Addictive Platforms*

Shota Ichihashi† Byung-Cheol Kim‡

March 23, 2021

Abstract

We study competition for consumer attention in which platforms choose the addictiveness of their services. A more addictive platform yields consumers a lower utility of participation but a higher marginal utility of allocating attention. The impact of competition depends on the scarcity of attention: If consumer attention is scarce, competition harms consumers compared to monopoly, because competing platforms have business stealing incentives and choose high addictiveness. Restricting consumers' platform usage could reduce addictiveness and improve their welfare. Platforms decrease addictiveness when they can charge for services, but consumers may be better off when services are free but monetize attention.

JEL codes: D40, L51, K21, M38

Keywords: platform competition, attention, addiction, merger

---

*We thank Alessandro Bonatti, Luis Cabral, Matthew Gentzkow, Doh-Shin Jeon, Martin Peitz, and seminar participants at MaCCI/EPoS Virtual IO Seminar and the SEA 90th Annual Meeting.

†Bank of Canada, 234 Wellington Street West, Ottawa, ON, K1A 0G9, Canada. Email: shotaichihashi@gmail.com. The opinions expressed in this article are the authors' own and do not reflect the views of the Bank of Canada.

‡Department of Economics, Finance, and Legal Studies, Culverhouse College of Business, University of Alabama. Tuscaloosa, AL, 35487, USA. E-mail: bkim34@ua.edu.

1 Introduction

Online platforms, such as Facebook, Google, and Twitter, monetize consumer attention. Because attention is finite, competition for attention may encourage firms to improve their services to attract consumers. At the same time, there is a growing concern for consumers and policymakers—that competition for attention could also incentivize a firm to sacrifice its service quality for attention. For example, a platform may adopt news feeds that display low quality content users are likely to watch; it may also adopt a certain user interface, such as an intrusive notification system or infinite scrolling (Scott Morton et al., 2019).[1]

We study a model of competition for consumer attention in which a platform can sacrifice the service quality for attention. The model consists of a consumer and platforms. First, platforms choose the addictiveness of their services. Second, the consumer chooses the set of platforms to join, then allocates her attention. A more addictive platform yields the consumer a lower utility of participation but a higher marginal utility of allocating attention. As a result, the consumer prefers to join less addictive platforms, but after joining, she allocates more attention to more addictive platforms. The consumer incurs a cost of allocating attention. She also faces an attention constraint, which caps the maximum attention she can allocate. The constraint captures the scarcity of attention. A platform provides the service for free and earns revenue that is increasing in the amount of attention the consumer allocates.

The addictiveness in our model captures a firm's choice to sacrifice the quality of a service to make it more capable of capturing consumer attention. For example, a platform may design the content selection algorithm or user interface in a certain way, or collect sensitive individual data for personalization. We model such a choice as the shift of service utilities and marginal utilities provided to the consumer.

Our main question is whether competition for attention benefits consumers. We show that competition may harm consumers, in particular when the attention is scarce. Competition affects platforms' incentives in two ways. On the one hand, competition encourages platforms to reduce

[1]For example, Scott Morton and Dinielli (2020) argue that "another reduction in quality that Facebook's market power allows is the serving of addictive and exploitative content to consumers. Facebook deploys various methods to maintain user attention—so that it can serve more ads—using techniques that the medical literature has begun to demonstrate are potentially addictive."

addictiveness: If a consumer faces competing platforms, she loses less by refusing to join a single platform and continuing to use other services. In such a case, platforms need to reduce addictiveness and offer high service quality to encourage participation. On the other hand, competition introduces business stealing incentives: A platform can increase its addictiveness to capture attention the consumer would allocate to its rivals.

The countervailing incentives derive our main insight: The impact of competition depends on the scarcity of attention; when attention is scarce, competition can harm consumers. If attention is scarce (i.e., the consumer faces a tight attention constraint), higher addictiveness does not increase total attention, but it only changes how the consumer divides her attention across platforms. In such a case, competing platforms, which have business stealing incentives, choose higher addictiveness than a monopolist. As a result, competition harms the consumer relative to monopoly. Conversely if the consumer does not face a tight attention constraint, a monopolist sets high addictiveness to capture more attention without discouraging consumer participation. Competition may then incentivize platforms to decrease addictiveness, leading to higher consumer welfare.

We relate our result to the impact of a merger. A merger to a monopoly could benefit or harm the consumer, depending on the scarcity of attention. We also identify a broader class of merger that harms the consumer by encouraging merged entities to increase addictiveness.

Our results have an implication on a digital curfew, which restricts the consumer's platform usage. For example, the Social Media Addiction Reduction Technology Act (the "SMART" Act) proposed in the US aims at curbing social media addiction by requesting that companies limit the time a user may spend on their services.[2] We capture a digital curfew as a reduction of the consumer's attention capacity. A digital curfew limits a platform's incentive to increase addictiveness to expand total attention, but it does not eliminate business stealing incentives. As a result, a digital curfew is more effective when attention is abundant and the market is less competitive.

The impact of competition also depends on the revenue models of platforms. To highlight the point, we compare the baseline model to a model of price competition, in which platforms earn

[2]See https://www.congress.gov/bill/116th-congress/senate-bill/2314 (accessed on November 24, 2020). Several other countries have implemented some restrictions to protect young students from addictive games. In 2003 Thailand implemented a shutdown law that banned young people from playing online games between 22:00 and 06:00. In 2011, South Korea passed a similar legislature, known as the Youth Protection Revision Act. In 2007, China introduced the so-called "fatigue" system under which game developers need to reduce or stop giving out rewards (e.g., game items, experience value) in games after a player reached certain hours of play.

revenue only by charging prices that are independent of the level of attention. In such a case, competition always benefits the consumer by reducing the equilibrium prices of services. Because platforms do not monetize attention, price competition also attains zero addictiveness. The consumer, however, can be better off under attention competition: The consumer faces high marginal utilities from addictive services, so she can earn a high incremental gain by refusing to join a platform and continuing to use other services. The better outside option encourages platforms to offer higher net utilities to the consumer under attention competition than price competition.

In our baseline model, the consumer correctly perceives the level of addictiveness. In practice, consumers may systematically underestimate the addictive features of platforms. We study such a naive consumer and extend our main insight that competition may harm the consumer. The naivete also increases equilibrium addictiveness, decreases her welfare, and render price competition more desirable than attention competition.

Related literature The paper relates to the literature on platform competition, in particular that for consumer attention (e.g., Rochet and Tirole 2003; Armstrong 2006; Anderson and De Palma 2012; Bordalo et al. 2016; Wu 2017; Evans 2017, 2019; Prat and Valletti 2019; Galperti and Trevino 2020; Anderson and Peitz 2020). Platforms in our model have a new strategic variable called addictiveness. It captures a firm's choice that degrades quality to capture attention, which we model as the increase in marginal utilities and the decrease in the level of utilities provided to consumers. The divergence between utilities and marginal utilities does not arise in competition on other dimensions, such as price, quality, and advertising load, in which they typically move in the same direction, or the allocation of attention is not explicitly modeled (e.g., Anderson and Coate 2005; de Corniere and Taylor 2020; Choi and Jeon 2020). Competition based on addictiveness generates our new insight: Competition for scarce attention could harm consumers. Finally, we abstract away from the two-sided aspect of the market, so the economic force that derives our insight differs from that of two-sided markets, in which competition for one side could harm other sides (e.g., Tan and Zhou 2020).

Second, the paper contributes to the nascent literature on possible negative impacts of digital services on consumers (Allcott and Gentzkow, 2017; Allcott et al., 2020; Mosquera et al., 2020). A recent discussion points out that technology companies may have an incentive to adopt features

(e.g., user interfaces) that increase user engagement at the expense of their welfare (Alter, 2017; Scott Morton et al., 2019; Newport, 2019; Rosenquist et al., 2020). We contribute to this literature by examining interactions between competition for attention and the addictiveness of digital services. Although we later motivate our model based on habit formation with a time-inconsistent agent, we largely abstract away from dynamics and behavioral biases relevant to addiction (Becker and Murphy, 1988; Gruber and Köszegi, 2001; Orphanides and Zervos, 1995). The abstraction allows us to provide a general intuition.

Finally, the recent policy and public debates recognize the problem that a firm that monetizes attention could distort its service quality to capture consumer attention (Crémer et al., 2019; U.K. Digital Competition Expert Panel, 2019; Scott Morton and Dinielli, 2020). We contribute to the discussion by providing a new intuition—that competition may mitigate or exacerbate the problem.

2 Model

There are $K \in \mathbb{N}$ platforms and a single consumer. We write K for the number and the set of the platforms. Suppose the consumer joins a set $K' \subset K$ of platforms, and allocates attention $a_k \geq 0$ to each platform $k \in K'$. If $K' = \emptyset$, she receives a payoff of zero. Otherwise, her payoff is

$$\sum_{k \in K'} u(a_k, d_k) - C\left(\sum_{k \in K'} a_k\right). \tag{1}$$

In the first term, $u(a_k, d_k)$ is the utility from platform k's service. The utility $u(a_k, d_k)$ depends on the *addictiveness* $d_k \in \mathbb{R}_+$ of platform k. We impose the following assumption (see Figure 1).

Assumption 1. The function $u(\cdot, \cdot) : \mathbb{R}^2_+ \to \mathbb{R}$ is continuously differentiable and satisfies the following:

(a) For every $d \geq 0$, utility $u(a, d)$ is strictly increasing and concave in a.

(b) For every $a \geq 0$, utility $u(a, d)$ is strictly decreasing in d, and $\max_{a \geq 0} u(a, d) - C(a) < 0$ for some d.

(c) For every $a \geq 0$, the marginal utility for attention $\frac{\partial u}{\partial a}(a, d)$ is strictly increasing in d.

(d) $u(0,0) \geq 0$.

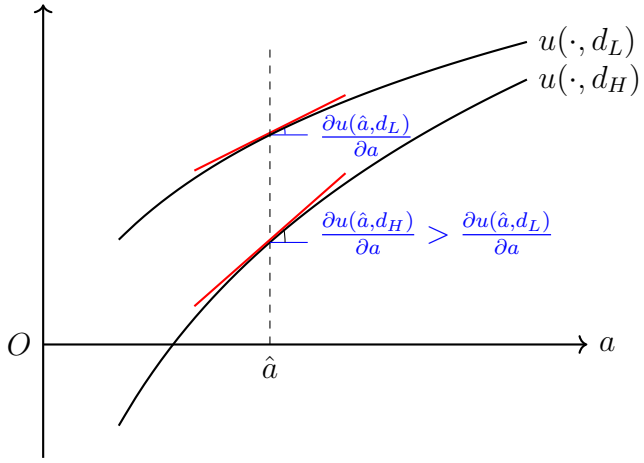

Figure 1: Utilities under d_L and $d_H > d_L$.

Points (b) and (c) imply that higher addictiveness decreases the consumer's utility of joining a platform but increases her marginal utility of allocating attention. Assumption 1 holds if, for example, $u(a,d) = (1+d)(1-e^{-\rho a}) - cd$ with $\rho > 0$ and $c > 1$ or $u(a,d) = v(a-d)$ with an increasing and concave $v(\cdot)$. Section 2.1 motivates the assumption.

The second term $C\left(\sum_{k \in K'} a_k\right)$ of the consumer's payoff (1) is the *attention cost*—e.g., the opportunity and cognitive costs of using services. We impose the following assumption.

Assumption 2. $C(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing, convex, and continuously differentiable.

The consumer also faces the *attention constraint*, which captures the scarcity of attention: The consumer can allocate the total attention of at most $\overline{A} \in (0, \infty]$ across platforms. The bound $\overline{A}$ comes from, for example, the consumer's preferences, physical constraints, and an exogenous restriction such as a digital curfew.

If the consumer allocates a to platform k, it earns a payoff of ra, where $r > 0$ is an exogenous value of attention to a platform (if the consumer does not join platform k, it receives a payoff of zero). For example, r is the unit price of attention in the (unmodeled) advertising market. Total surplus refers to the sum of the payoffs of the consumer and all platforms.

The timing of the game is as follows: First, each platform $k \in K$ simultaneously chooses its addictiveness, $d_k \geq 0$. Second, the consumer chooses which platforms to join and how much

attention to allocate. In equilibrium the consumer solves

$$\max_{\hat{K}\subset K,(a_k)_{k\in\hat{K}}} \sum_{k\in\hat{K}} u(a_k, d_k) - C\left(\sum_{k\in\hat{K}} a_k\right) \tag{2}$$
$$\text{s.t.} \quad \sum_{k\in\hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}.$$

Because $u(0, d) < 0$ for $d > 0$, the consumer's payoff of not joining platform k can differ from the payoff of joining but setting $a_k = 0$. Our solution concept is pure-strategy subgame perfect equilibrium, which we call *equilibrium*. Under monopoly, we study an equilibrium in which the platform breaks ties in favor of the consumer.[3]

2.1 Interpretation of Addictiveness d

The addictiveness d captures the choice of a firm that makes its service more capable of capturing attention at the expense of quality. We capture such choices as the changes of service utilities and marginal utilities provided to consumers. The paper is agnostic about a particular mechanism that cause such changes. However, we present two examples that illustrate how the changes of utilities and marginal utilities could occur.

2.1.1 Rational Addiction

We motivate our utility specification using a three-period model of rational addiction with a time-inconsistent consumer (e.g., Becker and Murphy 1988; Gruber and Köszegi 2001). Given addictiveness $(d_1, \ldots, d_K)$, consider the following problem (see Figure 2). In $t = 1$, the consumer chooses the set $K' \subset K$ of platforms to join. In $t = 2$, the consumer allocates attention $a_0 > 0$ and obtains utility $u_0 \geq 0$ on each platform in K'. This period is a "pre-addiction" stage—i.e., the consumer has yet to be addicted, and the service utilities and the optimal amount of attention

[3]Such a tie-breaking rule arises if the platform incurs a small cost of choosing positive addictiveness, which can be (unmodeled) costs of technological investment or reputational loss.

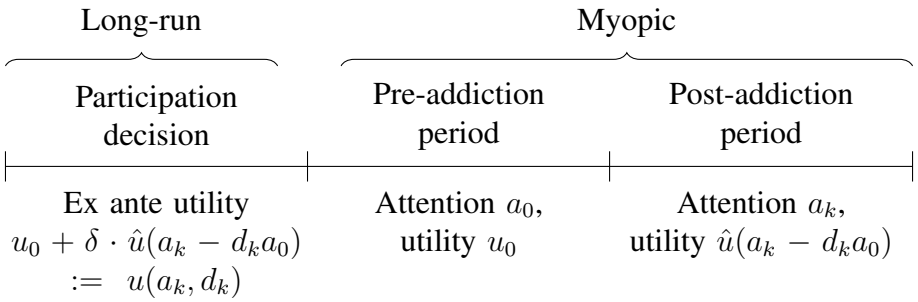

Figure 2: Three-period problem of the consumer

do not depend on $(d_1, \ldots, d_K)$.[4] In $t = 3$, the consumer allocates her attention across platforms in K'. This period is a "post-addiction" stage: If the consumer allocates attention a to platform k, she receives $\hat{u}(a - a_0 d_k)$, where $\hat{u}(\cdot)$ is an increasing concave function with $\hat{u}(0) \geq 0$. The payoff $\hat{u}(a - a_0 d_k)$ captures linear habit formation (e.g., Rozen, 2010). Here, $a_0 d_k$ is the reference point against which the consumer evaluates service consumption of platform k in $t = 3$. We can interpret $\frac{1}{d_k}$ as the "rate of disappearance of the physical and mental effects of past consumption" (Becker and Murphy, 1988). A higher d_k imposes a greater harm on the consumer in $t = 3$, and she needs to increase her attention in $t = 3$ to ensure the same payoff as in $t = 2$.

