Equivalence in Business Models for Informational Intermediaries*

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Abstract

An intermediary has the technology to provide information about a product to consumers and serves as a platform through which transactions between a monopoly and consumers take place. This paper explores the intermediary's revenue maximization problem across all possible business models. By examining the revenue maximizing solutions under three critical business models, I discover that the market outcomes—consumers' expected surplus, producer's expected profit and the intermediary's expected revenue—are equivalent across all business models if and only if the gains from trade are large enough, which provides some insights into, and implications for online selling platforms.

KEYWORDS: Monopolistic pricing, outcome equivalence, screening, targeting, mechanism design, information design.

JEL CLASSIFICATION: D42, D82, D83.

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1 Introduction

The information about a product that is provided to consumers plays a central role in defining profits and consumer surplus. In many real-world contexts, the channel through which consumers receive information relies on a third party, an *intermediary*, who possesses the technology to provide information about the products to the consumers. Consumers can be informed about the presence of a particular product and can further learn about its features via the intermediary.

For instance, online selling platforms such as Amazon or eBay have a well-publicized website on which the commodities are presented to the consumers and different information about the commodities can be provided via photos, certificates and descriptions of the product. Alternatively, television programs such as QVC also have well-known broadcasting channels through which the producers can present their products and information about the commodity can be provided. Owning such platforms, these intermediaries can exploit various business models to extract surplus from the market.

Moreover, due to rapid development of informational technology in the past decade, this kind of economic interaction has become more and more significant. For example, as of 2016 Q1, the number of active accounts on Amazon was 310 million; In December, 2017, the number of visits on Amazon exceeded 3 billion; As of 2017, Amazon's net revenue was more than 170 billion.[1] The sizes of these intermediaries and the volume of transactions made through these intermediaries amplify the impact of such economic interaction on the whole society. In addition, as they develop, more business models have been introduced and the intermediaries' options for entering the market have become more abundant. For instance, Amazon used to operate through the "Vendor Central" model, where it buys directly from producers and sells the product on its own website. In recent years, a new business model, "Seller Central", has become more significant. In particular, the share of "Seller Central" has grown from 26% to 52% over the past decade. Under the "Seller Central" model, Amazon serves only as a platform and producers (termed as "third party sellers") sell their products on the website directly and pay associated fees to Amazon.

Given the importance of consumers' information and the rapid growth of third party intermediaries, it is thus of great importance to understand how capable such intermediaries are and how would various business models affect market outcomes. More specifically, as more business models become available, it is crucial to understand how these business models differ, how the intermediaries operate, and what the impacts on the market are. Furthermore, the enhancement of computational powers and data collection and storage methods also enables these intermediaries to tailor their information provision according to

[1]Source: Statista: https://www.statista.com/topics/846/amazon/.

consumers' personal values via *targeting*. It is also crucial to understand how the intermediaries' targeting technology would affect market outcomes.

In this paper, I explore the above questions. Specifically, I consider a monopolistic pricing model, in which the producer can produce a product with some nonegative production cost. The consumers do not know their valuations for the product or the presence of the producer *a priori*, but can interact with the producer and learn about the product through an intermediary. More specifically, there is an intermediary who serves as a platform where consumers and the producer trade and can provide information about the product to consumers. The goal of the intermediary is to maximize revenue by exploiting this technology. Three major business models are considered: The *direct selling* model, the *full contracting model*, and the *weak contracting* model. Under the direct selling model, the intermediary buys directly from the producer and sells to the consumers while providing information about the product at the same time; Under the full contracting model, the intermediary signs a contract with the producer, which specifies the amount of payment the seller has to pay to the intermediary, the information that will be be provided to the consumers, and how the product should be sold; Under the weak contracting model, the intermediary delegates all the selling decision to the producer and only charges the producer for providing informational services. These three business models are the most relevant as they describe the ways that real-world intermediaries, such as Amazon, operate (c.f. the Amazon Vendor Central and the Amazon Seller Central). Moreover, they serve as a basis to extend the scope to incorporate *all* possible business models.

The main result of this paper is that the market outcomes—consumer surplus, producer profit, and the intermediary's revenue—are the same across all the business models if and only if gains from trade are large enough. As such, the exact form of the business model actually does not matter. Impacts of the intermediaries on the market are independent of what business model the intermediaries use to enter the market. Furthermore, I also show that even under the most extreme form of targeting—the intermediary knows exactly the consumers' value and can target them accordingly—the market outcomes are still the same and do not depend on the business models that are used, as long as the gains from trade are large enough. This outcome equivalence result addresses a significant concern that availability of various business models and better targeting technology might render the intermediary too much power and allow it to extract too much surplus from the market. The equivalence result implies that the exact form of business model and quality of targeting technology have no impacts on market outcomes, provided that gains from trade are large enough. Furthermore, using the equivalence result, this paper further provides welfare analyses and comparative statics that are robust to specifications of business models. I show that regardless of the business models that are used, total surplus and volume of trade when an intermediary has the informational technology are less than a benchmark case when the producer has control of the informational technology directly. Also, I show that regardless of the business models, total surplus and the producer's profit are larger when consumers' values are larger and when information rent of the producer is smaller.

To fix ideas, consider a parametric example that demonstrates the results of this paper. Suppose that the consumers' values are uniformly distributed on $[0, \bar{v}]$, for some $\bar{v} \geq 3$ (so that gains from trade are large enough) and that the producer's production cost is private information and is uniformly distributed on $[0, 1]$. If the intermediary uses the direct selling business model, then the under the revenue maximizing solution, the intermediary submits a demand to the producer via a nonlinear pricing schedule, in which a producer with cost c will sell $\bar{v}/2 - c$ units to the intermediary and receive payment $1-(1+c^2)/\bar{v}$ and the intermediary discloses binary information to the consumers so that they know only whether their value is above $2c$ and set a price at $\bar{v}/2 + c$. This yields revenue $7\bar{v}/6 - 1$. On the other hand, under the full contracting model, in the optimal contract, the intermediary still discloses binary information to consumers so that they know whether their value is above $2c$. The price that the producer sets is also contracted at $\bar{v}/2 + c$ and the amount of payment that a producer with cost c has to pay to the intermediary is $\alpha - \beta c^2$, for some $\alpha, \beta > 0$ that depend on $\bar{v}$, and the revenue is still $7\bar{v}/6 - 1$. Finally, under the weak contracting model, the intermediary sets a menu that has a continuum of items, each of them is indexed by a cutoff $k \in [0, 2]$. For each item indexed by k, the intermediary provides information to the consumers so that they perfectly learn the value whenever it is below k and learn nothing else whenever it is above k. The payment associated with this information structure is $\gamma + \delta(1 - k^2/4)$, for some $\gamma, \delta > 0$ that depend only on $\bar{v}$. The revenue that the intermediary has is still $7\bar{v}/6 - 1$.

The rest of this paper is organized as follows: The next section summarizes the related literature. Section 3 introduces the model and three major business models. Section 4 characterizes the revenue maximizing solutions for each of these three business models and establishes outcome equivalence among these three models. A necessary and sufficient condition for this equivalence result is characterized and interpreted. In section 5, targeting technology is introduced and the equivalence result with targeting is established. In section 6, abstract definition of an arbitrary business model is introduced and the outcome equivalence result is established when considering all possible business models. In addition, comparative statics and welfare analyses are provided. Section 7 contains discussions of the results and section 8 concludes. The proofs of the results can be found in the appendix.

2 Related Literature

This paper is related to several branches of literature in interplay between monopolistic pricing and information structure, selling information, and Bayesian persuasion. In the monopolistic pricing literature, Lewis and Sappington (1991) also examines how the change in consumer's information affects a monopolist's profit. They show that for a given monopolist with a constant production cost, the buyer having either full information or no information is optimal for the monopolist. The model in this paper differs from theirs in two major aspects. First, I examine how a third party would price information structures for a monopolist to purchase, instead of examining optimal information structure from a monopolist's perspective directly. Second, although the setting of Lewis and Sappington (1991) is close to a benchmark of the model here in which the monopolist can choose information structure that the buyer has directly, the model in this paper maintains an assumption that the commodity is indivisible so that buyers have 0-1 demand while the consumer's demand in their model can be more general. On the other hand, Lewis and Sappington (1991) restrict the information structures to vary within a one-dimensional family by assuming a particular disclosure rule, whereas the model here allows full flexibility of the choice of information structure. Johnson and Myatt (2006) also studies how change in the distribution of buyers' valuation, which is equivalent to the information that the buyer has in the model here, affects a monopolist's profit. In particular, they show that when the distributions are ordered by the rotational ordering, the monopolist will prefer two extremes of the order. Using the language of buyers' information, this result is similar to Lewis and Sappington (1991) in that it implies that under a particular one dimensional (and hence, totally-ordered) family of information structures, a monopolist will either prefer the most informative one or the least informative one. Again, the model here differs from theirs in that it focuses on a third party's surplus extraction and that complete flexibility in providing information structures is allowed. Recent developments, on the other hand, have adopted flexible information structures. Bergemann et al. (2015) characterizes all the possible surplus division that can arise by giving the monopolist different information about the buyer's valuation. Roesler and Szentes (2017) examines the buyer-optimal information structure when facing a monopoly.

There are several works that also study a problem of pricing information. Bergemann and Bonatti (2015) studies a pricing problem of a data provider who can provide information about the match value for a seller whose profit from trade depends on the match value of each consumer and the amount of investments the seller makes in each consumer. Bergemann et al. (2018) solves an optimal menu for a data provider to sell experiments to a decision maker who has a private estimate about the state. The model in this paper differs from the theirs in that the intermediary in this model sells information to affect the information of the *buyer* who is buying a product produced by the *seller*, which affects the seller's value function of different information indirectly, whereas the model in Bergemann et al. (2018) focuses on selling information structure to a decision maker whose value function depends on the information she purchases directly. Segura-Rodriguez (2019) considers a model where a data broker sells information for forecasting a multidimensional variable to firms who differ in the dimensions that are payoff-relevant. Yang (2019) studies the revenue-maximizing mechanisms for a data broker who sells data about the consumers value to a producer who has private information about her production cost and derive the economic implications. Although Yang (2019) also establishes an outcome equivalent result, the reasons and the techniques involved are distinct from this paper, since the information being sold in this paper is to inform the buyer instead of the seller.

Furthermore, the screening framework in the model for selling information structure is analogous to standard monopolistic screening problems as in Mussa and Rosen (1978), Myerson (1981) and Maskin and Riley (1984). The screening problem in the model here is more complicated in that the outcome space is infinite dimensional. Moreover, in some business models (e.g. the weak contracting model), the revenue maximizing problem is a mixture of screening and moral hazard problem from the intermediary's perspective. Also, the assumption that intermediary is able to commit to a menu and the characterization of information structure follows from the the Bayesian persuasion literature, as Kamenica and Gentzkow (2011), Gentzkow and Kamenica (2016).

3 Preliminaries

3.1 Notation

The following notation is used throughout the paper. For any real-valued functions f, g on an interval I, write $f \geq g$ if and only if $f(x) \geq g(x)$ for all $x \in I$; Write $f \geq 0$ if and only if $f(x) \geq 0$ for all $x \in I$. Define $f^+ := \max\{f, 0\}$. For any arbitrary function f, g defined on the same domain and codomain, write $f \equiv g$ if and only if $f(x) = g(x)$ for all x in the domain. For any topological space E, $\mathcal{B}(E)$ denotes the Borel σ-algebra on E induced by the topology and $\Delta(E)$ denotes the collection of probability measures (on the measurable space $(E, \mathcal{B}(E))$). For any topological space E, any random variable x on E with a law given by a probability measure $\mu \in \Delta(E)$ and any measurable function f on E, let

$$\mathbb{E}_\mu[f(x)] := \int_E f(x)\mu(\mathrm{d}x)$$

denote the expectation of the random variable $f(x)$. Similarly, for any (sub) σ-algebra $\mathcal{A} \subseteq \mathcal{B}(E)$, the conditional expectation of $f(x)$ is denoted as $\mathbb{E}_\mu[f(x)|\mathcal{A}]$, and for any non-empty, measurable $A \subseteq E$, $\mathbb{E}_\mu[f(x)|A] := \mathbb{E}_\mu[f(x)|\sigma(A)]$, where $\sigma(A)$ is the σ-algebra generated by

A. As notational conventions, when E is an interval $[a, b]$ with some $-\infty \leq a < b \leq \infty$, write

$$\int_b^a f(x)\mu(\mathrm{d}x) := -\int_a^b f(x)\mu(\mathrm{d}x).$$

Also, when $E = [a, b]$, $b' > b$ and $a' < a$, define

$$\mathbb{E}_\mu[x|x > b'] := b, \quad \mathbb{E}_\mu[x|x < a'] := a.$$

When there is no confusion, a probability measure on the real line and its associated CDF will be used interchangeably. For any measurable sets $(X, \mathcal{X})$, $(Y, \mathcal{Y})$ and any (sub) σ-algebra $\mathcal{Z} \subseteq \mathcal{X}$, $M(\mathcal{Z})$ denotes the collection of functions $f : X \to Y$ that are measurable with respect to $\mathcal{Z}$.

3.2 Model

There is a buyer (he), a seller (she) and an intermediary (it). The seller is selling an indivisible object to the buyer. The buyer has a quasi-linear preference with valuation v that follows a common prior F which has support on a compact interval $[\underline{v}, \bar{v}]$, for some $0 \leq \underline{v} < \bar{v}$. The buyer does not know his valuation *a priori*. Rather, he has to learn about his valuation through the information provided by the intermediary. More precisely, the intermediary has the technology to design (and commit to) *information structures*[2] in order to inform the buyer. Since the buyer has quasi-linear preference, the interim expected value is the only payoff relevant statistic for a given information structure. As such, the marginal distribution of the interim expected value conveys all the payoff-relevant aspect of a given information structure. As in Gentzkow and Kamenica (2016),[3] the information structures can be represented by the collection of CDFs that are mean-preserving contractions of F. Therefore, for any (integrable) function $H : \mathbb{R}_+ \to [0, 1]$, define

$$I_H(x) := \int_x^\infty (1 - H(z))\,\mathrm{d}z,$$

[2]The term information structure is used interchangeably with *disclosure rule* and *disclosure policy*.

[3]Similar characterizations appear in many recent developments in the literature of mechanism and information design, see for instance Neeman (2003), Bergemann and Pesendorfer (2007), Shi (2012), Roesler and Szentes (2017), Kolotilin et al. (2017), Bergemann and Morris (2017), Du (2018), Dworczak and Martini (2019), Brooks and Du (2019). Skreta and Perez-Richet (2018).

for all $x \geq 0$, the collection of information structures can then be represented by the set[4]

$$\mathcal{H}_F := \Big\{ H : \mathbb{R}_+ \to [0,1] \Big| I_H(x) \leq I_F(x),\ \forall x \in \mathbb{R}_+,\ \text{with equality at } x = 0,$$
$$H \text{ is nondecreasing, left-continuous, } H(0) = 0,\ \lim_{x\to\infty} H(x) = 1 \Big\},$$

which can be illustrated by Figure 1, where F_0 denotes the distribution that assigns probability 1 at $\mathbb{E}_F[v]$.

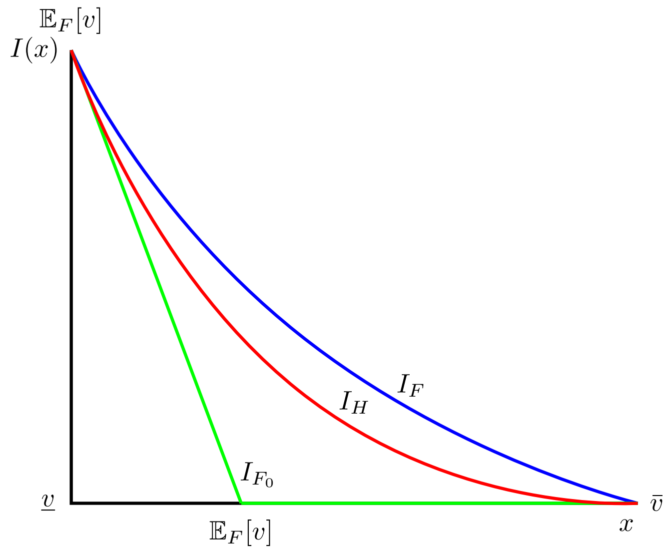

Figure 1: Feasible Set $\mathcal{H}_F$.

On the other hand, the seller has private information about her production cost $c \in [0, \bar{c}]$, for some $\bar{c} \geq 0$.[5] This private cost is drawn from a common prior G and is independent of the buyer's value v. Throughout the paper, the pair of priors (F, G) is assumed to be *regular*.

[4]For notational convenience, the information structures here are characterized by the left-limits of the distribution of interim expected values instead of the CDFs directly. This is because the buyer must break ties in favor of the seller in order for the equilibrium to exist. As any CDF is nondecreasing and thus has at most countably many jumps, this does not affect the value of (Lebesgue) integrals. Also for notational convenience, the second order stochastic dominance constraint is not stated in the canonical form. However, it is clear that for any CDF H such that $\mathbb{E}_H[v] = \mathbb{E}_F[v]$, for any $x \in [\underline{v}, \bar{v}]$,

$$\int_0^x H(z)\,\mathrm{d}z \leq \int_0^x F(z)\,\mathrm{d}z \iff I_H(x) \leq I_F(x).$$

As such, F is a mean preserving spread of H if and only if

$$I_H(x) \leq I_F(x),\ \forall x \in \mathbb{R}_+,\ \text{with “=” at } x = 0.$$

[5]The lower bound of seller's cost is normalized to zero. Under this normalization, the condition that

That is, both F and G admit densities $f > 0$ on $[\underline{v}, \bar{v}]$, $g > 0$ on $[0, \bar{c}]$, respectively, and the induced *virtual value*, $\phi(v) := v - (1 - F(v))/f(v)$, and *virtual cost*, $\psi(c) := c + G(c)/g(c)$, are increasing.[6] Finally, it is assumed that the seller cannot interact with the buyer without the intermediary. To exploit the information technology and extract surplus, the intermediary has different *business models* that it can use. Three major business models are introduced here, including the *direct selling model*, the *full contracting model* and the *weak contracting model* while section 6 provides an abstract definition for all business models. Each of these three models differs in the approach through which the intermediary enters the interaction between the buyer and the seller. Formal descriptions are as below:

Direct selling model: Under the direct selling model, the intermediary buys the ownership of the production technology from the seller and then sells and discloses information to the buyer by itself. That is, the intermediary pays the seller for producing the product and then chooses both the information structure and the selling mechanism to sell to the buyer. Notice that due to quasi-linearity, it is without loss to use a posted price mechanism when selling to the buyer. Therefore, by the revelation principle, a direct mechanism under the direct selling model can be formally defined as a trading plan $(q(c), t(c), p(c), H^c)_{c \in [0, \bar{c}]}$. The pair $(p(c), H^c)_{c \in [0, \bar{c}]}$ is the selling plan used to sell to the buyer, where for each report $c \in [0, \bar{c}]$, $p(c) \in [\underline{v}, \bar{v}]$ is the posted price and $H^c \in \mathcal{H}_F$ is the information structure provided to the buyer. On the other hand, $(q(c), t(c))_{c \in [0, \bar{c}]}$ is the production plan, where for each report $c \in [0, \bar{c}]$, $q(c) \in [0, 1]$ is the probability of asking the seller to produce the product and $t(c) \in \mathbb{R}$ is the associated payment from the intermediary to the seller. A trading plan must be such that the product is only produced after a sale is made. That is, $q(c) = (1 - H^c(p(c)))$ for all $c \in [0, \bar{c}]$. The timing of this model is as follows: First, Nature draws v and c from F and G, respectively. Then, the intermediary chooses an incentive compatible and individually rational trading plan $(q(c), t(c), p(c), H^c)_{c \in [0, \bar{c}]}$ such that $q(c) = (1 - H^c(p(c)))$ for all $c \in [0, \bar{c}]$. The seller then makes reports in the mechanism and outcomes are realized according to the

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$\underline{v} \geq 0$ is substantive, as it requires the minimum element of the support of buyer's values must be larger than that of seller's cost. The main results of this paper without such assumption remains unchanged but the characterization of the solutions is less compact.

[6] The regularity assumption is to simplify the expression of the solutions and to circumvent unnecessary complications. In the online appendix, I show that the main result still holds qualitatively even if the regularity assumption is relaxed.

reports. As such, to maximize revenue, the intermediary solves

$$\sup_{q,t,p,\{H^c\}} \mathbb{E}_G[p(c)q(c) - t(c)]$$

$$\text{s.t. } q(c) = (1 - H^c(p(c))),\ \forall c \in [0, \bar{c}] \tag{D-MC}$$

$$t(c) - cq(c) \geq t(c') - cq(c'),\ \forall c, c' \in [0, \bar{c}] \tag{D-IC}$$

$$t(c) - cq(c) \geq 0,\ \forall c \in [0, \bar{c}] \tag{D-IR}$$

Full contracting model: Under the full contracting model, the intermediary signs a contract with the seller. A contract specifies the information structure provided to the buyer, the price of the object when selling to the buyer, and the associated payments that the seller has to pay to the intermediary. Formally, by the revelation principle, a contract can be described by $(p(c), H^c, t(c))_{c\in[0,\bar{c}]}$, where for any report $c \in [0, \bar{c}]$, $p(c) \in [\underline{v}, \bar{v}]$ is the price of the object,[7] $H^c \in \mathcal{H}_F$ is the information structure that the intermediary provides to the buyer, and $t(c) \in \mathbb{R}$ is the amount of payment that the seller pays to the intermediary. Timing of this model is as follows: First, Nature draws v and c from F and G, respectively. Then the intermediary proposes an incentive compatible and individually rational contract $(p(c), H^c, t(c))_{c\in[0,\bar{c}]}$. The seller then reports her cost. Given the cost report, the contract is enforced: An information structure H^c is used to inform the buyer and the buyer makes purchasing decision after observing the signal realization and the posted price $p(c)$. As such, the revenue maximization problem for the intermediary under this model is given by:

$$\sup_{p,\{H^c\},t} \mathbb{E}_G[t(c)]$$

$$\text{s.t. } (p(c) - c)(1 - H^c(p(c))) - t(c) \geq (p(c') - c)(1 - H^{c'}(p(c'))),\ \forall c, c' \in [0, \bar{c}] \tag{F-IC}$$

$$(p(c) - c)(1 - H^c(p(c))) - t(c) \geq 0,\ \forall c \in [0, \bar{c}] \tag{F-IR}$$

Weak contracting model: Under the weak contracting model, the intermediary cannot contract on how the object is sold to the buyer, instead, it can only provide a service of information provision to the seller and delegates the choice of selling mechanism to the seller. By the revelation principle, a menu can be described by $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$, where for each report $c \in [0, \bar{c}]$, $\alpha(c) \in \{0, 1\}$ is the publicizing decision that determines whether the seller

[7] Again by quasi-linearity, it is without loss to consider contracts that specifies the posted price only instead of any other complex selling mechanisms. Formal arguments can be found in the online appendix.

can interact with the buyer,[8] $H^c \in \mathcal{H}_F$ is the information structure that the intermediary provides to the buyer, and $t(c) \in \mathbb{R}$ is the amount of payments from the seller to the intermediary. Timing of this model is similar: First, Nature draws v and c from F and G, respectively. Then the intermediary sets an incentive compatible and individually rational menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$. The seller then makes a report after seeing the menu. Given the reports, the seller makes payments to the intermediary and the intermediary provides information to the buyer according to the menu. If $\alpha(c) = 1$, the seller then design a selling mechanism to sell the object. Again, notice that regardless of the buyer's information about the value, quasi-linear preference implies that it is without loss for the seller to use posted price mechanisms. As such, the intermediary's revenue maximization problem is:[9]

$$\sup_{\alpha,\{H^c\},t} \mathbb{E}_G[t(c)]$$

$$\text{s.t. } \alpha(c) \max_{p\in[\underline{v},\bar{v}]}(p-c)(1-H^c(p)) - t(c) \geq \alpha(c') \max_{p\in[\underline{v},\bar{v}]}(p-c)(1-H^{c'}(p)),\ \forall c,c' \in [0,\bar{c}] \tag{W-IC}$$

$$\alpha(c) \max_{p\in[\underline{v},\bar{v}]}(p-c)(1-H^c(p)) - t(c) \geq 0,\ \forall c \in [0,\bar{c}] \tag{W-IR}$$

The revenue maximization problems above formalize three major business models an intermediary can operate with. These models differ in many aspects, especially the role that the intermediary plays in determining how the object is sold to the buyer. Despite the differences among these models, the main result of this paper shows that all these models are equivalent in terms of their expected outcomes—the expected revenue for the intermediary, the expected profit of the seller and the expected surplus of the buyer are all the same under optimal mechanism—if and only if gains from trade are large enough. In the next section, I will state this result formally and discuss the main intuition behind it by solving the revenue maximization problems associated with the aforementioned three business models.

4 Outcome Equivalence

The main result of this paper is that the market outcomes induced by the three business models are the same under a certain condition, which is formally stated below. Formally, say

[8]Recall that the seller cannot interact with the buyer without the intermediary. Thus, even though the intermediary does not have the ability to contract on price, it can still screen the seller by deciding whether to grant her access to the buyer. Under the full contracting model, this need not to be included because setting $p(c) = \bar{v}$ is equivalent to excluding the seller. Moreover, $\alpha(c) \in \{0, 1\}$ is a simplifying assumption. In the online appendix, I show that even when α is allowed to take values in $[0, 1]$, the optimal menu remains unchanged, indicating that this assumption is without loss.

