High-dimensional high-frequency retail price dynamics with missing data

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Section 1

Introduction

Introduction: data

- Scanner data on retail prices
    - Records information on about 15,000 households over time.
    - Observe every shopping trip to any supermarket/grocery store for 1-3 years.
    - Stores include 4 major supermarket chains, other large competitors, and a large set of small stores.
    - Observe quantity, price and characteristics of every item purchased.
    - More than 300 categories of products (e.g. shampoo, laundry detergent, fresh fruit, beer, etc.)
    - Observe prices and quantities for tens of thousands of distinct products.
    - Each shopping trip each consumer purchases nonzero quantities of 1 - 50 items.
- High frequency sales and promotions.
- Price levels and dynamics vary across categories, brands, pack sizes, stores, time.

Introduction: statistical problem

- High dimensional data on prices and quantities.
- Missing data issues
  - Promotional status not observed.
  - No price observed if no purchases recorded in the sample.
  - Products with moderate to small market shares have large missing data problems.
- Missing data complicates inference on:
  - Price levels of missing items.
  - Price dynamics and promotional dynamics of missing items.
  - Estimates of aggregate inflation.
  - Estimates of consumer response to prices.

Introduction: proposed solution

- Develop a nonlinear factor model **(Chen et al., 2021)** also known as a “generalized low rank” model **(Udell et al., 2016)** to capture the dynamics.
  - Assume dynamics of high dimensional prices and consumer demand driven by a common set of low-dimensional factors.
  - Factors evolve according to a simple VAR(K) model.
- Model price process as a switching model with switching between a “regular price” process and a “sales price” process.
- Promotional status is “missing” or unobserved.
- Price is also missing when observed demand is zero.
- Correct for missing data problem with consumer demand based model of sample selection.

Section 2

Model

Model

- Observe data on J products for T time periods.
  - For shampoo, $J = 820$ and $T = 365$ or $T = 52$.
  - Product is defined by brand, pack size, and store.
- Model time series evolution of prices using a low dimensional factor structure.
- Account for switching between "sales-price" and "regular-price" with a hidden Markov state.
- Account for missing price data with a model of "selection" based on consumer demand.

Model

- Let $q_{jt} \geq 0$ be the aggregate quantity of product j purchased by consumers in the sample in period t.
- Let p^*_{jt} be the (potentially unobserved) price of product j in period t.
    - If $q_{jt} > 0$, observe $p_{jt} = p^*_{jt}$.
    - if $q_{jt} = 0$, p^*_{jt} is not observed.
- Let s^*_{jt} be the unobserved sales status of product j in period t.
    - s^*_{jt} is always missing.
    - if $s^*_{jt} = 1$, then p^*_{jt} is drawn from the "sales price" distribution.
    - if $s^*_{jt} = 0$, then p^*_{jt} is drawn from the "regular price" distribution.
- Dynamics of sales status s^*_{jt} and of demand q^*_{jt} are driven by an R dimensional set of factors f_t with R known and $R < J$.

Model

1. Sales status

$$s^*_{jt} = 1\left[\alpha_{sj} + \lambda_{sj}^T f_t + \varepsilon_{sjt} \leq 0\right]$$

2. Price

$$\begin{aligned} p^*_{jt} &= \left(1 - s^*_{jt}\right) p_{Hjt} + s^*_{jt} p_{Ljt} \\ p_{Hjt} &= p_{0j} + \varepsilon_{Hjt} \\ p_{Ljt} &= p_{0j} - \delta_j + \varepsilon_{Ljt} \end{aligned}$$

3. Consumer demand

$$q^*_{jt} = \alpha_{qj} + \sum_{i=1}^{J} \beta_{ji} p^*_{it} + \lambda_{qj}^T f_t + \varepsilon_{qjt}$$

Identification

- The factors f_t capture time-varying demand and cost shocks.
- Our assumptions impose functional form restrictions:
  - p_t^* is not a linear function of f_t.
- Functional form restrictions based on observation:
  - Firms do not adjust prices continuously while consumers can choose to purchase at any time.
- Ongoing work (with Crawford and Myśliwski):
  - Incorporate information on cost shocks.
  - Develop model of price setting competition with adjustment costs (Kydland, 1975; Judd, 1996).