Motivated by dual-self models, we assume that the long-run self makes the participation decision and the short-run selves allocate attention (e.g., Thaler and Shefrin, 1981; Fudenberg and Levine, 2006). Specifically, in $t = 1$ the long-run self decides which platforms to join, anticipating the behavior of future selves: In $t = 2$ the short-run self allocates attention a_0 to each platform, then in $t = 3$ she allocates attention $(a_k^*)_{k \in K'}$ to maximize $\sum_{k \in K'} \hat{u}(a_k - a_0 d_k) - C\left(\sum_{k \in K'} a_k\right)$. Assume the long-run self has discount factor δ. The consumer's participation decision is based on the service utility $u(a_k, d_k) := u_0 + \delta \hat{u}(a_k - a_0 d_k)$, which satisfies Assumption 1.

Our model is suitable when a consumer is susceptible to addictive features of digital services, but she recognizes it and may avoid joining platforms as a commitment device. The model is not suitable for a consumer who joins platforms but can use them cautiously to avoid addiction. Such

[4] We do not need to specify the utility function the consumer faces in $t = 2$. However, to derive our functional form, we need to assume that a_0 does not depend on the number of platforms the consumer has joined. One way to endogenize such an outcome is to assume that the consumer's utility from each platform in $t = 2$ is $v(a)$ that is maximized at an interior optimum $a_0 \leq \overline{A}/K$. Alternatively, we can assume that the utility function in $t = 2$ is $u(a, 0)$, the attention cost is linear ($C(a) = ca$), and the attention constraint does not bind in the pre-addiction stage, i.e., $\arg\max_{a \geq 0} u(a, 0) - ca \leq \overline{A}/K$.

a situation would correspond to the consumer who is forward-looking in periods 2 and 3.

2.1.2 Data Collection and Personalization

The addictiveness d captures a firm's choice to degrade service quality for capturing attention, and it can apply to a situation that does not feature a typical "addiction." Suppose that a platform requests consumers to provide their personal data upon registration. Let d denote the amount of data the platform requests. To provide data, consumers incur a privacy cost—e.g., the risk of data leakage, identity theft, and discrimination. Suppose consumers incur a linear privacy cost, $\ell \cdot d$. The platform can use their data to personalize offerings, which increases the value of the service from the base value $w(a)$ to $(1+d)w(a)$, where $w(\cdot)$ is increasing, concave, and bounded. A consumer's utility from joining the platform is $u(a,d) := (1+d)w(a) - \ell d$. If $c > \sup_{a \geq 0} w(a)$, $u(a,d)$ satisfies Assumption 1. If consumers join platforms but do not use the services, they may receive a negative utility, because they experience the downside of data collection without enjoying the service.

2.2 Other Modeling Assumptions

Multi-homing. For any $K \geq 2$, the consumer can multi-home—e.g., they may divide time across social media, video streaming, and mobile applications, all of which monetize attention. If $K \geq 2$ but the consumer must single-home, all platforms set zero addictiveness in equilibrium.

Platform's revenue. A platform's payoff ra_k is an exogenous proportion of attention a_k from the consumer. The formulation allows us to focus on new economic forces by abstracting away from competition for the other side of the market. However, most of the results continue to hold in the following setting: If the consumer allocates attention $(a_1, \ldots, a_K)$, platform k earns a payoff of $r_k(a_1, \ldots, a_K)$ that is strictly increasing in a_k and may depend arbitrarily on $(a_j)_{j \in K \setminus \{k\}}$. For example, a platform's payoff captures its revenue in the advertising market, in which platforms can sell consumer attention at a market price.

Addictiveness reduces welfare. In practice, platforms may also adopt features that increase con-

sumer attention and their welfare.[5] To incorporate such features, suppose the consumer's utility from a platform is $u(a, d, b)$, where $u(a, d, b)$ and $\frac{\partial u}{\partial a}(a, d, b)$ are increasing in $b \in [0, 1]$. Since a higher b encourages the consumer to join a platform and allocates more attention, we can redefine $u(a, d) = u(a, d, 1)$ and apply our model.

2.3 The First-Best Outcomes

As a benchmark, we characterize the level of addictiveness that maximizes total or consumer surplus, when the consumer joins platforms and allocates attention to maximize her payoff. For any $d \geq 0$, let $CS(d)$ denote the maximized value of the consumer's problem (2) given $d_k = d$ for all k. It is the consumer's indirect utility when all platforms choose addictiveness d. Let $A(d)$ denote the total amount of attention she allocates to attain $CS(d)$. Note that for a high d, the consumer may not join some of the platforms. All omitted proofs are in the Appendix.

Claim 1. *Consumer surplus is maximized by $d_k = 0$ for all $k \in K$. Total surplus is maximized by $d^{TS} \in \arg\max_{d \geq 0} CS(d) + rA(d)$. If $A(0) < \overline{A}$, for a sufficiently large r, we have $d^{TS} > 0$.*

The consumer-optimal outcome is $d_k = 0$ because higher addictiveness lowers service quality. In contrast, total surplus may be maximized by $d > 0$.[6] Because the consumer does not internalize the value of attention to platforms, for a large r, she chooses an inefficiently low level of attention. In such a case, positive addictiveness increases attention and total surplus; the observation reflects the implicit two-sidedness of the market.

3 Equilibrium

We now characterize the equilibrium, compare it to the first-best, then examine the impact of competition. A monopoly platform may set zero addictiveness if attention is scarce. In contrast, competing platforms always set positive addictiveness because of the business stealing incentives.

[5] Hagiu and Wright (2020) study a model of dynamic competition with data-enabled learning. In one specification, higher past consumption leads to greater consumption utilities in the future, which resembles beneficial addiction.

[6] If the welfare-maximizing social planner could force the consumer to spend more attention than she would to maximize her utility, the planner would choose zero addictiveness and allocate consumer attention to maximize total surplus. However, a regulator would not have such control in practice.

3.1 Monopoly ($K = 1$)

A monopolist maximizes attention subject to the consumer's participation constraint. Let $d^P(\overline{A})$ denote the highest addictiveness that satisfies the participation constraint—i.e., $\max_{A\in[0,\overline{A}]} u(A,d) - C(A) = 0$. Also let $A(d) := \arg\max_{A\geq 0} u(A,d) - C(A)$ denote the consumer's unconstrained choice of attention, which is independent of $\overline{A}$. We then define $d^A(\overline{A}) := \min\left\{d \in [0,\infty] : A(d) \geq \overline{A}\right\}$, which is the lowest addictiveness under which the consumer exhausts her attention. The following result characterizes the monopoly equilibrium and presents comparative statics with respect to the scarcity of attention, $\overline{A}$.

Proposition 1. *In equilibrium, the monopolist sets addictiveness* $\min(d^A(\overline{A}), d^P(\overline{A}))$*, which increases in* $\overline{A}$*. There is an* $\overline{A}^M > A(0)$ *such that the following holds. As a function of* $\overline{A}$*, the consumer's equilibrium payoff is increasing on* $[0, A(0)]$*, decreasing on* $[A(0), \overline{A}^M]$*, and equal to zero on* $[\overline{A}^M, \infty]$*. In particular, if* $\overline{A} \leq A(0)$*, the equilibrium maximizes consumer and total surplus.*

In Figure 3, the blue solid line depicts the consumer's equilibrium payoff under monopoly as a function of her attention capacity $\overline{A}$. A monopolist's incentive depends on the scarcity of attention. If the attention constraint is tight, the consumer exhausts her attention capacity $\overline{A}$ at zero addictiveness. In such a case, the monopolist sets $d = 0$, which maximizes consumer and total surplus. As $\overline{A}$ increases beyond $A(0)$, the monopolist increases addictiveness to incentivize the consumer to spend more attention. Although a higher $\overline{A}$ expands the consumer's choice, the increased addictiveness reduces the service utility and harms the consumer. For a large $\overline{A} \geq \overline{A}^M$, the monopolist raises addictiveness to increase consumer attention until she becomes indifferent between joining and not joining the platform.

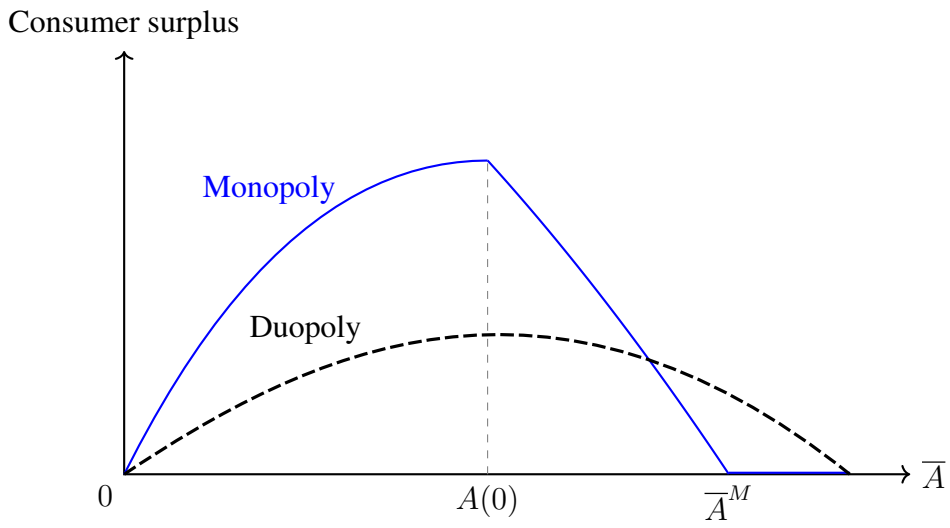

Figure 3: Consumer surplus under monopoly and duopoly

Note: The graph uses $u(a,d) = 1 - \rho e^{-\rho(a-d)}$ and $C(a) = \frac{c}{2}a^2$ with $(\rho, c) = (2, 1)$.

3.2 Competition ($K \geq 2$)

For each $K \geq 2$, define

$$A_K(d) := \arg \max_{A \in [0, \overline{A}]} K u\left(\frac{A}{K}, d\right) - C(A).$$

The consumer will choose total attention $A_K(d)$ if she joins K platforms with addictiveness d. The following result characterizes the equilibrium.

Proposition 2. *Fix any $K \geq 2$. In a unique equilibrium, all platforms choose positive addictiveness $d^* > 0$ that makes the consumer indifferent between joining and not joining each platform:*

$$K \cdot u\left(\frac{A_K(d^*)}{K}, d^*\right) - C\left(A_K(d^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(d^*)\right). \quad (3)$$

The equilibrium never maximizes consumer surplus.

The intuition is as follows. Upon choosing addictiveness, each platform faces a trade-off. On the one hand, higher addictiveness renders its service less attractive to the consumer. On the other hand, conditional on joining, she will allocate more attention to more addictive services. Each

platform then prefers to increase its addictiveness so long as the consumer joins it. The equilibrium addictiveness makes the consumer indifferent between joining and not joining each platform.

The equilibrium does not maximize consumer surplus because platforms choose positive addictiveness. In contrast, it is ambiguous whether the equilibrium addictiveness exceeds the welfare-maximizing level in Claim 1. The equilibrium addictiveness is determined by the consumer's indifference condition and is independent of r, but the welfare-maximizing level can depend on r. As a result, the equilibrium addictiveness can be higher or lower than the welfare-maximizing level. For example, if r is high but platforms decrease addictiveness, the consumer reduces total attention, which may decrease total surplus.

3.3 Monopoly vs. Duopoly

We now turn to the main question: Does competition benefit the consumer? To begin with, we compare monopoly to duopoly. Note that if monopoly and duopoly platforms set the same addictiveness, duopoly is trivially better for the consumer because she can use more services. However, in equilibrium the impact of competition depends on the scarcity of attention. Recall that $A(0)$ is the consumer's attention choice on a monopoly platform with zero addictiveness, and $\overline{A}^M$ is a threshold such that for any $\overline{A} \geq \overline{A}^M$, consumer surplus is zero under monopoly.

Proposition 3. *Compare monopoly to duopoly. If attention is so scarce that $\overline{A} \leq A(0)$ holds, the consumer is strictly better off under monopoly. If $\overline{A} \geq \overline{A}^M$, the consumer is weakly better off under duopoly.*

Proof. If $\overline{A} \leq A(0)$ the monopolist chooses zero addictiveness (Proposition 1). Under duopoly the consumer's payoff equals the payoff from joining a single platform, which now chooses positive addictiveness. As a result, the consumer is strictly better off under monopoly. If $\overline{A} \geq \overline{A}^M$, the consumer receives a payoff of zero under monopoly but a nonnegative payoff under duopoly. □

Figure 3 depicts consumer surpluses under monopoly and duopoly. When the attention constraint is tight, higher addictiveness does not increase total attention, so a monopoly platform sets zero addictiveness. Each of competing platforms, however, benefits from higher addictiveness because the consumer will allocate a greater fraction of her total attention $\overline{A}$ to more addictive

services. As a result, competition for attention can increase addictiveness and decrease consumer surplus, despite the benefit of providing more services to the consumer.

A natural question is how competition beyond duopoly affects addictiveness and welfare. For example, does a "sufficiently competitive" market attain zero addictiveness? To answer the question, we could compare the equilibria across different K's. The approach, however, has two problems. First, the limit economy ($K \to \infty$) may not be well-defined, because the equilibrium objects may diverge along the sequence. Second, a higher K, which implies more services, mechanically favors competition. The next section studies a sequence of markets with a fixed size, so that we can meaningfully talk about the competitive limit of our model.

4 The Impact of Competition with a Fixed Market Size

We study a sequence of markets, $(\mathcal{E}_K)_{K\in\mathbb{N}}$. The market $\mathcal{E}_1$ consists of a monopolist that provides service utility $u(a,d)$. For each $K \geq 2$ the market $\mathcal{E}_K$ consists of K platforms, each of which provides service utility $\frac{1}{K}u(aK,d)$. The markets $(\mathcal{E}_K)_{K\in\mathbb{K}}$ have the same size: If the consumer allocates total attention A equally across K platforms, she obtains total utility $u(A,d)$ regardless of K. The converse is also true: In any market with such a property, each platform provides utility $\frac{1}{K}u(aK,d)$.[7] Thus so long as we focus on symmetric markets with a constant size, our choice is unique. In all markets $(\mathcal{E}_K)_{K\in\mathbb{K}}$, the consumer faces the same attention cost and constraint $(C(\cdot),\overline{A})$. We fix the market size and study an increasing number of platforms. A similar approach appears in papers on the relation between perfectly and imperfectly competitive equilibria in product markets (e.g., Novshek, 1980, 1985; Allen and Hellwig, 1986).

The following result shows that competition beyond duopoly reduces but never eliminates addictive services. We define the consumer's best response as $A(d) := \arg\max_{A\in[0,\overline{A}]} u(A,d) - C(A)$.

Proposition 4. *For any $K \geq 2$, market $\mathcal{E}_K$ has a unique equilibrium, in which all platforms choose the same positive addictiveness that is decreasing in K. As $K \to \infty$, the equilibrium addictiveness*

[7] If utility function $v(a,d)$ has the property that the consumer obtains $u(A,d)$ by allocating A/K to each of K platforms, we have $Kv(\frac{a}{K},d) = u(a,d)$, which implies $v(a,d) = \frac{1}{K}u(aK,d)$.

converges to $d^\infty > 0$ *that solves*

$$u\left(A(d^\infty), d^\infty\right) = A\left(d^\infty\right) \cdot \frac{\partial u}{\partial a}\left(A(d^\infty), d^\infty\right). \tag{4}$$

Proposition 4 states that once we go beyond duopoly, competition reduces addictiveness but never eliminates it. If the market consists of many platforms, the consumer can avoid highly addictive services and allocate her attention to less addictive services. To attract the consumer who has better outside options, platforms need to reduce addictiveness and offer higher service quality. However, the business stealing incentives never vanish, so platforms set positive addictiveness in the limit. Equation (4) captures the intuition in the limiting case. The left-hand side $u\left(A(d^\infty), d^\infty\right)$ is the consumer's loss of not joining a platform, and the right-hand side $A\left(d^\infty\right) \cdot \frac{\partial u}{\partial a}\left(A(d^\infty), d^\infty\right)$ is the incremental gain of reallocating the saved attention. In equilibrium the two terms coincide.