[9]Again by quasi-linearity, it is without loss to restrict attention to the case in which seller sets a posted price optimally given any information structure.

that two business models are *outcome equivalent* if under any revenue-maximizing solution of both business models, the expected revenue of the intermediary, the expected surplus of the buyer, and the expected profit of the seller are the same.

Theorem 1 (Outcome Equivalence)**.** *For any regular prior (F, G), there exists $c^*(F, G) \geq 0$ such that the direct selling model, the full contracting model and the weak contracting model are outcome equivalent if and only if $c^*(F, G) \geq \bar{c}$.*

According to Theorem 1, for any regular prior (F, G), there exists a critical value $c^*(F, G) \geq 0$ such that market outcomes induced by the direct selling model, the full contracting model and the weak contracting model are the same if and only if the critical value $c^*(F, G)$ is greater than the highest production cost in the support of G. The exact formulae that define $c^*(F, G)$ can be found in Appendix B (see (14) and (15)). Whenever there is no confusion, I suppress the dependence of the critical value on the priors and denote the $c^*(F, G)$ as c^* hereafter.

The necessary and sufficient condition $c^*(F, G) \geq \bar{c}$ has an economically interpretable sufficient condition:

$$\psi(c) \leq \phi^{-1}(c) \tag{1}$$

for all $c \in [0, \bar{c}]$. By regularity, $\phi^{-1}(c)$ is the optimal monopoly price for a seller with cost c when the buyer has full information. Thus, $\psi(c) \leq \phi^{-1}(c)$ can be interpreted as the information rent retained by the seller with cost c being less than the monopoly mark-up she would charge when the buyer has full information. With this observation, Theorem 1 has an immediate corollary.

Corollary 1. *The direct selling model, the full contracting model and the weak contracting model are outcome equivalent if $\psi(c) \leq \phi^{-1}(c)$ for all $c \in [0, \bar{c}]$.*

For the rest of this section, I will illustrate the intuition behind Corollary 1. The general proof for Theorem 1 can be found in the appendix.

4.1 Equivalence between Direct Selling and Full Contracting

Below, I first discuss the equivalence between the direct selling model and the full contracting model. The equivalence between these two business model is in fact much stronger than that stated in Theorem 1. The selling mechanisms in these two models are actually isomorphic to each others. That is, for any incentive compatible and individually rational trading plan $(\hat{q}(c), \hat{t}(c), \hat{p}(c), \hat{H}^c)_{c \in [0, \bar{c}]}$, there exists an incentive compatible and individually rational full contract $(p(c), H^c, t(c))_{c \in [0, \bar{c}]}$ that yields the same outcome for any report $c \in [0, \bar{c}]$. Conversely, for any incentive compatible and individually rational full contract $(p(c), H^c, t(c))_{c \in [0, \bar{c}]}$, there exists an incentive compatible and individually rational trading plan $(\hat{q}(c), \hat{t}(c), \hat{p}(c), \hat{H}^c)_{c\in[0,\bar{c}]}$ that yields the same outcome for any report $c \in [0, \bar{c}]$. The proof of this claim can be found in the proof of Proposition 1, the intuition is simple: Consider any incentive compatible and individually rational trading plan under the direct selling model. Instead of buying and selling again directly, the intermediary can contract with the seller and requires the seller to post a price that would have been posted according to the trading plan, promise to provide information that would have been provided according to the trading plan, and ask for a payment that equals to the revenue that the intermediary would have obtained. As the original trading plan is incentive compatible and individually rational, the new contract must be incentive compatible as well. Furthermore, by construction, the outcomes must be the same. Analogous arguments can be used to establish the converse.

Due to this equivalence, solving the revenue maximization problem under the direct selling model is the same as solving the revenue maximization problem under the full contracting model. Furthermore, under the full contracting model, the revenue maximization problem is in fact a screening problem except that allocation space is infinite-dimensional. The role of "allocation" is replaced by the required posted price $p(c)$ and promised information structure H^c. As in classical mechanism design problems, the transfer is pinned down by allocations up to a constant due to local incentive constraints, and a monotonicity condition would ensure global incentive compatibility. This is formally stated in Lemma 1.

Lemma 1. *Under the full contracting model, a contract $(p(c), H^c, t(c))_{c\in[0,\bar{c}]}$ is incentive compatible if and only if:*

1. *There exists $\bar{t} \in \mathbb{R}$ such that*
$$t(c) = \bar{t} + (p(c) - c)(1 - H^c(p(c))) - \int_c^{\bar{c}} (1 - H^z(p(z)))\,\mathrm{d}z,$$
for all $c \in [0, \bar{c}]$.

2. *The function $c \mapsto (1 - H^c(p(c)))$ is nonincreasing.*

Using the characterization in Lemma 1, the revenue maximization problem under the full contracting model can be rewritten as (recall that $\psi(c) := c + G(c)/g(c)$ is the virtual cost):

$$\sup_{p,\{H^c\}} \int_0^{\bar{c}} (p(c) - \psi(c))(1 - H^c(p(c)))G(\mathrm{d}c) \tag{2}$$
$$\text{s.t. } c \mapsto (1 - H^c(p(c))) \text{ is nonincreasing.}$$

As noted above, the full contracting model is essentially a standard screening problem with quasi-linear preference. Therefore, under the regularity assumption, (2) can be solved by pointwise maximization. To construct a solution, consider any $c \in [0, \bar{c}]$. Notice that maximizing the integrand of (2) evaluated at c is equivalent to maximizing the seller's profit by choosing a price $p \geq 0$ and an information structure $H \in \mathcal{H}_F$ when her cost is $\psi(c)$. Furthermore, notice that the seller's profit must be bounded from above by the efficient surplus in an economy where the production cost is $\psi(c)$. That is,

$$(p - \psi(c))(1 - H(p)) \leq \int_{\psi(c)}^{\infty} (v - \psi(c))F(\mathrm{d}v) = \int_{\psi(c)}^{\infty} (1 - F(z))\,\mathrm{d}z = I_F(\psi(c)),$$

for all $p \geq 0$ and for all $H \in \mathcal{H}_F$, which can be also be seen geometrically from Figure 2.

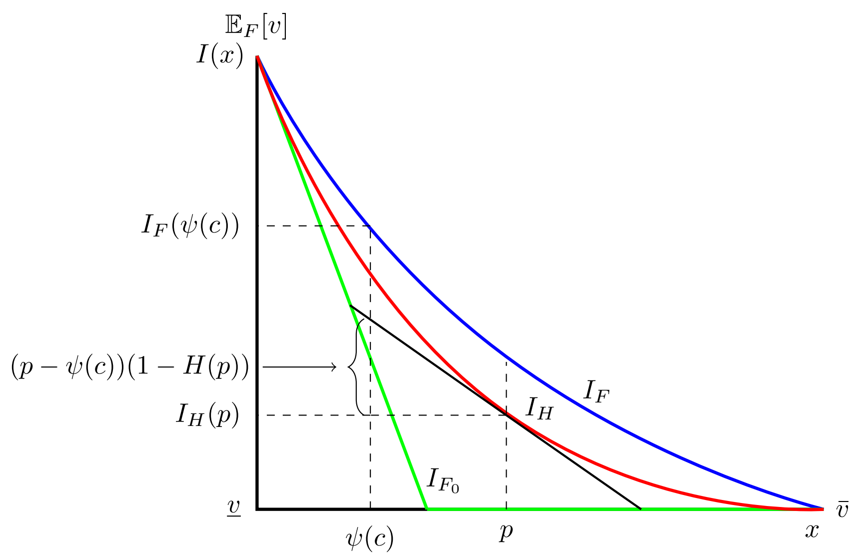

Figure 2: Pointwise Upper Bound

Now, consider the binary disclosure rule $H_{\mathrm{b}}^c \in \mathcal{H}_F$ that informs the buyer whether his value is above or below $\psi(c)$.[10] Together with a price equal to the buyer's interim expected value conditional on knowing that his value is above $\psi(c)$, $v(c) := \mathbb{E}_F[v|v > \psi(c)]$, the seller's profit is

$$(v(c) - \psi(c))(1 - F(\psi(c))) = (\mathbb{E}_F[v|v > \psi(c)] - \psi(c))(1 - F(\psi(c))) = I_F(\psi(c)),$$

as illustrated by Figure 3.

As a result, the seller's profit attains the upper bound $I_F(\psi(c))$ under price $p_{\mathrm{b}}(c) = \mathbb{E}_F[v|v \geq \psi(c)]$ and information structure H_{b}^c. Finally, the induced trading probability is $(1 - H_{\mathrm{b}}^c(p_{\mathrm{b}}(c))) = (1 - F(\psi(c)))$, which is nonincresing since ψ is increasing. Together

[10]Formally, H_{b}^c is defined as

$$H_{\mathrm{b}}^c(x) := \begin{cases} 0, & \text{if } x \in [0, \mathbb{E}_F[v|v \leq \psi(c)]] \\ F(\psi(c)), & \text{if } x \in (\mathbb{E}_F[v|v \leq \psi(c)], \mathbb{E}_F[v|v > \psi(c)]] \\ 1, & \text{if } x \in (\mathbb{E}_F[v|v > \psi(c)], \infty) \end{cases}.$$

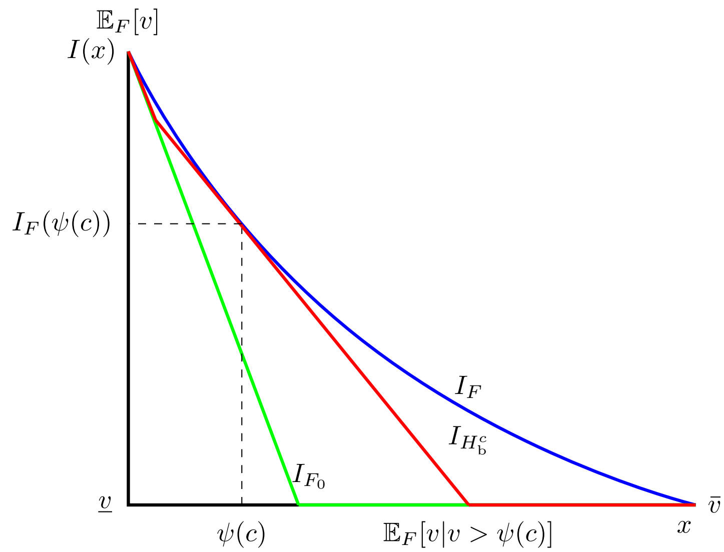

Figure 3: Profit under $p = \mathbb{E}_F[v|v > \psi(c)]$ and H_{b}^c

with Lemma 1, the revenue maximization problem under the full contracting model can be completely solved, which leads to Proposition 1 below.

Proposition 1. *Under both the direct selling model and the full contrating model, optimal revenue for the intermediary is:*

$$R^* := \int_0^{\bar{c}} \left(\int_{\psi(c)}^{\infty} (1 - F(z))\, \mathrm{d}z \right) G(\mathrm{d}c).$$

As demonstrated above, when the intermediary has control over selling mechanisms, a combination of tools in the mechanism design literature and the information design literature turns out to be extremely useful in solving this problem. Contractability of the price enables the intermediary to discipline the seller well so that the revenue maximization problem becomes separable. However, under the weak contracting model, the intermediary does not have control over selling mechanisms and hence the revenue maximization problem becomes more complicated. Nevertheless, the solution under the full contracting model would be a useful benchmark for this more complex problem, which will be examined in the next section.

4.2 Optimal Menu under the Weak Contracting model

The weak contracting model is substantially different from the full contracting model. Specifically, under the weak contracting model, the intermediary fully delegates the pricing decision to the seller and only provides the "informational service" by connecting the seller and the buyer on its platform and provide information to the buyer. The intermediary has no control over the sale and the seller retains full flexibility when selling the object. The inability to contract on price leads to a *mixture* problem between moral hazard and screening. As such, the solution constructed above may not be feasible due to the possibility of double deviations and thus the insufficiency of local incentive constraints.

To show that the full contracting model and the weak contracting model are equivalent in terms of their outcomes under condition (1), I first solve the revenue maximization problem under the weak contracting model with condition (1) (Proposition 2). The solution then implies that the intermediary's revenue is also R^*, which in turn implies that the buyer's surplus and the seller's profit are also the same as those under the full contracting model.

To begin with, first notice that despite the mixture nature, the envelope theorem is still valid and yields similar characterizations for incentive compatible menus. Specifically, by applying the envelope theorem twice, transfer in any incentive compatible menu is pinned down by the information structure and the publicizing policy up to a constant. The difference is that due to the possibility of double deviations, monotonicity is no longer sufficient for incentive compatibility and the entire integral constraint should be kept tracked. The formal statement is in Lemma 2.

Lemma 2. *Under the weak contracting model, a menu* $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$ *is incentive compatible if and only if there exists a selection*

$$p(c,c') \in \operatorname*{argmax}_{p\in[\underline{v},\bar{v}]} \alpha(c')(p-c)(1-H^{c'}(p)),$$

such that

1. *There exists* $\bar{t} \in \mathbb{R}$ *such that*

$$t(c) = \bar{t} + \alpha(c) \cdot \max_{p\in[\underline{v},\bar{v}]}(p-c)(1-H^c(p)) - \int_c^{\bar{c}} \alpha(z)(1-H^z(p(z,z)))\,\mathrm{d}z,$$

for all $c \in [0,\bar{c}]$.

2. $\int_c^{c'} [\alpha(z)(1-H^z(p(z,z))) - \alpha(c')(1-H^{c'}(p(z,c'))))]\,\mathrm{d}z \geq 0$ *for all* $c', c \in [0,\bar{c}]$.

Furthermore, the "only if" part holds for all selection for $\operatorname{argmax}_{p\in[\underline{v},\bar{v}]} \alpha(c')(p-c)(1-H^{c'}(p))$.

By Lemma 2 and individual rationality, after interchanging the order of integrals, it follows that optimal menus can be found by solving the following maximization problem:

$$\sup_{\{H^c\},\alpha} \int_0^{\bar{c}} \alpha(c)\left(\max_{p\in[\underline{v},\bar{v}]}(p-c)(1-H^c(p)) - (1-H^c(p(c,c)))\frac{G(c)}{g(c)}\right) G(\mathrm{d}c) \tag{3}$$

$$\text{s.t.} \int_c^{c'} [\alpha(z)(1-H^z(p(z,z))) - \alpha(c')(1-H^{c'}(p(z,c')))]\,\mathrm{d}z \geq 0,\ \forall c, c' \in [0,\bar{c}],$$

where $p(z,c')$ is the largest element in $\operatorname{argmax}_{p\in[\underline{v},\bar{v}]}(p-z)(1-H^{c'}(p))$ for all $z, c' \in [0,\bar{c}]$.

Below, I show that given (1), the value of (3) is exactly R^*, which is the same as that under the direct selling model and the full contracting model. To this end, I construct explicitly an optimal menu for the intermediary. This optimal menu features *upper censorship*, in the sense that for each reported cost c, the intermediary will provide an information structure so that whenever the buyer's value is below a certain report-dependent cutoff, he learns exactly his value, whereas when the buyer's value is above the cutoff, he learns nothing else than the fact that his value is above the cutoff. More specifically, an upper censorship information structure is defined below.

Definition 1. An information structure $H \in \mathcal{H}_F$ is an *upper censorship* with cutoff $k \in [\underline{v}, \bar{v}]$ if

$$H(x) = \begin{cases} F(x), & \text{if } x \in [0, k] \\ F(k), & \text{if } x \in (k, \mathbb{E}_F[v|v > k]] \\ 1, & \text{if } x \in (\mathbb{E}_F[v|v > k], \infty). \end{cases}, \forall x \in \mathbb{R}_+.$$

With the definitions above, the solution for the intermediary's problem under the weak contracting model can be stated as Proposition 2 below.

Proposition 2. *Under the weak contracting model and condtion* (1)*, for each* $c \in [0, \bar{c}]$*, let* H_{u}^c *be an upper censorship with cutoff* $\psi(c)$*, let* $\alpha_{\mathrm{u}}(c) = 1$ *and let*

$$t_{\mathrm{u}}(c) := (v(c) - c)(1 - F(\psi(c))) - \int_c^{\bar{c}} (1 - F(\psi(z)))\,\mathrm{d}z.$$

Then $(\alpha_{\mathrm{u}}(c), H_{\mathrm{u}}^c, t_{\mathrm{u}}(c))_{c \in [0,\bar{c}]}$ *is optimal for the intermediary. Moreover, the optimal revenue is* R^**.*

In other words, under the weak contracting model, the intermediary can achieve the same revenue as that under the full contracting model by revealing all the information to the buyer when it is below the cutoff $\psi(c)$, as illustrated by Figure 4 (where the dotted line depicts the optimal binary disclosure rule under the full contracting model). The formal proof of Proposition 2 is in the appendix. To see the intuition behind the proof, recall that the main distinction between the weak contracting model and the full contracting model is that the intermediary cannot contract on price under the weak contracting model, which creates additional constraints associated with the seller's pricing incentive and double deviation incentives. The presence of these extra constraints complicates the revenue maximization problem. Indeed, as price is not contractable under the weak contracting model, the menu consisting of binary disclosure information structures may not be feasible since it may not be optimal for a truthfully-reporting seller whose cost is c to set price at $v(c)$. Furthermore, there might be incentives for double deviations—a seller with cost c may have incentive to misreport a cost c' and then set a price distinct from $v(c')$ under the binary disclosure information structure.

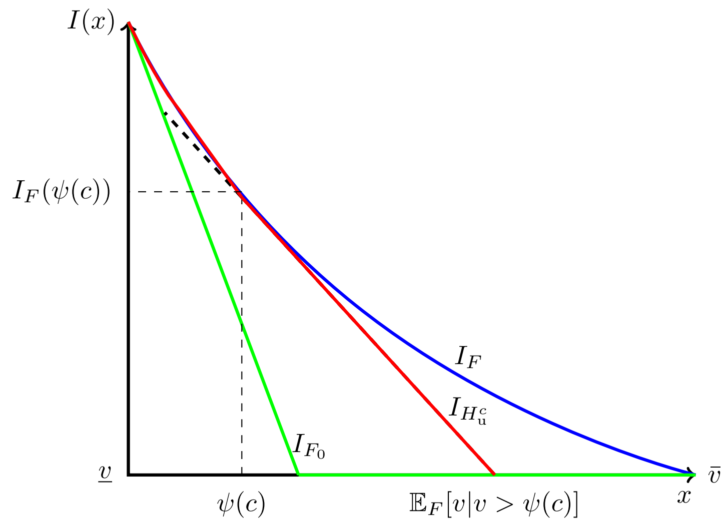

Figure 4: Upper Censorship H_u^c with Cutoff $\psi(c)$.

The property that the buyer learns exactly his value whenever it is below $\psi(c)$ under an upper censorship (with cutoff $\psi(c)$), however, deters the aforementioned deviations given that (1) holds. To see this, consider any $c \in [0, \bar{c}]$. Regularity of F and (1) imply that any price $p < \psi(c)$ yields a lower profit for the seller with cost c than the price $\psi(c)$, as $\psi(c)$ is below the optimal monopoly price under (1) and the profit function is singled-peaked by regularity. Moreover, charging price $v(c)$ would be better than charging price $\psi(c)$ under H_u^c, since the probability of trade would be the same but $v(c) > \psi(c)$. As a result, a truthfully reporting seller will optimally set a price at $v(c)$ under upper censorship H_u^c. Furthermore, by checking the integral constraint given in Lemma 2, under (1), such menu will indeed eliminate all the double deviation incentives. As a result, the menu $(\alpha_u(c), H_u^c, t_u(c))_{c\in[0,\bar{c}]}$ generates revenue R^* since

$$\max_{p\in[\underline{v},\bar{v}]}(p-c)(1-H_u^c(p)) - (1-H_u^c(p(c,c)))\frac{G(c)}{g(c)} = (p(c,c)-\psi(c))(1-H_u^c(p(c,c))) = I_F(\psi(c)),$$

for all $c \in [0, \bar{c}]$. As the revenue maximization problem under the full contracting model is a relaxation of the revenue maximization problem under the weak contracting model, this implies that $(\alpha_u(c), H_u^c, t_u(c))_{c\in[0,\bar{c}]}$ is optimal and the optimal revenue is R^*, which establishes Proposition 2.

On a higher level, the proof of Proposition 2 exploits the richness of information structures to accommodate additional incentives for the seller when the selling decision is fully delegated. Since the intermediary is allowed to use any information structure and the set of feasible information structures, $\mathcal{H}_F$, is rich, the intermediary can tailor the information structures in order to compensate the lack of ability of contracting on selling mechanisms. The optimal menu should provide correct incentives for the seller to set the desirable price and eliminate double deviation concerns. Under (1), the menu consisting of upper censorships is sufficient to achieve this goal and thus a modified pointwise maximization approach solves the problem (3). However, when this assumption does not hold, solving (3) would be more difficult and would entail more complex arguments than pointwise maximization. I now outline the solution for (3) without (1). In the appendix, I fully solve the intermediary's revenue maximization problem in this case, which would then lead to Theorem 1. Below, I summarize the optimal menu without (1).

As a benchmark, the optimal menu given by Proposition 2 has following features:

1. Given any report $c \in [0, \bar{c}]$, the buyer's value is *fully* disclosed when it is below $\psi(c)$ and is completely not disclosed otherwise.

2. The object is publicized for all reported cost.

3. For any seller with cost $c \in [0, \bar{c}]$, truth-telling and setting a price at $v(c)$ is optimal.

In contrast, the optimal menu without (1) constructed in the appendix differs from the optimal menu given by Proposition 2 in following ways:

1. Given any report $c \in [0, \bar{c}]$, the buyer's value is *partially* disclosed when it is below $\psi^*(c) := \{\psi(c), \psi(c^*)\}$ and is completely not disclosed otherwise.

2. The object is publicized for reported cost below a threshold $\hat{c} \geq c^*$ and is not publicized otherwise.

3. For any seller with cost $c \in [0, \bar{c}]$, truth-telling is optimal. For any seller with cost $c \in [0, \hat{c}]$, setting a price at $v(c)$ is optimal.

As a result, the intermediary's optimal revenue is still R^* if $c^* \geq \bar{c}$. Conversely, if $c^* < \bar{c}$, then the optimal revenue is strictly below R^*.

4.3 Outcome Equivalence and Interpretations

The outcome equivalence result (Theorem 1) follows from the characterizations given by Proposition 1 and Proposition 2. Equivalence of the intermediary's optimal revenue immediately follows from Proposition 1 and Proposition 2. To see that both the buyer's expected surplus and the seller's expected profit are the same across all models, a multiplicity issue has to be addressed. As Proposition 1 and Proposition 2 only provide one of the solutions to the revenue maximization problem of the intermediary, it is not immediate what buyer's surplus and seller's profit are under other solutions. Nevertheless, in the proof of Theorem 1 (which can be found in Appendix C), I use the fact that upper bound R^* is attained under all business models to show that buyer's surplus and seller's revenue must be the same in *all* solutions of each business models, which then implies that $c^* \geq \bar{c}$ is a necessary and sufficient condition for outcome equivalence.

Although Theorem 1 provides a necessary and sufficient condition for outcome equivalence among the business models, it is difficult to elicit economic interpretations direct from the condition. Below, the condition $c^* \geq \bar{c}$ is analyzed in further details and a more interpretable condition is derived. As Proposition 3 shows, the necessary and sufficient condition in Theorem 1 can be rewritten into another inequality that is more likely to be satisfied when the prior distribution of value is shifted to the right by translation. With this characterization, the main result can be understood as: Outcomes across business models are equivalent if and only if gains from trade are large enough.

Formally, fix a regular prior distribution of value F that has (full) support on $[0, \bar{v}]$, for each $v_0 \geq 0$, let $F_{v_0}(x) := F(x - v_0)$ for all $x \in \mathbb{R}_+$. $\{F_{v_0}\}_{v_0 \geq 0}$ is then a location family indexed by v_0. Proposition 3 summarizes the characterization.

Proposition 3. *For any $v_0 \geq 0$ and for any regular prior (F_{v_0}, G), there exists a unique $\tau(v_0, G)$ such that*

$$c^*(F_{v_0}, G) \geq \bar{c} \iff \psi(\bar{c}) \leq \tau(v_0, G).$$

Moreover, the function τ is increasing in v_0 for any given G.

Within a location family, it is then natural to define gains from trade by saying that the environment (F_{v_0}, G) *exhibits larger gains from trade* than the environment $(F_{v_0'}, G)$ if and only if $v_0 > v_0'$. With this interpretation, Theorem 1 can be restated as below.

Corollary 2. *The direct selling model, the full contracting model and the weak contracting model yield the same revenue for the intermediary, the same expected surplus for the buyer and the same expected profit for the seller if and only if gains from trade are large enough.*

5 Irrelevance of Targeting

Previous analyses rely on a premise that the intermediary has to provide the same information structure and make the same publicizing decision regardless of the valuation of the buyer. However, it is reasonable to argue that in practice, the intermediary can do more. For instance, in the Amazon example, the intermediary may have the technology to estimate the buyer's valuation—by collecting consumers' browsing history, personal information and purchasing behavior. Knowing the buyer's valuation, the intermediary can then design different publicizing and disclosure policy to different consumers in order to create more surplus. This section considers such extension in which a *targeting* technology is available for the intermediary.

