Platform Competition and Interoperability: The Net Fee Model*

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August 11, 2022

Abstract

Is more competition the key to mitigating dominance by large tech platforms? Could regulation of such markets be a better alternative? We study the effects of competition and interoperabilty regulation in platform markets. To do so, we propose an approach of competition in *net fees*, which is well-suited to situations where users pay additional charges, after joining, for on-platform interactions. Compared to existing approaches, the net fee model expands the tractable scope to allow variable total demand, platform asymmetry and merger analysis. Regarding competition, our findings raise two concerns: adding more platforms may lead to market contraction or the emergence of a dominant firm. In contrast, we find that interoperability can play a key role in boosting user participation and reducing market dominance. Broadly speaking, our results favor policy interventions that improve the quality of market competition, as opposed to those that merely give rise to more competitors.

Keywords: Platform Competition, Big Tech, Net Fees, Interoperability

JEL Codes: D43, L13, L15, L40

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*We thank Jacques Crémer, Xavier Gabaix, Marc Rysman, Lucy White and seminar participants at Boston University for helpful feedback and Baiyun Jing for excellent research assistance. Wu thanks the NET Institute (www.netinst.org) for financial support.

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1 Introduction

Large Internet platforms (e.g., Alibaba, Amazon, Apple, Facebook, Google, Tencent, etc.) are at the center of many of today's most important public policy debates. Platforms invite much criticism, including claims that they are too dominant and should, in some form, have their power reigned in.[1] Some argue that platforms should face more competition, while others focus on regulation. Behind this debate lies a set of basic economic questions. Can more competition in platform markets mitigate dominance? If not, what other remedies might work? One prominent proposal is regulation that would make competing platforms "interoperable" and thus less proprietary. Does this show promise?

This paper offers a modeling approach that sheds light on these two issues: the effects of increasing competition and requiring platform interoperability. Our approach, which we call the *net fee model*, brings about a high level of flexibility to the technically challenging topic of platform competition. The basic idea is that platforms compete by setting a kind of price that we refer to as their "net fee." (See Section 2 for details and motivation.) Importantly, using this approach allows us to solve a general discrete-choice model with network effects that accommodates asymmetries across platforms and variable total market participation. We provide three main sets of results, pertaining to (i) pricing and characterization of equilibrium, (ii) the effects of competition, and (iii) the consequences of interoperabilty. With regard to the latter two points we particularly focus on how these forces affect total platform demand and market dominance.

In the first category, we show that net fee competition leads to a straightforward pricing formula. We further show that this can be compared with numerous familiar pricing benchmarks, including from standard oligopoly competition and from earlier work on platform pricing. In line with pricing formulas in the existing literature on platforms, ours features the same three components – marginal cost, market power and network discount – while having the benefit of being more broadly applicable.

The second set of results deals with the effects of competition. Here, we first restrict

[1] For instance, see a trio of recent, high-profile policy reports addressing these issues (Crémer, de Montjoye and Schweitzer, 2019, Furman et al., 2019, Scott Morton et al., 2019).

attention to symmetric equilibrium. We show that increasing the number of competing platforms can lead to lower overall demand across all platforms. This expands upon the "perverse pattern" finding of Tan and Zhou (2021), whereby more competition leads to higher prices, in an environment with fixed total demand.

We then turn to the effects of competition on market dominance. We show that adding a new platform may lead one platform to become dominant or may enhance the position of an already dominant platform. Moreover, a potential merger between two small platforms can reduce the dominance of a large one, and the scope for this to occur grows as network effects become stronger. That is, in a potential merger, network effects can serve as a substitute for cost synergies. To see the underlying mechanism behind these results, consider a setting with one dominant platform and one or more niche platforms. The larger a platform's user base, the stronger its incentive is to discount its net fee. As competition increases, niche platforms' user bases get divided up, and the discounts they offer shrink. Thus, adding a new platform can lead to splintering among the niche players, enabling the dominant platform to capture a larger market share.

The third set of results studies the impact of allowing platforms to be at least partially *interoperable*. By this, we mean that, when two platforms are interoperable with one another, a user who joins either platform can enjoy the network externalities that come from the user bases of both platforms. Think, for instance, of the way a subscriber to one phone company can have conversations with subscribers to other companies. In our model, rather than being a discrete variable, interoperability can take on any value between 0 and 1.

Here, we once again focus first on the symmetric case. In this context, our findings tilt strongly in favor of interoperability. We show that, as the level of interoperability increases among platforms, their equilibrium net fee decreases, and more users participate, except in the special case of duopoly with no outside option. The main driving feature of this result is that higher interoperabilty acts like a technological improvement, and it is only in the aforementioned special case that platforms capture all of the benefits.

Finally, we analyze the effect of interoperability in a setting where one platform is more dominant than another. We show that increasing interoperability reduces the dominance of the bigger platform. This occurs because a higher degree of interoperability reduces the disparity between the network discounts that the two platforms offer, which is the driving force behind market dominance in the first place.

In the broad public policy debate regarding platforms, three topics that have received particular interest are, (i) what the effect might be of breaking up large firms, (ii) whether to promote entry by new competitors, and (iii) what the impact could be of requiring interoperability. Our finding of demand contraction under symmetric competition suggests the need for caution with respect to the first proposal. Our result that entry may increase an incumbent's dominance similarly urges caution with respect to the second. In contrast, our analysis of interoperability offers an encouraging view of such regulation, suggesting that such proposals should be explored in more detail.

1.1 Related Literature

This paper contributes to the broad literature that has come to be known as "platform economics." The earliest works in this area, which formalize the study of network effects in "one-sided" settings, include Rohlfs (1974), Katz and Shapiro (1985), Farrell and Saloner (1985). A significant step forward occurred when the concept of "multi-sidedness" was introduced, incorporating multiple groups of agents with interdependent demand. Pioneering works on multisided platforms include Caillaud and Jullien (2003), Evans (2003), Rochet and Tirole (2003, 2006), Rysman (2004), Anderson and Coate (2005), Parker and Van Alstyne (2005), Armstrong (2006), and Hagiu (2006). In the monopoly context, Weyl (2010) provides a general synthesis of the incentives influencing a platform's pricing.[2] Jullien, Pavan and Rysman (2021) offers an excellent, recent survey of the platform literature.

When appropriate, we compare the results obtaining under our net fee approach with those that arise under the most conventioanl approach, which we call "total pricing." Likely the best known example of total pricing is the Armstrong (2006) "two-sided single-homing" model, a Hotelling setup which has served as a workhorse in much subsequent literature. Recently, Tan and Zhou (2021) provides an important generalization of the total pricing approach and identify a so-called "perverse pattern," whereby competition

[2]Also see Veiga, Weyl and White (2017), which further generalizes the Weyl (2010) model to allow for selection effects as well as network effects.

increases prices. In Section 5, we provide a related result.

In addition to total pricing, the literature has considered several other assumptions on platforms' conduct, including the platform Cournot approach of Correia-da Silva et al. (2019) and the two-part tarriff competition of Reisinger (2014). Our approach is most closely related to the one developed in an earlier working paper, White and Weyl (2016), called "insulated equilibrium." Both embed, in an oligopoly framework, Dybvig and Spatt's (1983) insight regarding monopoly, further developed by Becker (1991) and Weyl (2010), that appropriately designed prices can alleviate potential coordination problems for users. Insulated equilibrium differs from our approach in that it applies a refinement to select from among multiple equilibria arising in a higher-dimensional strategy space, and it focuses on the effects of different forms of user heterogeneity. One separate but related line of research focuses on dynamic platform competition (Cabral, 2011) and explores the link between dynamic competition and static models of conduct (Cabral, 2019). Another related line of research focuses on multi-homing, following Armstrong (2006), Armstrong and Wright (2007), where Liu et al. (2022) provides a relevant recent contribution.[3]

On a technical level, our proof of equilibrium existence extends a result of Caplin and Nalebuff (1991). In order to derive results on uniqueness, we use aggregative game (Selten, 1970) techniques from Anderson, Erkal and Piccinin (2020).

2 The Model

Users can each choose to join at most one platform. Their options are indexed by $j \in \mathcal{J} \cup \{0\} = \{0, 1, ..., J\}$, where $J \geq 1$ is the number of platforms and 0 denotes the outside option. "Sides of the market" are indexed by $s \in \mathcal{S} = \{1, ..., S\}$, where $S \geq 1$. Each side of the market has a unit mass of users, each user belongs to exactly one side, and each platform serves all sides.

Users of a side s are identified by a type $\theta_s = (\theta_s^0, \theta_s^1, ..., \theta_s^J) \in \mathbb{R}^{J+1}$ which captures their *membership value* (standalone taste) for each platform as well as for the outside option. Types are distributed according to cumulative distribution function (CDF) F_s. We assume that F_s

[3] White (2022) discusses some ways in which interoperability and multi-homing are related.

admits a density f_s which is continuously differentiable and strictly positive on $\mathbb{R}^{J+1}$.

Payoffs from joining platform j may also depend on how many other users join j. Denote by n_s^j the fraction of side-s users that join platform j, and denote by p_s^j platform j's total side-s price. Users have quasilinear preferences with respect to money, and the payoff to user θ_s from joining platform j is

$$u_s^j := \theta_s^j + \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j - p_s^j, \tag{1}$$

where $\gamma_{s\hat{s}}^j$ denotes the *interaction value* with side-$\hat{s}$ users on the same platform.[4] That is, it measures the marginal externality that a user on side $\hat{s}$ of platform j contributes to users on side s of platform j. The payoff from choosing the outside option is $u_s^0 := \theta_s^0$.

Platforms compete by posting *net fees* to users, $t^j = (t_1^j, ..., t_S^j)$. A net fee t_s^j guarantees a user with type θ_s a payoff from joining platform j of

$$u_s^j = \theta_s^j - t_s^j. \tag{2}$$

This payoff does not depend on the joining decisions of other users. A net fee t^j implies a user who joins platform j pays a total transaction price of

$$p_s^j := t_s^j + \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j, \tag{3}$$

where the first term is the net fee and the second term is the interaction utility generated by the platform.

Given a profile of net fees $t_s = (t_s^1, ..., t_s^J)$ charged by platforms, a user on side s with type θ_s chooses the $j \in \mathcal{J} \cup \{0\}$ yielding the maximal u_s^j. The demand for platform j on side s is then

$$n_s^j(t_s) = \int \mathbf{1}_{\{u_s^j \geq u_s^k, \forall k \in \mathcal{J}\cup\{0\}\}} f_s(\theta_s) \mathrm{d}\theta_s. \tag{4}$$

[4]For simplicity, we assume interaction utility to be linear, but this is not essential for our approach.

Let $t \in \mathbb{R}^{J \times S}$ denote the vector of all net fees charged by all platforms on all sides. Platform j earns, from side-s users, profits of

$$\pi_s^j(t) = \left(p_s^j - c_s^j\right) n_s^j(t_s) \tag{5}$$

$$= \left(t_s^j + \sum_{\hat{s} \in \mathcal{S}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j(t_{\hat{s}}) - c_s^j\right) n_s^j(t_s), \tag{6}$$

which can be summed to give total profits of

$$\pi^j(t) = \sum_{s \in \mathcal{S}} \pi_s^j(t) \tag{7}$$

$$= \sum_{s \in \mathcal{S}} (t_s^j - c_s^j) n_s^j(t_s) + \sum_{s, \hat{s} \in \mathcal{S}} \gamma_{s\hat{s}}^j n_s^j(t_s) n_{\hat{s}}^j(t_{\hat{s}}). \tag{8}$$

We write consumer surplus for users on side s as

$$V_s(t_s) := \int \max_{j \in \mathcal{J} \cup \{0\}} \{\theta_s^j - t_s^j \cdot \mathbf{1}_{j \in \mathcal{J}}\} f_s(\theta_s) \mathrm{d}\theta_s, \tag{9}$$

noting that $\frac{\partial V_s}{\partial t_s^j} = -n_s^j$. Define total surplus by $W(t) := \sum_{j \in \mathcal{J}} \pi^j + \sum_{s \in \mathcal{S}} V_s$.