Model

4. Measurement equations

$$\begin{aligned} q_{jt} &= 1\left[q_{jt}^* > 0\right] q_t^* \\ p_{jt} &= 1\left[q_{jt}^* > 0\right] p_{jt}^* \end{aligned}$$

5. Factor dynamics

$$f_t = A_0 + \sum_{k=1}^{K} A_k f_{t-k} + \eta_t$$

Reduce dimension of β: exploit observable product characteristics

- Number of parameters increases with J and T.
- In particular, when J is large, β consists of J^2 parameters.
- Assume consumer substitution patterns depend on a lower dimensional vector of product characteristics (e.g. store, brand, "childrens' shampoo", or for alcohol, lager, ale, alcohol content, etc.)

$$\beta_{ji} = \sum_k \widetilde{\beta}_k z_{kji}$$

Model

- Joint distribution of the unobservables. For all (j, t)

$$\varepsilon_{jt} = \begin{bmatrix} \varepsilon_{sjt} \\ \varepsilon_{Hjt} \\ \varepsilon_{Ljt} \\ \varepsilon_{qjt} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sigma^2_{Hj} & 0 & \sigma_{Hyj} \\ 0 & 0 & \sigma^2_{Lj} & \sigma_{Lyj} \\ 0 & \sigma_{Hyj} & \sigma_{Lyj} & \sigma^2_{yj} \end{bmatrix} \right) \tag{1}$$

- Heteroscedasticity across products.
- Assume ε_{jt} independent over products and time periods.
- Write the above covariance matrix as $\Sigma_j = C_j C_j^T$.
- Also, assume $\eta_t \sim N\left(0, \Sigma_\eta\right)$.

Parameterisation of covariance matrix

- Assume

$$C_j = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{Hj} & 0 & 0 \\ 0 & 0 & c_{Lj} & 0 \\ 0 & c_{Hyj} & c_{Lyj} & c_{yj} \end{bmatrix}.$$

  with

$$\begin{aligned} c_{Hj} &= c_{H0} + \Delta c_{Hj} \\ c_{Lj} &= c_{L0} + \Delta c_{Lj} \\ c_{yj} &= c_{y0} + \Delta c_{yj} \\ c_{Hyj} &= c_{Hy0} + \Delta c_{Hyj} \\ c_{Lyj} &= c_{Ly0} + \Delta c_{Lyj}. \end{aligned}$$

- Add L1 penalty to “regularise” the Δ terms.

Asymptotic distribution

- Theorem's 1 and 2 in Chen et al. (2014) apply in the homoscedastic case.
- Model parameters $(\alpha_s, \alpha_y, \beta_{ji})$ and average partial effects (e.g. demand elasticities) can be consistently estimated.
- Due to incidental parameters problem, asymptotic bias exists.
- Remove bias either with analytic formula or with split-sample bias estimator.

Estimation: maximum likelihood

- Parameters are $\theta = \left\{ f, \alpha_s, \lambda_s, \alpha_y, p_0, \delta, \beta, C_j, A, C_\eta \right\}$.
- Likelihood estimation computationally costly due to cost of integration across 2^J sales states. Likelihood function is

$$
\begin{aligned}
L\left(p, q, f\right) &= T^{-1} \sum_{t=1}^{T} \log L_t\left(p_t, q_t \,|f_t\right) \\
&+ \left(T - K\right)^{-1} \sum_{t=1+K}^{T} \log \phi \left(f_t - A_0 - \sum_{k=1}^{K} A_k f_{t-k}, C_\eta \right).
\end{aligned}
$$

where

$$
\begin{aligned}
L_t\left(p_t, q_t \,|f_t\right) &= \sum_{s^* \in \mathcal{S}_J} \text{Ł}_{st}\left(p_t, q_t \,|f_t, s^*\right) \Pr\left(s_t^* | \, f_t\right) \\
\Pr\left(s_t^* | \, f_t\right) &= \prod_j \Phi\left(-a_{sjt}\right)^{s_j^*} \left[1 - \Phi\left(-a_{sjt}\right)\right]^{1-s_j^*} \\
a_{sjt} &= \alpha_{sj} + \lambda_{sj}^T f_t
\end{aligned}
$$

Estimation: EM algorithm (1)

- Use EM algorithm combined with numerical approximation:
  1. Guess θ_{w-1}.
  2. For M draws of $\{\varepsilon_{sjtm}\}$, compute $\pi_w(t,m) = \Pr\left(s_{tm} \middle| P, Q, F, \theta_{w-1}\right)$.
  3. Choose θ_w to maximize

$$L^{EM}(p,q,f) = T^{-1}\sum_{t=1}^{T} L_t^{EM}(p_t, q_t \,|f_t) \\ + (T-K)^{-1}\sum_{t=K+1}^{T} \log\phi\left(f_t - A_0 - \sum_{k=1}^{K} A_{t-k} f_{t-k}, \Sigma_\eta\right)$$

where

$$L_t^{EM}(p_t, q_t \,|f_t) = \sum_{m=1}^{M} \pi_w(t,m) \log L_{st}(p_t, q_t \,|f_t, s_{tm}). \\ + \sum_{m=1}^{M} \pi_w(t,m)\left(\sum_j \log\Pr\left(s_{jtm} \middle| f_t\right)\right)$$