For the ease of exposition, consider an alternative interpretation of the environment under which there is a unit mass of buyers that have differentiated values. The distribution of buyers is given by a CDF F that admits a density f and has full support on $[\underline{v}, \bar{v}]$.[11] Under this interpretation, availability of targeting means that the intermediary, when disclosing information, can select any (measurable) subset of buyers $A \subseteq [\underline{v}, \bar{v}]$, publicize the product to these consumers and provide arbitrary information structure to these buyers. The buyers who are not selected, $[\underline{v}, \bar{v}] \backslash A$, remain unaware of the object. As such, a *targeting policy* can be described by a measurable subset $A \subseteq [\underline{v}, \bar{v}]$. Let μ_F be the probability measure associated with the CDF F. If the intermediary targets a group A, the prior distribution of valuations effectively becomes:

$$\mu^A = \mu_F | A + \delta_{\{0\}} \cdot \mu_F([\underline{v}, \bar{v}] \backslash A).$$

That is, the consumers who are not in the target set A are effectively the consumers with zero values. For any targeting set A, let F^A be the CDF associated with μ^A. By the same arguments as it were used when targeting is not available, together with an additional requirement that whoever is not targeted cannot receive any information, the collection of information structures can then be characterized by the set

$$\mathcal{H}_A := \{H \in \mathcal{H}_{F^A} | H(0) = F^A(0)\}.$$

As such, under the direct selling model, a trading plan is given by $(q(c), t(c), A^c, p(c), H^c)_{c \in [0, \bar{c}]}$ with $q(c) = (1 - H^{A^c}(p(c)))$; Under the full contracting model, a contract is given by $(p(c), A^c, H^c, t(c))_{c \in [0, \bar{c}]}$; Under the weak contracting model, a menu is given by $(A^c, H^c, t(c))_{c \in [0, \bar{c}]}$, where $A^c \in \mathcal{B}([\underline{v}, \bar{v}])$ is the set of targeted consumer given the report c.

As in the previous section, the direct selling model and the full contracting model are essentially equivalent in the sense that any direct mechanism under the direct selling model can be represented by a contract under the full contracting model, and vice versa. Moreover, the envelope characterizations are still valid and Lemma 1 and Lemma 2 can be revised by requiring that $H^c \in \mathcal{H}_{A^c}$ for all $c \in [0, \bar{c}]$. Therefore, the revenue maximization problem under the full contracting model can still be written as:

$$\sup_{\{A^c\} \subseteq \mathcal{B}([\underline{v}, \bar{v}]), \{H^c\} \subseteq \prod_{c \in [0, \bar{c}]} \mathcal{H}_{A^c}} \int_0^{\bar{c}} (p(c) - \psi(c))(1 - H^c(p(c)))G(\mathrm{d}c) \tag{4}$$

$$\text{s.t. } c \mapsto (1 - H^c(p(c))) \text{ is nonincreasing,}$$

[11]Absolute continuity and regularity are in fact not necessary here. It is included for the main result in which a comparison between this and the previous environment where targeting is not available is made.

and the revenue maximization problem under the weak contracting model can be written as:

$$\sup_{\{A^c\}\subseteq\mathcal{B}([0,\bar{v}]),\,\{H^c\}\subseteq\prod_{c\in[0,\bar{c}]}\mathcal{H}_{A^c}} \int_0^{\bar{c}} \left(\max_{p\in[\underline{v},\bar{v}]}(p-c)(1-H^c(p)) - (1-H^c(p(c,c)))\frac{G(c)}{g(c)}\right) G(\mathrm{d}c)$$

$$\text{s.t.} \int_c^{c'} [(1-H^z(p(z,z))) - (1-H^{c'}(p(z,c'))]\,\mathrm{d}z \geq 0,\ \forall c,c'\in[0,\bar{c}]. \tag{5}$$

With the additional leverage of targeting, it is clear that the intermediary can (weakly) improve its revenue under any model.[12] Below, I show that even with targeting, the intermediary cannot do better than generating revenue R^*. As such, the outcome equivalent result can be strengthened so that regardless of the business models the intermediary uses *and* regardless of the availability of targeting technology, the (ex-ante) outcomes are equivalent if and only if gains from trade are large enough.

To see this, first notice that for any $A\in\mathcal{B}([\underline{v},\bar{v}])$, $F^A\geq F$ and therefore

$$\int_x^{\infty}(1-F^A(z))\,\mathrm{d}z \leq \int_x^{\infty}(1-F(z))\,\mathrm{d}z,$$

for any $x\in\mathbb{R}_+$ and thus any prior after targeting must induce that convex function I_{F^A}, which is always below I_F. By using the same arguments as in the proof of Proposition 1 and Proposition 2, R^* is still an upper bound of the objective of (4) and (5). It then suffices to construct a contract $(p_T(c),A_T^c,H_T^c,t_T(c))_{c\in[0,\bar{c}]}$ under the full contracting model and a menu $(A_T^c,H_T^c,t_T(c))_{c\in[0,\bar{c}]}$ under the weak contracting model that are incentive compatible and individually rational and attain R^*.

To the end, for any $c\in[0,\bar{c}]$, let $A_T^c:=[\psi(c),\bar{v}]$ and let

$$H_T^c(x):=\begin{cases} F(\psi(c)), & \text{if } x\in[0,v(c)) \\ 1, & \text{if } x\in[v(c),\bar{v}] \end{cases}.$$

That is, to target the buyers with value above the effective virtual cost $\psi(c)$ and disclose no more information to these targeted buyers (i.e. screen only on the *extensive margin*). Then $H_T^c\in\mathcal{H}_{A_T^c}$ for all $c\in[0,\bar{c}]$. Furthermore, let $p(c):=v(c)$ for all $c\in[0,\bar{c}]$. Finally, use the incentive constraints to back out the transfer t_T. That is, let $t_T(c):=(v(c)-c)(1-F(\psi(c)))-\int_c^{\bar{c}}(1-F(\psi(z)))\,\mathrm{d}z$. Then, as illustrated in Figure 5 and showed formally in the proof of Proposition 4, in which the original prior is represented by the blue curve, the prior after targeting A_T^c is represented by the red curve, and the distribution of interim expected value,

[12] To see this, under the full contracting model for each $c\in[0,\bar{c}]$, take $A^c=[\underline{v},\bar{v}]$. Then any contract that is available when there is no targeting is feasible. On the other hand, under the weak contracting model, for each $c\in[0,\bar{c}]$, taking $A^c=[\underline{v},\bar{v}]$ is equivalent to the class of menus with $\alpha(c)=1$ and taking $A^c=\emptyset$ is equivalent to the class of menus with $\alpha(c)=0$.

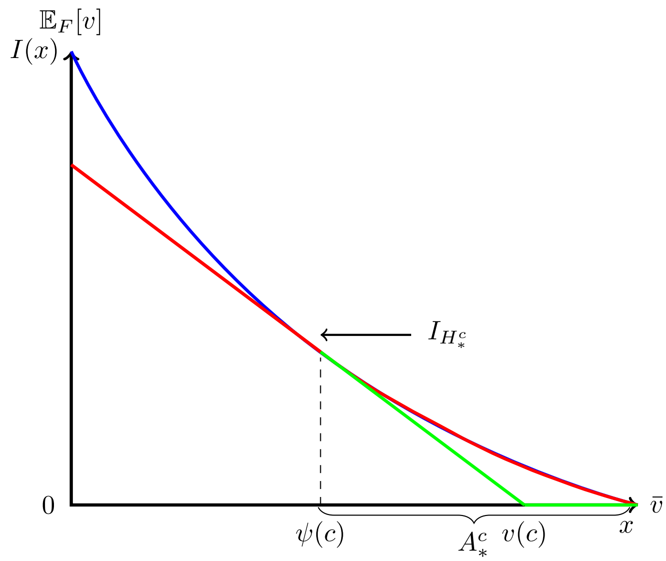

Figure 5: Optimal Menu with Targeting

H^c_T is represented by the green curve, under both the full contracting model and the weak contracting model, the contract $(p_T(c), A^c_T, H^c_T, t_T(c))_{c\in[0,\bar{c}]}$ and the menu $(A^c_T, H^c_T, t_T(c))_{c\in[0,\bar{c}]}$ are incentive compatible and individually rational and attain R^*. The key intuition is that by targeting only the buyers with value above $\psi(c)$ and give them no further information, *any* seller with cost below $v(c)$ will be willing to set a price at $v(c)$. This effectively eliminates the double deviation concerns and yield the ability to contract on price irrelevant. The result is formally recorded in Proposition 4.

Proposition 4. *Suppose that targeting is available. Then the direct selling model, the full contracting model and the weak contracting model are outcome equivalent.*

Combining Proposition 4 and Theorem 1, the main result is extended to the following:

Theorem 2 (Outcome Equivalence with Targeting)**.** *For any regular prior (F, G), there exists $c^*(F, G) \geq 0$ such that the direct selling model, the full contracting model and the weak contracting model, with targeting or not, are outcome equivalent if and only if $c^*(F, G) \geq \bar{c}$.*

6 Other Business Models and Comparative Statics

So far, the focus has been on the three major business models—the direct selling model, the full contracting model and the weak contracting model. In fact, the equivalence results can be even more general. Due to the nature of the models considered above, *any* reasonable business models can be thought of as an *intermediate* among these extreme models. Intuitively, given the premise that the intermediary has the technology to disclose information to the buyer and publicize the object, any kinds of reasonable business model should allow the intermediary to have control over the information structure and the publicizing policy. Determination of other aspects of the market, including the selling mechanism and transfer schedule, on the other hand, could vary across different business models. Conceptually, the full contracting model is an "upper bound" for all these models as it allows the intermediary to have control over every aspects, while the weak contracting model is a "lower bound" as it allows the intermediary to have control over nothing else than information structure and publicizing decisions. Below, I formalized the intuition above by introducing an abstract definition of business models and then use the results above to conclude that the market outcomes are the same across *all* business models.

To begin with, notice that the space for relevant outcomes is $\{0,1\} \times \mathcal{H}_F \times [\underline{v}, \bar{v}] \times \mathbb{R}$. A typical element is written as (α, H, p, r), where $\alpha \in \{0,1\}$ denotes whether the product is publicized, $H \in \mathcal{H}_F$ is the information structure provided to the buyer, $p \in [\underline{v}, \bar{v}]$ is the price of the product and $r \in \mathbb{R}$ is the intermediary's revenue extracted from the seller. A *business model* is a collection of mappings that take form of $(\alpha(c), H^c, p(\cdot|c), r(\cdot|c))_{c\in[0,\bar{c}]}$ such that for any $c, c', c'' \in [0, \bar{c}]$:

$$
\begin{aligned}
&(p(c|c) - c)(1 - H^c(p(c|c))) - r(c|c) \geq (p(c''|c') - c)(1 - H^{c'}(p(c''|c'))) - r(c''|c') \\
&(p(c|c) - c)(1 - H^c(p(c|c))) - r(c|c) \geq 0
\end{aligned}
$$

and for all $c' \in [0, \bar{c}]$

$$
p(\cdot|c') \in \mathcal{P}(\alpha(c'), H^{c'}), \quad r(\cdot|c') \in \mathcal{R}(\alpha(c'), H^{c'}),
$$

where for any $\alpha \in \{0,1\}$ and $H \in \mathcal{H}_F$,

$$
\Pi(\alpha, H) \subseteq \mathcal{P}(\alpha, H) \subseteq M(\mathcal{B}([0, \bar{c}])),
$$

with $\Pi(\alpha, H)$ being the collection of all selections of the correspondence $c \mapsto \text{argmax}_{x\in V}\, \alpha(x - c)(1 - H(x))$, is the set of possible prices given by the business model while

$$
M(\{[0, \bar{c}], \{\emptyset\}\}) \subseteq \mathcal{R}(\alpha, H) \subseteq M(\mathcal{B}([0, \bar{c}])).
$$

is the set of feasible transfer schedules.[13]

In other words, for each initial report $c \in [0, \bar{c}]$, $\alpha(c) \in \{0,1\}$ is a publicizing decision, $H^c \in \mathcal{H}_F$ is an information structure provided to the buyer, $p(\cdot|c) : [0, \bar{c}] \to [\underline{v}, \bar{v}]$ and

[13]At this stage, it is not obvious that it is without loss to restrict attention on deterministic mechanisms. Nevertheless, in the online appendix, I show that the equivalence result remains unchanged even if randomization is allowed. Since there is only one buyer, by allowing randomizations, this framework is generalized to incorporate all selling mechanisms instead of only posted prices.

$r(\cdot|c):[0,\bar{c}]\to\mathbb{R}$ together is a mechanism that determines the price for the object and the intermediary's revenue given the initial report. Notice that $\alpha(c)$ and H^c are constants while $p(\cdot|c), r(\cdot|c)$ are functions defined on $[0,\bar{c}]$. This reflects the assumption that the intermediary has control over the information structure and publicizing decision and therefore is able to contract on these two aspects. On the other hand, the price and the revenue may not be contractable and are determined by some re-negotiation process after the disclosure policy is chosen and the publicizing decision is made. By the revelation principle, any such re-negotiation processes can be represented by a direct mechanism $p(\cdot|c)$ and $r(\cdot|c)$ that maps a (second) report into outcomes given the initial report c. Different business models allow the intermediary to select different re-negotiation processes, which is modeled by the correspondences $\mathcal{P}:\{0,1\}\times\mathcal{H}_F\rightrightarrows M(\mathcal{B}([0,\bar{c}]))$, and $\mathcal{R}:\{0,1\}\times\mathcal{H}_F\rightrightarrows M(\mathcal{B}([0,\bar{c}]))$.[14] As the intermediary can always delegate all the selling decisions to the seller after providing information to the buyer, it can always guarantee a re-negotiation process that yields optimal prices for the seller and a payment from the seller that is independent of this process, which is the reason for the requirements $\Pi(\alpha,H)\subseteq\mathcal{P}(\alpha,H)$ and $M(\{V,\{\emptyset\}\})\subseteq\mathcal{R}(\alpha,H)$ for each $\alpha\in\{0,1\}$ and $H\in\mathcal{H}_F$.[15] As such, within a particular business model, the intermediary solves:

$$
\begin{aligned}
\sup_{\alpha,\{H^c\},p,r}\quad & \mathbb{E}_G[r(p(c)|c)] \\
\text{s.t.}\quad & (p(c|c)-c)(1-H^c(p(c|c)))-r(p(c|c)|c) \\
& \geq (p(c''|c')-c)(1-H^{c'}(p(c''|c')))-r(p(c''|c')|c'),\ \forall c,c',c''\in[0,\bar{c}] \\
& (p(c|c)-c)(1-H^c(p(c|c)))-r(p(c|c)|c)\geq 0,\ \forall c\in[0,\bar{c}] \\
& p(\cdot|c')\in\mathcal{P}(\alpha(c'),H^{c'}),\ r(\cdot|c')\in\mathcal{R}(\alpha(c'),H^{c'}),\ \forall c'\in[0,\bar{c}].
\end{aligned}
\tag{6}
$$

Notice that from the revenue perspective, a business model can be identified by the correspondences, $\mathcal{P}$ and $\mathcal{R}$. Indeed, as the intermediary has the technology to provide information and present the object to the buyer, a business model is essentially a set of ways

[14]The fact that $\mathcal{P}$ and $\mathcal{R}$ can depend on H and α is reflecting the possibility that such negotiating process can take place *after* the seller makes reports and the information structure and publicizing decision are realized. For instance, elements $p\in\mathcal{P}(\alpha,H)$ and $r\in\mathcal{R}(\alpha,H)$ can be thought of as an equilibrium outcome of some feasible bargaining protocol between the intermediary and the seller that is chosen by the intermediary and takes place after the publicizing decision α and information structure H is chosen, where $p(c)$ is the price that arises and $r(c)$ is the transfer paid from the seller to the intermediary in this equilibrium if the seller has cost c.

[15]For instance, if $\mathcal{R}(\alpha,H)$ the collection of all constant functions, then it means that the intermediary's revenue does not depend on the selected re-negotiation process but only the initial reports, whereas if $\mathcal{R}(\alpha,H)$ is all Borel measurable functions on $[0,\bar{c}]$, then it means that the set of feasible re-negotiation processes is rich enough so that the intermediary's revenue in equilibrium can depend on the process arbitrarily as long as it is incentive compatible and individually rational.

to exploit such technology to determine how the object should be sold and how the surplus should be divided between the intermediary and the seller. It is then clear that this definition incorporates all three business models considered above. For example, the full contracting model is incorporated by the business models in which $M(\{[0,\bar{c}],\{\emptyset\}\}) \subseteq \mathcal{P}(\alpha, H)$ so that the prices can be determined based only on the initial report. Similarly, the direct selling model is incorporated by any business model in which $M(\{[0,\bar{c}],\{\emptyset\}\}) \subseteq \mathcal{P}(\alpha, H)$: For any trading plan $(\hat{q}(c), \hat{t}(c), \hat{H}^c, \hat{p}(c))_{c\in[0,\bar{c}]}$. Let $p(\cdot|c)$ be a constant function with value $\hat{p}(c)$ and $r(\cdot|c)$ be a constant function with value $\hat{q}(c)\hat{p}(c) - \hat{t}(c)$. Then $(\hat{q}(c), \hat{t}(c), \hat{H}^c, \hat{p}(c))_{c\in[0,\bar{c}]}$ indeed corresponds to a mechanism under such business model. Also, the weak contracting model is the one in which $\mathcal{P}(\alpha, H) = \Pi(\alpha, H)$, $\mathcal{R}(\alpha, H) = M(\{[0,\bar{c}],\{\emptyset\}\})$ for all $\alpha \in \{0,1\}$ and $H \in \mathcal{H}_F$ so that prices are chosen by the seller after seeing α and H and this choice does not affect her payment to the intermediary.

Let $R(\mathcal{P}, \mathcal{R})$ be the value of the revenue maximization problem (6) under the business models identified by $(\mathcal{P}, \mathcal{R})$. Let $\overline{\mathcal{P}}$, $\overline{\mathcal{R}}$, $\underline{\mathcal{P}}$, $\underline{\mathcal{R}}$ be defined as: $\overline{\mathcal{P}}(\alpha, H) = \overline{\mathcal{R}}(\alpha, H) = M(\mathcal{B}([0,\bar{c}]))$, $\underline{\mathcal{P}}(\alpha, H) = \Pi(\alpha, H)$ and $\underline{\mathcal{R}}(\alpha, H) = M(\{[0,\bar{c}],\{\emptyset\}\})$, for all $\alpha \in \{0,1\}$ and $H \in \mathcal{H}_F$. The following observation then follows from the nature of (6).

Lemma 3. *For any business model* $(\mathcal{P}, \mathcal{R})$

$$R(\underline{\mathcal{P}}, \underline{\mathcal{R}}) \leq R(\mathcal{P}, \mathcal{R}) \leq R(\overline{\mathcal{P}}, \overline{\mathcal{R}}).$$

Using similar arguments as in the previous sections, it is not difficult to see that R^* is still an upper bound for the intermediary's revenue. As such, the binary disclosure solution given by Proposition 1 is a solution under any business model identified by $(\overline{\mathcal{P}}, \overline{\mathcal{R}})$ and thus $R(\overline{\mathcal{P}}, \overline{\mathcal{R}}) = R^*$. On the other hand, as the weak contracting model is a business model identified by $(\underline{\mathcal{P}}, \underline{\mathcal{R}})$, Proposition 7 implies that $R(\underline{\mathcal{P}}, \underline{\mathcal{R}}) = R^*$ if and only if $c^* \geq \bar{c}$. Together with some additional arguments, which can be found in the proof of Theorem 3, this gives the following generalized outcome equivalence result.

Theorem 3 (Generalized Outcome Equivalence)**.** *For any regular* (F, G)*, there exists* $c^*(F, G) \geq 0$ *such that all business models, with targeting or not, are outcome equivalent if and only if* $c^*(F, G) \geq \bar{c}$*.*

I conclude this section by providing some welfare analysis and comparative statics on the outcomes when the outcome equivalence result holds. To begin with, notice that for any pair of distributions (F, G) such that $c^* \geq \bar{c}$, total surplus generated by the sale is

$$(v(c) - c)(1 - F(\psi(c))) = \int_{\psi(c)}^{\infty} (1 - F(x))\,\mathrm{d}x + (\psi(c) - c)(1 - F(\psi(c)))$$

and the probability of trade is $(1 - F(\psi(c)))$ for all $c \in [0, \bar{c}]$. Comparing these with a benchmark case in which the seller has the technology to provide information to the buyer, where one of optimal information structures, as noted above, is a binary disclosure rule that gives total surplus generated by trade

$$(\mathbb{E}_F[v|v > c] - c)(1 - F(c)) = \int_c^\infty (1 - F(x))\,\mathrm{d}x$$

and a probability of trade $1 - F(c)$, this yields the following observation:

Proposition 5 (Welfare Comparison)**.** *Suppose that $c^* \geq \bar{c}$. Then the expected total surplus generated by trade and the probability of efficient trade are larger when the seller has control of the information technology than when the intermediary has control of the information technology under any business model.*

In brief, Proposition 5 shows that, regardless of the business model the intermediary uses, when the seller does not have the technology to provide information to the buyer directly but has to do so by interacting with an intermediary who has this technology, as the seller has private information about production cost, the ownership of such information technology matters. Indeed, when the seller has to buy such information technology from the intermediary, due to the presence of incomplete information, the seller would demand information rent from the intermediary and total surplus will be reduced since the intermediary has to provide information structures so that the seller would be willing to internalize her information rent when making pricing decisions.

Finally, I examine how the shifts of the distribution of valuation and the distribution of production cost affects the intermediary's revenue and the total surplus generated by trade. Specifically, take any two pairs of regular distributions (F_1, G_1) and (F_2, G_2) with $\mathrm{supp}(F_1) = \mathrm{supp}(F_2) = [\underline{v}, \bar{v}]$ and $\mathrm{supp}(G_1) = \mathrm{supp}(G_2) = [0, \bar{c}]$ such that $c^*(F_i, G_i) \geq \bar{c}$, for all $i \in \{1, 2\}$, the previous results then gives the following comparative statics analysis:

Proposition 6 (Comparative Statics)**.**

1. *Suppose that F_1 first order stochastic dominates F_2. That is, $F_1 \leq F_2$. Then under any business model, the total surplus, the intermediary's revenue and seller's expected net profit under (F_1, G_i) are larger than those under (F_2, G_i), $i \in \{1, 2\}$.*

2. *Suppose that F_1 is a mean preserving spread of F_2. Then under any business model, the intermediary revenue under (F_1, G_i) is larger than that under (F_2, G_i), $i \in \{1, 2\}$.*

3. *Suppose that G_2 dominates G_1 in the hazard rate order. That is $g_1/G_1 \geq g_2/G_2$. Then under any of the models business model, the total surplus, the intermediary's revenue and the seller's expected revenue are larger under (F_i, G_1) than those under (F_i, G_2), $i \in \{1, 2\}$.*

To summarize, regardless of the business model the intermediary uses, when the buyer's value becomes higher (in the sense of first order stochastic dominance), it becomes easier for the intermediary to generate trade surplus by providing proper information to the buyer and therefore total surplus, the intermediary's revenue and the seller's net profit all increase. When the distribution of valuation becomes more spread-out (in the sense of mean preserving spread), the informational tools for the intermediary becomes more flexible and therefore revenue increases. Finally, when the seller's cost shifts in hazard rate order, causing a reduction of information rent and the costs (in the sense of first order stochastic dominance), the seller retains less information rent and the distortion on information structure for her to internalize pricing decision reduces. These two factors jointly increase total surplus and intermediary's revenue as well. Furthermore, although the reduction of information rent and reduction of production has opposite effects on the seller's net profit, Proposition 6 shows that the gain in total surplus offsets the loss of information rent of the seller and hence also increases seller's expected net profit.

7 Discussions

The outcome equivalence among business models is surprising. As remarked above, this is essentially because of the richness of information structures. Even though the intermediary under some business models (e.g. the weak contracting model) has a far less active role on influencing the selling mechanism of the product comparing to others (e.g. the full contracting model and the direct selling model), by exploiting the details of the information provided, it can in fact incentivize the producers to behave as if it has control over the selling mechanism. As such, one of the insights of the equivalence result is that no matter what role the intermediary plays in determining the actual selling mechanism—be it by selling itself, by contracting on the selling process, by fully delegating the sale to the producer or by any other re-negotiation process—and regardless of whether the technology of targeting is possible, as long as it is the intermediary that provides the information about the product to the consumers and the gains from trade are large enough, these extra aspects are irrelevant.

As a result, even though there are many ways that the informational intermediaries can operate with and it has become more and more likely that these intermediaries have the ability to precisely estimate the consumers' value and design policies accordingly, in environments that exhibit large enough gains from trade, the market outcomes under all these possible business models and targeting technologies are in fact the same as if the intermediary plays a very passive role and only serves as a platform that provides a space for transactions and provides information about the products to the consumers.

Another insight of the equivalence result is that, as demonstrated in Proposition 5 and

Proposition 6, since the market outcomes are equivalent among the business models and do not depend on whether targeting is available, the comparative statics and welfare analyses can be done in a *robust* way that does not require further specifications of the actual business models and targeting technologies in this environment. As long as the gains from trade are large enough, analyzing the solutions of either one of the specifications is sufficient to provide predictions and implications for all other possible specifications.

Finally, from the intermediary's perspective, the equivalent result also sheds light on choosing the optimal business model to operate. As the equivalence result suggests, as long as gains from trade are large enough, there are no differences between these business models. This means that when deciding which business model to use, the intermediary only needs to consider the residual perspectives that are orthogonal to revenues, such as costs for each business models (e.g. the direct selling model might require more inventory costs than the weak contracting model) and how these models perform in terms of attracting participation. This observation is insightful in explaining the empirical fact that the share of third party sellers on Amazon has raised 26% in the past decades. After all, gains from trade of the products that are sold on Amazon have arguably increased due to the development of digital products and the weak contracting model can attract more retailers and has lower inventory costs, all these factors indicate a rise in the use of the weak contracting model.