In the game, platforms simultaneously announce net fees, which determine the demand and the profits. We focus on pure-strategy Nash equilibria, $t = (t^1, ..., t^J)$, where each platform $j \in \mathcal{J}$ chooses a vector of net fees, t^j, that maximizes $\pi^j(\cdot, t^{-j})$, where t^{-j} denotes the vector of net fees announced by platforms other than j.

2.1 Competition in Net Fees

The key feature of the net fee approach is an assumption on platforms' competitive conduct. In the model, the strategic variable that they set can be interpreted as the fee that users pay (or subsidy they receive) in order to join a given platform. This fee does not encompass all of the money that a platform earns per user. In addition, users that join a given platform interact with other users, generating interaction utility, all of which the platform extracts.

A simple story that corresponds to this assumption is one in which, after making a decision to join a platform, users become captive. Consider, for instance, the choice a user makes when deciding which video game console to purchase. At the time of the joining decision, users have the opportunity to shop around among different gaming platforms. They do so knowing that, after having joined, they will be at the mercy of the platform's fee structure for games. The gaming platforms also know this, and they compete to get users to sign up. In setting their prices, they take into account both the money users will pay to join and the subsequent stream of revenue that will generate. It is straightforward to write down an extensive form game with these features that boils down to exactly our net fee model.

The net fee approach contrasts with the more traditional *total price* modeling technique. (Armstrong, 2006, Tan and Zhou, 2021) That approach assumes platforms compete by setting single, all-inclusive prices capturing joining fees and interaction revenue in one variable. Such an assumption seems especially well suited to cases where, after users have joined a platform, there is little opportunity for the platform to earn additional revenue from users' interaction with one another. One such example might be a dating platform where users each pay in order to have access to the entire pool of potential romantic partners.

In reality, of course, platforms can employ more sophisticated pricing schemes. Any model hoping to capture platform competition inevitably must represent their behavior in a highly stylized way. Both the net fee approach and the total price approach are abstractions that simplify differently. Given the above discussion, the net fee model might a better fit in situations where, after joining, users typically make transactions involving the platforms. In cases where they do not, the total price model may be a better fit. Surveying the range of platform industries that are of particular interest to policymakers (e.g., search engine and social media advertising, software application stores, ride hailing), it is clear that interaction-based revenue (e.g., pay-per-click advertising, commissions, etc.) often plays an important role. Thus, we believe the net fee model is a helpful addition to the toolkit. This is especially true because, when comparable, the two approaches deliver qualitatively similar results (see Section 3.2), yet the net fee model turns out to be usable in an especially wide range of environments.

Remarks. We briefly highlight the following properties of the model.

1. Independently of the conduct assumed in the game, the vector of net fees is the relevant argument in the demand system we study.[5] That is, holding fixed an arbitrary profile of platform strategies (which might be net fees, total prices, or other, as long as there is no within-side price discrimination), the demand profile on side S depends precisely on the u_s^j's users receive. In this more general case, a net fee t_s^j can be defined as the difference between membership value θ_s^j and utility u_s^j, which is the same for all side-s users. Thus, the net fee is the relevant measure for demand and consumer surplus. Consequently, our framing of results in terms of t_s is not driven, *per se*, by the net fee conduct assumption but rather by the demand environment. Indeed, a point that we hope to convey is that, in this class of model, even when assuming total price conduct, comparative statics focused on net fees t_s can often be more informative that those focused on total prices.

2. Adopting net fee conduct contributes two features to the model that expand the scope of possible analysis.

   A. Net fees lead demand to be fully determined, so there is no problem of equilibrium multiplicity among users. Thus, for the sake of tying down demand, we invoke no constraints on the strength of network effects nor must we apply an equilibrium selection to the continuation game played by users.

   B. Under the net fee approach, the standard oligopoly demand system, as determined by the distribution of membership values, and network effects enter into platform j's profit function in a separable way. (See eq. (6).) In other words, given a demand system, our game with nonzero γ's and net fees has an analog involving the same demand system $n^j(t)$, no network effects, differentiated Bertrand competition, and variable marginal cost $c^j - \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j$ for each platform. This makes first-order conditions straightforward to express in a general environment. Also, it means that underlying properties of a given demand system are preserved.[6]

[5]This point extends to settings where users have nonlinear interaction values, as in Tan and Zhou (2021).

[6]For example, net fee conduct can preserve the aggregative property of demand, which Anderson, Erkal and Piccinin (2020) shows to be useful in analyzing oligopoly.

3 Pricing

3.1 Pricing under Net Fee Competition

First, we analyze the net fees that platform j chooses as a best response when the competing platforms choose t^{-j}. Consider the impact of a marginal effect on j's profits resulting from a change in t_s^j, holding fixed t^{-j}. This is given by

$$\frac{\partial \pi^j(t)}{\partial t_s^j} = \left(p_s^j - c_s^j\right)\frac{\partial n_s^j(t_s)}{\partial t_s^j} + n_s^j(t_s)\left(1 + \gamma_{ss}^j \frac{\partial n_s^j(t_s)}{\partial t_s^j}\right) + \frac{\partial\left(\sum_{\hat{s}\in\mathcal{S}\setminus\{s\}} \pi_{\hat{s}}^j(t)\right)}{\partial t_s^j}. \tag{10}$$

The first two terms capture $\frac{\partial \pi_s^j(t)}{\partial t_s^j}$, i.e., the effect of the fee increase on j's profits arising directly from side s, by taking the derivative of eq. (6). These contain the usual effects that appear under differentiated Bertrand competition without network effects as well as an additional factor, $\gamma_{ss}^j \frac{\partial n_s^j(t_s)}{\partial t_s^j}$, representing the within-side externality that j's side-s users exude on one another. The last term captures the impact that changing t_s^j has on j's profits from the other sides of the market. Plugging in $\frac{\partial \pi_{\hat{s}}^j(t)}{\partial t_s^j} = \frac{\partial n_s^j(t_s)}{\partial t_s^j}\gamma_{\hat{s}s}^j n_{\hat{s}}^j(t_{\hat{s}})$, the right-hand side of eq. (10) simplifies to

$$\left(p_s^j - c_s^j + \frac{n_s^j(t_s)}{\frac{\partial n_s^j(t_s)}{\partial t_s^j}} + \sum_{\hat{s}\in\mathcal{S}} \gamma_{\hat{s}s}^j n_{\hat{s}}^j(t_{\hat{s}})\right)\frac{\partial n_s^j(t_s)}{\partial t_s^j}. \tag{11}$$

The last term, $\frac{\partial n_s^j(t_s)}{\partial t_s^j}$, is strictly negative, because the density of types is strictly positive everywhere. Thus, the first-order condition that must hold in any best response implies that the bracketed term in eq. (11) must equal zero. Hence, we can immediately obtain the pricing formula of Proposition 1.

Proposition 1. *At any equilibrium, the net fee that platform j charges to users on side s satisfies*

$$t_s^j = c_s^j + \frac{n_s^j(t_s)}{-\frac{\partial n_s^j(t_s)}{\partial t_s^j}} - \sum_{\hat{s}\in\mathcal{S}} (\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j) n_{\hat{s}}^j(t_{\hat{s}}). \tag{12}$$

This proposition says that a platform's net fee is equal to the sum of (i) its marginal cost of serving a user, (ii) its standard "one-sided" market power, $\frac{n_s^j(t_s)}{-\frac{\partial n_s^j(t_s)}{\partial t_s^j}}$, and (iii) a term we refer to as the *network discount*. The last term captures the total interaction value that is generated when an additional side-s user joins platform j. It can be decomposed as follows. The first component, $\sum_{\hat{s}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j$, equals the additional money that a side-s user pays the platform, beyond the net fee. The second component, $\sum_{\hat{s}} \gamma_{\hat{s}s}^j n_{\hat{s}}^j$, measures the marginal interaction value that an additional side-s user creates for other users across all sides, which the platform extracts from them.

Example: Logit Demand. A particularly convenient functional form, which we use in Sections 5 and 6 on policy analysis, involves demand that takes on the Logit form, i.e.,

$$n_s^j(t_s) = \frac{e^{-t_s^j}}{e^{z_s} + \sum_{k \in \mathcal{J}} e^{-t_s^k}}, \tag{13}$$

where z_s parameterizes the outside option for side-s users.[7] Note that this gives $\frac{\partial n_s^j(t_s)}{\partial t_s^j} = -n_s^j(1 - n_s^j)$, and so the net fee in Proposition 1 becomes

$$t_s^j = c_s^j + \frac{1}{1 - n_s^j} - \sum_{\hat{s} \in \mathcal{S}} (\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j) n_{\hat{s}}^j(t_{\hat{s}}). \tag{14}$$

3.2 Relationship to Benchmarks

The pricing formula in Proposition 1 relates as follows to these notable benchmarks.

1. Compared to the net fee that maximizes total surplus, $W(t)$,

$$t_s^j = c_s^j - \sum_{\hat{s} \in \mathcal{S}} (\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j) n_{\hat{s}}^j(t_{\hat{s}}), \tag{15}$$

   it has the additional one-sided market power term.[8] The final terms in eqs. (12) and (15) coincide, because each platform fully internalizes the network effects that are

[7]Demand as in eq. (13) arises when the membership values of side-s users are drawn independently, with $\theta_s^1, \ldots, \theta_s^J \sim \text{Gumbel}(0,1)$ and $\theta_s^0 \sim \text{Gumbel}(z_s, 1)$.

[8]Equation (15) can be obtained by noting that the first-order condition for maximization of total surplus, $\frac{\partial \pi^j(t)}{\partial t_s^j} + \sum_{k \in \mathcal{J} \setminus \{j\}} \frac{\partial \pi^k(t)}{\partial t_s^j} + \frac{\partial V_s}{\partial t_s^j} = 0$, implies that $t_s^j = c_s^j - \sum_{\hat{s} \in \mathcal{S}} (\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j) n_{\hat{s}}^j$.

created by adding a marginal user.[9]

2. It generalizes the one from standard differentiated Bertrand competition, in discrete choice models without network effects, where all γ's are equal to zero.

3. In the special case of two-sided monopoly ($J = 1$, $S = 2$), it coincides with the "pure-membership" pricing formulas of Rochet and Tirole (2006) and Armstrong (2006).[10] Under S-sided monopoly, it coincides with the formula of Weyl (2010), and under two-sided oligopoly, it coincides with the formula of White and Weyl (2016), both specialized to affine, homogeneous-within-side interaction values.

4. Compared to Tan and Zhou's (2021) symmetric-equilibrium oligopoly pricing formula, our expression relates in the following way. Their paper makes a significant generalization of Armstrong (2006)'s classic "two-sided single-homing" Hotelling model.[11] In order to derive their formula, they assume that the distribution of membership values is symmetric across the platforms, marginal costs are identical across platforms and equal to c_s, interaction values are identical across platforms and equal to $\gamma_{s\hat{s}}$.[12] Furthermore, users have no outside option. For each $s \in \mathcal{S}$, let H_s and h_s denote the cumulative distribution function (CDF) and probability density function (PDF) of $\theta_s^1 - \max\{\theta_s^2, ..., \theta_s^J\}$.[13]

   At the symmetric equilibrium they study, the pricing formula is

$$p_s = c_s + \frac{1 - H_s(0)}{h_s(0)} - \frac{1}{J-1} \sum_{\hat{s} \in \mathcal{S}} \gamma_{\hat{s}s}. \tag{16}$$

[9]Note that, although these final terms coincide in the two expressions, they take on different values because the n_j^s's are endogenous. See Tan and Wright (2018) for discussion of this point in the case of monopoly.