Estimation: EM algorithm (2)

- Weights in L_t^{EM} are given by

$$\pi_w(t,m) = \Pr\left(s_{tm}\,|p_t, q_t, f_t, \theta_{w-1}\right)$$

$$\Pr\left(s_{tm}\,|p_t, q_t, f_t, \theta_{w-1}\right) = \frac{\Pr\left(s_{tm}, p_t, q_t\,|f_t, \theta_{w-1}\right)}{\sum_m \Pr\left(s_{tm}, p_t, q_t\,|f_t, \theta_{w-1}\right)}$$

$$\Pr\left(s_{tm}, p_t, q_t\,|f_t, \theta_{w-1}\right) = L_{st}\left(p_t, q_t\,|f_t, s_{tm}, \theta_{w-1}\right)\prod_j \Pr\left(s_{jtm}|\, f_t, \theta_{w-1}\right)$$

Penalisation

1. Assume most off-diagonal elements of matrix β equal zero. Add $\text{L1}\,(\beta)$ penalty to impose sparsity.
2. Need to normalise factors and loadings.
    1. Impose constraints $\frac{1}{T} ff^T = I$ and $\lambda^T \lambda =$ diagonal.
3. Impose sparsity on covariance matrix by adding penalties: $\text{L1}(\Delta C_{Hj})$, $\text{L1}(\Delta C_{Lj})$, $\text{L1}(\Delta C_{Yj})$, $\text{L1}(\Delta C_{HYj})$, and $\text{L1}(\Delta C_{LYj})$.

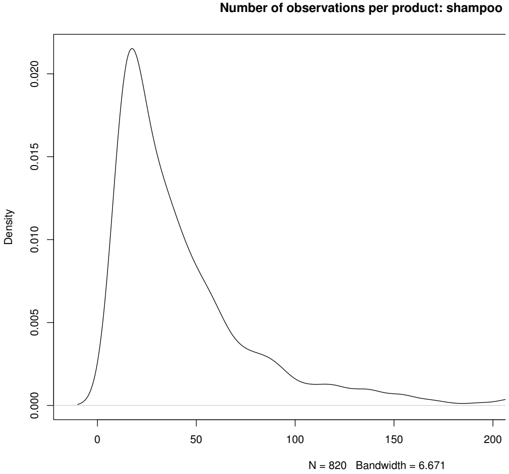
Section 3

Data

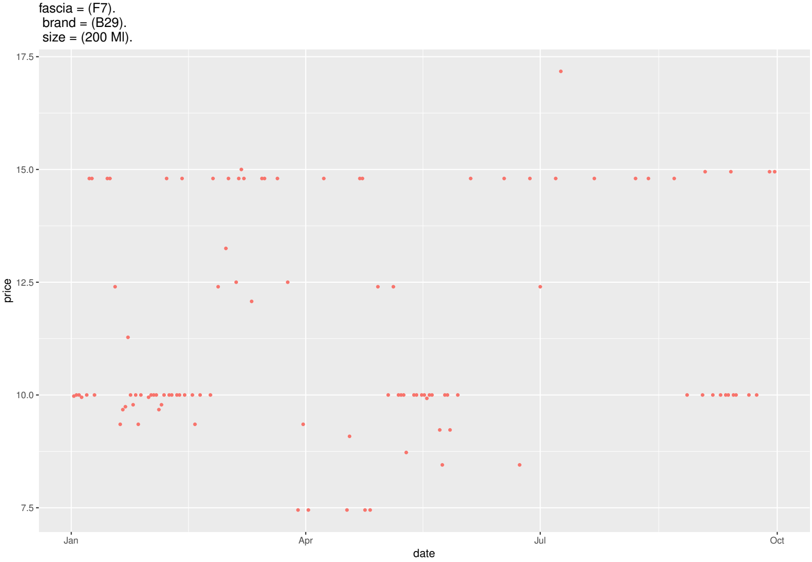
Shampoo data

- 13 fascia: Aldi, Asda, Co-Op, Costco, Iceland, Lidl, Morrisons, Ocado, other, Sainsbury's, Tesco, Tesco Metro, and Waitrose.
- 220 brands. For example, Alberto Balsam, Aldi Shampopo, Aussie Aussome Vol Shamp, Head & Shoulders, Tesco Standard Shampoo.
- Multiple bottle sizes. E.g. 150ml, 400 ml, 1000 ml.
- 820 "products" defined by fascia, brand and pack size.