Two final remarks are also noteworthy. First, the fact that the buyer always has zero surplus under any solution of any business models is a result of the assumption that buyer does not have any information about the value a priori and thus does not retain any rent when facing monopolies. It is natural to extend the model to generate positive buyer surplus by introducing some prior information to the buyer that is exclusive for himself. In the online appendix, I show that the main results of the paper still hold qualitatively—the expected outcomes are equivalent across three business models, regardless of the availability of targeting if and only if the gains from trade are large enough. The only difference is the characterizations of revenue maximizing solutions become less explicit. Second, although the paper focuses on the environment of monopolistic pricing in the spirit of Maskin and Riley (1984) and Myerson (1981), the same modeling techniques can also be applied in a setting where there is a regulator who wishes to regulate a monopolist's information disclosure of its product in spirit of Baron and Myerson (1982). The regulator can either buy the product altogether and sell by itself while disclosing information at the same time (the direct selling model), regulating information provision and price at the same time (the full contracting model), or regulating only the information provision (the weak contracting model). The payoffs between the monopoly and the consumers are transferable and thus taxes and subsidies are available. The regulator's goal is to maximize a weighed average surplus of the economy. Using similar arguments, with a modified definition for virtual cost function, the equivalence result suggests that all these kinds of regulations yield the same outcome if and only if gains from trade are large enough and gives optimal policies under each environment. This observation is also summarized in the online appendix.

8 Conclusion

In this paper, I explored how different business models an informational intermediary can use affect the market outcomes and how would the intermediary evaluate these different business models. I characterized the revenue maximizing solutions under the direct selling model, the full contracting model and the weak contracting model and use these solutions to show that the market outcomes are the same across all possible business models, regardless of the possibility of targeting, if and only if the gains from trade are large enough. Welfare analysis suggests that total surplus and probability of trade is smaller in this environment comparing to a benchmark in which the seller has the technology to disclose information, independent of the business models the intermediary uses, as long as gains from trade are large enough. Comparative statics show that, regardless of the business models the intermediary uses, total surplus and the seller's profit are higher when the buyer's value is higher and when the information rent of the seller is lower, as long as the gains from trade are large enough.

There are several aspects of the model that can be extended. First, the assumption that the seller is a monopoly can be reconsidered and competition can be introduced. Two natural extensions arise from this. One is to introduce different market structures to the seller. I suspect that as long as each seller has some sort of monopoly power(e.g. a monopolistic competition or a Cournot oligopoly model), the results will be similar, since the key aspect of the seller's behavior in the model above is the mark-up decisions rather than how much profit she can retain. This also allows an introduction of richer screening device to the intermediary. When there are multiple sellers and the buyer receives information through the intermediary, it is arguable that the *order* in which the information is presented matters. As a result, the intermediary can include the order of the products through which the buyer will see in its mechanisms. This is a topic for future research. Second, it is also plausible to consider an environment in which there are more than one intermediaries, these intermediaries compete in providing menus/contracts and information outlet to attract buyers and sellers. This is also a direction for future research.

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Appendix

A Envelope Characterizations

A1 Proof of Lemma 1

Proof of Lemma 1. For necessity, Given an incentive compatible contract $(p(c), H^c, t(c))_{c\in[0,\bar{c}]}$, let $V(c,c') := (p(c')-c)(1-H^{c'}(p(c'))) - t(c')$ for all $c, c' \in [0,\bar{c}]$ be the seller's net profit when her true cost is c and the report is c'. Incentive compatibility implies that

$$V^*(c) := \max_{c'\in[0,\bar{c}]} V(c,c') = V(c,c),\ \forall c \in [0,\bar{c}].$$

Notice that for any $c' \in [0,\bar{c}]$, the function $V(\cdot,c')$ is affine and thus absolutely continuous, and the family $\{V(\cdot,c')\}_{c'\in[0,\bar{c}]}$ is uniformly bounded. As such, by the envelope theorem (Milgrom and Segal, 2002), V^* is absolutely continuous and

$$V^*(c) = V^*(\bar{c}) + \int_c^{\bar{c}} (1-H^z(p(z)))\,\mathrm{d}z.$$

Furthermore, since V^* is a pointwise supremum of a family of affine functions, V^* is convex and therefore its subgradiant, $-(1-H^c(p(c)))$, is nondecreasing. This establishes assertion 2. On the other hand, since

$$V^*(c) = (p(c)-c)(1-H^c(p(c))) - t(c),$$

we have

$$t(c) = -V^*(\bar{c}) + (p(c)-c)(1-H^c(p(c))) - \int_c^{\bar{c}} (1-H^z(p(z)))\,\mathrm{d}z.$$

This then establishes assertion 1 after defining $\bar{t} := -V^*(\bar{c})$.

For sufficiency, suppose that a contract satisfies assertion 1 and 2. Again, define $(p(c')-c)(1-H^{c'}(p(c'))) - t(c')$, and let $\Pi(c,c') := (p(c')-c)(1-H^{c'}(p(c')))$. Notice that $\Pi(\cdot,c')$ is differentiable for all $c' \in [0,\bar{c}]$ and its derivative is $\Pi_1(c,c') = -(1-H^{c'}(p(c')))$ Then for any $c, c' \in [0,\bar{c}]$,

$$\begin{aligned}
&V(c,c) - V(c,c') \\
=&\int_c^{c'} (1-H^z(p(z)))\,\mathrm{d}z - (\Pi(c,c') - \Pi(c',c')) - \int_{c'}^{\bar{c}} (1-H^z(p(z)))\,\mathrm{d}z \\
=&\int_c^{c'} (1-H^z(p(z)))\,\mathrm{d}z - (\Pi(c,c') - \Pi(c',c')) \\
=&\int_c^{c'} (1-H^z(p(z)))\,\mathrm{d}z - \int_c^{c'} \Pi_1(z,c')\,\mathrm{d}z \\
=&\int_c^{c'} [(1-H^z(p(z))) - (1-H^{c'}(p(c')))] \\
\geq& 0,
\end{aligned}$$

where the first equality follows from assertion 1, the third equality follows from the fundamental theorem of calculus and the last inequality follows from assertion 2. This completes the proof. ∎

A2 Proof of Lemma 2

Proof of Lemma 2. For necessity, consider any incentive compatible and individually rational menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$. Let

$$\Pi(c,c') := \max_{p\in[\underline{v},\bar{v}]} \alpha(c')(p-c)(1-H^{c'}(p))$$

be the seller's expected profit under the information structure $H^{c'} \in \mathcal{H}_F$, publicizing policy $\alpha(c') \in \{0,1\}$ and cost c. By the envelope theorem (Milgrom and Segal, 2002), since the function

$$c \mapsto \alpha(c')(p-c)(1-H^{c'}(p))$$

is absolutely continuous with (almost everywhere) derivative uniformly bounded by $-\bar{c}$ and $\bar{v}$ for any fixed $p \in [\underline{v},\bar{v}]$, $c' \in [0,\bar{c}]$, $\Pi(\cdot,c')$ is absolutely continuous for all $c' \in [0,\bar{c}]$ and its derivative exists and equals to

$$\Pi_1(c,c') = -\alpha(c')(1-H^{c'}(p(c,c'))) \tag{7}$$

for any selection $p(c,c') \in \operatorname{argmax}_{p\in[\underline{v},\bar{v}]} \alpha(c')(p-c)(1-H^{c'}(p))$, for (Lebesgue) almost all $c \in [0,\bar{c}]$.

Now let

$$V(c,c') := \Pi(c,c') - t(c')$$

be the seller's profit net of transfer if the cost is c and the (mis)report c'. Incentive compatibility then implies

$$V^*(c) := V(c,c) = \max_{c'\in[0,\bar{c}]} V(c,c').$$

Since $\Pi(\cdot,c')$ is absolutely continuous and uniformly bounded by $-\bar{c}$ and $\bar{v}$, by the envelope theorem again,

$$V^*(c) = V(\bar{c}) - \int_c^{\bar{c}} \Pi_1(z,z)\,\mathrm{d}z = V(\bar{c}) + \int_c^{\bar{c}} (1-H^z(p(z,z)))\,\mathrm{d}z.$$

Rearranging, we have:

$$t(c) = -V(\bar{c}) + \alpha(c)\cdot \max_{p\in[\underline{v},\bar{v}]}(p-c)(1-H^c(p)) - \int_c^{\bar{c}} \alpha(z)(1-H^z(p(z,z)))\,\mathrm{d}z,$$

which established assertion 1.

In addition, by assertion 1, for any $c,c' \in [0,\bar{c}]$,

$$\begin{aligned}
&\int_c^{c'} [\alpha(z)(1-H^z(p(z,z))) - \alpha(c')(1-H^{c'}(p(z,c')))]\,\mathrm{d}z \\
=&\int_c^{c'} \alpha(z)(1-H^z(p(z,z)))\,\mathrm{d}z - \int_c^{c'} \Pi_1(z,c')\,\mathrm{d}z \\
=&\int_c^{c'} \alpha(z)(1-H^z(p(z,z)))\,\mathrm{d}z - (\Pi(c,c') - \Pi(c',c')) \\
=&\int_c^{\bar{c}} \alpha(z)(1-H^z(p(z,z)))\,\mathrm{d}z - (\Pi(c,c') - \Pi(c',c')) - \int_{c'}^{\bar{c}} \alpha(z)(1-H^z(p(z,z)))\,\mathrm{d}z \\
=&V(c,c) - V(c,c') \\
\geq&0,
\end{aligned} \tag{8}$$

where the first equality follows from (7), the second equality follows from the fundamental theorem of calculus and the last equality follows from assertion 1. This establishes assertion 3.

Finally, notice that for all $c' \in [0, \bar{c}]$, $\Pi(\cdot, c')$ is a pointwise supremun of a family of affine functions and thus is convex. Therefore, V^* is also convex as it is a pointwise supremum of a family of convex functions. Together, its subgradient $-(1 - H^c(p(c, c))$ is nondecreasing in c. This proves assertion 4.

Conversely, take any menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$ and any selection $p(c, c') \in \operatorname{argmax}_{p\in[\underline{v},\bar{v}]} \alpha(c')(p-c)(1-H^{c'}(p))$ that satisfy conditions 1 and 2. Again, let $\Pi(c, c') := \max_{p\in[\underline{v},\bar{v}]} \alpha(c')(p-c)(1-H^{c'}(p))$ and let $V(c, c') := \Pi(c, c') - t(c')$. By conditions 1 and 2, (7) and (8), for any $c, c' \in [0, \bar{c}]$,

$$V(c, c) - V(c, c') = \int_c^{c'} [\alpha(z)(1 - H^z(p(z, z))) - \alpha(c')(1 - H^{c'}(p(z, c'))]\, \mathrm{d}z \geq 0,$$

where the inequality follows from condition 2. This completes the proof. ■

B Optimal Mechanisms

B1 Proof of Proposition 1

Proof of Proposition 1. The fact that models (D) and (C) yield the same optimal revenue follows from the following equivalence between direct mechanisms. Take any incentive compatible and individually rational direct mechanism in the direct selling model $(\hat{q}, \hat{t})$, the associated selling plan $\hat{p}$ and the information disclosure policy $\{H^c\}_{c\in[0,\bar{c}]}$ let

$$t(c) := \hat{p}(c)\hat{q}(c) - \hat{t}(c),\ \forall c \in [0, \bar{c}]$$

and let $p := \hat{p}$. Then $(p(c), H^c, t(c))_{c\in[0,\bar{c}]}$ is a contract under the full contracting model. Moreover, for any $c, c' \in [0, \bar{c}]$,

$$\begin{aligned}
&(p(c) - c)(1 - H^c(p(c))) - t(c) \\
=&(\hat{p}(c) - c)(1 - H^c(\hat{p}(c))) - (\hat{p}(c)\hat{q}(c) - \hat{t}(c)) && \text{(definitions of } t \text{ and } p\text{)} \\
=&\hat{t}(c) - c\hat{q}(c) && \text{((MC) condition under the direct selling model)} \\
\geq&(\hat{t}(c') - c\hat{q}(c'))^+ && \text{((IC) and (IR) under the direct selling model)} \\
=&(\hat{p}(c') - c)(1 - H^{c'}(\hat{p}(c'))) - (\hat{p}(c')\hat{q}(c') - \hat{t}(c'))^+ \\
=&((p(c') - c)(1 - H^{c'}(p(c'))) - t(c'))^+
\end{aligned}$$

and hence $(p(c), H^c, t(c))_{c\in[0,\bar{c}]}$ is incentive compatible and individually rational. On the other hand, take any incentive compatible and individually rational contract under the full contracting model, $(p(c), H^c, t(c))_{c\in[0,\bar{c}]}$. Let $\hat{q}(c) := (1 - H^c(p(c)))$ and $\hat{t}(c) := p(c)(1 - H^c(p(c))) - t(c)$. Then $(\hat{q}, \hat{t})$ is a direct mechanism in the direct selling model and

$$\begin{aligned}
&\hat{t}(c) - c\hat{q}(c) \\
=&(p(c) - c)(1 - H^c(p(c))) - t(c) && \text{(definitions of } \hat{q} \text{ and } \hat{t}) \\
\geq&((p(c') - c)(1 - H^{c'}(p(c'))) - t(c'))^+ && \text{((IC) and (IR) under the full contracting model)} \\
=&(\hat{t}(c') - c\hat{q}(c'))^+
\end{aligned}$$

and thus the direct mechanism $(\hat{q}, \hat{t})$ is incentive compatible, individually rational and admits a selling policy p and information disclosure policy $\{H^c\}_{c\in[0,\bar{c}]}$ such that market clears. As such, models (D) and (C) are equivalent in the sense that the set of feasible direct mechanisms are isomorphic.

To see that optimal revenue under both models are R^*, it suffices to solve for the optimal revenue under the full contracting model. By Lemma 1, the intermediary's revenue maximizing problem can be rewritten as:

$$\sup_{p,\{H^c\}} \int_0^{\bar{c}} ((p(c) - \psi(c))(1 - H^c(p(c))))\, G(\mathrm{d}c) \tag{9}$$

$$\text{s.t. } c \mapsto (1 - H^c(p(c))) \text{ is nonincreasing} \tag{10}$$

To solve (9), a pointwise maximization approach can be adopted. Consider any $c \in [0, \bar{c}]$, $p \in [\underline{v}, \bar{v}]$ and any $H \in \mathcal{H}_F$. Notice that

$$\begin{aligned}
&(p - \psi(c))(1 - H(p)) \\
\leq&(p - \psi(c))(1 - H(p)) + \int_p^\infty (1 - H(z))\, \mathrm{d}z \\
\leq&\int_{\psi(c)}^\infty (1 - H(z))\, \mathrm{d}z \\
\leq&\int_{\psi(c)}^\infty (1 - F(z))\, \mathrm{d}z.
\end{aligned} \tag{11}$$

Furthermore, a binary information structure $H_{\mathrm{b}}^c \in \mathcal{H}_F$, defined as:

$$H_{\mathrm{b}}^c(x) := \begin{cases} 0, & \text{if } x \in [0, \mathbb{E}_F[v|v \leq \psi(c)]) \\ F(\psi(c)), & \text{if } x \in [\mathbb{E}_F[v|v \leq \psi(c)], \mathbb{E}_F[v|v > \psi(c)]] \\ 1, & \text{if } x \in (\mathbb{E}_F[v|v > \psi(c)], 1] \end{cases},$$

together with the price $v(c) := \mathbb{E}_F[v|v > \psi(c)]$, attains this upper bound. That is,

$$(v(c) - \psi(c))(1 - H_{\mathrm{b}}^c(v(c))) = \int_{\psi(c)}^{\bar{c}} (1 - F(z))\, \mathrm{d}z.$$

As such, for each $c \in [0, \bar{c}]$, the information structure H_{b}^c and the price $v(c)$ is a solution of

$$\max_{p,H} (p - \psi(c))(1 - H(p)).$$

Furthermore, consider the contract $(v(c), H_{\mathrm{b}}^c, t(c))_{c\in[0,\bar{c}]}$, where $t(c) := (v(c) - c)(1 - H_{\mathrm{b}}^c(v(c))) - \int_c^{\bar{c}}(1 - H_{\mathrm{b}}^z(v(z)))\,\mathrm{d}z$. Since ψ is increasing by regularity, $c \mapsto (1 - H_{\mathrm{b}}^c(v(c))) = (1 - F(\psi(c)))$ is nonincreasing, by Lemma 1, the menu $(v(c), H_{\mathrm{b}}^c, t(c))_{c\in[0,\bar{c}]}$ is incentive compatible and individually rational. Together, $(v(c), H_{\mathrm{b}}^c, t(c))_{c\in[0,\bar{c}]}$ maximizes the intermediary's revenue under the full contracting model and yields revenue R^*, as desired. ■

B2 Proof of Proposition 2

Proof of Proposition 2. I first show that R^* is an upper bound for the intermediary's revenue under all incentive compatible and individually rational menus. Then I show that the proposed upper censorship menu $(H_{\mathrm{u}}^c, t_{\mathrm{u}}(c), \alpha_{\mathrm{u}}(c))_{c\in[0,\bar{c}]}$ is incentive compatible, individually rational and attains the upper bound R^*.

First recall that given any incentive compatible and individually rational menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$. Lemma 2 gives the intermediary's revenue as:

$$\bar{t} + \int_0^{\bar{c}} \alpha(c)\left((p(c,c) - c)(1 - H^c(p(c,c))) - (1 - H^c(p(c,c)))\frac{G(c)}{g(c)}\right) G(\mathrm{d}c),$$

for some $\bar{t} \le 0$, where for each $c \in [0,\bar{c}]$, $p(c,c)$ is the largest selection of $\mathrm{argmax}_{p\in[\underline{v},\bar{v}]}(p - c)(1 - H^c(p))$. Notice that for each $c \in [0,\bar{c}]$,

$$\alpha(c)\left((p(c,c) - c)(1 - H^c(p(c,c))) - (1 - H^c(p(c,c)))\frac{G(c)}{g(c)}\right) \le \max_{p\in[\underline{v},\bar{v}]}(p - \psi(c))(1 - H^c(p)).$$

and thus for any incentive compatible and individually rational menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$, the intermediary's revenue is bounded from above by

$$\int_0^{\bar{c}} \max_{p\in[\underline{v},\bar{v}]}(p - \psi(c))(1 - H^c(p))G(\mathrm{d}c).$$

On the other hand, since $H^c \in \mathcal{H}_F$ for all $c \in [0,\bar{c}]$, for any $\tilde{p}(c,c) \in \mathrm{argmax}_{p\in[\underline{v},\bar{v}]}(p - \psi(c))(1 - H^c(p))$,

$$\begin{aligned}
&\max_{p\in[\underline{v},\bar{v}]}(p - \psi(c))(1 - H^c(p)) \\
\le&(\tilde{p}(c,c) - \psi(c))(1 - H^c(\tilde{p}(c,c)^-)) + \int_{\tilde{p}(c,c)}^{\infty}(1 - H^c(z))\,\mathrm{d}z \\
\le&\int_{\psi(c)}^{\infty}(1 - H^c(z))\,\mathrm{d}z \\
\le&\int_{\psi(c)}^{\infty}(1 - F(z))\,\mathrm{d}z \\
=&(\mathbb{E}_F[v|v > \psi(c)] - \psi(c))(1 - F(\psi(c))),
\end{aligned} \tag{12}$$

where second inequality follows from monotonicity of H^c, the third inequality follows from the fact that F is a mean-preserving spread of H^c and the last equality follows from integration by parts. Together, we have that for any incentive compatible and individually rational menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$,

$$
\begin{aligned}
R^* &:= \int_0^{\bar{c}} (v(c) - \psi(c))(1 - F(\psi(c)))G(\mathrm{d}c) \\
&\geq \int_0^{\bar{c}} \max_{p\in[\underline{v},\bar{v}]} (p - \psi(c))(1 - H^c(p))G(\mathrm{d}c). \\
&\geq \int_0^{\bar{c}} \alpha(c) \left((p(c,c) - c)(1 - H^c(p(c,c))) - (1 - H^c(p(c,c)))\frac{G(c)}{g(c)} \right) G(\mathrm{d}c)
\end{aligned}
$$

and therefore the intermediary's revenue given by any incentive compatible and individually rational menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$ must be no greater than R^*.

Now notice that under the upper censorship menu $(H^c_{\mathrm{u}}, t_{\mathrm{u}}(c), \alpha_{\mathrm{u}}(c))_{c\in[0,\bar{c}]}$, if each truthful-reporting seller whose cost is $c \in [0, \bar{c}]$ sets price optimally at $v(c)$, then for all $c \in [0, \bar{c}]$,

$$
\max_{x\in[0,\bar{c}]} (x - c)(1 - H^c_{\mathrm{u}}(x)) = (v(c) - \psi(c))(1 - F(\psi(c))). \tag{13}
$$

Therefore, it suffices to show that (13) holds for the upper censorship menu $(H^c_{\mathrm{u}}, t_{\mathrm{u}}(c), \alpha(c))_{c\in[0,\bar{c}]}$ and that this menu is incentive compatible and individually rational, as this would imply that the menu $(H^c_{\mathrm{u}}, t_{\mathrm{u}}(c), \alpha_{\mathrm{u}}(c))_{c\in[0,\bar{c}]}$ is feasible and attains the upper bound R^* of problem (3).

Indeed, first notice that $\phi^{-1}(c)$ is the unique element of $\mathrm{argmax}_{p\in[\underline{v},\bar{v}]}(p - c)(1 - F(p))$. Take and fix any $c \in [0, \bar{c}]$, under the upper censorship H^c_{u}, for a seller with cost c, setting price at any $x \in [0, \psi(c)]$ gives profit

$$
(x - c)(1 - F(x)) \leq (\psi(c) - c)(1 - F(\psi(c))),
$$

since the function $x \mapsto (x-c)(1-F(x))$ is single-peaked by regularity and since $x \leq \psi(c) \leq \phi^{-1}(c)$. Furthermore, setting any prices in $[\psi(c), v(c))$ must be worse than setting price at $v(c)$ since H^c_{u} is a constant on $(\psi(c), v(c))$. Finally, for any $x \in (v(c), \bar{v}]$, the seller gets zero profit by setting a price at x. Together, for the truthfully-reporting seller with cost c, setting price at $v(c)$ is indeed optimal.

Moreover, for any $c, c' \in [0, \bar{c}]$, if $c' \leq c$,

$$
\begin{aligned}
&\int_{c'}^{c} [(1 - H^z_{\mathrm{u}}(p(z,z))) - (1 - H^c_u(p(z,c)))]\,\mathrm{d}z \\
&= \int_{\tilde{c}}^{c} [F(\psi(c)) - F(\psi(z))]\,\mathrm{d}z + \int_{c'}^{\tilde{c}} [F(\phi^{-1}(z)) - F(\psi(z))]\,\mathrm{d}z \geq 0,
\end{aligned}
$$

for some $\tilde{c} \in [0, c]$, where the inequality follows from (1) and monotonocity of ψ. On the other hand, if $c' \in (c, v(c))$, then for any $z \in [c, c']$, $H^c_{\mathrm{u}}(p(z,c)) = F(\psi(c)) \leq F(\psi(z))$ by construction of H^u_c and by monotonocity of ψ. As such,

$$
\int_{c'}^{c} [(1 - H^z_{\mathrm{u}}(p(z,z))) - (1 - H^c_u(p(z,c)))]\,\mathrm{d}z = \int_{c}^{c'} [F(\psi(z)) - F(\psi(c))]\,\mathrm{d}z \geq 0.
$$

Finally, if $c' > v(c)$, optimal price under H^c_{u} gives zero profit and thus deviation gain must be negative. Together with Lemma 1, the upper censorship menu $(H^c_{\mathrm{u}}, t_{\mathrm{u}}(c), \alpha_{\mathrm{u}}(c))_{c\in[0,\bar{c}]}$ is indeed incentive compatible and individually rational. Moreover, under the menu $(H^c_{\mathrm{u}}, t_{\mathrm{u}}(c), \alpha_{\mathrm{u}}(c))_{c\in[0,\bar{c}]}$, the intermediary's expected revenue is

$$\int_0^{\bar{c}} (v(c) - \psi(c))(1 - F(\psi(c)))G(\mathrm{d}c).$$

Using integration by parts, since $v(c) = \mathbb{E}_F[v|v > \psi(c)]$, we have

$$(v(c) - \psi(c))(1 - F(\psi(c))) = \int_{\psi(c)}^{\infty} (1 - F(z))\,\mathrm{d}z$$

and therefore the optimal revenue is indeed R^*. This completes the proof. ■

B3 Optimal Mechanism without (1)

In this section, I construct an optimal mechanism for the intermediary under the weak contracting model. To this end, I first define the *obedient garbled upper censorship menus* below.

Definition 2. An information structure $H \in \mathcal{H}_F$ is a *garbled upper censorship* with cutoff $k \in \mathbb{R}_+$ if

$$H(x) = \begin{cases} F(k), & \text{if } x \in (k, \mathbb{E}_F[v|v > k]] \\ 1, & \text{if } x \in (\mathbb{E}_F[v|v > k], \bar{v}]. \end{cases}, \; \forall x \in [k, \bar{v}]$$

and

$$\int_k^{\infty} (1 - H(x))\,\mathrm{d}x = \int_k^{\infty} (1 - F(x))\,\mathrm{d}x.$$

Furthermore, a garbled upper censorship $H \in \mathcal{H}_F$ with cutoff k is said to be *obedient* at $c \in [0, \bar{c}]$ if

$$\mathbb{E}_F[v|v > k] \in \operatorname*{argmax}_{p\in[\underline{v},\bar{v}]} (p - c)(1 - H(p)).$$

Also, for any nondecreasing function $\widetilde{\psi} : [0, \bar{c}] \to \mathbb{R}_+$, a menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$ is called an *obedient garbled upper censorship menu* with cutoff function $\widetilde{\psi}$ if for all $c \in [0, \bar{c}]$ such that $\alpha(c) = 1$, H^c is a garbled upper censorship with cutoff $\widetilde{\psi}(c)$ that is obedient at c.