[10]See Rochet and Tirole's Proposition 1(iii), which encompasses Armstrong's Section 3 monopoly pricing formula. In RT's notation, their expression $p^i = \frac{p^i}{\eta^i} - b^j$ can be rewritten by substituting $p^i = \frac{A^i - C^i}{N^j}$, $\frac{p^i}{\eta^i} = \frac{N^i}{-\frac{\partial N^i}{\partial p^i}} = \frac{1}{N^j} \frac{N^i}{-\frac{\partial N^i}{\partial A^i}}$ and then translated into ours by noticing their generic A^i, C^i correspond to generic p^j, c^j in our notation.

[11]See Section 4 of Armstrong (2006).

[12]They also provide further generalization of allowing network effects to be nonlinear in demand.

[13]Notice that due to the symmetry of the distribution of membership values, H_s is independent of the platform.

In our model, under these assumptions, expressing eq. (12) as a total price gives

$$p_s = c_s + \frac{1 - H_s(0)}{h_s(0)} - \frac{1}{J} \sum_{\hat{s} \in \mathcal{S}} \gamma_{\hat{s}s}, \tag{17}$$

whose only difference from eq. (16) is in the denominator of the final term.[14]

4 Equilibrium Existence and Uniqueness

In order to guarantee the existence of equilibrium, we make two assumptions that ensure both the sufficiency of the first-order conditions for profit maximization and that the profit maximizing fees are bounded. We start with the following definition, followed by the two assumptions. In the following, we denote the marginal density of θ_s^j by $f_{s,j}\left(\theta_s^j\right)$, the conditional density of θ_s^k (conditioned on θ_s^j) by $f_{s,k|j}(\theta_s^k|\theta_s^j)$, and the conditional CDF of θ_s^0 (conditioned on θ_s^j) by $F_{s,0|j}\left(\theta_s^0|\theta_s^j\right)$.

Definition 1. For each side s, define $\overline{g}_s$ to be the supremum of the conditional density function of the membership values of any pair of alternatives, i.e.,

$$\overline{g}_s := \sup_{j \in \mathcal{J}, k \in \{0\} \cup \mathcal{J} \setminus \{j\}} \sup_{\theta_j, \theta_k} f_{s,k|j}\left(\theta_s^k|\theta_s^j\right). \tag{18}$$

Assumption A1. $\forall s \in \mathcal{S}$, there exists $\rho_s \geq -\frac{1}{J+2}$, such that

(a) the joint distribution of side-s users' membership values, $f_s(\theta_s)$, is ρ_s-concave;

(b) $\left(\gamma_{ss}^j + \sum_{\hat{s} \neq s} \left|\frac{\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j}{2}\right|\right) \cdot \overline{g}_s \leq \frac{1}{2J}\left[1 + \frac{\rho_s}{1+(J+1)\rho_s}\right], \forall j \in \mathcal{J}.$

[14] In terms of net fees, Tan and Zhou's formula becomes

$$t_s = c_s + \frac{1 - H_s(0)}{h_s(0)} - \frac{1}{J-1} \sum_{\hat{s} \in \mathcal{S}} \gamma_{\hat{s}s} - \frac{1}{J} \sum_{\hat{s} \in \mathcal{S}} \gamma_{s\hat{s}},$$

and our eq. (12) specializes to

$$t_s = c_s + \frac{1 - H_s(0)}{h_s(0)} - \frac{1}{J} \sum_{\hat{s} \in \mathcal{S}} \gamma_{\hat{s}s} - \frac{1}{J} \sum_{\hat{s} \in \mathcal{S}} \gamma_{s\hat{s}}.$$

Also note, in particular, that eqs. (16) and (17) share the same limit behavior as the number of platforms grows large.

Assumption A2. $\forall s \in \mathcal{S},\ j \in \mathcal{J}$, we have $\lim_{t_s^j \to \infty} t_s^j \cdot \int_{-\infty}^{\infty} F_{s,0|j}\left(\theta_s^j - t_s^j | \theta_s^j\right) f_{s,j}\left(\theta_s^j\right) \mathrm{d}\theta_s^j = 0$.

Assumption A1 pertains to both the membership value distribution and the magnitudes of interaction values. It ensures that each platform's profit function, $\pi^j(t^j, t^{-j})$, is quasiconcave in t^j, for every t^{-j}. Roughly speaking, part (b) says that, as network effects grow larger, the degree of concavity that must be imposed in part (a) on the distribution of membership values becomes more stringent. In the special case without network effects, i.e., when $\gamma_{s\hat{s}}^j = 0$ for all $s, \hat{s} \in \mathcal{S}$, this assumption reduces to Assumption A2 in Caplin and Nalebuff's (1991) seminal work on the existence of equilibrium in oligopoly.[15]

Assumption A2 ensures that the measure of users who prefer platform j to the outside option goes to zero sufficiently fast as j's net fee, t_s^j, grows large. It implies that platforms do not charge arbitrarily high fees, and thus it provides a bound for the set of best responses. Using standard techniques, including the Brouwer's fixed point theorem, we obtain the following result. Note that this assumption requires the existence of an outside option.

Proposition 2. *Under Assumptions A1 and A2, there exists a pure-strategy Nash equilibrium.*

The following result, dealing with Logit demand, provides an existence condition that applies even when users do not have an outside option.

Proposition 2′. *Assume demand takes the Logit form as specified in eq. (13), and either $J \geq 2$ or users have an outside option. If $\gamma_{ss}^j + \sum_{\hat{s} \neq s} \left| \frac{\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j}{2} \right| \leq 3.375, \forall j \in \mathcal{J}, s \in \mathcal{S}$, then there exists a pure-strategy Nash equilibrium.*

4.1 Uniqueness under Logit

Proposition 3 contains a result on uniqueness of equilibrium in the Logit case.

Proposition 3. *Assume demand takes on the Logit form, either $J \geq 2$ or users have an outside option, and the market is one-sided ($S = 1$). If $\gamma^j < 2.610, \forall j \in \mathcal{J}$, there exists a unique pure-strategy Nash equilibrium.*

[15] A slight difference in accounting is that Caplin and Nalebuff label the dimensionality of users' types as n, whereas we label it as $J + 1$.

Although this result applies only in the case of one-sided markets, it is the first of its kind in the literature of which we are aware. In order to obtain it, we make use of two features. The first is that, in a single market without network effects, when demand is of the Logit form, standard differentiated Bertrand competition gives rise to profits for each firm that have the *aggregative* property. That is, to calculate a given firm's profits, it is sufficient to know a sum that depends on all competitors' prices; it is not necessary to know each of their prices individually. The second feature we make use of is the one discussed in remark 2B of Section 2. This is the fact that, in markets with nonzero γ's, when platforms compete in net fees, the demand system and the network effects enter into each platform's profits in a separable manner. Consequently, in a one-sided platform context, the aforementioned aggregative property is preserved. We can thus make use of the technique provided by Anderson, Erkal and Piccinin (2020) to establish our bound for equilibrium uniqueness.[16]

Together, Propositions 2′ and 3 establish that the following straightforward configuration holds in one-sided markets with Logit demand. When network effects are not too strong ($\gamma^j \leq 3.375$, $\forall j$), equilibrium exists; if they also satisfy a tighter upper bound ($\gamma^j < 2.610$, $\forall j$), then it is unique. We now turn to the effects of competition, studying both settings with potential equilibrium multiplicity and with guaranteed uniqueness.

5 Effects of Competition

A much-discussed concern regarding platform industries is the dominance of one firm in a given market (e.g., Google, Facebook, Amazon, etc.). In the context of such discussion, it is sometimes proposed that such dominance could be alleviated by the entry of more players into the given market. In this section, we use our model to address three questions related to the effects of competition. In a setting with strong network effects, we first study symmetric equilibrium and examine the effect of competition on prices and user participation. Second, we expand the scope to consider potential asymmetric equilibria,

[16] An idea that at first blush seems tempting is to apply the multi-product technique of Nocke and Schutz (2018) to establish uniqueness in the case of $S \geq 2$. Unfortunately, such an intuition is misguided, because multi-sidedness in platform competition corresponds to an oligopoly setting with multiple markets, not a setting, like the one they study, where a given firm can sell multiple products within one market.

and we ask whether more competition could help tip the market in favor of one dominant platform. Third, we analyze a potential merger, in a setting where network effects are not too strong, so that equilibrium is unique. We show that, when a merger occurs between two smaller platforms competing in the same market as a dominant one, network effects play a substitutable role to standard cost synergies.

Throughout this section we assume that demand takes on the Logit form, as specified in eq. (13), and we further assume that $S = 1$. We are thus in an environment covered by Propositions 2′ and 3. We also assume that network effect strength is common across all platforms, i.e., $\gamma^j = \gamma$, $\forall j$.

5.1 Market Contraction Under Symmetric Competition

Here and in Section 5.2, we assume that platforms are *ex ante* identical. Thus, without loss of generality, we normalize platforms' common marginal cost c to zero, since the sum of cost and outside option, $c + z$, is what matters for equilibrium market shares. Proposition 4 shows that, under strong network effects, increasing the number of competing platforms can lead to *market contraction*, i.e., lower total participation by users.

Proposition 4 (Market contraction)**.** *Assume* $\gamma \in (2.71, 3.375]$*. There exists an interval of outside option* z *such that total demand is lower at the symmetric equilibrium of the duopoly model than it is under monopoly.*

This proposition complements the "perverse pattern" pricing result of Tan and Zhou (2021), which takes place in an environment of fixed total demand. It stands in contrast with the standard pattern in oligopoly whereby more competition drives prices down and demand up. Of course, in our model such behavior also obtains for low enough values γ.

We now discuss the underlying mechanism and its generality beyond the environment of the proposition. Observe that, in the pricing formula of a generic platform j,

$$t^j = \frac{1}{1 - n^j} - 2\gamma n^j,$$

the network discount, $2\gamma n^j$, is increasing in j's market share n^j. Suppose a monopoly receives a market share of N. Upon the arrival of another platform, if the total demand were to stay the same, each platform would receive a lower market share $N/2$. A lower market share would incentivize everyone to offer a lower network discount and thus a higher net fee. The strength of this effect is increasing in the network externality γ. Moreover, a higher net fee leads to a lower market share for each platform. If this self-reinforcing effect is strong, namely if γ is high, there is a symmetric equilibrium under duopoly in which the total demand is lower than under monopoly.

In general, at a symmetric equilibrium of the model with J platforms, the market share of platform j relative to the outside option satisfies

$$\frac{N/J}{1-N} = \frac{e^{-t^j}}{e^z},$$

which gives rise to an inverse symmetric demand function

$$t^j = -z - \ln \frac{N/J}{1-N}.$$

Combining this with the pricing formula yields a characterization of the total demand N among J platforms, as

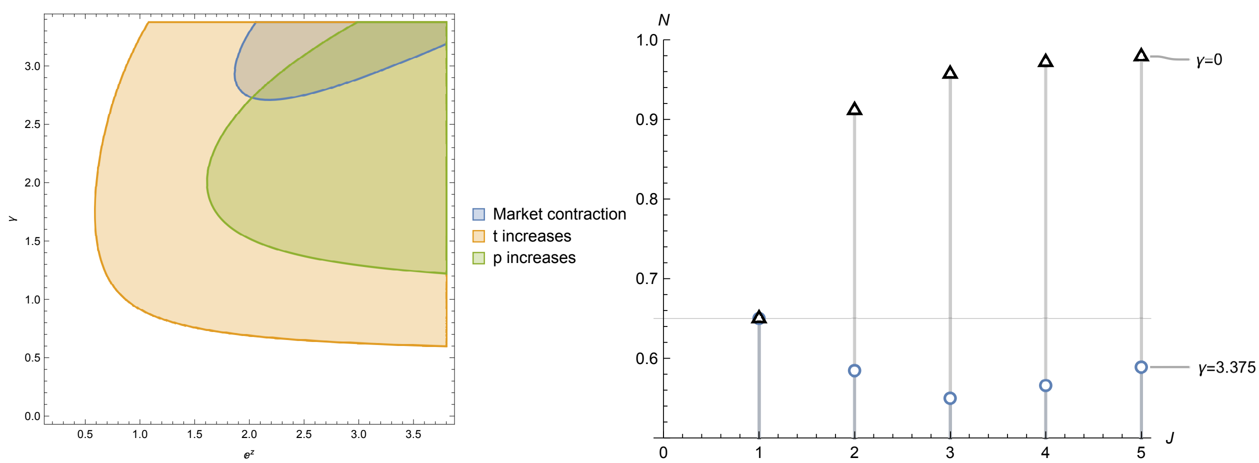
$$\frac{1}{1-N/J} - 2\gamma\frac{N}{J} + z + \ln\frac{N/J}{1-N} = 0. \tag{19}$$

With more platforms, the network discount $2\gamma N/J$ is lower, and there is an equilibrium where the total demand N will be lower too.