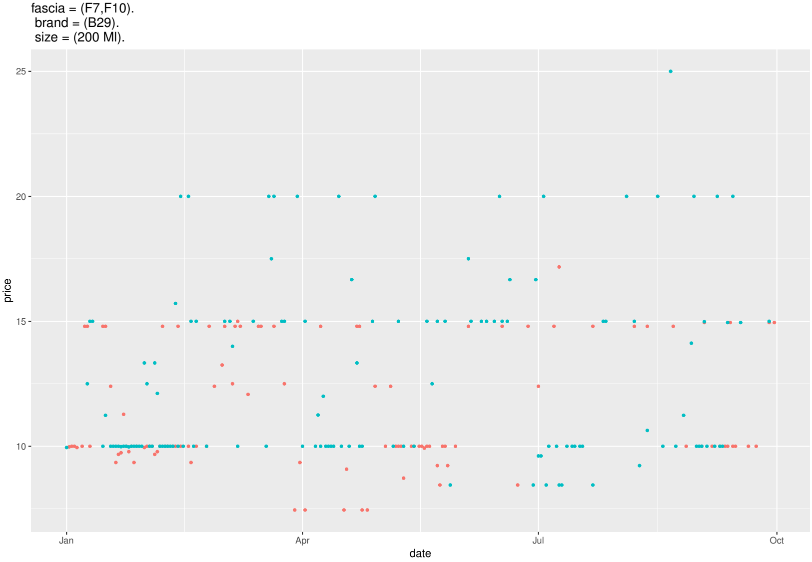
Number of observations per product: shampoo 2016



