Intermediaries in the Online Advertising Market *

[1], Anna D'Annunzio[1], and Antonio Russo[2]

[1]TBS Business School

[2]Loughborough University and CESifo

October 6, 2021

Abstract

A large share of the ads displayed by digital publishers (e.g., newspapers and blogs) are sold via intermediaries (e.g., Google), that have large market power and reportedly allocate the ads in an opaque way. We study the incentives of an intermediary to disclose consumer information to advertisers when auctioning ad impressions. In turn, we study how disclosure affects the incentives of publishers to outsource the sale of their ads to an intermediary, and relate these incentives to the extent of consumer multi-homing, the competitiveness of advertising markets and the ability of platforms to profile consumers. We show that disclosing information that enables advertisers to optimize the allocation of ads on multi-homing consumers is profitable to the intermediary only if advertising markets are sufficiently thick. Even when most consumers multi-home, the publishers may be worse off by outsourcing to the intermediary, in particular if they operate in thin advertising markets. Finally, we study how the intermediary responds to policies designed to enhance transparency or consumer privacy, and the implications of these policies for the online advertising market.

Keywords: online advertising, intermediary, multi-homing, privacy, transparency

JEL Classification: D43, D62, L82, M37

*D'Annunzio: dannunzio.anna@gmail.com, Russo: a.russo@lboro.ac.uk. We gratefully acknowledge the contribution by Christian Peukert to the first stages of this work. We also thank Bruno Jullien, Leonardo Madio and participants to presentations at Nottingham University Business School, the Louvain and Paris Online Seminar, the Scientific Seminar on the Economics, Law and Policy of Communications and Media at EUI in Florence, the Workshop on the economics of platforms (ECOP) in Bologna, the OsloMet seminar for valuable comments. Part of this research was carried out while Anna D'Annunzio was at the University Federico II, Naples.

1 Introduction

Online advertising is one of the most dynamic markets in the economy and is central to the business model of a vast array of digital content providers and publishers that sell consumers' attention to advertisers (e.g. newspapers, blogs, review websites, etc.).[1] Thus, the functioning of the online advertising market and the distribution of surplus among its players affect a key part of the digital economy. Furthermore, there are important spillovers on society at large. For instance, advertising revenue drives investment decisions in the quality of content by digital publishers, including in crucial domains such as journalism.[2] In addition, advertising affects the prices that consumers pay for goods and services (Bagwell, 2007).

Digital publishers typically provide "display" ads.[3] A striking feature of the market for this kind of advertising is the major role played by intermediaries. While digital publishers sell some of their ads through internal agencies, more than 60 percent of display ad spend is traded via intermediaries (IAB, 2017). Furthermore, although there is a complex chain of intermediaries connecting advertisers to publishers, one major platform (Google) has a dominant position in every link of this chain (see Figure 1 for an illustration).[4] In this paper, we study the behavior of large advertising intermediaries, particularly regarding their transparency when auctioning the ad impressions. In addition, we study whether and when the publishers gain by using an intermediary, and consider the implications for the online advertising market.

Conceivably, a large intermediary like Google presents several attractive features to digital publishers, particularly when consumers visit multiple publishers in a short time frame (multi-home), as they commonly do online. The intermediary centralizes the sale of ads and thus softens competition for attracting advertisers, which would otherwise be intense with consumer multi-homing (Ambrus et al., 2016). Furthermore, the intermediary can typically achieve a more precise profiling of consumers and allocate ad impressions more effectively. In addition,

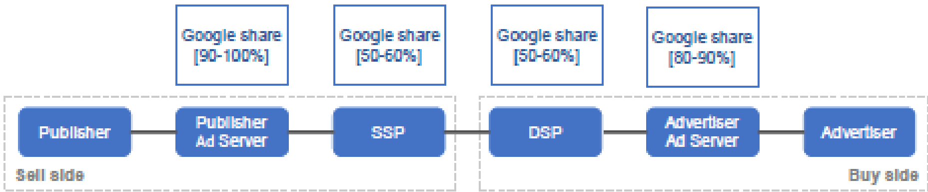
[1] Global digital advertising spending amounted to about USD 280 billion in 2018, and about USD 330 billion in 2019. `https://www.statista.com/statistics/237974/online-advertising-spending-worldwide/`.

[2] As news consumption increasingly takes place online, the link between online advertising and the quality of journalism has drawn the attention of policymakers. On the relation between competition in the online advertising market - with particular regard to the role of large platforms - and the viability of high quality journalism, see chapter 4 of the Cairncross Review (Cairncross, 2019).

[3] Display ads represent one of the three main segments of the digital advertising market (the other two being "search" ads and "classified" ads). In the UK, the display advertising market was worth GBP 5.5 billion in 2019, compared to GBP 7.3 billion for the search advertising market (CMA, 2020)

[4] As shown in the figure, the chain of intermediaries includes supply-side platforms (SSPs) that collect ad inventories from publishers and run ad auctions; demand-side platforms (DSPs) that allow advertisers to buy ad inventories; publisher ad servers, that manage publishers' inventory and decide which ad to serve, based on the bids received from SSPs and direct deals between the publisher and advertisers. Google has virtually a monopoly in the ad server market, and also dominates the SSP and DSP segments (CMA, 2020).

Figure 1: Google as the dominant intermediary in online display advertising. Source: CMA (2020)



the intermediary can coordinate consumers' exposure to ads on each publisher, limiting the cross-outlet ad repetition that hinders the reach of advertising campaigns (Athey et al., 2018), or allowing more effective re-targeting.[5] Potentially, these features can increase total surplus in the advertising market, as well as the share of such surplus captured by the platforms on the supply side of the market.

However, the functioning of intermediated advertising markets is quite complex and obscure, with substantial information asymmetries between the intermediaries and their customers, i.e. publishers and advertisers. Regulators and market operators have pointed out the lack of transparency in how intermediaries run the auctions for ad impressions and, more generally, in how they allocate these impressions. One of the main concerns is that intermediaries strategically retain valuable information from advertisers, for example by making it difficult to assess the effectiveness of ad impressions and whether such impressions end up on consumers that have already been exposed to them. Hence, the advertisers find it difficult to evaluate the reach and frequency of campaigns taking place on multiple publishers (CMA, 2020).

The above observations raise several interesting and thus far largely unexplored questions. What drives the incentives of an intermediary to disclose information to advertisers when auctioning ad impressions? Under which conditions do digital publishers gain by using an intermediary to sell their advertising space, and how do these conditions depend on information disclosure? How does transparency (or lack thereof) by the intermediary affect the overall efficiency of the market and the distribution of profits? What are the possible effects of regulatory interventions that affect the information disclosure by intermediaries, e.g. transparency and privacy rules? We address these questions by proposing a model that endogenizes the intermediary's decision on transparency in advertising auctions as well as the publishers' decision to outsource the sale of ads. In our model, these decisions are driven by fundamental attributes of advertising markets, such as thickness (or competitiveness), advertising returns and multi-homing.

[5]The relevance of repetition on the same consumers for the reach of digital advertising campaigns is testified by the emphasis on *unique* users and impression repetition in Google's own campaign evaluation tools. See `https://support.google.com/google-ads/answer/2472714?hl=en&ref_topic=3123050`.

We consider a setting with two publishers and an intermediary. Consumers either visit one publisher or multi-home, being exposed to one ad impression per visit. Each consumer is characterized by a type, that corresponds to an advertising market, i.e. a given set of advertisers that intend to reach (only) this specific consumer. We characterize the advertising markets by their thickness (i.e., the number of advertisers) and marginal returns to advertising. Ad impressions are allocated to advertisers via auctions, either directly by the publishers or by the intermediary if the publishers outsource their ad inventories. The publishers choose whether to outsource at onset of the game, based on the advertising revenue that the intermediary can generate. If the publishers outsource, the intermediary centralizes the allocation of ads and can gather more accurate information about consumers than the publishers, which allows to sell a greater volume of targeted ads. Moreover, the intermediary observes whether a consumer is multi-homing and thus receives ad impressions on different publishers. This information enables the advertisers to manage consumers' frequency of exposure to their messages across different outlets.

In the baseline model, we assume there are diminishing returns to advertising to the same consumer (e.g., because each impression increases the probability that the consumer is already informed about the advertised product). The possibility that their ad is repeated multiple times on multi-homers reduces the willingness to pay by advertisers. Consequently, disclosing information about the browsing history of a consumer when auctioning an impression entails a key trade-off for the intermediary: advertisers that are not already reaching the consumer with another impression raise their bid, but the others lose interest. When the consumer multi-homes, therefore, disclosing the above information raises the equilibrium price of impressions if and only if the advertising market is thick enough. Otherwise, the price of impressions drops sharply because there are few bidders in the auction. Hence, in thin advertising markets, the intermediary chooses not to be transparent to the advertisers about whether their impressions end up on already exposed consumers, conflating repeated and not repeated impressions.

The value that the publishers obtain from outsourcing the sale of ads depends crucially on the competitiveness of the advertising markets that their audience belongs to. Outsourcing is more likely to be profitable to the publishers if their audience belongs to thick advertising markets. The reason is that in such markets the intermediary can not only sell a higher volume of targeted impressions (given its superior ability to profile consumers), but also sell them at a higher price, by disclosing information that allows the advertisers to avoid repetition.

However, if the publishers' audience belongs primarily to thin advertising markets, outsourcing to the intermediary may reduce the equilibrium price of targeted impressions. The reason is that the probability of repetition on multi-homers can increase with the intermediary. To see why, recall that the intermediary does not provide the advertisers with information that allows to avoid repetition in thin markets, and consider that it is sufficient for the intermediary to profile a multi-homer on one outlet to expose her to targeted ads on both outlets. Without the intermediary, instead, the same ad can hit a multi-homer twice only if she is profiled by both publishers independently. Consequently, the publishers are better off not outsourcing to the intermediary if they tend to serve thin advertising markets and if their ability to profile consumers independently is not exceedingly large.

Outsourcing to the intermediary affects the size and distribution of surplus in the advertising market. In our model, the publishers outsource in equilibrium if and only if the intermediary increases the revenue generated from their ad inventory. However, the advertisers are worse off when the publishers outsource, because competition among advertising outlets weakens. We also find that the intermediary does not necessarily increase the total surplus in the advertising market, despite its superior ability to target consumers and allocate ads. This is because the intermediary's choice to limit transparency in thin markets does not allow the advertisers to avoid wasteful repetition of targeted impressions on multi-homers. By contrast, when the publishers do not outsource, advertisers single-home in equilibrium, so there is no repetition. As a result, outsourcing to the intermediary may reduce the total surplus in the advertising market.

We evaluate the potential implications of two different kinds of regulation, both of which have been recently discussed in policy circles. First, we assume the regulator imposes transparency to the intermediary, by forcing it to disclose all the information at its disposal to the advertisers (see, e.g., the proposed remedies in CMA, 2020, p. 395). As explained above, with full disclosure the price of ads in thin markets drops substantially. Hence, the intermediary and the publishers lose while advertisers benefit. This regulation may either increase or reduce total surplus: on the one hand, transparency raises the efficiency of targeted ads. On the other, if the regulation induces the publishers not to use the intermediary, the industry cannot benefit from the intermediary's superior ability to target ads.

Secondly, we consider the implications of privacy policy, such as the European Union's General Data Protection Regulation (GDPR) and California's Consumer Privacy Act. We model the implications of this policy as a restriction on the intermediary's ability to profile consumers and to share information about their browsing behavior. The regulation would reduce the profits earned by the intermediary in thick markets, and thus make the publishers less likely to outsource.[6] Interestingly, this outcome can be beneficial to the advertisers, as competition among advertising outlets increases. Moreover, since repetition may increase with the intermediary, the regulation may result in higher total surplus in the advertising market, particularly if the publishers serve thin advertising markets and the intermediary's ability to profile consumers is not much greater than the publishers'.

In the final part of the analysis, we present several extensions to the baseline model. These include reserve prices in advertising auctions, increasing returns to advertising, conflation of impressions by consumer preferences and heterogeneous advertising returns within each advertising market.

The remainder of the paper is organized as follows. Section 2 discusses previous related literature. Section 3 presents the model, that we solve in Section 4. We provide a welfare analysis and an evaluation of the effects of different regulatory policies in Section 5. Section 6 provides an overview of the extensions. We discuss the policy and managerial implications of our analysis in Section 7 and 8, respectively. Section 9 concludes. Proofs of lemmas and propositions not given in the text are in the Appendix.

2 Literature

This paper contributes to several strands of literature. A recent literature studies the incentives of platforms to disclose information about consumers to advertisers. Bergemann and Bonatti (2015) consider an online data provider that sells information to advertisers targeting specific consumers. The authors study the demand for this information by advertisers and the optimal pricing policy of the data provider. Xiang and Sarvary (2013) consider competing data providers and analyze how competition among advertisers affects the price of data. Gu et al. (2019) show that the nature of competition between data brokers depends on the value of merged versus separate data-sets. Unlike these papers, we focus on platforms that sell advertising impressions, as well as information, to advertisers. de Cornière and de Nijs (2016) consider an advertising-financed publisher that has detailed information about consumers. When disclosed to an advertiser, this information puts the advertiser in a monopolist position when selling

[6]This finding is in line with evidence documenting a reduction in websites' use of third-party web technologies (including intermediaries) after the GDPR (e.g. Peukert et al., 2020; Johnson and Shriver, 2020).

its products. The publisher thus discloses the information if and only if there is sufficient competition among advertisers, which enables the publisher to capture the ensuing monopoly rents.[7] Although information disclosure by platforms is central in our paper as well, we focus on an intermediary that connects publishers to advertisers and study how the managing of information by the intermediary affects the structure of the market, i.e. whether the publishers choose to use an intermediary to sell their ad inventories.

Few previous papers have considered the disclosure of information by third-party platforms that connect advertisers to publishers and how this disclosure affects the market for ad impressions. Levin and Milgrom (2010) discuss informally the incentives by platforms to conflate ad impressions, referring to the case where "similar but distinct products are treated as identical in order to make markets thick or reduce cherry-picking". We model the relation between the attributes of advertising markets (e.g., thickness and multi-homing) and the platform's choice of information disclosure. Furthermore, we embed this relation in a model that studies the interplay between digital publishers, the intermediary platform and advertisers. Ghosh et al. (2015) consider a setting with digital publishers and a third-party platform that collects information about consumers in the publishers' audience. They show that the platform can use the information gathered about consumers visiting a publisher to enable the advertisers to target the same consumer on other publishers at a lower price (information leakage). This result is somewhat related to our findings, although the mechanism we focus on is different. Furthermore, we explore the conditions such that the publishers rely on the third-party platform to sell their ad inventories and the ensuing effects on the volume and distribution of surplus in the advertising market.