While we present the market contraction result in the special case of monopoly to duopoly under Logit demand, the discussion above applies more broadly, if one analyzes an increase from J platforms to J' platforms under general demand form. It is also implied that, in order for the presence of an additional platform to lead to market contraction, its arrival into the market must lead to a sufficiently large increase in the equilibrium net fee.

In Figure 1, the left-hand panel illustrates Proposition 4 as well as the associated changes in the equilibrium net fee and total price. On the horizontal axis is e^z, an increasing function of the outside option parameter. On the vertical axis is γ, measuring the strength of network effects. The blue region represents the set of parameter values such that a shift from monopoly to symmetric duopoly leads to market contraction. Note that this arises only when network effects are relatively strong. As the above discussion implies, the blue area lies completely within the orange area, which represents parameter values such that the net fee t increases with the addition of a new platform. In contrast, the blue region's overlap with the green one reflects the fact that total price p and total user participation may move together or in opposite directions.

The right-hand panel considers specific parameter values and allows the number of platforms, J, to vary from 1 to 5. In it, the triangles, which increase monotonically with J, represent total user participation in a non-platform market where $\gamma = 0$. The circles represent the same in a platform market with $\gamma = 3.375$. In this latter case, user participation drops significantly when the monopoly is split into duopoly, and it does not regain its original level even with 5 platforms.



(a) Reactions to a shift from monopoly to duopoly (b) Participation trajectory under low and high γ[17]

Figure 1: Effects of competition on platforms' fees and user participation

[17] The parameters underlying two lines in the right panel are $\gamma = 0, z = -3.48$ and $\gamma = 3.375, z = 0.91$ respectively, such that a monopoly has a market share of 65% in both cases.

5.2 Competition and Market Dominance

A frequent concern regarding markets with network effects is the idea that they are prone to "tipping" towards dominance by one, or perhaps a small number, of platforms. Here, we state our second result on the potential unintended consequences of competition.

Proposition 5 (Competition may increase dominance)**.** *Assume no outside option and* $\gamma \in (2.71, 3.375]$*. There exists an equilibrium under triopoly in which a dominant platform's market share is greater than the market share of any platform in any duopoly equilibrium.*

When γ is in the lower region of the assumed interval, duopoly has a unique equilibrium which is symmetric, where each platform has a market share of 1/2. Under triopoly, there is an asymmetric equilibrium in which the dominant platform's market share is greater than 1/2. When γ is in the upper region of the assumed interval, duopoly has an asymmetric equilibrium. Nevertheless, adding a new platform to the market can lead the dominant platform to become even more dominant. Figure 2 shows the largest possible equilibrium market share of any platform under duopoly and triopoly, with different strengths of network externality γ.

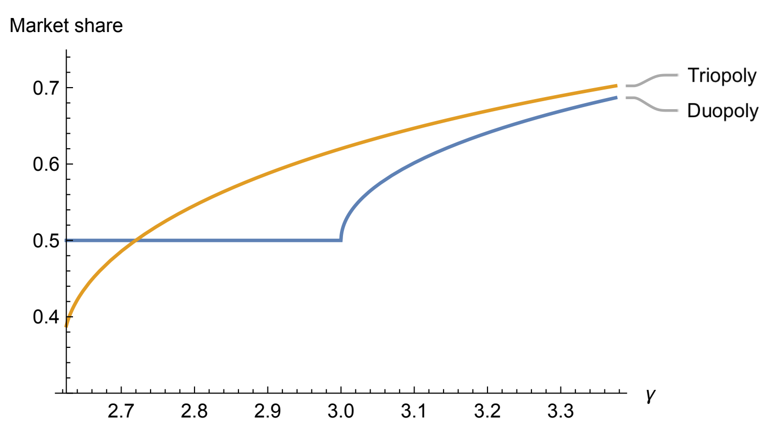


Figure 2: Largest possible equilibrium market share of any platform

The underlying mechanism that allows competition to increase dominance is similar to the one driving market contraction, except that, here, the dominant platform plays a role similar to the outside option in the symmetric case. Suppose that under duopoly, there is one smaller platform that receives a market share $n < 1/2$, and the dominant platform serves the remaining users. Upon the arrival of another platform, if the combined demand of the entrant and the smaller incumbent stayed at the same level as the initial small platform's had been, they each would receive a lower market share $n/2$. This incentivizes them to lower their network discounts and thus raise their net fees, which contributes to a further decline in their total market share. As a consequence, the large platform grows even more dominant. As in the case of market contraction under symmetric competition, this mechanism is at play for an arbitrary increase in the number of platforms under general demand.

5.3 Merger Analysis

We now study the effects of a possible merger between two smaller platforms in a triopoly market that includes a dominant platform. In this analysis, we restrict attention to an environment in which equilibrium is unique (see Proposition 3), but we allow for the platforms to be *ex ante* asymmetric. In particular, we take into account cost synergies (i.e., reductions in marginal costs) brought about by the potential merger, which play a central role in standard merger analysis. We address the question of how the strength of network effects in a given market influences the amount of cost synergy that is needed in order for a merger between two smaller platforms to help reduce the large platform's dominance.

The environment is as follows. In the pre-merger setting, the dominant platform has some market share of at least 1/2, and the remaining users, who have no outside option, are equally split between the two non-dominant platforms. The dominant platform's marginal cost is assumed to be zero, whereas the smaller platforms have some positive marginal cost, $c > 0$. The particular demand profile in question can be supported by some combination of cost difference, c, and network effect strength, γ.[18] In the event of a merger, the two smaller platforms become one entity, which enjoys both combined network effects and cost synergies, given by $\Delta c \in (0, c)$.[19] We now state Proposition 6.

[18]The condition for the large platform to have market share of at least 1/2 is $c + \gamma/2 > 1.36$.

[19]There are different possible ways to model a merger. We take the approach of assuming one of the merged platforms shuts down, allowing the two user bases to be combined. An alternative assumption, which is also compatible with the net fee modeling approach, allows the two merged platforms to continue to operate as separate entities but with a single agent setting both of their prices.

Proposition 6. *Assume no outside option and* $\gamma < 2.610$. *In a merger between the two non-dominant platforms, the minimum cost synergy needed to reduce the market share of the dominant platform decreases with the strength of network effects.*

To interpret this proposition, first consider a traditional oligopoly setting without network effects. There, following a merger, if there were no cost synergies, the merged firm would have an incentive to raise its price compared to the pre-merger level. This decreases its market share and thus increases the dominance of the non-merging firm. Hence, a significant cost synergy would be necessary in order for a merger not to cause the large firm to become more dominant. In a market with network effects, however, since the merged entity benefits from a larger user base, post-merger it incorporates a larger network discount into its pricing. This larger network discount plays a role that can substitute for the one played by cost synergies. Thus, the stronger the network externality, the smaller the required cost synergy to prevent the dominant firm from growing.

To conclude this section, note the following theme that is present in Propositions 4 to 6. When network effects are present, policies that intend to be procompetitive may have the opposite effect. This is because the presence of more platforms can interfere with existing competitive pressures by dividing up user bases.

6 Interoperability

In policy debates on platform governance, it is sometimes argued that regulation, not more competition, is a better approach to tempering the dominance of large platforms. In this vein, a particular policy that is sometimes proposed is a requirement that competing platforms be (at least partially) compatible or "interoperable" with one another. The basic idea is that a user who joins one platform could be able to interact with not just other users of the same platform but also with users of its competitors.[20] This section explores the effects of such a requirement. It first shows that, under symmetric equilibrium, interoperability lowers net fees and boosts participation. Second, it shows that, in asymetric situations,

[20]To fix ideas, contrast the case of Facebook, on which users can be friends only with other Facebook users, with phone service, where subscribers can call one another, regardless of their respective networks.

interoperability tends to reduce market dominance.

The key ingredient we add to the model here is the parameter, $\lambda \in [0,1]$, denoting the *degree of interoperability*. For simplicity, we assume that this single parameter captures the level of interoperability between any two platforms in the market, although a more complicated configuration would be consistent with our framework. We continue to assume there is one side, that is, $S = 1$. When there are J platforms, the expression (updated from eq. (1)) for the gross utility derived by a user who joins platform j is

$$u^j := \theta^j + \gamma n^j + \lambda \sum_{k \in \mathcal{J} \setminus \{j\}} \gamma n^k - p^j. \tag{20}$$

The notion of the net fee extends naturally to cover all externalities that the user receives from joining the platform, i.e.,

$$p^j := t^j + \gamma n^j + \lambda \sum_{k \in \mathcal{J} \setminus \{j\}} \gamma n^k.$$

6.1 Pricing

Here we extend the pricing formula of Proposition 1 and we state a comparative statics result regarding the impact of interoperability on net fees. Equation (21) below, which we derive in Appendix A.2, pins down equilibrium net fees under interoperability. It makes use of notation $\varphi^j(t) := \frac{\sum_{k \in \mathcal{J} \setminus \{j\}} \frac{\partial n^k(t)}{\partial t^j}}{-\frac{\partial n^j(t)}{\partial t^j}} \in [0,1]$, denoting platform j's *diversion ratio*, under general demand form. This captures the share of new users that platform j would attract from other platforms, rather than from the outside option, if it were to decrease its net fee by a small amount.

$$t^j = c^j + \frac{n^j}{-\frac{\partial n^j}{\partial t^j}} - \left(2 + \lambda \left(\frac{\sum_{k \in \mathcal{J} \setminus \{j\}} n^k}{n^j} - \varphi^j\right)\right) \gamma n^j. \tag{21}$$

Under Logit demand, the diversion ratio φ^j equals

$$\varphi^j = \frac{\sum_{k \in \mathcal{J} \setminus \{j\}} n^k}{1 - n^j} = \frac{\sum_{k \in \mathcal{J} \setminus \{j\}} \mathrm{e}^{-t^k}}{\mathrm{e}^z + \sum_{k \in \mathcal{J} \setminus \{j\}} \mathrm{e}^{-t^k}}, \tag{22}$$

which is independent of t^j.

With interoperability, under general demand there exists a pure-strategy Nash equilibrium under Assumptions A1 and A2, as long as $\frac{\partial \varphi^j}{\partial t^j} \leq 0$ holds globally, extending our baseline existence result (Proposition 2). Under Logit demand, this condition $\frac{\partial \varphi^j}{\partial t^j} \leq 0$ is trivially met as φ^j is independent of t^j, and we can dispense with Assumption A2 as before.

We now consider the effects of interoperability on net fees, under symmetric competition. In this case, without loss of generality we again normalize c to be zero.