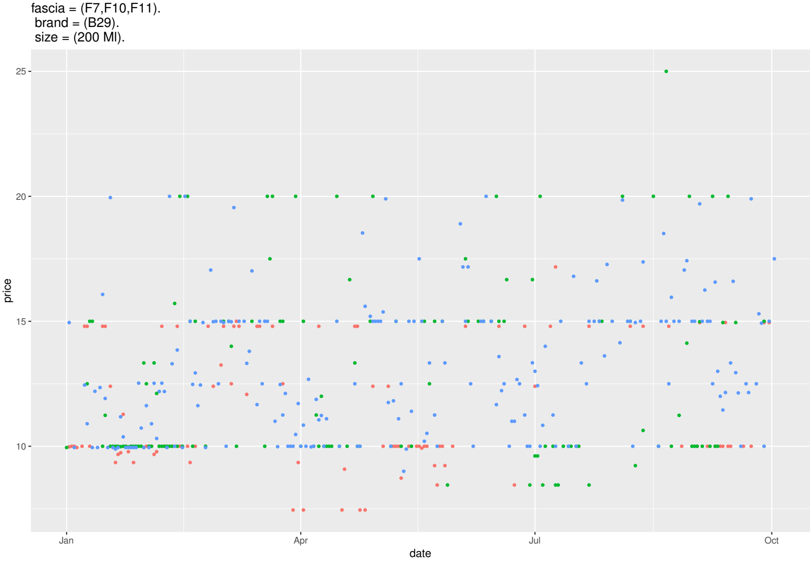
Shampoo: data



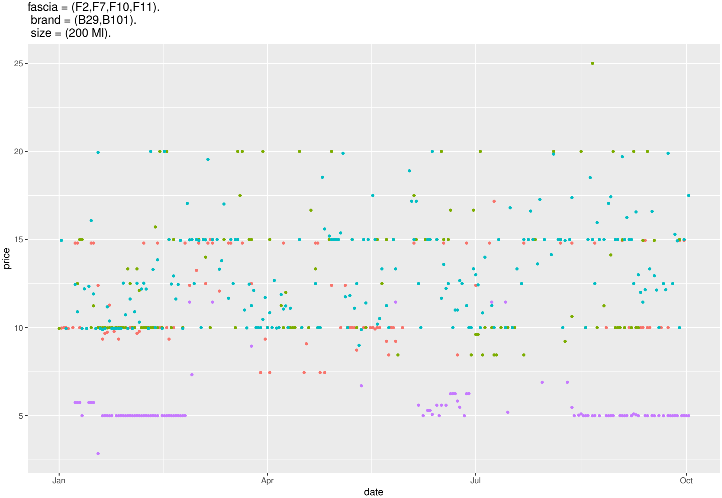
Shampoo: data



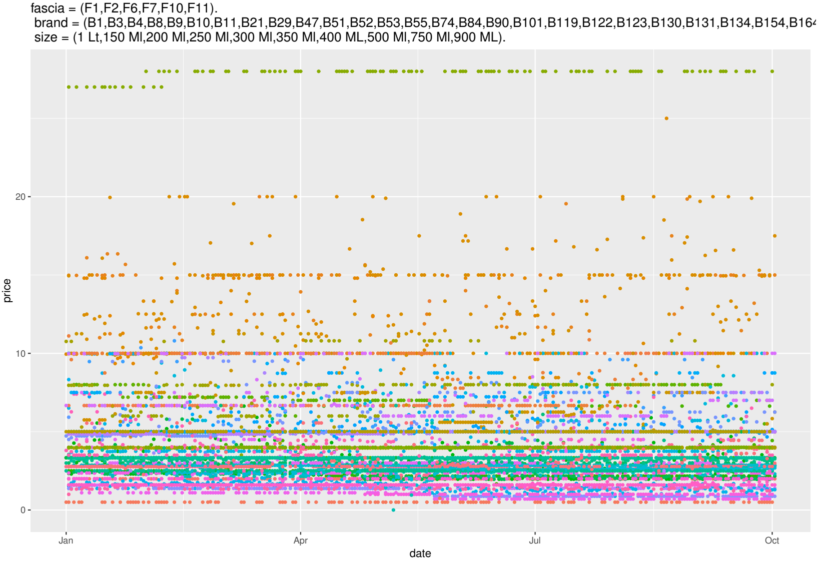
Shampoo: data



Shampoo: data



Shampoo: data



Shampoo data summary

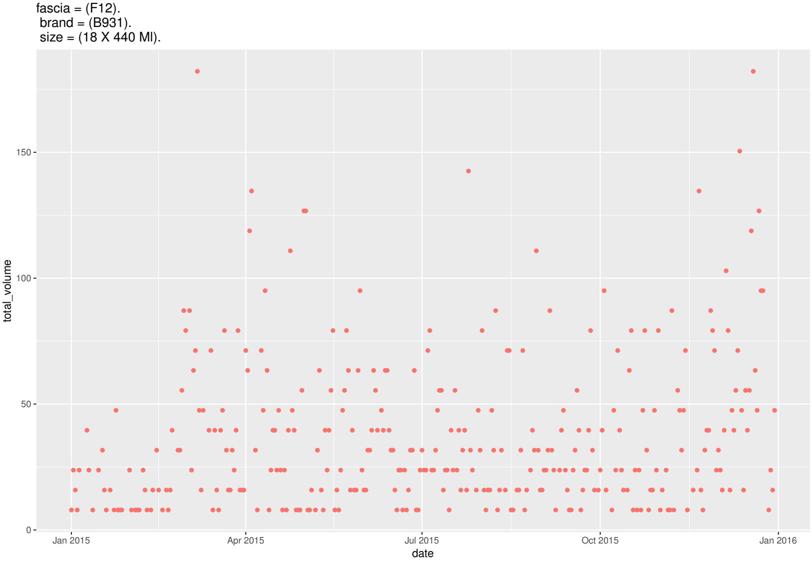
- Large volume of missing prices.
- Current model does not allow for "sticky prices" nor for prices that never move.
- Work in progress: "sticky price" model.
  - Prices only move when sales state changes.
  - Otherwise, price equals price from previous period.

$$\begin{aligned} p^*_{jt} &= \left|\Delta s^*_{jt}\right| \left(s^*_{jt} p_{Hjt} + \left(1 - s^*_{jt}\right) p_{Ljt}\right) \\ &\quad + \left(1 - \left|\Delta s^*_{jt}\right|\right) p^*_{j,t-1} \end{aligned}$$

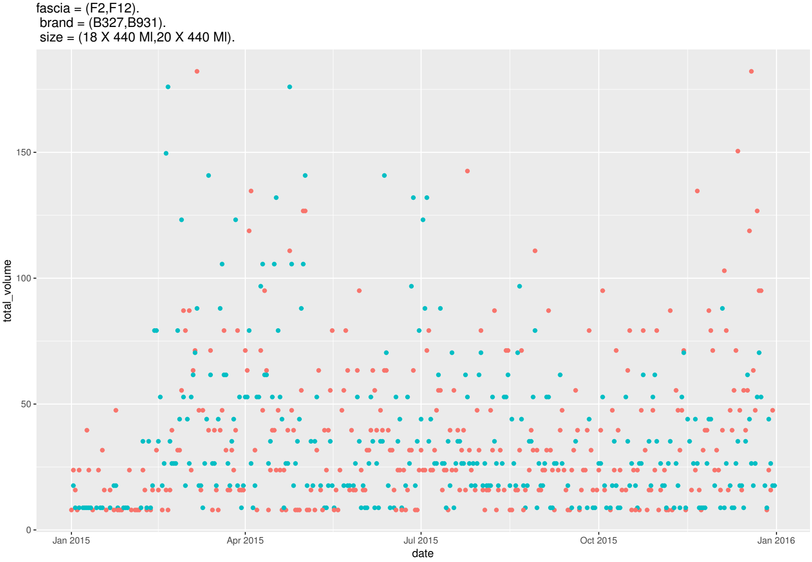
Beer data

- More than 1,000 brands including Stella Artois, Adnams Broadside Ale, Tesco lager, etc.
- Multiple sizes and pack types: can vs. bottle, single item vs multi-pack, 250ml, 500ml, etc.
- Submarkets for lager, ale, cider, bitter, etc.
- Missing data and price plots look similar to shampoo.