Only a limited number of papers study how large intermediaries affect key outcomes in the online advertising market. Marotta et al. (2021) study the welfare implications of consumer data sharing in targeted advertising, evaluating the role of an advertising intermediary as well the effects of restricting data sharing. The authors model the effects of information on competition among the advertisers on the product market, but do not consider digital publishers. Sharma et al. (2019) consider an asymmetric duopoly of ad networks. The authors focus on the analysis the contractual arrangements between publishers and ad networks, and on how the publishers sort across the different networks. Furthermore, they study the implications of data frictions (such as data sharing regulations or digital ad taxes). However, they do not consider the

[7]There is also a literature studying the effects of targeted advertising on consumers and product markets without considering platforms. See, e.g., Johnson (2013) and Karle and Reisinger (2019).

allocation of ads and the disclosure of information by the ad networks. D'Annunzio and Russo (2020) consider an ad network that centralizes the sale of ads in presence of multi-homing by consumers and advertisers, focusing on how the ad network affects the equilibrium quantity of ads and on the implications of consumers' avoidance of third-party tracking. Unlike the above papers, we explore the type and the extent of information disclosed by a dominant intermediary to advertisers, and how this decision relates to market fundamentals. We can therefore study the interplay between transparency and the choice of digital publishers to rely on intermediary for the sale of ads. Furthermore, we focus on how the intermediary affects the allocation and price, rather than the quantity of ads displayed by digital publishers. In a recent paper, Peitz and Reisinger (2020) show that an ad network that tracks consumers across outlets can have a negative effect on the equilibrium price of impressions on multi-homers, because advertisers do not want to reach the same consumer too many times. We take a different approach to modeling advertising auctions, and find a similar outcome only if advertising markets are thin, in particular when the intermediary fully discloses information to advertisers. However, we let the intermediary choose the degree of information disclosure, and show that full disclosure is suboptimal in thin advertising markets.

Our analysis also relates to the literature on information disclosure in auctions. Ganuza and Penalva (2010) show that disclosing more information about the object for sale increases some buyers' willingness to pay, but may also increase the buyers' information rents. As a result, they find that disclosing information is not profitable when there are few bidders.[8] Bourreau et al. (2017) find similar results in a setting with competing auctioneers. Hummel and McAfee (2016) analyze the incentives to disclose information for different auction formats. Rafieian and Yoganarasimhan (2021) show theoretically and empirically an ad network's revenues may decrease when user information is used to target advertising. We confirm these fundamental insights in a context where platforms choose whether to disclose not only information on consumers' characteristics, but also their browsing history and exposure to ads on multiple outlets. Moreover, we analyze how the intermediary's choice to disclose information affects publishers' choice to outsource the sale of ads.

Finally, in a broader perspective, our analysis relates to the literature that analyzes the incentives of platforms to deter advertising click fraud and to disclose related information to advertisers when auctioning ad impressions. Wilbur and Zhu (2009) show that the incentives to

[8] Chen and Stallaert (2014) obtain a similar result in a setting where a publisher decides whether to use behavioral information when selling ads.

disclose click fraud by a platform that sells search ads increase with the competitiveness of the advertising market, which is consistent with our findings.

3 The Model

3.1 Setup

We consider three platforms: two digital publishers, $i = 1, 2$, and an intermediary, IN. The publishers provide free content to consumers and sell ad impressions to advertisers, either directly or via IN.

Consumers. There is a unit mass of consumers. Let m be the (exogenous) quantity of multi-homers and $\frac{1-m}{2}$ the quantity of single-homers on each publisher. Each consumer is characterized by a type θ, i.e. a set of characteristics such as interests (in, e.g., culture, sports, news, etc.), geographic location and demographics, which determine the consumer's relevance to the advertisers. We let θ be distributed among consumers according to the uniform distribution $G(\theta)$ with support $[0, 1]$ (so that $g(\theta) = 1$, $\forall\theta$) and assume this distribution is independent of the allocation of consumers on the publishers. The quantity of impressions that each publisher exposes a consumer to is given and set to one. Therefore, single-homing consumers receive one impression and multi-homers receive two impressions in total.

Advertisers. Ads inform consumers about products. Let $k(\theta)$ be the set of advertisers that intend to reach type-θ consumers. An ad generates a positive return to an advertiser in $k(\theta)$ only when informing a type-θ consumer. The return from informing a consumer of a different type is zero. We refer to each type θ as a separate *advertising market*, because only advertisers in $k(\theta)$ intend to reach consumers of that type. We assume an advertiser may belong to more than one market, but each consumer belongs to one and only one market (for instance, the consumer can only be in one location at a time). We say that an ad impression is *targeted* if the advertisers are made aware of the consumer's θ by the platform that sells the impression. Given a continuum of types, the expected return from informing a consumer unconditional on θ is zero. Hence, non-targeted impressions are worthless to the advertisers.

Each advertising market is characterized by two parameters. First, the number of advertisers, n, that we refer to as the market's *thickness*. We refer to markets such that $n = 2$ as "thin", to markets such that $n = 3$ as "intermediate" and to markets such that $n \geq 4$ as "thick".[9] Let x, y and $1 - x - y$ be the share, respectively, of thin, intermediate and thick markets. At times, we will refer to this parameter also as the market's competitiveness. The thickness of advertising markets may depend, for example, on the degree of concentration of supply for the product or service in question.

The second parameter characterizing advertising markets is the advertisers' marginal return from informing a relevant consumer. We denote this return by v and assume it is distributed according to a distribution $F(v)$ with smooth density, $f(v)$, and support $[0, v_H]$. We denote by $\overline{v} \equiv \int_0^{v_H} v\, dF(v)$ the mean of this distribution. The advertising return may vary across markets, for example, because products sell at different margins, which in turn determine the profitability of marginal sales. In the baseline model, v is homogeneous among advertisers within a market.

The distributions of n and v across markets are independent and common knowledge. We assume the number of advertisers in each market, n, is observable. However, v private information of each advertiser.[10]

We assume impressing a consumer with one ad is enough to inform her. Sending the same ad twice to the consumer is thus wasteful. Therefore, as will become clear, an advertiser's willingness to pay for an ad impression depends on whether the consumer (i) belongs to the relevant market and (ii) may receive the same ad while visiting another publisher.

Publishers. The publishers earn all of their revenue from the sale of ads and incur no costs. If a publisher does not outsource to IN, it sells each impression in a second-price auction. All auctions take place simultaneously. If there is more than one winning bid for an impression, the publisher allocates the impression randomly to one of the top bidders.

Based on the information collected on its website (e.g. with first-party cookies), each publisher generates a signal σ for each consumer, that conveys information about the consumer's type. This signal is perfectly informative (i.e., $\sigma = \theta$) with probability q and is pure noise otherwise. When $\sigma = \theta$, we say that the publisher *profiles* the consumer. We assume σ is i.i.d. across consumers and publishers. The publishers always reveal σ to the advertisers when selling an impression (no advertiser would place a positive bid otherwise). However, each publisher

[9] The definition of market thickness is relative to the maximum quantity of impressions available on each consumer, which is equal to two in our model.

[10] The platforms can uncover n quite easily. Suppose a platform reveals the consumer's θ and ask the advertisers to register in order to bid in the ensuing auction. Given our assumptions, the advertisers would register if and only if they belong to the corresponding market.

does not observe whether a consumer visits -and is thus exposed to ads on- the other publisher.

Intermediary. IN makes to each publisher $i = 1,2$ a simultaneous take-it-or-leave-it offer specifying a transfer T_i for the publisher's ad inventory. If a publisher outsources to the intermediary, the latter sells each available impression in simultaneous second-price auctions. Similarly to the publishers, the intermediary generates a signal, σ^{IN}, for each consumer (i.i.d. across consumers), that is perfectly informative about the consumer's θ with probability $\tilde{q}$ and uninformative otherwise. If only one publisher outsources, IN obtains the same information about consumers as the publisher and so profiles each consumer with the same probability, $\tilde{q} = q$. If both publishers outsource, however, the intermediary can track consumers on both outlets, which enables it to gather superior information. That is, the intermediary profiles each consumer with higher probability, i.e. $\tilde{q} > q$. In addition, the intermediary can observe which publishers a consumer visits and which ads she is exposed to during the visit.

In the baseline model, we assume the intermediary reveals σ^{IN} to the advertisers when selling an impression. However, if it manages the ad inventory of both publishers, the intermediary must decide whether to disclose information about consumers' browsing behavior. Specifically, for each impression, the intermediary chooses among two information disclosure regimes. In the *partial disclosure* regime (PD), the intermediary only reveals σ^{IN} to the advertisers. In the *full disclosure* regime (FD), the intermediary also reveals whether the consumer visits a different publisher and thus receives another impression there. This piece of information allows the advertisers to bid for each impression opportunity knowing whether they are already acquiring another impression on the same consumer. This is valuable knowledge since a repeated impression is wasted, and thus reduces the efficiency and the reach of the ad campaign.

Timing. We summarize the model by describing the timing of moves:

1. IN offers T_i to publisher $i = 1,2$ in exchange for the publisher's ad inventory. Each publisher accepts or refuses.
2. Consumers visit the publishers and all impressions opportunities are generated.
    (a) If one or no publisher outsourced, the platforms profile each consumer with probability q;
    (b) If both publishers outsourced, the intermediary profiles each consumer with probability $\tilde{q}$.

3. If both publishers outsourced, IN chooses among FD and PD for each impression. The platforms (publishers or intermediary) sell each available impression in simultaneous second-price auctions. The advertisers bid for each impression simultaneously.

4. Impressions take place and all payoffs are realized.

3.2 Discussion of the setup

We briefly discuss some of our assumptions. We assume the intermediary is a monopolist consistently with stylized facts of the market for "open display" digital ads. Impressions in this market are either sold directly by the publishers or, more commonly, via an intermediary. Google is by far the largest intermediary, with a market share estimated to be as high as 90% (CMA, 2020). We focus on second-price auctions as the mechanism for allocating the ad impressions because most digital platforms (e.g., Google Ad Sense) run auctions that are based on this format.[11] We ignore reserve prices in the baseline model, but allow the platforms to introduce such prices in an extension (see Section 6.1).

In keeping with the literature on advertising-financed platforms (e.g., Anderson and Coate, 2005; Ambrus et al., 2016; Athey et al., 2018), we assume there are diminishing returns to advertising. To economize on notation, we set to zero the marginal return to sending an ad to a consumer more than once. Letting this return be positive would have no fundamental impact on our results. In an extension (see Section 6.3), we assume increasing returns to advertising on the same consumer (as in, e.g., the case of re-targeting). The baseline model also assumes that the return from informing consumers is homogeneous across advertisers within the same market. We relax this assumption in Section 6.4.

By normalizing the number of impressions received by a consumer on each publisher to one, we effectively rule out the possibility that a consumer receives the same ad multiple times on the same publisher. This possibility is typically not a major concern in realty, since digital publishers have the means to manage the frequency of ads within their own domains, e.g. using first-party cookies. We thus focus on cross-outlet repetition, which is a more relevant challenge in the management of ad campaigns (Athey et al., 2018).

To streamline the exposition, we assume the signal σ, which informs the intermediary and the publishers about the products or services that the consumer is interested in, is either per-

[11] More precisely, AdSense runs generalized second price auctions. See `https://support.google.com/adsense/answer/160525/about-the-ad-auction?hl=en`.

fectly informative (i.e., not noisy) or completely uninformative. If we allowed for some noise in the signal (e.g., allowing the signal to show an incorrect value of θ), such noise would reduce the advertisers' willingness to pay for a targeted impression, since the impression could end up on the "wrong" consumer. However, the structure of the equilibrium bids, and the allocation of ads would not be affected. We also assume that, when available, the platforms always disclose information about the consumer's type, θ, to the advertisers. Retaining this information is not profitable, since advertisers do not care for impressions that are not targeted given the large number of consumer types. However, as will become clear in the course of the analysis, the intermediary may have an incentive to increase the number of bidders for impressions in some markets. Conceivably, to achieve this objective, the intermediary may introduce some uncertainty about the type of consumer associated to some impressions. We evaluate this possibility in a robustness check (see Section 6.2).

We model the contractual arrangements between the publishers and the intermediary in a stylized way: the only condition for the publishers to outsource to the intermediary in equilibrium is that the intermediary generates greater total advertising surplus than the publishers. Given our assumptions, the intermediary captures all this extra surplus. Assuming a different split of this surplus between the parties (i.e., letting the publishers have greater bargaining power) would not change our main results.

Finally, we assume the intermediary can apply its superior targeting technology ($\tilde{q} > q$) only if it can gather data about consumers from both publishers. Alternatively, one could assume that the probability the intermediary profiles consumers is $\tilde{q}$ regardless of how many publishers outsource. This assumption would make the analysis slightly more involved without yielding different results (details are available upon request).

4 Solving the model

We first study the equilibrium that emerges if the publishers do not outsource to the intermediary, and then consider the equilibrium in the case where they do outsource. In each case, we first characterize the equilibrium bidding strategies of the advertisers (conditional on the characteristics of their market and the information available). We then determine the optimal information disclosure-regime for the intermediary and the equilibrium prices and revenue earned by the platforms.

At the auction stage, we consider only equilibria in undominated strategies, where advertisers bid truthfully. Conditional on the publishers not outsourcing at Stage 1, the game admits multiple equilibria for some parameter values. We restrict attention to equilibria such that no advertiser can maintain the same level of profit but acquire a larger volume of impressions by deviating. As we clarify below, this refinement comes at no significant loss of generality.

4.1 No intermediary

We begin by characterizing the equilibrium that emerges conditional on no publisher outsourcing to IN. Note that if only one publisher outsourced, the equilibrium would be identical, because the intermediary would be exactly in the same position as the publisher, i.e. have the same probability of profiling consumers, q, and unable to track consumers across outlets.

4.1.1 Advertisers' willingness to pay for impressions

Consider an advertiser $j \in k(\theta)$ and let w_{ij} be the advertiser's willingness to pay for a targeted impression delivered by publisher i. The advertiser anticipates a positive return from informing the consumer but, since repeated impressions are wasteful, the value of w_{ij} depends on the probability the consumer is already informed while visiting the other publisher, $i' \neq i$. This probability is zero if the consumer visits i only, in which case the impression is worth v. A multi-homer, however, may be exposed to the same ad on i'. The likelihood of repetition depends on three factors. First, the likelihood that an impression falls on a multi-homer. Given each publisher sells $(1+m)/2$ impressions in total, this probability is $\frac{m}{\frac{1+m}{2}}$ (whereas the probability of falling on a single-homer is $\frac{\frac{1-m}{2}}{\frac{1+m}{2}}$). Second, the probability that the other publisher profiles the same multi-homer, q (the consumer would otherwise receive an impression from the same advertiser with zero probability). Third, the likelihood of repetition depends on the probability that the targeted impression the multi-homer receives on the other publisher is from advertiser j. This probability is captured by $S_{i'j}$, i.e. the share of targeted impressions that advertiser j acquires on i'. Hence, we have

$$w_{ij} = v\left(\frac{\frac{1-m}{2} + m(1 - qS_{i'j})}{\frac{1+m}{2}}\right) = v\left(1 - q\frac{2mS_{i'j}}{1+m}\right), \quad i = 1, 2; i' \neq i; j \in k(\theta). \tag{1}$$

Observe that w_{ij} decreases in $S_{i'j}$: due to consumers multi-homing, each advertiser perceives the impressions on the two publishers as substitutes. Finally, recall that the advertiser's willingness to pay for an impression on a consumer whose θ is unknown is zero.