In what follows, I will show that an obedient garbled upper censorship menu with cutoff function $\psi^* := \min\{\psi, \psi(c^*)\}$ is optimal. To fully characterize this solution, it then remains to determine the threshold value c^*, the exact information to disclose conditional on the buyer's value being below $\psi^*(c)$, and the range of reports for which the product is publicized. For the buyer's information conditional on his value being below $\psi^*(c)$, define

$$H^z_\pi(x) := \begin{cases} 0, & \text{if } x \le \pi + z \\ 1 - \frac{\pi}{x - z}, & \text{if } x > \pi + z \end{cases},$$

for any $x, z, \pi \ge 0$. For the critical value c^*, for each $c, c' \in [0, \bar{c}]$, let

$$k(c) := c - \frac{1}{F(\psi(c))} \int_0^c F(\psi(z))\,\mathrm{d}z, \tag{14}$$

and let

$$\pi(c,c') := (v(c) - c')(1 - F(\psi(c))).$$

By regularity of F, the equation

$$F(x) = H^{k(c)}_{\pi(c,k(c))}(x)$$

has at most three solutions. Let $P(\psi(c), k(c))$ denote the largest one and let

$$c^* := \sup\left\{ c \in [0,\bar{c}] \,\middle|\, \int_0^{P(\psi(c),k(c))} H^{k(c)}_{\pi(c,k(c))}\,\mathrm{d}x \leq \int_0^{P(\psi(c),k(c))} F(x)\,\mathrm{d}x \right\}. \tag{15}$$

Notice that the function

$$c \mapsto \int_0^{P(\psi(c),k(c))} H^{k(c)}_{\pi(c,k(c))}(x)\,\mathrm{d}x - \int_0^{P(\psi(c),k(c))} F(x)\,\mathrm{d}x$$

is increasing and hence

$$\int_0^{P(\psi(c),k(c))} H^{k(c)}_{\pi(k(c))}\,\mathrm{d}x \leq \int_0^{P(\psi(c),k(c))} F(x)\,\mathrm{d}x$$

if and only if $c \leq c^*$. This definition is illustrated by Figure 6 below

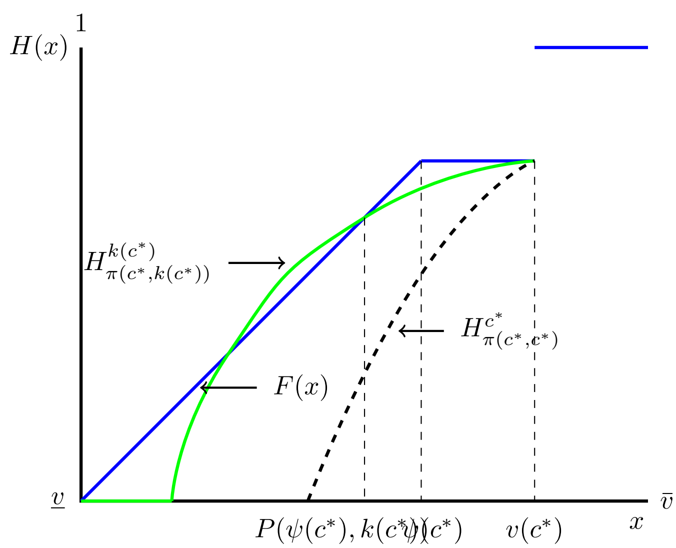

Figure 6: Constructing c^*

In Figure 6, the blue curve represents an upper censorship with cutoff $\psi(c^*)$. To incentivize seller with cost c^* to set the price at $v(c^*)$, the distribution to the left of $\psi(c^*)$ must be garbled so that it lies above the Pareto-shaped iso-profit curve for cost c^* seller,[16] as represented by the dotted curve. The green curve represents a Pareto distribution $H^{k(c^*)}_{\pi(c^*,k(c^*))}$. The threshold cost c^* is the

[16]The fact that the seller with cost c^* is indifferent in charging any price on the support of $H^{c^*}_{\pi(c^*,c^*)}$ is well-known.

cost at which the second order stochastic dominance condition binds. That is, the cost at which the area below the green curve agrees with the area below the blue curve on $[0, P(\psi(c^*), k(c^*))]$.[17]

Finally, for the publicizing range, let $v^*(c) := \mathbb{E}_F[v|v > \psi^*(c)]$ and let $\hat{c}$ be the smallest $c \in [0, \bar{c}]$ such that $\psi(c) \geq v^*(c)$. Then, as stated by Proposition 7, an obedient garbled upper censorship menu with cutoff function ψ^* and publicizing policy $\mathbf{1}_{[0,\hat{c}]}$ is optimal.

Proposition 7. *Under the weak contracting model, for each $c \in [0, \bar{c}]$, let*

$$\alpha_{\text{gu}}(c) := \mathbf{1}_{[0,\hat{c}]}(c)$$

and let

$$t_{\text{gu}}(c) := (v^*(c) - c)(1 - F(\psi^*(c))) - \int_c^{\hat{c}} (1 - F(\psi^*(z)))\,\mathrm{d}z.$$

Then there exists a family of information structures $\{H^c_{\text{gu}}\}_{c\in[0,\bar{c}]} \subset \mathcal{H}_F$ such that the obedient garbled upper censorship menu $(\alpha_{\text{gu}}(c), H^c_{\text{gu}}, t_{\text{gu}}(c))_{c\in[0,\bar{c}]}$ with cutoff function ψ^* is a solution for the intermediary's revenue maximization problem. In particular, optimal revenue is R^* if and only if $c^* \geq \bar{c}$.

Before stating the formal proof of Proposition 7, I first outline each step and sketch the structure of the proof. First, I will show that to maximize the expected revenue by choosing among all the possible incentive compatible and individually rational menu, it is without loss to restrict attention to incentive compatible, individually rational and obedient garbled upper censorship menus. This will be done by Lemma 4. Second, I develop a characterization for incentive compatible, individually rational and obedient garbled upper censorship menus that allows us to represent them by a family of inequalities so that the intermediary's problem can then be expressed as a constraint optimization problem. This will be done by Lemma 2 and Lemma 6. Next, I will then identify a class of critical constraints for the constraint optimization problem just obtained and show that for any feasible choice in the constraint optimization, two types of constraints must be met with equality. This is the content of Lemma 7. Finally, I will use the critical constraints to write down the dual of the constraint optimization problem and find the proper Lagrange multipliers so that the proposed menu indeed solves the dual problem, which effectively closes the duality gap and establishes optimality.

Proof of Proposition 7.

Step 1: I first show that it is without loss to restrict attention to the family of incentive compatible, individually rational and obedient garbled upper censorship menus. This is implied by the following Lemma.

Lemma 4. *For any incentive compatible and individually rational menu $(H^c, t(c), \alpha(c))_{c\in[0,\bar{c}]}$, there exists a nondecreasing function $\widetilde{\psi} : [0, \bar{c}] \to [0, \bar{v}]$ and an incentive compatible and individually*

[17] For all costs $c \leq c^*$, the construction is similar except that when the second order stochastic dominance constraint hold with strict inequality, a procedure that resembles the "ironing" technique is required to ensure the mean constraint.

rational menu $(\widehat{H}^c, \widehat{t}(c), \widehat{\alpha}(c))_{c\in[0,\bar{c}]}$ *such that* $\widehat{H}^c$ *is an obedient garbled upper-censorship with cutoff* $\widetilde{\psi}(c)$ *for all* $c \in [0, \bar{c}]$ *such that* $\widehat{\alpha}(c) = 1$ *and that*

$$\int_0^{\bar{c}} t(c)G(\mathrm{d}c) \leq \int_0^{\bar{c}} \widehat{t}(c)G(\mathrm{d}c).$$

Proof. Let $(H^c, t(c), \alpha(c))_{c\in[0,\bar{c}]}$ be an incentive compatible and individually rational menu. For each $c, z \in [0, \bar{c}]$, denote $p(z, c)$ by the largest element of

$$\operatorname*{argmax}_{p\in[\underline{v},\bar{v}]} \alpha(c)(p - z)(1 - H^c(p))$$

and let $p(c) := p(c, c)$. Notice that by Lemma 1, the function $c \mapsto \alpha(c)(1 - H^c(x(c)))$ must be nonincreasing. Therefore, the set $\{c \in [0, \bar{c}] | \alpha(c)(1 - H^c(x(c))) = 0\}$ must be an interval with supremum $\bar{c}$ and infimum $c_0 \in [0, \bar{c}]$. Notice that this implies $\alpha(c) = 1$ and $H^c(x(c)) < 1$ for all $c \in [0, c_0]$.

Thus, by Lemma 1 and Fubini's theorem,

$$\begin{aligned} \int_0^{\bar{c}} t(c)G(\mathrm{d}c) &= t(\bar{c}) + \int_0^{c_0} (x(c) - \psi(c))(1 - H^c(x(c)))G(\mathrm{d}c) \\ &\leq \int_0^{c_0} (x(c) - \psi(c))(1 - H^c(x(c)))G(\mathrm{d}c). \end{aligned} \tag{16}$$

For any $c \in [0, c_0]$, let $\widetilde{\psi}(c) := F^{-1}(H^c(x(c)))$. Notice first that by Lemma 1, $\widetilde{\psi}$ is nondecreasing. Now define an information structure $\widehat{H}^c$ as follows:

$$\widehat{H}^c(x) := \begin{cases} H^c(x), & \text{if } x \in [0, \underline{v}(c)] \\ F(\widetilde{\psi}(c)), & \text{if } x \in (\underline{v}(c), \widetilde{v}(c)] \\ 1, & \text{if } x \in (\widetilde{v}(c), \bar{v}] \end{cases},$$

where $\underline{v}(c)$ is uniquely determined by the equation

$$\int_0^{\widetilde{\psi}(c)} F(x)\,\mathrm{d}x = \int_0^{\underline{v}(c)} H^c(x)\,\mathrm{d}x + (\widetilde{\psi}(c) - \underline{v}(c))F(\widetilde{\psi}(c))$$

and

$$\widetilde{v}(c) := \mathbb{E}_F[v | v > \widetilde{\psi}(c)].$$

I claim that $\widetilde{v}(c)$ is the largest optimal price for a seller with cost c under the information structure $\widehat{H}^c$. That is,

$$\widetilde{v}(c) = \max\left\{ \operatorname*{argmax}_{p\in[\underline{v},\bar{v}]} (p - c)(1 - \widehat{H}^c(p)) \right\}.$$

Indeed, any $x \in (\underline{v}(c), \widetilde{v}(c))$ cannot be optimal since the function is strictly increasing on $(\underline{v}(c), \widetilde{v}(c))$. Also, any $x \in (\widetilde{v}(c), \bar{v}]$ cannot be optimal either since it gives zero profit to the seller. Finally, for any $x \in [0, \underline{v}(c)]$,

$$\begin{aligned} (x - c)(1 - \widehat{H}^c(x)) &= (x - c)(1 - H^c(x)) \\ &\leq (x(c) - c)(1 - H(x(c))) \\ &\leq (\mathbb{E}_{H^c}[v | v > x(c)] - c)(1 - H(x(c))) \\ &\leq (\mathbb{E}_F[v | v > \widetilde{\psi}(c)] - c)(1 - F(\widetilde{\psi}(c))), \end{aligned}$$

where the first equality follows from the fact that $x \in [0, \underline{v}(c)]$, the first inequality follows from optimality of $x(c)$ under H^c and the last inequality follows from the construction of $\widetilde{\psi}$ do that $F(\widetilde{\psi}(c)) = H^c(x(c))$ and that $\mathbb{E}_{H^c}[v|v > x(c)] \leq \mathbb{E}_F[v|v > \widetilde{\psi}(c)]$, which follows from $F(\widetilde{\psi}(c)) = H^c(x(c))$ and the fact that $H^c \in \mathcal{H}_F$.

For any $c \in [c_0, \bar{c}]$, select any $H \in \mathcal{H}_F$ and let $\widehat{H}^c := H$. I will now verify that there exists some transfer $\widehat{t} : [0, \bar{c}] \to \mathbb{R}$ such that the menu $(\widehat{H}^c, \widehat{t}(c), \mathbf{1}_{[0,c_0]}(c))_{c \in [0,\bar{c}]}$ is incentive compatible. By Lemma 1, it suffices to show that for any $c', c \in [0, c_0]$ with $c' < c$,

$$\int_{c'}^{c} [\widehat{H}^c(\widehat{p}(z,c)) - \widehat{H}^z(\widehat{p}(z,z))] \,\mathrm{d}z \geq 0,$$

where $\widehat{p}(z,c)$ is the largest element of

$$\underset{p \in [\underline{v},\bar{v}]}{\operatorname{argmax}} (p - z)(1 - \widehat{H}^c(p)).$$

Take and fix any $c', c \in [0, c_0]$ with $c' < c$. Notice that as argued above, since $\widetilde{v}(c)$ is the largest optimal price for a seller with cost c under $\widehat{H}^c$, $\widehat{H}^c(\widehat{p}(c,c)) = F(\widetilde{\psi}(c)) = H^c(x(c))$ for all $c \in [0, c_0]$. On the other hand, notice that by construction of $\widehat{H}^c$, for any $z \in [c', c]$, either $\widehat{p}(z,c) = \widetilde{v}(c)$ or $\widehat{p}(z,c) = p(z,c) \in [0, \underline{v}(c))$. In addition, since the original menu $(H^c, t(c), \alpha(c))_{c \in [0,\bar{c}]}$ is incentive compatible, Lemma 1 implies that

$$\int_{c'}^{c} [H^c(p(z,c)) - H^z(p(z,z))] \,\mathrm{d}z \geq 0.$$

Together,

$$\begin{aligned}
\int_{c'}^{c} [\widehat{H}^c(\widehat{p}(z,c)) - \widehat{H}^z(\widehat{p}(z,z))] \,\mathrm{d}z &= \int_{c'}^{c} [\widehat{H}^c(\widehat{p}(z,c)) - F(\widetilde{\psi}(z))] \,\mathrm{d}z \\
&\geq \int_{c'}^{c} [H^c(p(z,c)) - F(\widetilde{\psi}(z))] \,\mathrm{d}z \\
&= \int_{c'}^{c} [H^c(p(z,c)) - H^z(p(z,z))] \,\mathrm{d}z \\
&\geq 0,
\end{aligned}$$

where the two equalities follows from the constructions of $\widehat{H}^z$ and $\widetilde{\psi}$ so that $\widehat{H}^z(\widehat{p}(z,z)) = F(\widetilde{\psi}(z)) = H^z(p(z,z))$, the first inequality follows from the observation that $\widehat{H}^c(\widetilde{v}(c)) = F(\widetilde{\psi}(c)) \geq H^c(x)$ for any $x \in [0, \underline{v}(c))$.

Finally, since as noted above, $F(\widetilde{\psi}(c)) = H^c(x(c))$ and $\mathbb{E}_{H^c}[v|v > x(c)] \leq \mathbb{E}_F[v|v > \widetilde{\psi}(c)]$, for any $c \in [0, c_0]$, we have:

$$\begin{aligned}
(x(c) - \psi(c))(1 - H(x(c))) &\leq (\mathbb{E}_{H^c}[v|v > x(c)] - \psi(c))(1 - H(x(c))) \\
&\leq (\mathbb{E}_F[v|v > \widetilde{\psi}(c)] - \psi(c))(1 - F(\widetilde{\psi}(c))) \qquad (17) \\
&= (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c))).
\end{aligned}$$

Together, by Lemma 1, let

$$\widehat{t}(c) := \mathbf{1}_{[0,c_0]}(c) \left[(\widetilde{v}(c) - c)(1 - F(\widetilde{\psi}(c))) - \int_{c}^{c_0} (1 - F(\widetilde{\psi}(z))) \,\mathrm{d}z \right].$$

Then $(\widehat{H}^c, \widehat{t}(c), \mathbf{1}_{[0,c_0]}(c))_{c\in[0,\bar{c}]}$ is an incentive compatible, individually rational and is an obedient garbled upper censorship with cutoff function $\widetilde{\psi}$.

$$\begin{aligned}\int_0^{c_0} \widehat{t}(c)G(\mathrm{d}c) &= \int_0^{\bar{c}} (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))G(\mathrm{d}c) \\ &\geq \int_0^{\bar{c}} (x(c) - \psi(c))(1 - H(x(c)))G(\mathrm{d}c) \\ &\geq \int_0^{\bar{c}} t(c)G(\mathrm{d}c),\end{aligned}$$

where the last two inequalities follow from (16) and (17). This completes the proof. ■

Step 2: I now characterize the family of incentive compatible, individually rational and obedient garbled upper censorship menus by a collection of inequalities. This is the result of Lemma 6. To show Lemma 6, Lemma 5 is needed to simplify the procedure.

Lemma 5. *Fix any $\tilde{c} \in [0, \bar{c}]$. Given any functions $p : [0, \tilde{c}] \to [\underline{v}, \bar{v}]$ and $q : [0, \tilde{c}] \to [0, 1]$. Then*

$$(p(c) - c)(1 - q(c)) \geq (p(c') - c)(1 - q(c')), \forall c, c' \in [0, \tilde{c}] \tag{18}$$

if and only if

1. $(p(c) - c)(1 - q(c)) = (p(\tilde{c}) - \tilde{c})(1 - q(\tilde{c})) + \int_c^{\tilde{c}} (1 - q(z))\, \mathrm{d}z$ *for all* $c \in [0, \tilde{c}]$.
2. q *is nondecreasing.*

Proof. For necessity, consider and pair of functions p, q satisfying (18). Let $\pi(c, c') := (p(c') - c)(1 - q(c'))$, for any $c, c' \in [0, \tilde{c}]$. Notice that for each $c' \in [0, \tilde{c}]$, the function $\pi(\cdot, c')$ is absolutely continuous and uniformly bounded by $-\tilde{c}$ and $\bar{v}$. By the envelope theorem, the function

$$\pi^*(c) := \pi(c, c) = \max_{c'\in[0,\bar{c}]} \pi(c, c')$$

is also absolutely continuous and the derivative exists and equals to $-(1 - q(c))$ for almost all $c \in [0, \tilde{c}]$. Thus,

$$(p(c) - c)(1 - q(c)) = \pi^*(c) = \pi^*(\tilde{c}) + \int_c^{\tilde{c}} (1 - q(z))\, \mathrm{d}z,$$

which establishes assertion 1. Moreover, since $\pi(\cdot, c')$ is an affine function, π^* is a pointwise supremum of a family of affine functions and thus is convex. As a result, its derivative, $-(1 - q)$, is nondecreasing.

For sufficiency, consider a pair of functions p, q satisfying 1 and 2. Then, for any $c, c' \in [0, \tilde{c}]$,

$$\begin{aligned}
&(p(c)-c)(1-q(c)) - (p(c')-c)(1-q(c'))\\
=&(p(c)-c)(1-q(c)) - (p(c')-c')(1-q(c')) - (c'-c)(1-q(c'))\\
=&\int_c^{c'} (1-q(z))\,\mathrm{d}z - (c'-c)(1-q(c'))\\
=&\int_c^{c'} (q(c')-q(z))\,\mathrm{d}z\\
\geq&0,
\end{aligned}$$

where the second equality follows from 1 and the inequality follows from 2. ■

Lemma 6. *For any* $c_\alpha \in [0, \bar{c}]$*, any nondecreasing function* $\widetilde{\psi} : [0, c_\alpha] \to [0, \bar{v}]$*, let* $\widetilde{v}(c) := \mathbb{E}_F[v|v > \widetilde{\psi}(c)]$*,* $\widetilde{\pi}(c) := (\widetilde{v}(c) - c)(1 - F(\widetilde{\psi}(c))), \forall c \in [0, \bar{c}]$*. There exists* $t : [0, \bar{c}] \to \mathbb{R}$*, such that* $(H^c, t(c), \mathbf{1}_{[0,c_\alpha](c)})_{c\in[0,\bar{c}]}$ *is an incentive compatible, individually rational and obedient garbled upper censorship menu with cutoff function* $\widetilde{\psi}$ *if and only if there exists* $q : [0, c_\alpha]^2 \to [0, 1]$ *such that:*

1. $H^c(\widetilde{\psi}(c)) = F(\widetilde{\psi}(c))$*,* $\int_0^x [H^c(z) - F(z)]\,\mathrm{d}z \geq 0$*, for all* $x \in [0, \widetilde{\psi}(c)]$*, with equality at* $x = \widetilde{\psi}(c)$*, for all* $c \in [0, c_\alpha]$*.*
2. $\int_{c'}^c [q(z|c) - F(\widetilde{\psi}(z))] \geq 0$*, for all* $c, c' \in [0, c_\alpha]$ *with* $c' \leq c$*.*
3. *For any* $c \in [0, c_\alpha]$*,* $x \in [0, \widetilde{\psi}(c)]$*,* $H^c(x) \geq \max_{c'\in[0,c]}[1 - (\widetilde{\pi}(c) + \int_{c'}^c (1 - q(z|c)\,\mathrm{d}z))/(x - c')^+]^+$ *with equality if and only if* $(x - c')(1 - q(c'|z)) = \widetilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))\,\mathrm{d}z$ *for some* $c' \in \operatorname{argmax}_{c'\in[0,c]}[1 - (\widetilde{\pi}(c) + \int_{c'}^c (1 - q(z|c)\,\mathrm{d}z))/(x - c')^+]^+$*.*
4. $q(\cdot|c)$ *is nondecreasing and* $q(c|c) = F(\widetilde{\psi}(c))$ *for all* $c \in [0, c_\alpha]$*.*

Proof. For necessity, consider any incentive compatible, individually rational and obedient garbled upper censorship menu $(H^c, t(c), \mathbf{1}_{[0,c_\alpha]}(c))_{c\in[0,\bar{c}]}$ with cutoff function $\widetilde{\psi}$. Since H^c is a garbled upper censorship, assertion 1 is clearly satisfied.

Since H^c is obedient, for any $c \in [0, c_\alpha]$,

$$(x - c)(1 - H^c(x)) \leq (\widetilde{v}(c) - c)(1 - F(\widetilde{\psi}(c))) = \widetilde{\pi}(c), \forall x \in [0, \bar{v}]$$

and $H^c(x) = 1$ for all $x \in (\widetilde{v}(c), \bar{v}]$. Therefore,

$$\widetilde{v}(c) \in \operatorname{argmax}(x - c)(1 - H^c(x)), \forall c \in [0, c_\alpha].$$

On the other hand, for any $c', c \in [0, c_\alpha]$ with $c' \leq c$, take any selection

$$p(c', c) \in \operatorname*{argmax}_{p\in[\underline{v},\bar{v}]}(p - c')(H^c(p))$$

and let

$$p(c'|c) := p(c', c)$$
$$q(c'|c) = H^c(p(c', c)).$$

Since $(H^c, t(c), \mathbf{1}_{[0,c_\alpha]}(c))_{c\in[0,\bar{c}]}$ is incentive compatible, Lemma 1 then ensures that

$$\int_{c'}^{c} [q(z|c) - F(\widetilde{\psi}(z))] \, \mathrm{d}z = \int_{c'}^{c} [H^c(p(z, c)) - F(\widetilde{\psi}(z)))] \, \mathrm{d}z \geq 0,$$

which establishes assertion 2.

Now notice that as shown above, $q(c|c) = F(\widetilde{\psi}(c))$ and $(p(c|c) - c)(1 - q(c|c)) = \widetilde{\pi}(c)$ for all $c \in [0, c_\alpha]$. Since $p(c', c) \in \operatorname{argmax}_{x\in[0,c_\alpha]}(x - c')(H^c(x))$, for any such c, c' we have:

$$(x - c')(1 - H^c(x)) \leq (p(c'|c) - c')(1 - q(c'|c)), \forall x \in [0, \bar{v}]. \tag{19}$$

Rearranging, we have:

$$H^c(x) \geq 1 - \frac{(p(c'|c) - c')(1 - q(c'|c))}{(x - c')^+}, \forall x \in [0, \bar{v}]. \tag{20}$$

Notice that the right hand side of (20) does not depend on c', we thus have:

$$H^c(x) \geq \max_{c'\in[0,c]} \left[1 - \frac{(p(c'|c) - c')(1 - q(c'|c))}{(x - c')^+}\right]^+, \forall x \in [0, \widetilde{\psi}(c)], \forall c \in [0, c_\alpha]. \tag{21}$$

Moreover, by $p(c', c) \in \operatorname{argmax}_{p\in[\underline{v},\bar{v}]}(p-c')(H^c(p))$ again, for any $c', c'' \in [0, c]$ and for any $c \in [0, c_\alpha]$,

$$(p(c'|c) - c')(1 - q(c'|c)) \geq (p(c''|c) - c')(1 - q(c''|c)). \tag{22}$$

By Lemma 5, $\hat{q}(\cdot|c)$ is nondecreasing, which establishes assertion 4.