Proposition 7 (Stronger interoperability reduces net fees)**.** *Consider any two levels of interoperability, $\underline{\lambda} < \overline{\lambda}$. Under duopoly with an outside option or when $J \geq 3$, let $\underline{t}$ denote a net fee that arises at some symmetric equilibrium when $\lambda = \underline{\lambda}$. When $\lambda = \overline{\lambda}$, there is a symmetric equilibrium with $\overline{t} < \underline{t}$. Under duopoly with no outside option, the same statement is true except that $\overline{t} = \underline{t}$.*

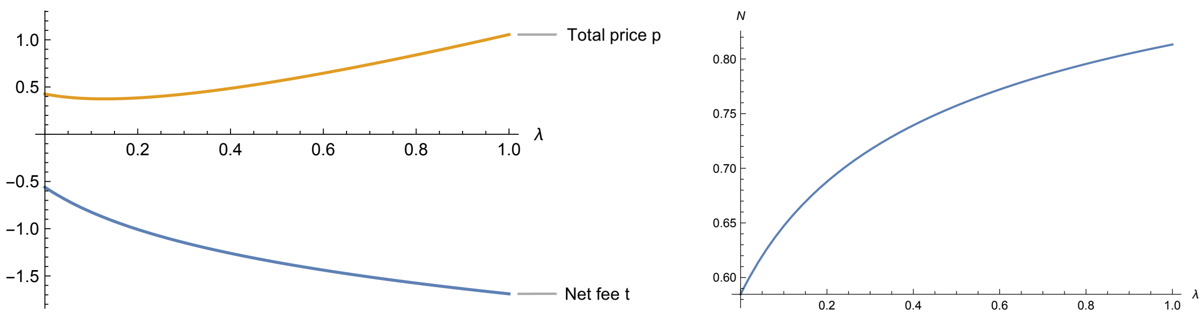
At a symmetric equilibrium, the pricing formula eq. (21) specializes to

$$t = \frac{1}{1 - n^j} - \left(2 + \lambda(J - 1 - \varphi^j)\right)\gamma n^j. \tag{23}$$

Comparing this to its counterpart in the no-interoperability case, the factor $\lambda(J - 1 - \varphi^j)$ is new, and it reflects the following tradeoff. When adding a small mass of additional users, platform j extracts the "off network" interaction utility that they will derive. That is, the new users enjoy a per-interaction benefit of $\lambda\gamma$ with each of the $(J-1)n^j$ users that join other platforms. This is included in the platform's marginal gain from adding an additional user, but it must be excluded when calculating the platform's net fee. On the other hand, when platform j adds this small mass of new users, a fraction of these, measured by φ^j, switch to j from other platforms, rather than from the outside option. This flow of φ^j users from other platforms to j eats away at the revenue from "off network" interaction that j can extract from its existing n^j users, at a rate of γ per interaction.

Proposition 7 regards the relative magnitudes of these two effects. It says that, following a shift from some $\underline{\lambda}$ to a greater $\overline{\lambda}$, the former effect dominates, in terms of its effect on the equilibrium net fee, except in the case of duopoly with no outside option, when these two effects balance each other out. A further implication is that, as the net fees charged by platforms go down due to strengthened interoperability, the total demand will go up, unless there is no outside option. This is in contrast to the perverse pattern of market contraction induced by competition presented in Proposition 4.

Figure 3 plots the decreasing net fee and increasing user participation under duopoly, as we gradually strengthen interoperability λ, under $\gamma = 3.375$.



(a) Net fee t and total price p (b) User participation N

Figure 3: Effects of interoperability on fees and user participation under duopoly[21]

6.2 Interoperability and Market Dominance

Here we examine the effect of interoperability on market dominance.

Proposition 8 (Interoperability may mitigate dominance)**.** *Assume no outside option. Consider any two levels of interoperability $\underline{\lambda} < \overline{\lambda}$. For any duopoly equilibrium under $\overline{\lambda}$ in which the dominant platform has market share $\overline{n}^1 > 1/2$, when $\lambda = \underline{\lambda}$, there is an equilibrium with $\underline{n}^1 > \overline{n}^1$.*

The logic behind this result is that an increase in interoperability, λ, changes the equilibrium in a way similar to a decrease in the network externality, γ. Note that at the asymmetric equilibrium with no outside option, the pricing formula eq. (21) becomes, since the diversion ratio $\varphi^j = 1$,

$$t^j = c + \frac{1}{1-n^j} - 2\gamma n^j - \gamma\lambda(1-n^j) + \gamma\lambda n^j \tag{24}$$

$$= (c - \gamma\lambda) + \frac{1}{1-n^j} - 2\gamma(1-\lambda)n^j. \tag{25}$$

[21]The parameters to generate the plots are $\gamma = 3.375, z = 0.91$.

A platform market with marginal cost c, network externality γ, and interoperability λ is equivalent to a market with marginal cost $\tilde{c} := c - \gamma\lambda$, network externality $\tilde{\gamma} := \gamma(1 - \lambda)$, and no interoperability. Hence, increasing λ is equivalent to decreasing γ, leading to equilibrium market shares that are closer to one another.

In fact, we can revisit Figure 2, whose blue line plots the largest market share of any platform under duopoly. As network externality γ increases, the larger platform grows even larger. At a given level of externality γ, increasing interoperability λ lowers the effective externality $\tilde{\gamma}$, which brings the two market shares closer to each other. Indeed, once λ goes beyond a certain threshold, there no longer exists any asymmetric equilibrium. Figure 4 illustrates this, plotting the market share of the largest platform under duopoly under different levels of interoperability λ, holding fixed $\gamma = 3.375$.

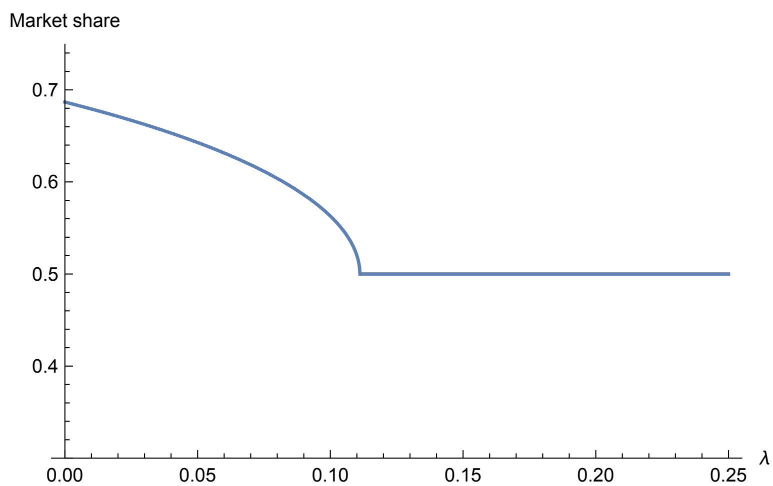


Figure 4: Largest possible duopoly equilibrium market share, as a function of interoperability

As in the analysis on competition, the basic mechanisms driving our results on interoperability remain at play generally, both in terms of number of platforms and demand form.

7 Conclusion

Better understanding the way platform markets operate is of great importance in the era of big tech. In this area, an overarching issue that is widely perceived to need further clarification regards the relative merits of interventions that are more competition-driven versus those that are more regulatory. Using a novel approach, this paper provides a set of results that shed light on some key questions within this broad topic.

The distinguishing feature of our approach is that we assume platforms compete by setting *net fees*. This stands in contrast to the more common assumption that they set *total prices*. While both of these approaches are abstract representations of a more complicated reality, we argue that the net fee approach fits particularly well in settings where *ex ante*, platforms lack the power to commit not to extract the surplus that users generate from interacting with one another. A benefit of using the net fee approach is that it brings about a great degree of analytical tractability to the study of platform competition. As such, we incorporate arbitrary asymmetry among platforms and variable total demand into a general discrete choice setting and derive a straightforward pricing formula.

Using our modeling approach we address a set of policy questions that attract significant debate. We show that increasing competition may have the unintended consequences of reducing total demand or tipping the market towards a dominant platform. We also show that, in the context of mergers, strong network effects can act as a substitute for cost synergies. We also study the effects of interoperability regulation. There, we show that, in contrast to competition, it reliably increases total demand and mitigates market dominance. Within the policy analysis, our focus has been on identifying the key mechanisms driving these results. However, we believe our framework can be useful in addressing further, related questions in a wide range of platform settings.

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A Proofs

A.1 Proofs of Equilibrium Existence

We first establish that the pricing formula in Proposition 1 is the best response of each platform, by showing that each platform's profit is quasiconcave in t^j. Then we prove that there exists a fixed point to the set of pricing formulas. Thus there exists an equilibrium. Towards the end, we briefly discuss how the existence condition extends to the case with interoperability.

A.1.1 General Demand

Lemma 1. *Under Assumption A1, given any t_{-j}, we have $-\frac{\partial n_s^j}{\partial t_s^j} \in \left(0, J \cdot \overline{g}_s\right)$, for any $s \in S$.*

Proof of Lemma 1. The demand n_s^j is the mass under probability measure $f_s(\theta_s)$ of set

$$A = \{\theta_s | \theta_s^j - t_s^j \geq \max\{\theta_s^0, \max_{k \neq j} \theta_s^k - t_s^k\}\} \tag{26}$$

$$= \cap_{k \neq j} \{\theta_s | \theta_s^j - t_s^j \geq \theta_s^k - t_s^k\} \cap \{\theta_s | \theta_s^j - t_s^j \geq \theta_s^0\}. \tag{27}$$

We have $-\frac{\partial n_s^j}{\partial t_s^j} > 0$ since f_s has full support. The shrinkage of this set A resulting from a marginal increase in t_s^j satisfies

$$\frac{\partial A}{\partial t_s^j} \subset \bar{A} = \cup_{k \neq j} \{\theta_s | \theta_s^j - t_s^j = \theta_s^k - t_s^k\} \cup \{\theta_s | \theta_s^j - t_s^j = \theta_s^0\}, \tag{28}$$

and thus the slope of demand satisfies

$$-\frac{\partial n_s^j}{\partial t_s^j} \leq \int_{\bar{A}} f_s(\theta_s) \mathrm{d}\theta_s \tag{29}$$

$$= \sum_{k \neq j} \int f_{s,k|j}(\theta_s^j - t_s^j + t_s^k | \theta_s^j) f_{s,j}(\theta_s^j) \mathrm{d}\theta_s^j + \int f_{s,0|j}(\theta_s^j - t_s^j | \theta_s^j) f_{s,j}(\theta_s^j) \mathrm{d}\theta_s^j \tag{30}$$

$$\leq \sum_{k \neq j} \int \overline{g}_s f_{s,j}(\theta_s^j) \mathrm{d}\theta_s^j + \int \overline{g}_s f_{s,j}(\theta_s^j) \mathrm{d}\theta_s^j \tag{31}$$

$$\leq J \cdot \overline{g}_s. \tag{32}$$

□

Lemma 2. *Under Assumption A1, $\pi^j(n^j, t^{-j})$ is concave in n^j, given any $t^{-j} := \left(t^k\right)_{k \in \mathcal{J} \setminus \{j\}}$.*

Proof of Lemma 2. Suppress t^{-j} for brevity, acknowledging that we are holding t^{-j} fixed. First, we show the mapping $n^j(t^j)$ is globally univalent, and thus we can think of platform j's optimization problem as choosing n^j. Second, we show π^j is concave in n^j.

First, the Jacobian of $n^j(t^j)$ is a $S \times S$ diagonal matrix with negative diagonals $\frac{\partial n_s^j}{\partial t_s^j} < 0$ from Lemma 1. Thus the Jacobian is negative definite and thus globally univalent. (Gale and Nikaido, 1965).