The standard antitrust argument proxies welfare with output. The result suggests that the argument could fail in the attention economy if platforms can sacrifice service quality to capture consumer attention. Indeed, so long as $K \geq 2$ the consumer spends less time on services and receives higher payoffs in a more competitive market.

Proposition 4 allows us to establish an analogue of Proposition 3: The consumer is better off under monopoly than the limit economy if the attention is scarce. To state the result, for any $\overline{A} > 0$ and $K \in \mathbb{N}$, let $CS_K(\overline{A})$ denote the consumer surplus in the equilibrium of $\mathcal{E}_K$, and let $CS_\infty(\overline{A})$ denote the one in the limit economy, i.e., $CS_\infty(\overline{A}) = \max_{A \in \left[0, \overline{A}\right]} u\left(A, d^\infty\right) - C(A)$. Recall that $A(0)$ is the consumer's choice of attention on a monopoly platform with zero addictiveness.

Corollary 1. *Compare monopoly to the limit economy.*

1. *If* $\overline{A} < A(0)$*, the consumer is strictly better off under monopoly:* $CS_1(\overline{A}) > CS_\infty(\overline{A})$.
2. *There is an* $A^* > 0$ *such that if* $\overline{A} > A^*$*, the consumer is weakly better off in the limit economy:* $CS_1(\overline{A}) \leq CS_\infty(\overline{A})$.

4.1 Monopoly vs. Competition: Full Comparison

We now restrict consumer preferences to obtain sharper conclusions. The following results present the welfare implications, and we relegate the analytical expression of the equilibrium objects to

the appendix. First, we provide a condition under which competition never benefits the consumer.

Proposition 5 (Linear Attention Cost). *Assume $C(a) = ca$ for some $c > 0$, and $u(a,d) = v(a-d)$ for an increasing concave $v(\cdot)$ with $v'(0) > c$. Competition never benefits the consumer. Formally, let $g = (v')^{-1}$ denote the inverse of v'. Then the following holds.*

1. *If $\overline{A} < \frac{v(g(c))}{c}$, consumer surplus is strictly higher under monopoly than any other market $(\mathcal{E})_{K\in\mathbb{N}}$.*
2. *If $\overline{A} \geq \frac{v(g(c))}{c}$, consumer surplus is zero in any market $(\mathcal{E})_{K\in\mathbb{N}}$.*

Figure 4 depicts the consumer's equilibrium payoffs under monopoly and the limit economy. Corollary 1 suggests that the consumer could benefit or lose from competition, depending on her attention capacity $\overline{A}$. Proposition 5 shows that the welfare comparison could be unambiguous, i.e., competition may weakly harm the consumer across all parameters.

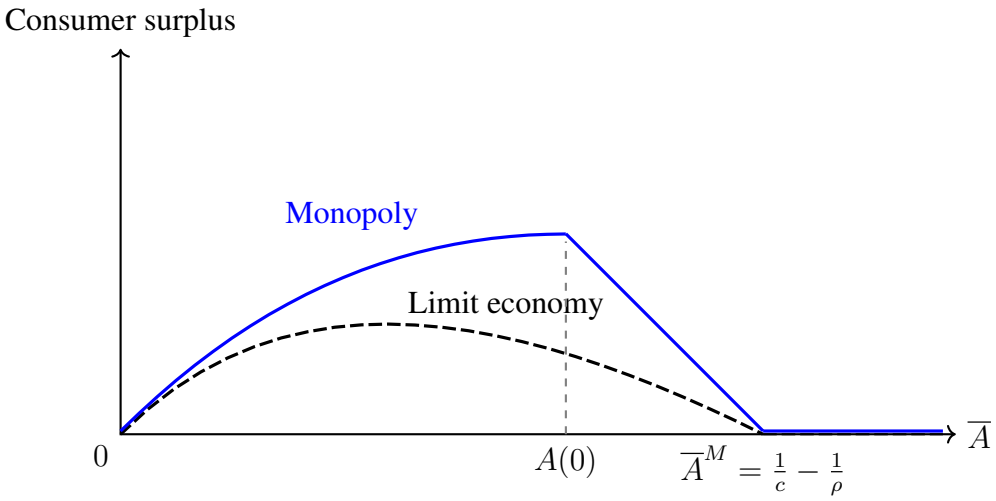

Figure 4: Consumer surpluses in the linear environment

Note: The graph uses $u(a,d) = 1 - e^{\rho(a-d)}$ and $C(a) = ca$ with $(\rho, c) = (2, 1)$, where $A(0) \approx 0.346$ and $\frac{1}{c} - \frac{1}{\rho} = 0.5$.

The following result presents a case in which we obtain a strict welfare comparison for almost all parameters (see Figure 5).

Proposition 6 (Quadratic Attention Cost with Exponential Utility). *Assume $C(a) = \frac{ca^2}{2}$ for some $c > 0$, and $u(a,d) = 1 - e^{-\rho(a-d)}$ for some $\rho > c$. Consumer surplus is greater under*

monopoly than the limit economy if and only if $\overline{A} \leq \overline{A}^* := \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}$. *The welfare comparison is strict whenever* $\overline{A} \neq \overline{A}^*$.

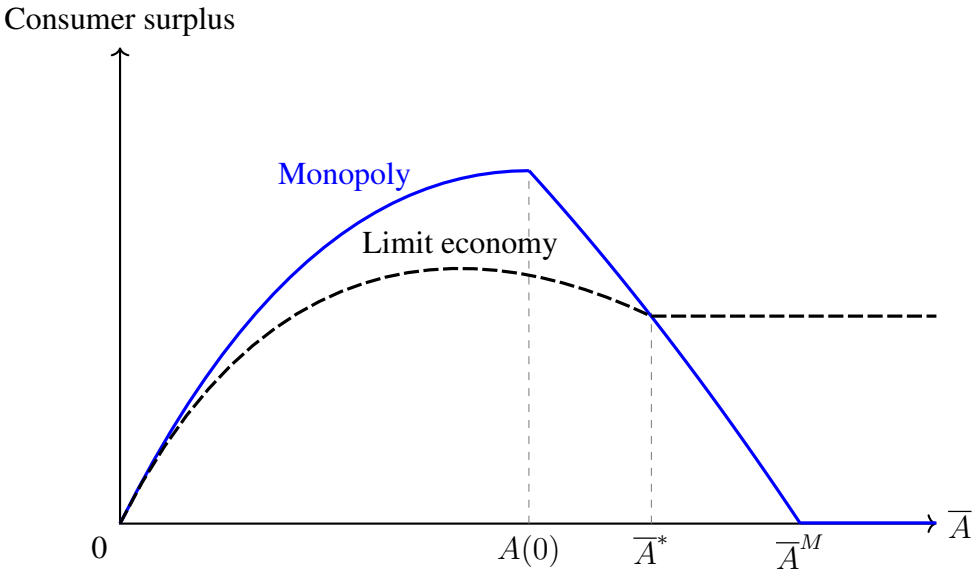

Figure 5: Consumer surpluses under the quadratic attention cost

Note: The graph uses $(\rho, c) = (2, 1)$, where $A(0) \approx 0.601$, $\overline{A}^* \approx 0.781$, and $\overline{A}^M = 1$.

Recall that under monopoly, the consumer's equilibrium payoff is non-monotone in her attention capacity $\overline{A}$ (Proposition 1). The results of this section show that the same non-monotonicity holds in the limit economy. Competition does not eliminate a platform's incentive to raise addictiveness to influence the consumer's choice of total attention.

4.2 The Implication on Platform Merger

A merger may reduce business stealing incentives of platforms and increase the number of higher quality services that are less addictive. Our results capture the idea by showing that a merger to a monopoly benefits the consumer if the attention is scarce. The conclusion is stark, but the intuition—that a merger could reduce firms' incentive to sacrifice service quality to capture attention—would be relevant in a broader context. The observation contrasts with the idea that a

merger to monopoly is more harmful than other types of mergers.[8]

Our results also capture the standard negative impact of a merger: A merged platform can bundle their services and request consumers to choose between joining all services or none. Such a merger may reduce the consumer's outside option, allow the merged platform to increase addictiveness, and reduce consumer welfare. Appendix E generally defines pre-merger and post-merger games, then provides a class of mergers that harms consumers. For example, if $K-1$ out of K platform merge to form a single platform, the merged entity increases addictiveness, the non-merged entity reduces addictiveness, and consumer surplus decreases.

5 Digital Curfew

Our results have an implication on a digital curfew, which restricts the consumer's platform usage. Under a digital curfew at A, the consumer's attention cap becomes $\overline{A} = A$. Recall $A(0)$ denotes the consumer's optimal attention on a monopoly platform with zero addictiveness.

Proposition 7. *A digital curfew is more effective in a less competitive market:*

1. *In a monopoly market, a digital curfew at $\overline{A} = A(0)$ attains the consumer-optimal outcome.*
2. *For any $K \geq 2$, no digital curfew attains the consumer-optimal outcome.*
3. *Take any $K \geq 2$ and $\overline{A}$. Suppose the attention constraint holds with a strict inequality in equilibrium. Then a digital curfew at some $A_D < \overline{A}$ strictly benefits the consumer.*

The intuition is that a digital curfew reduces a platform's incentive to increase addictiveness to expand the consumer's total attention, but it does not eliminate business stealing incentives. For example, consider a digital curfew at $\overline{A} = A(0)$, which prevents the consumer from spending longer time on digital services than how much she would have spent if the services had zero addictiveness. Under monopoly, such a digital curfew makes it optimal for the platform to set zero addictiveness. The same digital curfew, however, does not eliminate business stealing incentives, because the

[8]For example, Competition Policy Guidance of the Federal Trade Commission states that a unilateral anticompetitive effect of a merger is most obvious in the case of a merger to monopoly. See https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/mergers/competitive-effects (accessed on February 10, 2021).

consumer will allocate a greater fraction of her attention to more addictiveness platforms, even if the total attention is fixed.

Two remarks are in order. First, we have examined a policy that limits total attention across platforms, but it is not the only way to define a digital curfew. For example, suppose a regulator could require that the consumer spend at most $A(0)/K$ unit of attention on *each* platform in $\mathcal{E}_K$. The regulator can then induce zero addictiveness.

Second, we assume a digital curfew is exogenous to the consumer, but we could ask whether consumers are willing to adopt a digital curfew voluntarily. Suppose that there is a continuum of consumers, each of whom $i \in [0,1]$ chooses the maximum amount of attention $A_i \in [0, A_{max}]$ she can spend on platforms ($A_{max} > 0$ is an exogenous cap on possible attention constraints). After consumers choose $(A_i)_{i\in[0,1]}$, the original game of attention competition is played.[9] In equilibrium, all consumers choose the maximum attention A_{max}, because each consumer is atomless and her choice does not affect the behavior of platforms. Consumers cannot voluntarily enforce a digital curfew, even though they could benefit from collectively reducing A_i's.

6 Price Competition and Attention Competition

We have shown that competition for attention could harm the consumer. The result depends on the revenue models of platforms. To highlight the idea, we study the following model of price competition. As in Section 4, we describe the model of price competition by keeping the market size constant. The game of price competition in market $\mathcal{E}_K$ is as follows. First, each platform $k \in K$ simultaneously chooses its addictiveness $d_k \geq 0$ and price $p_k \in \mathbb{R}$. The consumer observes $(d_k, p_k)_{k\in K}$, then chooses the set $\hat{K} \subset K$ of platforms to join and how much attention to allocate. The consumer has to pay price p_k to join platform k. Each platform $k \in \hat{K}$ receives a payoff of p_k, and any platform $k \notin \hat{K}$ obtains a payoff of zero. The consumer receives a payoff of $\sum_{k\in\hat{K}} \frac{1}{\hat{K}} [u(Ka_k, d_k) - p_k] - C\left(\sum_{k\in\hat{K}} a_k\right)$ if $\hat{K} \neq \emptyset$ and zero if $\hat{K} = \emptyset$ (recall that in $\mathcal{E}_K$, the

[9] If platform k obtains attention a_k^i from each consumer i, then k's profit is $\int_{i\in[0,1]} a_k^i$.

service utility of each platform is $\frac{1}{K}u(Ka,d))$. In equilibrium, the consumer solves

$$\max_{\hat{K}\subset K,(a_k)_{k\in\hat{K}}} \sum_{k\in\hat{K}} \left[\frac{1}{K}u(Ka_k,d_k)-p_k\right] - C\left(\sum_{k\in\hat{K}} a_k\right) \tag{5}$$
$$\text{s.t.} \quad \sum_{k\in\hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K},$$

where the objective is zero if $\hat{K}=\emptyset$.

Under price competition, platforms do not monetize attention, and they charge prices that are independent of consumer attention. The model captures digital services not supported by advertising, such as Netflix and YouTube Premium.[10] The following result characterizes the equilibrium under price competition.

Lemma 1. *The game of price competition has a unique equilibrium, in which all platforms choose zero addictiveness and set the same positive price that makes the consumer indifferent between joining K and $K-1$ platforms.*

Under price competition the profits of platforms do not depend on attention, so they prefer to decrease addictiveness and charge higher prices. In equilibrium all platforms set zero addictiveness, and price p^* equals the incremental contribution of each platform to the consumer's total payoff. In particular, a monopoly platform extracts full surplus from the consumer, so competition always benefits the consumer.

Corollary 2. *Consider price competition. For any $K \geq 2$, consume surplus in market $\mathcal{E}_K$ is weakly greater than the one under monopoly.*

We now compare different business models from the consumer's perspective. The consumer can use services for free under attention competition, but the service quality is typically lower than under price competition. The following result provides sufficient conditions under which the consumer is better off under attention competition.

[10] We do not consider the endogenous choice of business models or richer pricing instruments that may use allocated attention to determine a price. For recent studies on business models in two-sided markets, see, e.g., Gomes and Pavan (2016), Lin (2020), Carroni and Paolini (2020), and Jeon et al. (2021).

Proposition 8. *The consumer is strictly better off under attention competition than price competition in market $\mathcal{E}_K$ if any of the following holds:*

1. *The attention cost $C(\cdot)$ is strictly convex and the market is sufficiently competitive—i.e., K is greater than some cutoff $K^* \in \mathbb{N}$.*

2. *The market is monopoly and the attention is scarce—i.e., $K = 1$ and $\overline{A} \leq A(0)$.*

The intuition for Point 1 is as follows. Under attention competition, platforms set positive addictiveness, so the consumer faces higher marginal utilities of allocating attention. The consumer then faces a higher gain of refusing to join a platform and continuing to use other $K-1$ platforms. For example, if the consumer spends total attention A on platforms, she can increase her attention to each platform $j \neq 1$ from $\frac{A}{K}$ to $\frac{A}{K-1}$ by not joining platform 1. The gain of doing so is higher when services are more addictive. If the consumer faces a higher gain of not joining each platform under attention competition, platforms must provide high utilities to ensure consumer participation. As a result, consumer surplus is higher under attention competition. The actual proof is more subtle, because the consumer faces a steeper utility function under attention than price competition, but she evaluates these functions at different levels of attention. However, in a sufficiently competitive market, higher marginal utilities ensure a higher consumer surplus under attention competition.

Attention competition dominates price competition in terms of consumer welfare also in the monopoly market with scarce attention. In such a case, the equilibrium attains the consumer-optimal outcome under attention competition and the worst outcome under price competition.