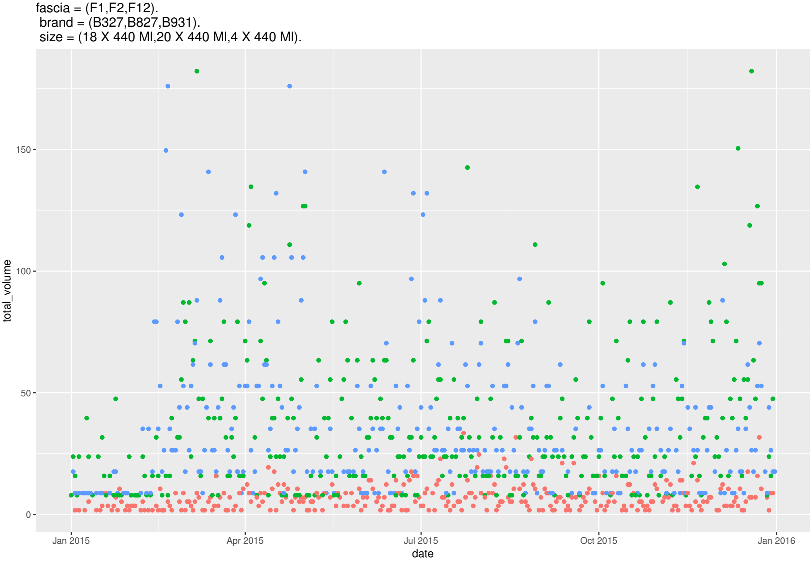
Data: beer quantities



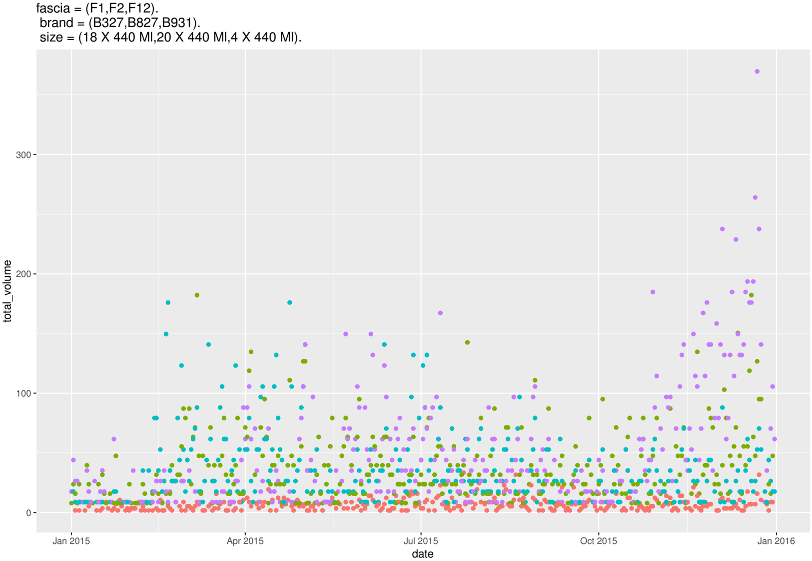
Data: beer quantities



Data: beer quantities

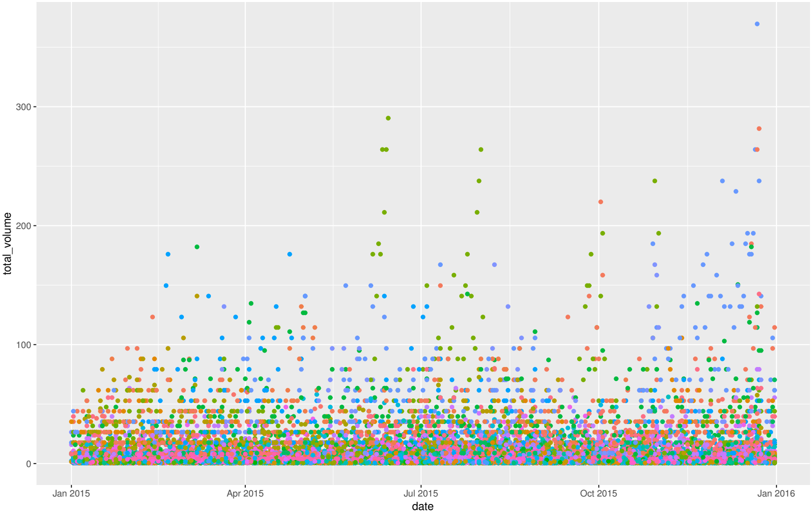


Data: beer quantities



Data: beer quantities

fascia = (F1,F2,F4,F6,F9,F10,F12).
brand = (B26,B28,B62,B76,B84,B93,B109,B125,B129,B160,B173,B211,B219,B220,B318,B327,B357,B358,B415,B416,B418,B435,B
size = (10 X 250 Ml,12 X 440 Ml,15 X 440 Ml,18 X 440 Ml,20 X 440 Ml,4 X 440 Ml,4 X 500 Ml,4 X 568 Ml,500 Ml,6 X 330 Ml,660 ML,8