Second, we have

$$\frac{\partial \pi^j}{\partial n_s^j} = \frac{\frac{\partial \pi^j}{\partial t_s^j}}{\frac{\partial n_s^j}{\partial t_s^j}} = \frac{n_s^j}{\frac{\partial n_s^j}{\partial t_s^j}} + \sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right) n_{\hat{s}}^j + t_s^j - c_s^j, \tag{33}$$

and

$$\frac{\partial^2 \pi^j}{\partial n_s^j \partial n_{\hat{s}}^j} = \frac{\partial \frac{\partial \pi^j}{\partial n_s^j} \big/ \partial t_{\hat{s}}^j}{\frac{\partial n_{\hat{s}}^j}{\partial t_{\hat{s}}^j}} = \left(2 - \frac{n_s^j \frac{\partial^2 n_s^j}{\partial (t_s^j)^2}}{\left(\frac{\partial n_s^j}{\partial t_s^j}\right)^2}\right) \frac{1}{\frac{\partial n_s^j}{\partial t_s^j}} \cdot 1_{s=\hat{s}} + \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right). \tag{34}$$

For the Hessian $\left(\frac{\partial^2 \pi^j}{\partial n_s^j \partial n_{\hat{s}}^j}\right)_{s,\hat{s} \in S}$ to be globally negative semi-definite, it suffices for it to be a diagonally dominant matrix with non-positive diagonals, i.e.

$$\frac{\partial^2 \pi^j}{\partial (n_s^j)^2} + \sum_{\hat{s} \neq s} \left| \frac{\partial^2 \pi^j}{\partial n_s^j \partial n_{\hat{s}}^j} \right| \leq 0, \ \forall s \in S. \tag{35}$$

The LHS of (35) equals,

$$\left(2 - \frac{n_s^j \frac{\partial^2 n_s^j}{\partial (t_s^j)^2}}{\left(\frac{\partial n_s^j}{\partial t_s^j}\right)^2}\right) \frac{1}{\frac{\partial n_s^j}{\partial t_s^j}} + 2\gamma_{ss}^j + \sum_{\hat{s} \neq s} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right|. \tag{36}$$

Thus the inequality (35) simplifies to

$$2\gamma_{ss}^j + \sum_{\hat{s} \neq s} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| \leq \left(2 - \frac{n_s^j \frac{\partial^2 n_s^j}{\partial (t_s^j)^2}}{\left(\frac{\partial n_s^j}{\partial t_s^j}\right)^2}\right) \frac{1}{-\frac{\partial n_s^j}{\partial t_s^j}}. \tag{37}$$

By Theorem 1 in the Appendix of Caplin and Nalebuff (1991), Assumption A1 implies $n_s^j(t_s^j)$

is $\left(\frac{\rho_s}{1+(J+1)\rho_s}\right)$-concave, further implying

$$-\frac{n_s^j \frac{\partial^2 n_s^j}{\partial (t_s^j)^2}}{\left(\frac{\partial n_s^j}{\partial t_s^j}\right)^2} \geq \frac{\rho_s}{1+(J+1)\rho_s} - 1, \tag{38}$$

and Lemma 1 shows

$$-\frac{\partial n_s^j}{\partial t_s^j} \in \left(0, J \cdot \overline{g}_s\right). \tag{39}$$

Thus the inequality (37) holds if

$$\left(\gamma_{ss}^j + \sum_{\hat{s} \neq s} \left| \frac{\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j}{2} \right| \right) \cdot \overline{g}_s \leq \frac{1}{2J} \left(1 + \frac{\rho_s}{1+(J+1)\rho_s}\right). \tag{40}$$

Under Logit demand, the inequality (37) takes the form of

$$2\gamma_{ss}^j + \sum_{\hat{s} \neq s} \left| \gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j \right| \leq \frac{1}{n_s^j \left(1 - n_s^j\right)^2}, \tag{41}$$

the RHS of which is minimized at $n_s^j = \frac{1}{3}$ to obtain 6.75. This is a weaker bound, as we make use of the specific demand functional form. □

Proof of Proposition 2. Lemma 2 implies that given any t^{-j}, $\pi^j(t^j, t^{-j})$ is maximized at at most one t^j under Assumption A1, though it could be monotonic in t_s^j for some $s \in S$. The full set of pure-strategy equilibria is thus the set of solutions to the system of pricing formulas for all J platforms. Now we claim that, with Assumption A2 in addition, there exists a solution. It suffices to show that it is without loss of generality to restrict best responses to $[L, U]^{JS}$, so that we can apply Brouwer's fixed point theorem.

We can write

$$\pi^j(t^j, t^{-j}) = \sum_s \left(t_s^j + \sum_{\hat{s}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j - c_s^j\right) n_s^j \tag{42}$$

$$= (t_s^j - c_s^j) n_s^j + \sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right) n_s^j n_{\hat{s}}^j + g(t_{-s}^j, t^{-j}), \tag{43}$$

in which $g(t^j_{-s}, t^{-j})$ is independent of t^j_s. Define

$$h(t^j_s, t^j_{-s}, t^{-j}) := \pi^j(t^j, t^{-j}) - g(t^j_{-s}, t^{-j}) = \left[t^j_s - c^j_s + \sum_{\hat{s}} \left(\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s} \right) n^j_{\hat{s}}(t^j_{\hat{s}}, t^{-j}_{\hat{s}}) \right] n^j_s(t^j_s, t^{-j}_s). \tag{44}$$

For the lower bound, let $L := \min_{j,s} \left(c^j_s - \sum_{\hat{s}} \left| \gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s} \right| \right) - 1$. It follows that $h(t^j_s, t^j_{-s}, t^{-j}) < 0$ as long as $t^j_s < L$. However, if the platform j sets $t^j_s = \check{t}^j_s := c^j_s + \sum_{\hat{s}} \left| \gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s} \right| + 1$, it ensures $h(t^j_s, t^j_{-s}, t^{-j}) > 0$. Formally, we have, for any t^j_{-s}, t^{-j}, if $t^j_s < L$, then

$$\pi^j(t^j_s, t^j_{-s}, t^{-j}) < \pi^j(\check{t}^j_s, t^j_{-s}, t^{-j}), \tag{45}$$

Thus j would never set $t^j_s < L$. This also implies that it is without loss of generality to restrict to t^j_s such that $h(t^j_s, t^j_{-s}, t^{-j}) > 0$.

For the upper bound, we notice that when $h(t^j_s, t^j_{-s}, t^{-j})$ is positive, since n^j_s is increasing in each competitor's side-s net fee, it is bounded between

$$h(t^j_s, t^j_{-s}, t^{-j}) \geq h^L(t^j_s, t^j_{-s}, t^{-j}) := \left[t^j_s - c^j_s + \sum_{\hat{s}} \left(\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s} \right) n^j_{\hat{s}}(t^j_{\hat{s}}, t^{-j}_{\hat{s}}) \right] n^j_s(t^j_s, L), \tag{46}$$

and

$$h(t^j_s, t^j_{-s}, t^{-j}) \leq h^\infty(t^j_s, t^j_{-s}, t^{-j}) := \left[t^j_s - c^j_s + \sum_{\hat{s}} \left(\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s} \right) n^j_{\hat{s}}(t^j_{\hat{s}}, t^{-j}_{\hat{s}}) \right] n^j_s(t^j_s, \infty). \tag{47}$$

It is guaranteed that at $t^j_s = \check{t}^j_s$, the lower bound $h^L(\check{t}^j_s, t^j_{-s}, t^{-j}) \geq \underline{h} := n^j_s(\check{t}^j_s, L) > 0$. Meanwhile, the upper bound at any t^j_s satisfies $h^\infty(t^j_s, t^j_{-s}, t^{-j}) \leq \overline{h}(t^j_s) := \left(t^j_s - c^j_s + \sum_{\hat{s}} \left| \gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s} \right| \right) n^j_s(t^j_s, \infty)$. Under Assumption A2, $\lim_{t^j_s \to \infty} \overline{h}(t^j_s) = 0$, and hence there exists U such that, for any $t^j_s > U$, we have $\overline{h}(t^j_s) < \underline{h}$, implying $h(t^j_s, t^j_{-s}, t^{-j}) < h(\check{t}^j_s, t^j_{-s}, t^{-j})$. Consequently, we have, for any t^j_{-s}, t^{-j}, if $t^j_s > U$, then

$$\pi^j(t^j_s, t^j_{-s}, t^{-j}) < \pi^j(\check{t}^j_s, t^j_{-s}, t^{-j}). \tag{48}$$

Therefore, j would never set $t^j_s > U$, completing our proof. □

A.1.2 Logit Demand

Proof of Proposition 2′. The extension we accommodate here is that there is no outside option but multiple platforms. We have already shown that the first-order condition is the best-response function, when inequality (41) holds. Now we show that there is a fixed point to the system of best response functions, even when there is no outside option ($e^z = 0$).

The system of best response functions is

$$t_s^j = T_s^j(t) := c_s^j + \frac{1}{1 - n_s^j} - \sum_{\hat{s} \in \mathcal{S}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right) n_{\hat{s}}^j, \ \forall j \in \mathcal{J}, s \in \mathcal{S}, \tag{49}$$

with

$$n_s^j(t) = \frac{e^{-t_s^j}}{e^z + \sum_{k \in \mathcal{J}} e^{-t_s^k}}. \tag{50}$$

Use Schaefer's fixed point theorem recited as follows. Assume that X is a Banach space and that $T : X \to X$ is a continuous compact mapping. Moreover assume that the set

$$\cup_{0 \le \lambda \le 1} \{x \in X : x = \lambda T(x)\} \tag{51}$$

is bounded. Then T has a fixed point.

Our $T(t)$ is continuous. And in Euclidean space, a continuous mapping is a compact mapping. Thus it suffices to show that for our $T(t)$, $\Lambda := \cup_{0 \le \lambda \le 1} \{t \in \mathbb{R}^{JS} : t = \lambda T(t)\}$ is bounded. Claim that there exists $L \le 0, U \ge 0$ such that $\Lambda \subset [L, U]^{JS}$, which would imply our existence result.

For the lower bound, as

$$T_s^j(t) = \frac{1}{1 - n_s^j} - \sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right) n_{\hat{s}}^j + c_s^j \tag{52}$$

$$\ge -\sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| + c_s^j, \tag{53}$$

letting $L := \min_{j,s} \{0, -\sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| + c_s^j\}$, we have $\lambda T_s^j(t) \ge L, \forall \lambda \in [0, 1], t \in \mathbb{R}^{JS}$. Thus any t with $t_s^j < L$ will not be in Λ.

For the upper bound, we study a candidate t that satisfies $t = \lambda T(t)$ for some $\lambda \in [0, 1]$, and we show that there exists a constant U such that $t_s^j \le U, \forall j, s$. From $t = \lambda T(t)$, we have

$$t_s^j = \lambda \frac{1}{1 - n_s^j} + \lambda \left[-\sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right) n_{\hat{s}}^j + c_s^j \right] \tag{54}$$

$$\le \frac{1}{1 - n_s^j} + c_s^j + \sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| \tag{55}$$

$$= \frac{n^j}{1 - n_s^j} + 1 + c_s^j + \sum_{\hat{s}} \left| \gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j \right| \tag{56}$$

$$= \frac{e^{-t_s^j}}{e^z + \sum_{l \neq j} e^{-t_s^l}} + 1 + c_s^j + \sum_{\hat{s}} \left| \gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j \right| \tag{57}$$

$$= \frac{e^{-t_s^j}}{e^z + \sum_{l \neq j} e^{-t_s^l}} + d_s^j, \tag{58}$$

with $d_s^j := c_s^j + \sum_{\hat{s}} \left| \gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j \right|$ as a constant. For any side s, pick two generic platforms indexed by j, k, there are three possible possibilities: 1. Two inequalities $t_s^j > d_s^j, t_s^k > d_s^k$ both hold; 2. only one of them holds; 3. neither holds. We claim that all three cases lead to some upper bound for t.

In the first case where $t_s^j > d_s^j, t_s^k > d_s^k$, from inequality (58) we have

$$t_s^j \leq e^{-t_s^j} e^{t_s^k} + d_s^j, \tag{59}$$

$$t_s^k \leq e^{-t_s^k} e^{t_s^j} + d_s^k. \tag{60}$$

Move the d terms to the LHS and multiply two inequalities. We get

$$(t_s^j - d_s^j)(t_s^k - d_s^k) \leq 1, \tag{61}$$

$$t_s^k \leq \frac{1}{t_s^j - d_s^j} + d_s^k. \tag{62}$$

Plugging this back into inequality (59) gives a new inequality solely dependent on t_s^j, d_s^j, d_s^k,

$$t_s^j \leq e^{-t_s^j} \exp\left(\frac{1}{t_s^j - d_s^j} + d_s^k \right) + d_s^j. \tag{63}$$

The LHS increases to ∞ and the RHS decreases to d_s^j as $t_s^j \to \infty$. Thus there exists a threshold $u_{j,k,s}^1$ such that $t_s^j \leq u_{j,k,s}^1$. We set $U^1 = \max_{j,k,s} u_{j,k,s}^1$.

