Content-hosting platforms: discovery, membership, or both?*

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Abstract

We propose a model that classifies platforms in the so-called "creator economy", such as Youtube, Patreon, TikTok, and Twitch, into three broad business models: pure discovery mode (provides recommendations to help viewers to discover creators); pure membership mode (enables individual creators to monetize their viewers directly); and hybrid mode that combines both. Creators respond to platforms' decisions by individually choosing to supply content designed along a niche-broad spectrum, which involves a trade-off between viewership size and per-viewer revenue. Such endogenous responses create a link between two sources of platform revenue (advertising and transaction commission). Compared to the pure modes, the hybrid mode can lead to negative spillovers across the two sources of platform revenue so that it is not necessarily more profitable. In the case of competing platforms, incentives to avoid the negative spillovers from competition in transaction commissions to advertising revenue results in platforms choosing different equilibrium business models.

JEL classification: D26, D4, L2, L5

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1 Introduction

Online platforms such as YouTube, Facebook, Instagram, Snap, and Twitter have traditionally made the majority of their revenue by facilitating content discovery and selling the attention attracted by content creators hosted on their platforms to advertisers. As online content creation matures as an industry, firms like Patreon, Subbable, Substack, and Clubhouse have sprung up and operate as "membership portals". These portals provide an infrastructure for creators to directly monetize their viewers or readers through, e.g., subscriptions to an individual creator's channel, tipping/donations, and sales of exclusive content and merchandise. Meanwhile, Twitch (a live-streaming platform owned by Amazon) and TikTok (a short-video platform) have operated in a "hybrid mode" that includes both a content discovery component and an integrated membership component since their respective inceptions in 2013 and 2016. These contrasting modes of operation (content discovery mode, membership mode, and hybrid mode) motivate our theoretical study of platforms' choice of business models in the content creation market.

In recent years, a growing number of platforms in the content creation market have switched to operating as hybrid platforms. Leading content discovery platforms such as YouTube, Facebook, and Twitter have started to include membership portals in their operations. In 2018, YouTube and Facebook rolled out "channel membership" and "creator membership", which allows selected creators to offer per-month membership plans to their fans that provide access to exclusive live streams, exclusive content, fan badges, and members-only community posts, among other perks. In 2021, Twitter acquired a Substack-like newsletter startup and added Clubhouse-like audio features. The Economist (2021) noted that

> *"...Twitter was in danger of becoming a promotional tool for Substack writers and Clubhouse broadcasters,"*

and described Twitter's move as "trying to beat both (platforms) at their own game." Meanwhile, emerging platforms such as Teachable (for online courses) and Playbook (for fitness creators) launched as membership portals before starting to include content discovery components in their operations.

Is the hybrid mode necessarily more profitable than the other two business models? What are the trade-offs faced by a platform when it switches to the hybrid mode? How do the trade-offs change when there are multiple platforms? Can asymmetric platform business models arise in equilibrium? How do these affect the content design by creators in the equilibrium?

To explore these questions, we develop a model where creators choose the "design" of their content while consumers sequentially search for creators, realize a match value, and decide whether to become viewers of a creator. To model design decisions, we adopt a modified version of Johnson and Myatt (2006): the designs range from "broad" designs that are highly likely to match each consumer's taste but generate relatively low willingness to pay (conditional on a match realizing) to more "niche" designs where the opposite is true. This modeling choice captures the idea that creators primarily compete through content designs rather than solely rely on pricing strategies. Creators' revenue comes in two parts: a fixed payment for each viewer (e.g., advertising revenue) and additional revenue from pricing and selling exclusive content (e.g., channel subscriptions, additional content, and merchandise). Consumers initiate search if and only if the expected surplus is higher than their outside option, which in equilibrium increases with the broadness of content design chosen on average by the creators.

In our benchmark model we consider a monopoly platform that can include one or both of the following components: (i) a discovery portal and (ii) a membership portal. A discovery portal facilitates consumers' search by providing a recommendation at each step of their search process. We model the recommendation process as a Tullock contest where the probability of each creator being recommended is increasing in the match likelihood of the creator (which in turn increases with the broadness of the content design).[1] As an endogenous governance decision, the platform chooses the sensitivity of its recommendation to the match likelihood of creators. Another feature of the discovery portal is that it generates additional ad revenue for creators and the platform per visiting consumer. Meanwhile, a membership portal facilitates transaction convenience between consumers and creators, raising each creator's expected revenue from exclusive content. The platform charges a transaction commission on each unit of exclusive content revenue.

Our first finding shows that switching from the pure discovery mode (operating only the discovery portal) to the hybrid mode (operating both portals) is profitable for the platform and leads the platform to induce creators to shift toward niche content design. Intuitively, a niche design raises consumers' (conditional) willingness to pay for exclusive content, which a creator extracts through exclusive content pricing. The introduction of membership portal allows for a more effective transfer of consumer surplus, thus raising creators' marginal gain from choosing a niche content design. Even though the shift toward niche content may result in fewer consumers visiting the platform and thus harming the platform's profit, the platform can more than compensate for that loss via its optimal choice of transaction commission.

Our second finding shows that the hybrid mode is not necessarily more profitable than the pure membership mode (operating only the membership portal) even though the hybrid mode gives the platform an additional source of income in the form of ad revenue. All else being equal, introducing the discovery portal "distracts" creators in that it causes them to shift toward a broader content design due to (i) the competition for recommendations, and (ii) the weakly higher advertising revenue for creators (i.e., a greater marginal gain from increased viewership). This is sometimes counterproductive if the platform's revenue comes primarily from taxing creators' exclusive content revenue or if the number of consumers visiting the platform is already high. Moreover, the platform is generally unable to fully mitigate this "distraction effect" if the choice of feasible recommendation sensitivity is restricted or if creators are earning ad revenues that are unobserved and hence not taxable by the platform.

Allowing for multiple platforms (that make endogenous business model decisions), our next set of results show that a platform's choice of business model generates externalities on its rival

1. In practice, creators often complain of having to "chase the algorithm" where they compete with each other indirectly to get recommended to each consumer, much like a contest. A contest function provides a tractable reduced-form formulation to capture this type of competition. Following the contest literature, we adopt the Tullock contest function due to its desirable analytical properties and strong microfoundation (Jia, Skaperdas, and Vaidya 2013).

platform and affects the trade-offs across the choices of business models. This is true even if the platforms are not competing with each other directly, e.g., a pure discovery mode platform coexisting with a pure membership mode platform. Specifically, there are two key changes to the insights from the monopoly benchmark.

First, the hybrid mode is no longer necessarily more profitable than the pure discovery mode when there are competing platforms. Specifically, whenever the rival platform operates a membership portal, the switch to the hybrid mode results in competition between the two platforms for content creators, lowering the transaction commission. Unlike in the monopoly benchmark, the competitive constraint on the commission means that platforms are unable to mitigate (through its choice of commission) the resulting shift toward niche content design. Thus, our model describes a consequence of the competition that industry observers have been describing as "an arms race to acquire creators" (Culliford and Dang 2021): when a platform introduces a membership portal that competes with its rival, not only does it reduce the industry profit from transaction commissions, but the shift in creator production towards niche content design may also generate a negative spillover effect on the platform's existing advertising revenue.

Second, whenever the rival platform operates a discovery portal, the distraction effect that arises when a monopoly platform switches from the pure membership mode to the hybrid mode is no longer relevant in the profit comparison. Intuitively, when the rival platform is already operating a discovery portal, the distraction effect is always present regardless of another platform's choice of mode. Nonetheless, this does not necessarily imply that the profit comparison shifts in favor of the hybrid mode (relative to the monopoly benchmark) because the competition between the discovery portals of the two platforms also reduces the ad revenue that each platform earns from introducing a discovery portal.

Analyzing competing platforms' choices of business models, we find asymmetric business models arise in the equilibrium, with a mixture of a pure discovery platform with either a pure membership platform or a hybrid platform. The asymmetry arises as long as competing membership portals would be viewed as sufficiently close substitutes, and it reflects platforms' strategic incentive to avoid the spillover from competition in transaction commission to advertising revenue. Symmetric business model arise in the equilibrium only if the commission competition is sufficiently weak.

A corollary of the previous result is that having multiple platforms induces content design with higher broadness in equilibrium relative to the monopoly benchmark. Intuitively, the competition for creators means that only one platform can access infra-marginal consumers' value for niche content. Thus, direct competition between platforms (which only occurs in equilibrium when one platform is a hybrid) focuses on marginal consumers. This leads to the discovery portal(s) setting maximal sensitivity to match probability in its (their) recommendation algorithm(s), even when one platform operates in hybrid mode.

Finally, we consider a number of extensions of our framework: (i) allowing creators to be heterogeneous in terms of the profitability of their advertising revenue; (ii) allowing for cross-group network effects in creator and consumer participation. Our main insights remain valid in these extensions, but with richer mechanisms.

2 Related literature

Media platforms and user-generated content. Most of the existing literature on media platforms has focused on two-sided intermediaries between consumers and advertisers (Anderson and Coate 2005; Armstrong 2006; Crampes, Haritchabalet, and Jullien 2009; Peitz and Valletti 2008; Athey, Calvano, and Gans 2018; Anderson and Peitz 2020).[2] We simplify the advertiser side of the market, as we take platform and creators' advertising revenue as given. Our focus is instead on the side of independent creators who contribute content to media platforms and relate this feature with platforms' choice of business model.

A number of recent works in economics, strategy, and marketing literature explore the role of user-generated content on media platforms. Among the issues explored in this branch of the literature are: quality investments by independent news or content contributors (Jeon and Nasr 2016; Dellarocas, Katona, and Rand 2013; De Cornière and Sarvary 2020), user-generated ratings (Luca 2015), user-generated content and endogenous horizontal differentiation among media platforms (Zhang and Sarvary 2015), bias in media provision (Yildirim, Gal-Or, and Geylani 2013), and how the competitive environment on platforms affects the behavior of independent creators or influencers (Pei and Mayzlin 2021; Fainmesser and Galeotti 2020; Kerkhof 2020). These works do not consider the implications of media platforms' choices of business models nor do they consider the possibility of platforms facilitating creators' direct monetization of their viewers.

Business models of media platforms. Our work is closely related to recent works that analyze media platforms' endogenous choice between two types of business models: a subscription model (or Pay-TV) in which the platform raises most (if not all) of its revenue from the consumers, and an ad-funded (or free-to-air) model in which the platform raises most of its revenue from advertisers.[3] Kind, Nilssen, and Sørgard (2009) relate symmetric business model choices to the extent of content differentiation among media firms. Calvano and Polo (2020) show that asymmetric business models can coexist in broadcasting market even when firms are ex-ante symmetric, reflecting a strong substitutability in firms' advertising quantity decisions. Carroni and Paolini (2020) link a monopoly platform's business model choice with its incentive to price discriminate between "non-paying" users and "premium" users. These papers do not consider the role of independent content creators and the governance design options available to the platform in each business model, both of which are the main drivers of our results.

The only exceptions are the recent contributions by Liu, Yildirim, and Zhang (forthcoming) and Bhargava (forthcoming). Liu, Yildirim, and Zhang (forthcoming) consider a platform that make content moderation decisions that affects users willingness to participate and their intensity of posting content. Among other things, they show how a monopoly platform's choice

2. For a comprehensive textbook treatment on this large literature, see Anderson, Waldfogel, and Strömberg (2016).

3. In a slightly different vein, a number of contributions focus on intermediaries that connect between buyers and sellers, and compare between business models such as: marketplace, reseller, or a combination of both (Hagiu and Wright 2015; Anderson and Bedre-Defolie 2021; Hagiu, Teh, and Wright, forthcoming), price-dependent profit sharing (Foros, Hagen, and Kind 2009), platform or vertically integrated firm (Hagiu and Wright 2018). These comparisons involve trade-offs such as double marginalization, price coordination in vertical channels, asymmetric costs and information, and moral hazard, which are less prominent in our context of content platforms.

between subscription and ad-funded models depend crucially on users' utility from posting content. Bhargava (forthcoming) focuses on a monopoly content platform that is ad-financed with endogenous creators' participation and supply decisions. He derives the implications of various platform design choices, e.g., tools that lower consumers' distaste for ads and creators' creation costs. Our analysis differ from these in many respects, including the following: we model the emerging business model of membership portal that enables direct transaction between creators and consumers, we focus on creators' content design decision along a broad-niche spectrum, and we consider platform recommendation design. Furthermore, we show that models of monopoly platform and competing platforms lead to substantially different insights on the profit comparison of business models.

Discovery portal and platform governance design. One novelty of our formulation of a discovery mode, which distinguishes it from the ad-funded business model considered in the media literature, is that a discovery mode platform makes content recommendations that facilitates consumer search. Thus, our paper broadly relates to recent contributions on platform incentives in governance design decisions. Casner (2020) and Teh (forthcoming) focus on the role of governance (e.g., screening, search design, and information provision) as an instrument that trades off between competition among sellers and gross value generated from transactions. Choi and Jeon (2021) and Madio and Quinn (2021) consider technology adoption and content moderation as tools for a platform to balance between the interests of consumers and advertisers. These papers take as given platform business model and focus on the welfare distortions in platform design decisions that arise due to the platform's profit-maximization motive.

Membership portal and crowdfunding: Finally, like crowdfunding platforms, membership portals act as a coordination device to help consumers who like a specific product (in this case a creator's content) agree to fund the creation of that product. Our paper thus has a loose connection to the crowdfunding literature (Deb, Oery, and Williams 2019; Ellman and Hurkens 2019). Notably however, that literature tends to focus on the mechanism design aspects of one-shot project-based crowdfunding efforts, whereas the business model of membership portals is based around support for content creators who produce content on a continuing basis so long as doing so is more appealing than their outside option.

3 Benchmark model

There is a monopoly platform P, a continuum of consumers and a continuum of ex-ante symmetric content creators, both of measure one. The platform can include one or both of the following functionalities in its operation: (i) a discovery portal; (ii) a membership portal. We say that the platform is hybrid if it operates both portals.

Consumers. For each consumer j, creator i matches j's taste with probability λ_i. In this case, if the consumer becomes a viewer of the creator, she obtains utility

$$b_j + \beta_j \max\{v_i - p_i, 0\} > 0.$$

Here, b_j is the consumer-specific preference intensity for watching generic content, while β_j is the likelihood (conditioned on being a viewer) that the consumer is interested in purchasing and accessing exclusive content, which gives gross utility v_i at price p_i. This exclusive content does not have to be of the same type as the generic content. More generally, it includes other direct value transfers from viewers to creators such as membership subscription and donation in exchange for virtual items.[4] The consumer can choose not to access exclusive content. With the remaining probability $1 - \lambda_i$ there is a taste mismatch and the consumer gets zero utility.

Consumers have heterogeneous β_j and b_j, which are independently realized with smooth distributions. Denote $\beta_0 = E[\beta_j] > 0$ as the unconditional average value of β_j, and denote

$$G(x) = \Pr(b_j > 1/x). \tag{1}$$

which is assumed to be an increasing, continuous differentiable, and concave function. As will be seen later, function $G(x)$ is the measure of the extensive margin of the market, i.e., the mass of consumers that are active.

Creator content design. Each creator i chooses a single content design strategy (λ_i, v_i) subject to the constraint that $v_i \leq v(\lambda_i)$, where $\lambda_i \in [0,1]$ and $v(.)$ is a decreasing function. This feature reflects the broad-versus-niche design trade-off in the spirit of Johnson and Myatt (2006) and Bar-Isaac, Caruana, and Cuñat (2012). A high λ_i corresponds to a "broad" design: the content has mass-market appeal and is likely to match the taste of many consumers but consumers have low willingness to pay for exclusive content (conditioned on a match); a low λ_i corresponds to a "niche" design: the creator tailors and targets content to a small group of viewers such that these viewers have high willingness to pay for exclusive content.[5] We assume function $v(\lambda_i)$ is continuously differentiable with boundary condition $v(1) = 0$.

Per-viewer revenue. On top of the content design decision, each creator i also chooses the access price p_i for their exclusive contents. Without the discovery and membership portals, a creator i's default per-viewer revenue is

$$a_0 + \beta_0 p_i.$$

We loosely interpret $a_0 \geq 0$ as the ad or sponsorship revenues but it can also include creators' intrinsic and image-related utility from gaining viewers and followers (Toubia and Stephen 2013). Meanwhile, $\beta_0 p_i$ is the expected exclusive content revenue. Creators face no fixed costs, while the value of their outside option of being inactive (i.e., producing no content) is normalized to zero.[6]

4. For example, in the context of Twitch, which is primarily a video game streaming website, this can take the form of add-on content such as specially designed chat emotes or events where the streamer plays games with viewers. Creators on Patreon will sometimes offer small chat sessions with supporters and some will even send "thank you" postcards, while creators on Youtube can sell their official branded merchandise through Youtube's integrated portal.

5. We are only assuming that there is a trade-off at the "design possibility frontier". The assumption does not imply that a broader design always leads to a lower willingness to pay.

6. The zero fixed cost assumption for creators guarantees that all creators are active (produce content) in equilibrium. The same holds even if we allow creators to face a strictly positive fixed cost $c > 0$ of being active as long as c is not too large. In Section 6.2, we extend our analysis to the case where creators face heterogeneous net gain from being active, so that the platform can influence the mass of active creators through its decisions.