Section 4

Simulation results

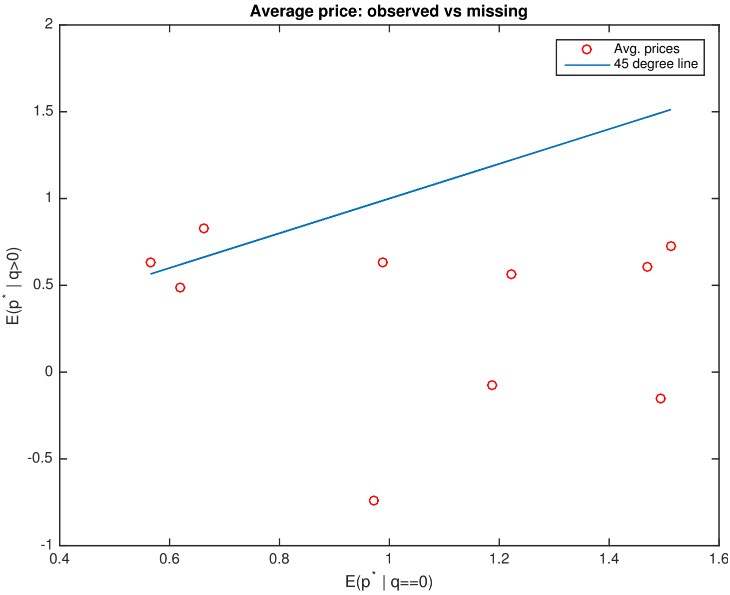
Preliminary simulation results

- Simulate data for an economy with J=10, T=100, R = 2.
- Estimated parameters using EM algorithm and penalised likelihood.
- Analysis of Monte Carlo results in progress.
- Some limited experiments with J = 50 and J=100. Requires parallel computation to speed up estimation. Work in progress.
- Below, I show some results on missing data bias from small scale experiment.

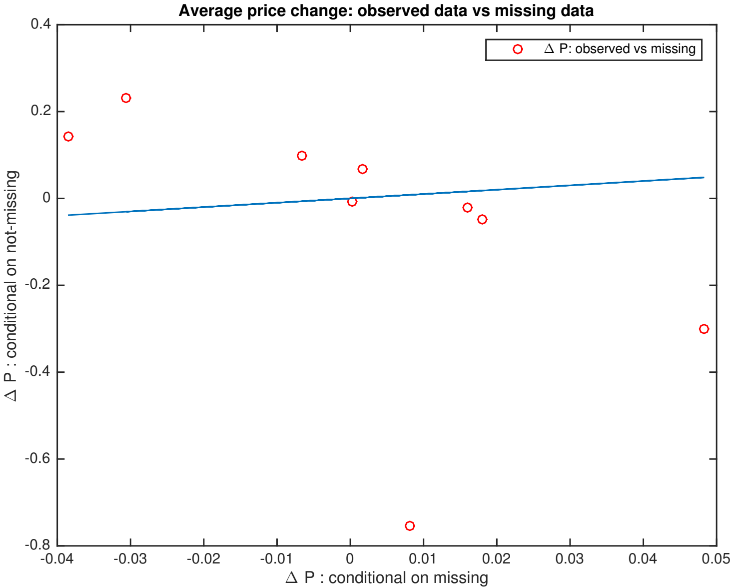
Bias due to missing data

- Plot $E\left(p_{jt}^{*}\middle|\, q_{jt} > 0\right)$ vs. $E\left(p_{jt}^{*}\middle|\, q_{jt} = 0\right)$.
- Plot $E\left(s_{jt}^{*}\middle|\, q_{jt} > 0\right)$ vs. $E\left(s_{jt}^{*}\middle|\, q_{jt} = 0\right)$.
- Plot $E\left(\Delta p_{jt}^{*}\middle|\, q_{jt} > 0 \text{ and } q_{j,t-1} > 0\right)$ vs. $E\left(\Delta p_{jt}^{*}\middle|\, q_{jt} = 0 \text{ or } q_{j,t-1} = 0\right)$.
- Plot true β_{jj} vs estimate from OLS regression of q_{jt} on (p_{jt}, f_t).