4.1.2 Market equilibrium without the intermediary

Given the bids placed for impressions generated by publisher $i' \neq i$, it is a dominant strategy for each advertiser to "truthfully" place bids equal to w_{ij} for each targeted impression (on consumers in her own market) generated by publisher i. The advertiser bids zero for all the other impressions on that publisher. Thus, as anticipated, no publisher could gain by not revealing σ when selling an impression.

The substitutability of impressions on different publishers is key to characterize the equilibrium bidding strategies of the advertisers for the targeted impressions available in the market. As expression (1) suggests, w_{ij} depends on how successful the advertiser's bids are on the other publisher. Consider two advertisers, a and b, in the same market. Advertiser a outbids b for every targeted impression on publisher i if and only if b acquires a larger share of targeted impressions on publisher i' than a. Consequently, there can only be two sets of bidding strategies in equilibrium. The first is such that all advertisers in a market single-home, i.e. each places winning bids on one publisher only. The second possible set of equilibrium strategies is such that all advertisers in a market place equal bids (given by (1)) for each targeted impression and thus acquire identical shares of such impressions from each publisher (i.e. $S_{ij} = 1/n, \forall i, j$). However, in this candidate equilibrium, the net payoff of any advertiser would be zero because, in a second-price auction, each advertiser pays exactly w_{ij} for each impression on both publishers. Hence, any advertiser can deviate profitably by bidding v for all impressions on one publisher and zero on the other. We can therefore make the following claim (see Appendix A.1 for the proof):

Lemma 1. *If neither or only one of the publishers outsources to the intermediary, advertisers in each market single-home on different outlets.*

This finding is in line with Athey et al. (2018), who show that, with overlapping audiences among multiple publishers, an advertiser multi-homes only if its return from informing consumers is sufficiently larger than that of its competitors.

We are now in a position to characterize the equilibrium bids, the ensuing allocation of impressions and the profits earned by the publishers in the case where neither outsources to the intermediary. In the following, we distinguish advertising markets only according to their thickness

Table 1: Equilibrium bids and publisher revenues without the intermediary

	Publisher 1	Publisher 2
Thin markets ($n = 2$)		
adv.a	v	$v(1 - \frac{2mq}{1+m})$
adv.b	$v(1 - \frac{2mq}{1+m})$	v
$R_{i,n=2}$	$\frac{vq}{2}(1 + m(1 - 2q))$	$\frac{vq}{2}(1 + m(1 - 2q))$
Intermediate markets ($n = 3$)		
adv.a	v	$v(1 - \frac{mq}{1+m})$
adv.b	v	$v(1 - \frac{mq}{1+m})$
adv.c	$v(1 - \frac{2mq}{1+m})$	v
$R_{i,n=3}$	$\frac{vq}{2}(1 + m)$	$\frac{vq}{2}(1 + m(1 - q))$
Thick markets ($n = 4$)		
adv.a	v	$v(1 - \frac{mq}{1+m})$
adv.b	v	$v(1 - \frac{mq}{1+m})$
adv.c	$v(1 - \frac{mq}{1+m})$	v
adv.d	$v(1 - \frac{mq}{1+m})$	v
$R_{i,n=4}$	$\frac{vq}{2}(1 + m)$	$\frac{vq}{2}(1 + m)$

because, as will become clear, the allocation of impressions does not depend on the realization of v. Furthermore, to ease exposition we just present one of the symmetric equilibria in each market, since the equilibrium prices and profits for all parties are identical.

Thin markets. Consider a market such that $n = 2$. Given Lemma 1 and expression (1), advertiser a (resp. b) bids v for each targeted impression on publisher 1 (resp. 2), and acquires all such impressions. Furthermore, a (resp. b) bids the following for each targeted impression on 2 (resp. 1)

$$b_{n=2} = v\left(1 - q\frac{2m}{1+m}\right). \tag{2}$$

To understand this expression, consider that, given $S_{1a} = 1$, advertiser a informs all the consumers in this market profiled by publisher 1. Hence, any targeted impression on publisher 2 is worthless to the advertiser if it hits a multi-homer profiled by 1 already. Given second-price auctions, (2) is also the equilibrium price of each impression on both publishers. Each publisher thus earns

$$R_{i,n=2} = \frac{vq}{2}(1 + m(1 - 2q)), \quad i = 1, 2. \tag{3}$$

This expression shows that, when the advertising market is thin, the publishers cannot extract all the advertising surplus. We summarize the equilibrium bids and profits in the top panel of Table 1.

Intermediate markets. Consider a market such that $n = 3$ and focus on the equilibrium market configuration such that two advertisers (say, a and b) single-home on publisher 1, whereas advertiser c single-homes on 2. The winning bids for targeted impressions on each publisher equal v. Furthermore, advertisers a and b bid the following for each targeted impression on publisher 2:

$$b_{n=3} = v\left(\frac{\frac{1-m}{2} + m(1-\frac{q}{2})}{\frac{1+m}{2}}\right) = v\left(1 - \frac{mq}{1+m}\right). \tag{4}$$

This expression is derived from (1), noting that a and b each win half the available targeted impressions on publisher 1, so $S_{1a} = S_{1b} = 1/2$. Instead, advertiser c places a bid equal to (2) for each targeted impression on publisher 2, since $S_{2c} = 1$. Consequently, the equilibrium price of these impressions on publisher 1 is v, whereas it equals (4) on publisher 2. Publishers 1 and 2 thus earn, respectively

$$R_{1,n=3} = \frac{vq}{2}(1+m) \quad \text{and} \quad R_{2,n=3} = \frac{vq}{2}(1+m(1-q)). \tag{5}$$

The publisher that ends up serving two advertisers extracts the full advertising surplus, but the other publisher does not. We summarize the bids and profits of the publishers in the middle panel of Table 1.

Thick markets. Consider now a market such that $n = 4$. Given Lemma 1, we focus on the equilibrium such that two advertisers (say, a and b) single-home on publisher 1 while the other two (say, c and d) single-home on 2. The winning bids on each publisher equal v and each advertiser receives half the targeted impressions supplied by the respective publisher.[12] Given (1) and $S_{ij} = 1/2$, $\forall i, j$, advertisers a and b (resp. c and d) place a bid equal to (4) for each impression on publisher 2 (resp. 1). The equilibrium price of impressions on both publishers is v. Hence, each publisher earns

$$R_{i,n=4} = \frac{vq}{2}(1+m), \quad i = 1, 2. \tag{6}$$

When advertising markets are thick, therefore, the publishers extract the full value of targeted impressions from the advertisers. We summarize the bids and profits of the publishers in the

[12] Equilibria with more than $n/2$ advertisers on the same publisher would not satisfy our requirement that no advertiser can maintain the same level of profit but acquire a larger volume of impressions by deviating. Suppose three advertisers single-home on publisher 1 and one advertiser single-homes on 2. All advertisers bid v for each relevant impression on the respective publisher. Hence, if one of the advertisers on 1 deviates and single-homes on 2, it gets the same net surplus (zero) but a larger volume of impressions.

bottom panel of Table 1. The outcome is similar for markets such that $n > 4$.

We can now compute the aggregate profits earned by the publishers. Given the profits in Table 1 and recalling that v and n are independently distributed, we get the following:

Lemma 2. *If neither publisher joins the intermediary, the equilibrium is such that the publishers collect the following total revenue*

$$\begin{aligned} R_1 + R_2 = 2x \left[\frac{\overline{v}q}{2}(1 + m(1 - 2q))\right] + y \left[\frac{\overline{v}q}{2}(1 + m) + \frac{\overline{v}q}{2}(1 + m(1 - q))\right] + \\ + 2(1 - x - y) \left[\frac{\overline{v}q}{2}(1 + m)\right] = \overline{v}q \left[1 + m \left(1 - q \left(2x + \frac{y}{2}\right)\right)\right]. \end{aligned} \tag{7}$$

4.2 Intermediary

We now consider the equilibrium that takes emerges conditional on both publishers outsourcing to the intermediary.

4.2.1 The willingness to pay for impressions by advertisers

Let w_j^{PD} and w_j^{FD} be advertiser j's willingness to pay for a relevant impression sold by the intermediary under the Partial and Full Disclosure regime, respectively.

Partial Disclosure. Suppose the intermediary adopts PD for impressions in a given advertising market. Recall that, under PD, the intermediary conflates all the targeted impressions in that market by not distinguishing between those that fall on single-homers from those that fall on multi-homers. Hence, just like when the publishers sell the impressions directly, the advertisers are unaware of whether the consumer is hit by two, possibly identical, impressions. Each targeted impression is worth v to advertiser j if the consumer is a single-homer (probability $(1 - m)/(1 + m)$). If the consumer is a multi-homer (probability $2m/(1 + m)$), she already receives the same impression with probability S_j^{PD}, i.e. the share of targeted impressions acquired by j. Hence, the impression is worth $v(1 - S_j^{PD})$ to the advertiser and we have

$$w_j^{PD} = v \left(\frac{1 - m + 2m(1 - S_j^{PD})}{1 + m}\right) = v \left(1 - \frac{2mS_j^{PD}}{1 + m}\right), \quad \forall j \in k(\theta). \tag{8}$$

The impression is instead worth zero if it is not targeted.[13]

An important observation can be made by comparing w_j^{PD} to the willingness to pay for relevant impressions when the publishers do not outsource, given in (1). The comparison reveals that, for a given share of targeted impressions acquired on a platform, an advertiser may face a *higher* probability of repetition when the intermediary adopts PD than when the publishers do not outsource. In the latter case, a multi-homer can receive the same ad twice only if she is identified by *both* publishers independently. By contrast, profiling the consumer on a single outlet is sufficient for the intermediary to expose her to two targeted impressions from the same market. As a result, the price of targeted impressions in a thin market can be higher when sold by the publishers independently than by the intermediary. We return to this point below.

Full disclosure. Suppose now the intermediary adopts FD for impressions in a given market. In this regime, the advertisers bid for each targeted impression knowing whether the consumer is a single- or a multi-homer, i.e. whether another impression is available on the same consumer. Furthermore, given the bids of the other advertisers, in equilibrium each advertiser can anticipate whether it is already acquiring this other impression. If the consumer is a single-homer, there is no chance of repetition. Hence, each advertiser's willingness to pay for a targeted impression on a single-homer is $w_j^{FD} = v$. If the consumer is a multi-homer, however, each advertiser is willing to pay $w^{FD} = v$ for one of the two targeted impressions available if and only if it is not already placing a winning bid for the other impression. Otherwise, the impression is certainly repeated and thus worthless.

4.2.2 Information disclosure by the intermediary and market equilibria

Expression (8) shows that, under the PD regime, the bigger the share of targeted impressions in a given market that an advertiser expects to acquire, the smaller its willingness to pay for each such impression. This aspect is key to characterize the bidding strategies under PD. In a second-price auction, each advertiser in a given market bids w_j^{PD} for each targeted impression, so there cannot be equilibrium strategies such that one or more advertisers outbid the rivals within the same market. To see why, suppose an advertiser outbids the others and thus acquires a larger share of targeted impressions. Given (8), the advertiser's bid for each impression

[13] In principle, the intermediary could avoid the risk of repetition under PD by revealing a profiled multi-homer's θ for only one of the two impressions available. In our model, this choice is essentially the same as selling only one of these impressions. The probability of repetition in equilibrium is low enough to make this alternative course of action not profitable.

would be smaller than that of other advertisers' in the same market, which is a contradiction. Consequently, the equilibrium bidding strategies must be such that all advertisers in a market bid (8) for each relevant impression and receive an equal share of such impressions, so that $S_j^{PD} = 1/n, \forall j \in k(\theta)$.

Lemma 3. *If the intermediary adopts Partial Disclosure, in equilibrium all advertisers in a market bid the following for each targeted impression:*

$$w^{PD} = v\left(1 - \frac{2m}{n(1+m)}\right). \tag{9}$$

Consider now the FD regime. As we established above, no advertiser is willing to acquire more than one of the two targeted impressions available on a multi-homer. Hence, in a second price auction, each advertiser is willing to bid the full value of an impression as long as they can be certain it will not be wastefully repeated. However, the advertiser will bid zero for any additional impression on the same consumer.

Lemma 4. *If the intermediary adopts Full Disclosure, in equilibrium all advertisers in a market bid v for each targeted impression that falls on a single-homer. Furthermore, they bid v for one of the two targeted impressions that fall on a multi-homer, and zero for the other.*

With FD, the intermediary enables the advertisers to cap the frequency whereby their message hits the same consumer. FD thus boosts the advertisers' willingness to pay for impressions that certainly do not fall on a consumer that they already reach. However, FD also trims the number of advertisers willing to place a positive bid for each targeted impression that falls on multi-homers. As we shall now see, this effect can result in a drastic reduction in the equilibrium price of targeted impressions in thin advertising markets.

Thin markets. Suppose the intermediary adopts PD. As claimed above, in a given market each advertiser bids (9) for all relevant impressions. With $n = 2$, this expression simplifies to

$$b_{n=2}^{PD} = v\left(1 - \frac{m}{1+m}\right). \tag{10}$$

The equilibrium price of each impression on identified consumers in that market is equal to $b_{n=2}^{PD}$ as well. Given a total of $\tilde{q}(1+m)$ identified impressions, the revenue collected by the

Table 2: Equilibrium bids for impressions on multi-homers, full disclosure, $n = 2$

	Impression 1	Impression 2
adv.a	v	0
adv.b	0	v

intermediary in that market is therefore

$$R^{PD}_{n=2} = v\tilde{q}. \tag{11}$$

Each impression falling on a multi-homer generates half of its potential value. The reason is that each advertiser acquires half the available targeted impressions in its market, so an impression that hits a multi-homer is repeated and thus wasteful with 50 percent probability.

Suppose now the intermediary adopts FD. Given Lemma 4, in a second-price auction the equilibrium price of impressions on single-homers in a given market is v, whereas the price of the impressions on multi-homers is zero (see Table 2). Full disclosure of information about consumers' browsing behavior (and, implicitly, exposure to ads) effectively reduces the number of advertisers that compete for impressions on multi-homers. If the market is thin, the result is a sharp reduction in the equilibrium price of impressions. The intermediary collects the following revenue in that market

$$R^{FD}_{n=2} = v\tilde{q}\,(1 - m)\,. \tag{12}$$

Comparing (11) and (12), we conclude that, when the advertising market is thin the intermediary's optimal disclosure regime is PD. By conflating impressions on single- and multi-homers, the intermediary preserves the number of bidders for each impression. Despite the loss in the value of impressions due to repetition taking place in equilibrium, PD is advantageous to the intermediary since it avoids the large drop in the price of impressions that would take place with FD.