Whenever the platform operates a discovery portal, it increases the per-viewer ad revenue of the creator to $\bar{a}$ while generating ad revenue $A \geq 0$ for itself. We assume $\bar{a} \geq a_0$, which reflects potential economies of scale in advertising opportunities that raises the ad or external sponsorship revenue earned by each creator. Following Gabszewicz, Laussel, and Sonnac (2004) and Casadesus-Masanell and Zhu (2010), we assume that a_0, $\bar{a}$ and A are endogenously determined by a competitive advertising sector.

Whenever the platform operates a membership portal, it increases β_0, the average likelihood that each consumer is interested in accessing exclusive content, to $\bar{\beta} > \beta_0$. This reflects that a membership portal facilitates trust and convenience of direct transactions between creators and viewers, making them more likely to purchase exclusive content. For each unit of exclusive content revenue, the platform takes a share (or commission rate) τ while the creator receives the remaining $1 - \tau$.

Search process. A consumer incurs a positive search cost to learn whether there is a match in taste with any particular creator i and i's price for exclusive content. Consumers search sequentially, and at any point of the search process they can choose to (i) search through the platform if the platform operates a discovery portal (with per-search cost $s > 0$); or (ii) search directly (with per-search cost $s_0 \geq s$). Consumers stop when they reach a creator that matches their taste and they always have an outside option of doing nothing, which yields zero utility.

If a consumer searches directly, the search process is random as in Wolinsky (1986) with each creator being drawn at equal probability. If a consumer searches through the platform (whenever possible), then at every step the platform recommends a creator to the consumer and the consumer decides whether to incur a search cost to learn about the creator. Borrowing from the huge literature on the economics of contests, we model platform's recommendation as a Tullock contest. Suppose a set $\mathcal{I}$ of creators join the platform, then the probability of a given creator $i \in \mathcal{I}$ being recommended is

$$D(\lambda_i; \lambda_{-i}) = \frac{\lambda_i^r}{\int_{k \in \mathcal{I}} \lambda_k^r \, dk}, \tag{2}$$

where the exponent $r \in [\underline{r}, \bar{r}] \subseteq [0, \infty)$ is the standard noise parameter of Tullock contest function.

We call $D(\lambda_i; \lambda_{-i})$ the *recommendation function* and r the *sensitivity* of the platform's recommendation algorithm to each creator's λ_i. The platform chooses r as its governance decision. The recommendation becomes completely random if $r = 0$ and perfectly discriminative if $r \to \infty$. Operating a discovery portal involves a fixed setup-up cost $C \geq 0$, which is assumed to be not too large relative to A so that operating a pure discovery portal is never loss-making.[7]

Timing. The timing of the model is the following:

1. The platform chooses its mode of operation

7. The exact sufficient condition is, $G\left(\frac{\lambda^*(\tau,r)}{s}\right)|_{\tau=0,r=\bar{r}} \geq \frac{C}{A}$ where $\lambda^*(\tau, r)$ is defined in (6). In addition, our model easily extends to the case where $C(r)$ increases with r, capturing the idea that a more informative recommendation may involve costly investments. The amendment affects the comparison of the equilibrium broadness between the pure discovery platform and the hybrid platform (i.e., the second inequality in Proposition 1), but does not otherwise affect the main insights.

2. The platform sets its recommendation design r (if it operates a discovery portal) and its transaction commission τ (if it operates a membership portal).

3. Creators simultaneously make participation decisions and choose λ_i and p_i.

4. Consumers observe r and τ, do not observe decisions of creators, and then choose whether and where to search.[8]

The equilibrium concept we adopt is symmetric perfect Bayesian equilibrium (PBE) with all creators adopting the same strategy in the equilibrium. We rule out trivial equilibria in which no creators join the platform's discovery portal, and, expecting that, no consumers discover through the discovery portal. Notice that consumers observe only decisions by the platform and do not observe decisions of the creators (λ_i, p_i, and participation). When deciding whether and where to search, consumers form rational beliefs on these decision variables (conditioned on the observed variables). As is standard in the search literature, we impose that consumers keep the same (passive) beliefs about creators' decisions off the equilibrium path whenever applicable.

3.1 Discussions of modeling features

The assumption of symmetric creators simplifies the exposition, but it is not a necessary ingredient to derive the main insights. As an extension, we consider creators who are heterogeneous in terms of the profitability of their advertisement revenue. For example, creators that focus on fashion or boutique related content may find it easier to secure advertisement or sponsorship deals from merchants (e.g., higher willingness to pay for "eyeballs" for such content categories) compared to creators who focus on educational content. Our main results continue to hold in this setting but with richer mechanisms, which we discuss in Section 6.1.

Instead of a broad-niche trade-off, an alternative interpretation for creators' design variable λ_i is that each creator has one unit of fixed time endowment, which can be allocated between public or ungated content (λ_i) and exclusive content ($1 - \lambda_i$). Investing in public content allows the creator (hence the platform) attract consumers and expand the size of viewership, while investing in exclusive content allows the creator to raise consumers' willingness to pay for exclusive content.

There are a few possible interpretations for the platform's probabilistic recommendation rule (2) and the recommendation sensitivity r. Suppose the platform can only condition its recommendation on the "popularity" of each creator (as measured by λ_i). Then, (2) means that the platform recommends the most popular creators, subject to an (inverse) noise factor r that reflects the precision of the platform's knowledge of the true λ_i of each creator. Alternatively, (2) could reflect that the platform gives personalized recommendations to heterogeneous consumers, where r indicates the weight that the platform assigns to λ_i relative to idiosyncratic consumer attributes.

8. Alternatively, we can assume that consumers observe the "average design" λ_i of creators before searching. Given that there is a continuum of creators (so that decision of each creator's unilateral decision does not affect consumers' search decisions), all of our analysis remains unaffected in this case regardless of whether consumers observe r and τ or not.

	Creator's ad revenue	Creator's exclusive content revenue	Recommendation sensitivity	Search cost
Pure discovery	$\bar{a}$	β_0	$r \in [\underline{r}, \bar{r}]$	s
Pure membership	a_0	$(1-\tau)\bar{\beta}$	$r = 0$	s_0
Hybrid	$\bar{a}$	$(1-\tau)\bar{\beta}$	$r \in [\underline{r}, \bar{r}]$	s

Table 1: Three modes of operation.

Our model can easily accommodate the case of preference intensity b_j being creator-consumer specific, that is, $b_j = b_{ij}$ (creator i and consumer j). In this case, consumers may continue searching even after finding a match (instead of stopping at the first positive-match creator. Nonetheless, if we assume that consumers observe each creator i's price p_i for exclusive content only after becoming a viewer,[9] then creators' maximization problem (hence their equilibrium choices of design λ_i) would remain unaffected based on a Diamond Paradox argument. It is then easily verified that all our results below remain valid after modifying the exact expression for consumers' ex-ante expected net gain from initiating search.

4 Analysis of monopoly benchmark

4.1 Creators and consumers decisions

We start by characterizing the equilibria in the subgame under each of the three business modes of platform P: pure membership mode, pure discovery mode, or hybrid mode. We focus on analyzing the subgame between creators and consumers for each given design r and commission τ chosen by P, assuming that P operates as a hybrid platform. Notice that the analysis of the subgame in hybrid platform nests the cases of pure membership portal and pure discovery platform as special parametric cases as stated in Table 1.

Whenever P is a hybrid platform, we assume throughout that two of its component functionalities are *unbundled*, in line with our motivating examples. This means that creators can choose to join its membership portal, its discovery portal, or both. Consequently, it is easy to see that each creator's dominant strategy is to join the membership portal component as long as $(1-\tau)\bar{\beta} \geq \beta_0$, and not to join this component otherwise. This implies M's commission is bounded by this participation constraint (on the membership portal)

$$\tau \leq 1 - \frac{\beta_0}{\bar{\beta}}.$$

In equilibrium, M never sets τ that violates this constraint. Then, the equilibrium of the subgame following each given $\tau \leq 1 - \beta_0/\bar{\beta}$ and r can be stated as:

1. Each creator i joins both portals of the platform, sets design

$$\lambda^* = \arg\max_{\lambda_i} \left\{ \lambda_i^{1+r} \times \left(\bar{a} + (1-\tau)\bar{\beta} v(\lambda_i) \right) \right\}. \tag{3}$$

9. In the baseline with creator-invariant b_j, the logic of Diamond Paradox implies that whether consumers observe p_i before or after becoming viewers would not affect the analysis because creators will always set $p_i = v_i$ in the equilibrium.

and exclusive content price $p^* = v(\lambda^*)$.

2. Each consumer believes that all creators adopt strategy (λ^*, p^*) stated above, initiates search if and only if

$$b_j > \frac{s}{\lambda^*} \tag{4}$$

and do so through the platform. The consumer continues searching on the platform until finding a match. The mass of consumers who search is $G\left(\frac{\lambda^*}{s}\right)$.

We now describe the equilibrium construction. We first note that, upon receiving the recommendation, a consumer has no incentive to deviate by not following the recommendation given the belief of symmetric equilibrium. This feature of the equilibrium remains robust even if creators choose asymmetric design in the equilibrium, because (i) the recommendation rule (2) and (ii) creators' exclusive content pricing strategies, together, imply that the recommended creator is more likely to result in a positive surplus (higher λ_i) than the non-recommended creators.[10]

The analysis of consumers' search decision follows from the standard analysis of Weitzman (1979). Any consumer j who has found a match at creator i will not search further because the surplus from stopping is higher than the continuation value:

$$b_j + \beta_j \max\left\{v\left(\lambda_i\right) - p_i, 0\right\} > \lambda^* b_j - s$$

given the equilibrium strategies of creators. Meanwhile, any consumer j who has not found a match will continue searching if

$$\lambda^* b_j - s \geq 0.$$

With a continuum of creators, all consumers who have initiated search will continue searching until they have found a match. Finally, all consumers search through the platform (whenever a discovery portal is available) in every step of search given the weakly lower search cost.

Consider a creator i's decisions, expecting that all other creators are choosing the equilibrium strategy. Denote ρ as the expected probability that a random creator $k \neq i$ is recommended and successfully results in a match with a consumer (i.e., successful viewer conversion); this is exogenous from creator i's perspective given that there is a continuum of creators. In particular, i's decision does not affect the denominator of recommendation function (2). Denote the expected number of consumers who are recommended i and join i's audience in the first round of search as

$$m_i = G\left(\frac{\lambda^*}{s}\right) \times D\left(\lambda_i; \lambda^*\right) \times \lambda_i.$$

In the second round of search, a further $(1-\rho)m_i$ consumers do not find a match in the first round of search are recommended i and become i's viewers; In the third round, a further $(1-\rho)^2 m_i$ consumers become i's viewers; and so on. We can, therefore, write creator i's profit as

$$\pi_i = \frac{m_i}{\rho}\left(\bar{a} + (1-\tau)\bar{\beta} p_i\right).$$

10. We formally show this in the extended model with asymmetric creators in Section 6.1.

Clearly, $p_i = v(\lambda_i)$ is optimal because consumers will not purchase the exclusive content at higher prices. Expanding m_i, we get

$$\pi_i = \frac{G\left(\frac{\lambda^*}{s}\right)}{\rho} \times \frac{\lambda_i^r}{\int_{k\in\mathcal{I}} \lambda_k^r dk} \times \lambda_i \times \left(\bar{a} + (1-\tau)\bar{\beta} v\left(\lambda_i\right)\right).$$

Finally, no creators have incentive to deviate by not participating because doing so does not influence consumers' search decisions, meaning that the deviation profit is zero.

After dropping the multiplicative factors that are exogenous from i's viewpoint, maximizing π_i with respect to λ_i yields the optimal content design decision λ^* in (3). If $\lambda^* > 0$ is interior, then the corresponding first-order condition is

$$\left(\frac{\bar{a}}{(1-\tau)\bar{\beta}} + v\left(\lambda^*\right)\right)(1+r) + v'(\lambda^*)\lambda^* = 0; \tag{5}$$

and otherwise $\lambda^* = 1$. Expression (5) reflects the standard trade-off between marginal revenue from expanding viewer size (through a higher broadness) and the inframarginal loss from a lower per-viewer exclusive content revenue (given $v'(\lambda^*) < 0$).

The following comparative statics exercise describes how the platform's choice of business model and decisions affect the equilibrium design decisions of content creators in the subgame as follows:

Lemma 1. *In the equilibrium of the creator subgame, λ^* is non-decreasing in r, $\bar{a}$, and τ and strictly increasing if λ^* is interior.*

The result is intuitive. A more sensitive recommendation design (higher r) intensifies creators' competition for recommendation, inducing them to raise their broadness in the equilibrium (higher λ^*). A higher creator advertising revenue $\bar{a}$ or a lower creator exclusive content revenue $(1-\tau)\bar{\beta}$ shifts the profitability of a broad content design strategy relative to those of a niche design, thereby inducing creators to raise their broadness.

Finally, from the characterization of the equilibrium of the subgame under the hybrid mode, we can recover the equilibrium of the subgame under the pure discovery mode and pure membership mode by substituting the parameters according to Table 1. Moreover, Lemma 1 extends immediately to these two modes.

4.2 Platform decisions in each mode

In this section, we analyze the platform's decision in each of the three business modes, and then compare platform profit and market outcomes across them.

Pure membership mode. From the previous subsection, we know that the equilibrium design of creators (as a function of platform commission τ) in this case is:

$$\lambda_M = \lambda_M(\tau) = \arg\max_{\lambda_i} \left\{\lambda_i \times \left(a_0 + (1-\tau)\bar{\beta} v\left(\lambda_i\right)\right)\right\},$$

which is increasing in τ (Lemma 1). We also know that every consumer that initiates search would eventually find a match. Thus, the platform chooses $\tau \leq 1 - \beta_0/\bar{\beta}$ (creators' participation constraint) to maximize

$$\Pi_M(\tau) = G\left(\frac{\lambda_M}{s_0}\right)\tau\bar{\beta}v(\lambda_M).$$

If creators' design were exogenous, then the platform optimally sets the highest possible τ. However, the endogeneity of design generates an additional trade-off: a higher τ induces creators to shift towards broader designs, which expands the total number of viewers but reduces the per-viewer transaction revenue of the platform. Denote the platform's optimal commission as $\tau_M^* > 0$ and the induced design as $\lambda_M^* = \lambda_M(\tau_M^*)$.

Pure discovery mode. The equilibrium design of creators (as a function of platform recommendation design r) in this case is:

$$\lambda_D = \lambda_D(r) = \arg\max_{\lambda_i} \left\{\lambda_i \times (\bar{a} + \beta_0 v(\lambda_i))\right\},$$

which is increasing in r (Lemma 1). Then, the platform chooses $r \in [\underline{r}, \bar{r}]$ to maximize

$$\Pi_D(r) = G\left(\frac{\lambda_D}{s}\right)A - C.$$

Given that a higher r induces creators to shift towards broader design, expanding the viewer base participating on the platform, we conclude that the platform optimally chooses $r_D^* = \bar{r}$ (i.e., the most sensitive recommendation design that is possible) and induces $\lambda_D^* = \lambda_D(r_M^*)$.

Hybrid mode. For each given r and τ, denote the equilibrium design of creators as $\lambda_H = \lambda_H(\tau, r)$, which is exactly λ^* in equation (3). The platform chooses r and $\tau \leq 1 - \beta_0/\bar{\beta}$ to maximize

$$\Pi_H(\tau, r) = G\left(\frac{\lambda_H}{s}\right)\left(A + \tau\bar{\beta}v(\lambda_H)\right) - C.$$

Denote the solution as r_H^* and $\tau_H^* > 0$, and let $\lambda_H^* = \lambda_H(\tau_H^*, r_H^*)$. Compared to the pure discovery mode, a platform in the hybrid mode has to balance the viewership expansion effect of a higher r against its negative effect on the transaction revenue (as creators shift away from niche designs). Hence, in the hybrid mode the platform chooses an algorithm that is less sensitive and induces weaker competition between creators, i.e., $r_H^* \leq r_D^*$ in general.

Lemma 2. *Consumer search costs have the following effects:*

1. τ_M^* *and* λ_M^* *are weakly increasing in* s_0*;*
2. r_D^* *and* λ_D^* *are independent of* s*;*
3. τ_H^**,* r_H^**, and* λ_H^* *are weakly increasing in* s*.*

Intuitively, a higher search cost (a higher s or s_0) shrinks the viewer size thus increases the platform's marginal gain from expanding the viewer size. As such, the platform adopts decisions that are more conducive for broad content, that is, a higher τ and r. The independence result in the pure discovery mode is an artifact of absence of any trade-off in platforms' recommendation design r. In more general models where raising r involves additional operating costs to the pure discovery platform, one can easily show that r_D^* is increasing in s, following the same intuition discussed in this paragraph.

Lemma 2 is reminiscent of those results obtained by Bar-Isaac, Caruana, and Cuñat (2012), who show a higher search cost induces competing sellers to adopt broad product designs (analogous to a higher equilibrium λ in our setting). The mechanism of our result is different from theirs because, in our setting, search cost affects creators' equilibrium content design exclusively through the platforms' decisions on recommendation design and commission that favors broader designs. Our result thus generates a testable implication on how search cost affects the recommendation design and commission strategies employed by platforms.