7 Extension: Naive Consumer

In practice, consumers may be unaware of the (part of) addictive features of platforms. We now consider such a naive consumer and show the robustness of our results.[11] We extend the model of Section 2 as follows (Appendix H provides details). In the first stage, all platforms simultaneously

[11] Our extension is different from a model in which the consumer does not observe addictiveness but correctly anticipates it in equilibrium. Our consumer naivete relates to "uninformed myopes" in Gabaix and Laibson (2006), whereby consumers do not recognize the full price of a product before making actual purchase decisions.

choose addictiveness, $(d_k)_{k\in K}$. In the second stage, the consumer decides which platforms to join, *based on the perceived addictiveness,* $(sd_k)_{k\in K}$. The parameter $s \in (0, 1]$ is exogenous and captures the degree of consumer sophistication. If (and only if) $s < 1$, the consumer falsely thinks that the addictiveness of each platform is lower than the true value. In the third stage, after joining platforms the consumer allocates attention to maximize her utility based on the true addictiveness. The consumer's attention allocation problem is the same as (2) except she has chosen the set of platforms to join based on the perceived addictiveness. Consumer surplus now refers to her payoff based on the true addictiveness.

If the consumer is naive, competing platforms find it more profitable to increase its addictiveness: A platform can then capture a greater share of attention without much affecting the consumer's participation decision, because she does not recognize the full change of addictiveness. The following result shows that if attention is scarce, the consumer naivete leads to higher addictiveness and lower consumer welfare under competition but not under monopoly.

Proposition 9. *Suppose the attention is scarce—i.e.,* $\overline{A} \leq A(0)$. *In equilibrium, a monopoly platform chooses zero addictiveness, but competing platforms choose addictiveness* $\frac{d^*}{s} > 0$*, where* d^* *is the equilibrium addictiveness under the sophisticated consumer in Proposition 2. In such case, for any* $K \geq 2$ *a consumer with a sufficiently small* s *is strictly better off under monopoly than under the market with* K *platforms (even if the market size is not constant).*

Appendix H extends other results to the case of a naive consumer. First, consumer surplus is increasing in s under attention competition but independent of s under price competition. As a result, for any $K \geq 2$, a sufficiently naive consumer is better off under price competition than attention competition when there are multiple platforms. Second, the consumer's naivete could eliminate the beneficial impact of a digital curfew that reduces the equilibrium addictiveness. Naive consumers think that a digital curfew does not affect her payoff, because they believe that they will not spend much time on platforms. Such a wrong expectation reduces platforms' incentive to lower addictiveness to encourage participation.

8 Conclusion

We study how firms compete for consumer attention by choosing the addictiveness of their services. The model captures a firm's choices that sacrifice service quality to capture attention. The main takeaway is that the impact of competition depends on the scarcity of attention, and when attention is scarce, competition could increase addictiveness and lower consumer welfare because of business stealing incentives. We demonstrate that the negative impact of competition does not arise under the standard price competition. A digital curfew could improve consumer welfare because such a policy eliminates part of a platform's incentive to increase addictiveness. However, such a policy may not eliminate business stealing incentive, in which case our qualitative insight continues to be relevant. To our best knowledge, the paper is the first to theoretically study competition and digital addiction, which have important policy implications.

We close the paper with several directions for future research. First, the literature points to behavioral biases that are relevant to addiction, so it would be promising to incorporate them into a model of competition for attention in which firms may exploit behavioral biases to capture attention. Second, it appears worth studying platforms' choices of business models and their implications on addictiveness and welfare. For example, a firm may offer both ad-supported and subscription plans to screen consumers who have different preferences. From the empirical perspective, the most relevant question is what features of digital services correspond to the "addictiveness" of our model. Also, little seems known about how consumers allocate their attention across multiple digital services and how their attention responds to various features of platforms. Building upon our work, we anticipate further studies on various intriguing questions on theoretical and empirical fronts.

References

Allcott, H., L. Braghieri, S. Eichmeyer, and M. Gentzkow (2020). The welfare effects of social media. *American Economic Review 110*(3), 629–76.

Allcott, H. and M. Gentzkow (2017). Social media and fake news in the 2016 election. *Journal of Economic Perspectives 31*(2), 211–36.

Allen, B. and M. Hellwig (1986). Bertrand-edgeworth oligopoly in large markets. *Review of Economic Studies 53*(2), 175–204.

Alter, A. (2017). *Irresistible: The rise of addictive technology and the business of keeping us hooked*. Penguin.

Anderson, S. P. and S. Coate (2005). Market provision of broadcasting: A welfare analysis. *Review of Economic Studies 72*(4), 947–972.

Anderson, S. P. and A. De Palma (2012). Competition for attention in the information (overload) age. *RAND Journal of Economics 43*(1), 1–25.

Anderson, S. P. and M. Peitz (2020). Ad clutter, time use and media diversity. *Anderson, Simon P. and Peitz, Martin, Ad Clutter, Time Use, and Media Diversity (August 2020). CEPR Discussion Paper No. DP15130, Available at SSRN: https://ssrn.com/abstract=3674907*.

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics 37*(3), 668–691.

Becker, G. S. and K. M. Murphy (1988). A theory of rational addiction. *Journal of Political Economy 96*(4), 675–700.

Bordalo, P., N. Gennaioli, and A. Shleifer (2016). Competition for attention. *Review of Economic Studies 83*(2), 481–513.

Carroni, E. and D. Paolini (2020). Business models for streaming platforms: Content acquisition, advertising and users. *Information Economics and Policy 52*(C).

Choi, J. P. and D.-S. Jeon (2020). Two-sided platforms and biases in technology adoption. *TES Working Paper n.20-1143*.

Crémer, J., Y.-A. de Montjoye, and H. Schweitzer (2019). Competition policy for the digital era. *Report for the European Commission*.

de Corniere, A. and G. Taylor (2020, February). Data and competition: a general framework with applications to mergers, market structure, and privacy policy. *TES Working Paper n.20-1076*.

Evans, D. S. (2017). The economics of attention markets. *Available at SSRN 3044858*.

Evans, D. S. (2019). Attention platforms, the value of content, and public policy. *Review of Industrial Organization 54*(4), 775–792.

Fudenberg, D. and D. K. Levine (2006). A dual-self model of impulse control. *American Economic Review 96*(5), 1449–1476.

Gabaix, X. and D. Laibson (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics 121*(2), 505–540.

Galperti, S. and I. Trevino (2020). Coordination motives and competition for attention in information markets. *Journal of Economic Theory*, 105–139.

Gomes, R. and A. Pavan (2016). Many-to-many matching and price discrimination. *Theoretical Economics 11*(3), 1005–1052.

Gruber, J. and B. Köszegi (2001). Is addiction "rational"? theory and evidence. *The Quarterly Journal of Economics 116*(4), 1261–1303.

Hagiu, A. and J. Wright (2020). Data-enabled learning, network effects and competitive advantage. *Working paper*.

Jeon, D.-S., B.-C. Kim, and D. Menicucci (2021, forthcoming). Second-degree price discrimination by a two-sided monopoly platform. *American Economic Journal: Microeconomics*.

Lin, S. (2020). Two-sided price discrimination by media platforms. *Marketing Science 39*(2), 317–338.

Milgrom, P. and I. Segal (2002). Envelope theorems for arbitrary choice sets. *Econometrica 70*(2), 583–601.

Mosquera, R., M. Odunowo, T. McNamara, X. Guo, and R. Petrie (2020). The economic effects of facebook. *Experimental Economics 23*(2), 575–602.

Newport, C. (2019). *Digital minimalism: Choosing a focused life in a noisy world*. Penguin.

Novshek, W. (1980). Cournot equilibrium with free entry. *Review of Economic Studies 47*(3), 473–486.

Novshek, W. (1985). Perfectly competitive markets as the limits of cournot markets. *Journal of Economic Theory 35*(1), 72–82.

Orphanides, A. and D. Zervos (1995). Rational addiction with learning and regret. *Journal of Political Economy 103*(4), 739–758.

Prat, A. and T. M. Valletti (2019). Attention oligopoly. *Available at SSRN 3197930*.

Rochet, J.-C. and J. Tirole (2003). Platform competition in two-sided markets. *Journal of the European Economic Association 1*(4), 990–1029.

Rosenquist, J. N., F. Scott Morton, and S. N. Weinstein (2020). Addictive technology and its implications for antitrust enforcement. *Available at* https://som.yale.edu/sites/default/files/Addictive-Technology.pdf.

Rozen, K. (2010). Foundations of intrinsic habit formation. *Econometrica 78*(4), 1341–1373.

Rudin, W. (1976). *Principles of mathematical analysis*, Volume 3. McGraw-hill New York.

Scott Morton, F., P. Bouvier, A. Ezrachi, B. Jullien, R. Katz, G. Kimmelman, A. D. Melamed, and J. Morgenstern (2019). Report: Committee for the study of digital platforms-market structure and antitrust subcommittee. *George J. Stigler Center for the Study of the Economy and the State, The University of Chicago Booth School of Business*.

Scott Morton, F. and D. Dinielli (2020). Roadmap for an antitrust case against facebook. Technical report, Omidyar Network.

Tan, G. and J. Zhou (2020). The effects of competition and entry in multi-sided markets. *The Review of Economic Studies*.

Thaler, R. H. and H. M. Shefrin (1981). An economic theory of self-control. *Journal of Political Economy 89*(2), 392–406.

U.K. Digital Competition Expert Panel (2019). Unlocking digital competition: Report of the digital competition expert panel. *Report prepared for the Government of the United Kingdom, March*.

Wu, T. (2017). Blind spot: The attention economy and the law. *Antitrust Law Journal 82*.

Appendix

A Proof of Proposition 1

Proof. Suppose the monopolist chooses $d^M := \min(d^A, d^P)$. Because $d^M \leq d^P$, it is optimal for the consumer to join the platform. If $d^M = d^P$ and the monopolist increases addictiveness, the consumer will not join it. If $d^M = d^A$ and the monopolist increases addictiveness, the consumer will continue to choose $\overline{A}$ because her marginal utility of allocating attention is increasing in d. The monopolist then continues to earn the same payoff, $r\overline{A}$. In either case the monopolist does not strictly benefit from changing d^M. Because $d^P(\overline{A})$ and $d^A(\overline{A})$ are increasing in $\overline{A}$, d^M is increasing in $\overline{A}$.

To show the remaining part, we say that the participation constraint binds if the consumer's payoff of zero. We also say that the attention constraint is slack if the consumer chooses $A(d^M(\overline{A})) \leq \overline{A}$, that is, the consumer's unconstrained choice of attention at $d^M(\overline{A})$ satisfies the attention constraint. Note that the attention constraint can hold with equality and be slack at the same time.

First, take any $\overline{A}^1$ at which the participation constraint binds. If the attention constraint is not slack (i.e., $A(d^M(\overline{A}^1)) > \overline{A}^1$), the monopoly could slightly lower addictiveness to attain the same payoff $r\overline{A}^1$. This contradicts the tie-breaking rule of the monopolist (see Section 2). Thus if the participation constraint binds, the attention constraint is slack. As a result we have $\max_{a\geq 0} u(a, d^M(\overline{A}^1)) - C(a) = 0$, i.e., the consumer's optimal payoff from the (hypothetical) unconstrained problem is equal to zero. Take any $\overline{A}^2 > \overline{A}^1$. We have

$$\max_{a\in[0,\overline{A}^2]} u(a, d^M(\overline{A}^2)) - C(a) \leq \max_{a\geq 0} u(a, d^M(\overline{A}^1)) - C(a) = 0,$$

because in the right-hand side, the consumer does not face the attention constraint and the platform

chooses lower addictiveness. As a result the participation constraint also binds at $\overline{A}^2$.

Let $\overline{A}^M$ denote the cutoff such that the participation constraint binds if and only if $\overline{A} \geq \overline{A}^M$. First, for any $\overline{A} \leq A(0)$, the consumer chooses $\overline{A}$ at $d^M = 0$, so it is indeed the monopolist's equilibrium choice. The consumer's equilibrium payoff is increasing in $\overline{A}$ whenever $\overline{A} \leq A(0)$ because the consumer faces the same addictiveness but the more relaxed constraint under a higher $\overline{A}$. Because the consumer earns a positive payoff at $d^M = 0$, we have $\overline{A}^M > A(0)$. Second, for any $\overline{A} \in [A(0), \overline{A}^M)$, the participation constraint is not binding. In such a case we have $d^M(\overline{A}) = d^A(\overline{A})$, i.e., the monopolist chooses the lowest addictiveness at which the consumer exhausts her attention. Given such a choice, the consumer's unconstrained choice of attention $A(d^M(\overline{A}))$ satisfies the attention constraint with equality. Take any $\overline{A}^3$ and $\overline{A}^4$ such that $A(0) \leq \overline{A}^3 < \overline{A}^4 < \overline{A}^M$. We have

$$\begin{aligned}
&\max_{a \in [0, \overline{A}^4]} u(a, d^M(\overline{A}^4)) - C(a) \\
=&\max_{a \geq 0} u(a, d^M(\overline{A}^4)) - C(a) \\
\geq&\max_{a \geq 0} u(a, d^M(\overline{A}^3)) - C(a) \\
=&\max_{a [0, \overline{A}^3]} u(a, d^M(\overline{A}^3)) - C(a)
\end{aligned}$$

Thus the consumer's payoff is lower under $\overline{A}^4$ than $\overline{A}^3$. □

B Proof of Claim 1

Proof. Consumer surplus is maximized by $d_k = 0$ because service utilities decrease in addictiveness. To show $d^{TS} > 0$ for a large r, suppose all platforms choose $d = 0$. The consumer optimally joins all platforms. Denoting $TS(d) = CS(d) + rA(d)$, the envelope formula implies $TS'(0) = Ku_2\left(\frac{A(0)}{K}, 0\right) + rA'(0)$. Because $A'(0) > 0$, we have $TS'(0) > 0$ for a large r. For such an r, if all platforms choose a small but positive d, the consumer strictly increases her attention, and the total surplus strictly increases. □

C Proof of Proposition 2

To show the result, we first prove some lemmas.

Lemma 2. *Take any increasing, strictly concave, and differentiable function, $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}$. Then, $u(x) - xu'(x)$ is strictly increasing in x.*

Proof. For any x and $y > x$, we have

$$\begin{aligned}
&\frac{u(y) - u(x)}{y - x} > u'(y) \\
\Rightarrow &u(y) - u(x) > u'(y)(y - x) \\
\Rightarrow &u(y) - u(x) > u'(y)y - u'(x)x \\
\Rightarrow &u(y) - yu'(y) > u(x) - xu'(x).
\end{aligned}$$

□

Lemma 3. *Consider the problem*

$$\max_{A \in [0, \overline{A}]} y \cdot u\left(\frac{A}{y}, d\right) - C(A). \tag{A.1}$$

Let $A^*(y)$ and $V^*(y)$ denote the maximizer and the maximized value, respectively. Then, $A^*(y)$ is increasing in y, $\frac{A^*(y)}{y}$ is decreasing in y, $V^*(y)$ is strictly concave in y, and $\frac{\partial V^*}{\partial y}$ is decreasing in d.

Proof. Define $V(A, y) := y \cdot u\left(\frac{A}{y}, d\right) - C(A)$. We have $\frac{\partial^2 V}{\partial A \partial y} = -\frac{A}{y^2} u_{11}\left(\frac{A}{y}, d\right) > 0$, and thus $A^*(y)$ is increasing in y. To show $\frac{A^*(y)}{y}$ is decreasing, we rewrite (A.1) as

$$\max_{a \in [0, \overline{A}/y]} y \cdot u\left(a, d\right) - C(ay). \tag{A.2}$$

The maximizer of (A.2) is $a^*(y) := \frac{A^*(y)}{y}$. If a^* is an interior solution (or in other words, when $A^*(y) < \overline{A}$), it satisfies the first order condition $u_1(a, d) - C'(ay) = 0$, whose solution is decreasing in y. If y is so large that $A^*(y) = \overline{A}$, then for any such y, we have $a^*(y) = \frac{\overline{A}}{y}$, which is decreasing in y. Overall, $\frac{A^*(y)}{y}$ is decreasing in $y > 0$.