Intermediate and thick markets. Suppose the intermediary adopts PD for impressions in a given market. The equilibrium when $n \geq 3$ is qualitatively identical to that described for thin markets, except that the share of relevant ads acquired by each advertiser, S^{PD} is $1/n$. Therefore, the equilibrium price of each identified impression in that market is

$$b^{PD}_{n>2} = v\left(1 - \frac{2m}{n(1+m)}\right). \tag{13}$$

and IN earns the following revenue in that market

$$R_{n>2}^{PD} = v\tilde{q}\left(1 + m\left(1 - \frac{2}{n}\right)\right). \tag{14}$$

Suppose now the intermediary adopts FD for impressions in a given market. For each impressions on a multi-homer, there are at least two advertisers willing to bid v. That is, the market is thick enough to generate competition for all impressions on multi-homers, even with FD. As a result, all the identified impressions are sold at their full value, v. This is a radically different outcome than in thin markets. The intermediary earns the following revenue in that market

$$R_{n>2}^{FD} = v\tilde{q}\,(1 + m)\,. \tag{15}$$

It is clear from (14) and (15) that the intermediary's optimal disclosure regime is FD whenever the market is sufficiently thick, i.e. $n \geq 3$.

The above discussion establishes that the intermediary's incentives to disclose information about consumers' browsing behavior depends on the competitiveness of each advertising market. Specifically, when the advertising market is thin, the intermediary discloses information about the consumer's type (targeting the impression), but does not reveal whether the consumer is exposed to ads when visiting a different publisher. Therefore, the amount of information revealed by the intermediary increases with the number of bidders (Ganuza, 2004, Ganuza and Penalva, 2010, Bourreau et al., 2017).

Proposition 1. *When both publishers outsource, the intermediary sells the impressions under Partial Disclosure if and only if the consumer belongs to a thin advertising market.*

We can now compute the intermediary's total revenue. Given that the distributions of v and n are independent, we get the following

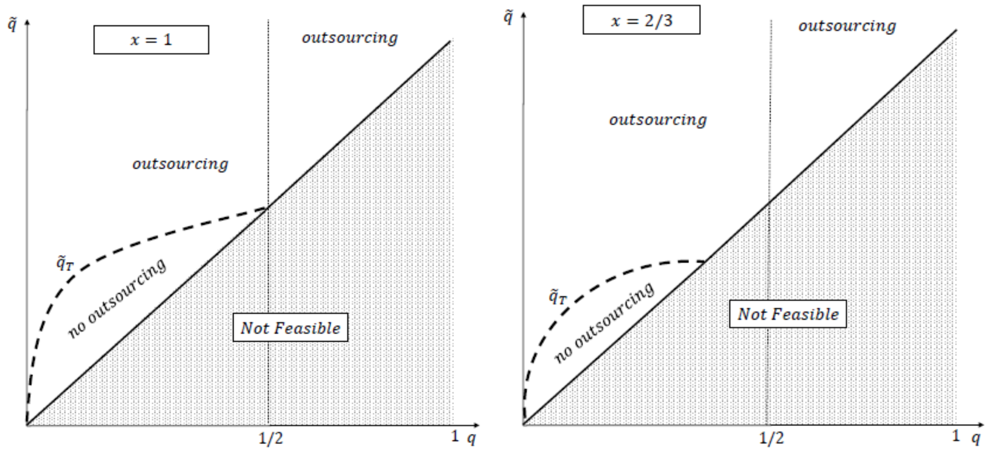
Lemma 5. *When both publishers outsource, the intermediary's total revenue is*

$$R_{IN} = x\,[\overline{v}\tilde{q}] + y\,[\overline{v}\tilde{q}(1+m)] + (1 - x - y)\,[\overline{v}\tilde{q}(1+m)] = \overline{v}\tilde{q}\,[1 + m(1-x)]\,. \tag{16}$$

4.3 The publishers' decision to join the intermediary

We can now study the decision by the publishers to outsource to the intermediary at stage 1. The intermediary can profitably induce each publisher to outsource if and only if the intermediary's

Figure 2: Threshold $\tilde{q}_T$, variation with respect to q.



Note: figures obtained setting $m = 1/2$ and $y = 0$.

total revenue from selling ads (characterized in Lemma 5) is greater than the aggregate revenue of the two publishers when they do not outsource (characterized in Lemma 2).[14] Rearranging (7) and (16), we find that this condition holds if and only if the intermediary's ability to profile consumers is above a threshold that we denote by $\tilde{q}_T$:

$$\tilde{q} \geq \tilde{q}_T \equiv q\left[1 + \frac{m\left(x(1-2q) - \frac{yq}{2}\right)}{1 + m(1-x)}\right]. \tag{17}$$

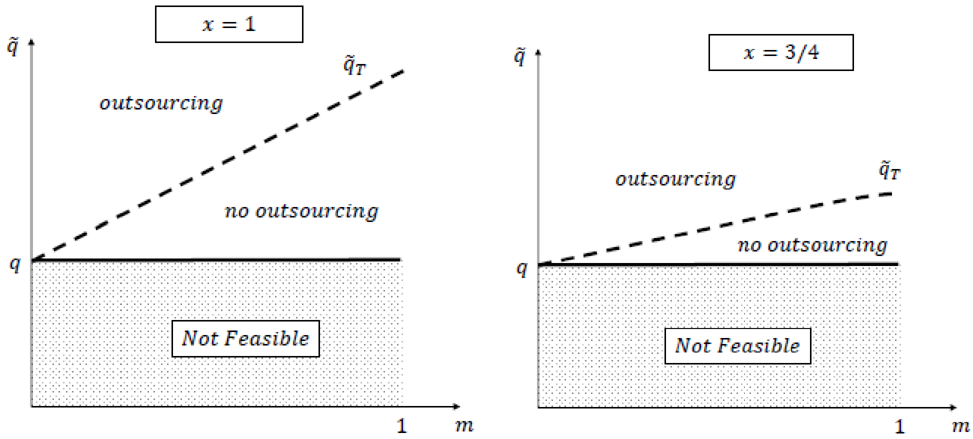
The term in square parenthesis in (17) exceeds one only if $q < 1/2$. Since $\tilde{q} \geq q$ by assumption, the equilibrium must be such that the publishers outsource if $q \geq 1/2$. The following proposition summarizes these findings and describes how $\tilde{q}_T$ varies with the parameters of the model (see Figures 2 and 3 for an illustration).

Proposition 2. *The equilibrium is such the publishers outsource to the intermediary if their ability to profile consumers is large enough, i.e. $q \geq 1/2$. If $q < 1/2$, there exists a threshold $\tilde{q}_T$, defined in (17), such that the publishers outsource if and only if $\tilde{q} \geq \tilde{q}_T$ holds. This threshold increases with q and the share of consumers belonging to thin markets, x. Furthermore, $\tilde{q}_T$ increases with the share of multi-homing consumers, m, if and only if x is small enough.*

Considering some polar cases helps with the intuition. Suppose first that there are no thin markets (i.e. $x = 0$). It is clear from (17) that $\tilde{q}_T \leq q$. The intermediary can profile at least as many consumers as the publishers can, given $\tilde{q} \geq q$. Furthermore, when advertising markets are sufficiently thick ($n \geq 3$), the intermediary can extract the full value from targeted

[14] The scenario where only one publisher outsources cannot arise in equilibrium: as explained at the beginning of Section 4.1, given our assumptions the intermediary cannot raise more revenue than the publisher itself if the other does not outsource.

Figure 3: Threshold $\tilde{q}_T$, variation with respect to m.



Note: figures obtained setting $q = 1/3$ and $y = 0$.

impressions by adopting the FD regime (see (15)). The publishers are instead unable to extract this value in intermediate markets (see (5)), particularly when the share of multi-homers, m, increases. Thus, outsourcing is necessarily advantageous to the publishers when the share of thin advertising markets is small.

We focus now on the opposite polar case, i.e. $x = 1$ (and $y = 0$). Given this condition, $\tilde{q}_T$ boils down to $q(1+m(1-2q))$, which implies that $\tilde{q} < \tilde{q}_T$ can hold only if $q < 1/2$. To understand this condition, recall that the intermediary adopts partial disclosure when the advertising market is thin. Under this regime, although the intermediary can profile more consumers than the publishers can ($\tilde{q} \geq q$) and thus sell a larger volume of targeted impressions, it may have to sell each such impression at a lower price. This difference can be seen by comparing (2) to (9) when $n = 2$. The reason is that, for a given share of impressions acquired, the advertisers may face a higher chance of hitting a multi-homer twice with partial disclosure than when the publishers sell the impressions separately. To see this, consider that if the intermediary profiles a multi-homer, this consumer receives two targeted ads. Without the intermediary, instead, a multi-homer profiled by one publisher receives a targeted ad on the other publisher only if that publisher profiles the consumer as well (which happens with probability q). Thus, when q is below a threshold (specifically, $q < 1/2$), the probability of repetition across publishers is small enough that the expected value of a targeted impression to the advertisers is higher than it would be with the intermediary. The intermediary can thus collect more revenue than the publishers on aggregate only by profiling a sufficiently larger number of consumers, i.e. if and only if $\tilde{q} \geq \tilde{q}_T$ holds. On the other hand, if $q \geq 1/2$ not only the volume, but also the equilibrium price of targeted impressions fall short of the level guaranteed by the intermediary.

The two polar cases considered above also provide the intuition for how the share of multi-homers, m, affects the threshold $\tilde{q}_T$. If the advertising market is sufficiently thick, the intermediary can fully exploit the information collected by tracking consumers across outlets. Thus, each additional multi-homer implies a bigger advantage for the intermediary. However, if the advertising market is thin and $q < 1/2$, the publishers sell each targeted impression at a higher price than the intermediary, and this gap increases with m. Consequently, $\tilde{q}_T$ increases with m if and only if the share of thin markets, x, is sufficiently large.

5 Welfare analysis and the effects of regulating the intermediary

In this section, we evaluate how the intermediary affects publishers, advertisers and the total surplus in the advertising market. Furthermore, we evaluate the implications of two regulatory policies: one that mandates full transparency to the intermediary and one that restricts the amount of information that the intermediary can share with the advertisers, to protect the privacy of consumers.

5.1 How does the intermediary affect total surplus and its distribution?

We consider how the intermediary affects the total surplus generated on the advertising market (i.e., the sum of profits of publishers, intermediary and advertisers) and the distribution of this surplus. Given our assumptions, the necessary and sufficient conditions for the publishers to outsource to the intermediary (Proposition 2) are also necessary and sufficient for the revenue generated by the intermediary to exceed the total revenue the publishers could earn when operating independently. That is, the total revenue on the supply side of the advertising market increases when the publishers use the intermediary. However, despite improving the targeting of impressions, the intermediary makes advertisers worse off, because competition between advertising outlets is weakened by the centralization of the sale of ads. The expressions in the upper panel of Table 1 indicate that each advertiser in thin markets gets the following net surplus per each targeted impression in equilibrium when the publishers do not outsource

$$v - v\left(1 - \frac{2mq}{1+m}\right) = v\frac{2mq}{1+m}. \tag{18}$$

In contrast, the net surplus with the intermediary is zero, because the value of each targeted impression with PD, given in (10), equals the equilibrium price. To continue, the advertisers get zero net surplus in intermediate and thick markets when the publishers outsource to the intermediary, but retain some surplus in intermediate markets otherwise (see Table 1).

Finally, total surplus in the advertising market may either increase or decrease when the publishers outsource to the intermediary. Outsourcing to IN results in a higher volume of targeted impressions ($q \leq \tilde{q}$). On the other hand, by adopting PD in thin markets, the intermediary allows for some wasteful repetition of these impressions in equilibrium, which instead advertisers avoid by single-homing on different publishers when the latter operate independently (see Lemma 1). We thus obtain the following result (see Appendix A.8 for the proof):

Proposition 3. *There exists a threshold* $\tilde{q}_W \equiv q(1 + \frac{xqm}{1+m})$ *such that total welfare increases when the publishers outsource if and only if* $\tilde{q} > \tilde{q}_W$. *This threshold increases with the share of thin advertising markets,* x.

5.2 The effects of transparency regulation

There is a vibrant policy debate regarding regulatory interventions to address the lack of transparency by intermediaries in the display advertising market. In particular, regulators have considered the possibility of mandating greater transparency by intermediaries towards advertisers. See, for example, the proposed draft of the Digital Market Act (European Commission, 2020) and the remedies discussed by the UK Competition and Markets Authority (CMA, 2020, pag. 395).[15] We can study the possible implications of these proposals by means of our model, assuming that regulation imposes full disclosure to the intermediary. Proposition 1 shows that the intermediary may not want to share valuable information with advertisers when auctioning ads when advertising markets are thin. Because the intermediary adopts full disclosure in thick markets regardless of the regulation, we focus on the effect of regulation in thin markets.

Perhaps not surprisingly, our model suggests that mandating greater transparency reduces the profits earned by the intermediary and the publishers, to the benefit of advertisers. In the region of parameters where the publishers outsource to the intermediary despite this regulation, full disclosure makes the price of impressions on multi-homers drop to zero in thin markets. As

[15] In the proposal for the Digital Market Act, the European Commission states that gatekeepers should "provide advertisers and publishers, upon their request and free of charge, with access to the performance measuring tools of the gatekeeper and the information necessary for advertisers and publishers to carry out their own independent verification of the ad inventory".

a consequence, the regulation also tightens the necessary conditions such that the publishers outsource. The advertisers are also better off if the regulation does deter the publishers from outsourcing, because competition between advertising outlets increases.

Consider now the effects of transparency regulation on total surplus. If the publishers outsource even when the regulation is imposed, full disclosure avoids wasteful repetition of targeted impressions on multi-homers in thin markets, which instead would take place when the intermediary adopts partial disclosure. Hence, total surplus increases. However, if the regulation induces the publishers not to use the intermediary, the effect on welfare can be either positive or negative (the conditions are the same as described in Proposition 3): there are fewer targeted ads without the intermediary, but also less repetition of targeted ads because the advertisers single-home in equilibrium. We summarize the results in the following proposition (see Appendix A.9 for the proof).

Proposition 4. *Regulation imposing full disclosure is detrimental to the supply side of the advertising market (publishers and intermediary), but beneficial to the advertisers. Welfare increases if the publishers use the intermediary despite the regulation. Otherwise, welfare decreases if and only if $\tilde{q}$ exceeds the threshold $\tilde{q}_W$ characterized in Proposition 3.*

5.3 The effects of privacy regulation

We now consider some possible implications of privacy regulation, such as the European Union's GDPR (European Parliament, 2016), and similar laws adopted in some US States (e.g., California), Chile, Japan, Brazil, and South Korea. These laws affected the ability of digital intermediaries to share with advertisers information about consumers' browsing behavior.[16] In our model, the effects of privacy policy can be captured as a requirement to adopt PD indiscriminately, i.e. not to share with the advertisers information about which publishers a consumer visits. We discuss here the main results and provide proofs in Appendix A.10.

