integration of Amazon in nested Logit frameworks, respectively with endogenous entry of sellers for given commissions and with endogenous commissions for given sellers. It would be important to account for endogenous entry, prices and commissions under more general demand conditions.

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Appendix: Advertising for product discovery

In this Appendix we augment the version of the baseline model with logarithmic preferences (4) introducing a surplus function $z(a_i)v_i(p_i)$ for each third party seller i, where the scale function $z(a_i)$ determines the probability of sale and depends on the amount of ads a_i, so that the price aggregator becomes $D(\mathbf{p}) = \sum_j z(a_j)v_j(p_j) + H$, and each demand function:

$$q_i(a_i, p_i) = \frac{z(a_i)\,|v_i'(p_i)|}{\sum_j z(a_j)v_j(p_j) + H}$$

is affected by all ads. In practice ads influence the frequency of "clicks" that expand demand according to the function $z(a) > 0$, assumed to satisfy $z'(a) > 0$ and $z''(a) < 0$ in $a \in (0, \bar{a}]$ for some upperbound $\bar{a}$, and for simplicity we set $z(a) = z$ exogenously for the products of the marketplace.

Each third party seller i endogenously selects ads a_i by investing a fraction $a_i\tau_a \in (0, 1)$ of its revenues, where the ad fee τ_a determines how costly is advertising.[25] The ad fee is regarded as exogenous if the willingness to pay of sellers depends on ad returns on alternative platforms and is not biased by the platform, but we will later consider the case in which the marketplace can exploit its market power and select the ad fee.

Given percentage commission and ad fee, each seller i faces the profits:

$$\pi_i = [(1-\tau)p_i - c - a_i\tau_a p_i]\frac{z(a_i)\,|v'(p_i)|}{D(\mathbf{p})}$$

and selects price and ads under monopolistic competition taking as given the price aggregator as usual. This provides rules $p = p(\tau, \tau_a)$ and $a = a(\tau, \tau_a)$ that satisfy:

$$p = \frac{\varepsilon(p)c}{(1-\tau-\tau_a a)(\varepsilon(p)-1)} \quad \text{and} \quad \frac{\tau_a a}{1-\tau-\tau_a a} = \frac{\rho(a)}{\varepsilon(p)}$$

where $\rho(a) \equiv z'(a)a/z(a)$ is the elasticity of demand with respect to ads. The markup is increased to take into account ad costs, while the profit-maximizing spending in ads as a fraction of net earnings per sale is selected to equate the ratio of demand elasticities with respect to ads and price. For instance, in the case of power functions $v(p) = p^{1-\varepsilon}$ and $z(a) = a^\rho$, with $\rho > 0$, we can solve interior solutions for:

$$p = \frac{(\varepsilon+\rho)c}{(\varepsilon-1)(1-\tau)} \quad \text{and} \quad a = \frac{\rho(1-\tau)}{(\varepsilon+\rho)\tau_a}$$

[25] For the case of Amazon, estimates for spending in ad fees are around 5% of the selling price on third party sales (Gutierrez, 2021).

both of which increase in ρ (reverting to the baseline model when this is approximately null). In this example and more generally under regularity conditions assumed here, a higher commission is shifted into higher prices and fewer ads, and a higher ad fee reduces spending in ads.

The free entry condition determines the price aggregator, and therefore welfare, as a decreasing function of the two rates:

$$D(\tau,\tau_a)=\frac{[p(\tau,\tau_a)(1-\tau-\tau_a a(\tau,\tau_a))-c]z(a(\tau,\tau_a))\left|v'\left(p(\tau,\tau_a)\right)\right|}{f}$$

and, once again, independently from the products introduced by the marketplace.

Let us define $r(\tau,\tau_a)=[\tau+\tau_a a(\tau,\tau_a)]\zeta(p(\tau,\tau_a))$ as an index of revenues per product, assumed weakly increasing in the rates. Using the definition of the aggregator we can rewrite the profits of the marketplace as:

$$\Pi(\tau,\tau_a)=r(\tau,\tau_a)\left(1-\frac{H}{D(\tau,\tau_a)}\right)+\frac{\Delta(\tau,\tau_a)}{D(\tau,\tau_a)}$$

where we defined the index of profitability:

$$\Delta(\tau,\tau_a)=\sum_{j=1}^{m}\left[(\bar{p}_j-\bar{c}_j)z\left|v_j'(\bar{p}_j)\right|-r(\tau,\tau_a)zv_j(\bar{p}_j)\right]$$

taking into account the prices set by the marketplace according to:[26]

$$\bar{p}_j=\frac{\varepsilon_j(\bar{p}_j)\bar{c}_j}{\varepsilon_j(\bar{p}_j)-1-r(\tau,\tau_a)}$$

From now on we will denote derivatives through pedeces. Let us consider the choice of the commission rate taking as given the ad fee. The optimal commission τ for a hybrid marketplace satisfies:

$$r_\tau(\tau,\tau_a)=\frac{|D_\tau(\tau,\tau_a)|\left[r(\tau,\tau_a)H-\Delta(\tau,\tau_a)\right]}{D(\tau,\tau_a)\left[D(\tau,\tau_a)-\Psi(\tau,\tau_a)-H\right]}$$

where $\Psi(\tau,\tau_a)=\sum_{j=1}^{m}v(\bar{p}_j)$ is evaluated at the equilibrium prices. The monetization through ads tends to reduce the marginal commission revenues on the left hand side (to the extent that a higher commission reduces the ads) and therefore the optimal commission rate: intuitively the marketplace is aware of the impact of higher ad costs on prices and therefore sales. However, the introduction of products by the marketplace exerts the usual ambiguous impact on the marginal cost on the right hand side, and therefore on the optimal commission rate.[27]

[26] In the example above with common cost and demand elasticity, this price would be the same as that of the sellers and $r(\tau,\tau_a)=\frac{(\rho+\varepsilon\tau)(\varepsilon-1)}{\varepsilon+\rho}$ would be independent from τ_a.

[27] In our example with power functions we can generalize (29) to:

$$\tau=\frac{D-H-(\varepsilon+\rho)\frac{\rho}{\varepsilon}H+\frac{\rho}{\varepsilon}\Psi}{D-H+(\varepsilon+\rho)H+\frac{\rho}{\varepsilon}\Psi}$$

which is increasing in Ψ: now the introduction of own products induces an increase of the commission rate.

Let us finally consider the case where the platform can also set the ad fee τ_a. If the impact on revenues is small, it is optimal to set $\tau_a = 0$ providing free ads to all the sellers at the upperbound $\bar{a}$, which brings back to the benchmark model: this is what happens in the example with power functions. Otherwise, an interior equilibrium is characterized also by a first order condition with respect to τ_a. Combining the two conditions and deriving $D_\tau(\tau, \tau_a) = D_{\tau_a}(\tau, \tau_a)/a$ from the equilibrium aggregator, we obtain the optimality rule:

$$\frac{a\tau_a}{\tau} = \frac{\tau_a r_{\tau_a}(\tau, \tau_a)}{r(\tau, \tau_a)} \Big/ \frac{\tau r_\tau(\tau, \tau_a)}{r(\tau, \tau_a)}$$

The ratio of ad revenues and commission revenues per sale should equate the ratio of the elasticities of revenue with respect to ad fee and commission rate. Since this rule is independent from the introduction of own products, it implies that pure and hybrid marketplace should decide on the structure of monetization on third party sellers independently from the source of revenues.