4.3 Comparing platform business models

Starting from either pure discovery mode or pure membership mode, we consider how introducing additional functionalities (and thus switching to the hybrid mode) affects platform's profit and market outcome.

Proposition 1. *(Pure discovery versus hybrid).* $\Pi_H^* > \Pi_D^*$ *and* $\lambda_H^* \leq \lambda_D^*$ *with the inequality strict if* $\lambda_H^* < 1$.

Starting from the pure discovery mode, adding the membership portal raises creators' marginal exclusive content revenue ($(1-\tau)\bar{\beta} \geq \beta_0$), which induces a shift towards niche content design and so $\lambda_H^* \leq \lambda_D^*$. This effect is further cemented by the fact that in the equilibrium the platform optimally adopts a less selective algorithm after becoming hybrid ($r_H^* \leq r_D^*$).

One might worry that the shift towards niche designs may be undesirable for the platform if viewership expansion is important (i.e. if G is highly elastic). However, the hybrid platform can exactly replicate the equilibrium design of the pure discovery mode by setting $r = r_D^*$ and $\tau = 1 - \beta_0/\bar{\beta}$, and so a profit replication argument implies $\Pi_H^* > \Pi_D^*$. Notably, this result heavily relies on the fact that the platform only faces a participation constraint in its commission choice. As will be seen in the next section, the replication argument no longer holds when the platform faces competition, in which case the hybrid mode is not necessarily more profitable than the pure discovery mode.

Proposition 2. *(Pure membership versus hybrid).*

- *There exists a threshold* $A_1 \geq 0$ *such that* $\Pi_H^* > \Pi_M^*$ *if and only if the platform's advertising revenue* $A > A_1$*; Threshold* A_1 *becomes lower when* s *decreases or* s_0 *increases.*
- *There exists a threshold* $A_2 \geq 0$ *such that* $\lambda_H^* \geq \lambda_M^*$ *if and only if the platform's advertising revenue* $A > A_2$*; Threshold* A_2 *becomes lower when* s *increases or* s_0 *decreases.*

The first part of Proposition 2 says that the hybrid mode is sometimes less profitable than the pure membership mode, and this is true even if the setup cost of the discovery portal is $C = 0$. Intuitively, by adding a discovery portal, the platform benefits from: (i) earning additional advertising revenue A; (ii) lowering consumer search costs to $s \leq s_0$ thus attracting more consumers to search for content. However, all else being equal, the increased creator advertising revenue $\bar{a} \geq a_0$ (due to the addition of discovery portal) and the competition for recommendation ($r \geq 0$) would "distract" creators and induce them to choose broader designs. This distraction effect helps to increase total platform viewership, but it may be counterproductive if the platform is primarily earning from taxing creators' exclusive content revenue.

As opposed to Proposition 1, the profit replication argument is not applicable here. First, the platform may not be able to completely eliminate the increased creator advertising revenue $\bar{a} - a_0$ even if it is allowed to tax those revenues. In practice, creators often engage in external advertising sponsorship and affiliated marketing that are unobservable (hence not taxable) by the platform, meaning that the same mechanism remains valid even if we allow the platform to partially tax the advertising revenue.[11] Second, operating the discovery portal may entail a minimum level of recommendation precision $\underline{r} > 0$, which prevents the hybrid platform from exactly replicating the outcome of the pure membership mode.[12]

Proposition 2 helps explain why pure membership platforms like Patreon do not offer more extensive discovery services. If they were to do so, then this would lead creators adjusting their content design to appeal to whatever recommendation algorithm Patreon's hypothetical discovery portal would use, leading to a reduction in value for the exclusive content given to users of the platform.[13]

As noted in the proposition, it is possible under some parameter sets that the benefits of adding a discovery portal outweigh the costs and so $A_1 = 0$. However, we show in Remark 1 below an example of where adding a discovery portal reduces profit. Consistent with the intuition above, the possibility of $\Pi_M^* > \Pi_H^*$ is driven by $\bar{a} \geq a_0$ and $r \geq 0$[14]

Remark 1. *Suppose $v(.)$ is linear, $A \to 0$, $C = 0$, $s \to s_0$, and $\tau_M^* < 1 - \beta_0/\bar{\beta}$. If $\bar{a} > a_0$, then $\Pi_M^* > \Pi_H^*$ and the difference $\Pi_M^* - \Pi_H^*$ becomes larger as $\bar{a}$ increases or $\underline{r}$ increases.*

The second part of Proposition 2 says that the equilibrium content design may become more niche after the platform adds the discovery portal. This may be surprising given that the portal creates "competition for recommendations", so that standard intuition suggests that content design should become broader. Indeed, the intuition would be true were the platform's commission exogenous. With the commission set endogenously however, the lower search cost

11. See, e.g., https://medium.com/writers-blokke/why-youtube-adsense-shouldnt-be-your-main-source-of-income-312c9674e518.

12. In some settings, combining the functionalities of search and membership portals on a single platform leads to a synergistic effect because consumers may be more willing to purchase through the membership portal of the same platform that they search on due to, e.g., behavioral inertia or exogenous switching costs. Our model can easily incorporate this feature by assuming that consumers' average likelihood to purchase through the membership portal is $\bar{\beta} + \epsilon$ if they purchase through the membership portal of the same platform that they search on where $\epsilon \geq 0$. Otherwise, the likelihood remains at $\bar{\beta}$ in pure membership mode. All else being equal, a higher $\epsilon \geq 0$ simply makes the hybrid platform more profitable, thus shifting the comparisons in Proposition 2, lowering A_1.

13. See https://web.archive.org/web/20190410012719/https://blog.patreon.com/why-isnt-patreon-discovery-platform for some discussion of this tradeoff by Patreon itself.

14. Alternatively, it is easily verified that if $\bar{a} = a_0$, then $\Pi_H^* \geq \Pi_M^*$.

$s \leq s_0$ and the associated expansion in viewership size means that the platform may want to lower its commission (Lemma 2) after becoming hybrid.

As a case in point, the following remark shows that the possibility of $\lambda_M^* \geq \lambda_H^*$ is indeed driven by $s \leq s_0$:

Remark 2. *If $A \to 0$, $\bar{r} \to 0$, and $\bar{a} \to a$, then $\tau_M^* \geq \tau_H^*$ and $\lambda_M^* \geq \lambda_H^*$ where both differences become larger as s_0 increases or s decreases.*

Finally, the comparison between pure discovery and pure membership modes is essentially a special case of Proposition 2, hence omitted here. Following the same logic as above, one can easily show that $\Pi_D^* > \Pi_M^*$ if A is sufficiently large and that $\lambda_D^* \geq \lambda_M^*$, with the inequality strict if $\lambda_M^* < 1$.

5 Multiple platforms

Suppose there are two homogeneous platforms P_l , $l = 1, 2$, each deciding its whether to operate a discovery portal, a membership portal, or both. Creators are free to multihome: they can join multiple discovery portals and multiple membership portals. Consistent with the benchmark model, each creator makes a single content design strategy that is not contingent on how each consumer finds the creator.

Consumers are free to choose where to search in each step. Whenever a consumer finds a creator that matches her taste, it generates platform advertising revenue only on the platform where the match occurs. Then, if the consumers wants to purchase exclusive content from the creator, she randomly chooses to do so through one of the membership portals (that the creator has joined) given that these portals are homogeneous from her viewpoint.[15] Thus, consumers are multihoming in the sense that they do not incur additional cost for using different platforms to discover content and purchase exclusive content.

Timing and tie-breaking. The timing of this model is the same as the monopoly benchmark, except that the platforms simultaneously choose their modes of operation in Stage 1, and then simultaneously choose design r_l and/or commission τ_l in Stage 2.

We start by stating the equilibrium of the creator-consumer subgame for given $(\tau_l, r_l)_{l=1,2}$, where $r_l = 0$ if platform P_l does not operate a discovery portal and $\tau_l = 1$ if P_l does not operate a membership portal. As a tie-breaking rule, creators join each given portal (membership or discovery) whenever they are indifferent between joining and not joining. Meanwhile, whenever consumers are indifferent between searching through P_1 and P_2's discovery portal, we assume that they break tie in favor on the portal with the highest r_l (and randomize with equal probability if $r_1 = r_2$). In Section 6.1, we show that this search tie-breaking rule can be obtained as a special case when creators are asymmetric and the extent of asymmetry approaches zero.[16]

15. Suppose we allow the platforms to charge exclusive content commission on the consumers (on top of the commission on creators) and assume that the net commission has to be non-negative. Then, the tax neutrality principle and the fact that sellers can influence each consumer's choice of purchase medium through their participation decisions imply that the analysis below remains unaffected.

16. In particular, consumers expects a strictly higher probability to find a match when searching through the portal with the highest r_l because the associated recommendation algorithm favors creators with the higher λ_i.

The following equilibrium can be derived as in the monopoly benchmark, after accounting for the tie-breaking rules stated in Section 5. Let $r = \max\{r_1, r_2\}$ and $\tau = \min\{\tau_1, \tau_2\} \leq 1 - \beta_0/\bar{\beta}$. The equilibrium of the subgame following $(\tau_l, r_l)_{l=1,2}$ is given by:[17]

1. Each creator i joins both discovery portals but joins only the membership portal with the lowest τ_l (or both if $\tau_1 = \tau_2$). Then, the creator sets design

$$\lambda^* = \arg\max_{\lambda_i} \left\{\lambda_i^{1+r} \times \left(\bar{a} + (1-\tau)\bar{\beta}v\left(\lambda_i\right)\right)\right\} \tag{6}$$

   and exclusive content price $p^* = v\left(\lambda^*\right)$

2. Each consumer believes that all creators adopt strategy (λ^*, p^*), and initiates search if and only if

$$b_j > \frac{s}{\lambda^*},$$

   They do so through the discovery portal with the highest r_l (randomizing if $r_1 = r_2$) . The consumer continues searching on the same portal until finding a match. The mass of consumers who search is $G\left(\frac{\lambda^*}{s}\right)$.

5.1 Equilibrium business models

With slight abuse of notation, we use $P_l \in \{M, D, H\}$ to denote platform P_l as operating in pure membership, pure discovery, and hybrid modes respectively. Given that platforms are ex-ante symmetric, without loss of generality, we focus on characterizing platform P_2's optimal choice of mode in response to that of platform P_1.

Proposition 3. *(Best-responding business mode). There are thresholds $A_3 \geq A_3' \geq 0$ such that:*

- *If $P_1 = M$, then platform P_2 optimally chooses D;*
- *If $P_1 = H$, then platform P_2 optimally chooses D if $A \geq A_3'$ and chooses M if $A \leq A_3'$*
- *If $P_1 = D$, then platform P_2 optimally chooses H if $A \geq A_3$ and chooses M if $A \leq A_3$.*

Moreover, both thresholds increase with C and equal zero when $C = 0$.[18]

Proposition 3 says that when the opponent platform is operating a membership portal component (modes M or H), it is unprofitable for platform P_2 to switch from pure discovery to hybrid That is, strategic considerations overturn the monopoly result in Proposition 1. Intuitively, the intense competition between two homogeneous membership portal components drives down platforms' commissions τ on exclusive content, which raises creators' revenue from exclusive content. In response, creators shift towards niche content designs, resulting in fewer visiting consumers and hence a negative "spillover" on P_2's existing total revenue from advertisement.

17. Our proposition statements implicitly assume at least one platform operates a discovery portal, which is true on the equilibrium path. For completeness, if both platforms have no discovery portal, then we can simply replace $\bar{a}$ with a_0 and s with s_0, as in Section 4.

18. We verified that the set of $A < A_3$ that still satisfies the cost condition in footnote 7 is generally non-empty.

By staying in pure discovery mode, platform P_2 avoids the competition in commissions τ and the resulting negative spillover on its advertising revenue.

When the opponent platform is in pure discovery mode (mode D), the logic of Proposition 1 implies $P_2 = D$ is never a best response. Then, in choosing between M and H, platform P_2 faces a trade-off between the fixed cost C of introducing a discovery portal and the new revenue source from advertisement $A/2$ (which is half due to the coexistence of two discovery portals). Notice that is in contrast to Proposition 2, where $C = 0$ is a sufficient condition for $P_2 = H$ to be a best response. The key difference with the monopoly case is that the potential downside of the hybrid mode — the "distraction effect" — is absent because creators always earn advertising revenue $\bar{a}$ and face a recommendation system with sensitivity $r \geq \underline{r}$, no matter P_2' choice of business model.

From the best response functions in Proposition 3, we yield the following overall equilibrium:

Proposition 4. *In the equilibrium of the overall game,*[19]

- *If $A \leq A_3$, one platform operates in pure discovery mode and the other platform operates in pure membership mode.*
- *If $A \geq A_3$, one platform operates in pure discovery mode and the other platform operates in hybrid mode.*

When advertisement revenue is small relative to the cost of operating a discovery portal, the proposition predicts the coexistence of pure discovery mode and membership mode platforms, and otherwise predicts the coexistence of pure discovery mode and hybrid mode platforms. The equilibrium characterization allows us to evaluate the implication of platform competition (relative to the monopoly case) on the equilibrium content design.

Corollary 1. *Suppose $\bar{r}$ is sufficiently large, then platform competition induces a weakly broader equilibrium content design, i.e., a weakly higher λ^*.*

The platforms are competing for both the consumers and content creators, but the avenues by which platforms compete for each side differ. Competition for creators pushes commissions down, which would lead to more niche content design, except that competition for consumers means that even when one platform operates as a hybrid, any recommendation system must be maximally sensitive to match probability in equilibrium. Because platforms can mitigate competition for creators by choosing a business model where competition in transaction commissions does not arise, and a pure discovery platform will set $r = \bar{r}$ even when it does not face direct competition, Corollary 1 says that the latter effect wins out if the recommendation algorithm is sufficiently capable of highlighting content with a high match probability.

19. If $A = A_3$, then both types of equilibria coexist.

5.2 Differentiated membership portals

In the equilibrium characterized by Proposition 4, there is no coexistence of two membership portals because platforms are homogenous and would compete intensely in commission to attract creators to use their respective portals. This suggests that to obtain richer equilibrium configurations of business models, horizontal differentiation between membership portals is a necessary ingredient, which we now introduce as an extension. We will focus on describing the main insights in what follows, and relegate the details to Section C of the Online Appendix.

For the ease of exposition, we assume linear $v(\cdot)$ and focus on the case of completely differentiated membership portals. Whenever two membership portals are available in the creator subgame, the extent of horizontal differentiation is large enough such that they are local monopolies from the point of view of creators: creators always split themselves equally between the two membership portals, regardless of the commission difference between the two portals.

Denote

$$\eta(x) = \frac{xg(x)}{G(x)} \geq 0, \text{ where } x > 0$$

as the elasticity of consumer participation. The following result is analogous to Proposition 3

Proposition 5. *(Best-responding business mode with differentiated membership portals). Suppose membership portals are local monopolies. There exist thresholds η^* and $\tilde{r}$ such that if $r > \tilde{r}$ and $\max_x \eta(x) < \eta^*$, then there are thresholds A_3^*, A_3', A_3'' such that:*

- *If $P_1 = M$, then platform P_2 optimally chooses M if $A \leq A_3'$ and chooses H if $A \geq A_3'$;*
- *If $P_1 = H$, then platform P_2 optimally chooses M if $A \leq A_3''$ and chooses H if $A \geq A_3''$;*
- *If $P_1 = D$, then platform P_2 optimally chooses M if $A \leq A_3^*$ and chooses H if $A \geq A_3^*$.*

  Moreover, $A_3'' > A_3^*$.

When membership portals are local monopolies, introducing a second membership portal when a competing platform's current strategy includes one has two effects on commissions: First, it pushes commissions down because consumers' participation decision is based on the *average* λ in the market, and an increase in τ affects that average less when half of the creators are participating in a different membership portal than when there is only one. Second, when there are two membership portals and at least one discovery portal, then the introduction of the second membership portal creates competition for recommendations on the part of the platforms as well as the creators, which pushes τ up as the platforms want to induce content design that appeals to the recommendation algorithm. If η is small, then the second effect dominates and τ is higher when there are two membership portals and at least one discovery portal, meaning participation is greater and D is never a best response to any business model strategy when membership portals are local monopolies.[20]

The best responses from Proposition 5 lead immediately to Proposition 6:

20. We discuss larger values of η in Section C of the Online Appendix.

Proposition 6. *Suppose membership portals are local monopolies. Under the conditions stated in Proposition 5:*

- *If $A \leq A_3'$, both platforms operate in pure membership mode;*
- *If $A_3' \leq A \leq A_3''$, one platform operates in pure membership mode and the other platform operates in hybrid mode;*
- *If $A_3'' \geq A$ both platforms operate in hybrid mode.*

We have not shown that $A_3'' > A_3'$ in general, so under some parameter sets the second bullet point may refer to an empty parameter set. In that case, coexistence of two platforms operating in pure membership mode and coexistence of two hybrid mode platforms would both be equilibria for $A_3'' < A < A_3'$.

6 Extensions

6.1 Asymmetric creators

In our baseline model, creators are ex-ante symmetric and make symmetric content design decisions in the equilibrium. In this section, we extend the monopoly platform model by introducing asymmetric creators to show that the main insights on the comparison between the three business models of the platform remain unchanged.