As argued in Proposition 1, without regulation the intermediary would adopt full disclosure in thick and intermediate markets. Full disclosure eliminates the risk of repetition and thus increases the overall value generated by targeted ad impressions, but also maximizes the revenue that the intermediary can extract from the advertisers. Imposing partial disclosure would thus reduce the profits earned by the intermediary and tighten the conditions such that the publishers

[16]For instance, citing compliance with the GDPR as the main motive, in 2018 Google restricted access to its DoubleClick IDs to advertisers and curtailed the availability of user-level exposure data from ad campaigns.

outsource their ad inventories in equilibrium.

It is interesting to note that the effect of privacy regulation on advertisers is not necessarily negative. Advertisers are left with zero surplus with partial as well as full disclosure, since the intermediary extracts the entire value of impressions in both cases. However, when privacy regulation induces the publishers not to outsource, the advertisers benefit because of the increased competition between advertising outlets, which increases the advertisers' net surplus in thin and intermediate markets.

We also point out that privacy regulation does not necessarily reduce the total surplus in the advertising market. As explained above, partial disclosure by the intermediary increases the likelihood of wasteful repetition, reducing the value of targeted impressions to the advertisers. Hence, if the publishers outsource to the intermediary despite the regulation, total surplus decreases. However, if the regulation induces the publishers not to outsource, total surplus can either increase or decrease (the conditions are the same as described in Proposition 3). On the one hand, the market does not benefit from the intermediary's superior ability to profile consumers and target ads. On the other hand, when the publishers do not outsource, no targeted impression is wasted due to repetition because advertisers single-home in equilibrium. In contrast, repetition of targeted ads does take place under partial disclosure. We summarize in the following proposition.

Proposition 5. *Regulation imposing Partial Disclosure reduces the profits of the intermediary and the publishers. However, profits of the advertisers increase if the regulation induces the publishers not to use the intermediary. In that case, welfare decreases if and only if* $\tilde{q}$ *exceeds the threshold* $\tilde{q}_W$ *characterized in Proposition 3.*

Privacy regulation may also result in a restriction to the intermediary's ability to target ads, i.e. to $\tilde{q}$. If the publishers outsource to the intermediary despite this reduction, the result is a loss in total surplus with a negative effect on the profits of advertisers, publishers and the intermediary. However, a reduction in $\tilde{q}$ may also result in the publishers not outsourcing anymore. As discussed above, this change reduces the profit on the supply side of the market and benefits the demand side, while total surplus may increase or decrease. Hence, Proposition 5 would not change in a qualitative sense.

6 Extensions

6.1 Reserve prices in advertising auctions

In a first extension, we allow the intermediary to introduce reserve prices in advertising auctions.[17] We have shown in Section 4.2.2 that the intermediary adopts partial disclosure when selling impressions on consumers that belong to thin advertising markets (Proposition 1). Full disclosure results in a sharp drop in the equilibrium price of impressions if these consumers multi-home (without a reserve price). In our model, given full disclosure, a reserve price is redundant for impressions in thick markets (where the intermediary can extract the whole advertising surplus regardless), but can limit the price drop for impressions on multi-homers in thin markets. The drawback is that the higher the reserve price, the higher the share of markets where advertisers drop out from the auction because their return from informing consumers, v, is too low. Due to this trade-off, we show in Appendix B.1 that, even if the intermediary sets the revenue-maximizing reserve price (given the available information), it is not obvious that the revenue earned exceeds the revenue with partial disclosure in thin markets. For instance, full disclosure with a reserve price is weakly dominated by partial disclosure given some notable distributions of v (e.g., uniform and continuous Bernoulli distributions). We conclude that the possibility to adopt reserve prices does not necessarily induce the platform to be more transparent when auctioning ad impressions.

6.2 Conflation of impressions by consumer preferences

Proposition 1 shows that, to maintain a sufficient number of bidders for impressions on multi-homers, IN conflates impressions on single and multi-homers in thin advertising markets. To pursue the same objective, the intermediary may adopt an alternative kind of conflation. Namely, retain some information about the product preferences of multi-homers, while fully disclosing information about their exposure to ads. We consider this possibility in Appendix B.2. We assume that, rather than disclosing the consumer's θ, IN only lets the advertisers know that her type belongs to a discrete set of values. The uncertainty regarding the consumer's true type implies that, although they place positive bid, the corresponding advertisers bid less than the full value, v. Thus, it is in the intermediary's interest to conflate impressions from as

[17] Reserve prices are sometimes adopted in advertising auctions ran by intermediaries, but anecdotal evidence suggests that the platforms do not necessarily set them at revenue-maximizing levels. See, e.g., Ostrovsky and Schwarz (2016). Google lets each publisher decide the reserve price (if any) for the impressions on its own webpages (`https://support.google.com/admanager/answer/9298008?hl=en`).

little different markets as possible. In equilibrium, the intermediary conflates impressions by consumer preferences only in thin advertising markets, and never from more than two such markets. Furthermore, there is no conflation for impressions on single-homers.

In equilibrium, the number of bidders for each impression in thin markets is $2n = 4$. Hence, there are at least two bidders for each impression on a multi-homer and the equilibrium price is positive. However, each advertiser that is not already acquiring an impression on a given consumer bids half of the advertising return, v, since with probability 1/2 the consumer belongs to an irrelevant market. In expectation, therefore, IN's aggregate revenue in thin markets is $x\bar{v}\tilde{q}\left((1-m)+2m/2\right) = x\bar{v}\tilde{q}$ (recall that IN does not observe v prior to auctioning the impressions). This revenue equals the revenue obtained by adopting partial disclosure (see (11)). In our model, therefore, conflating impressions by preferences is not superior to partial disclosure. The intuition for this result is as follows: with conflation by preferences, there are no repeated impressions on multi-homers, but each impression has a 50 percent chance of being sent to an irrelevant consumer. On the other hand, with partial disclosure all the impressions are correctly targeted, but each impression on a multi-homer is repeated with 50 percent probability.

6.3 Increasing returns to advertising on the same consumer and re-targeting

In the baseline model, we assumed diminishing returns to advertising on the same consumer. However, some advertisers may want to propose an ad containing a specific offer on a certain product to a consumer who has previously shown interest in it. Also, advertisers might want to send ads in sequence to tell a brand story. In both cases, advertisers put a premium on hitting the same consumer more than once (re-targeting). To capture this possibility, in Appendix B.3 we assume each advertiser gets a higher return from the second impression that hits a consumer than from the first one. Under this assumption, we find the intermediary always benefits from adopting full disclosure, which enables it to extract all the advertising surplus even in thin markets. Consequently the publishers always benefit from outsourcing to the intermediary. Furthermore, selling impressions independently gets less profitable to the publishers as the number of advertisers in a market increases. The reason is that, without a centralized platform allocating the impressions, the equilibrium probability of hitting the same consumer twice decreases with the number of advertisers.

6.4 Heterogeneous advertising returns within markets

In Appendix B.4, we relax the assumption that the return from informing consumers is homogeneous across advertisers within the same market. We allow for a subset of markets where one of the advertisers (say, a) gets a larger return from informing a profiled consumer, v^+, than the other advertisers in the market. With homogeneous returns, all advertisers in a market single-home when the publishers do not outsource to IN (see Lemma 1). However, if v^+ is large enough compared to v, advertiser a dominates, i.e. outbids the other advertisers for each targeted impression on *both* publishers.[18]

The presence of a dominant advertiser does not affect the incentives of the intermediary to disclose information. In particular, the intermediary still chooses partial disclosure if and only if markets are thin (Proposition 1). However, the fact that some advertisers multi-home when the digital publishers operate independently could, in principle, change the publishers' incentives to outsource to the intermediary. We therefore check the robustness of Proposition 2 to the presence of a dominant advertiser. More precisely, we assume there is a share x' of thin markets where one of the advertisers has a return from informing consumers large enough that it acquires all the impressions on all platforms in equilibrium, while there is a share x of thin markets with homogeneous advertisers. Similarly, we assume there is a share y' and z of, respectively, intermediate and thick markets with a dominant advertiser.

When advertising markets are thin or intermediate, the equilibrium with one advertiser acquiring all the targeted impressions in its market entails a higher revenue for the publishers than when the advertisers are homogeneous (all else given). The reason is that if one advertiser acquires all the targeted impressions, the other advertisers in the market place no ads in equilibrium. Thus, they are willing to bid v for each impression, since they do not discount their bids by the risk of repetition. The price of targeted impressions increases from the expressions given in (2) and (4) to v. By the same token, though, the presence of a dominant advertiser increases the equilibrium price of targeted impressions sold by the intermediary under partial disclosure. This price increases from the expression given in (10) to v. On the other hand, the presence of a dominant advertiser does not change the revenue earned by the platforms in thick markets, due to the more intense competition among advertisers. As a result, Proposition 2 does not change.

[18]This finding is consistent with Athey et al. (2018). In a setting with two publishers and no third-party platform, the authors show that only advertisers with the highest returns from informing consumers multi-home.

7 Policy implications

Our analysis has relevant policy implications. First, it contributes to the debate on regulating transparency in the digital advertising market. For instance, the European Commission is considering imposing transparency requirements to advertising gatekeepers (European Commission, 2020). We have found that the intermediary prefers not to disclose all the available information on consumers when auctioning impressions in thin advertising markets. Hence, imposing transparency would be detrimental to the supply-side of the market (publishers as well the intermediary). However, this regulation would benefit the advertisers, by allowing not only a more efficient allocation of ads, but also reducing their equilibrium price (Proposition 4).

Another important ongoing debate concerns the possibility of restricting third-party tracking to protect consumer privacy. For example, the European Union's GDPR of 2018 and the California Consumer Privacy Act of 2020 restrict the use of information collected across different publishers for the purpose of targeted advertising. We capture the effects of these policies by assuming that the intermediary must adopt Partial Disclosure indiscriminately. We have shown that there is little effect on the profits of the intermediary and of the publishers (the "supply side" of the market), if their audience belongs primarily to thin advertising markets. However, the regulation clearly decreases the profits of the supply side if the audience belongs to thicker markets, because under Partial Disclosure the intermediary is unable to extract the whole advertising surplus (Proposition 5). Furthermore, the effect of this regulation on the advertisers is not necessarily negative, in particular if the regulation discourages the publishers from outsourcing to the intermediary, which would increase competition among advertising outlets. Although this change in market structure would benefit the advertisers, the overall effect on the total surplus in the advertising market could be negative. The model suggests that this is the case if the intermediary's ability to profile consumers is large enough, and is more likely when the publishers serve predominantly thin advertising markets (Proposition 3).

Before concluding this section, we emphasize that our measure of welfare only focuses on surplus on the advertising market, including the surplus of the intermediary, publishers and advertisers. We do not consider consumer surplus and do not include any potential privacy cost for consumers. Studying how these aspects could be affected by the policies just discussed would require a more detailed model of consumer behavior, which is outside the scope of this paper.

8 Managerial implications

The analysis has interesting managerial implications for several players in the digital advertising market. Our results suggest that the pervasiveness of multi-homing by consumers should not necessarily induce digital publishers to rely on intermediaries for selling their advertising space. We have found that, especially when multi-homing is highly pervasive, the publishers could end up selling the impressions in thin markets at a higher price when operating independently. The intermediary can use the data collected on one publisher to profile a consumer and sell targeted impressions on such consumer when she visits another publisher. This sort of "information leakage" can result in a higher risk of repetition and, hence, lower ad prices. More generally, if there are diminishing returns to advertising, e.g. if advertisers prefer to avoid excessive repetition on the same consumer (frequency capping), the thickness of the advertising market should be a key parameter driving the decision whether to use an intermediary (Proposition 2). On the other hand, if the advertisers value impressing the same consumer multiple times (re-targeting), these conclusions change: the publishers unambiguously benefit from outsourcing to the intermediary, because its ability to track consumers across outlets results not only in a higher volume of targeted impressions, but also in higher prices of such impressions.

The fact that digital publishers sell their impressions via an intermediary has mixed implications for the advertisers. On the one hand, the intermediary has a technological and informational advantage compared to the publishers in terms of profiling consumers, targeting the impressions to the right consumer and managing the frequency of exposure to the same ad. On the other hand, competition on the supply side is significantly weakened when the publishers outsource to the intermediary. In our model, the net effect of these forces on the surplus of advertisers is negative. Overall, the advertisers should benefit from (and should thus be supportive of) regulation that encourages greater transparency. The advertisers should also favor regulation enhancing privacy, as long as the latter discourages digital publishers from relying on the intermediary.

Our model also provides some insights concerning the optimal disclosure of information about the behavior and preferences of consumers in advertising auctions. We have shown that the thickness of advertising markets is an important determinant of this disclosure (Proposition 1). Disclosing information to advertisers is not necessarily beneficial to the intermediary and, more generally, to the supply side of the advertising market. If the market is thin, disclosing information on the browsing history of consumers, that allows advertisers to avoid repetition on multi-homers, can reduce the equilibrium price of ad impressions. By contrast, disclosure is advantageous to all players when advertising markets are sufficiently thick.

9 Concluding remarks

We have studied the incentives of a large online advertising intermediary to disclose consumer information to advertisers when auctioning ad impressions. We have shown that disclosing information that enables advertisers to optimize the allocation of ads on multi-homing consumers is profitable to the intermediary only if advertising markets are sufficiently thick. We have also considered how disclosure affects the incentives of publishers to outsource the sale of their ads to an intermediary, and related these incentives to the extent of consumer multi-homing, the competitiveness of advertising markets and the ability of platforms to profile consumers. We have shown that, even when most consumers multi-home, the publishers do not necessarily benefit from outsourcing to the intermediary, and may be worse off in thin advertising markets. Finally, we have studied how the intermediary responds to policies designed to enhance transparency or consumer privacy, and the implications of these policies for the online advertising market.

References

Ambrus, A., Calvano, E., and Reisinger, M. (2016). “Either or Both Competition: a Two-Sided Theory of Advertising With Overlapping Viewerships.” *American Economic Journal: Microeconomics*, *8*, 189?–222.

Anderson, S. P., and Coate, S. (2005). “Market provision of broadcasting: A welfare analysis.” *Review of Economic Studies*, *72*(4), 947–972.

Athey, S., Calvano, E., and Gans, J. (2018). “The Impact of Consumer Multi-homing on Advertising Markets and Media Competition.” *Management Science*, *64*(4), 1574–1590.