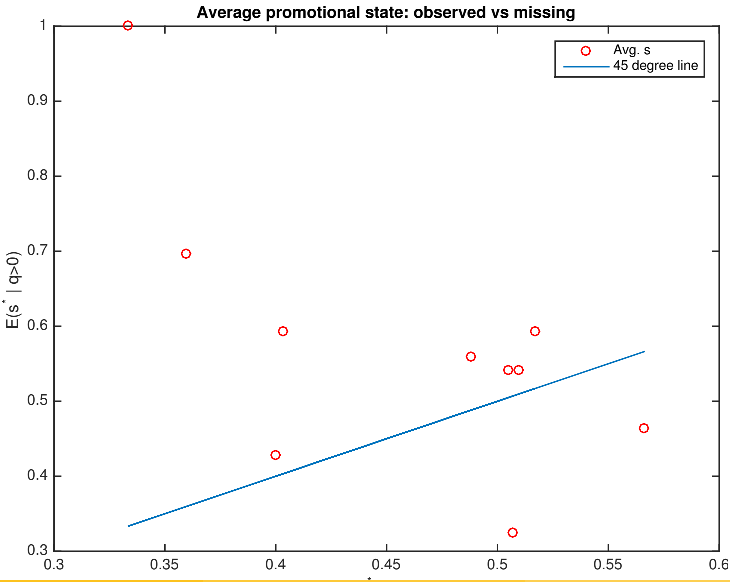
Bias in price levels: simulation



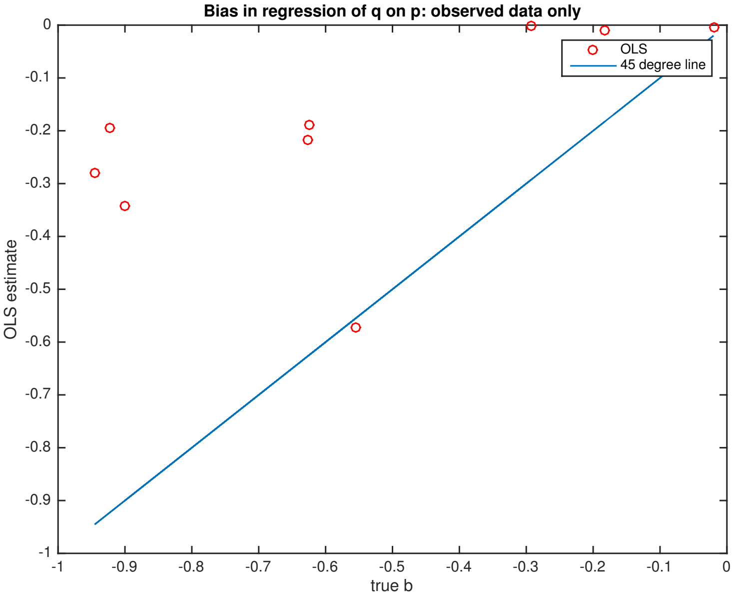
Bias in price changes: simulation



Average sales state: conditional on missing status: simulation



Bias in slope of demand function: simulation



Section 5

Conclusions and extensions

Price index

- From 2023, ONS will integrate scanner data with web-scraped and traditional data sources.
- Our method allows to construct multilateral price index (Diewert and Fox, 2018) that accounts for missing data.
- For new product introductions, our method can be used to estimate the virtual price at which demand equals zero.

Ongoing work

- Estimate model for a few categories of food (shampoo, beer, laundry detergent)
- Empirically analyse how results affect price indices.
- Complete work on sticky price model.
- Applications to competition policy, optimal taxation.
- Developing model of firms' high frequency price setting competition.

Mingli Chen, Iván Fernández-Val, and Martin Weidner. Nonlinear panel models with interactive effects. *arXiv preprint arXiv:1412.5647*, 2014.

Mingli Chen, Iván Fernández-Val, and Martin Weidner. Nonlinear factor models for network and panel data. *Journal of Econometrics*, 220(2): 296–324, 2021.

W Erwin Diewert and Kevin J Fox. Substitution bias in multilateral methods for cpi construction using scanner data. *UNSW Business School Research Paper*, (2018-13), 2018.

Kenneth L Judd. Cournot versus bertrand: A dynamic resolution. *Hoover Institution*, 1996.

Finn Kydland. Noncooperative and dominant player solutions in discrete dynamic games. *International economic review*, pages 321–335, 1975.

Madeleine Udell, Corinne Horn, Reza Zadeh, Stephen Boyd, et al. Generalized low rank models. *Foundations and Trends® in Machine Learning*, 9(1):1–118, 2016.