Suppose that the continuum of creators are indexed by type $t_i \geq 0$ which scales the profitability of their advertising revenue. One interpretation is that differing types correspond to content categories (e.g., education, video games, toys, fashion) that differ in terms of advertisers' willingness to pay for the "eye balls" of viewers in each category. A creator i earns per-viewer advertising revenue $a_0 t_i$ (or $\bar{a} t_i$ if the creator joins a discovery portal).[21] We assume that t_i is distributed according to a CDF F with compact support $[\underline{t}, \bar{t}]$. Note that we recover the baseline model if $\underline{t} = \bar{t} = 1$.

Consider the hybrid mode (recall that the analysis nests the case of pure membership and pure discovery modes as special cases). Following the analysis in Section 4,in the equilibrium of the creator-consumer subgame, each creator of type t_i joins both portals of the platform, and sets price $p_i^* = v\left(\lambda^*\left(t_i\right)\right)$ and design

$$\lambda^*\left(t_i\right) = \arg\max_{\lambda_i} \left\{\lambda_i^{1+r} \times \left(\bar{a} t_i + (1-\tau)\bar{\beta} v\left(\lambda_i\right)\right)\right\}.$$

Notice that $\lambda^*\left(t_i\right)$ is increasing: higher-type creators opt for broader designs than lower-type creators.

21. We would obtain the same insights if we instead introduce scaling heterogeneity on the profitability of creators' exclusive content revenue. This can be easily seen from (3), where the maximizer depends only on the ratio $\bar{a}/\bar{\beta}$ after applying a multiplicative transformation.

To describe consumers' search pattern, define the recommendation-weighted "average broadness" (or the expected match probability) as

$$\Lambda = \int_{\underline{t}}^{\bar{t}} \lambda^* (t_i) \left(\frac{\lambda^* (t_i)^r}{\int_{\underline{t}}^{\bar{t}} \lambda^* (t_i)^r \, dF(t_i)} \right) dF(t_i). \tag{7}$$

That is, Λ is the ex-ante probability that a consumer eventually finds a match from searching on the platform and following the recommendation. Notice that if $r = 0$ then Λ becomes the unweighted average broadness $\int_{\underline{t}}^{\bar{t}} \lambda^* (t_i) \, dF(t_i)$, i.e., the expected match probability under random search. Given the exclusive content pricing by creators, a consumer j obtains expected surplus $b_j \Lambda$ from searching each creator, and so she initiates search if and only if $b_j \geq s/\Lambda$.

Importantly, consumers optimally follow the platform's recommendation in each step of the search in the equilibrium because

$$\Lambda > \int_{\underline{t}}^{\bar{t}} \lambda^* (t_i) \, dF(t_i),$$

which can be proven with a simple first-order stochastic dominance argument. Intuitively, the platform's recommendation rule (2) implies that the probability of finding a match from following the recommendation is higher than the corresponding probability with a random search.

To ensure that the analysis of the platform's problem remains tractable, for the rest of this subsection we assume that function $v(\lambda)$ is a linear function and that the parameters are such that $\lambda^* (t_i) \in (0, 1)$ for all $r \in [\underline{r}, \bar{r}]$ and $\tau \leq 1 - \beta_0/\bar{\beta}$. Then, the following lemma is analogous to Lemma 1 in the baseline model.

Lemma 3. *The recommendation-weighted average broadness Λ in (7) is strictly increasing in r and $\bar{a}$, and τ.*

To understand the intuition of Lemma 3, consider how a higher recommendation sensitivity (r) affects the weighted average content broadness in the equilibrium. From the baseline model, we already know that a higher r induces all creators to raise their individual broadness $\lambda^*(t_i)$.

With asymmetric creators, there are two additional channels that reflect how a higher r *improves the recommendations* received by consumers. First, holding $\lambda^*(t_i)$ constant, a higher r means that creators with a higher $\lambda^*(t_i)$ are more likely to be recommended; Second, it can be shown that the higher a creator's type t_i is, the more elastic her content broadness is towards r (formally, $\frac{\partial \lambda^*(t_i)/\partial r}{\lambda^*(t_i)/r} > 0$ is increasing in t_i). In other words, creators with high t_i raises their content broadness more than creators with low t_i, which further raises the recommendation probability of high t_i creators (whose broadness $\lambda^*(t_i)$ is higher). Similar intuitions apply for the results on advertising revenue $\bar{a}$ and commission rate τ, except that the second channel above is absent in these cases.

Based on Lemma 3, in Section A of the Online Appendix we verify that our results comparing profits and equilibrium designs in hybrid mode with the two pure modes (Propositions 1 and 2) continue to hold. The only difference is that, when creators make asymmetric content design decisions in the equilibrium, the platform's choice of mode now influences the market outcome through the additional recommendation-improving effect described above.

6.2 Elastic creator participation and cross-group network effects

In our model, all creators are active and join the platform in the equilibrium. As such, participation by consumers and creators are essentially independent (as long as we rule out the trivial equilibrium with no participation), meaning that in our model there is no cross-group network effects emphasized by the literature of two-sided markets (Rochet and Tirole 2003; Armstrong 2006). Allowing for elastic creator participation (that is, platform decisions affect the mass of active creators in a continuous manner) does not affect our results so long as a strictly positive mass of creators are always active. This is due to the assumptions of: (i) a continuum of symmetric creators and (ii) consumers unit demand by consumers.

In Section B of the Online Appendix, we expand the asymmetric creators model of Section 6.1 by exploring the impact of elastic creator participation. We assume that creators face a fixed cost $c > 0$ for being active (regardless of whether the creator is joining the discovery portal, the membership portal, or both) so that their participation is elastic. In this case, participation decisions are consumers and creators are interdependent, thus generating cross-group network effects.

To see this point, let us focus on the most general case of a hybrid mode. It can be shown that there exists a unique creator participation threshold $T \in [\underline{t}, \bar{t}]$ such that the marginal creator with type $t_i = T$ is indifferent between being active (and joining the monopoly platform) and being inactive:

$$G\left(\frac{\Lambda_T}{s}\right)\left(\frac{\lambda^*(T)^r}{\int_T^{\bar{t}} \lambda^*(t_i)^{1+r}\, dF(t_i)}\right)\pi(T) = c. \tag{8}$$

where $\pi(T) = \bar{a}T + (1-\tau)\bar{\beta}v(\lambda^*(T))$ is creator T's per-viewer revenue and Λ_T is the counterpart of average broadness in (7):

$$\Lambda_T = \int_T^{\bar{t}} \lambda^*(t_i)\left(\frac{\lambda^*(t_i)^r}{\int_T^{\bar{t}} \lambda^*(t_i)^r\, dF(t_i)}\right) dF(t_i). \tag{9}$$

All creators with type $t_i \geq T$ are active while those with type $t_i < T$ are inactive. Notice from (8) that creator participation depends on consumer participation $G(\Lambda_T/s)$, which in turns depends on creator participation through the average broadness Λ_T in (9). Taking into account this interdependency, we have:

Lemma 4. *The recommendation-weighted average broadness Λ_T jointly pinned down by (8) and (9) is strictly increasing in r and τ.*

To illustrate the intuition of the Lemma 4, consider a hybrid platform that raise r (the same logic applies to raising τ). Then, totally differentiating (9):

$$\frac{d\Lambda_T}{dr} = \frac{\partial\Lambda_T}{\partial r} + \frac{\partial\Lambda_T}{\partial T}\frac{dT}{dr}.$$

We know that $\partial\Lambda_T/\partial r \geq 0$ (Lemma 3) and $\partial\Lambda_T/\partial T \geq 0$ (from (9), whenever the participation threshold increases, it implies that the composition of active creators is now of higher type t_i). However, the sign of dT/dr, which reflects how the raise in r affects the *composition of active creators*, is generally ambiguous due to two opposing effects. On the one hand, a higher r intensifies the competition for recommendation. By the logic discussed below Lemma 3, this implies that the marginal (threshold) creator with type T becomes relatively less likely to be recommended than the creators with type $t_i > T$. This competition effect decreases the profit of the marginal creator, thus raising T. On the other hand, a higher r induces a greater content broadness, thus raising the mass of consumers who initiate search. This market expansion effect increases the marginal creator's profit, thus lowering T. Nonetheless, regardless of the sign of the composition effect dT/dr, an incomplete pass through argument bounds its magnitude such that $d\Lambda_T/dr$ is unambiguously positive.

Based on Lemma 4, we verify that results in Proposition 1 and Proposition 2 remain valid. The only exception is the result on equilibrium broadness of content design, i.e., on $\Lambda^*_H \geq \Lambda^*_M$. This reflects that, holding commission rate τ constant, the hybrid mode (relative to the pure membership mode) raises the per-viewer revenue of each creator through the directly raising the advertising revenue ($\bar{a} > a_0$). If the difference $\bar{a} - a_0$ is sufficiently large, it is possible that the switch to the hybrid mode attracts a large number of lower-type creators (i.e., a significant lower threshold T) whose content broadness is low. This composition effect lowers the average broadness of all creators in the hybrid mode, and it may dominates other broadness-inducing effects of the hybrid mode (even if the condition in $A > A_2$ in Proposition 2 holds).

7 Discussion and conclusion

This paper presents a tractable model of platform-intermediated content creation market, whereby content creators endogenously make design decisions positioning their content along the niche-broad spectrum. We analyze three distinct platform business models: (i) pure discovery mode (which facilitates audiences' discovery of creators' content and earns advertising revenue), (ii) pure membership mode (which enables creators to profit from providing exclusive content and earns revenue from transaction commissions), and (iii) hybrid mode platform (combining both business models).

Our results yield several implications for platform businesses in content creation markets. Importantly, these insights are driven by creators' endogenous content design responses to the platform's decisions.

- First, an existing pure discovery platform can always benefit from introducing a discovery portal (thus going hybrid) and choosing an appropriate level of commission if it is a monopolist. However, this is not necessarily true when the platform faces a competing platform that operates a membership portal as the competition in platform commissions creates a negative spillover on the platform's existing advertising revenue.
- Second, an existing pure membership platform does not always benefit from introducing a discovery portal. Doing so distracts creators from focusing on raising the value of their

exclusive content (thus harming the platform's commission revenue) due to the competition for recommendation and the additional advertisement revenue for creators.

- Third, strategic differentiation in platform business model can arise in the equilibrium in which one platform operates in pure discovery while the other pure membership or hybrid. The differentiation occurs when the competition in transaction commission among membership portals is sufficient intense.

At a high level, our paper also echoes the recent interest in understanding the welfare and social implications of different business models of digital platforms (Caffarra et al. 2020). The growing prominence of content platforms like Youtube and Facebook as sources for media consumption, means that these platforms have considerable influence on the type of media content being created. We identify conditions under which changes in platform business model increase or decrease the equilibrium level of content broadness chosen by the creators. These implications are empirically testable in principle if a proper notion of "content broadness" can be defined (see, e.g., Gong (2021)). We leave this as a promising direction for future research.

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8 Appendix

8.1 Proofs in Section 4

Proof. **(Lemma 1).** Whenever λ^* is interior, we apply implicit function theorem on (5) and $v'(.) < 0$ to get

$$\begin{aligned}\frac{d\lambda^*}{dr} &= \frac{1}{-(2+r)v'(\lambda^*) - v''(\lambda^*)}\left(\frac{\bar{a}}{(1-\tau)\bar{\beta}} + v(\lambda^*)\right) > 0;\\ \frac{d\lambda^*}{d\bar{a}} &= \frac{1}{-(2+r)v'(\lambda^*) - v''(\lambda^*)}\left(\frac{(1+r)}{(1-\tau)\,\bar{\beta}}\right) > 0;\\ \frac{d\lambda^*}{d\tau} &= \frac{1}{-(2+r)v'(\lambda^*) - v''(\lambda^*)}\left(\frac{\bar{a}\,(1+r)}{(1-\tau)^2\,\bar{\beta}}\right) > 0,\end{aligned}$$

where concavity of v ensures the denominators are all positive. ■

Proof. **(Lemma 2).** (i) If $\tau_M^* = 1 - \beta_0/\bar{\beta}$, then it is independent of s_0. Otherwise, τ_M^* satisfies FOC

$$\frac{s_0 G\left(\lambda_M/s_0\right)}{g\left(\lambda_M/s_0\right)}\left(\frac{v(\lambda_M)}{d\lambda_M/d\tau} + \tau v'(\lambda_M)\right) + \tau v(\lambda_M) = 0. \tag{8.10}$$

and the local stability condition for interior solution at $\tau = \tau_M^*$ implies the left-hand side of (8.10) is decreasing in τ. Next,

$$\frac{d}{dx}\frac{G(x)}{xg(x)} = \frac{1}{x}\left(\frac{G(x)}{xg(x)} - 1\right) - \frac{G(x)}{xg(x)}g'(x) > 0$$

where both components are positive due to concavity of G. Thus, the left-hand side of (8.10) is increasing in s_0 because (8.10) implies $\frac{v(\lambda_M)}{d\lambda_M/d\tau} + \tau v' < 0$. Implicit function theorem gives $d\tau_M^*/ds_0 > 0$. The result on λ_M^* then follows from Lemma 1 because $\lambda_M^* = \lambda_M(\tau_M^*)$ is independent of s_0 except through τ_M^*. ■

(ii) $r_D^* = \bar{r}$ is obviously independent of s.

(iii) If τ_H^* and r_H^* are non-interior, then they are independent of s. Suppose τ_H^* satisfies FOC

$$\frac{sG\left(\lambda_H/s\right)}{g\left(\lambda_H/s\right)}\left(\frac{v(\lambda_H)}{d\lambda_H/d\tau} + \tau v'(\lambda_H)\right) + \left(\frac{A}{\bar{\beta}} + \tau v(\lambda_H)\right) = 0.$$

Similar to the proof of case (i), implicit function theorem gives $d\tau_H^*/ds > 0$. Finally, if r_H^* satisfies FOC

$$\frac{sG\left(\lambda_H/s\right)}{g\left(\lambda_H/s\right)}\tau v'(\lambda_H) + \left(\frac{A}{\bar{\beta}} + \tau v(\lambda_H)\right) = 0,$$

where the left-hand side is increasing in s. Clearly $dr_H^*/ds > 0$, because the expression is decreasing in λ_H (for arbitrary r) and $d\lambda_H/dr \geq 0$ by Lemma 1.

Proof. **(Proposition 1).** We know $\lambda_H(1 - \beta_0/\bar{\beta}, r_D^*) = \lambda_D^*$, so $\Pi_H^* \geq \Pi_H\left(1 - \beta_0/\bar{\beta}, r_D^*\right) > \Pi_D^*$, where the last inequality is due to $1 - \beta_0/\bar{\beta} > 0$. The second result follows from Lemma 1 (given $r_H^* \leq r_D^* = \bar{r}$ and $\tau_H^* \leq 1 - \beta_0/\bar{\beta}$). ■

Proof. (**Proposition 2**). By envelope theorem, $\Pi_H^* - \Pi_M^*$ is (i) monotone increasing in A and $\Pi_H^* \to \infty$ when $A \to \infty$; (ii) decreasing in s; and (iii) increasing in s_0. The intermediate value theorem and implicit function theorem proves the result on threshold A_1 (if $\Pi_H^* > \Pi_M^*$ for all A then $A_1 = 0$). ■

From Lemma 1, $\bar{a} \geq a_0$ and $r \geq 0$ implies $\lambda_H(\tau) \geq \lambda_M(\tau)$ for the same τ (equality arises only when both λ are corner solutions). Hence, if $\tau_H^* = 1 - \beta_0/\bar{\beta}$ then

$$\lambda_H^* \geq \lambda_H(\tau_M^*) \geq \lambda_M^*$$

and we are done. Otherwise, τ_H^* satisfies FOC

$$\frac{sG\left(\lambda_H/s\right)}{g\left(\lambda_H/s\right)}\left(\frac{v(\lambda_H)}{d\lambda_H/d\tau} + \tau v'(\lambda_H)\right) + \left(\frac{A}{\bar{\beta}} + \tau v(\lambda_H)\right) = 0,$$

and the same steps as in the proof of Lemma 2 implies τ_H^* is increasing in A and reaches $\tau_H^* = 1 - \beta_0/\bar{\beta}$ when A becomes sufficiently large. The intermediate value theorem establishes the existence of threshold A_2 (if $\lambda_H^* \geq \lambda_D^*$ for all A then $A_2 = 0$). The relation between A_2 and s and s_0 follows from Lemma 2.