In the second case, we let $t_s^j > d_s^j$ but $t^k \leq d_s^k$. Then from inequality (58) we have

$$t_s^j \leq e^{-t_s^j} e^{t_s^k} + d_s^j \tag{64}$$

$$\leq e^{-t_s^j} e^{d_s^k} + d_s^j. \tag{65}$$

Once again, we observe that the LHS is increasing to ∞ and the RHS is decreasing to d_s^j as $t_s^j \to \infty$, and hence there exists $u_{j,k,s}^2$ such that $t_s^j \leq u_{j,k,s}^2$. We set $U^2 = \max_{j,k,s} u_{j,k,s}^2$.

In the third case where $t_s^j \leq d_s^j, t_s^k \leq d_s^k$, we simply set $U^3 = \max_{j,s} d_s^j$.

Taking stock, we let $U := \max\{U^1, U^2, U^3\}$, which would guarantee that any candidate t that satisfies $t = \lambda T(t)$ would have $t_s^j \leq U, \forall j, s$, completing our proof. □

A.1.3 Interoperability

Here we sketch how the existence condition extends to the case with interoperability, covered in Section 6. As before, first, each platform's best response is characterized by its first-order condition, under the additional assumption that $\frac{\partial \varphi^j}{\partial t^j} \leq 0$ holds globally. Second, there is a fixed point.

In one-sided markets ($S = 1$), we have $\pi^j = (t^j + \gamma^j n^j + \gamma^j \lambda n^{-j} - c^j) n^j$, with $n^{-j} := \sum_{k \in \mathcal{J} \setminus \{j\}} n^k$. The marginal profit is

$$\frac{\partial \pi^j}{\partial n^j} = \frac{\frac{\partial \pi^j}{\partial t^j}}{\frac{\partial n^j}{\partial t^j}} = \frac{n^j}{\frac{\partial n^j}{\partial t^j}} + 2\gamma^j n^j + \lambda \gamma^j n^{-j} - \lambda \gamma^j \varphi^j n^j + t^j - c^j, \tag{66}$$

and

$$\frac{\partial^2 \pi^j}{\partial (n^j)^2} = \frac{\frac{\partial \frac{\partial \pi^j}{\partial n^j}}{\partial t^j}}{\frac{\partial n^j}{\partial t^j}} = \left(2 - \frac{n^j \frac{\partial^2 n^j}{\partial (t^j)^2}}{\left(\frac{\partial n^j}{\partial t^j}\right)^2} \right) \frac{1}{\frac{\partial n^j}{\partial t^j}} - \lambda \gamma^j n^j \frac{\frac{\partial \varphi^j}{\partial t^j}}{\frac{\partial n^j}{\partial t^j}} + 2\gamma^j (1 - \lambda \varphi^j). \tag{67}$$

It is non-positive, if $\frac{\partial \varphi^j}{\partial t^j} \leq 0$ holds globally, in addition to Assumptions A1 and A2. (See the proof of Lemma 2 for bounds on terms unrelated to λ.) Under Logit demand, the diversion ratio φ^j is independent of t^j as shown in eq. (22), and thus this condition is met.

In terms of a fixed point, the pricing formula under interoperability is, reciting eq. (21),

$$t^j = c^j + \frac{n^j}{-\frac{\partial n^j}{\partial t^j}} - \left(2 + \lambda \left(\frac{\sum_{k \in \mathcal{J} \setminus \{j\}} n^k}{n^j} - \varphi^j \right) \right) \gamma n^j. \tag{68}$$

Compared to the no-interoperability case, γ is now multiplied by $2 + \lambda \left(\frac{\sum_{k \in \mathcal{J} \setminus \{j\}} n^k}{n^j} - \varphi^j \right)$ instead of simply 2. However, since $\lambda, \frac{\sum_{k \in \mathcal{J} \setminus \{j\}} n^k}{n^j}, \varphi^j$ are all bounded between 0 and 1, it is straightforward to repeat the proofs of Propositions 2 and 2′ to establish a fixed point.

A.2 Other Proofs

Proof of Proposition 3. We use the aggregative games approach from Anderson, Erkal and Piccinin (2020) to establish uniqueness. We recap their assumptions; A1-A3 ensure existence and A4 ensures uniqueness. Each platform plays $a^j \geq 0$ and the aggregate is $A = \sum_j a^j$, including the outside option if any. And let $A^{-j} = A - a^j$. In the Logit specification, we have $a^j = \mathrm{e}^{-t^j}$. The *best response* (br) function is defined as $r^j(A^{-j})$. As $(r^j)' > -1$ and $A^{-j} + r^j(A^{-j})$ strictly increases in A^{-j} implied by A3 shown below, define the *inclusive best response* (ibr) as $\tilde{r}^j(A)$.

- A1 (competitiveness): $\pi^j(A^{-j} + a^j, a^j)$ strictly decreases in A^{-j} for $a^j > 0$.
- A2 (payoffs): (a) $\pi^j(A^{-j} + a^j, a^j)$ is twice differentiable, and strictly quasi-concave in a^j, with a strictly negative second derivative with respect to a^j at an interior maximum.

  (b) $\pi^j(A, a^j)$ is twice differentiable, and strictly quasi-concave in a^j, with a strictly negative second derivative with respect to a^j at an interior maximum.
- A3 (reaction function slope): $\frac{\mathrm{d}^2\pi^j}{\mathrm{d}(a^j)^2} < \frac{\mathrm{d}^2\pi^j}{\mathrm{d}a^j \mathrm{d}A^{-j}}$.
- A4 (slope condition): $(\tilde{r}^j)'(A) < \frac{\tilde{r}^j(A)}{A}$.

Using br $r^j(A^{-j})$ instead of ibr $\tilde{r}^j(A)$, A4 is equivalently expressed as[22]

- A4′ (slope condition): $(r^j)'(A^{-j}) < \frac{r^j(A^{-j})}{A^{-j}}$.

We now proceed to characterize the threshold on γ^j that satisfies A3 and A4′, so as to show equilibrium uniqueness.

Our FOC is

$$t^j - c^j = \frac{1}{1 - n^j} - 2\gamma n^j, \tag{69}$$

$$-\ln r^j(A^{-j}) - c^j = \frac{r^j(A^{-j}) + A^{-j}}{A^{-j}} - 2\gamma^j \frac{r^j(A^{-j})}{r^j(A^{-j}) + A^{-j}}, \tag{70}$$

which implies

$$(r^j)'(A^{-j}) = \frac{\left[\frac{1}{(A^{-j})^2} - \frac{2\gamma^j}{\left(r^j(A^{-j})+A^{-j}\right)^2}\right] r^j(A^{-j})}{\left[\frac{1}{(A^{-j})^2} - \frac{2\gamma^j}{\left(r^j(A^{-j})+A^{-j}\right)^2}\right] r^j(A^{-j}) + \frac{1}{A^{-j}}} \frac{r^j(A^{-j})}{A^{-j}}. \tag{71}$$

[22] Rewrite $(\tilde{r}^j)'(A) = \frac{(r^j)'}{1+(r^j)'}$ and $\frac{\tilde{r}^j}{A} = \frac{r^j}{A^{-j}+r^j}$. We have $\frac{(r^j)'}{1+(r^j)'} < \frac{r^j}{A^{-j}+r^j}$ if and only if $(r^j)' < \frac{r^j}{A^{-j}}$. the equivalence of A4 and A4′ holds only when there is a well-defined ibr, which is true when $(r^j)'(A^{-j}) > -1$.

Simplify the denominator of the first term,

$$\left[\frac{1}{(A^{-j})^2} - \frac{2\gamma^j}{(r^j(A^{-j}) + A^{-j})^2}\right] r^j(A^{-j}) + \frac{1}{A^{-j}} = \frac{r^j}{(r^j + A^{-j})^2}\left[\frac{(r^j + A^{-j})^3}{r^j(A^{-j})^2} - 2\gamma^j\right], \tag{72}$$

$$= \frac{r^j}{(r^j + A^{-j})^2}\left[\frac{1}{n^j(1-n^j)^2} - 2\gamma^j\right]. \tag{73}$$

As $\frac{1}{n^j(1-n^j)^2}$ obtains its minimum of 6.75 when $n^j = 1/3$, $\gamma^j \leq 3.375$ ensures the denominator is positive. Since the numerator of the first term is smaller than the denominator, it is ensured the first term is smaller than 1 and thus A4′ holds.

Our Logit game with generic γ^j admits an ibr if $(r^j)'(A^{-j}) > -1$, which is the Lemma 1 in Anderson, Erkal and Piccinin (2020) implied by their A3. When $\gamma^j \leq 3.375$, the denominator is positive, and thus $(r^i)'(A^{-j}) \geq -1$ is simplified to

$$\left[\frac{1}{(A^{-j})^2} - \frac{2\gamma^j}{(r^j + A^{-j})^2}\right](r^j + A^{-j}) + \frac{1}{r^j} \geq 0 \tag{74}$$

$$\frac{1}{n^j} + \frac{1}{(1-n^j)^2} \geq 2\gamma^j, \tag{75}$$

the LHS of which obtains its minimum of ≈ 5.219 when $n^j \approx 0.361$. Thus when $\gamma^j \lesssim 2.610, \forall j$, our Logit game admits an ibr, which combined with A4′ yields equilibrium uniqueness. □

Proof of Proposition 4. With J platforms, a symmetric equilibrium with total demand N is characterized by the demand function

$$\frac{e^{-t}}{e^z} = \frac{n^j}{n^0} = \frac{N/J}{1-N}, \tag{76}$$

together with the pricing formula,

$$t = c + \frac{1}{1-n^j} - 2\gamma n^j = c + \frac{1}{1-N/J} - 2\gamma\frac{N}{J}. \tag{77}$$

We can combine these two and arrive at a characterization function $g(N;J)$ whose zeros are symmetric equilibria,

$$g(N;J) = z + c + \ln N - \ln J - \ln(1-N) + \frac{1}{1-N/J} - 2\gamma\frac{N}{J}. \tag{78}$$

We notice $\lim_{N\to 0} g(N;J) = -\infty$.

Suppose the monopoly has a market share of N, which solves $g(N;1) = 0$. With 2 platforms, if $g(N;2) > 0$, then by the intermediate value theorem, there must exist $N' < N$ that satisfies $g(N';2) = 0$. That is, the total demand under duopoly is lower than under monopoly. Since $g(N;1) = 0$, an equivalent condition to $g(N;2) > 0$ is $g(N;2) - g(N;1) > 0$, and we have

$$g(N;2) - g(N;1) = -\ln 2 + \frac{1}{1-N/2} - \frac{1}{1-N} + \gamma N, \tag{79}$$

which is positive if and only if γ is larger than a threshold,

$$\gamma \geq \frac{\ln 2}{N} + \frac{1}{(2-N)(1-N)}. \tag{80}$$

The RHS is convex in N. Numerically, we can find that the RHS obtains its minimum of 2.708 when $N \approx 0.470$. Thus, for any $\gamma \gtrsim 2.708$, there exists an interval of N such that this inequality holds. For any N in this interval, the $z + c$ that supports it as a monopoly equilibrium can be found from $g(N, 1) = 0$. Therefore, there exists an interval of $z + c$ such that total demand is lower at the symmetric equilibrium of the duopoly model than it is under monopoly. □

Proof of Proposition 5. The proof has a similar idea to the proof of Proposition 4. Here we study an asymmetric equilibrium in which there is one dominant platform, and $(J-1)$ symmetric smaller platforms. We use superscript 1 to denote the dominant platform and 2 for a generic smaller platform. We define $n := (J-1)n^2$ as the total demand of all smaller platforms, with $n^1 = 1 - n, n^2 = n/(J-1)$.