Furthermore, also by Lemma 5,

$$(p(c'|c)-c')(1-q(c'|c)) = (p(c|c)-c)(1-q(c|c))+\int_{c'}^{c}(1-q(z|c)) \, \mathrm{d}z = \widetilde{\pi}(c)+\int_{c'}^{c}(1-q(z|c)) \, \mathrm{d}z, \tag{23}$$

for any $c \in [0, c_\alpha]$ and any $c' \in [0, c]$. Combining (21) and (23), we obtain

$$H^c(x) \leq \max_{c'\in[0,x]} \left[1 - \frac{\widetilde{\pi}(c) + \int_{c'}^{c}(1 - \hat{q}(z|c)}{(x - c')^+}\right]^+, \forall x \in [0, \bar{v}].$$

Moreover, by (23), for any $c \in [0, c_\alpha]$, any $x \in [0, \widetilde{\psi}(c)]$,

$$(x - c')(1 - q(c'|c)) = \widetilde{\pi}(c) + \int_{c'}^{c} (1 - q(z|c)) \, \mathrm{d}z$$

for some $c' \in \operatorname{argmax}_{c'\in[0,c]}[1-(\widetilde{\pi}(c)+\int_{c'}^{c}(1-q(z|c) \, \mathrm{d}z))/(x-c')]^+$, if and only if $x = p(c', c) = p(c'|c)$ and

$$H^c(x) = \max_{c'\in[0,c]} \left[1 - \frac{(p(c'|c) - c')(1 - q(c'|c))}{(x - c')^+}\right]^+,$$

which, together with (22), establishes assertion 3.

Conversely, for sufficiency, take any $q : [0, c_\alpha]^2 \to [0, 1]$ and $\{H^c\}_{c\in[0,c_\alpha]}$ satisfying conditions 1, 2, 3 and 4. Notice that if H^c is not a garbled upper censorship, then for each $c \in [0, c_\alpha]$, $x \in [0, \bar{v}]$, let

$$\widehat{H}^c(x) := \begin{cases} H^c(x), & \text{if } x \in [0, \widetilde{\psi}(c)] \\ F(\widetilde{\psi}(c)), & \text{if } x \in (\widetilde{\psi}(c), \widetilde{v}(c)] \\ 1, & \text{if } x \in (\widetilde{v}(c), \bar{v}] \end{cases} .$$

By assertion 1, $\widehat{H}^c \in \mathcal{H}_F$ and is a garbled upper censorship with cutoff $\widetilde{\psi}(c)$ for all $c \in [0, c_\alpha]$ and $\widehat{H}^c \equiv H^c$ on $[0, \widetilde{\psi}]$. Thus suffices to take H^c as a garbled upper censorship and verify that there exists $t : [0, \bar{c}] \to \mathbb{R}$ such that $(H^c, t(c), \mathbf{1}_{[0,c_\alpha]}(c))_{c\in[0,\bar{c}]}$ is an incentive compatible, individually rational and obedient menu.

For obedience, notice that by assertion 3, for any $x \in [0, \widetilde{\psi}(c)]$,

$$H^c(x) \geq \max_{c'\in[0,c]} \left[1 - \frac{\widetilde{\pi}(c) + \int_{c'}^{c}(1 - q(z|c))\,\mathrm{d}z}{(x - c')^+}\right]^+ \geq \left[1 - \frac{\widetilde{\pi}(c)}{(x - c)^+}\right]^+ .$$

Rearranging, we have:

$$(x - c)(1 - H^c(x)) \leq \widetilde{\pi}(c) = (\widetilde{v}(c) - c)(1 - H^c(\widetilde{v}(c))).$$

On the other hand, any $x \in (\widetilde{v}(c), \bar{v}]$ gives zero profit under $\widehat{H}^c$. Thus,

$$\widetilde{v}(c) \in \operatorname*{argmax}_{p\in[\underline{v},\bar{v}]}(p - c)(1 - H^c(p)), \forall c \in [0, c_\alpha].$$

Therefore, $\{H^c\}_{c\in[0,c_\alpha]}$ is indeed obedient.

For incentive compatibility and individual rationality, by Lemma 5, since $q(\cdot|c)$ is increasing and $q(c|c) = F(\widetilde{\psi}(c))$ for all $c \in [0, c_\alpha]$, there exists $p : [0, c_\alpha]^2 \to [0, \bar{v}]$ such that

$$(p(c'|c) - c')(1 - q(c'|c)) = \widetilde{\pi}(c) + \int_{c'}^{c}(1 - q(z|c))\,\mathrm{d}z, \forall c' \in [0, c],\ c \in [0, c_\alpha]$$

and that

$$(p(c'|c) - c')(1 - q(c'|c)) \geq (p(c''|c) - c')(1 - q(c''|c)), \forall c', c'' \in [0, c],\ c \in [0, c_\alpha].$$

As such, for any $c \in [0, c_\alpha]$, $x \in [0, \widetilde{\psi}(c)]$ and any $c' \in [0, c]$, whenever

$$H^c(x) = \max_{c'\in[0,c]} \left[1 - \frac{\widetilde{\pi}(c) + \int_{c'}^{c}(1 - q(z|c))\,\mathrm{d}z}{(x - c')^+}\right]^+, \tag{24}$$

by assertion 3 and (5), there exists c' such that $H^c(x) = q(c'|c)$

$$(x - c')(1 - H^c(x)) \geq \widetilde{\pi}(c) + \int_{c''}^{c}(1 - q(z|c)) = (p(c''|c) - c')(1 - q(c''|c)), \forall c'' \in [0, c],$$

for some $c' \in [0, c_\alpha]$. Assertion 3 then implies that whenever (24) holds for some $x \in [0, \widetilde{\psi}(c)]$, $c' \in [0, c_\alpha]$

$$(x - c')(1 - H^c(x)) \geq (y - c')(1 - H^c(y))$$

for any y such that (24) holds for some (possibly distinct) $c'' \in [0, c]$.

On the other hand, for any $x \in [0, \widetilde{\psi}(c)]$ such that (24) does not hold, there must be some $c' \in [0, c_\alpha]$ such that

$$(x - c')(1 - H^c(x)) < \widetilde{\pi}(c) + \int_{c'}^{c} (1 - q(z|c))\,\mathrm{d}z$$

and therefore

$$(x - c')(1 - H^c(x)) < (y - c')(1 - H^c(y))$$

for $y = p(c'|c)$. Together, it must be that $\operatorname{argmax}_{p\in[0,\widetilde{\psi}(c)]}(p - c')(1 - H^c(p))$ is exactly the set where (24) holds for some $c' \in [0, c_\alpha]$. Thus, we may take a selection $p(c', c) \in \operatorname{argmax}_{p\in[0,\widetilde{\psi}(c)]}(p - c')(1 - H^c(p))$ such that $H^c(p(c', c)) = q(c'|c)$ for all $c \in [0, c_\alpha]$, $c' \in [0, c]$. Then, by Lemma 1 and assertion 2, there exists $t : [0, \bar{c}] \to \mathbb{R}$ such that $(H^c, t(c), \mathbf{1}_{[0,c_\alpha]}(c))_{c\in[0,\bar{c}]}$ is incentive compatible and individually rational. ∎

Step 3: Now, I identify two critical constraints for the intermediary's problem. Lemma 7, together with Lemma 6, show that for any incentive compatible, individually rational and obedient garbled upper censorship menu, two families of equality constraints must be satisfied, which will later be used in forming the dual problem.

Lemma 7. *For any $c_\alpha \in [0, \bar{c}]$, any nondecreasing function $\widetilde{\psi} : [0, c_\alpha] \to [0, \bar{v}]$, any $\{H^c\}_{c\in[0,\bar{c}]} \subset \mathcal{H}_F$ and any $q : [0, c_\alpha]^2 \to [0, 1]$ satisfying conditions 1-4 in Lemma 6, there exists some $\hat{q} : [0, c_\alpha]^2 \to [0, 1]$ such that $\hat{q}(c|c) = F(\widetilde{\psi}(c))$, $\hat{q}(\cdot|c)$ is increasing and*

$$\int_0^{\widetilde{\psi}(c)} [\Gamma^c_{\hat{q},\widetilde{\psi}}(x) - F(x)]\,\mathrm{d}x = 0$$
$$\int_0^{c} [\hat{q}(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z = 0,$$

where

$$\Gamma^c_{q,\widetilde{\psi}}(x) := \begin{cases} q(0|c), & \text{if } x \in [0, \underline{x}_q(c)) \\ \max_{c'\in[0,c_\alpha]} \left[1 - \frac{\widetilde{\pi}(c) + \int_{c'}^{c}(1-q(z|c))\,\mathrm{d}z}{(x-c')^+}\right]^+, & \text{if } x \in [\underline{x}_q(c), \bar{x}_q(c)) \\ F(x), & \text{if } x \in [\bar{x}_q(c), \widetilde{\psi}(c)) \\ F(\widetilde{\psi}(c)), & \text{if } x \in [\widetilde{\psi}(c), \widetilde{v}(c)) \\ 1, & \text{if } x \in [\widetilde{v}(c), 1] \end{cases},$$

for any $q : [0, c_\alpha]^2 \to [0, 1]$ with $q(\cdot|c)$ being nondecreasing , any $c \in [0, c_\alpha]$ and any $x \in [0, \widetilde{\psi}(c)]$, with

$$\underline{x}_q(c) := \frac{1}{(1 - q(0|c))}\left[\widetilde{\pi}(c) + \int_0^c (1 - q(z|c))\,\mathrm{d}z\right]$$

$$\bar{x}_q(c) := \begin{cases} \frac{1}{(1-q(\bar{z}_q(c)|c))}\left[\widetilde{\pi}(c) + \int_{\bar{z}_q(c)}^{c}(1 - q(z|c))\,\mathrm{d}z\right] + \bar{z}_q(c), & \text{if } \Phi^{-1}_{\bar{z}_q(c)}\left(\widetilde{\pi}(c) + \int_{\bar{z}_q(c)}^{c}(1 - q(z|c))\,\mathrm{d}z\right) = \emptyset \\ \max \Phi^{-1}_{\bar{z}_q(c)}\left(\widetilde{\pi}(c) + \int_{\bar{z}_q(c)}^{c}(1 - q(z|c))\,\mathrm{d}z\right), & \text{if } \Phi^{-1}_{\bar{z}_q(c)}\left(\widetilde{\pi}(c) + \int_{\bar{z}_q(c)}^{c}(1 - q(z|c))\,\mathrm{d}z\right) \neq \emptyset \end{cases}$$

$$\bar{z}_q(c) := \inf\{z \in [0, c] | q(z|c) = F(\widetilde{\psi}(c))\},$$

where $\Phi_z(x) := (x - z)(1 - F(x))$ is the profit function under prior F when cost is z.

Proof. Take any $\widetilde{\psi}$, $\{H^c\}_{c\in[0,c_\alpha]}$ and q satisfying conditions 1-4 in Lemma 6. Fix any $c \in [0, c_\alpha]$. By Lemma 6, we have

$$\int_0^{\widetilde{\psi}(c)} [H^c(x) - F(x)]\,\mathrm{d}x = 0.$$

Moreover, since $\widetilde{v}(c) > \widetilde{\psi}(c)$, we must have $\bar{z}_q(c) < c$ and $F(\widetilde{\psi}(c)) > q(\bar{z}_q(c)|c)$.

Consider first the case where

$$\int_0^c [q(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z > 0,$$

first define $\underline{z}_q(c)$ as the unique z that solves to the equation

$$\int_z^c q(t|c)dt = \int_0^c F(\widetilde{\psi}(t))dt.$$

For this $\underline{z}_q(c)$, consider the following construction: Define

$$\hat{q}(z|c) := \begin{cases} 0, & \text{if } z \in [0, \underline{z}_q(c)) \\ q(z|c), & \text{if } z \in [\underline{z}_q(c), c] \end{cases} .$$

Notice that we then have:

$$\int_0^c [\hat{q}(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z = 0.$$

If, furthermore,

$$\int_0^{\underline{x}_{\hat{q}}(c)} \left[H^c(x) - \left(1 - \frac{\widetilde{\pi}(c) + \int_0^c (1-\hat{q}(z|c))\,\mathrm{d}z}{x}\right)^+ \right] \mathrm{d}x$$
$$\leq \int_{\bar{x}_q(c)}^{\widetilde{v}(c)} \left[F(\widetilde{\psi}(c)) - \left(1 - \frac{\widetilde{\pi}(c) + \int_{\bar{z}_{\hat{q}}(c)}^c (1-\hat{q}(z|c))\,\mathrm{d}z}{(x - \bar{z}_q(c))^+}\right)^+ \right] \mathrm{d}x,$$

then there exists some $q_0 \in [q(\bar{z}_q(c)|c), F(\widetilde{\psi}(c))]$ such that

$$\int_0^{\underline{x}_{\hat{q}}(c)} \left[H^c(x) - \left(1 - \frac{\widetilde{\pi}(c) + \int_0^c (1-\hat{q}(z|c))\,\mathrm{d}z}{x}\right)^+ \right] \mathrm{d}x$$
$$= (\widetilde{v}(c) - \hat{x})F(\widetilde{\psi}(c)) - \int_{\bar{x}_q(c)}^{\widetilde{v}(c)} \left(1 - \frac{\widetilde{\pi}(c) + \int_{\bar{z}_{\hat{q}}(c)}^c (1-\hat{q}(z|c))\,\mathrm{d}z}{(x - \bar{z}_{\hat{q}}(c))^+}\right)^+ \mathrm{d}x,$$

where

$$\hat{x} := \frac{1}{1-q_0}\left[\widetilde{\pi}(c) + \int_{\bar{z}_{\hat{q}}}^c (1-\hat{q}(z|c))\,\mathrm{d}z\right] + \bar{z}_q(c).$$

Thus, by possibly redefining $\hat{q}(\bar{z}_{\hat{q}}|c)$ as q_0, we then have:

$$\int_0^{\widetilde{\psi}(c)} \Gamma^c_{\hat{q},\widetilde{\psi}}(x)\,\mathrm{d}x = \int_0^{\widetilde{\psi}(c)} H^c(x)\,\mathrm{d}x = \int_0^{\widetilde{\psi}(c)} F(x)\,\mathrm{d}x.$$

Furthermore, since the the point $\bar{z}_q(c)$ has Lebesgue measure zero, we still have

$$\int_0^c [\hat{q}(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z = 0.$$

On the other hand, if

$$\int_0^{\underline{x}_{\hat{q}}(c)} \left[H^c(x) - \left(1 - \frac{\widetilde{\pi}(c) + \int_0^c (1 - \hat{q}(z|c))\,\mathrm{d}z}{x} \right)^+ \right] \mathrm{d}x$$
$$> \int_{\bar{x}_q(c)}^{\widetilde{v}(c)} \left[F(\widetilde{\psi}(c)) - \left(1 - \frac{\widetilde{\pi}(c) + \int_{\bar{z}_{\hat{q}}(c)}^c (1 - \hat{q}(z|c))\,\mathrm{d}z}{(x - \bar{z}_q(c))^+} \right)^+ \right] \mathrm{d}x,$$

I construct a similar procedure for any $x \in [\underline{x}_q(c), \bar{x}_q(c)]$. Specifically, consider any selection

$$c_q(x) \in \operatorname*{argmax}_{c' \in [0,c]} \left[1 - \frac{\widetilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))\,\mathrm{d}z}{(x - c')^+} \right]^+.$$

Observe that for any such selection c_q and any $x \in [\underline{x}_q(c), \bar{x}_q(c)]$, $c_q(x) \geq \underline{z}_q(c)$ and therefore there exists $q_x \geq 0$ such that

$$\int_{c_q(x)}^c [q(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z + \int_0^{c_q(x)} [q_x - F(\widetilde{\psi}(z))]\,\mathrm{d}z = 0$$

Now define

$$q_x(z|c) := \begin{cases} q_x, & \text{if } z \in [0, c_q(x)) \\ q(z|c), & \text{if } z \in [c_q(x), c] \end{cases}$$

and let

$$\hat{x}(x) := \frac{1}{1 - q_x} \left[\widetilde{\pi}(c) + \int_{c_q(x)}^c (1 - q_x(z|c))\,\mathrm{d}z \right].$$

Notice that for each $x \in [\underline{x}_q(c), \bar{x}_q(c)]$, the set

$$\left\{ \int_0^{\hat{x}(x)} \left[H^c(x) - \left(1 - \frac{\widetilde{\pi}(c) + \int_{\bar{z}_q(c)}^c (1 - q_x(z|c))\,\mathrm{d}z}{(x - \bar{z}_q(c))^+} \right)^+ \right] \mathrm{d}x \;\middle|\; c_q(x) \in \operatorname*{argmax}_{c' \in [0,c]} \left[1 - \frac{\widetilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))\,\mathrm{d}z}{(x - c')^+} \right]^+ \right\}$$

is a (possibly degenerate) interval by monotonicity of $q(\cdot|c)$. As such, the correspondence

$$x \mapsto \left\{ \int_0^{\hat{x}(x)} \left[H^c(x) - \left(1 - \frac{\widetilde{\pi}(c) + \int_{\bar{z}_q(c)}^c (1 - q_x(z|c))\,\mathrm{d}z}{(x - \bar{z}_q(c))^+} \right)^+ \right] \mathrm{d}x \;\middle|\; c_q(x) \in \operatorname*{argmax}_{c' \in [0,c]} \left[1 - \frac{\widetilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))\,\mathrm{d}z}{(x - c')^+} \right]^+ \right\}$$

is a Kakutani correspondence[18] since the set

$$\underset{c'\in[0,c]}{\operatorname{argmax}}\left[1-\frac{\widetilde{\pi}(c)+\int_{c'}^{c}(1-q(z|c))\,\mathrm{d}z}{(x-c')^{+}}\right]^{+}$$

is upper-hemicontinuous by the Theorem of Maximum. Then, as long as

$$\int_{0}^{\hat{x}(\bar{x}_q(c))}\left[H^{c}(x)-\left(1-\frac{\widetilde{\pi}(c)+\int_{\bar{z}_q(c)}^{c}(1-q_x(z|c))\,\mathrm{d}z}{(x-\bar{z}_q(c))^{+}}\right)^{+}\right]\mathrm{d}x$$
$$\leq\int_{\bar{x}_q(c)}^{\widetilde{v}(c)}\left[F(\widetilde{\psi}(c))-\left(1-\frac{\widetilde{\pi}(c)+\int_{\bar{z}_{\hat{q}}(c)}^{c}(1-q_x(z|c))\,\mathrm{d}z}{(x-\bar{z}_q(c))^{+}}\right)^{+}\right]\mathrm{d}x,$$

by the generalized intermediate value theorem (c.f. de Clippel (2008), Lemma 2 and Lipnowski and Ravid (2019), Lemma 3.), since $\bar{z}_q(c)\in\operatorname{argmax}_{c'\in[0,c]}[1-\widetilde{\pi}(c)+\int_{c'}^{c}(1-q(z|c))\,\mathrm{d}z/(x-c')^{+}]^{+}$ by construction, there must be some $x^{*}\in[\underline{x}_q(c),\bar{x}_q(c)]$ and some

$$c_q(x^{*})\in\underset{c'\in[0,c]}{\operatorname{argmax}}\left[1-\frac{\widetilde{\pi}(c)+\int_{c'}^{c}(1-q(z|c))\,\mathrm{d}z}{(x^{*}-c')^{+}}\right]^{+}$$

such that

$$\int_{0}^{\hat{x}(x^{*})}\left[H^{c}(x)-\left(1-\frac{\widetilde{\pi}(c)+\int_{c_q(x^{*})}^{c}(1-q_x(z|c))\,\mathrm{d}z}{(x-c_q(x^{*}))^{+}}\right)^{+}\right]\mathrm{d}x$$
$$=\int_{\bar{x}_q(c)}^{\widetilde{v}(c)}\left[F(\widetilde{\psi}(c))-\left(1-\frac{\widetilde{\pi}(c)+\int_{\bar{z}_{\hat{q}}(c)}^{c}(1-q_x(z|c))\,\mathrm{d}z}{(x-\bar{z}_q(c))^{+}}\right)^{+}\right]\mathrm{d}x.$$

We then have

$$\int_{0}^{\widetilde{\psi}(c)}\Gamma^{c}_{q_{x^{*}},\widetilde{\psi}}(x)\,\mathrm{d}x=\int_{0}^{\widetilde{\psi}(c)}H^{c}(x)\,\mathrm{d}x=\int_{0}^{\widetilde{\psi}(c)}F(x)\,\mathrm{d}x.$$

Therefore, by defining $\hat{q}(\cdot|c)\equiv q_{x^{*}}(\cdot|c)$, we then have the desired $\hat{q}$.

Finally, if

$$\int_{0}^{\hat{x}(\bar{x}_q(c))}\left[H^{c}(x)-\left(1-\frac{\widetilde{\pi}(c)+\int_{\bar{z}_q(c)}^{c}(1-q_x(z|c))\,\mathrm{d}z}{(x-\bar{z}_q(c))^{+}}\right)^{+}\right]\mathrm{d}x$$
$$>\int_{\bar{x}_q(c)}^{\widetilde{v}(c)}\left[F(\widetilde{\psi}(c))-\left(1-\frac{\widetilde{\pi}(c)+\int_{\bar{z}_{\hat{q}}(c)}^{c}(1-q_x(z|c))\,\mathrm{d}z}{(x-\bar{z}_q(c))^{+}}\right)^{+}\right]\mathrm{d}x,$$

then there exists $x^{*}\in[0,\bar{x}_q(c)]$ such that

$$\int_{0}^{x^{*}}\max\left\{q_{\bar{x}_q(c)},\left(1-\frac{\widetilde{\pi}(c)+\int_{\bar{z}_q(c)}^{c}(1-q(z|c))\,\mathrm{d}z}{(x-\bar{z}_q(c))^{+}}\right)^{+}\right\}\mathrm{d}x+(\widetilde{v}(c)-x^{*})F(\widetilde{\psi}(c))=\int_{0}^{\widetilde{\psi}(c)}H^{c}(x)\,\mathrm{d}x.$$

[18]i.e. Is nonempty, convex valued and upper-hemicontinuous.

Then, by defining

$$\hat{q}(z|c) := \begin{cases} q_{\bar{x}_q}, & \text{if } z \in [0, \bar{z}_q(c)] \\ F(\widetilde{\psi}(c)), & \text{if } z \in (\bar{z}_q(c), c] \end{cases},$$

we then have

$$\int_0^{\widetilde{\psi}(c)} \Gamma^c_{\hat{q},\widetilde{\psi}}(x)\,\mathrm{d}x = \int_0^{\widetilde{\psi}(c)} H^c(x)\,\mathrm{d}x = \int_0^{\widetilde{\psi}(c)} F(x)\,\mathrm{d}x,$$

as desired.

Finally, if, on the other hand,

$$\int_0^c [q(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z = 0,$$

then it must be that $q(0|c) = 0$ since by Lemma 6,

$$\int_{c'}^c [q(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z \geq 0$$

and $F(\widetilde{\psi}(0)) = 0$. Thus, by selecting a proper $\tilde{q} \in (0, q(\underline{z}_q(c)^+|c))$ and letting $\tilde{q}(z|c) := \tilde{q}$ for all $z \in [0, \underline{z}_q c]$ and $\tilde{q}(z|c) := q(z|c)$ otherwise, we will have

$$\tilde{H}^c(x) \geq \max_{c' \in [0,c_\alpha]} \left[1 - \frac{\widetilde{\pi}(c) + \int_{c'}^c (1 - \tilde{q}(z|c))\,\mathrm{d}z}{(x - c')^+}\right]^+,$$

$$\int_0^{\widetilde{\psi}(c)} \tilde{H}^c(x)\,\mathrm{d}x = \int_0^{\widetilde{\psi}(c)} H^c(x)\,\mathrm{d}x = \int_0^{\widetilde{\psi}(c)} F(x)\,\mathrm{d}x$$

and

$$\int_0^c [\hat{q}(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z > 0,$$

where

$$\tilde{H}^c(x) := \begin{cases} \tilde{q}, & \text{if } x \in [0, \tilde{x}) \\ H^c(x), & \text{if } x \in [\tilde{x}, \widetilde{\psi}(c)] \end{cases},$$

and

$$\tilde{x} := \frac{1}{1 - \tilde{q}} \left[\widetilde{\pi}(c) + \int_{\underline{z}_q(c)}^c (1 - \tilde{q}(z|c))\,\mathrm{d}z\right].$$

Since the previous arguments hinge only on the property that

$$\int_0^{\widetilde{\psi}(c)} H^c(x)\,\mathrm{d}x = \int_0^{\widetilde{\psi}(c)} F(x)\,\mathrm{d}x$$

and that $H^c(x) \geq \max_{c' \in [0,c_\alpha]}[1 - (\widetilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))\,\mathrm{d}z)/(x - c')^+]^+$ for all $x \in [0, \widetilde{\psi}(c)]$, by replacing H^c with $\tilde{H}^c$, q with $\tilde{q}$ and repeating the procedures, the desired $\hat{q}$ can then be found. ■

Step 4: I now show that for any fixed $c_\alpha \in [0, \bar{c}]$, the menu induced by the family of information structure $\{H^c_{\text{gu}}\}_{c \in [0,c_\alpha]} \subset \mathcal{H}_F$ and the publicizing policy $\mathbf{1}_{[0,c_\alpha]}$, with H^c_{gu} being an obedient garbled upper censorship with cutoff $\psi^*(c)$, is incentive compatible and individually rational and maximized the intermediary's revenue among all incentive compatible, individually rational and obedient garbled upper censorship menu with the same publicizing policy. To this end, first notice that when $c_\alpha \in [0, c^*]$, since $\psi^*(c) = \psi(c)$, the proof of Theorem 1 then ensures the desired property. Below I discuss the case when $c_\alpha \in (c^*, \bar{c}]$

Using the characterizations above, a dual for the original problem with a fixed c_α can be formed. Fix any Borel measures μ, ν on the measurable space $[0, c_\alpha]$ (endowed with the Borel algebra). Let

$$\begin{aligned} D(\mu,\nu) :=& \sup_{\widetilde{\psi},q} \Bigg[\int_0^{c_\alpha} (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))G(\mathrm{d}c) - \int_0^{c_\alpha} \left(\int_0^{\bar{v}} (1 - \Gamma^c_{q,\widetilde{\psi}}(x)) - (1 - F(x))\,\mathrm{d}x \right) \mu(\mathrm{d}c) \\ & - \int_0^{c_\alpha} \left(\int_0^c [q(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z \right) \nu(\mathrm{d}c) \Bigg], \end{aligned} \tag{25}$$

where the supremum is taken over all nondecreasing function $\widetilde{\psi} : [0, c_\alpha] \to [0, \bar{v}]$ and all $q : [0, c_\alpha]^2 \to [0, 1]$ such that $q(\cdot|c)$ is nondecreasing and $q(c|c) = F(\widetilde{\psi}(c))$, for all $c \in [0, c_\alpha]$.