We now show that $V^*(y)$ is concave. Let y^* denote the smallest value that satisfies $A^*(y) = \overline{A}$. First, we show $V^*(y)$ is concave on $[0, y^*)$. For any $y \in [0, y^*)$, $A^*(y)$ is an interior solution. The

envelope theorem implies

$$\frac{dV^*}{dy} = u\left(\frac{A^*(y)}{y}, d\right) - \frac{A^*(y)}{y} u_1\left(\frac{A^*(y)}{y}, d\right).$$

This expression is decreasing in y, because $u(x,d) - xu'(x,d)$ is increasing in x (Lemma 2) and $\frac{A^*(y)}{y}$ is decreasing in y. Second, we show $V^*(y)$ is concave on $[y^*, \infty)$. After $A^*(y)$ hits $\overline{A}$, the maximized value is $V^*(y) := y \cdot u\left(\frac{\overline{A}}{y}, d\right) - C(\overline{A})$. We have $\frac{dV^*}{dy} = u\left(\frac{\overline{A}}{y}, d\right) - \frac{\overline{A}}{y} u_1\left(\frac{\overline{A}}{y}, d\right)$. By the same argument as above, $\frac{dV^*}{dy}$ is decreasing in y. Finally, at $y = y^*$, the right and the left limits of $\frac{dV^*}{dy}$ coincides. Thus, $V^*(y)$ is globally concave.

Finally, $\frac{\partial^2 V^*}{\partial y \partial d} = u_2\left(\frac{A^*(y)}{y}, d\right) + y \cdot u_{12}\left(\frac{A^*(y)}{y}, d\right) \cdot \frac{\partial}{\partial y}\left(\frac{A^*(y)}{y}\right) < 0$. The cross derivative $\frac{\partial^2 V^*}{\partial y \partial d}$ is well-defined (at least) for all $y \neq y^*$. Thus, $\frac{\partial V^*}{\partial y} = \int_0^d \frac{\partial^2 V^*}{\partial y \partial d}(y,t)dt + c$ (with some constant c) is decreasing in d. □

Lemma 4. *Fix any $d' \geq 0$, and consider the problem*

$$U(x,y,d) := \max_{A \in [0,\overline{A}], A_y \in [0,A]} x \cdot u\left(\frac{A - A_y}{x}, d\right) + y \cdot u\left(\frac{A_y}{y}, d'\right) - C(A). \tag{A.3}$$

Then, $U_2(x,y,d)$ is decreasing in d.

Proof. The envelope theorem implies $U_2(x,y,d) = u\left(\frac{A_y^*}{y}, d'\right) - \frac{A_y^*}{y} u_1\left(\frac{A_y^*}{y}, d'\right)$, where A_y^* is a part of the maximizer (A^*, A_y^*) of (A.3). Because the objective function in (A.3) is supermodular in $(A, -A_y^*, d)$, A_y^* is decreasing in d. Also, Lemma 2 implies $u(a,d') - a \cdot u_1(a,d')$ is increasing in a. Thus, $U_2(x,y,d)$ is decreasing in d. □

The following result shows that the consumer faces a decreasing incremental gain of joining platforms for any choices of addictiveness. We later use this lemma to establish the uniqueness of the equilibrium.

Lemma 5. *Take any $S, S' \subset K_{-1} := \{2, 3, \ldots, K\}$ such that $S' \subset S$. For any choice of addictiveness, the consumer's incremental gain of joining platform 1 is greater when she has already joined platforms S' than S. Formally, the following holds. Fix any $(d_1, \ldots, d_K) \in \mathbb{R}_+^K$. For any $y \in [0,1]$*

and $S \subset K_{-1}$, define

$$V(y,S) := \max_{(a_k)_{k\in S\cup\{1\}}} \sum_{k\in S} u(a_k,d_k) + y\cdot u(a_1,d_1) - C\left(\sum_{k\in S\cup\{1\}} a_k\right) \tag{A.4}$$
$$\text{s.t.} \quad \sum_{k\in S\cup\{1\}} a_k \le \overline{A} \quad \text{and} \quad a_k \ge 0, \forall k \in S\cup\{1\}\,.$$

Then for any $S', S \subset K_{-1}$ such that $S' \subsetneq S$,

$$\frac{\partial V}{\partial y}(y,S) \le \frac{\partial V}{\partial y}(y,S'). \tag{A.5}$$

In particular, $V(1,S)-V(0,S) \le V(1,S')-V(0,S')$. These inequalities are strict whenever the consumer allocates positive attention to every platform in S and S' upon solving (A.4)*.*

Proof. Let $a_1(y,S)$ denote the optimal value of a_1 in (A.4). The envelope theorem implies

$$\frac{\partial V}{\partial y}(y,S) = u(a_1(y,S),d_1).$$

Thus, to show (A.5), we first show that $a_1(y,S) \le a_1(y,S')$ for any S' and $S \supset S'$.

Suppose to the contrary that $a_1(y,S) > a_1(y,S')$. Note that in the problem (A.4), the marginal utilities from any two platforms in S are equal whenever the consumer allocates positive attention to them. Thus, for any $k \in S'$ such that $a_k(y,S') > 0$, we have $a_k(y,S) > a_k(y,S')$. This inequality leads to a contradiction if the attention constraint is binding under S', i.e., $\sum_{k\in S'\cup\{1\}} a_k(y,S') = \overline{A}$. Suppose the attention constraint is not binding under S'. Then for any $j \in S'$, we have

$$u_1(a_j(y,S),d_j) < u_1(a_j(y,S'),d_j) = C'\left(\sum_{k\in S'\cup\{1\}} a_k(y,S')\right) < C'\left(\sum_{k\in S\cup\{1\}} a_k(y,S)\right).$$

These inequalities imply that $(a_k(y,S))_{k\in S\cup\{1\}}$ does not solve (A.4), because the marginal cost exceeds the marginal utility from any platform $j \in S$. This is a contradiction. Thus, we obtain $a_1(y,S) \le a_1(y,S')$. Integrating both sides of (A.5) from $y=0$ to $y=1$, we have $V(1,S) - V(0,S) \le V(1,S') - V(0,S')$.

Now, suppose the consumer allocates positive attention to every platform in S and S' upon

solving (A.4). Then, we can use the same argument to show that $a_1(y,S) \geq a_1(y,S')$ leads to a contradiction. Thus, we have $a_1(y,S) < a_1(y,S')$ and obtain (A.5) as a strict inequality. □

We are now ready to prove Proposition 2.

Proof of Proposition 2. STEP 1:*There is a unique d^* that satisfies (3).* To show this, define

$$f(K,d) := K \cdot u\left(\frac{A_K(d)}{K}, d\right) - C(A_K(d)) - \left[(K-1)\cdot u\left(\frac{A_{K-1}(d)}{K-1}, d\right) - C(A_{K-1}(d))\right].$$

The function $f(K,d)$ is the difference between payoffs when the consumer uses K platforms and when she uses $K-1$ platforms, given optimally allocating attention. Hereafter, we use the notation $V^*(y,d)$ for $V^*(y)$ of Lemma 3 to make the dependence of $V^*(y)$ on d explicit. We can write $f(K,d) = V^*(K,d) - V^*(K-1,d)$. Lemma 3 implies $V_1^*(y,d)$ is decreasing in d. Thus, $f(K,d) = \int_{K-1}^{K} V_1^*(y,d)dy$ is decreasing in d. Also, $f(K,0) > 0$, and $f(K,d) < 0$ for a sufficiently large d. Thus, there is a unique d^* that solves (3) (i.e., $f(K,d^*) = 0$).

STEP 2: *There is an equilibrium in which each platform sets d^*.* Suppose all platforms choose d^*. First, we show that the consumer prefers to joins all the platforms. Given $d_k = d^*$ for all k, the consumer's payoff from joining $J \leq K$ platforms is $V^*(J,d)$, which is strictly concave in J (Lemma 3). Also, we have $V^*(K,d^*) = V^*(K-1,d^*)$ by construction. As a result, $V^*(J,d^*)$ is strictly increasing in $y \leq K-1$. Thus, the consumer prefers to join all platforms.

Second, we show that no platform has a profitable deviation. Without loss of generality, we consider the incentive of platform 1. If it increases d_1, the consumer joins only platforms $2,\ldots,K$ to achieve the same payoff as without platform 1's deviation. Suppose platform 1 decreases d_1 from d^* to d. The consumer joins platform 1. If she additionally joins other y platforms, her payoff becomes $U(1,y,d)$ according to the notation of Lemma 4 (with $d' = d^*$). When $d_1 = d^* > d$, $U(1,y,d^*)$ is maximized at $y = K-1$ and $y = K$. Because $U_{23}(1,y,d) < 0$ by Lemma 4, the consumer's marginal gain from joining platforms increases after platform 1's deviation. As a result, $U(1,y,d)$ is uniquely maximized at $y = K$ across all $y \in \{1,\ldots,K\}$. However, the consumer will then allocate a smaller amount of attention to platform 1 compared to without deviation, because platform 1 now offers a lower marginal utility. Thus, platform 1 does not strictly benefit from the deviation to $d < d^*$.

STEP 3: *The above equilibrium is a unique one.* To show this, take any pure-strategy subgame perfect equilibrium. Because any platform can set $d_k = 0$ to ensure participation, the consumer joins all platforms in equilibrium. First, we show all platforms choose the same addictiveness. Suppose to the contrary that there is an equilibrium in which platforms choose $(d_k^*)_{k\in K}$ such that (without loss of generality) $d_2 = \max_k d_k^* > \min_k d_k^* = d_1$. Suppose now that platform 1 deviates and increases its addictiveness to $d_1 = d_1^* + \varepsilon < d_2^*$. We show that the consumer joins platform 1. Suppose to the contrary that she does not join platform 1. If she joins platform 2, it is a contradiction, because she could obtain a strictly higher payoff by replacing platform 2 with 1. Thus, the consumer does not join platform 2. Lemma 5 implies that the consumer's incremental gain of joining platform 1 is strictly higher when (i) she has joined some set of platforms $K' \subset \{1,\dots,K\} \setminus \{1,2\}$ than when (ii) she has joined platforms $2,\dots,K$. Because the consumer weakly prefers to join platform 1 under (ii) at $d_1 = d_1^*$, she strictly prefers to join it under (i) at $d_1 = d_1^*$. As a result, the consumer strictly prefers to join platform 1 under (i) at $d_1 = d_1^* + \varepsilon$ for a small $\varepsilon > 0$. To sum up, if platform 1 deviates to $d_1^* + \varepsilon$ with small $\varepsilon > 0$, the consumer joins platform 1 and allocates strictly greater attention to it. This contradicts $(d_k^*)_{k\in K}$ being an equilibrium.

Therefore, in any equilibrium, d_k^* is the same for all $k \in K$. Finally, take any equilibrium in which all platforms choose the same addictiveness. If the consumer's indifference condition (3) fails, then one of the following holds: (i) the left-hand side is strictly greater, in which case a platform prefers to deviate and increase its addictiveness, or (ii) the right-hand side is greater, in which case the consumer does not join at least one platform, which is a contradiction. □

D Proofs for Section 4: The Impact of Competition

Proof of Proposition 4. The first part of the result follows from Proposition 2. To show the second part, fix any $K \geq 2$ and let d^* denote the equilibrium addictiveness. Each platform provides a service utility of $\frac{1}{K}u(Ka, d)$. Let $A_x(d)$ denote the unique maximizer of the problem

$$V^*(x,d) := \max_{A\in[0,\overline{A}]} x \cdot u\left(\frac{A}{x}, d\right) - C(A). \tag{A.6}$$

If the consumer joins K platforms with addictiveness d, she allocates total attention $A_1(d)$. If the consumers joins $K-1$ platforms, she allocates total attention $A_{\frac{K-1}{K}}(d)$. In equilibrium, the consumer is indifferent between joining K and $K-1$ platforms. Thus, we have

$$u(A_1(d^*), d^*) - C(A_1(d^*)) = \frac{K-1}{K} u\left(\frac{K}{K-1} A_{\frac{K-1}{K}}(d^*), d^*\right) - C\left(A_{\frac{K-1}{K}}(d^*)\right). \tag{A.7}$$

Suppose to the contrary that for some K, the equilibrium addictiveness weakly increases from d^* to d^{**} as we move from K platforms to $K+1$ platforms. Equation (A.7) implies that $V^*(1, d^*) = V^*(\frac{K-1}{K}, d^*)$. Because $V^*(x, d)$ is strictly concave in x, this equation implies

$$u(A_1(d^*), d^*) - C(A_1(d^*)) < \frac{K}{K+1} u\left(\frac{K+1}{K} A_{\frac{K}{K+1}}(d^*), d^*\right) - C\left(A_{\frac{K}{K+1}}(d^*)\right). \tag{A.8}$$

If d^* increases, the left-hand side decreases more than the right-hand side. To see this, first, note that

$$\frac{\partial}{\partial d} V^*(x, d) = x u_2\left(\frac{A_x(d)}{x}, d\right), \tag{A.9}$$

$$\frac{\partial^2}{\partial x \partial d} V^*(x, d) = u_2\left(\frac{A_x(d)}{x}, d\right) + x \cdot \frac{\partial}{\partial x}\left(\frac{A_x(d)}{x}\right) \cdot u_{12}\left(\frac{A_x(d)}{x}, d\right) < 0. \tag{A.10}$$

The inequality uses $\frac{\partial}{\partial x}\left(\frac{A_x(d)}{x}\right) < 0$, which follows from Lemma 3. Now, we can write (A.8) as $V^*(1, d^*) < V^*\left(\frac{K}{K+1}, d^*\right)$, or equivalently, $\int_{\frac{K}{K+1}}^1 \frac{\partial}{\partial x} V^*(x, d^*) dx < 0$. Because $\frac{\partial}{\partial x} V^*(x, d)$ is decreasing in d, we have $\int_{\frac{K}{K+1}}^1 \frac{\partial}{\partial x} V^*(x, d^{**}) dx < 0$, or equivalently, $V^*(1, d^{**}) < V^*\left(\frac{K}{K+1}, d^{**}\right)$. As a result, we have

$$u(A_1(d^{**}), d^{**}) - C(A_1(d^{**})) < \frac{K}{K+1} u\left(\frac{K+1}{K} A_{\frac{K}{K+1}}(d^{**}), d^{**}\right) - C\left(A_{\frac{K}{K+1}}(d^{**})\right),$$

which contradicts that the consumer joins all platforms in equilibrium even when there are $K+1$ platforms.

We show the last part. We write the equilibrium addictiveness as d_x^* to emphasize that it depends

on $x = \frac{K}{K+1}$, or equivalently, K. we write (A.7) as

$$\frac{u(A_1(d_x^*), d_x^*) - C(A_1(d_x^*)) - \left[xu\left(\frac{1}{x}A_x(d_x^*), d_x^*\right) - C\left(A_x(d_x^*)\right)\right]}{1-x} = 0, \quad \forall x \in \left\{\frac{K}{K+1}\right\}_{K\in\mathbb{N}} \tag{A.11}$$

Define

$$f_x(d) = \frac{u(A_1(d), d) - C(A_1(d)) - \left[xu\left(\frac{1}{x}A_x(d), d\right) - C\left(A_x(d)\right)\right]}{1-x}. \tag{A.12}$$

We can write equation (A.11) as $f_x(d_x^*) = 0$.