Bagwell, K. (2007). “The economic analysis of advertising.” *Handbook of Industrial Organization*, vol. 3, 1701–1844, Elsevier.

Bergemann, D., and Bonatti, A. (2015). “Selling Cookies.” *American Economic Journal: Microeconomics*, *7*(3), 259?–294.

Bourreau, M., Caillaud, B., and de Nijs, R. (2017). “The Value of Consumer Data in Online Advertising.” *Review of Network Economics*, *16*(3), 269–289.

Cairncross, F. (2019). “The Cairncross Review: a sustainable future for journalism.”, UK Department for Digital, Culture, Media and Sport.

Chen, J., and Stallaert, J. (2014). “An Economic Analysis of Online Advertising Using Behavioral Targeting.” *MIS Quarterly*, *38*(2), 429–450.

CMA (2020). “Online platforms and digital advertising market study.”, market study final report, Competition and Markets Authority.

D’Annunzio, A., and Russo, A. (2020). “Ad Networks and Consumer Tracking.” *Management Science*, *66*(11), 5040–5058.

de Cornière, A., and de Nijs, R. (2016). “Online Advertising and Privacy.” *RAND Journal of Economics*, *15*(3), 311–327.

European Commission (2020). “Proposal for the Digital Markets Act.”, Communication 842-2020.

European Parliament (2016). “General Data Protection Regulation.”, Regulation 2016/679.

Ganuza, J.-J. (2004). “Ignorance Promotes Competition: An Auction Model with Endogenous Private Valuations.” *RAND Journal of Economics*, *35*(3), 583–598.

Ganuza, J.-J., and Penalva, J. S. (2010). “Signal Orderings Based on Dispersion and the Supply of Private Information in Auctions.” *Econometrica*, *78*(3), 1007–1030.

Ghosh, A., Mahdian, M., McAfee, R. P., and Vassilvitskii, S. (2015). “To Match or Not to Match: Economics of Cookie Matching in Online Advertising.” *ACM Transactions on Economics and Computation*, *3*(2), article 12.

Gu, Y., Madio, L., and Reggiani, C. (2019). “Data brokers co-opetition.”, available at SSRN: `https://ssrn.com/abstract=3308384`.

Hummel, P., and McAfee, R. P. (2016). “When Does Improved Targeting Increase Revenue?” *ACM Transactions on Economics and Computation*, *5*(1).

IAB (2017). “IAB Europe Report: European Programmatic Market Sizing 2017.”, available at `https://iabeurope.eu/research-thought-leadership/iab-europe-report-european-programmatic-market-sizing-2017/`.

Johnson, G., and Shriver, S. (2020). “Privacy & market concentration: Intended & unintended consequences of the gdpr.”

Johnson, J. P. (2013). “Targeted advertising and advertising avoidance.” *The RAND Journal of Economics*, *44*(1), 128–144.

Karle, H., and Reisinger, M. (2019). “Excessive Targeting.”, working Paper.

Levin, J., and Milgrom, P. (2010). “Online Advertising: Heterogeneity and Conflation in Market Design.” *American Economic Review*, *100*, 603–07.

Marotta, V., Wu, Y., Zhang, K., and Acquisti, A. (2021). “The welfare impact of targeted advertising technologies.”, available at SSRN: `https://ssrn.com/abstract=2951322`.

Ostrovsky, M., and Schwarz, M. (2016). “Reserve prices in internet advertising auctions: A field experiment.”, Working Paper, Stanford University.

Peitz, M., and Reisinger, M. (2020). “Advertising networks and consumer tracking.”, mimeo.

Peukert, C., Batikas, M., Bechtold, S., and Kretschmer, T. (2020). “European privacy law and global markets for data.”, CEPR Discussion Paper No. DP14475.

Rafieian, O., and Yoganarasimhan, H. (2021). “Targeting and Privacy in Mobile Advertising.” *Marketing Science*, *4*(2), 193–218.

Sharma, P., Sun, Y., and Wagman, L. (2019). "The Differential Effects of New Privacy Protections onPublisher and Advertiser Profitability.", Working Paper, Illinois Institute of Technology. Available at SSRN: https://ssrn.com/abstract=3503065.

Wilbur, K. C., and Zhu, Y. (2009). "Click Fraud." *Marketing Science*, *28*(2), 293–308.

Xiang, Y., and Sarvary, M. (2013). "Buying and Selling Information Under Competition." *Quantitative Marketing and Economics*, *11*, 321–351.

Online Appendix

A Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

We focus on the subgame played by the advertisers given that the publishers have not outsourced to IN. Consider an advertiser's bidding strategy for each relevant impression on publisher i. The advertiser gets an expected return of (1) from each such impression. This return is independent on the volume of impressions acquired on publisher i, but depends on $S_{i'}$, that is, the share of the impressions on relevant consumers in the same market the advertiser acquires on i'. In a second-price auction, the advertiser cannot do better than bid (1) for each relevant impression on i. Hence, there is no loss in restricting attention to bidding strategies such that the advertiser bids (1) for each relevant impression on publisher i. Because these strategies are conditional on $S_{i'}$, there are potentially multiple equilibrium strategies for each advertiser. To characterize them, we have to establish which values of $S_{i'}$ can emerge in any equilibrium of the subgame.

Thin markets. Consider a market such that $n = 2$. Advertisers in the market are denoted by $j = a, b$ and S_{ij} is the share of relevant impressions acquired by advertiser j on publisher i. Focus, without loss of generality, on the relation between the share of relevant impressions acquired by an advertiser in this market on publisher 2 and the value of (1) for relevant impressions on publisher 1. Consider the bidding strategy of the advertisers on publisher 1. There are two possible cases:

- A: if $S_{2a} > S_{2b}$, the advertisers' bid for each relevant impression on 1 are such that $v(1 - \frac{2mqS_{2a}}{1+m}) < v(1 - \frac{2mqS_{2b}}{1+m})$. Hence, b outbids a for all such impressions, i.e. $S_{1a} = 0 < S_{1b} = 1$.
- B: if $S_{2a} = S_{2b}$, the advertisers' bid for each relevant impression on 1 are such that $v(1 - \frac{2mqS_{2a}}{1+m}) = v(1 - \frac{2mqS_{2b}}{1+m})$. Hence, a and b place equal bids for all such impressions, so $S_{1a} = S_{1b} = 1/2$.

In equilibrium, these bidding strategies must be consistent with the bidding strategies (and the ensuing shares S_{2j}) on publisher 2. If case A applies, since $S_{1a} < S_{1b}$, by the same reasoning as above we must have $S_{2a} = 1 > S_{2b} = 0$. This case constitutes an equilibrium candidate such that the advertisers single-home, i.e. place winning bids on a single publisher. Given (1), the winning advertiser on publisher i bids v while the other one bids (2). The latter is also the price of relevant impressions on both publishers (second highest bid). Therefore, each advertiser earns $v\frac{2mq}{1+m}$ per impression acquired. Given there are $q\frac{1+m}{2}$ relevant impressions per publisher in this market, each advertiser earns vmq^2 in total.

If Case B applies, all advertisers place identical bids on both publishers. Given (1) each advertiser bids $v(1 - \frac{mq}{1+m})$ for all relevant impressions on each publisher. The latter is also the price of relevant impressions on both publishers (second highest bid). This price equals the expected return from each impression for all advertisers. Therefore, the advertisers make zero profit in this equilibrium candidate.

We have thus identified two equilibrium candidates and must now establish whether these are indeed equilibria. The candidate associated to case B cannot be an equilibrium because, given the bids placed by the rival, an advertiser can deviate by bidding v for all relevant impressions on one publisher (and thus win them all) and zero for all impressions on the other. The advertiser would earn $\left[v - \left(v(1 - \frac{mq}{1+m}\right)\right]\left(q\frac{1-m}{2}\right) = \frac{vmq^2}{2}$ by deviating, so the deviation is profitable.

As for the candidate associated to case A, there is no profitable deviation: an advertiser cannot profitably outbid the other on the publisher where it is not acquiring any impression, because the willing bid is v on that publisher. Nor would the advertiser gain by reducing its bid for the impressions it is winning, because the auction is a second price. Hence, the candidate associated to case A are indeed equilibria, as we claim in the text. There exists also the symmetric equilibrium obtained from swapping a and b.

Intermediate markets. Consider a market such that $n = 3$. Let $j = a, b, c$ be the set of advertisers in the market. Consider, without loss of generality, the bidding strategy of the advertisers on publisher 1 and focus on the relation between S_{2j} and the value of (1) for relevant impressions on publisher 1. There are the following possible cases

- A: if $S_{2c} > S_{2b} > S_{2a}$, the advertisers' bid for each relevant impression on 1 are such that $v(1 - \frac{2mqS_{2c}}{1+m}) < v(1 - \frac{2mqS_{2b}}{1+m}) < v(1 - \frac{2mqS_{2a}}{1+m})$. Hence, a outbids the other advertisers for all such impressions, i.e. $S_{1c} = S_{1b} = 0 < S_{1a} = 1$.

- B: if $S_{2c} > S_{2a} = S_{2b}$, the advertisers' bid for each relevant impression on 1 are such that $v(1 - \frac{2mqS_{2a}}{1+m}) = v(1 - \frac{2mqS_{2b}}{1+m}) > v(1 - \frac{2mqS_{2c}}{1+m})$. Hence, a and b bid equally and outbid c for for all such impressions, i.e. $S_{1a} = S_{1b} = 1/2 > S_{1c} = 0$.

- C: if $S_{2a} = S_{2b} = S_{2c}$, the advertisers' bid for each relevant impression on 1 are all equal to $v(1 - \frac{2mqS_{2j}}{1+m})$. So $S_{1a} = S_{1b} = S_{1c} = 1/3$.

In equilibrium, these bidding strategies (and the ensuing shares S_{2j}) must be consistent with the bidding strategies on publisher 2. If case A applies, since $S_{1c} = S_{1b} = 0 < S_{1a} = 1$, by the same reasoning as above we must have $S_{2c} = S_{2b} = 1/2 > S_{2a} = 0$. As this outcome is inconsistent with the assumption that $S_{2c} > S_{2b} > S_{2a}$, we can disregard this case.

If Case B applies, we have $S_{1a} = S_{1b} = 1/2 > S_{1c} = 0$ and thus $S_{2c} = 1 > S_{2a} = S_{2b} = 0$. This case constitutes an equilibrium candidate such that all advertisers single-home, i.e. place winning bids on a single publisher. Given (1), advertisers a and b bid v on publisher 1 while c bids (2). The price of relevant impressions on publishers 1 (second highest bid) is thus v. As for publisher 2, c bids v while a and b bid (4). The latter is also the price of relevant impressions on 2 (second-highest bid). Therefore, advertisers a and b earn zero while advertiser c earns $v\frac{mq}{1+m}$ for each impression acquired. Given there are $q\frac{1+m}{2}$ relevant impressions per publisher in this market, advertiser c advertiser earns $vmq^2/2$ in total.

If Case C applies, all advertisers place identical bids on both publishers. Given (1) each advertiser bids $v(1 - \frac{2mq}{3(1+m)})$ for each relevant impression on each publisher. The latter is also the price of relevant impressions on both publishers (second highest bid). This price equals the expected return from each impression for all advertisers. Therefore, the advertisers make zero profit in this equilibrium candidate.

We have thus identified two equilibrium candidates (case B and case C) and must now establish whether these are indeed equilibria. The candidate associated to case C cannot be an equilibrium because, given the bids placed by the rivals, each advertiser can deviate by bidding v for all relevant impressions on one publisher (and thus win them all) and zero for all impressions on the other. The advertiser would earn a strictly positive profit by deviating, so the deviation is profitable.

As for the candidate associated to case B, there is no profitable deviation: no advertiser can profitably outbid the others on the publisher where it is not acquiring any impression, because the winning bid equals v. Nor would the advertiser gain by reducing its bid for the impressions it is winning. Hence, the candidate associated to case B is indeed an equilibrium. Note that also all the other candidates associated to case B, obtained by permutations of a, b, c and $1, 2$, are equilibria.

Thick markets. Consider a market such that $n = 4$. Let a, b, c, d denote the set of advertisers in the market. Focus again on the relation between S_{2j} and the value of (1) for relevant impressions on publisher 1. Consider the bidding strategy of the advertisers on publisher 1. There are the following possible cases:

- A: if $min(S_{2d}, S_{2c}) > S_{2b} > S_{2a}$, the advertisers' bid for each relevant impression on 1 are such that $max(v(1 - \frac{2mqS_{2d}}{1+m}), v(1 - \frac{2mqS_{2c}}{1+m})) < v(1 - \frac{2mqS_{2b}}{1+m}) < v(1 - \frac{2mqS_{2a}}{1+m})$. Hence, a outbids the other advertisers for all such impressions, i.e. $S_{1d} = S_{1c} = S_{1b} = 0 < S_{1a} = 1$.
- B: if $min(S_{2d}, S_{2c}) > S_{2b} = S_{2a}$, the advertisers' bid for each relevant impression on 1 are such that $max(v(1 - \frac{2mqS_{2d}}{1+m}), v(1 - \frac{2mqS_{2c}}{1+m})) < v(1 - \frac{2mqS_{2b}}{1+m}) = v(1 - \frac{2mqS_{2a}}{1+m})$. Hence, a and b bid equally and outbid c and d for for all such impressions, i.e. $S_{1a} = S_{1b} = 1/2 > S_{1c} = S_{1d} = 0$.
- C: if $S_{2d} > S_{2c} = S_{2b} = S_{2a}$, the advertisers' bid for each relevant impression on 1 are such that $v(1 - \frac{2mqS_{2d}}{1+m}) < v(1 - \frac{2mqS_{2c}}{1+m})) = v(1 - \frac{2mqS_{2b}}{1+m}) = v(1 - \frac{2mqS_{2a}}{1+m})$. Hence, a,b and c bid equally and outbid d for for all such impressions, i.e. $S_{1a} = S_{1b} = S_{1c} = 1/3 > S_{1d} = 0$.
- D: if $S_{2a} = S_{2b} = S_{2c} = S_{2d}$, the advertisers' bid for each relevant impression on 1 are all equal to $v(1 - \frac{2mqS_{2j}}{1+m})$. So $S_{1a} = S_{1b} = S_{1c} = S_{1d} = 1/4$.

In any equilibrium, these bidding strategies must be consistent with the bidding strategies on publisher 2. If case A applies, since $S_{1d} = S_{1c} = S_{1b} = 0 < S_{1a} = 1$, by the same reasoning as above we must have $S_{2d} = S_{2c} = S_{2b} = 1/3 > S_{2a} = 0$. As this outcome is inconsistent with the assumption that $min(S_{2d}, S_{2c}) > S_{2b} > S_{2a}$, we can disregard this case.