Notice that for any incentive compatible, individually rational and obedient garbled upper censorship menu $(H^c, t(c), \mathbf{1}_{[0,c_\alpha]}(c))_{c \in [0,\bar{c}]}$ with cutoff function $\widetilde{\psi}$, the expected revenue for the intermediary is

$$R(\widetilde{\psi}) := \int_0^{c_\alpha} (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))G(\mathrm{d}c).$$

By Lemma 6 and Lemma 7, for any such menu, there exists $q_{\widetilde{\psi},H} : [0, c_\alpha]^2 \to [0, c_\alpha]$ such that $q_{\widetilde{\psi},H}(\cdot|c)$ is nondecreasing, $q_{\widetilde{\psi},H}(c|c) = F(\widetilde{\psi}(c))$,

$$\begin{aligned} &\int_0^{\bar{v}} [(1 - \Gamma^c_{q_{\widetilde{\psi},H},\widetilde{\psi}}(x)) - (1 - F(x))] = 0, \\ &\int_0^c [q_{\widetilde{\psi},H}(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z = 0, \end{aligned}$$

for all $c \in [0, c_\alpha]$.

As such, for any such menu, for any Borel measures μ, ν on $[0, c_\alpha]$,

$$\begin{aligned} R(\widetilde{\psi}) =& \int_0^{c_\alpha} (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))G(\mathrm{d}c). \\ =& \int_0^{c_\alpha} (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))G(\mathrm{d}c) - \int_0^{c_\alpha} \left(\int_0^{\bar{v}} (1 - \Gamma^c_{q_{\widetilde{\psi},H},\widetilde{\psi}}(x)) - (1 - F(x))\,\mathrm{d}x \right) \mu(\mathrm{d}c) \\ & - \int_0^{c_\alpha} \left(\int_0^c [q_{\widetilde{\psi},H}(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z \right) \nu(\mathrm{d}c) \\ \leq& \sup_{\widetilde{\psi},q} \Bigg[\int_0^{c_\alpha} (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))G(\mathrm{d}c) - \int_0^{c_\alpha} \left(\int_0^{\bar{v}} (1 - \Gamma^c_{q,\widetilde{\psi}}(x)) - (1 - F(x))\,\mathrm{d}x \right) \mu(\mathrm{d}c) \\ & - \int_0^{c_\alpha} \left(\int_0^c [q(z|c) - F(\widetilde{\psi}(z))]\,\mathrm{d}z \right) \nu(\mathrm{d}c) \Bigg] \\ =& D(\mu,\nu), \end{aligned}$$

and therefore, if R^* is the supremum of the expected revenue among all possible incentive compatible, individually rational and obedient garbled upper censorship menus with publicizing policy

$\mathbf{1}_{[0,c_\alpha]}$, we have

$$R^* \le D(\mu, \nu).$$

It then suffices to show that there exists Borel measures μ^*, ν^* and a function $q^* : [0, c_\alpha]^2 \to [0,1]$ with $q^*(\cdot|c)$ being nondecreasing and $q^*(c|c) = F(\psi^*(c))$ such that (ψ^*, q^*) solves the dual problem (25), that

$$\int_0^{c_\alpha} \left(\int_0^{\bar{v}} [(1 - \Gamma^c_{q^*,\psi^*}(x)) - (1 - F(x)) \, \mathrm{d}x \right) \mu^*(\mathrm{d}c) = 0,$$
$$\int_0^{c_\alpha} \left(\int_0^{c} [q^*(z|c) - F(\psi^*(z))] \, \mathrm{d}z \right) \nu^*(\mathrm{d}c) = 0$$

for all $c \in [0, c_\alpha]$ and that there exists an incentive compatible, individually rational and obedient garbled upper censorship menu with cutoff function ψ^* and publicizing policy $\mathbf{1}_{[0,c_\alpha]}$, as this would imply:

$$R^* \le D(\mu^*, \nu^*) = R(\psi^*) \le R^*.$$

Together, we will then have

$$R^* = R(\psi^*),$$

as desired.

Indeed, for any Borel measures μ, ν on $[0, c_\alpha]$, (25) can be written as:

$$\sup_{\widetilde{\psi}} \Bigg[\int_0^{c_\alpha} (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))G(\mathrm{d}c)$$
$$+ \sup_q \left(\int_0^{c_\alpha} \left(\int_0^{\bar{v}} [\Gamma^c_{q,\widetilde{\psi}}(x) - F(x)] \, \mathrm{d}x \right) \mu(\mathrm{d}c) - \int_0^{c_\alpha} \left(\int_0^{c} [q(z|c) - F(\widetilde{\psi}(z))] \, \mathrm{d}z \right) \nu(\mathrm{d}c) \right) \Bigg].$$

Notice that for any fixed $\widetilde{\psi}$, the functional

$$q \mapsto \Gamma^c_{q,\widetilde{\psi}}$$

is convex, as it is essentially a pointwise supremum of a family of affine functionals. Also, since for each $c \in [0, c_\alpha]$, the collection of nondecreasing functions $q(\cdot|c) : [0, c] \to [0, \widetilde{\psi}(c)]$, $\mathcal{Q}(c)$, is convex, for each $c \in [0, c_\alpha]$, and any fixed nondecreasing function $\widetilde{\psi} : [0, c_\alpha] \to [0, \bar{v}]$, the maximization problem

$$\max_{q(\cdot|c) \in \mathcal{Q}(c)} \int_0^{\bar{v}} [\Gamma^c_{q,\widetilde{\psi}}(x) - F(\widetilde{\psi})] \, \mathrm{d}x - \int_0^{c} [q(z|c) - F(\widetilde{\psi}(z))] \, \mathrm{d}z$$

has a solution and one of the extreme points of $\mathcal{Q}(c)$, which take form of $q(z|c) \in \{0, F(\widetilde{\psi}(c))\}$ for all $z \in [0, c]$, attains the maximum. Therefore, (25) can be reduced to choosing cutoff points of the extreme points of $\mathcal{Q}(c)$, denoted as $k(c)$, instead of choosing among all nondecreasing functions for each $c \in [0, c_\alpha]$. That is:

$$
\begin{aligned}
D(\mu,\nu) = & \sup_{\widetilde{\psi}} \Bigg[\int_0^{c_\alpha} (\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))G(\mathrm{d}c) \\
& + \sup_{k:[0,c_\alpha]\to[0,c_\alpha],\, k(c)\le c} \Bigg(\int_0^{c_\alpha} \left(\int_0^{\bar{v}} [\Gamma^c_{F(\widetilde{\psi}(c))\mathbf{1}\{z\ge k(c)\},\widetilde{\psi}}(x) - F(x)]\,\mathrm{d}x \right) \mu(\mathrm{d}c) \\
& - \int_0^{c_\alpha} \left(\int_0^c [F(\widetilde{\psi}(c))\mathbf{1}\{z \ge k(c)\} - F(\widetilde{\psi}(z))]\,\mathrm{d}z \right) \nu(\mathrm{d}c) \Bigg) \Bigg].
\end{aligned}
$$

Notice that by definition of $\Gamma^c_{F(\widetilde{\psi}(c))\mathbf{1}\{z\ge k(c)\},\widetilde{\psi}}$, fix any $\widetilde{\psi}$ and k,

$$
\int_0^{\bar{v}} [\Gamma^c_{F(\widetilde{\psi}(c))\mathbf{1}\{z\ge k(c)\},\widetilde{\psi}}(x) - F(x)]\,\mathrm{d}x \ge \int_0^{P(\widetilde{\psi}(c),k(c))} [\gamma(x,\widetilde{\psi}(c),k(c)) - F(x)]\,\mathrm{d}x,
$$

where

$$
\gamma(x,\widetilde{\psi}(c),k(c)) := \left(1 - \frac{(\widetilde{v}(c) - k(c))(1 - F(\widetilde{\psi}(c)))}{(x - k(c))^+} \right)^+, \forall x \in [0, \widetilde{\psi}(c)]
$$

and

$$
P(\widetilde{\psi}(c),k(c)) := \begin{cases} \max \Phi^{-1}_{k(c)}((\widetilde{v}(c) - k(c))(1 - F(\widetilde{\psi}(c)))), & \text{if } \Phi^{-1}_{k(c)}((\widetilde{v}(c) - k(c))(1 - F(\widetilde{\psi}(c)))) \neq \emptyset \\ 0, & \text{if } \Phi^{-1}_{k(c)}((\widetilde{v}(c) - k(c))(1 - F(\widetilde{\psi}(c)))) = \emptyset \end{cases},
$$

with $\Phi_z(x) := (x - z)(1 - F(x))$ being the profit function of a seller with a cost $z \in [0, c_\alpha]$.

Now fix Borel measures μ, ν on $[0, c_\alpha]$ that are absolutely continuous with respect to the Lebesgue measure with densities m, n, respectively, and consider an auxiliary problem:

$$
\begin{aligned}
D'(\mu,\nu) := & \sup_{\widetilde{\psi},k} \Bigg[\int_0^{c_\alpha} \Bigg((\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c)))g(c) \\
& - \left(\int_0^{P(\widetilde{\psi}(c),k(c))} [(1 - \gamma(x,\widetilde{\psi}(c),k(c))) - (1 - F(x))]\,\mathrm{d}x \right) m(c) \\
& - \left((c - k(c))F(\widetilde{\psi}(c)) - \int_0^c F(\widetilde{\psi}(z))\,\mathrm{d}z \right) n(c) \Bigg) \mathrm{d}c \Bigg].
\end{aligned} \tag{26}
$$

Notice that for any fixed $\widetilde{\psi}$, (26) is a (concave) pointwise maximization problem of choosing $k(c) \le c$, whereas for any fixed k, (26) is a (concave) variational problem.[19] As such, for ψ^*, k^* to be optimal under the Borel measures μ, ν, it is equivalent to that $k^*(c)$ solves the pointwise first order condition given ψ^*

$$
\begin{aligned}
& m(c) \int_0^{P(\psi^*(c),k^*(c))} \gamma_3(x,\psi^*(c),k^*(c))\,\mathrm{d}x = -F(\psi^*(c))n(c) \\
\iff & m(c) = -\frac{F(\psi^*(c))}{\int_0^{P(\psi^*(c),k^*(c))} \gamma_3(x,\psi^*(c),k^*(c))\,\mathrm{d}x} n(c),
\end{aligned} \tag{27}
$$

[19] This can be seen by observing that $(\widetilde{v}(c) - \psi(c))(1 - F(\widetilde{\psi}(c))) = \int_{\widetilde{\psi}(c)}^{\bar{v}} (1 - F(x))\,\mathrm{d}x - (\psi(c) - \widetilde{\psi}(c))(1 - F(\widetilde{\psi}(c)))$, $(\widetilde{v}(c) - k(c))(1 - F(\widetilde{\psi}(c))) = \int_{\widetilde{\psi}(c)}^{\bar{v}} (1 - F(x))\,\mathrm{d}x + (\widetilde{\psi}(c) - k(c))(1 - F(\widetilde{\psi}(c)))$ and by letting $\eta(c) := \frac{1}{F(\widetilde{\psi}(c))} \int_0^c F(\widetilde{\psi}(z))\,\mathrm{d}z$.

whenever $P(\psi^*(c), k^*(c)) > 0$ and that ψ^* solves the Euler-Largrange equation given k^*:

$$n(c) = \frac{d}{\mathrm{d}c}\left[-(\psi(c)-\psi^*(c))g(c) - m(c)\int_0^{P(\psi^*(c),k^*(c))} \gamma_2(x,\psi^*(c),k^*(c))\,\mathrm{d}x + (c-k^*(c))n(c)\right], \tag{28}$$

for (Lebesgue) almost all $c \in [0, c_\alpha]$.

Substituting (27) into (28) and integrating both sides with the fact that $\psi^*(0) = 0$, we then have:

$$N(c) := \int_0^c n(z)\,\mathrm{d}z = -(\psi(c)-\psi^*(c))g(c) + \Omega^*(c)n(c), \tag{29}$$

where

$$\Omega^*(c) := -\frac{\int_0^{P(\psi^*(c),k^*(c))} \gamma_2(x,\psi^*(c),k^*(c))\,\mathrm{d}x F(\psi^*(c))}{\int_0^{P(\psi^*(c),k^*(c))} \gamma_3(x,\psi^*(c),k^*(c))\,\mathrm{d}x f(\psi^*(c))} - (c-k^*(c))$$

CLAIM 1: Ω^* is strictly decreasing on $[c^*, c_\alpha]$ and $\Omega^*(c) \geq 0$ for all $c \in [c^*, c_\alpha]$.

Notice that (29) is a first order linear differential equation and therefore the initial value problem with $N(c^*) = 0$ has a unique solution

$$N^*(c) = \frac{1}{\zeta(c)}\left(\int_{c^*}^c \zeta(z)\frac{(\psi(z)-\psi(c^*))g(z)}{\Omega^*(z)}\,\mathrm{d}z\right),$$

where

$$\zeta(c) := \exp\left(\int_0^c -\frac{1}{\Omega^*(z)}\,\mathrm{d}z\right), \forall c \in [0, c_\alpha].$$

It is then clear that N^* is increasing since $N^*(c) \geq 0$ and hence $n^*(c) = (\psi(c)-\psi(c^*))g(c) + N^*(c)$ for all $c \in [c^*, c_\alpha]$. Therefore, N^* is indeed a CDF of a Borel measure with density n^*. As such, let

$$m^*(c) := -\frac{F(\psi^*(c))}{\int_0^{P(\psi^*(c),k^*(c))} \gamma_3(x,\psi^*(c),k^*(c))\,\mathrm{d}x} n^*(c), \forall c \in [0, c_\alpha],$$

and let μ^*, ν^* be the Borel measures induced by m^* and n^*. Notice that $\mathrm{supp}(\mu^*) = \mathrm{supp}(\nu^*) = [c^*, c_\alpha]$ and that ψ^*, k^* solves the auxiliary problem (26). Moreover, by construction, for any $c \in [c^*, c_\alpha]$,

$$\int_0^{\bar{v}} [\Gamma^c_{F(\psi^*(c))\mathbf{1}\{z\geq k(c)\},\psi^*}(x) - F(x)]\,\mathrm{d}x = 0 = \int_0^{P(\psi^*(c),k^*(c))} [\gamma(x,\psi^*(c),k(c)) - F(x)]\,\mathrm{d}x$$

and therefore,

$$D'(\mu^*,\nu^*) = D(\mu^*,\nu^*).$$

Furthermore, let $q^*(z|c) := F(\psi^*(c))\mathbf{1}\{z \geq k(c)\}$. Then (ψ^*, q^*) indeed solves the dual problem (25) and

$$\int_0^{c_\alpha}\left(\int_0^{\bar{v}} [(1-\Gamma^c_{q^*,\psi^*}(x)) - (1-F(x))]\,\mathrm{d}x\right)\mu^*(\mathrm{d}c) = 0,$$
$$\int_0^{c_\alpha}\left(\int_0^{c} [q^*(z|c) - F(\psi^*(z))]\,\mathrm{d}z\right)\nu^*(\mathrm{d}c) = 0$$

for all $c \in [0, c_\alpha]$.

Finally, for any $c \in [0, c_\alpha]$, any $x \in [0, \bar{v}]$, let

$$\widehat{H}^c(x) := \begin{cases} \left[1 - \frac{(v^*(c)-k^*(c))(1-F(\psi^*(\psi(c)))}{(x-k^*(c))^+}\right]^+, & \text{if } x \in [0, P(\psi^*(c), k^*(c))] \\ F(x), & \text{if } x \in (P(\psi^*(c), k^*(c)), \psi^*(c)] \\ F(\psi^*(c)), & \text{if } x \in (\psi^*(c), v^*(c)] \\ 1, & \text{if } x \in (v^*(c), \bar{v}] \end{cases},$$

where $v^*(c) := \mathbb{E}_F[v | v \geq \psi^*(c)]$. Now let

$$I^c_{gu}(x) := \text{conv}\Big(\min\{I_F(x), I_{\widehat{H}^c}(x)\} \Big).$$

By construction, for all $c \in [0, c_\alpha]$ I^c_{gu} is convex and thus its subdifferential, $\partial I^c_{gu}(x)$, is nonempty for all $x \in [0, \bar{v}]$. Finally, let

$$H^c_{gu}(x) := \sup \partial I^c_{gu}(x),\ \forall x \in [0, \bar{v}], c \in [0, c_\alpha].$$

It is straightforward to verify that $H^c_{gu} \in \mathcal{H}_F$ for all $c \in [0, c_\alpha]$ and that $\{H^c_{gu}\}_{c \in [0, c_\alpha]}$ satisfies the sufficient conditions 1 and 2 in Lemma 2. Then by Lemma 6 and Lemma 2, there exists a transfer $t_{\text{gu}} : [0, c_\alpha] \to \mathbb{R}$ such that $(H^c_{gu}, t_{\text{gu}}(c), \mathbf{1}_{[0,c_\alpha]}(c))_{c \in [0, c_\alpha]}$ is indeed an incentive compatible, individually rational and obedient garbled upper censorship menu with cutoff function ψ^*, as desired.

Step 5: Finally, it remains to find c_α that maximizes

$$\int_0^{c_\alpha} (v^*(c) - \psi(c))(1 - F(\psi^*(c)))G(\mathrm{d}c).$$

By definition of $\hat{c}$ and by monotonocity of ψ, and thus of ψ^*,

$$(v^*(c) - \psi(c))(1 - F(\psi^*(c))) \geq 0 \iff c \leq \hat{c}.$$

Since G has full support, the function

$$c_\alpha \to \int_0^{c_\alpha} (v^*(c) - \psi(c))(1 - F(\psi^*(c)))G(\mathrm{d}c)$$

is indeed maximized at $\hat{c}$.

To see that the intermediary's optimal revenue is R^* if and only if $c^* \geq \bar{c}$, notice that under the solution $(H^c_{gu}, t_{\text{gu}}(c), \alpha_{\text{gu}}(c))_{c \in [0, c_\alpha]}$ the intermediary's revenue is

$$\int_0^{\hat{c}} (v^*(c) - \psi(c))(1 - F(\psi^*(c)))G(\mathrm{d}c).$$

Now suppose that $c^* \geq \bar{c}$. Then $\psi^* \equiv \psi$, $v^* \equiv v$ and $\hat{c} = \bar{c}$. As such, optimal revenue is

$$\int_0^{\bar{c}} (v(c) - \psi(c))(1 - F(\psi(c)))G(\mathrm{d}c) = \int_0^{\bar{c}} \left(\int_{\psi(c)}^{\infty} (1 - F(z))\,\mathrm{d}z \right) G(\mathrm{d}c) = R^*$$

Conversely, suppose that $c^* < \bar{c}$. By definition of ψ^*, $\psi^*(c) = \psi(c)$, $v^*(c) = v(c)$ for all $c \in [0, c^*]$. On the other hand, since ψ is increasing, for any $c \in (c^*, \bar{c}]$, $\psi^*(c) = \psi(c^*) < \psi(c)$. As such, for any $c \in (c^*, \bar{c}]$,

$$(v^*(c) - \psi(c))(1 - F(\psi^*(c))) = (v(c^*) - \psi(c))(1 - F(\psi(c^*))) < (v(c) - \psi(c))(1 - F(\psi(c))).$$

Moreover, since G has full support,

$$\int_0^{\bar{c}} (v^*(c) - \psi(c))(1 - F(\psi^*(c)))G(\mathrm{d}c) < \int_0^{\bar{c}} (v(c) - \psi(c))(1 - F(\psi(c)))G(\mathrm{d}c) = R^*.$$

This completes the proof.

■

B4 Proof of Proposition 4

Proof of Proposition 4. As in the proof of Proposition 1 and Proposition 2, I first find upper bounds for the objective in (4) and (5). Under the direct selling model and (C), for any $A \in \mathcal{B}([\underline{v}, \bar{v}])$, $H \in \mathcal{H}_A$ and any $p \in [\underline{v}, \bar{v}]$,

$$\begin{aligned}
&(p - \psi(c))(1 - H(p)) \\
\leq &(p - \psi(c))(1 - H(p)) + \int_p^{\bar{v}} (1 - H(z))\,\mathrm{d}z \\
\leq &\int_{\psi(c)}^{\infty} (1 - H(z))\,\mathrm{d}z \\
\leq &\int_{\psi(c)}^{\infty} (1 - F^A(z))\,\mathrm{d}z \\
\leq &\int_{\psi(c)}^{\infty} (1 - F(z))\,\mathrm{d}z.
\end{aligned} \tag{30}$$

On the other hand, under the weak contracting model, for any for any $A \in \mathcal{B}([\underline{v}, \bar{v}])$, $H \in \mathcal{H}_A$ and any $p(c, c) \in \mathrm{argmax}_{p \in [\underline{v}, \bar{v}]}(p - c)(1 - H(p))$,

$$\begin{aligned}
&\max_{p \in [\underline{v}, \bar{v}]} (p - c)(1 - H(p)) - \frac{G(c)}{g(c)}(1 - H(p(c, c))) \\
&\leq \max_{x \in [\underline{v}, \bar{v}} (x - \psi(c))(1 - H(x)) \\
&\leq \max_{x \in [\underline{v}, \bar{v}} (x - \psi(c))(1 - H(x)) + \int_{p(c,c)}^{\infty} (1 - H(z))\,\mathrm{d}z \\
&\leq \int_{\psi(c)}^{\infty} (1 - H(z))\,\mathrm{d}z \\
&\leq \int_{\psi(c)}^{\infty} (1 - F^A(z))\,\mathrm{d}z \\
&\leq \int_{\psi(c)}^{\infty} (1 - F(z))\,\mathrm{d}z.
\end{aligned} \tag{31}$$

Therefore, under all both models (D), (C) and the weak contracting model, R^* is still an upper bound of the intermediary's revenue. It it then clear that the contract $(p_T(c), A_T, H_T^c, t_T(c))_{c\in[0,\bar{c}]}$ attains R^* and is feasible under the direct selling model and (C). Furthermore, under the weak contracting model and the menu $(A_T^c, H_T^c, t_T(c))_{c\in[0,\bar{c}]}$, any seller whose report is c will be willing to set the price at $v(c)$ and thus, by the envelope characterization of incentive compatibility, $(A_T^c, H_T^c, t_T(c))_{c\in[0,\bar{c}]}$ is incentive compatible. Together with the fact that $(A_T^c, H^c, t_T(c))_{c\in[0,\bar{c}]}$ attains R^*, this implies that optimal revenue for the intermediary under all three models are R^*.

For buyer's surplus and seller's profit, notice that under models (D) and (C) and any optimal mechanism, the weak inequalities in (30) must hold with equality and thus, as argued in the proof of Theorem 1, it must be that buyer's expected surplus is zero and the seller's expected profit is

$$\int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F^{A^z}(\psi(z)))\, \mathrm{d}z \right) G(\mathrm{d}c) = \int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F(\psi(z)))\, \mathrm{d}z \right) G(\mathrm{d}c).$$

On the other hand, under the weak contracting model, for any optimal menu, all the weak inequalities in (31) must hold with equality and thus, as in the proof of Theorem 1, the buyer's expected surplus is zero and the seller's profit is

$$\int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F^{A^z}(\psi(z)))\, \mathrm{d}z \right) G(\mathrm{d}c) = \int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F(\psi(z)))\, \mathrm{d}z \right) G(\mathrm{d}c).$$

This completes the proof. ■

C Outcome Equivalence

C1 Proof of Theorem 1

Proof of Theorem 1. Equivalence in the intermediary's revenue is a direct consequence of Proposition 1, Proposition 2 and Proposition 7.