We make several observations. First, the equilibrium addictiveness is decreasing in K. Thus, across all $x \in \left\{\frac{K}{K+1}\right\}_{K\in\mathbb{N}}$, the set of possible levels of equilibrium addictiveness is a subset of a compact set $[0, d_{\frac{2}{3}}^*]$, where $d_{\frac{2}{3}}^*$ is the one for duopoly. Second, for each x, $f_x(d)$ is continuous in d. As $x \to 1$, it converges pointwise to

$$\lim_{x\to 1} \frac{u(A_1(d), d) - C(A_1(d)) - \left[xu\left(\frac{1}{x}A_x(d), d\right) - C\left(A_x(d)\right)\right]}{1-x} = u\left(A_1(d), d\right) - A_1(d)u_1\left(A_1(d), d\right) := f_1(d).$$

Here we use the envelope theorem. Third, the function $xu\left(\frac{1}{x}A_x(d), d\right) - C\left(A_x(d)\right)$ is concave in x (Lemma 3). As a result for any d, $f_x(d)$ is decreasing in x.

We have shown that $(f_x(\cdot))_x$ is a monotonically decreasing sequence of continuous functions defined on a compact set, and the sequence converges pointwise to $f_1(\cdot)$. By Dini's Theorem, $f_x(\cdot)$ uniformly converges to $f_1(\cdot)$ (e.g., Theorem 7.13 in Rudin (1976)). Recall that we have $f_x(d_x^*) = 0$ for any x. Because $f_x(\cdot)$ uniformly converges to $f_1(\cdot)$ and $d_x^* \to d^\infty$, we have $\lim_{x\to\infty} f_x(d_x^*) = f_1(d^\infty) = 0$.[12] As a result, we have $u\left(A_1(d^\infty), d^\infty\right) - A_1(d^\infty)u_1\left(A_1(d^\infty), d^\infty\right) = 0$. Finally, we show $d^\infty > 0$. If $d^\infty = 0$, we have $u(A, 0) - Au_1(A, 0) = 0$ for $A = A(0)$, which implies $\frac{u(A,0)-u(0,0)}{A-0} = u_1(A, 0)$. This is a contradiction, because $u(x, 0)$ is strictly concave and $A > 0$. □

Proof of Corollary 1. For any $\overline{A} \leq A(0)$, the consumer is strictly better off under monopoly than the limit economy, because the monopoly platform sets zero addictiveness. Also, Proposition 1 implies that the consumer's payoff is decreasing in $\overline{A}$ for $\overline{A} \geq A(0)$. Thus, it suffices to show that

[12] If $f_n(\cdot)$ uniformly converges to a continuous function $f(\cdot)$ and x_n converges to x, then $f_n(x_n)$ converges to $f(x)$. Indeed, we have $|f_n(x_n) - f(x)| \leq |f_n(x_n) - f(x_n)| + |f(x_n) - f(x)|$. Then $|f_n(x_n) - f(x_n)| \to 0$ because of the uniform convergence, and $|f(x_n) - f(x)| \to 0$ because f is continuous.

the consumer's equilibrium payoff is strictly increasing in $\overline{A} \geq A(0)$. In the limit economy the consumer's payoff is $A_1(d^\infty)C'(A_1(d^\infty)) - C(A_1(d^\infty))$, because $u(A_1(d^\infty)) = A_1(d^\infty)C'(d^\infty)$. If $\overline{A}$ increases, d^∞ and $A_1(d^\infty)$ increase. Because $xC'(x) - C(x)$ is increasing in x, the consumer surplus increases. Under monopoly, Proposition 1 implies that the consumer obtains the first-best payoff for $\overline{A} \leq A(0)$, and a payoff of zero for $\overline{A} \geq A(d^0)$. On $[A(0), A(d^0)]$, the monopolist chooses d^1, which is the lowest addictiveness that makes it optimal for the consumer to choose $\overline{A}$. Because $\overline{A}$ globally maximizes the consumer's payoff given d^1, we have $u_1(\overline{A}, d^1) - C'(\overline{A}) = 0$. Because the left-hand side is strictly decreasing in $\overline{A}$ and strictly increasing in d^1, it follows that d^1 increases in $\overline{A}$. Now, for $\overline{A} \in [A(0), A(d^0)]$, the consumer's payoff is $u(\overline{A}, d^1) - C(\overline{A})$. Differentiating this expression in $\overline{A}$ and using the first-order condition, the change in consumer surplus with $\overline{A}$ is equal to $u_2(\overline{A}, d^1) \cdot \frac{d}{d\overline{A}} d^1 < 0$. As a result, the consumer surplus under monopoly is strictly decreasing in $\overline{A} \geq A(0)$. We can define A^* as the unique value at which the consumer is indifferent between monopoly and the limit economy. □

Proof of Proposition 5. First we characterize the equilibrium in the limit economy. Let $A^U(d)$ denote the consumer's unconstrained choice of total attention when she faces platforms with addictiveness d. Because $A^U(d)$ solves the first-order condition $v'(a - d) = c$, we have

$$A^U(d) = d + g(c), \quad \text{where} \quad g = (v')^{-1}.$$

The consumer's objective is concave, so the consumer's constrained choice of total attention is $A^*(d, \overline{A}) := \min\left\{\overline{A}, d + g(c)\right\}$. The equilibrium addictiveness d^∞ in the limit economy solves $u(A^*(d^\infty, \overline{A}), d^\infty) = A^*(d^\infty, \overline{A}) \cdot u_1(A^*(d^\infty, \overline{A}), d^\infty)$, which is equivalent to

$$\begin{aligned} & v(A^*(d^\infty, \overline{A}) - d) = A^*(d, \overline{A}) \cdot v'(A^*(d^\infty, \overline{A}) - d) \\ \iff & d^\infty = A^*(d^\infty, \overline{A}) - h\left(A^*(d^\infty, \overline{A})\right), \quad \text{where} \quad h = \left(\frac{v}{v'}\right)^{-1}. \end{aligned}$$

Suppose $A^*(d^\infty, \overline{A}) = \overline{A}$, which implies $d^\infty = \overline{A} - h(\overline{A})$. Then we have $A^U(d^\infty) = \overline{A} - h(\overline{A}) + g(c)$. The attention constraint indeed binds if and only if

$$\overline{A} - h(\overline{A}) + g(c) \geq \overline{A} \iff g(c) \geq h(\overline{A}) \iff \frac{v(g(c))}{c} \geq \overline{A}. \tag{A.13}$$

As a result, if $\frac{v(g(c))}{c} \geq \overline{A}$, the equilibrium total attention, addictiveness, and consumer surplus in the limit economy are as follows:

$$A^\infty = \overline{A}, \tag{A.14}$$

$$d^\infty = \overline{A} - h(\overline{A}), \quad \text{and} \tag{A.15}$$

$$CS^\infty = v(\overline{A} - d^*) - c\overline{A} = v(h(\overline{A})) - c\overline{A}. \tag{A.16}$$

We now consider the other case: $\frac{v(g(c))}{c} < \overline{A}$. Suppose the consumer's choice is interior given the equilibrium addictiveness d^∞. The addictiveness d^∞ satisfies

$$\begin{aligned} d^\infty &= d^\infty + g(c) - h(d^\infty + g(c)) \\ \iff d^\infty &= h^{-1}(g(c)) - g(c) = \frac{v(g(c))}{c} - g(c). \end{aligned}$$

Because $A^U(d^\infty) = \frac{v(g(c))}{c} < \overline{A}$, the consumer's choice is interior. As a result, if $\frac{v(g(c))}{c} < \overline{A}$, the equilibrium is as follows:

$$A^\infty = \frac{v(g(c))}{c}, \tag{A.17}$$

$$d^\infty = \frac{v(g(c))}{c} - g(c), \tag{A.18}$$

$$CS^\infty = v(A^\infty) - c \cdot A^\infty = 0. \tag{A.19}$$

We now turn to monopoly. Take any $\overline{A}$ and suppose the monopoly chooses d such that the consumer exhausts her attention:

$$\overline{A} = d + g(c) \iff d = \overline{A} - g(c). \tag{A.20}$$

Thus the monopolist sets the addictiveness of $\max(0, \overline{A} - g(c))$ to make the consumer choose $\overline{A}$. The consumer's payoff is then

$$v(\overline{A} - \max(0, \overline{A} - g(c))) - c\overline{A}. \tag{A.21}$$

Because the consumer's payoff is positive for $\overline{A} < g(c)$, the payoff (A.21) becomes non-positive if and only if $\overline{A} \geq \frac{v(g(c))}{c}$, which is the same threshold at which the consumer's equilibrium payoff becomes zero in the limit economy.

We now compare consumer surpluses under monopoly and the limit economy. If $\overline{A} > \frac{v(g(c))}{c}$, the consumer's equilibrium payoff is zero in either case. Suppose $\overline{A} < \frac{u(g(c))}{c}$. If $\overline{A} \leq A(0)$, the monopolist sets zero addictiveness. Otherwise, the monopoly is strictly better if and only if

$$\begin{aligned} & v(g(c)) - c\overline{A} > v(h(\overline{A})) - c\overline{A} \\ \iff & \overline{A} < \frac{v(g(c))}{c}. \end{aligned}$$

Thus, for any $\overline{A} < \frac{v(g(c))}{c}$, the monopoly is strictly better. Because monopoly dominates the limit economy, it dominates any other market $\mathcal{E}_K$ with $K \geq 2$. □

Proof of Proposition 5. The consumer's unconstrained attention allocation problem solves the first-order condition:

$$\rho e^{-\rho(A^U(d)-d)} = cA^U(d) \iff A^U(d) = g\left(\frac{c}{\rho}e^{-\rho d}\right), \quad \text{where} \quad g^{-1}(x) = \frac{e^{-\rho x}}{x}.$$

The solution of the consumer's constrained problem is

$$A^*(d, \overline{A}) = \min\left\{\overline{A}, g\left(\frac{c}{\rho}e^{-\rho d}\right)\right\}.$$

The equilibrium addictiveness d^∞ satisfies

$$\begin{aligned} & 1 - e^{-\rho(A^*(d^\infty,\overline{A})-d^\infty)} = A^*(d^\infty, \overline{A})\rho e^{-\rho(A^*(d^\infty,\overline{A})-d^\infty)} \\ \iff \quad & 1 = (1 + \rho A^*(d^\infty, \overline{A})) \cdot e^{-\rho(A^*(d^\infty,\overline{A})-d^\infty)} \\ \iff \quad & d^\infty = A^*(d^\infty, \overline{A}) - \frac{1}{\rho}\ln\left(1 + \rho A^*(d^\infty, \overline{A})\right). \end{aligned}$$

Suppose $A^*(d^\infty, \overline{A}) = \overline{A}$ in equilibrium. Then,

$$d^\infty = \overline{A} - \frac{1}{\rho}\ln\left(1 + \rho\overline{A}\right).$$

The attention constraint binds at d^∞ if and only if

$$g\left(\frac{c}{\rho}e^{-\rho d^\infty}\right) \geq \overline{A} \iff \frac{c}{\rho}e^{-\rho d^\infty} \leq g^{-1}(\overline{A}) \quad (\because g^{-1} \text{ is decreasing}) \iff \frac{c}{\rho}e^{-\rho d^\infty} \leq \frac{e^{-\rho\overline{A}}}{\overline{A}}$$
$$\iff \frac{c}{\rho}e^{-\rho\left[\overline{A}-\frac{1}{\rho}\ln\left(1+\rho\overline{A}\right)\right]} \leq \frac{e^{-\rho\overline{A}}}{\overline{A}} \iff \frac{c}{\rho}\left(1+\rho\overline{A}\right) \leq \frac{1}{\overline{A}} \iff 0 \geq c\rho\overline{A}^2 + c\overline{A} - \rho$$
$$\iff \overline{A} \leq \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}.$$

As a result, if $\overline{A} \leq \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}$ the equilibrium in the limit economy is as follows:

$$\begin{aligned} A^\infty &= \overline{A} \\ d^\infty &= \overline{A} - \frac{1}{\rho}\ln\left(1+\rho\overline{A}\right) \\ CS^\infty &= 1 - \frac{1}{1+\rho\overline{A}} - \frac{c}{2}\overline{A}^2. \end{aligned} \tag{A.22}$$

We now consider the other case: $\overline{A} > \frac{-c+\sqrt{c^2+4c\rho}}{2c\rho}$. Suppose the consumer's choice is interior. The addictiveness d^∞ satisfies

$$\begin{aligned} & 1 = (1+\rho A^U(d^\infty))e^{-\rho(A^U(d^\infty)-d^\infty)} \\ \iff \quad & 1 = (1+\rho A^U(d^\infty))\frac{cA^U(d^\infty)}{\rho} \\ \iff \quad & c\rho(A^U(d^\infty))^2 + cA^U(d^\infty) - \rho = 0. \end{aligned}$$

Thus,

$$A^U(d^\infty) = \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho} \tag{A.23}$$

The consumer surplus is

$$\begin{aligned} CS^\infty &= A^\infty(d,\overline{A})\rho e^{-\rho(A^\infty(d,\overline{A})-d)} - \frac{c}{2}A^\infty(d,\overline{A})^2 \\ &= cA^U(d)^2 - \frac{c}{2}A^U(d)^2 \\ &= \frac{c}{2}A^U(d)^2 \\ &= \frac{c}{2}\left[\frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}\right]^2 > 0 \end{aligned}$$

Finally, we show CS^{∞} is non-monotone in $\overline{A}$. When the attention constraint binds, consumer surplus is (A.22). We have

$$\frac{\partial CS^{\infty}}{\partial \overline{A}} = \frac{\rho}{(1+\rho\overline{A})^2} - c\overline{A}.$$

Because the right-hand side is decreasing in $\overline{A}$, CS^{∞} is concave in $\overline{A}$ for $\overline{A} \in [0, \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}]$. To show CS^{∞} is non-monotone in $\overline{A}$ on $[0, \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}]$, it suffices to show $\frac{\partial CS^{\infty}}{\partial \overline{A}} < 0$ at the cutoff $\overline{A}^* = \frac{-c+\sqrt{c^2+4c\rho^2}}{2c\rho}$. Recall that the cutoff $\overline{A}^*$ satisfies $\frac{c}{\rho}\left(1+\rho\overline{A}^*\right) = \frac{1}{\overline{A}^*}$, so $\frac{1}{1+\rho\overline{A}^*} = \frac{c\overline{A}^*}{\rho}$. As a result

$$\frac{\partial CS^{\infty}}{\partial \overline{A}}\Big|_{\overline{A}=\overline{A}^*} = \rho \cdot \left(\frac{c\overline{A}^*}{\rho}\right)^2 - c\overline{A}^* = c\overline{A}^* \cdot \left(\frac{c\overline{A}^*}{\rho} - 1\right) < 0.$$

Next, consider monopoly. Suppose the attention constraint binds and the monopolist sets positive addictiveness d^*. Note that d^* satisfies the consumer's first order condition at $\overline{A}$:

$$\rho e^{-\rho(\overline{A}-d^*)} = c\overline{A} \iff d^* = \overline{A} + \frac{1}{\rho}\ln\left(\frac{c\overline{A}}{\rho}\right).$$

Consumer surplus is

$$CS^M = 1 - \frac{c\overline{A}}{\rho} - \frac{c\overline{A}^2}{2}.$$

The attention constraint bids at

$$\begin{aligned}
&1 - \frac{c\overline{A}}{\rho} - \frac{c\overline{A}^2}{2} = 0 \\
\iff &c\rho\overline{A}^2 + 2c\overline{A} - 2\rho = 0 \\
\Rightarrow &\overline{A}^M = \frac{-2c + \sqrt{4c^2 + 8c\rho^2}}{2c\rho} \\
\iff &\overline{A}^M = \frac{-c + \sqrt{c^2 + 2c\rho^2}}{c\rho}.
\end{aligned}$$

In the limit economy, if the attention constraint binds, we have

$$\frac{\partial CS^{\infty}}{\partial \overline{A}} = \frac{\rho}{(1+\rho\overline{A})^2} - c\overline{A}. \tag{A.24}$$

If the attention constraint does not bind, $\frac{\partial CS^{\infty}}{\partial \overline{A}} = 0$. Under monopoly, if the attention constraint binds,

$$\frac{\partial CS^{M}}{\partial \overline{A}} = -\frac{c}{\rho} - c\overline{A}. \tag{A.25}$$

As a result, if $\overline{A}$ is such that the attention constraint binds under monopoly, we have $\frac{\partial CS^{\infty}}{\partial \overline{A}} > \frac{\partial CS^{M}}{\partial \overline{A}}$.