If Case B applies, we have $S_{1a} = S_{1b} = 1/2 > S_{1c} = S_{1d} = 0$ and thus $S_{2c} = S_{2d} = 1/2 > S_{2a} = S_{2b} = 0$. This case constitutes an equilibrium candidate such that all advertisers single-home, i.e. place winning bids on a single publisher. Given (1), advertisers a and b bid v on publisher 1 while c and d bid (4). The price of relevant impressions on publisher 1 (second highest bid) is thus v. Symmetrically, on publisher 2, c and d bid v while a and b bid (4). The price of relevant impressions on publisher 2 (second highest bid) is thus v. Therefore, the advertisers make zero profit in this equilibrium candidate.

If Case C applies, we have $S_{1a} = S_{1b} = S_{1c} = 1/3 > S_{1d} = 0$ and thus $S_{2d} = 1 > S_{2a} = S_{2b} = S_{2c} = 0$. This case constitutes an equilibrium candidate such that all advertisers single-home, i.e. place winning bids on a single publisher. Given (1), advertisers a, b and c bid v on publisher 1 while d bids (2). The price of relevant impressions on publishers 1 (second highest bid) is thus v. On publisher 2, d bids v while a, b and c bid $v(1 - \frac{2mq}{3(1+m)}))$. The latter is the price of relevant impressions on publishers 2 (second highest bid). Therefore, in this equilibrium candidate advertisers a, b and c make zero profit, while d makes $v(\frac{2mq}{3(1+m)}))$ per each of the $q\frac{1+m}{2}$ impressions acquired on publisher 2.

If Case D applies, all advertisers place identical bids on both publishers. Given (1) each advertiser bids $v(1 - \frac{mq}{2(1+m)})$ for all relevant impressions on each publisher. The latter is also the equilibrium price of relevant impressions on both publishers (second highest bid). This price equals the expected return from each impression for all advertisers. Therefore, the advertisers make zero profit in this equilibrium candidate.

We have thus identified possible equilibrium candidates in cases B, C and D, and must now establish whether these are indeed equilibria. The candidate associated to case D cannot be an equilibrium because, given the bids placed by the rivals, each advertiser can deviate by bidding v for all relevant impressions on one publisher (and thus win them all) and zero for all impressions on the other. The advertiser would earn a strictly positive profit by deviating, so the deviation is profitable.

Similarly, the candidate associated to case C cannot be an equilibrium because, given the bids placed by the rivals, each advertiser among a, b and c can deviate by bidding v for all relevant impressions on publisher 2 (and thus split the impressions on this publisher with d) and zero for all impressions on the other. The advertiser would still earn zero, but obtain strictly more impressions. Hence, by the assumption that advertisers prefer to acquire as many impressions as possible for a given profit level, the deviation is profitable.

As for the candidate associated to case B, there is no profitable deviation: no advertiser can profitably outbid the others on the publisher where it is not acquiring any impression, for the winning bid there equals v already. Nor would the advertiser gain by reducing its bid for the impressions it is winning. Hence, the candidate associated to case B is indeed an equilibrium. Note that also all the other candidates associated to case B, obtained by permutations of a, b, c, d and $1, 2$, are equilibria.

Finally, following a similar reasoning as above one can show that, if $n > 4$, the equilibria are again such that advertisers single-home and there are at least two advertisers winning impressions and bidding v on each publisher. Hence, the equilibrium price of all impressions is still equal to v, as in the case where $n = 4$.

A.2 Proof of Lemma 2

Proof is in the text.

A.3 Proof of Lemma 3

Proof is in the text.

A.4 Proof of Lemma 4

Proof is in the text.

A.5 Proof of Proposition 1

Consider thin markets ($n = 2$). As shown in the text, profits of the intermediary under the Full disclosure and the Partial disclosure regimes respectively are

$$\pi_{n=2}^{FD} = v\tilde{q}\,(1-m)\,, \pi_{n=2}^{PD} = v\tilde{q}. \tag{A.1}$$

Hence, $\pi_{n=2}^{FD} < \pi_{n=2}^{PD}$. Consider now intermediate and thick markets ($n > 2$). In the text we have shown that profits of the intermediary under the Full Disclosure and the Partial Disclosure regimes respectively are

$$\pi_{n>2}^{FD} = v\tilde{q}\,(1+m)\,, \quad \pi_{n>2}^{PD} = v\tilde{q}\left(1 + m\left(1 - \frac{2}{n}\right)\right). \tag{A.2}$$

Comparing the above equations, we find that $\pi_{n>2}^{FD} > \pi_{n>2}^{PD}$, because $n > 2$. Hence, we conclude that the intermediary sells the impressions under Partial Disclosure (respectively, Full Disclosure) if the consumer belongs to a thin (respectively, intermediate or thick) advertising market.

A.6 Proof of Lemma 5

Proof is in the text.

A.7 Proof of Proposition 2

Lemmas 2 and 5 provides the revenues of the publishers and the intermediary respectively. Comparing the expressions, we obtain that profits of the intermediary are bigger than those of the publishers if and only if

$$\tilde{q} \geq \tilde{q}_T \equiv q\left[1+\frac{m\left(x(1-2q)-\frac{yq}{2}\right)}{1+m(1-x)}\right]. \tag{A.3}$$

It is easy to show that the term in parenthesis $1+\frac{m\left(x(1-2q)-\frac{yq}{2}\right)}{1+m(1-x)} > 1$ if and only if $(x(1-2q) > \frac{yq}{2}$, that can occur if and only if $q < 1/2$. Hence, since $\tilde{q} \geq q$ by assumption, the equilibrium must be such that the publishers outsource if $q \geq 1/2$. Then, if $q < 1/2$, the publishers outsource if and only if $\tilde{q} \geq \tilde{q}_T$.

Now, we look at the comparative statics of $\tilde{q}_T$. We find that $\frac{\partial \tilde{q}_T}{\partial q} = \frac{1+m(1-q(4x+y))}{1+m(1-x)}$. The denominator is always positive (because $x \leq 1$). The numerator is positive if $m(-1+4qx+yq) < 1$. Note that the $m(-1+4qx+yq)$ is decreasing in q and the threshold $\tilde{q}_T$ exists only if $q < \frac{1}{2}$. Hence, we compute $m(-1+4qx+yq) < 1$ in $q = \frac{1}{2}$, finding $m(-1+2x+\frac{y}{2}) \leq 1$. This inequality holds if and only if $x + \frac{y}{4} \leq 1$. This is always true because $x + \frac{y}{4} < x + y \leq 1$. Hence, the numerator is always positive, implying that $\tilde{q}_T$ increases in q.

We find that $\tilde{q}_T$ always increases in x. Because $x \leq 1$ the denominator is always positive. Then, the higher x, the higher the numerator is, implying that $\tilde{q}_T$ increases in x.

We find that $\frac{\partial \tilde{q}_T}{\partial m} = \frac{q(x(2-4q)-qy)}{2(1+m(1-x))^2}$. The denominator of this expression is always positive. Instead, the numerator is positive if and only if $x > \frac{qy}{2-4y}$ with $q < 1/2$.

A.8 Proof of Proposition 3

To characterize the threshold $\tilde{q}_w$, consider first the total surplus generated by identified impressions when the publishers do not outsource. Given Lemma 1, no impression is wastefully repeated and thus generates a value v in equilibrium. Therefore, the total advertising surplus when the publishers do not outsource is

$$\bar{v}q(1+m). \tag{A.4}$$

Consider now the total surplus generated when the publishers outsource. Given Proposition 1, no identified impression is repeated in thick and intermediate markets. However, each identified impression on a multi-homer is repeated with probability 1/2 in thin markets. Hence, the total advertising surplus when the publishers outsource is

$$\bar{v}\tilde{q}(1+m(1-x)). \tag{A.5}$$

We can therefore obtain the threshold $\tilde{q}_w$ by comparing the two above expressions. The rest of the claim follows directly from our statements in the main text.

A.9 Proof of Proposition 4

When restricted to the FD regime in all markets, the IN earns the following total revenue

$$y\bar{v}\tilde{q}\,(1+m) + (1-x-y)\bar{v}\tilde{q}\,(1+m) = (1-x)\bar{v}\tilde{q}\,(1+m) \tag{A.6}$$

Comparing the above with (16), it is straightforward to conclude that the intermediary's total revenue is smaller when restricted to FD. Since the regulation does not affect the revenue the publishers can collect independently (given in (7)), it follows that the region of parameters such that the publishers outsource gets smaller when the regulation is adopted. The rest of the claim follows directly from our statements in the main text.

A.10 Proof of Proposition 5

When restricted to the PD regime in all markets, the IN earns the following total revenue

$$x\overline{v}\tilde{q} + y\overline{v}\tilde{q}\left(1 + m - \frac{2m}{3}\right) + (1 - x - y)\overline{v}\tilde{q}\left(1 + m - \frac{2m}{n}\right) \tag{A.7}$$

Comparing the above with (16), it is straightforward to conclude that the intermediary's total revenue is smaller when restricted to PD. Since the regulation does not affect the revenue the publishers can collect independently (given in (7)), it follows that the region of parameters such that the publishers outsource gets smaller when the regulation is adopted. The rest of the claim follows directly from our statements in the main text.

B Proofs of robustness checks

B.1 Reserve price

We establish that adopting FD with a revenue-maximizing reserve price does not necessarily increase IN's revenue. As shown in our baseline model, the intermediary earns the maximum advertising revenue by adopting FD without reserve prices in all markets where $n \geq 3$. Hence, reserve prices are redundant in such markets. Therefore, to establish our claim it is sufficient to focus on thin advertising markets. We proceed as follows. We characterize the equilibrium bidding strategies of advertisers conditional on the reserve price p. Next, we compute the equilibrium value of p chosen by IN. Note that p can only be made conditional on n and on whether the consumer is a single- or multi-homer, since the platforms do not observe v. Finally, we compare the revenue earned with FD and the revenue-maximizing reservation price in thin markets to the revenue earned with PD.

Consider an impression on an identified consumer and assume IN adopts FD. If the consumer is a single-homer, all the advertisers in the given market bid v. Hence, the equilibrium price of the impression is v and IN cannot do better by imposing a reserve price (in other words, the revenue-maximizing reservation price for these impressions is zero). If the consumer is a multi-homer, the equilibrium bids are as characterized in Table 2 for the couple of impressions on this consumer. Hence, the equilibrium price of each such impression is zero if there is no reserve price. With a reserve price p, the equilibrium price is p, but the impressions are sold only if the market is such that $v \geq p$.

Let us now compute IN's expected revenue by market when adopting FD and setting a reserve price p for any impression on a multi-homer. This revenue is $\tilde{q}(v(1-m)+2mp)$ if $v \geq p$, and $\tilde{q}v(1-m)$ if $v < p$, since in the latter case IN sells only the impressions on single-homers. Recall that the intermediary's revenue in a thin market is $\tilde{q}v(1-m)$ if it does impose any p. It follows that, conditional on adopting FD, the intermediary is better off imposing a reserve price such that $v_H \geq p > 0$.

Let us now calculate the revenue-maximizing p. The total revenue in thin advertising markets is

$$\pi^{FD}_{n=2}(p) = x\tilde{q}\left((1-m)\int_0^{v_H} v\,dF(v) + 2mp\int_p^{v_H} dF(v)\right) = x\tilde{q}\left((1-m)\overline{v} + 2pm\left(1 - F(p)\right)\right). \tag{B.1}$$

The revenue-maximizing reserve price, p^*, is such that

$$\frac{d\pi^{FD}_{n=2}(p)}{dp} = 1 - F(p) - pf(p) = 0 \tag{B.2}$$

and therefore

$$p^* = \frac{1 - F(p^*)}{f(p^*)}. \tag{B.3}$$

Suppose now IN adopts PD. There is no scope for a reserve price to increase revenues under PD, because all advertisers bid their true valuation and the two bids received are equal. Hence, conditional on choosing PD, IN does not impose any reserve price. Given the expression (11) in the main text, which characterizes the revenue earned under PD in a given thin market, the total revenue in thin markets under PD, $\pi^{PD}_{n=2}$, is equal to $x\tilde{q}\bar{v}$.

Given the above findings, if and only if $\pi^{FD}_{n=2}(p^*) > \pi^{PD}_{n=2}$ IN adopts FD with the reserve price p^* in (B.4) for each impression on multi-homers. Otherwise, IN adopts PD without a reserve price for each such impression. We find that

$$\pi^{FD}_{n=2}(p^*) > \pi^{PD}_{n=2} \iff xm\tilde{q}\left(-\bar{v} + 2\frac{(1 - F(p^*))^2}{f(p^*)}\right) > 0. \tag{B.4}$$

At this level of generality, it is difficult to establish whether the above condition holds, as this ultimately depends on the distribution of advertising returns, $F(v)$. However, the above inequality does not hold for two well known distributions:

- Suppose $v \sim U(0, v_H)$. Hence, $E(v) = \frac{v_H}{2}$, $F(p) = \frac{p}{v_H}$ and $f(p) = \frac{1}{v_H}$. Replacing these values in (B.3), we obtain that $p^* = \frac{v_H}{2}$. Replacing this value in (B.4), we obtain that $\pi^{FD}_{n=2}(p^*) = \pi^{PD}_{n=2}$.
- Suppose that $F(v)$ is a continuous Bernoulli distribution with $\lambda = \frac{1}{2}$. Hence, $E(v) = \frac{1}{2}$, $F(p) = p$ and $f(p) = 2$. Replacing these values in (B.3), we obtain $p^* = \frac{1}{3}$. Replacing this value in (B.4), we obtain that $\pi^{FD}_{n=2}(p^*) < \pi^{PD}_{n=2}$.

B.2 Conflation of impressions by consumer preferences

For simplicity, let us assume all markets are thin (recall that IN can extract the full value of impressions in thick markets adopting FD, so conflation is unnecessary in such markets). Assume that IN adopts FD when selling an impression. If the impression falls on a single-homer (SH), the equilibrium price is v. Suppose the impression falls on a multi-homer (MH) and suppose the IN reveals to the advertisers that the consumer's θ belongs to a set Θ_l comprising $l \geq 1$ values, where l is a positive integer. In this setting, therefore, l represents the number of advertising markets conflated. As we established in our baseline model, if $l = 1$ the equilibrium price of the impression on a MH is zero. Suppose $l > 1$. Each advertiser is willing to bid zero if already acquiring another impression on the consumer, and v/l otherwise, since with probability $1/l$ the impression does not fall on a consumer in the advertiser's own market. Let $v_{\theta l}$ be the return of the advertiser within the set Θ_l that places the second-highest bid for the impression. With $2l - 1$ advertisers bidding for each impression on a MH, the equilibrium price of the impression is $v_{\theta l}/l$. Note that the conflated markets can only be chosen randomly, since the platform does not observe v in any market. Hence, the mean of $v_{\theta l}$ is the mean of v, i.e. $\bar{v}$.