Proof. (**Remark 1**). Construct a fictitious profit function,

$$\tilde{\Pi}(\tau, r, a) = G\left(\frac{\lambda^*(\tau, r, a)}{s}\right)\left(\tau\bar{\beta}v(\lambda^*(\tau, r, a))\right)$$

where $\lambda^*(\tau, r, a)$ is the solution of (5) (replacing $\bar{a}$ with a). The stated parametric conditions ($A \to 0$, $C = 0$, and $s \to s_0$) imply $\tilde{\Pi}(\tau, 0, a_0) = \Pi_M(\tau)$ and $\tilde{\Pi}(\tau, r, \bar{a}) = \Pi_H(\tau, r)$. We claim that

$$\max_{\tau \leq 1 - \beta_0/\bar{\beta}} \tilde{\Pi}(\tau, r, a) \tag{8.11}$$

is decreasing in a and r for all $a \geq a_0$ and $r \geq 0$, which then implies the proposition statement. The first derivative, $\frac{d\tilde{\Pi}(\tau, r, a)}{d\tau}$, has the same sign as

$$\phi(\tau, r, a) = \frac{sG\left(\lambda^*/s\right)}{g\left(\lambda^*/s\right)}\left(\frac{v(\lambda^*)}{d\lambda^*/d\tau} + \tau v'\right) + \tau v(\lambda^*).$$

From the proof of Lemma 1, note that the assumption of $v\left(.\right)$ being linear means that λ is increasing and convex in τ. Hence, ϕ is decreasing in τ. Utilizing $\phi(0, r, a) > 0$ and $\phi(1, r, a) < 0$ (a consequence of $\lambda^*(1, r, a) = 1$ and $v(1) = 0$), there exists $\tilde{\tau}(r, a) \in [0, 1]$ which solves $\phi(\tilde{\tau}, r, a) = 0$. Moreover,

$$\left[\frac{v(\lambda^*)}{d\lambda^*/d\tau} + \tau v'\right]_{\tau = \tilde{\tau}} < 0.$$

From the proof of Lemma 1, observe that if $v(.)$ is linear then λ^* and $\frac{d\lambda^*}{d\tau}$ are both increasing in a and r. Thus, $\frac{\partial \phi}{\partial a}|_{\tau = \tilde{\tau}} \leq 0$ and $\frac{\partial \phi}{\partial r}|_{\tau = \tilde{\tau}} \leq 0$, so $\tilde{\tau}(r, a)$ is decreasing in a and r.

We know $\tilde{\tau}(0, a_0) = \tau_M^* < 1 - \beta_0/\bar{\beta}$, hence $\tilde{\tau}(r, a) < 1 - \beta_0/\bar{\beta}$ for all $a > a_0$ and $r > 0$, i.e., the FOC is satisfied. Applying envelope theorem on (8.11):

$$\begin{aligned}
\frac{d\tilde{\Pi}(\tau, r, a)}{dr} &= \left[\frac{sG\left(\lambda^*/s\right)}{g\left(\lambda^*/s\right)}\tau v' + \tau v(\lambda^*)\right] g\left(\lambda^*/s\right)\frac{\partial \lambda^*}{\partial r} < 0, \\
\frac{d\tilde{\Pi}(\tau, r, a)}{da} &= \left[\frac{sG\left(\lambda^*/s\right)}{g\left(\lambda^*/s\right)}\tau v' + \tau v(\lambda^*)\right] g\left(\lambda^*/s\right)\frac{\partial \lambda^*}{\partial a} < 0.
\end{aligned}$$

where the inequalities follow from the FOC of $\tilde{\tau}(r, a)$. ■

Proof. **(Remark 2).** When $\bar{r} \to 0$, and $\bar{a} \to 0$, then $\lambda_H(\tau) = \lambda_M(\tau)$ for the same τ. Thus, by Lemma 1, if $\tau_M^* \geq \tau_H^*$ then $\lambda_M^* \geq \lambda_H^*$ and so it suffices to prove the former. Given the stated parametric conditions, we know that $\Pi_M(\tau)$ and $\Pi_H(\tau, 0)$ are exactly the same except $s < s_0$. Hence, $(\tau_M^* - \tau_H^*)_{s_0 \to s} = 0$, and Lemma 2 implies that τ_M^* increases when s_0 increases (and becomes larger than s) while τ_H^* decreases when s decreases. ∎

8.2 Proofs in Section 5

Proof. (**Proposition 3**). We use $\lambda^*(\tau, r)$ to denote (6) as functions of τ and r, and note that $\lambda^*(\tau, r)$ satisfies all the properties in Lemma 1. A useful observation is that the Bertrand competition between two homogeneous membership portal components to attract creators imply zero equilibrium exclusive content commission.

Case 1: $P_1 = M$**.** Consider $P_2 = D$, and let $\tau_{1,MD} > 0$ be the equilibrium commission of platform 1 in this subgame (the alphabetical subscript denotes the mode choices of P_1 and P_2 in successive order). Then, platform P_2 earns

$$\Pi_{2,MD} = G\left(\frac{\lambda^*(\tau_{1,DM}, \bar{r})}{s}\right) A - C > 0,$$

where the equilibrium design is $r_{2,MD} = \bar{r}$ and the inequality is due to the cost condition in footnote 7. If $P_2 = M$, the Bertrand logic implies $\tau_{1,MM} = \tau_{1,MM} = 0$ so that $\Pi_{2,MM} = 0 < \Pi_{2,MD}$. If $P_2 = H$, $\tau_{1,MH} = \tau_{2,MH} = 0$ so

$$\Pi_{2,MH} = G\left(\frac{\lambda^*(0, \bar{r})}{s}\right) A - C < G\left(\frac{\lambda^*(\tau_{1,MD}, \bar{r})}{s}\right) A - C = \Pi_{2,MD}$$

given that $\lambda^*(\tau, r)$ is increasing in τ. Hence, $P_2 = D$ is the best response.

Case 2: $P_1 = H$. By the analysis of Case 1, we know (i) $\Pi_{2,HM} = 0$; and (ii)

$$\Pi_{2,HD} = G\left(\frac{\lambda^*(\tau_{1,HD}, \bar{r})}{s}\right) \frac{A}{2} - C,$$

where $\tau_{1,HD} > 0$ while the advertisement revenue $A/2$ reflects that consumers split between searching through the discovery portals of the two platforms. Notice the cost condition in footnote 7 does not imply $\Pi_{2,HD} > 0$ due to $A/2 < A$. Define

$$A_3' = \frac{2C}{G\left(\frac{\lambda^*(\tau_{1,HD}, \bar{r})}{s}\right)} \tag{8.12}$$

so that $\Pi_{2,HD} \geq 0 = \Pi_{2,HM}$ if and only if $A \geq A_3'$. It remains to show $P_2 = H$ is never a best response. When $P_1 = P_2 = H$, the Bertrand-like competition for consumer search (to earn the platform advertising revenue $A > 0$) and creators' participation on membership portal mean that both platforms set $r_{1,HH} = r_{2,HH} = \bar{r}$ and $\tau_{1,HH} = \tau_{2,HH} = 0$ in the equilibrium. There is no incentive to deviate by lowering r_l or raising τ_l because that doing so does not affect creators' content design hence does not affect consumers' search decisions. Equilibrium profits are

$$\Pi_{2,HH} = \Pi_{1,HH} = G\left(\frac{\lambda^*(0, \bar{r})}{s}\right) \frac{A}{2} - C < \Pi_{2,HD}.$$

given that $\lambda^*(\tau, r)$ is increasing in τ.

Case 3: $P_1 = D$. We know platform P_1 always set $r = \bar{r}$. Proposition 1 means $P_2 = D$ is never a best response. Then, If $P_2 = M$, we have

$$\Pi_{2,DM} = \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G\left(\frac{\lambda^*(\tau,\bar{r})}{s}\right) \tau\bar{\beta}v(\lambda^*(\tau,\bar{r})) \right\}$$

with the solution denoted as $\tau_{2,DM}$. If $P_2 = H$, we have

$$\Pi_{2,DH} = \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G\left(\frac{\lambda^*(\tau,\bar{r})}{s}\right) \left(\frac{A}{2} + \tau\bar{\beta}v(\lambda^*(\tau,\bar{r}))\right) - C \right\}.$$

with the solution denoted as $\tau_{2,DH}$. Let $A_3 \geq 0$ be the solution to $\Pi_{2,DH} = \Pi_{2,DM}$, which exists and is unique given $\Pi_{2,DH}$ is strictly increasing in A by the envelope theorem. Then, $\Pi_{2,DH} \geq \Pi_{2,DM}$ if and only if $A \geq A_3$.

Next, observe that for all $A < A_3'$ as defined in (8.12), then

$$\begin{aligned}
\Pi_{2,DH} &< G\left(\frac{\lambda^*(\tau_{2,DH},\bar{r})}{s}\right) \left(\frac{A_3'}{2} + \tau\bar{\beta}v(\lambda^*(\tau_{2,DH},\bar{r}))\right) - C \\
&= G\left(\frac{\lambda^*(\tau_{2,DH},\bar{r})}{s}\right) \tau\bar{\beta}v(\lambda^*(\tau_{2,DH},\bar{r})) \\
&\leq \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G\left(\frac{\lambda^*(\tau,\bar{r})}{s}\right) \tau\bar{\beta}v(\lambda^*(\tau,\bar{r})) \right\} = \Pi_{2,DM}
\end{aligned}$$

where we invoked symmetry $\tau_{2,DH} = \tau_{1,HD}$ in the second equality. Thus, $A < A_3'$ implies $A < A_3$, hence we conclude $A_3' \leq A_3$. Finally, from the definitions of A_3' and A_3, it is clear that both are increasing in C and equal zero when $C = 0$. ■

Proof. (**Proposition 4**). When $A < A_3$, Proposition 3 implies that $P_l = M$ and $P_{-l} = D$ are best responses to each other for $l = 1, 2$. When $A \geq A_3 \geq A_3'$, Proposition 3 implies that $P_l = H$ and $P_{-l} = D$ are best responses to each other. ■

Proof. (**Corollary 1**). For sufficiently large $\bar{r}$, the equilibrium λ in the monopoly scenario is independent of $\bar{r}$ (recall the monopolist operates either in pure membership mode of hybrid mode). Meanwhile, when there are multiple platforms, at least one of them operates as pure discovery portal and chooses the maximum $r = \bar{r}$. If $\bar{r} \to \infty$, then in the creator subgame we have $\lambda^*(\tau, r) = 1$ for all τ. ■

Online Appendix

Ben Casner and Tat-How Teh

A Asymmetric creators

In this section, we analyze the monopoly platform's decisions when creators are asymmetric with type $t_i \in [\underline{t}, \bar{t}]$. As stated in the main text, we assume that function $v(.)$ is linear with

$$v(\lambda) = (1 - \lambda)\, v_0,$$

where $v_0 > 0$ is constant. We also assume that the parameters are such that $\lambda^*(t_i) \in (0, 1)$ for all $r \in [\underline{r}, \bar{r}]$ and $\tau \leq 1 - \beta_0/\bar{\beta}$ such that it is always pinned down first-order condition:

$$\lambda^*(t_i) = \left(\frac{\bar{a} t_i}{(1-\tau)\bar{\beta} v_0} + 1\right)\left(\frac{1+r}{2+r}\right) \in (0, 1), \tag{A.1}$$

Observe that $\lambda^*(t_i)$ is strictly increasing in t_i, τ, r, and $\bar{a}$. Formally, the required parametric restriction for $\lambda^*(t_i) \in (0, 1)$ is

$$\frac{\bar{a}\bar{t}}{\beta_0 v_0} < \frac{1}{1+\bar{r}}.$$

Define random variable $\tilde{t}_i$ as the t_i of the creator recommended by the discovery portal of the platform, where the corresponding cumulative distribution function is

$$H(\tilde{t}_i) \equiv \frac{\int_{\underline{t}}^{\tilde{t}_i} \lambda^*(t_i)^r \, dF(t_i)}{\int_{\underline{t}}^{\bar{t}} \lambda^*(t_i)^r \, dF(t_i)} \quad \text{for } \tilde{t}_i \in [\underline{t}, \bar{t}]\,. \tag{A.2}$$

If $r = 0$ (i.e., no recommendation or consumers doing a random search) then $\tilde{t}_i$ has the same distribution as t_i.

The following lemma is analogous to Lemma 1 in the baseline model.

Lemma A.1. *In the equilibrium of the creator subgame with asymmetric creators,*

$$\Lambda = \int_{\underline{t}}^{\bar{t}} \lambda^*\left(\tilde{t}_i\right) dH(\tilde{t}_i) \tag{A.3}$$

is strictly increasing in τ, r and $\bar{a}$.

Proof. Consider the comparative statics with respect to τ, where we write $\lambda^* = \lambda^*(t_i; \tau)$ and rewrite (A.2) as

$$H(x; \tau) \equiv \frac{\int_{\underline{t}}^{x} \lambda^*(t_i; \tau)^r \, dF(t_i)}{\int_{\underline{t}}^{\bar{t}} \lambda^*(t_i; \tau)^r \, dF(t_i)} \quad \text{for } x \in [\underline{t}, \bar{t}]$$

to make explicit the dependence on τ. In what follows, we prove that $\tilde{t}_i$ is increasing in τ in the sense of first-order stochastic dominance (FOSD); that is, $H(x;\tau)$ is decreasing in τ. Given that $\lambda^*\left(\tilde{t}_i; \tau\right)$ is strictly increasing in both of its arguments, it then follows that Λ is increasing in τ.

Taking derivative, $\frac{dH(x;\tau)}{d\tau}$ is negative if and only if

$$\frac{\int_{\underline{t}}^{x} \frac{d}{d\tau}\left(\lambda^*(t_i;\tau)^r\right) dF(t_i)}{\int_{\underline{t}}^{x} \lambda^*(t_i;\tau)^r \, dF(t_i)} \leq \frac{\int_{\underline{t}}^{\bar{t}} \frac{d}{d\tau}\left(\lambda^*(t_i;\tau)^r\right) dF(t_i)}{\int_{\underline{t}}^{\bar{t}} \lambda^*(t_i;\tau)^r \, dF(t_i)}, \tag{A.4}$$

which holds if and only if the LHS of (A.4) is increasing in x. The corresponding derivative of the LHS of (A.4) is positive if and only if

$$\begin{aligned}\frac{\frac{d}{d\tau}\left(\lambda^*\left(x;\tau\right)^r\right)}{\lambda^*\left(x;\tau\right)^r} &\geq \frac{\int_{\underline{t}}^{x}\frac{d}{d\tau}\left(\lambda^*\left(t_i;\tau\right)^r\right)dF(t_i)}{\int_{\underline{t}}^{x}\lambda^*\left(t_i;\tau\right)^r dF(t_i)} \\ &= \int_{\underline{t}}^{x}\frac{\frac{d}{d\tau}\left(\lambda^*\left(t_i;\tau\right)^r\right)}{\lambda^*\left(\tilde{t}_i;\tau\right)^r}\frac{\lambda^*\left(\tilde{t}_i;\tau\right)^r}{\int_{\underline{t}}^{x}\lambda^*\left(t_i;\tau\right)^r dF(t_i)}dF(t_i) \\ &= \int_{\underline{t}}^{x}\frac{\frac{d}{d\tau}\left(\lambda^*\left(\tilde{t}_i;\tau\right)^r\right)}{\lambda^*\left(\tilde{t}_i;\tau\right)^r}dH(\tilde{t}_i).\end{aligned} \tag{A.5}$$

Hence, a sufficient condition for (A.5) is $\frac{\frac{d}{d\tau}(\lambda^*(x;\tau)^r)}{\lambda^*(x;\tau)^r}$ being increasing in x. From (A.1), we have

$$\frac{\frac{d}{d\tau}\left(\lambda^*\left(x;\tau\right)^r\right)}{\lambda^*\left(x;\tau\right)^r} = r\frac{\frac{d\lambda^*(x;\tau)}{d\tau}}{\lambda^*\left(x;\tau\right)} = \frac{r}{(1-\tau)}\left(\frac{\frac{\bar{a}x}{(1-\tau)\bar{\beta}v_0}}{\frac{\bar{a}x}{(1-\tau)\bar{\beta}v_0}+1}\right) \tag{A.6}$$

which is obviously increasing in x. A similar proof applies for the results with respect to r and $\bar{a}$, whereby the counterparts of (A.6) are, respectively,

$$\frac{\frac{d}{d\bar{a}}\left(\lambda^*\left(x;\bar{a}\right)^r\right)}{\lambda^*\left(x;\bar{a}\right)^r} = r\left(\frac{\frac{x}{(1-\tau)\bar{\beta}v_0}}{\frac{\bar{a}x}{(1-\tau)\bar{\beta}v_0}+1}\right) \tag{A.7}$$

and

$$\frac{\frac{d}{dr}\left(\lambda^*\left(x;r\right)^r\right)}{\lambda^*\left(x;r\right)^r} = \frac{r}{(1+r)(2+r)} + \ln\left(\lambda^*\left(x;\tau\right)\right) \tag{A.8}$$

both of which are increasing in x. ■

We use $\lambda^*(t_i;\tau,r,\bar{a})$ and $\Lambda(\tau,r,\bar{a})$ to denote (A.1) and (A.3) as functions of τ, r, and $\bar{a}$. Then, the profit function of the monopoly platform in the pure membership, pure discovery, and hybrid modes are, respectively,

$$\begin{aligned}\Pi_M(\tau) &= G\left(\frac{\Lambda(\tau,0,a_0)}{s_0}\right)\tau\bar{\beta}\mathbf{E}[v(\lambda^*(t_i;\tau,0,a_0))]; \\ \Pi_D(r) &= G\left(\frac{\Lambda(1-\beta_0/\bar{\beta},r,\bar{a})}{s}\right)A - C; \\ \Pi_H(\tau,r) &= G\left(\frac{\Lambda(\tau,r,\bar{a})}{s}\right)\left(A+\tau\bar{\beta}\mathbf{E}[v(\lambda^*(\tilde{t}_i;\tau,r,\bar{a}))|r]\right) - C.\end{aligned}$$

where

$$\begin{aligned}\mathbf{E}[v(\lambda^*(t_i;\tau,0,a_0))] &= \int_{\underline{t}}^{\bar{t}} v(\lambda^*(t_i;\tau,0,a_0))dF(t_i) \\ \mathbf{E}[v(\lambda^*(\tilde{t}_i;\tau,r,\bar{a}))|r] &= \int_{\underline{t}}^{\bar{t}} v(\lambda^*(\tilde{t}_i;\tau,r,\bar{a}))dH(\tilde{t}_i)\end{aligned}$$

Denote Π_M^*, Π_D^*, and Π_H^* as the respective maximized profit, and the equilibrium recommendation-weighted match probability (or broadness) in each mode as Λ_M^*, Λ_D^*, and Λ_H^*. The following result is analogous to Proposition **??**, showing that the equilibrium match probability in each mode increases with search cost even when creators are asymmetric.