We now establish the welfare comparison. If $\overline{A} \leq A(0)$, the monopolist sets $d = 0$, so the consumer is strictly better off under monopoly. If $A(0) < \overline{A} < \overline{A}^M$, consumer surplus under monopoly decreases faster than consumer surplus in the limit economy. At $\overline{A}^M$, the consumer gets a payoff of zero under monopoly and a positive payoff in the limit economy. Thus there is a unique cutoff $A^{**} \in (A(0), \overline{A}^M)$ such that the consumer is better off under monopoly if and only if $\overline{A} \leq A^{**}$.

Finally, we show that $A^{**} = \overline{A}^*$. First, we show $\overline{A}^* < \overline{A}^M$. We have

$$\begin{aligned}
\overline{A}^* < \overline{A}^M &\iff \frac{-c+\sqrt{c^2+4c\rho}}{2c\rho} < \frac{-c+\sqrt{c^2+2c\rho^2}}{c\rho} \\
&\iff c+\sqrt{c^2+4c\rho^2} < 2\sqrt{c^2+2c\rho^2} \\
&\iff 1+\sqrt{1+4x} < 2\sqrt{1+2x}, \quad \text{where} \quad x = \frac{\rho^2}{c} \\
&\iff 2\sqrt{1+4x} < 2+4x \\
&\iff 0 < 4x^2.
\end{aligned}$$

As a result, the cutoff at which the participation constrain binds under monopoly is strictly greater than the cutoff $\overline{A}^*$ at which the attention constraint binds in the limit economy. It implies that when $\overline{A} = \overline{A}^*$, the monopolist and platforms in the limit economy set the same addictiveness, i.e., all of them set the lowest addictiveness at which the attention constraint binds. Thus, the consumer obtains the same equilibrium payoff in the two cases when $\overline{A} = \overline{A}^*$. Thus we conclude $A^{**} = \overline{A}^*$. □

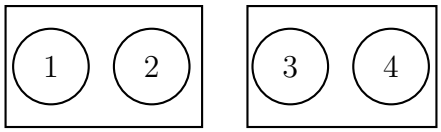

Figure A.6: Symmetric merger

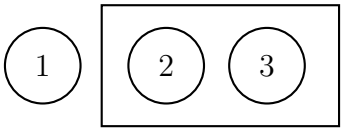

Figure A.7: All-but-one merger

E Appendix for Section 4.2: Platform Merger

Another way to examine the effect of competition is to study the welfare impact of platform merger. We compare the original game to the *post-merger* game. It is characterized by a *market structure* $\mathcal{M}$, which is a partition of $\mathcal{K} := \{1, \dots, K\}$. We write $\mathcal{M} = \{P_1, \dots, P_M\}$, where each $P_m \subset \mathcal{K}$ is a merged platform that consists of platforms (or services) $k \in P_m$. The original game corresponds to $\mathcal{M} = \{\{k\}\}_{k \in \mathcal{K}}$. We write M for the set and the number of platforms in the post-merger game. Given any $\mathcal{M}$, the post-merger game works as follows. First, each platform $m \in M$ simultaneously chooses its addictiveness, d_m. The consumer observes $(d_m)_{m \in M}$, then chooses the set $M' \subset M$ of platforms to join and the allocation $(a_k)_{k \in \cup_{m \in M'} P_m}$ of attention, in order to maximize her payoff $\sum_{m \in M'} \sum_{k \in P_m} \frac{1}{K} u(K a_k, s d_m) - C(\sum_{m \in M'} \sum_{k \in P_m})$. The payoff of each platform m is $r \sum_{k \in P_m} a_k$ if $m \in M'$, and zero if $m \notin M'$.

A merger changes the game in two ways. First, each platform $m \in M$ chooses a single level of addictiveness for all services in P_m. Second, services operated by the same platform are tied—i.e., the consumer cannot join a nonempty strict subset of services in P_m. We examine two types of mergers, which Figure A.7 illustrates: Circles and rectangles are platforms before and after the merger, respectively.

Definition 1. A *symmetric merger* refers to $\mathcal{M}$ such that $|P_m| = |P_\ell| \geq 2$ for all $m, \ell \in M$ and $M \geq 2$. An *all-but-one merger* refers to $\mathcal{M}$ such that $\mathcal{M} = \{\mathcal{K} \setminus \{k\}, \{k\}\}$ for some $k \in K$.

All-but-one merger is unique up to the identity of the non-merged platform. In contrast, there can be multiple symmetric mergers. The following result presents the welfare impacts of a merger.

Proposition 10. *A symmetric merger increases the addictiveness of all of the K services. An all-but-one merger increases the addictiveness of $K-1$ merged services and decreases that of the non-merged platform. In either case, a merger strictly decreases consumer surplus.*

To see the intuition, suppose two out of three firms merge to form a single platform M. After the merger if the consumer refuses to join platform M, she loses access to two services. Because the consumer faces a lower outside option, platform M can set higher addictiveness for its services than before the merger. The merger also encourages the non-merged platform to decrease its addictiveness; the consumer has a stronger incentive to stay with platform M, so the non-merged platform has to offer a higher service utility to ensure participation. On balance, the merger harms the consumer because her payoff is equal to the payoff from platform M alone.

Proof for Proposition 10. The case of a symmetric merger follows Proposition 4: A symmetric merger that changes the number of platforms from K to L is equivalent to the change from $\mathcal{E}_K$ to $\mathcal{E}_L$.

We consider an all-but-one merger. To simplify notation, we use the original service utility function $u(a,d)$ instead of the normalized one. Suppose platforms $2,\dots,K$ merge and become platform M. Let d_1 and d_M denote the addictiveness of platforms 1 and M. We denote their equilibrium values as d_1^* and d_M^*. In equilibrium the consumer joins all platforms, because any platform can choose $d_k=0$ to obtain a positive amount of attention. Let $a_1(d_1,d_M)$ and $A_M(d_1,d_M)$ denote the attention allocated to platform 1 and M, respectively, when platforms 1 and M choose (d_1,d_M) and the consumer joins both in the post-merger market. Let $A_k(d)$ denote the total attention allocated when the consumer joins k platforms with addictiveness d in the pre-merger market. In equilibrium, we have

$$u\left(a_1(d_1^*,d_M^*),d_1^*\right)+(K-1)u\left(\frac{A_M(d_1^*,d_M^*)}{K-1},d_M^*\right)-C\left(a_1(d_1^*,d_M^*)+A_M(d_1^*,d_M^*)\right) \tag{A.26}$$

$$=u\left(A_1(d_1^*),d_1^*\right)-C\left(A_1(d_1^*)\right) \tag{A.27}$$

$$=(K-1)u\left(\frac{A_{K-1}(d_M^*)}{K-1},d_M^*\right)-C\left(A_{K-1}(d_M^*)\right). \tag{A.28}$$

The expression (A.26) is the consumer's payoff of joining platforms 1 and M: Platform M consists of $K-1$ symmetric services with decreasing marginal utilities, so the consumer allocates $\frac{A_M(d_1^*,d_M^*)}{K-1}$

to each of the $K-1$ services. The expressions (A.27) and (A.28) are the consumer's payoffs of joining only platform 1 and M, respectively. If any of the equalities fails, some platform will have a profitable deviation.

First, we show that the merger of $K-1$ platforms increases the addictiveness of the merged services and decreases that of the non-merged platform. Let d_0 denote the equilibrium addictiveness before the merger. First, we show $d_M^* > d_0$. Suppose to the contrary that $d_M^* \leq d_0$. Lemma 3 implies that when all platforms set the same addictiveness in the pre-merger market, the consumer's optimal payoff (i.e., $V^*(y)$ in the lemma) is strictly concave in the number of platforms she joins. Also, given d_0 the consumer is indifferent between joining K and $K-1$ platforms. As a result, the consumer strictly prefers joining K platforms to a single platform, i.e.,

$$\begin{aligned}
&u\left(a_1(d_0,d_0),d_0\right)+(K-1)u\left(\frac{A_M(d_0,d_0)}{K-1},d_0\right)-C\left(a_1(d_0,d_0)+A_M(d_0,d_0)\right)\\
>&u\left(A_1(d_0),d_0\right)-C\left(A_1(d_0)\right).
\end{aligned}$$

Using $d_M^* \leq d_0$, we have

$$\begin{aligned}
f(d_0) :=&u\left(a_1(d_0,d_M^*),d_0\right)+(K-1)u\left(\frac{A_M(d_0,d_M^*)}{K-1},d_M^*\right)-C\left(a_1(d_0,d_M^*)+A_M(d_0,d_M^*)\right)\\
&-\left[u\left(A_1(d_0),d_0\right)-C\left(A_1(d_0)\right)\right]>0.
\end{aligned}$$

Because (A.26) equals (A.27), we need $f(d_1^*)=0$. Because $u_{12}>0$ and $a_1(d,d_M)<A_1(d)$, the envelope theorem implies

$$f'(d)=u_2\left(a_1(d,d_M),d\right)-u_2\left(A_1(d),d\right)<0.$$

As a result, we need $d_1^* > d_0$ to satisfy $f(d_1^*)=0$. However this is a contradiction. To see this, suppose $d_1^* > d_0$ and $d_M^* = d_0$. The consumer will join only platform M because it consists of $K-1$ services and the consumer is indifferent between joining K services and $K-1$ services when they all choose $d_0 < d_1^*$. If $d_M^* < d_0$, platform M can profitably deviate to d_0, because the consumer will then join platform M alone. Therefore we obtain $d_M^* > d_0$.

Next, we show platform 1 reduces its addictiveness after the merger, i.e., $d_1^* < d_0$. If all services

have d_0, the consumer is indifferent between joining K and $K-1$ platforms:

$$\begin{aligned}&u\left(a_1(d_0,d_0),d_0\right)+(K-1)u\left(\frac{A_M(d_0,d_0)}{K-1},d_0\right)-C\left(a_1(d_0,d_0)+A_M(d_0,d_0)\right)\\=&(K-1)u\left(\frac{A_{K-1}(d_0)}{K-1},d_0\right)-C\left(A_{K-1}(d_0)\right).\end{aligned}$$

Using $d_M^*>d_0$ and $u_{12}>0$, we obtain

$$\begin{aligned}&u\left(a_1(d_0,d_M^*),d_0\right)+(K-1)u\left(\frac{A_M(d_0,d_M^*)}{K-1},d_M^*\right)-C\left(a_1(d_0,d_M^*)+A_M(d_0,d_M^*)\right)\\<&(K-1)u\left(\frac{A_{K-1}(d_M^*)}{K-1},d_M^*\right)-C\left(A_{K-1}(d_M^*)\right).\end{aligned}\tag{A.29}$$

If we replace d_0 in the left-hand side of (A.29) with d_1^*, it is equal to the right-hand side of (A.29), because (A.26) equals (A.28). Therefore, we have $d_1^*<d_0$.

Consumer surplus in the post-merger game is $(K-1)u\left(\frac{A_{K-1}(d_M^*)}{K-1},d_M^*\right)-C\left(A_{K-1}(d_M^*)\right)$, and the one in the pre-merger game is $(K-1)u\left(\frac{A_{K-1}(d_0)}{K-1},d_0\right)-C\left(A_{K-1}(d_0)\right)$. Because $d_M^*>d_0$, the merger decreases consumer surplus.

Finally, we show that there is a pure-strategy subgame perfect equilibrium in the post-merger game. First, let $\bar{d}$ be the unique value that satisfies $\max_{A\in[0,\overline{A}]}u\left(A,\bar{d}\right)-C(A)=0$. Consider the equation

$$\begin{aligned}&u\left(a_1(d_1,d_M),d_1\right)+(K-1)u\left(\frac{A_M(d_1,d_M)}{K-1},d_M\right)-C\left(a_1(d_1,d_M)+A_M(d_1,d_M)\right)\\=&\,u\left(A_1(d_1),d_1\right)-C\left(A_1(d_1)\right).\end{aligned}\tag{A.30}$$

Fix $d_1\in[0,\bar{d}]$. If $d_M=0$, the left-hand side is weakly greater. As $d_M\to\infty$, the left-hand side goes to $-\infty$. Also, the left-hand side is continuous and strictly decreasing in d_M. As a result, there is a unique d_M that satisfies the above equation. Let $d_M(d_1)$ denote such a d_M. Note that $d_M(d_1)$ is continuous. To show $d_M(d_1)$ is decreasing, define

$$\begin{aligned}g(d_1,d_M):=&\,u\left(A_1(d_1),d_1\right)-C\left(A_1(d_1)\right)\\&-\left[u\left(a_1(d_1,d_M),d_1\right)+(K-1)u\left(\frac{A_M(d_1,d_M)}{K-1},d_M\right)-C\left(a_1(d_1,d_M)+A_M(d_1,d_M)\right)\right].\end{aligned}$$

Note that $d_M(d_1)$ satisfies $g(d_1, d_M(d_1)) = 0$. By the envelope theorem,

$$g_1(d_1, d_M) = u_2\left(A_1(d_1), d_1\right) - u_2\left(a_1(d_1, d_M), d_1\right) \geq 0,$$

because $u_{12}(a, d) > 0$ and $A_1(d_1) > a_1(d_1, d_M)$. Because $g_2(d_1, d_M) > 0$, to satisfy equation $g(d_1, d_M) = 0$, d_M must decrease whenever d_1 increases. As a result, $d_M(d_1)$ is weakly decreasing.

Next, for each $d_1 \in [0, \bar{d}_1]$, let $\hat{d}_M(d_1)$ solve

$$(K-1)u\left(\frac{A_{K-1}(d_M)}{K-1}, d_M\right) - C\left(A_{K-1}(d_M)\right) = u\left(A_1(d_1), d_1\right) - C\left(A_1(d_1)\right). \tag{A.31}$$

By the similar argument as above, we can show that $\hat{d}_M(d_1)$ is unique, continuous, and strictly increasing. At $d_1 = 0$, the left-hand side of (A.30) is weakly greater than that of (A.31). Thus, $d_M(0) \geq \hat{d}_M(0)$. At $d_1 = \bar{d}$, the left-hand side of (A.30) is weakly smaller than that of (A.31), because

$$\begin{aligned}
&u\left(a_1(d_1, d_M), d_1\right) + (K-1)u\left(\frac{A_M(d_1, d_M)}{K-1}, d_M\right) - C\left(a_1(d_1, d_M) + A_M(d_1, d_M)\right) \\
&\leq (K-1)u\left(\frac{A_M(d_1, d_M)}{K-1}, d_M\right) - C\left(A_M(d_1, d_M)\right) \\
&\leq (K-1)u\left(\frac{A_{k-1}(d_M)}{K-1}, d_M\right) - C\left(A_{K-1}(d_M)\right).
\end{aligned}$$

Here, the first inequality holds because at $d_1 = \bar{d}$, the consumer's payoff decreases by joining platform 1. Thus, $d_M(\bar{d}) \leq \hat{d}_M(\bar{d})$. Because $d_M(d_1)$ is weakly decreasing and $\hat{d}_M(d_1)$ is strictly increasing, they have a unique crossing point, which corresponds to an equilibrium. □

F Proof of Proposition 7: The Impact of Digital Curfew

Proof. Point 1 follows from Proposition 1, and Point 2 follows from Proposition 2. We show Point 3. We adopt the baseline setting in which we do not keep the market size constant. For d and K, let $A_K(d)$ denote the consumer's total attention when she joins K platforms with addictiveness d. Note that $A_K(d)$ implicitly depends on $\overline{A}$. Let $d^*(A)$ denote the equilibrium addictiveness with

$\overline{A} = A$. The equilibrium addictiveness satisfies

$$K \cdot u\left(\frac{A_K(d^*)}{K}, d^*\right) - C\left(A_K(d^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(d^*)\right). \quad \text{(A.32)}$$

If we cap the maximum attention at $X < A_K(d^*)$, we have

$$K \cdot u\left(\frac{X}{K}, d^*\right) - C\left(X\right) < (K-1) \cdot u\left(\frac{A^X_{K-1}(d^*)}{K-1}, d^*\right) - C\left(A^X_{K-1}(d^*)\right). \quad \text{(A.33)}$$

Generally, if the consumer joins y platforms with addictiveness d at cap X, her optimal payoff is $U(y,d) := \max_{A\in[0,X]} yu\left(\frac{A}{y}, d\right) - C(A)$. The envelope formula implies $U_2(y,d) = yu_2\left(\frac{A(y,d)}{y}, d\right)$. Now, $u_2\left(x,d\right)$ is negative and increasing in x. Also, $\frac{A(y,d)}{y}$ is decreasing in y. Thus, $U_2(y,d) = yu_2\left(\frac{A(y,d)}{y}, d\right)$ is decreasing in y.