Consider now the total revenue IN can earn by adopting this form of conflation. This is given by $\bar{v}\tilde{q}(1 - m)$ for the impressions on SHs. As for the impressions on MHs, the revenue is $2m(\bar{v}/l)\tilde{q}$. Therefore, setting $l = 2$ is optimal. The overall revenue is therefore $\bar{v}\tilde{q}$. This is the same revenue earned with PD (see expression (11)).

B.3 Increasing returns to advertising on the same consumer and re-targeting

We here solve the model where advertisers value repeated impressions on a consumer. For simplicity, we assume that the first impression by an advertiser on a consumer is worth v and the second impression is woth $v_s = v + y$, with $y > 0$.

	Publisher 1	Publisher 2
adv.a	v	$v\left(\frac{1}{1+m}\right)+v_s\left(\frac{m}{1+m}\right)$
adv.b	v	$v\left(\frac{1}{1+m}\right)+v_s\left(\frac{m}{1+m}\right)$
$\pi_i(n=2)$	$\frac{vq}{2}(1+m)$	$\frac{q}{2}(v+v_s m)$

Table 3: Retargeting. Equilibrium bids without intermediary and with $n = 2$

	Publisher 1	Publisher 2
adv.α	v	$\frac{1}{n(1+m)}\left(v(n+m(6-n))+2mv_s\right)$
$\pi_i(n=4)$	$\frac{vq}{2}(1+m)$	$\frac{vq}{2}(1+m)$

Table 4: Re-targeting. Equilibrium bids without intermediary and with $n >= 4$

B.3.1 No outsourcing by the publishers

We now characterize the equilibrium conditional on the publishers not outsourcing to the intermediary.

Thin markets. Suppose $n = 2$. The equilibrium bids of both advertisers on a publisher are equal. Therefore, each advertiser acquires half of all the relevant impressions on each publisher. The equilibrium bids for such impressions equal v on one publisher. On the other publisher, each advertiser takes into account that some impressions will be repeated and bids

$$b \equiv v\left(\frac{1}{1+m}\right)+v_s\left(\frac{m}{1+m}\right). \tag{B.5}$$

Given second-price auctions, these bids are also the equilibrium price of each impression on the publishers. Table 3 summarizes the bids and profits of the publishers.

Intermediate and Thick markets. Assume $n \geq 3$. The equilibrium bids of the advertisers follow the same logic as in the previous cases. The market configuration is such that all advertisers multi-home and each advertiser wins a share $1/n$ impressions on each publisher. The winning bids on one publisher equal v and on the other publisher is equal to

$$b \equiv \frac{1}{n(1+m)}\left(v(n+m(6-n))+2mv_s\right). \tag{B.6}$$

Consequently, the equilibrium price of impressions is equal to the bids on each publisher. We summarize the bids and profits of the publishers in Table 4. Because all advertisers have the same behavior, the values we report in the table refer to a generic advertiser a.

B.3.2 Outsourcing to the Intermediary

We now characterize the equilibrium conditional on the publishers outsourcing to the intermediary.

Full Disclosure. For each impression in a given market, the intermediary discloses to the advertiser whether the impression is on a single- or a multi-homer, and her type θ. In this case, the equilibrium bids for both impressions on a single-homer and on a multi-homer is v. The equilibrium bid for a second impression on a multi-homer is v_s. Hence, the intermediary can extract all advertising surplus, and earn the following revenue in the given market:

$$v + v_s m. \tag{B.7}$$

Partial Disclosure. Under this information disclosure regime, for each impression in a given market the intermediary shares with the advertiser only the preference of the consumer. The equilibrium bids for all targeted impression in this market is

$$v \left(\frac{\frac{1+m}{n}}{1+m} \right) + v_s \left(\frac{\frac{m}{n}}{1+m} \right), \tag{B.8}$$

and the intermediary earns the following profit in the given market

$$v + \frac{m}{n} \left(v_s - v \right). \tag{B.9}$$

We conclude that full disclosure dominates partial disclosure, irrespective of the value of n in the given market.

B.3.3 The choice by the publishers whether to outsource

Differently from the baseline model, $n = 2$ is enough to have full profit extraction under outsourcing. Instead, $n = 4$ is not enough to observe full profits extraction under competition between platforms. On the contrary, the bigger n is, the lower the profits are when platforms compete independently: when the number of bidders goes to infinity, the probability of impressing twice the same consumers (henceforth, the revenues from second impressions) goes to zero. We conclude that outsourcing to the intermediary is always optimal for the publishers and they always outsource in equilibrium.

B.4 Heterogeneous returns to advertising within each advertising market

We modify the baseline setting by allowing for markets where one advertiser (that we take to be a without loss of generality) has a higher valuation, v^+, than the remaining $n-1$ advertisers, whose valuation is $v < v^+$. Specifically, we assume there is a share x of thin markets such that advertisers are homogeneous and a share x' of thin markets where advertiser a (the "dominant" one) has a return v^+ from informing consumers, large enough that it acquires all the impressions on all platforms. Similarly, there is a share y of intermediate markets with homogeneous advertisers and a share y' with a dominant advertiser. Finally, there is a share z of thick markets where advertisers are homogeneous, and a share $1 - x - x' - y - y' - z$ of thick markets with a dominant advertiser. For simplicity, we assume advertisers within each market are aware of the presence of a dominant advertiser (if any) and of the value of v^+.

Our objective is to establish that Proposition 2 still holds. The analysis proceeds as follows. We first focus on thin markets with heterogeneous advertisers. We then consider thin markets where advertisers are homogeneous. Next, we turn to the case of thick and intermediate markets. Finally, we compare the total revenue earned by the publishers to the revenue earned by the intermediary.

B.4.1 Thin markets with heterogeneous advertisers

No intermediary. We first focus on the subgame played by the advertisers given that the publishers have not outsourced to IN. Consider advertiser j's bidding strategy for each targeted impression on publisher i. The advertiser gets an expected return of w_{ij} from each such impression. This return is independent on the volume of targeted impressions acquired on publisher i, but depends on $S_{i'j}$. We can characterize advertiser j's willingness to pay for each impression on publisher i as follows

$$w_{ia} = v^+ \left(1 - q\frac{2mS_{i'a}}{1+m}\right), \quad w_{ib} = v\left(1 - q\frac{2mS_{i'b}}{1+m}\right), \quad i = 1, 2. \tag{B.10}$$

In a second-price auction, the advertiser cannot do better than bid (1) for each targeted impression on i. Hence, there is no loss in restricting attention to bidding strategies such that the advertiser bids w_{ij} for each targeted impression on publisher i. However, because these strategies are conditional on $S_{i'j}$, there are potentially multiple equilibrium strategies for each advertiser. Potentially, the following configurations emerge in equilibrium

- A: $S_{2b} = 0$ and $S_{1b} = 0$. The bids for each targeted impression on each publisher must be such that $w_{ia} = v^+(1 - \frac{2mq}{1+m}) > w_{ib} = v$. That is, a outbids b for every targeted impression on every publisher and thus multi-homes.
- B: $S_{ib} = 0$ and $S_{i'b} = 1$. The bids for each targeted impression on i' must be such that $w_{ia} = v^+(1 - \frac{2mq}{1+m}) < w_{ib} = v$. Hence, a and b single-home on different publishers. The same condition is necessary and sufficient for an equilibrium that entails the symmetric configuration, $S_{i'b} = 0$ and $S_{ib} = 1$.

All other configurations can be ruled out as follows. Consider any equilibrium candidate bidding strategy such that $S_{1b} > 0$ and $S_{2b} > 0$ (i.e. advertiser b multi-homes). Suppose $S_{ib} = 1$ for either $i = 1$ or $i = 2$. Then $w_{i'a} = v^+$ must exceed $w_{i'b} \leq v$. This implies that $S_{i'a} = 1$, which contradicts the assumption that both S_{1b} and S_{2b} are strictly positive. Suppose now that $1 > S_{1b} > 0$ and $1 > S_{2b} > 0$. These inequalities can hold if and only if all advertisers bid $w_{ia} = w_{ib}, \forall i$. However, if the latter equalities hold, both advertisers get a surplus equal to zero in the candidate equilibrium. Hence, advertiser a can deviate by bidding zero for each targeted impression on publisher i' and bidding $w_{ia} = v^+$ for each targeted impression on i. This deviation would be profitable because, given $S_{i'b} > 0$, then $p_i = w_{ib} < v < v^+$.

Summing up, the subgame that takes place conditional on the publishers not outsourcing to IN admits two possible equilibrium configurations. If and only if $v^+(1 - \frac{2mq}{1+m}) \geq v$ holds, the equilibrium is such that a multi-homes and buys all the targeted impressions. Otherwise, the equilibrium is such that each advertiser single-homes on a different publisher. In the former equilibrium, the price of each targeted impression, p_i equals v, so each publisher earns $vq\frac{1+m}{2}$. In the latter equilibrium, we have $p_1 = v^+(1 - \frac{2mq}{1+m})$ and $p_2 = v(1 - \frac{2mq}{1+m})$. Hence, the publishers earn $v^+q\frac{1+m(1-2q)}{2}$ and $vq\frac{1+m(1-2q)}{2}$ respectively.

Intermediary. Consider now the subgame that follows when the publishers decide to outsource to IN. Given Proposition 1, we can focus without loss on equilibria such that IN chooses PD. The advertisers' return from each targeted impression can be characterized as follows

$$w_a^{PD} = v^+ \left(1 - q\frac{2mS_a^{PD}}{1+m}\right), \quad w_b^{PD} = v\left(1 - q\frac{2mS_b^{PD}}{1+m}\right), \quad i = 1, 2. \tag{B.11}$$

The following configurations can potentially emerge in equilibrium

- A: $S_a^{PD} = 1$. The bids for each targeted impression must be such that $w_a^{PD} = v^+(1 - \frac{2m}{1+m}) > w_b^{PD} = v$. That is, a outbids b for every targeted impression.
- B: $1 > S_a^{PD} > 0$ and $1 > S_b^{PD} > 0$. The bids for each targeted impression on 1 must be such that $w_a^{PD} = v^+(1 - \frac{2mS_a^{PD}}{1+m}) = w_b^{PD} = v(1 - \frac{2mS_b^{PD}}{1+m})$. Observe that this configuration can emerge only if IN allocates the targeted impressions in such a way that the shares

S_j^{PD} satisfy the above equality. We assume it is the case (otherwise, there is no equilibrium when $v^+(1-\frac{2m}{1+m}) \le v$).

Summing up, if and only if $v^+(1-\frac{2m}{1+m}) > v$, the equilibrium is such that advertiser a acquires all the targeted impressions in the given market. The price of each such impression is $p^{PD} = v$ and the revenue of the intermediary is $v\tilde{q}(1+m)$. If $v^+(1-\frac{2m}{1+m}) \le v$, the equilibrium is such that the advertisers share the targeted impressions and $1/2 \ge S_b^{PD} \ge 0$. This share decreases with v^+. The price of impressions is $p^{PD} = w_b^{PD} = v(1-\frac{2mS_b^{PD}}{1+m})$. This price increases with v^+ and the revenue earned by IN is $v(1+m(1-2S_b^{PD}))\tilde{q}$.

Comparing revenue from impressions with and without IN.

- If $v \le v^+(1-\frac{2m}{1+m})$, IN's revenue is $v\tilde{q}(1+m)$ and the publishers' joint revenue is $vq(1+m)$.
- If $v^+(1-\frac{2m}{1+m}) < v \le v^+(1-\frac{2qm}{1+m})$, IN's revenue is $v(1+m(1-2S_b^{PD}))\tilde{q}$ and the publishers' joint revenue is $vq(1+m)$.
- If $v > v^+(1-\frac{2m}{1+m})$, IN's revenue is $v(1+m(1-2S_b^{PD}))\tilde{q}$, with $S_b^{PD} = 1/2$ when $v = v^+$. The publishers' joint revenue is $\left(\frac{v^+ + v}{2}\right) q(1+m(1-2q))$.

B.4.2 Thin markets with homogeneous advertisers

The analysis in these markets is identical to the baseline model and therefore not repeated here. Each publisher earns $\frac{vq}{2}(1+m(1-2q))$ in each such market, whereas IN's revenue is $v\tilde{q}$.

B.4.3 Intermediate markets with heterogeneous advertisers

With a dominant advertiser that acquires all the impressions on both publishers, and two remaining advertisers that bid v for each targeted impression, each publisher earns $\frac{vq}{2}(1+m)$ in each such market when operating independently. With $n = 3$, following the same analysis as in the baseline model one can show that the intermediary always prefers FD to PD and thus earns a total revenue of $v\tilde{q}(1+m)$ in each market, given second-price auctions.

B.4.4 Intermediate markets with homogeneous advertisers

The analysis in these markets is identical to the baseline model and therefore not repeated here. The publishers earn a joint revenue of $\frac{vq}{2}(1+m) + \frac{vq}{2}(1+m(1-q))$, whereas IN's revenue is $v\tilde{q}(1+m)$.

B.4.5 Thick markets

As in the baseline model, each publisher earns $\frac{vq}{2}(1+m)$ in each such market when operating independently. This holds regardless of the presence of a dominant advertiser. With $n \ge 4$, following the same analysis as in the baseline model one can show that the intermediary always prefers FD to PD and thus earns a total revenue of $v\tilde{q}(1+m)$ in each market, given second-price auctions. Again, this holds regardless of the presence of a dominant advertiser.

B.4.6 When do the publishers outsource?

As claimed above, we restrict our attention to the case where $v \le v^+(1-\frac{2m}{1+m})$, which is necessary and sufficient for the "high valuation" advertiser a to acquire all the impressions on each platform, whether the publishers outsource or not. Under this condition, given what we have established above, we can write the total revenue earned by the publishers without the intermediary as follows

$$\pi_1 + \pi_2 = 2x\left[\frac{\overline{v}q}{2}(1+m(1-2q))\right] + 2(x'+y')\left[\frac{\overline{v}q}{2}(1+m)\right] + y\left[\frac{\overline{v}q}{2}(1+m) + \frac{\overline{v}q}{2}(1+m(1-q))\right] + \\ + 2(1-x-x'-y'-y)\left[\frac{\overline{v}q}{2}(1+m)\right] = \overline{v}q\left[1+m\left(1-q\left(2x+\frac{y}{2}\right)\right)\right]. \tag{B.12}$$

Note that $\overline{v}$ is the mean of the same distribution as in the baseline model, since v^+ never enters the expression for the equilibrium revenue earned by the platforms. The revenue earned by the intermediary is

$$\pi_{IN} = x\left[\overline{v}\tilde{q}\right] + (x'+y'+y)\left[\overline{v}\tilde{q}(1+m)\right] + (1-x-x'-y'-y)\left[\overline{v}\tilde{q}(1+m)\right] = \overline{v}\tilde{q}\left[1+m(1-x)\right]. \tag{B.13}$$

Expressions (B.12) and (B.13) boil down to (7) and (16), respectively. Therefore, the conditions such that the publishers outsource in equilibrium are identical to those provided in Proposition 2.