The demand of the dominant platform relative to a smaller platform satisfies,

$$\frac{e^{-t^1}}{e^{-t^2}} = \frac{n^1}{n^2} = \frac{1-n}{\frac{n}{J-1}}, \tag{81}$$

and the pricing formulas are,

$$t^1 = c + \frac{1}{1-n^1} - 2\gamma n^1 = c + \frac{1}{n} - 2\gamma(1-n) \tag{82}$$

$$t^2 = c + \frac{1}{1-n^2} - 2\gamma n^2 = c + \frac{1}{1-\frac{n}{J-1}} - 2\gamma\frac{n}{J-1}. \tag{83}$$

We can combine these three and arrive at a characterization function $g(n;J)$ whose zeros are equilibria,

$$g(n;J) = \ln n - \ln(J-1) - \ln(1-n) + \frac{1}{1-\frac{n}{J-1}} - 2\gamma\frac{n}{J-1} - \left(\frac{1}{n} - 2\gamma(1-n)\right). \tag{84}$$

We notice $\lim_{n\to 0} g(n;J) = -\infty$.

As before, suppose the duopoly has an equilibrium featuring a demand of the smaller platform n, which solves $g(n;2) = 0$. With 3 platforms, if $g(n;3) > 0$, then by the intermediate value theorem, there must exists $n' < n$ that satisfies $g(n';3) = 0$. As the dominant platform's market share is $1 - n'$, that means, the dominant platform is more dominant under triopoly than under duopoly. Since $g(n;2) = 0$, an equivalent condition to $g(n;3) > 0$ is $g(n;3) - g(n;2) > 0$, and we have

$$g(n;3) - g(n;2) = -\ln 2 + \frac{1}{1-n/2} - \frac{1}{1-n} + \gamma n. \tag{85}$$

This shows that, at a given n, the network externality γ has to be relatively strong for the difference to be larger than zero, very similar to the previous market contraction result.

However, as we restrict attention to *ex ante* identical platforms, the asymmetric equilibrium market outcome is also solely driven by γ. We rewrite $g(n;J) = 0$ as

$$\gamma = f(n,J) := \frac{1}{2}\frac{\ln\frac{n}{1-n} - \ln(J-1) + \frac{J-1}{J-1-n} - \frac{1}{n}}{\frac{n}{J-1} - (1-n)}, \tag{86}$$

which takes on a U-shape in n when $J = 2, 3$. In the relevant parameter range $\gamma \in (2.71, 3.375]$ that we are interested in, we can verify that at any n that solves $f(n,2) = \gamma$, we have $f(n,3) < f(n,2)$, suggesting that there exists $n' < n$ that solves $f(n',3) = \gamma$. We conclude that there exists an equilibrium under triopoly in which a dominant platform's market share is greater than the market share of any platform in any duopoly equilibrium. □

Proof of Proposition 6. Let 1 denote one of the two symmetric firms with marginal cost c to be merged, and 2 denote the other firm with zero marginal cost. Before the merger, with a market share of $N/2$, firm 1's FOC is

$$t^1 = c + \frac{1}{1-\frac{N}{2}} - 2\gamma\frac{N}{2}, \tag{87}$$

and firm 2's FOC is

$$t^2 = \frac{1}{N} - 2\gamma(1-N). \tag{88}$$

Their relative demand satisfies

$$\frac{e^{-t^1}}{e^{-t^2}} = \frac{N/2}{1-N}. \tag{89}$$

Combining these 3 equations to cancel t^1, t^2, we arrive at a characterization of the equilibrium N in terms of c, γ,

$$f(N) = c + \frac{1}{1-\frac{N}{2}} - \gamma N - \frac{1}{N} + 2\gamma(1-N) + \ln\frac{N}{2(1-N)} = 0. \tag{90}$$

Notice that $\lim_{N\to 0} f(N) = -\infty$, $f(\frac{2}{3}) = c > 0$, $\lim_{N\to 1} f(N) = \infty$, Further, when $\gamma < 2.62$ which we assume, $f(N)$ is increasing in N. Thus there is a unique solution $N \in \left(0, \frac{2}{3}\right)$, which is decreasing in c. When $N \in \left(0, \frac{2}{3}\right)$, $f(N)$ is also increasing in γ, suggesting N is decreasing in γ. As long as $c + \frac{\gamma}{2} > \frac{2}{3} + \ln 2 \approx 1.36$ so that $f\left(\frac{1}{2}\right) > 0$, it is guaranteed that $N < \frac{1}{2}$. That is, the most efficient firm (with zero marginal cost) has a market share that is larger than one half.

After the merger, if the merged identity has a marginal cost of $c' > 0$, its equilibrium market share N' is characterized by

$$g(N') = c' + \frac{1}{1-N'} - 2\gamma N' - \frac{1}{N'} + 2\gamma(1-N') + \ln\frac{N'}{1-N'} = 0. \tag{91}$$

Similarly, we observe that $g(0) = -\infty$, $g(\frac{1}{2}) = c' > 0$, $g(1) = \infty$. Further, when $\gamma \le 3$, $g(N')$ is increasing in N'. There is a unique solution $N' \in \left(0, \frac{1}{2}\right)$ that is decreasing in both c' and γ.

Suppose the pre-merger equilibrium features N, i.e. $f(N) = 0$. We have

$$g(N) = g(N) - f(N) = -\gamma N + \frac{N}{(1-N)(2-N)} + \ln 2 + c' - c. \tag{92}$$

The post-merger equilibrium entails $N' > N$ if and only if $g(N) < 0$, i.e.

$$\Delta c := c - c' > \frac{N}{(1-N)(2-N)} - \gamma N + \ln 2. \tag{93}$$

Here we see that, given N, a larger γ leads to a smaller threshold of Δc. □

Derivation of eq. (21). Denote $\pi(t^j, t^{-j})$ as platform j's profit, and denote $n^{-j} = \sum_{k\in\mathcal{J}\setminus\{j\}} n^k$. We have

$$\pi(t^j, t^{-j}) = (t^j - c^j + \gamma n^j + \gamma\lambda n^{-j})n^j, \tag{94}$$

and thus

$$\pi_1(t^j, t^{-j}) = -\frac{\partial n^j}{\partial t^j} \cdot \left(-t^j + c^j + \frac{n^j}{-\frac{\partial n^j}{\partial t^j}} - 2\gamma n^j - \gamma\lambda n^{-j} + \gamma\lambda\varphi^j n^j \right), \tag{95}$$

in which $\varphi^j(t) := \frac{\sum_{k \in \mathcal{J}\setminus\{j\}} \frac{\partial n^k(t)}{\partial t^j}}{-\frac{\partial n^j(t)}{\partial t^j}} \in [0,1]$, denoting platform j's *diversion ratio*. □

Proof of Proposition 7. For a symmetric equilibrium among symmetric platforms, we have $n^{-1} = (J-1)n^1$ and all platforms charge the same t^1. We define

$$\xi(t;\lambda) = -t + c + \frac{1}{1-n^j} - 2\gamma n^j - \gamma\lambda(J - 1 - \varphi^j)n^j, \tag{96}$$

with all platforms charging the same t. Any solution to $\xi(t;\lambda) = 0$ is a symmetric equilibrium, and conversely any symmetric equilibrium would satisfy $\xi = 0$.

Suppose there is a symmetric equilibrium with interoperability $\underline{\lambda}$ featuring $t^j = \underline{t}$, and we are to find a new symmetric equilibrium with higher interoperability $\overline{\lambda}$ featuring $t^j = \bar{t}$. We can write

$$\xi(t;\overline{\lambda}) = \xi(t;\underline{\lambda}) - \gamma(\overline{\lambda} - \underline{\lambda})(J - 1 - \varphi^j)n^j, \tag{97}$$

the latter term of which enters negatively unless $J = 2$ and $\varphi^j = 1$, in which case $\xi(t;\overline{\lambda}) = \xi(t;\underline{\lambda}), \forall t$ and thus there exists a new equilibrium with $\bar{t} = \underline{t}$.

If, however, $J > 2$ or $\varphi^j < 1$, then given any $\overline{\lambda} > \underline{\lambda}$, we have $\xi(t;\overline{\lambda}) < \xi(t;\underline{\lambda}), \forall t$. In this case, $\xi(\underline{t};\overline{\lambda}) < 0$, since $\xi(\underline{t};\underline{\lambda}) = 0$. To show that there exists $\bar{t} < \underline{t}$ satisfying $\xi(\bar{t};\overline{\lambda}) = 0$, it suffices to show that there exists t_- such that $\xi(t;\overline{\lambda}) > 0, \forall t < t_-$ and then the intermediate value theorem establishes the existence of such a $\bar{t} \in (t_-, \underline{t})$. We can choose any t_- such that

$$t_- < c - 2\gamma - \gamma\overline{\lambda}(J-1), \tag{98}$$

which implies, $\forall t < t_-$,

$$\xi(t;\overline{\lambda}) = -t + c + \frac{1}{1-n^j} - 2\gamma n^j - \gamma\overline{\lambda}(J - 1 - \varphi^j)n^j \tag{99}$$

$$> \frac{1}{1-n^j} + 2\gamma(1-n^j) + \gamma\overline{\lambda}\varphi^j n^j \tag{100}$$

$$> 0, \tag{101}$$

completing our proof.

Since we are studying symmetric equilibria with a fixed number of platforms, total market participation $N = Jn^j$ is inversely related to t, unless there is no outside option, in which case N is always equal to 1. □

Proof of Proposition 8. At the asymmetric equilibrium with no outside option, the pricing formula eq. (21) simplifies to, since the diversion ratio $\varphi^j = 1$,

$$t^j = c + \frac{1}{1-n^j} - 2\gamma n^j - \gamma\lambda(1-n^j) + \gamma\lambda n^j \tag{102}$$

$$= (c-\gamma\lambda) + \frac{1}{1-n^j} - 2\gamma(1-\lambda)n^j. \tag{103}$$

We use superscript 1 for the larger platform and 2 for the smaller one. The demand function gives that

$$\frac{e^{-t^1}}{e^{-t^2}} = \frac{n^1}{n^2}. \tag{104}$$

Combining these to cancel t^1, t^2 and plugging in $n^2 = 1 - n^1$, we get

$$\zeta(n^1;\lambda) := \ln\frac{n^1}{1-n^1} + \frac{1}{1-n^1} - \frac{1}{n^1} - 2\gamma(1-\lambda)(2n^1-1) = 0 \tag{105}$$

Observe that $\lim_{n^1\to 1}\zeta(n^1;\lambda) = \infty, \forall\lambda$. Consider two levels of interoperability $\underline{\lambda}, \overline{\lambda}$, and suppose the dominant platform has a market share $\overline{n}^1 > 1/2$ under $\overline{\lambda}$, i.e. $\zeta(\overline{n}^1;\overline{\lambda}) = 0$. At this $\overline{n}^1$ under a lower level of interoperability $\underline{\lambda}$, we have

$$\zeta(\overline{n}^1;\underline{\lambda}) = \zeta(\overline{n}^1;\underline{\lambda}) - \zeta(\overline{n}^1;\overline{\lambda}) \tag{106}$$

$$= 4(\underline{\lambda} - \overline{\lambda})\gamma\overline{n}^1 < 0. \tag{107}$$

As $\lim_{n^1\to 1}\zeta(n^1;\underline{\lambda}) = \infty$, by the intermediate value theorem, there exists $\underline{n}^1 > \overline{n}^1$ that solves $\zeta(\underline{n}^1;\underline{\lambda}) = 0$. That is, when the level of interoperability is lower, there exists an equilibrium in which the dominant platform has an even larger market share. □