The above observation implies that if platforms increased addictiveness after a cap of X, the consumer continues to join at most $K-1$ platforms, which contradicts the equilibrium condition. Thus, after a curfew, the platforms set a strictly lower addictiveness.

If $A^*(K) < \overline{A}$, we have $A^*(K-1) < A^*(K)$ (Lemma 3 implies $A^*(K-1) \leq A^*(K)$, and the comparative statics strictly holds if the consumer's choice is interior). Consider a digital curfew with a cap $A_D = A^*(K-1)$. Before platforms adjust addictiveness, this digital curfew does not change the consumer's payoff, because she can join $K-1$ platforms and allocates attention $A^*(K-1)$ optimally. After the cap, the platforms strictly decrease their addictiveness. Thus, the consumer is strictly better off than without the digital curfew. □

G Proofs for Section 6: Price Competition and Attention Competition

Proof of Lemma 1. Throughout the proof, we fix K and use the notations and the results in Lemma 3 with $u(a,d)$ replaced by $\hat{u}(a,d) := \frac{1}{K}u(Ka,d)$. Define

$$p^* := K\hat{u}\left(\frac{A^*_K(0)}{K}, 0\right) - C(A^*_K(0)) - \left[(K-1)\hat{u}\left(\frac{A^*_{K-1}(0)}{K-1}, 0\right) - C(A^*_{K-1}(0))\right].$$

Then, we show that there is an equilibrium in which each platform k sets $d_k = 0$ and $p_k = p^*$.

Suppose each platform k sets $(d_k, p_k) = (0, p^*)$. The consumer chooses the number K' of

platforms to join to maximize $V(K')$, where

$$V(K') := \max_{A \in [0,\overline{A}]} K' \hat{u}\left(\frac{A}{K'}, 0\right) - C(A) - K' p^*$$

Lemma 3 implies that $V(K')$ is concave on $[0, K]$. Because p^* makes the consumer indifferent between joining K and $K-1$ platforms, it is optimal for her to join all platforms.

Suppose platform k deviates and chooses (d'_k, p'_k). If $d'_k > 0$, platform k has to set $p'_k < p^*$; otherwise, the consumer strictly prefers to join only platforms $2, \ldots, K$. In this case the deviation reduces k's payoff. Conditional on $d'_k = 0$, p^* is the maximum price that platform k can charge, because the consumer is indifferent between joining $K-1$ and K platforms at price p^*. Thus, platform k has no profitable deviation.

The above equilibrium is unique. To show this, take any equilibrium, and suppose each platform k chooses (d^*_k, p^*_k). First, we show that the consumer joins all platforms in equilibrium. Fix $\hat{k} \in K$, and suppose platform $\hat{k}$ sets $(d_{\hat{k}}, p_{\hat{k}}) = (0, 0)$, which may or may not be a deviation. Let K_0 denote the set of platforms the consumer joins, following $(d_{\hat{k}}, p_{\hat{k}}) = (0, 0)$. Take any $K' \subset K$ such that $\hat{k} \notin K'$. First, if $d^*_j > 0$ for some $j \in K'$, then the consumer strictly prefers joining $(K' \setminus \{j\}) \cup \left\{\hat{k}\right\}$ to joining K'. Second, if $d^*_j = 0$ for all $j \in K'$ or $K' = \emptyset$, then the consumer strictly prefers $K' \cup \left\{\hat{k}\right\}$ to K'. Thus, for any set K' of platforms such that $\hat{k} \notin K'$, we can find some set S of platforms such that $\hat{k} \in S$ and the consumer strictly prefers S to K'. As a result, for a sufficiently small $p_{\hat{k}} > 0$ and $d_{\hat{k}} = 0$, the consumer still joins platform $\hat{k}$. This argument implies that any platform earns a positive profit in any equilibrium. Therefore, the consumer joins all platforms.

Second, we show all platforms set zero addictiveness in any equilibrium. Suppose to the contrary that $d^*_k > 0$ for some k. Suppose platform k deviates and chooses $(d_k, p_k) = (0, p^*_k)$. Before the deviation, the consumer weakly prefers joining all platforms to joining any set K' of platforms that does not contain k. Thus, after the deviation to $(0, p^*_k)$, the consumer strictly prefers to joining platform k. As a result, platform k can slightly increase its price while retaining the consumer. This is a contradiction.

We have shown that in any equilibrium, the consumer joins all platforms, which set zero addictiveness. The price of each platform makes the consumer indifferent between joining and not

joining the platform; otherwise, the platform can deviate by slightly increasing its price. Therefore, $(d_k^*, p_k^*) = (0, p^*)$ is a unique equilibrium. □

Proof of Proposition 8. First, we show Point 1. Under price competition, all platforms choose zero addictiveness. To simplify notation, we write $u(a)$ instead of $u(a, 0)$, and A_x instead of $A_x(0)$. In equilibrium, the consumer is indifferent between joining K and $K-1$ platforms that choose zero addictiveness. Thus, we have

$$u(A_1) - C(A_1) - Kp^* = \frac{K-1}{K} u\left(\frac{K}{K-1} A_{\frac{K-1}{K}}\right) - C\left(A_{\frac{K-1}{K}}\right) - (K-1)p^*. \tag{A.34}$$

The equation implies

$$Kp^* = K(1-x) \cdot \frac{u(A_1) - C(A_1) - \left[xu\left(\frac{A_x}{x}\right) - C\left(A_x\right)\right]}{1-x} \tag{A.35}$$

for any $x \in \left\{\frac{K-1}{K}\right\}_{K\in\mathbb{N}}$. Now, define $f(x) := xu\left(\frac{A_x}{x}\right) - C\left(A_x\right)$. Since $K(1-x) = 1$ for any $x \in \left\{\frac{K-1}{K}\right\}_{K\in\mathbb{N}}$, the right-hand side of (A.35), as $K \to \infty$, converges to $f'(1)$. Corollary 4 of Milgrom and Segal (2002) implies $f'(1) = u(A_1) - A_1 u'\left(A_1\right)$. Thus, by taking $K \to \infty$, we obtain $\lim_{K\to\infty} Kp^* = u(A_1) - A_1 u'\left(A_1\right)$.

Thus, the consumer's payoff converges to

$$u(A_1) - C(A_1) - [u(A_1) - A_1 u'(A_1)] = A_1 u'(A_1) - C(A_1).$$

In the limit $K \to \infty$, the consumer's payoffs under attention competition and price competition are $A_1(d^*)u_1(A_1(d^*), d^*) - C(A_1(d^*))$ and $A_1(0)u'(A_1(0), 0) - C(A_1(0))$, respectively.

To show $A_1(d^*)u_1(A_1(d^*), d^*) - C(A_1(d^*)) > A_1(0)u'(A_1(0), 0) - C(A_1(0))$, we consider three cases. Note that we always have $A_1(0) \leq A_1(d^*)$. First, suppose $A_1(d^*) < \overline{A}$. Then by the first-order conditions, these payoffs are respectively equal to $A_1(d^*)C'(A_1(d^*)) - C(A_1(d^*))$ and $A_1(0)C'(A_1(0)) - C(A_1(0))$. The function $xC'(x) - C(x)$ is increasing because its first derivative is $xC''(x) \geq 0$. As a result,

$$A_1(d^*)C'(A_1(d^*)) - C(A_1(d^*)) \geq A_1(0)C'(A_1(0)) - C(A_1(0)).$$

If $C''(\cdot) > 0$, the inequality is strict.

Second, suppose $A_1(0) = A_1(d^*) = \overline{A}$. Then, the consumer's payoffs under attention competition and price competition are $\overline{A}u_1(\overline{A}, d^*) - C(\overline{A})$ and $\overline{A}u_1(\overline{A}, 0) - C(\overline{A})$, respectively. The former is strictly greater than the latter as $u_{12} > 0$.

Third, suppose $A_1(0) < A_1(d^*) = \overline{A}$. Then, the consumer's payoffs under attention competition is $\overline{A}u_1(\overline{A}, d^*) - C(\overline{A}) \geq \overline{A}C'(\overline{A}) - C(\overline{A}) > A_1(0)C'(A_1(0)) - C(A_1(0))$. Thus, the consumer is strictly better off under attention competition in the limit, so the same welfare comparison holds for a sufficiently large K.

Point 2 follows from the observation that a monopoly platform yields zero consumer surplus under price competition, but it chooses zero addictiveness under attention competition when $\overline{A} \leq A(0)$. □

H Appendix for Section 7: Naive Consumer

Let us formally describe the timing of the game and the optimization problems of the consumer. First, each platform $k \in K$ simultaneously chooses its addictiveness, $d_k \geq 0$. Second, given the perceived addictiveness $(sd_k)_{k \in K}$, the consumer chooses the set $\hat{K} \subset K$ of platforms to join. In equilibrium, $\hat{K}$ maximizes the perceived indirect utility $V(K')$ across all $K' \in 2^K$, where

$$V(K') := \max_{(a_k)_{k \in K'} \in \mathbb{R}_+^{K'}} \sum_{k \in K'} u(a_k, sd_k) - C\left(\sum_{k \in K'} a_k\right) \tag{A.36}$$
$$\text{s.t.} \quad \sum_{k \in \hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}.$$

If $\hat{K} = \emptyset$, all players obtain a payoff of zero, and the game ends. After joining platforms $\hat{K} \neq \emptyset$, the consumer observes the true addictiveness of each platform, then allocates her attention. In equilibrium, the consumer solves

$$\max_{(a_k)_{k \in \hat{K}} \in \mathbb{R}_+^{\hat{K}}} \sum_{k \in \hat{K}} u(a_k, d_k) - C\left(\sum_{k \in \hat{K}} a_k\right) \tag{A.37}$$
$$\text{s.t.} \quad \sum_{k \in \hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}.$$

The above two maximization problems coincide if $s = 1$. Our solution concept continues to be pure-strategy subgame perfect equilibrium. Even if $s < 1$, we can use SPE by treating the consumer who solves (A.36) and the consumer who solves (A.37) as different players who have different objectives.

First, we prove Proposition 9, which characterizes the equilibrium and conducts comparative statics in s.

Proof of Proposition 9. For now, we assume $K \geq 2$. Define $d_s^* := \frac{d^*}{s}$, where d^* is the equilibrium addictiveness of the original model (i.e., $s = 1$). Recall that d^* satisfies the sophisticated consumer's indifference condition, which we can rewrite as

$$K \cdot u\left(\frac{A_K(sd_s^*)}{K}, sd_s^*\right) - C\left(A_K(sd_s^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(sd_s^*)}{K-1}, sd_s^*\right) - C\left(A_{K-1}(sd_s^*)\right). \tag{A.38}$$

The equation means that the consumer with s is indifferent between joining K and $K-1$ platforms that choose addictiveness d_s^*. Note that the participation decision uses the perceived addictiveness, sd_s^*. By the same argument as the proof of Proposition 2, we can use this indifference condition to show the following: (i) given addictiveness d_s^*, the consumer joins all platforms; (ii) if platform k deviates and increases its addictiveness, the consumer joins all platforms but k; and (iii) if platform k deviates and decreases its addictiveness, she joins all platforms. Points (i) and (ii) imply that any platform cannot profitably deviate by increasing its addictiveness. For Point (iii), although the consumer's attention allocation is based on d_s^* (not on sd_s^*), she still allocates less attention to less addictive platforms. Thus, Point (iii) implies that any platform cannot profitably deviate by decreasing its addictiveness. The equilibrium addictiveness $\frac{d^*}{s}$ is decreasing in s, and the consumer joins all platforms for any s. In contrast, a monopoly platform sets zero addictiveness when $\overline{A} \leq A(0)$, because the consumer exhausts her attention at $d = 0$ regardless of the level of sophistication. □

Under price competition, the platforms first set addictiveness and prices. Then, the consumer

decides which platforms to join by maximizing $V^P(K')$, where

$$V^P(K') := \max_{(a_k)_{k\in K'}\in\mathbb{R}_+^{K'}} \sum_{k\in K'} [u(a_k, sd_k) - p_k] - C\left(\sum_{k\in K'} a_k\right) \quad \text{(A.39)}$$
$$\text{s.t.} \quad \sum_{k\in\hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}.$$

Note that the consumer now pays p_k to join platform k. After joining platforms, the consumer allocates her attention by solving (A.37). As before, the payoff of platform k is p_k and 0 if the consumer does and does not join platform k, respectively.

Claim 2. *For any $K \geq 2$, there is an $s^* \in (0, 1]$ such that the following holds: The consumer is better off under price competition than attention competition if and only if $s \leq s^*$.*

Proof. For any $s \in (0, 1]$ the same argument as Lemma 1 implies that all platforms set zero addictiveness in a unique equilibrium under price competition. Thus the consumer's payoff is independent of s under price competition, and it is increasing in s under attention competition. Also, for a small s the consumer's payoff under attention competition is negative because of Point (b) of Assumption 1. As a result, price competition gives the consumer a greater payoff if s is below some $s^* \in (0, 1]$. □

We now turn to the impact of digital curfew. Fix K, and let d_s^* denote the equilibrium addictiveness given $s \in (0, 1]$. For each $J \leq K$, let $A^*(d)$ denote the total amount of attention the consumer allocates to platforms, conditional on joining all platforms with the true addictiveness d.

Claim 3. *Take any $A \geq A^*(K, d_1^*)$. If $s = 1$, a digital curfew at $\overline{A} = A$ (as opposed to $\overline{A} = \infty$) does not affect consumer surplus. If $s = s^* \in (0, 1)$, the digital curfew weakly decreases consumer surplus. In particular, it strictly decreases consumer surplus if $\overline{A} \in [A^*(K, d_1^*), A^*(K, d_{s^*}^*))$.*

Proof. If $A \geq A^*(K, d_1^*)$, the cap at A has no impact on the equilibrium addictiveness or the consumer's participation decision, because the consumer's perceived addictiveness is $sd_s^* = d_1^*$ for any s and thus the maximum attention she believes she will allocate is $A^*(K, d_1^*)$. However, if $s < 1$ and $A \in [A^*(K, d_1^*), A^*(K, d_{s^*}^*))$, the cap strictly reduces the total attention she can allocate after joining the platforms. Such a digital curfew harms the consumer with $s < 1$. □