Proposition A.1. *Λ_M^* is weakly increasing in s_0, while Λ_D^* and Λ_H^* are increasing in s.*

Proof. A slight modification of the proof of Proposition **??** delivers the result. ■

We are now ready to compare across the modes of operations.

Proposition A.2. *(Pure discovery versus hybrid).* $\Pi_H^* > \Pi_D^*$ *and* $\Lambda_H^* < \Lambda_D^*$.

Proof. The first part of the proposition follows from the same profit replication argument as in the proof of Proposition 1. Next, clearly $r_D^* \geq r_H^*$ by Lemma A.1. Then, the second result follows from Lemma A.1, where the strict inequality is due to the interiority assumption on $\lambda^*(t_i)$. ■

Proposition A.3. *(Pure membership versus hybrid).*

- *There exists a threshold* $A_1 \geq 0$ *such that* $\Pi_H^* > \Pi_M^*$ *if and only if the platform's advertising revenue* $A > A_1$*; Threshold* A_1 *becomes lower when* s *decreases.*
- *There exists a threshold* $A_2 \geq 0$ *such that* $\Lambda_H^* \geq \Lambda_M^*$ *if and only if the platform's advertising revenue* $A > A_2$*; Threshold* A_2 *becomes lower when* s *increases.*

Proof. The first part follows from the same envelope theorem argument as in the proof of Proposition 2. For the second part, we note from Lemma A.1, $\bar{a} > a_0$ and $r_H^* \geq 0$ implies $\Lambda(\tau, r_H^*, \bar{a}) > \Lambda(\tau, 0, a_0)$ for the same τ (the strict inequality is due to the interiority assumption on $\lambda^*(t_i)$. Hence, if $\tau_H^* = 1 - \beta_0/\bar{\beta}$ then

$$\Lambda_H^* \geq \Lambda(\tau_M^*, r_H^*, \bar{a}) \geq \Lambda(\tau_M^*, 0, a_0) = \lambda_M^*$$

and we are done. Otherwise, τ_H^* satisfies FOC

$$\begin{aligned} & \frac{sG\left(\Lambda(\tau, r_H^*, \bar{a})/s\right)}{g\left(\Lambda(\tau, r_H^*, \bar{a})/s\right)} \left(\mathbf{E}[v(\lambda^*(\tilde{t}_i; \tau, r, \bar{a}))|r] + \tau \frac{d\mathbf{E}[v(\lambda^*(\tilde{t}_i; \tau, r, \bar{a}))|r]}{d\tau} \right) \\ + \quad & \left(\frac{A}{\bar{\beta}} + \tau \mathbf{E}[v(\lambda^*(\tilde{t}_i; \tau, r, \bar{a}))|r] \right) \frac{d\Lambda(\tau, r_H^*, \bar{a})}{d\tau} = 0, \end{aligned}$$

where the left-hand side is increasing in A and s (recall that for the FOC to hold, we must have $\mathbf{E}[v(\lambda^*(\tilde{t}_i; \tau, r, \bar{a}))|r] + \tau \frac{d\mathbf{E}[v(\lambda^*(\tilde{t}_i;\tau,r,\bar{a}))|r]}{d\tau} < 0$). Thus, τ_H^* is increasing in A and s by the implicit function theorem. The intermediate value theorem establishes the existence of threshold A_2 (if $\Lambda_H^* \geq \Lambda_M^*$ for all A then $A_2 = 0$) that is decreasing in s. ■

B Elastic creator participation and cross-group network effect

Consider the hybrid mode. Let $T \in [\underline{t}, \bar{t}]$ be the threshold such that all creators with type $t_i \geq T$ are active (and joins the platform) while those with type $t_i \geq T$ are inactive. Denote

$$\Lambda_T = \int_T^{\bar{t}} \lambda^*\left(\tilde{t}_i\right) dH(\tilde{t}_i | \tilde{t}_i \geq T)$$

where

$$H(\tilde{t}_i | \tilde{t}_i \geq T) \equiv \frac{\int_T^{\tilde{t}_i} \lambda^*\left(t_i\right)^r dF(t_i)}{\int_T^{\bar{t}} \lambda^*\left(t_i\right)^r dF(t_i)} \quad \text{for } \tilde{t}_i \in [T, \bar{t}]\,.$$

The random variable $\tilde{t}_{i|\tilde{t}_i \geq T}$ is FOSD increasing in T, because $\frac{dH(\tilde{t}_i;T)}{dT}$ has the same sign as

$$-\left[\int_{\tilde{t}_i}^{\bar{t}} \lambda^*\left(t_i\right)^r dF(t_i)\right] \lambda^*\left(T\right)^r f(T) < 0.$$

Thus, as claimed in the main text

$$\frac{\partial \Lambda_T}{\partial T} \geq 0. \tag{B.1}$$

From the analysis in the previous section, we know that each consumer j makes her search decision accordingly and initiates search if and only if $b_j \Lambda(T) \geq s$.

As stated in the maintext, the threshold type T is pinned down by the indifference condition:

$$G\left(\frac{\Lambda_T}{s}\right)\left(\frac{\lambda^*(T)^r}{\int_T^{\bar{t}} \lambda^*(t_i)^r \, dF(t_i)}\right)\lambda^*(T)\left(\bar{a}T + (1-\tau)\bar{\beta}v(\lambda^*(T))\right) = c. \tag{B.2}$$

Since Λ_T and $1/\int_T^{\bar{t}} \lambda^*(t_i)^r \, dF(t_i)$ are increasing in T, we conclude that the left-hand side of (B.2) is increasing in T. Thus, the solution $T \in [\underline{t}, \bar{t}]$ must be unique whenever it exists. If the solution $T \in [\underline{t}, \bar{t}]$ to (B.2) does not exist, then we either set $T = \underline{t}$ (all creators are active) or $T = \bar{t}$ (all creators are inactive).

Denote

$$\eta(x) = \frac{xg(x)}{G(x)} \geq 0, \text{ where } x > 0$$

as the elasticity of consumer participation. As a case in point, if G has the standard constant elasticity form, then $\eta(x)$ is independent of x. The following lemma describe how the creator participation threshold T changes with platform design decisions r and τ, and the level of advertising revenue $\bar{a}$.

Lemma B.1. *Consider T implicitly defined by (B.2).*

1. *T is increasing in r and τ if $\max_x \eta(x)$ is sufficiently small;*
2. *T is decreasing in r and τ if $\min_x \eta(x)$ is sufficiently large;*

Proof. Denote

$$D(T) = \frac{\lambda^*(T)^r}{\int_T^{\bar{t}} \lambda^*(t_i)^r \, dF(t_i)}.$$

and $\tilde{\pi} = \lambda^*(T)^{1+r}\left(\bar{a}T + (1-\tau)\bar{\beta}v(\lambda^*(T))\right)$. Then, denote the left-hand side of (B.2) as

$$\phi(T) \equiv G\left(\frac{\Lambda_T}{s}\right) D(T) \frac{\tilde{\pi}}{\lambda^*(T)^r}.$$

By implicit function theorem,

$$\frac{dT}{d\tau} = \frac{\frac{\partial \phi(T)}{\partial \tau}}{-\frac{\partial \phi(T)}{\partial T}}$$

We already know from the main text that $\frac{\partial \phi(T)}{\partial T} > 0$. Meanwhile,

$$\begin{aligned} \frac{\partial \phi(T)}{\partial \tau} &= g\left(\frac{\Lambda_T}{s}\right) D(T) \frac{\tilde{\pi}}{\lambda^*(T)^r} \frac{1}{s} \frac{\partial \Lambda_T}{\partial \tau} \\ &\quad + G\left(\frac{\Lambda_T}{s}\right) \frac{\tilde{\pi}}{\lambda^*(T)^r} \frac{\partial D(T)}{\partial \tau} \\ &\quad + G\left(\frac{\Lambda_T}{s}\right) D(T) \frac{\partial}{\partial \tau}\left(\frac{\tilde{\pi}}{\lambda^*(T)^r}\right). \end{aligned} \tag{B.3}$$

Dividing by $\frac{\tilde{\pi} D(T)}{\lambda^*(T)^r} G\left(\frac{\Lambda_T}{s}\right)$, then $\frac{\partial \phi(T)}{\partial \tau}$ has the same sign as

$$\eta\left(\frac{\Lambda_T}{s}\right) \underbrace{\frac{\partial \Lambda_T / \partial \tau}{\Lambda_T}}_{>0} + \underbrace{\frac{\partial D(T)/\partial \tau}{D(T)}}_{\leq 0} + \underbrace{\frac{\lambda^*(T)^r}{\tilde{\pi}} \frac{\partial}{\partial \tau}\left(\frac{\tilde{\pi}}{\lambda^*(T)^r}\right)}_{<0}. \tag{B.4}$$

The first term in (B.4) is positive by Lemma A.1 (given we are holding T constant). The second term in (B.4) is negative because $\frac{\partial D(T)}{\partial \tau} \leq 0$ if and only if

$$\frac{\frac{\partial}{\partial \tau}\left(\lambda^*(T)^r\right)}{\lambda^*(T)^r} \leq \frac{\int_T^{\bar{t}} \frac{\partial}{\partial \tau}\left(\lambda^*\left(t_i\right)^r\right) dF(t_i)}{\int_T^{\bar{t}} \lambda^*\left(t_i\right)^r dF(t_i)} = \int_T^{\bar{t}} \frac{\frac{\partial}{\partial \tau}\left(\lambda^*\left(\tilde{t}_i\right)^r\right)}{\lambda^*\left(\tilde{t}_i\right)^r} dH(\tilde{t}_i|\tilde{t}_i \geq T),$$

which holds because $\frac{\frac{\partial}{\partial \tau}(\lambda^*(x)^r)}{\lambda^*(x)^r}$ is increasing in x from (A.6); The last term in (B.4) is negative because the envelope theorem on creator's maximization problem implies

$$\frac{\partial}{\partial \tau}\left(\frac{\tilde{\pi}}{\lambda^*\left(T\right)^r}\right) = -\lambda^*\left(T\right)\bar{\beta} v\left(\lambda^*\left(T\right)\right) - \frac{r\tilde{\pi}}{\lambda^*\left(T\right)^{1+r}}\frac{\partial \lambda^*\left(T\right)}{\partial r} \leq 0.$$

Finally, note that the determination of λ^* and Λ are independent of consumer participation G, hence any changes to $\eta(.)$ affects (B.4) only by scaling the first term. It follows that $\frac{\partial \phi(T)}{\partial \tau} < 0$ (so $\frac{dT}{d\tau} > 0$) if $\max_x \eta(x)$ is small, and $\frac{\partial \phi(T)}{\partial \tau} > 0$ (so $\frac{dT}{d\tau} < 0$) if $\max_x \eta(x)$ is large. The result on $\frac{dT}{dr}$ follows from a similar proof after utilizing Lemma A.1 and (A.8), hence omitted here. ■

The following is the same as Lemma 4.

Lemma B.2. *The recommendation-weighted average broadness Λ_T in (9) is strictly increasing in r and τ.*

Proof. Lemma A.1 and B.1 imply

$$\frac{d\Lambda(T)}{d\tau} = \underbrace{\frac{\partial \Lambda(T)}{\partial \tau}}_{>0} + \underbrace{\frac{\partial \Lambda(T)}{\partial T}}_{>0}\frac{dT}{d\tau} \tag{B.5}$$

Hence, if $dT/d\tau \geq 0$ then we are done. Suppose instead $\frac{dT}{d\tau} = -\frac{\partial \phi(T)}{\partial \tau}/\frac{\partial \phi(T)}{\partial T} < 0$ where we recall $\phi(T)$ is the LHS of (B.2). Continue from the proof of Lemma B.1, it is easily verified that the positive denominator of $\frac{dT}{d\tau}$ is bounded above by

$$\frac{\partial \phi(T)}{\partial T} < g\left(\frac{\Lambda_T}{s}\right) D(T)\frac{\tilde{\pi}}{\lambda^*\left(T\right)^r}\frac{1}{s}\frac{\partial \Lambda_T}{\partial T};$$

while (B.3) implies that the negative numerator is bounded below by

$$-\frac{\partial \phi(T)}{\partial \tau} > -g\left(\frac{\Lambda_T}{s}\right) D(T)\frac{\tilde{\pi}}{\lambda^*\left(T\right)^r}\frac{1}{s}\frac{\partial \Lambda_T}{\partial \tau}.$$

Hence,

$$\frac{dT}{d\tau} > -\frac{\partial \Lambda(T)/\partial \tau}{\partial \Lambda(T)/\partial T}$$

so that $\frac{d\Lambda(T)}{d\tau} > 0$ by (B.5). The result on $\frac{d\Lambda(T)}{dr}$ follows from the same steps, where we utilize

$$-\frac{\partial \phi(T)}{\partial r} > -g\left(\frac{\Lambda_T}{s}\right) D(T)\frac{\tilde{\pi}}{\lambda^*\left(T\right)^r}\frac{1}{s}\frac{\partial \Lambda_T}{\partial r}.$$

■

We are now ready to compare across the three modes of operations.

Proposition B.1. *(Pure discovery versus hybrid). $\Pi_H^* > \Pi_D^*$ and $\Lambda_H^* < \Lambda_D^*$.*

Proof. Let $T_H = T(\tau, r)$ and $T_D(r)$ denote the participation threshold in the hybrid mode and pure discovery mode respectively. Notice $T_H = T\left(1 - \beta_0/\bar{\beta}, r\right) = T_D(r)$. Hence, a profit replication argument shows $\Pi_H^* > \Pi_D^*$. Meanwhile, the result $\Lambda_H^* < \Lambda_D^*$ follows from (i) the pure discovery platform chooses the highest r to maximize $\Lambda(T)$ (Lemma B.1); (ii) $\Lambda(T)$ is increasing in τ and $\tau_H^* < 1 - \beta_0/\bar{\beta}$; and (iii) the interiority assumption on $\lambda^*(t_i)$. ■

Proposition B.2. *(Pure membership versus hybrid). There exists a threshold $A_1 \geq 0$ such that $\Pi_H^* > \Pi_M^*$ if and only if the platform's advertising revenue $A > A_1$; Threshold A_1 becomes lower when s decreases.*

Proof. The result follows from the same envelope theorem argument as in the proof of Proposition 2. ■

C Differentiated Membership Platforms

This section elaborates on the multi-platform section in the main paper by allowing for creators to view membership portals as being horizontally differentiated. The timing is identical to the multi-platform model in the main paper. In addition, we introduce the following features to the model:

- Denote $\eta(x) = \frac{xg(x)}{G(x)} \geq 0$, where $x > 0$ as the elasticity of consumer participation.
- We adopt the content differentiation scheme of Wang and Wright (2020)
  - 1/2 of creators join each membership portal as a "default". If they consider switching to the alternative portal they face a switching cost σz where z $Q[\underline{z}, \bar{z}]$. $\sigma \in (0, \infty]$ represents the degree of differentiation between platforms and Q is is a CDF with associated density function q.
  - We assume Q to be continuously differentiable and $D_j v(\lambda_j)$ non-increasing in τ_j and $\max_x \eta(x)$ sufficiently small such that platform profits are concave in τ (discussed further in the proof of Proposition C.1).
- If both platforms choose symmetric business models we assume symmetric equilibrium.
- In the sub-game with one hybrid and one pure membership platform, we assume that the pure membership platform believes the hybrid will choose the level of r which leads to the hybrid platform profit-maximizing equilibrium.