For buyer's surplus and seller's profit. First notice under the direct selling model and (C), as optimal revenue is R^*, which is attained by a pointwise maximum of the objective of (9). Thus, for any other contract $(p(c), H^c, t(c))_{c\in[0,\bar{c}]}$ that attains revenue R^*, by (11), all the weak inequalities must hold with equality. In particular, for G-almost all $c \in [0, \bar{c}]$

$$\int_{p(c)}^{\bar{v}} (1 - H^c(z))\, \mathrm{d}z = 0$$

and thus the buyer's expected surplus must be zero under any optimal mechanism. On the other hand, also from (11), for G-almost all $c \in [0, \bar{c}]$, it must be that $H^c(z) = H^c(p(c))$ for all $z \in [\psi(c), p(c))$ and that

$$\int_{\psi(c)}^{\infty} (1 - H^c(z))\, \mathrm{d}z = \int_{\psi(c)}^{\infty} (1 - F(z))\, \mathrm{d}z.$$

Together with $H^c \in \mathcal{H}_F$ for all $c \in [0, \bar{c}]$, this implies that for G-almost all $c \in [0, \bar{c}]$, $H^c(p(c)) = F(\psi(c))$. Therefore, together with Lemma 1, the seller's profit must be

$$\int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F(\psi(z)))\, \mathrm{d}z \right) G(\mathrm{d}c)$$

under any optimal mechanism. Contrarily, under the weak contracting model, when $\phi(\psi(c)) \leq c$ for all $c \in [0, \bar{c}]$ or when $c^* \geq \bar{c}$, Proposition 2 and Proposition 7 shows that optimal revenue is R^*. Therefore, if a menu $(\alpha(c), H^c, t(c))_{c\in[0,\bar{c}]}$ is optimal, then all the weak inequalities in (12) must be equalities. In particular, it must be that $\alpha \equiv 1$,

$$\int_{p(c,c)}^{\infty} (1 - H^c(z))\,\mathrm{d}z = 0,$$

$$H^c(z) = H^c(\psi(c)), \forall z \in [\psi(c), p(c,c)),$$

and

$$\int_{\psi(c)}^{\infty} (1 - H^c(z))\,\mathrm{d}z = \int_{\psi(c)}^{\infty} (1 - F(z))\,\mathrm{d}z,$$

for G-almost all $c \in [0, \bar{c}]$, where $p(c,c)$ is the largest element of $\mathrm{argmax}_{p\in[\underline{v},\bar{v}]}(p - c)(1 - H^c(p))$. Together with $H^c \in \mathcal{H}_F$ for all $c \in [0, \bar{c}]$, we have

$$H^c(\psi(c)) = H^c(p(c,c)) = F(\psi(c)),$$

for G-almost all $c \in [0, \bar{c}]$, which in turns implies that

$$p(c,c) = \mathbb{E}_F[v|v > \psi(c)],$$

for G-almost all $c \in [0, \bar{c}]$. Together, the buyer's expected surplus under any optimal menu is zero, and the seller's expected profit under and optimal menu is

$$\int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F(\psi(z)))\,\mathrm{d}z \right) G(\mathrm{d}c),$$

the same as in the direct selling model and (C). This completes the proof. ■

C2 Proof of Theorem 3

Proof of Theorem 3. In the case in which the intermediary cannot target, I first show that $R(\overline{\mathcal{P}}, \overline{\mathcal{R}}) = R^*$. Take any feasible mechanism in (6), let

$$V(c', c'', c) := (p(c''|c') - c)(1 - H^{c'}(p(c''|c'))) - r(c''|c').$$

Incentive compatibility implies that

$$V^*(c) := V(c, c, c) = \max_{c', c'' \in [0, \bar{c}]} V(c', c'', c).$$

Since the function $V(\cdot, \cdot, c)$ is absolutely continuous and its derivative is uniformly bounded, by the Envelope theorem, we have:

$$V^*(c) = V^*(\bar{c}) + \int_c^{\bar{c}} (1 - H^z(p(z|z)))\,\mathrm{d}z$$

for all $c \in [0, \bar{c}]$. As such,

$$\mathbb{E}_G[r(c|c)] = -V^*(\bar{c}) + \int_0^{\bar{c}} (p(c|c) - \psi(c))(1 - H^c(p(c|c)))G(\mathrm{d}c).$$

By individual rationality and by (11), we still have that $\mathbb{E}_G[r(c|c)] \leq R^*$.

On the other hand, notice that the solution given in the proof of Proposition 1 is feasible under the any business model identified by $(\overline{\mathcal{P}}, \overline{\mathcal{R}})$ and it always gives revenue R^*. As such, we have $R(\overline{\mathcal{P}}, \overline{\mathcal{R}}) = R^*$.

Finally, since the weak contracting model is identified by $(\underline{\mathcal{P}}, \underline{\mathcal{R}})$, Proposition 7 implies that $R(\underline{\mathcal{P}}, \underline{\mathcal{R}}) = R^*$ if and only if $c^* \geq \bar{c}$. Together with Lemma 3, for any $(\mathcal{P}, \mathcal{R})$,

$$R^* = R(\underline{\mathcal{P}}, \underline{\mathcal{R}}) \leq R(\mathcal{P}, \mathcal{R}) \leq R(\overline{\mathcal{P}}, \overline{\mathcal{R}}) = R^*,$$

if and only if $c^* \geq \bar{c}$. This establishes the revenue equivalence.

For the seller's profit and the buyer's surplus, take any $(\mathcal{P}, \mathcal{R})$, as the intermediary's revenue given by any mechanism under any business model identified by $(\mathcal{P}, \mathcal{R})$ is

$$-V^*(\bar{c}) + \int_0^{\bar{c}} (p(c|c) - \psi(c))(1 - H^c(p(c|c)))G(\mathrm{d}c).$$

Together with (11), for any solution $(\alpha(c), H^c, p(\cdot|c), r(\cdot|c))_{c \in [0,\bar{c}]}$ of (6) under any business model identified by $(\mathcal{P}, \mathcal{R})$ that attains the optimal revenue $R(\mathcal{P}, \mathcal{R}) = R^*$, it must be that $V^*(\bar{c}) = 0$ and

$$(p(c|c)) - \psi(c))(1 - H^c(p(c|c))) = \int_{\psi(c)}^{\infty} (1 - F(z))\, \mathrm{d}z.$$

for G-almost all $c \in [0, \bar{c}]$, which in turn means that for G-almost all $c \in [0, \bar{c}]$, $p(c|c) > \psi(c)$, $H^c(z) = H^c(p(c|c))$ for all $z \in [\psi(c), p(c|c))$,

$$\int_{p(c|c)}^{\bar{v}} (1 - H^c(z))\, \mathrm{d}z = 0$$

and

$$\int_{\psi(c)}^{\bar{c}} (1 - H^z(z))\, \mathrm{d}z = \int_{\psi(c)}^{\infty} (1 - F(z))\, \mathrm{d}z.$$

As such, the buyer's expected surplus is

$$\int_0^{\bar{c}} \left(\int_{p(c|c)}^{\bar{v}} (1 - H^c(z)) \right) G(\mathrm{d}c) = 0$$

under any solution of (6) if and only if $c^* \geq \bar{c}$.

Furthermore, as $H^c \in \mathcal{H}_F$ for all $c \in [0, \bar{c}]$, $H^c(z) = H^c(p(c|c))$ and

$$\int_{\psi(c)}^{\bar{c}} (1 - H^z(z))\, \mathrm{d}z = \int_{\psi(c)}^{\infty} (1 - F(z))\, \mathrm{d}z$$

imply that $H^c(p(c|c)) = F(\psi(c))$ for G-almost all $c \in [0, \bar{c}]$. Hence, the seller's profit must be

$$\int_0^{\bar{c}} V^*(c)G(\mathrm{d}c) = \int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - H^z(p(z|z))\, \mathrm{d}z \right) G(\mathrm{d}c) = \int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F(\psi(z))\, \mathrm{d}z \right) G(\mathrm{d}c)$$

under any solution of (6) if and only if $c^* \geq \bar{c}$.

In the case when targeting is available, Proposition 4 shows that both the weak contracting model and the full contracting model always yield optimal revenue R^*. Outcome equivalence then follows from arguments exactly as above. This completes the proof. ∎

D Interpretations and Comparative Statics

D1 Proof Proposition 3

Proof of Proposition 3. Fix $v_0 \geq 0$, regular F and G. Recall that $c^*(v_0)$ is defined as:

$$c^*(v_0) := \sup\left\{c \in [0,\bar{c}] \middle| \int_0^{P_{v_0}(\psi(c),k_{v_0}(c))} \left(1 - \frac{\pi(k_{v_0}(c))}{(x-k_{v_0}(c))^+}\right)^+ \mathrm{d}x \leq \int_0^{P_{v_0}(\psi(c),k_{v_0}(c))} F_{v_0}(x)\,\mathrm{d}x\right\}$$

, where

$$\begin{aligned}
k_{v_0}(c) &:= c - \frac{1}{F_{v_0}(\psi(c))}\int_0^c F_{v_0}(\psi(z))\,\mathrm{d}z \\
\pi(k_{v_0}(c)) &:= (v_{v_0}(c) - k_{v_0}(c))(1 - F_{v_0}(\psi(c))) \\
v_{v_0}(c) &:= \mathbb{E}_{F_{v_0}}[v|v > \psi(c)],
\end{aligned}$$

for all $c \in [0,\bar{c}]$. Moreover, $P_{v_0}(\psi(c), k_{v_0}(c))$ is the largest solution of

$$\int_0^P \left(1 - \frac{\pi(k_{v_0}(c))}{(x-k_{v_0}(c))^+}\right)^+ \mathrm{d}x = \int_0^P F_{v_0}(x)\,\mathrm{d}x.$$

Let $\hat{v}(v_0) := \psi^{-1}|_{[v_0,\bar{v}+v_0]}(c^*(v_0))$, we then have

$$c^*(v_0) \geq \bar{c} \iff \psi(\bar{c}) \leq \hat{v}(v_0).$$

Furthermore, let

$$\gamma(x,\psi(c),k_{v_0}(c)) := \left(1 - \frac{(v_{v_0}(c) - \psi(c))(1 - F_{v_0}(\psi(c)))}{(x-k_{v_0}(c))^+}\right)^+,$$

for all $x \in [v_0, \bar{v}+v_0]$. Notice that since F_{v_0} is in a location family of F for all $v_0 > 0$,

$$\int_0^{P_{v_0}(\psi(c),k_{v_0}(c))} (\gamma(x,\psi(c),k_{v_0}(c)) - F_{v_0}(x))\,\mathrm{d}x = \int_0^{P_0(\psi(c)-v_0,k_{v_0}(c))} (\gamma(x,\psi(c)-v_0,k_{v_0}(c)) - F(x))\,\mathrm{d}x.$$

As such, for any $v_0' > v_0 \geq 0$ and for any $c \in [0,\bar{c}]$, if $k_{v_0'}(c) \geq k_{v_0}(c)$, then as γ is decreasing in its third argument and increasing in its second argument.

$$\int_0^{P_0(\psi(c)-v_0',k_{v_0'}(c))} (\gamma(x,\psi(c)-v_0',k_{v_0'}(c)) - F(x))\,\mathrm{d}x < \int_0^{P_0(\psi(c)-v_0,k_{v_0}(c))} (\gamma(x,\psi(c)-v_0,k_{v_0}(c)) - F(x))\,\mathrm{d}x.$$

On the other hand, if $k_{v_0'}(c) < k_{v_0}(c)$, then since k_{v_0} and ψ is increasing in c, there exists $c^1, c^2 < c$ such that $\psi(c^1) - v_0 = \psi(c) - v_0'$ and $k_{v_0}(c^2) = k_{v_0'}(c)$. Let $\tilde{c} := \max\{c^1, c^2\}$. Then, since

$$c \mapsto \int_0^{P_{v_0}(\psi(c),k_{v_0}(c))} (\gamma(x,\psi(c),k_{v_0}(c)) - F(x))\,\mathrm{d}x$$

is increasing for all $v_0 \geq 0$, we have:

$$\begin{aligned}
&\int_0^{P_0(\psi(c)-v_0', k_{v_0'}(c))} (\gamma(x, \psi(c) - v_0';, k_{v_0'}(c)) - F(x))\,\mathrm{d}x \\
\leq &\int_0^{P_0(\psi(\tilde{c})-v_0, k_{v_0}(\tilde{c}))} (\gamma(x, \psi(\tilde{c}) - v_0, k_{v_0}(\tilde{c})) - F(x))\,\mathrm{d}x \\
< &\int_0^{P_0(\psi(c)-v_0, k_{v_0}(c))} (\gamma(x, \psi(c) - v_0, k_{v_0}(c)) - F(x))\,\mathrm{d}x.
\end{aligned}$$

Together, the function

$$v_0 \mapsto \int_0^{P_{v_0}(\psi(c), k_{v_0}(c))} (\gamma(x, \psi(c), k_{v_0}(c)) - F_{v_0}(x))\,\mathrm{d}x$$

is decreasing in v_0. Therefore, by definition of $c^*(v_0)$ and monotonicity of ψ^{-1}, $\hat{v}(v_0)$ is increasing v_0. ■

D2 Proof of Proposition 5

Proof of Proposition 5. Notice that for each $c \in [0, \bar{c}]$, probability of efficient trade is the probability of the event that trade occurs when the buyer's value is greater than the seller's cost. Since $\psi(c) > c$ for all $c \in [0, \bar{c}]$,

$$\int_0^{\bar{c}} (1 - F(c))G(\mathrm{d}c) > \int_0^{\bar{c}} (1 - F(\psi(c)))G(\mathrm{d}c),$$

which implies that the probability of efficient trade is larger when the seller has control of the information technology.

On the other hand, since ψ is increasing and $\psi(c) > c$ for all $c \in [0, \bar{c}]$,

$$\begin{aligned}
&\int_{\psi(c)}^{\infty} (1 - F(x))\,\mathrm{d}x + (\psi(c) - c)(1 - F(\psi(c))) \\
< &\int_{\psi(c)}^{\infty} (1 - F(x))\,\mathrm{d}x + \int_c^{\psi(c)} (1 - F(x))\,\mathrm{d}x \\
= &\int_c^{\infty} (1 - F(x))\,\mathrm{d}x,
\end{aligned}$$

for all $c \in [0, \bar{c}]$. Thus,

$$\int_0^{\bar{c}} (v(c) - c)(1 - F(\psi(c)))G(\mathrm{d}c) < \int_0^{\bar{c}} (\mathbb{E}_F[v|v > c] - c)(1 - F(c))G(\mathrm{d}c).$$

This completes the proof. ■

D3 Proof Proposition 6

Proof of Proposition 6. For 1., notice that the intermediary's revenue is given by

$$\int_0^{\bar{c}} (v(c) - \psi(c))(1 - F(\psi(c)))G(\mathrm{d}c),$$

and total surplus is

$$\int_0^{\bar{c}} (v(c) - c)(1 - F(\psi(c)))G(\mathrm{d}c),$$

and the seller's expected net profit is

$$\int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F(\psi(z)))\,\mathrm{d}z \right) G(\mathrm{d}c)$$

for any distributions F, G such that $c^* \geq \bar{c}$. For any $i, j \in \{1, 2\}$, let $v_i^j(c) := \mathbb{E}_{F_i}[v | v > \psi_j(c)]$ for all $c \in [0, \bar{c}]$. As such, for any $i \in \{1, 2\}$,

$$\begin{aligned}(v_1^i(c) - \psi_i(c))(1 - F_1(\psi_i(c))) &= \int_{\psi_i(c)}^{\infty} (1 - F_1(x))\,\mathrm{d}x \\ \geq \int_{\psi_i(c)}^{\infty} (1 - F_2(x))\,\mathrm{d}x &= (v_2^i(c) - \psi_i(c))(1 - F_2(\psi_i(c))),\end{aligned}$$

and

$$\begin{aligned}(v_1^i(c) - c)(1 - F_1(\psi_i(c))) &= \int_{\psi_i(c)}^{\infty} (1 - F_1(x))\,\mathrm{d}x + (\psi_i(c) - c)(1 - F_1(\psi_i(c))) \\ \geq (v_2^i(c) - c)(1 - F_2(\psi_i(c))) &= \int_{\psi_i(c)}^{\infty} (1 - F_2(x))\,\mathrm{d}x + (\psi_i(c) - c)(1 - F_2(\psi_i(c))),\end{aligned}$$

and also

$$\int_c^{\bar{c}} (1 - F_1(\psi_i(z)))\,\mathrm{d}z \geq \int_c^{\bar{c}} (1 - F_2(\psi_i(z)))\,\mathrm{d}z,$$

for all $c \in [0, \bar{c}]$ and therefore

$$\int_0^{\bar{c}} (v_1^i(c) - \psi_i(c))(1 - F_1(\psi_i(c)))G_i(\mathrm{d}c) \geq \int_0^{\bar{c}} (v_2^i(c) - \psi_i(c))(1 - F_2(\psi_i(c)))G_i(\mathrm{d}c)$$

and

$$\int_0^{\bar{c}} (v_1^i(c) - c)(1 - F_1(\psi_i(c)))G_i(\mathrm{d}c) \geq \int_0^{\bar{c}} (v_2^i(c) - c)(1 - F_2(\psi_i(c)))G_i(\mathrm{d}c),$$

and also

$$\int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F_1(\psi_i(z)))\,\mathrm{d}z \right) G_i(\mathrm{d}c) \geq \int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F_2(\psi_i(z)))\,\mathrm{d}z \right) G_i(\mathrm{d}c),$$

for any $i \in \{1, 2\}$.

For 2., notice that by using integration by parts, for all $c \in [0, \bar{c}]$, $i, j \in \{1, 2\}$,

$$\int_{\psi_i(c)}^{\infty} (1 - F_j(x))\,\mathrm{d}x = \int_0^{\infty} \mathbf{1}\{x \geq \psi_i(c)\}(1 - F_j(x))\,\mathrm{d}x = \int_0^{\infty} (x - \psi_i(c))^+ F_j(\mathrm{d}x).$$

Therefore, since the function $x \mapsto (x - \psi(c))^+$ is convex and since F_1 is a mean preserving spread of F_2, for any $i \in \{1,2\}$,

$$\begin{aligned}
&\int_0^{\bar{c}} (v_1^i(c) - \psi_i(c))(1 - F_1(\psi_i(c)))G_i(\mathrm{d}c) \\
&= \int_0^{\bar{c}} \left(\int_0^{\infty} (x - \psi_i(c))^+ F_1(\mathrm{d}x) \right) \\
&\geq \int_0^{\bar{c}} \left(\int_0^{\infty} (x - \psi_i(c))^+ F_1(\mathrm{d}x) \right) \\
&= \int_0^{\bar{c}} (v_2^i(c) - \psi_i(c))(1 - F_2(\psi_i(c)))G_i(\mathrm{d}c)
\end{aligned}$$

For 3., first notice that the hazard rate dominance implies that $\psi_1 \leq \psi_2$ and that

$$G_1(c) = \exp\left(- \int_c^{\bar{c}} \frac{1}{\psi_1(z)} \,\mathrm{d}z \right) \geq \exp\left(- \int_c^{\bar{c}} \frac{1}{\psi_2(z)} \,\mathrm{d}z \right) = G_2(c).$$

That is, G_2 first order stochastic dominates G_1. As such, for each $i \in \{1,2\}$,

$$\begin{aligned}
&\int_0^{\bar{c}} (v_i^1(c) - \psi_1(c))(1 - F_i(\psi_1(c)))G_1(\mathrm{d}c) \\
&= \int_0^{\bar{c}} \left(\int_{\psi_1(c)}^{\infty} (1 - F_i(x)) \,\mathrm{d}x \right) G_1(\mathrm{d}c) \\
&\geq \int_0^{\bar{c}} \left(\int_{\psi_2(c)}^{\infty} (1 - F_i(x)) \,\mathrm{d}x \right) G_1(\mathrm{d}c) \\
&\geq \int_0^{\bar{c}} \left(\int_{\psi_2(c)}^{\infty} (1 - F_i(x)) \,\mathrm{d}x \right) G_2(\mathrm{d}c) \\
&= \int_0^{\bar{c}} (v_i^2(c) - \psi_2(c))(1 - F_i(\psi_1(c)))G_2(\mathrm{d}c),
\end{aligned}$$

where the first inequality follows from $\psi_1 \leq \psi_2$ and the second inequality follows from the fact that G_2 first order stochastic dominates G_1 and that ψ_2 is increasing. Similarly, for each $i \in \{1,2\}$,

$$\begin{aligned}
&\int_0^{\bar{c}} (v_i^1(c) - c)(1 - F_i(\psi_1(c)))G_1(\mathrm{d}c) \\
&= \int_0^{\bar{c}} \left(\int_{\psi_1(c)}^{\infty} (1 - F_i(x)) \,\mathrm{d}x + (\psi_1(c) - c)(1 - F(\psi_1(c))) \right) G_1(\mathrm{d}c) \\
&\geq \int_0^{\bar{c}} \left(\int_{\psi_2(c)}^{\infty} (1 - F_i(x)) \,\mathrm{d}x + (\psi_2(c) - c)(1 - F(\psi_2(c))) \right) G_1(\mathrm{d}c) \\
&\geq \int_0^{\bar{c}} \left(\int_{\psi_2(c)}^{\infty} (1 - F_i(x)) \,\mathrm{d}x + (\psi_2(c) - c)(1 - F(\psi_2(c))) \right) G_1(\mathrm{d}c) \\
&= \int_0^{\bar{c}} (v_i^2(c) - c)(1 - F_i(\psi_2(c)))G_2(\mathrm{d}c).
\end{aligned}$$

Finally, for the same reasons,

$$\int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F_i(\psi_1(z))) \,\mathrm{d}z \right) G_1(\mathrm{d}c) \geq \int_0^{\bar{c}} \left(\int_c^{\bar{c}} (1 - F_i(\psi_2(z))) \,\mathrm{d}z \right) G_2(\mathrm{d}c)$$

This completes that proof. ■

E Proof of Claim

Proof of Claim 1. Take a sequence of strictly increasing and differentiable functions ψ_n such that $\{\psi_n\} \to \psi^*$ pointwisely. For each $n \in \mathbb{N}$, let

$$k_n(c) := c - \frac{1}{F(\psi_n(c))} \int_0^c F(\psi_n(z))\,\mathrm{d}z$$
$$\pi_n(c) := \int_{\psi_n(c)}^{\infty} (1 - F(x))\,\mathrm{d}x + (\psi_n(c) - k_n(c))(1 - F(\psi_n(c))),$$

and let

$$k^*(c) := c - \frac{1}{F(\psi^*(c))} \int_0^c F(\psi^*(z))\,\mathrm{d}z$$
$$\pi^*(c) := \int_{\psi^*(c)}^{\infty} (1 - F(x))\,\mathrm{d}x + (\psi^*(c) - k^*(c))(1 - F(\psi^*(c))),$$

for all $c \in [0, \bar{c}]$. Notice that since $\{\psi_n\} \to \psi^*$ pointwisely, by the dominated convergence theorem, $\{\pi_n\} \to \{\pi^*\}$ pointwisely as well and $(\psi_n(c) - k_n(c)) \geq 0$ for all $c \in [0, \bar{c}]$ for n large enough. As such, for n large enough,

$$\pi_n'(c) = -(\psi_n(c) - k_n(c))(1 - F(\psi_n(c))) - \frac{(1 - F(\psi_n(c)))}{F(\psi_n(c))^2} \int_0^c F(\psi_n(z))\,\mathrm{d}z < 0, \forall c \in [0, \bar{c}]$$

This then implies that for n large enough,

$$\frac{d}{\mathrm{d}c}\left[\int_0^{P(\psi_n(c),k_n(c))} [(1 - \gamma(x, \psi_n(c), k_n(c)) - (1 - F(x))]\,\mathrm{d}x\right] \leq 0, \forall c \in [0, \bar{c}].$$

Therefore,

$$\begin{aligned}
0 \geq & \frac{d}{\mathrm{d}c}\left[\int_0^{P(\psi_n(c),k_n(c))} [(1 - \gamma(x, \psi_n(c), k_n(c))) - (1 - F(x))]\,\mathrm{d}x\right] \\
= & f(\psi_n(c))\psi_n'(c)\Bigg[- \frac{1}{f(\psi_n(c))} \int_0^{P(\psi_n(c),k_n(c))} \gamma_2(x, \psi_n(c), k_n(c))\,\mathrm{d}x \\
& - \int_0^{P(\psi_n(c),k_n(c))} \gamma_3(x, \psi_n(c), k_n(c))\,\mathrm{d}x \frac{\int_0^c F(\psi_n(z))\,\mathrm{d}z}{F(\psi_n(c))^2}\Bigg] \\
= & f(\psi_n(c))\psi_n'(c)\left[- \frac{\int_0^{P(\psi_n(c),k_n(c))} \gamma_2(x, \psi_n(c), k_n(c))\,\mathrm{d}x F(\psi_n(c))}{\int_0^{P(\psi_n(c),k_n(c))} \gamma_3(x, \psi_n(c), k_n(c))\,\mathrm{d}x f(\psi_n(c))} - (c - k_n(c))\right] \\
& \times \frac{\int_0^{P(\psi_n(c),k_n(c))} \gamma_3(x, \psi_n(c), k_n(c))\,\mathrm{d}x}{F(\psi_n(c))},
\end{aligned}$$

for all $c \in [0, \bar{c}]$, $n \in \mathbb{N}$. Furthermore, direct calculation shows that

$$\frac{\int_0^{P(\psi_n(c),k_n(c))} \gamma_3(x, \psi_n(c), k_n(c))\,\mathrm{d}x}{F(\psi_n(c))} < 0, \forall c \in [0, \bar{c}],\ n \in \mathbb{N}.$$

Together, since $f(\psi_n(c))\psi_n'(c) > 0$ for all $c \in [0, \bar{c}]$ and $n \in \mathbb{N}$, for n large enough,

$$\Omega_n(c) := -\frac{\int_0^{P(\psi_n(c),k_n(c))} \gamma_2(x, \psi_n(c), k_n(c)) \, \mathrm{d}x F(\psi_n(c))}{\int_0^{P(\psi_n(c),k_n(c))} \gamma_3(x, \psi_n(c), k_n(c)) \, \mathrm{d}x f(\psi_n(c))} - (c - k_n(c)) \geq 0,$$

for all $c \in [0, \bar{c}]$. Finally, by the dominated convergence theorem, $\{\Omega_n\} \to \Omega^*$ pointwisely and therefore

$$\Omega^*(c) \geq 0, \forall c \in [0, \bar{c}].$$

Direct computation then shows that Ω^* is strictly decreasing on $[c^*, \bar{c}]$. This completes the proof. ■