Denote by the subscript m creators using the pure membership platform and h creators using the hybrid platform's portal and $\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)$ as the proportion of creators participating on the pure membership platform. Consumers' expected value of a single search (assuming sellers price at $p = v(\lambda)$) is then

$$(\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_m\lambda_m + (1 - \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h\lambda_h)b - s$$

Let $\bar{\lambda} = \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_m\lambda_m + (1 - \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h\lambda_h$, then consumers will search on the discovery platform if $b > \frac{s}{\bar{\lambda}}$, which leads to equilibrium participation $G(\frac{\bar{\lambda}}{s})$

$\bar{\beta}$ is the same across platforms so creators will stay with their default j if

$$\bar{\beta}(1 - \tau_j) > \bar{\beta}(1 - \tau_{-j}) - \sigma z$$
$$\implies z > \frac{\bar{\beta}}{\sigma}(\tau_j - \tau_{-j})$$

Define $\Delta\tau \equiv (\tau_m - \tau_h)$ Therefore the mass of creators on the pure membership platform is

$$\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau) \equiv \begin{cases} \frac{1}{2}(1 - Q(\frac{\bar{\beta}}{\sigma}\Delta\tau)) & \Delta\tau \geq 0 \\ \frac{1}{2} + \frac{1}{2}(Q(-\frac{\bar{\beta}}{\sigma}\Delta\tau)) & \Delta\tau \leq 0 \end{cases}$$

Otherwise the creator's maximization problem is essentially unchanged, so once they have chosen a platform they will still set λ according to

$$\lambda(\tau) = \arg\max_{\lambda_i} \left\{ \lambda_i^{1+r} \times \left(\bar{a} + (1-\tau)\bar{\beta}v\left(\lambda_i\right)\right) \right\}$$

Note that this means that the design choices of creators on one platform's membership portal are not affected by the commissions of a different portal in subgames with multiple membership portals. The membership platform's profit is

$$\Pi_m = \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\bar{\lambda}}{s})\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)\bar{\beta}D_m\tau_m v(\lambda_m) \right\}$$

The pure membership platform's tradeoff is similar to the monopoly problem, except that the addition of the other platform means that it must balance the shift in creator participation as well as the effect of changing λ_m on D_m in addition to balancing increasing margin against the distraction effect.

The hybrid platform's profit is

$$\begin{aligned} \Pi_h &= \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\bar{\lambda}}{s}) \left[(1 - \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h \left(\bar{\beta}\tau_h v(\lambda_h) + A\right) + \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_m A \right] - C \right\} \\ &= \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\bar{\lambda}}{s}) \left[(1 - \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h\bar{\beta}\tau_h v(\lambda_h) + A \right] - C \right\} \end{aligned}$$

Denote

$$\eta(x) = \frac{xg(x)}{G(x)} \geq 0, \text{ where } x > 0$$

as the elasticity of consumer participation. Additionally we impose the following assumption:

Assumption 1. *In the duopoly model with differentiated membership portals*

- $\bar{a}$ *is not too small and* $v'^*(0, \bar{r}))$ *is not too negative such that* $\lim_{\sigma\to\infty} \tau_j > 0 \ \forall j$

With this assumption we are ready to state Proposition C.1:

Proposition C.1. *Under Assumption 1, for any combination of competing membership portals we have that*

- $\lim_{\sigma\to\infty} \tau_j > 0 \ \forall j$
- $\tau_j \to 0 \ \forall j$ *as* $\sigma \to 0$

Proof. Consider the FOC for the Pure membership platform:

$$
\begin{aligned}
&g(\frac{\bar{\lambda}}{s})\frac{1}{s}\frac{d\bar{\lambda}}{d\tau_m}\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)\bar{\beta}D_m\tau_m v(\lambda_m) \\
+&G(\frac{\bar{\lambda}}{s})\bar{\beta}\Bigg[\\
&\quad \frac{\bar{\beta}}{\sigma}\bar{Q}'(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_m\tau_m v(\lambda_m) \\
&\quad + \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_m v(\lambda_m) \\
&\quad + \frac{d\lambda_m}{d\tau_m}\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_m\tau_m v'(\lambda_m) \\
&\quad + \frac{dD_m}{d\tau_m}\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)\tau_m v(\lambda_m) \\
&\quad \Bigg] = 0
\end{aligned}
\tag{C.1}
$$

And the hybrid

$$
\begin{aligned}
&g(\frac{\bar{\lambda}}{s})\frac{1}{s}\frac{d\bar{\lambda}}{d\tau_h}\left[(1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h\bar{\beta}\tau_h v(\lambda_h) + A\right] \\
+&G(\frac{\bar{\lambda}}{s})\bar{\beta}\Bigg[\\
&\quad \frac{\bar{\beta}}{\sigma}\bar{Q}'(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_h\tau_h v(\lambda_h) \\
&\quad + (1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h v(\lambda_h) \\
&\quad + \frac{d\lambda_h}{d\tau_h}(1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h\tau_h v'(\lambda_h) \\
&\quad + \frac{dD_h}{d\tau_h}(1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))\tau_h v(\lambda_h) \\
&\quad \Bigg] = 0
\end{aligned}
\tag{C.2}
$$

Note that apart from the first line, each other line of the FOC is identical up to $\tau_h,\ \tau_m$ being reversed. Consider the following normalization of the hybrid FOC:

$$
\begin{aligned}
&\frac{g(\frac{\bar{\lambda}}{s})\frac{\bar{\lambda}}{s}}{G(\frac{\bar{\lambda}}{s})}\frac{1}{\bar{\lambda}}\frac{d\bar{\lambda}}{d\tau_h}\left[(1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h\bar{\beta}\tau_h v(\lambda_h) + A\right] \\
+&\bar{\beta}\Bigg[\\
&\quad \frac{\bar{\beta}}{\sigma}\bar{Q}'(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_h\tau_h v(\lambda_h) \\
&\quad + (1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h v(\lambda_h) \\
&\quad + \frac{d\lambda_h}{d\tau_h}(1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_h\tau_h v'(\lambda_h) \\
&\quad + \frac{dD_h}{d\tau_h}(1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))\tau_h v(\lambda_h) \\
&\quad \Bigg] = 0
\end{aligned}
\tag{C.3}
$$

$\max_x \eta(x)$ not too large, $\frac{dD_j}{d\tau_j}v(\lambda_j)$ decreasing in τ and $\bar{Q}''(\cdot) \leq 0$ are together sufficient assumptions to give concavity of profits in τ (and hence ensure sufficiency of the FOC). From the creators' problem and the definition of $\bar{\lambda}$, $\bar{\lambda} \in [\lambda^*(0, \underline{r}), 1]$, so the first line will go to 0 as $\max_{\frac{\bar{\lambda}}{s}} \eta(\frac{\bar{\lambda}}{s}) = \frac{g(\frac{\bar{\lambda}}{s})\frac{\bar{\lambda}}{s}}{G(\frac{\bar{\lambda}}{s})}$ goes to 0. We can apply a similar normalization and come to the same conclusion for the pure membership FOC. Thus, as $\max_{\frac{\bar{\lambda}}{s}} \eta(\frac{\bar{\lambda}}{s}) \to 0$ the first order conditions are almost identical. Now consider $\frac{dD_j}{d\tau_j}$ (assuming j's creator share is given by $1 - \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)$):

$$\begin{aligned}\frac{dD_j}{d\tau_j} =& \frac{r\lambda_j^r\lambda_{-j}^r \frac{\frac{d\lambda_j}{d\tau_j}}{\lambda_j}\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau) + \lambda_j^r\frac{\bar{\beta}}{\sigma}\bar{Q}'(\frac{\bar{\beta}}{\sigma}\Delta\tau)\left[\lambda_j^r - \lambda_{-j}^r\right]}{\left((1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))\lambda_j^r + \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)\lambda_{-j}^r\right)^2} \\ =& r\frac{\frac{d\lambda_j}{d\tau_j}}{\lambda_j}\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_jD_{-j} + \frac{\lambda_j^r\frac{\bar{\beta}}{\sigma}\bar{Q}'(\frac{\bar{\beta}}{\sigma}\Delta\tau)\left[\lambda_j^r - \lambda_{-j}^r\right]}{\left((1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))\lambda_j^r + \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)\lambda_{-j}^r\right)^2}\end{aligned} \tag{C.4}$$

If $\frac{d\lambda_j}{d\tau_j}$ is increasing in τ_j, then if the first term increases, it does so more slowly than $\frac{d\lambda_j}{d\tau_j}D_h$. Together with concavity of v this implies that $D_jv(\lambda_j)$ non-increasing in τ_j combined with $\max_x \eta(x)$ not too large is a sufficient (if much stronger than necessary) condition to ensure concavity of profits in τ_j.

Commissions positive for large σ

$\lim_{\sigma\to\infty} \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau) = \frac{1}{2}$. , while the term $\frac{\bar{\beta}}{\sigma}\bar{Q}'(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_h\tau_hv(\lambda_h)$ tends to 0.

From the definition of $\bar{\lambda}$, $\frac{d\bar{\lambda}}{d\tau_j} = \frac{\partial\bar{\lambda}}{\partial\lambda_j}\frac{d\lambda_j}{d\tau_j}$ and simple calculus shows that $\lim_{\sigma\to\infty}\frac{\partial\bar{\lambda}}{\partial\lambda_j}$ has the same sign as

$$\lambda_j^{2r} + \lambda_j^r\lambda_{-j}^r\left(1 + r - r\frac{\lambda_{-j}}{\lambda_j}\right) \tag{C.5}$$

The solution to the creator's problem sets $\lambda = \left\{\frac{(1+\bar{r})(\bar{a}+(1-\tau)v(\lambda)}{-(1-\tau)v'(\lambda)}, 1\right\}$, which is minimized at $\tau = 0$, so $\bar{a}$ not too small ensures that $\frac{\lambda_{-j}}{\lambda_j}$ is not too far from 1, meaning that the first term in each profit derivative for both the hybrid and pure membership platforms is either positive or not too negative. This condition combined with $v'(\lambda^*(0,\bar{r}))$ not too negative gives that the derivative of both profit functions will be positive when $\tau = 0$ as concavity of $v(\cdot)$ and λ increasing in r together mean that $v'(\lambda^*(0,\bar{r})) < v'(\lambda^*(0,r)$ for any $r < \bar{r}$.

Commissions converging to 0 as $\frac{\bar{\beta}}{\sigma} \to \infty$

This follows directly from the FOCS. As σ approaches 0, the term

$$\frac{\bar{\beta}}{\sigma}\bar{Q}'(\frac{\bar{\beta}}{\sigma}\Delta\tau)D_h\tau_hv(\lambda_h)$$

becomes unboundedly negative, and so the FOCs cannot be satisfied for any interior commission.

These results follow for any combination of business model modes which include two competing membership portals as the only change in the FOCs for two pure membership platforms competing is that we substitute s_0 for s, and the only change for competition between two hybrids is substituting $A/2$ for A. ∎

The intuition behind this proposition is mostly straightforward. As $\sigma \to 0$ the membership portals are viewed by creators as increasingly close substitutes, so the degree of competition increases and commissions decrease. While if $\sigma \to \infty$ the platforms are local monopolies from the perspective of creators, and a change in τ does not affect creator participation on a platform.

Lemma C.1. *If neither platform operates on a pure membership model, then both platforms set $r = \bar{r}$.*

Proof. If one platform is a pure discovery platform and the other a pure membership platform, then because τ and r are set simultaneously, the pure discovery platform cannot influence τ through its choice of r, and so by logic similar to the equilibrium in the monopoly model, its best response is always $\bar{r}$. For all other cases, because consumers discontinuously choose the discovery portal with maximal r, or divide evenly between portals when the platforms set r equal to each other, the platforms face discontinuities in their profits.

Case 1: $r_j < r_{-j}$

r_j has no influence on creator behavior as all consumers are finding creators through $-j$'s discovery portal. If $r_j < r_{-j}$ then j receives no advertising revenue, while it receives half of the advertising revenue if $r_j = r_{-j}$ so the platform can profitably deviate to $r_j = r_{-j}$.

Case 2: $r_j = r_{-j} < \bar{r}$

Because consumers' behavior is discontinuous, j can deviate to set $r_j = r_{-j} + \epsilon$, have an infinitesimal impact on creator behavior, and capture all of the advertising revenue.

The only equilibrium is $r_j = r_{-j} = \bar{r}$ ■

Essentially, competition for consumers leads platforms to induce the maximal broadness of content they can achieve. The main proposition of this section concerns the choice of business models as a best response to another platform's choice of business model. Denote by the subscript j, XY a variable attributed to platform when platform j chooses business model X and platform 2 chooses business model Y.

Assumption 2. *The following conditions ensure that that $\tau_{1,MH}$ is increasing in r.*

1. $(2 + \bar{r})v''(\lambda) + v'''(\lambda) \geq 0$
2. *$v''(\cdot)$ is sufficiently close to 0 such that $\tau_{1,MH}$ is increasing in r.*

For the following proposition we focus on best responses of platform 2 to a business model decision by platform 1. In this proof we focus on polar values of σ, i.e. $\sigma = 0$ or $\sigma \to \infty$, which we refer to as the "competitive" regime and the "local monopoly" regime respectively.

Proposition C.2. *Under Assumption 2, there exist thresholds η^* and $\tilde{r}$ such that if $r > \tilde{r}$ and $\max_x \eta(x) < \eta^*$, then in the local monopoly regime:*

- *There exists a threshold A_3^* such that the best response of a platform to D is M if $A < A_3^*$ and H if $A > A_3^*$.*
- *There exists a threshold A_3' such that the best response of a platform to M is M if $A < A_3'$ and H if $A > A_3'$.*
- *There exists a threshold A_3'' such that the best response of a platform to H is M if $A < A_3''$ and H if $A > A_3''$. Further, $A_3'' > A_3^*$.*

Proof. *Case 1: $P_1 = D$*

Given P_1's choice to operate as a pure discovery platform, the profit replication argument of Proposition 1 still applies, so P_2's best response is either M or H. P_2's profits under these two options are given by:

$$\Pi_{2,DM} = \max_{\tau \leq 1 - \beta_0/\bar{\beta}} \left\{ G\left(\frac{\lambda^*(\tau, \bar{r})}{s}\right) \tau \bar{\beta} v(\lambda^*(\tau, \bar{r})) \right\}$$

If $P_2 = M$, with the optimal τ denoted as $\tau^*_{2,DM}$. Alternatively

$$\Pi_{2,DH} = \max_{\tau \leq 1 - \beta_0/\bar{\beta}} \left\{ G\left(\frac{\lambda^*(\tau, \bar{r})}{s}\right) \left(\frac{A}{2} + \tau \bar{\beta} v(\lambda^*(\tau, \bar{r}))\right) - C \right\}.$$

If $P_2 = H$. with the solution denoted as $\tau^*_{2,DH}$. r does not appear in the platform's decision for this maximization problem due to lemma C.1. Also from Lemma C.1 the distraction effect that appeared in the monopoly model does not apply in this case, therefore if P_2 is a hybrid, the set of λ^* it it can induce is precisely the same as it could as a pure membership platform. Therefore, for $A > A^*_3 \equiv \frac{2C}{G\left(\frac{\lambda^*(\tau^*_{DH},\bar{r})}{s}\right)}$, P_2 finds operating as a hybrid more profitable than operating as a pure membership platform. Otherwise $P_2 = M$ is platform 2's best response.

$\underline{P_1 = M}$

With P_1 operating as a pure membership platform, if $P_2 = D$ P_2's profits are then:

$$\Pi_{2,MD} = G\left(\frac{\lambda^*(\tau_{1,MD},\bar{r})}{s}\right)A - C$$

Let $\boldsymbol{\tau} = \{\tau_1, \tau_2\}$. Then if $P_2 = M$

$$\Pi_{2,MM} = \max_{\tau \le 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\bar{\lambda}(\boldsymbol{\tau},0)}{s_0})\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau)\bar{\beta}\tau v(\lambda(\tau,0)) \right\}$$

and if $P_2 = H$

$$\Pi_{2,MH} = \max_{\tau \le 1-\beta_0/\bar{\beta}, r\in[\underline{r},\bar{r}]} \left\{ G(\frac{\bar{\lambda}(\boldsymbol{\tau},r)}{s}) \left[(1-\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_{2,MH}\bar{\beta}\tau v(\lambda(\tau,r)) + A \right] - C \right\}$$

If $\sigma = 0$, then from Proposition C.1, $\tau_j = 0$ for both j if $P_2 \in \{M, H\}$. This would then imply that $0 = \Pi_{2,MM} < \Pi_{2,MH} < \Pi_{2,MD}$ by the same logic as in section 5. On the other hand, if $\sigma \to \infty$, then $\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau) = 1 - \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau) = \frac{1}{2}$ for any vector $\boldsymbol{\tau}$. It then follows that for A large $\Pi_{2,MD} > \Pi_{2,MM}$, while for A small, the inequality is reversed. The question is then how $\Pi_{2,MH}$ compares to the other two .

$$\Pi_{1,MD} = \max_{\tau \le 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\lambda(\tau,\bar{r})}{s})\bar{\beta}\tau v(\lambda(\tau,\bar{r})) \right\}$$

$$\Pi_{1,MM} = \max_{\tau \le 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\bar{\lambda}(\boldsymbol{\tau},0)}{s_0})\frac{1}{2}\bar{\beta}\tau v(\lambda(\tau,0) \right\}$$

and

$$\Pi_{1,MH} = \max_{\tau \le 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\bar{\lambda}(\boldsymbol{\tau},r^*)}{s})\frac{1}{2}D_{1,MH}\bar{\beta}\tau v(\lambda(\tau,r^*_{1,MH}) \right\}$$

When $P_2 = D$ both regimes P_1's optimal τ solves

$$\bar{\beta}\left(g(\frac{\lambda}{s})\frac{1}{s}\frac{d\lambda}{d\tau}(\tau v(\lambda)) + G(\frac{\lambda}{s})\left[v(\lambda + \tau v'(\lambda)\frac{d\lambda}{d\tau} \right] \right) = 0$$

Which can be rearranged to

$$\eta(\frac{\lambda}{s})\frac{\frac{d\lambda}{d\tau}}{\lambda}\tau + 1 = \frac{-\tau v'(\lambda)\frac{d\lambda}{d\tau}}{v(\lambda)}$$

While if $P_2 = H$, P_1's FOC is

$$g(\frac{\bar{\lambda}}{s})\frac{1}{s}\frac{d\bar{\lambda}}{d\tau_1}[D_1\tau_1 v(\lambda_1)] + G(\frac{\bar{\lambda}}{s})\left[\frac{dD_1}{d\tau_1}\tau_1 v(\lambda_1) + D_1 v(\lambda_1) + D_1\tau_1 v'(\lambda_1)\frac{d\lambda_1}{d\tau_1} \right] = 0$$

From (C.4) In the local monopoly regime we have that

$$\frac{dD_j}{d\tau_j} = \frac{r\frac{1}{2}\lambda_j^r\lambda_{-j}^r\frac{\frac{d\lambda_j}{d\tau_j}}{\lambda_j}}{\left(\frac{1}{2}\lambda_j^r+\frac{1}{2}\lambda_{-j}^r\right)^2} = \frac{r}{2}\frac{\frac{d\lambda_j}{d\tau_j}}{\lambda_j}D_jD_{-j} \tag{C.6}$$

and

$$\bar{\lambda} = \frac{\lambda_1^{r+1}+\lambda_2^{r+1}}{\lambda_1^r+\lambda_2^r}$$

Taking the derivative and then applying some algebra

$$\frac{d\bar{\lambda}}{d\tau_1} = \frac{d\lambda_1}{d\tau_1}\frac{1}{2}D_1\frac{\lambda_1^r+\lambda_2^r[(r+1)-r\frac{\lambda_2}{\lambda_1}]}{\lambda_1^r+\lambda_2^r} \tag{C.7}$$

Plugging these identities into the FOC

$$G(\frac{\bar{\lambda}}{s})\left[rD_2\frac{\frac{d\lambda_1}{d\tau_1}}{\lambda_1}\tau_1 v(\lambda_1)+v(\lambda)+\tau_1 v'(\lambda_1)\frac{d\lambda_1}{d\tau_1}\right]+g(\frac{\bar{\lambda}}{s})\frac{1}{s}\frac{1}{2}\frac{d\lambda_1}{d\tau_1}\left(\frac{\lambda_1^r+\lambda_2^r[(r+1)-r\frac{\lambda_2}{\lambda_1}]}{\lambda_1^r+\lambda_2^r}\right)D_1\tau_1 v(\lambda_1)=0$$

Which can then be rearranged to

$$\eta(\frac{\bar{\lambda}}{s})\frac{1}{2\bar{\lambda}}\frac{d\lambda_1}{d\tau_1}\left(\frac{\lambda_1^r+\lambda_2^r[(r+1)-r\frac{\lambda_2}{\lambda_1}]}{\lambda_1^r+\lambda_2^r}\right)D_1\tau_1+rD_2\frac{\frac{d\lambda_1}{d\tau_1}}{\lambda_1}\tau_1+1=\frac{-\tau_1 v'(\lambda_1)\frac{d\lambda_1}{d\tau_1}}{v(\lambda_1)} \tag{C.8}$$

The term $rD_2\frac{\frac{d\lambda_1}{d\tau_1}}{\lambda_1}\tau_1$ is positive and not present in P_1's FOC when $P_2 = D$. The comparison of $\eta(\frac{\bar{\lambda}}{s})\frac{1}{2\bar{\lambda}}\frac{d\lambda_1}{d\tau_1}\left(\frac{\lambda_1^r+\lambda_2^r[(r+1)-r\frac{\lambda_2}{\lambda_1}]}{\lambda_1^r+\lambda_2^r}\right)D_1\tau_1$ and $\eta(\frac{\lambda}{s})\frac{\frac{d\lambda}{d\tau}}{\lambda}\tau$ is not so straightforward as the former depends on the endogenous value of τ_2. However, if we note that as $\max_x \eta(x) \to 0$, then there must be some value such that $rD_2\frac{\frac{d\lambda_1}{d\tau_1}}{\lambda_1}\tau_1 > \eta(\frac{\lambda}{s})\frac{\frac{d\lambda}{d\tau}}{\lambda}\tau$ under any parameter space. Therefore there exists a cutoff value η^* such that $\tau_{1,MD} < \tau_1 MH$ if $\max_x \eta(x) < \eta^*$.[1]

Taking the derivative of $\frac{d\lambda}{d\tau}$ with regard to r from the proof of Lemma 1, it is easy to show that $\frac{d\lambda}{d\tau}$ is increasing in r if $(2+r)v''(\lambda)+v'''(\lambda)$ is close to 0 or positive. Thus $(2+\bar{r})v''(\lambda)+v'''(\lambda) \geq 0$ is a sufficient assumption to ensure that $\frac{d\lambda}{d\tau}$ is increasing in r. Dividing Equation C.8 through by $\frac{d\lambda_1}{d\tau_1}$, we can see from inspection that the LHS is increasing in r if $\frac{d\lambda_1}{d\tau_1}$ is decreasing in r. Therefore the equilibrium τ_1 is increasing in r if the RHS is not increasing too quickly (i.e. $v(\cdot)$ not too concave).

Therefore if P_1 expects P_2 to set $r < \bar{r}$ in equilibrium of the MH subgame, τ_1 is greater than τ in the MD subgame, so P_2 could deviate to setting $r = \bar{r}$ and receive greater advertising revenue (through larger λ) than what it received as a pure discovery platform, and it also receives membership revenue. Therefore, operating as a hybrid is strictly more profitable than operating as a pure discovery platform.

For A small, when P_2 is deciding between operating as a pure membership platform and a hybrid. The distraction effect from the monopoly model is still present so long as $\underline{r} > 0$, so $\tau_m v(\lambda_m) > \tau_m v(\lambda_h)$ for any τ. Given our assumption of inelastic consumer participation, the platform cannot compensate for this with additional consumer participation. Therefore for A sufficiently small, the reduction in membership revenue is not compensated by the additional revenue from advertising and $\Pi_{2,MH} < \Pi_{2,MM}$.

Equation C.8 implies that P_1's best response function in the MH subgame does not depend on A.

1. It would be relatively simple to find a lower bound for this cutoff if we assume λ_2 is the argument that minimizes the maximum of the LHS of P_1's FOC in the MH subgame. But this exercise would involve a great deal of effort and would likely not yield significant insight.

Therefore, if A increases, P_2 could set the same r and τ as it would for lower A and receive strictly more profit via greater advertising revenue. It is likely that P_2 could increase profits further by adjusting its strategy. Given our assumptions about P_1's beliefs, we can conclude from this result that Π_2, MH is increasing in A. Further, it is trivial to show that for A sufficiently large $\Pi_{2,MH} > \Pi_{2,MM}$.[2] Therefore, in the local monopoly regime, if $\max_x \eta(x)$ is not too large, then there must be some threshold A_3' such that P_2's best response to $P_1 = M$ is M for $A \leq A_3'$ and H for $A > A_3'$.

$P_1 = H$

In this case:

$$\Pi_{2,HD} = G\left(\frac{\lambda^*(\tau_{1,HD}, \bar{r})}{s}\right)\frac{A}{2} - C$$

When $P_2 = D$

$$\Pi_{2,HM} = \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\bar{\lambda}(\boldsymbol{\tau}, r^*)}{s})\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_{2,HM}\bar{\beta}\tau v(\lambda(\tau_{2,HM}, r^*)) \right\}$$

When $P_2 = M$, and

$$\Pi_{2,HH} = \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G(\frac{\bar{\lambda}(\boldsymbol{\tau}, \bar{r})}{s}) \left[(1 - \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau))D_{2,HH}\bar{\beta}\tau v(\lambda(\tau_{2,HH}, \bar{r})) + \frac{1}{2}A \right] - C \right\}$$

When $P_2 = H$.

Once again, if $\sigma = 0$ the results mirror Section 5. Focusing then on the local monopoly regime: Under HH, P_1's problem is identical to P_2, but under HD it solves

$$\Pi_{1,HD} = \max_{\tau \leq 1-\beta_0/\bar{\beta}} \left\{ G\left(\frac{\lambda^*(\tau, \bar{r})}{s}\right)\left(\frac{A}{2} + \tau\bar{\beta}v(\lambda^*(\tau, \bar{r}))\right) - C \right\}.$$

Following similar logic to the previous case, the FOC for $\tau_{1,HD}$ is (after some simplification)

$$0 = \frac{G\left(\frac{\lambda(\tau)}{s}\right)}{g\left(\frac{\lambda(\tau)}{s}\right)}\bar{\beta}s\left(\frac{v(\lambda(\tau))}{d\lambda(\tau)/d\tau} + \tau v'(\lambda(\tau)\frac{d\lambda(\tau)}{d\tau}\right) + \bar{\beta}\tau v(\lambda(\tau)) + \frac{A}{2} \tag{C.9}$$

While the FOC for $\tau_{1,HH}$ is (using the fact that $\bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau) = 1 - \bar{Q}(\frac{\bar{\beta}}{\sigma}\Delta\tau) = 1/2$)

$$\frac{d}{d\tau}\left(G\left(\frac{1}{s}\left(\frac{\lambda(\tau)^{1+r} + \lambda(\tau_{HH}^*)^{1+r}}{\lambda(\tau)^r + \lambda(\tau_{HH}^*)^r}\right)\right)\left(\bar{\beta}\tau\frac{\lambda(\tau)^r}{\lambda(\tau)^r + \lambda(\tau_{HH}^*)^r}v(\lambda(\tau)) + \frac{A}{2}\right)\right) = 0$$

Equations C.7 and C.6 remain valid in the HH configuration, so imposing symmetry we get

$$\frac{d}{d\tau_1}\bar{\lambda}_{\tau_1=\tau_2} = \frac{1}{2}\frac{d\lambda_1}{d\tau_1}$$

and

$$\frac{d}{d\tau_1}\left(\frac{\lambda(\tau)^r}{\lambda(\tau)^r + \lambda(\tau_{HH}^*)^r}\right)_{\tau_1=\tau_2} = \frac{d}{d\tau_1}\left(\frac{1}{2}D_1\right)_{\tau_1=\tau_2} = \frac{r}{4\lambda(\tau)}\frac{d\lambda(\tau)}{d\tau}$$

2. One example would be to set $A = \Pi_{2,MM}$

Hence the FOC for $\tau_{1,HH}$ after imposing symmetry, can be written as

$$\begin{aligned} 0 &= G\left(\frac{\bar{\lambda}}{s}\right)\bar{\beta}\left(\frac{1}{2}\frac{v(\lambda(\tau))}{d\lambda(\tau)/d\tau}+\tau\left(\frac{1}{2}v'(\lambda(\tau))+\frac{rv(\lambda(\tau))}{4\lambda(\tau)}\right)\right) \\ &\quad +\left(\frac{\bar{\beta}}{2}\tau v(\lambda(\tau))+\frac{A}{2}\right)\frac{1}{2}g\left(\frac{\bar{\lambda}}{s}\right)\frac{1}{s} \end{aligned}$$

Divide everything by $\frac{1}{2s}g\left(\frac{\bar{\lambda}}{s}\right)$,

$$\begin{aligned} 0 &= \frac{G\left(\frac{\bar{\lambda}}{s}\right)}{g\left(\frac{\bar{\lambda}}{s}\right)}s\bar{\beta}\left(\frac{v(\lambda(\tau))}{d\lambda(\tau)/d\tau}+\tau\left(v'(\lambda(\tau))+\frac{rv(\lambda(\tau))}{2\lambda(\tau)}\right)\right) \\ &\quad +\frac{\bar{\beta}}{2}\tau v(\lambda(\tau))+\frac{A}{2} \end{aligned} \tag{C.10}$$

Evaluating the RHS of (C.9) and (C.10) at the same τ. As before, compared to (C.9), the first line in (C.10) is larger because $\frac{rv(\lambda(\tau))}{2\lambda(\tau)}>0$. However, the second line in (C.10) is smaller. Hence, we cannot directly conclude $\tau^*_{HH}\geq\tau^*_{HD}$. A sufficient condition for the RHS of (C.10) to be larger (which implies $\tau_{1,HH}\geq\tau_{1,HD}$) is

$$\tau\bar{\beta}s\frac{G\left(\frac{\bar{\lambda}}{s}\right)}{g\left(\frac{\bar{\lambda}}{s}\right)}\frac{rv(\lambda_1)}{2\lambda_1}\geq\frac{\bar{\beta}}{2}\tau v(\lambda(\tau))$$

or simply

$$\bar{r}>\eta\left(\frac{\bar{\lambda}}{s}\right)\quad\text{for all }\tau$$

Where we use $\lambda_1=\bar{\lambda}$ in the symmetric equilibrium to get $\frac{G(\frac{\bar{\lambda}}{s})}{g(\frac{\bar{\lambda}}{s})}\frac{s}{\lambda_1}=\eta(\frac{\bar{\lambda}}{s})^{-1}$. From Lemma C.1 $r=\bar{r}$ for both platforms, so once again competition for recommendations drives up equilibrium τ and advertising revenue is greater in the HH subgame. This then implies that P_2 finds operating as a hybrid more profitable than operating as a pure discovery platform.

Let $\tau_{\bar{r}}$ be the τ a monopolist would set were it constrained to set $r=\bar{r}$. Comparing C.10 with the hybrid monopolist's FOC from the benchmark model, it must be the case that $\tau^*_{HH}<\tau_{\bar{r}}$ because $\frac{d\bar{\lambda}}{d\tau_j}<\frac{d\lambda_j}{d\tau_j}$, and each platform only receives half of the advertising revenue. Again from Equation C.10, it must be the case that τ^*_{HH} is increasing in A, which combined with $\tau^*_{HH}<\tau_{\bar{r}}$ implies that industry profit is increasing in A. Because in equilibrium each platform receives half of industry profit, $\Pi_{j,HH}$ is increasing in A for all j.

From Lemma C.1, when P_2 adds a discovery portal $r=\bar{r}$ for both platforms. Then as $\bar{r}\to\infty$ $\lambda(0,\bar{r})\to 1$. Since $\lambda(\tau,\bar{r})$ is increasing in τ and $v(1)=0$, this implies that for $\bar{r}$ sufficiently large $D_2\bar{\beta}\tau v(\lambda(\tau,\bar{r}))<D_2\bar{\beta}\tau v(\lambda(\tau,r^*))$ even if D_2 is larger when $P_2=H$ than when $P_2=M$. This then implies that unless $r^*=\bar{r}$, membership revenue must decrease if P_2 switches to operating as a hybrid. By the same logic as the previous sections, there exists a threshold A''_3 such that $\Pi_{2,HH}<\Pi_{2,HM}$ for $A<A''_3$ and the inequality is reversed for $A>A''_3$. Note further that if $A=A^*_3$, the platform receives no net advertising revenue and has less membership revenue than if it were a pure membership platform, so $A''_3>A^*_3$. ■

The best responses from Proposition C.2 lead immediately to Corollary C.1:

Corollary C.1. *Under Assumption 2, there exist thresholds η^* and $\tilde{r}$ such that if $r>\tilde{r}$ and $\max_x\eta(x)<\eta^*$, then in the local monopoly regime there exist thresholds A^*_3, A'_3, A''_3 such that*

- *If $P_1=M$, P_2 chooses M if $A<A'_3$ and H if $A>A'_3$.*

- *If $P_1 = H$, P_2 chooses M if $A < A_3''$ and H if $A > A_3''$. Further, $A_3'' > A_3^*$.*
- *If $P_1 = D$, P_2 optimally chooses M if $A < A_3^*$ and chooses H if $A > A_3^*$.*

We have not shown that $A_3'' > A_3'$ in general, so under some parameter sets the second bullet point may refer to an empty parameter set. The equilibria under various parameter sets, as implied by the benchmark duopoly model and Corollary C.1, are laid out in the table below

Competitive regime		Local monopoly regime	
$A > A_3$	D, H	$A > A_3''$	H, H
$A > A_3$	D, H	$A_3'' > A > A_3'$	M, H
$A < A_3$	D, M	$A < A_3'$	M, M

Introducing a second membership portal when a competing platform's current strategy includes one has two effects on commissions: first, it pushes commissions down because consumers' participation decision is based on the *average* λ in the market, and an increase in τ affects that average less when half of the creators are participating in a different membership portal than when there is only one. On the other hand, when there are two membership portals and at least one discovery portal, then the introduction of the second membership portal creates competition for recommendations on the part of the platforms as well as the creators, which pushes τ up as the platforms want to induce content design that appeals to the recommendation algorithm. If η is small, then the second effect dominates and *tau* is higher when there are two membership portals and at least one discovery portal, meaning participation is greater and D is never a best response to any business model strategy in the local monopoly regime.

We conjecture that if we allow for $\eta > \eta^*$ in the local monopoly regime then there would be a cutoff value of A above which the best response to H is D. The reason being that with large η the first effect could dominate if the platform choosing D were to deviate to H, lowering participation and thus advertising revenue. For large A, the reduction in advertising revenue would then not be compensated for by the addition of exclusive content revenue, and so D would then be a best response to H.